A Model with Spillovers in the Adaptation of New Renewable Technologies

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Abstract

We study the adaptation of new technologies by renewable energy-producing firms in a dynamic general equilibrium model where energy is an input in the production of a final good. Energy can come from fossil or renewable sources. Both require the use of capital, which is also needed for the production of the final good. Renewable energy firms can invest in improving the productivity of their capital stock. The actual improvement is random and subject to spillovers, as it depends on the aggregate level of investment. Renewable firms finance this investment by "scrapping" some of their existing capital. Spill-overs lead to an overall under-investment in the productivity of renewable energy production. We discuss the implementation of the efficient level of investment through policy.

1 Introduction

Economic growth in both developed and developing nations generates a tremendous demand for energy. Historically, this need has been met largely through the use of fossil fuel. In recent decades, renewable energy sources (such as solar and wind) have been increasing their representation in many nations' energy supply. As concerns about the consequences of climate change become more prevalent and fossil fuel becomes more scarce, it is likely that renewable energy will become even more widely used. Unfortunately, renewable energy is still too costly to directly compete with fossil fuel sources. Yet, the transition towards renewable energy supplies is expected to accelerate as investment in this sector increases and the resulting technological progress reduces costs.

A widely held view holds that societies currently under-invest in renewable energy. This argument can take many different forms. Under-investment might refer to resources spent on R&D, or to actual installation and usage of facilities that harvest renewable energy, such as wind mills and solar PVs. Similarly, the reasons cited in connection to under-investment range from the externalities associated with climate change, to spillovers and related externalities associated with innovation. An important question concerns the projected rate at which declining costs will lead to increased competitiveness for renewables. What determines the productivity improvements in renewable energy production? How does the rate of productivity improvement respond to policy? What are the consequences for the fossil fuel sector and the macroeconomy? Our paper attempts to study these and related questions in the context of a structural economic model.

We study adaptation of new technologies in renewable energy in a dynamic general equilibrium model where energy is an input in the production of a consumption good. Energy can be produced from either fossil or renewable sources. Both require capital, which is also needed for the production of the final good. At each point in time, renewable energy-producing firms can invest in improving the productivity of their capital stock. The actual improvement is random and subject to a spillover: it depends on the aggregate investment in the renewable sector. Renewable firms finance this investment by "scrapping" some of their existing, less productive capital. The spill-over effect leads to an overall under-investment in the productivity of renewable energy-related capital. We study implementation of the efficient level of investment through policy and demonstrate that a policy which subsidizes(taxes) firms proportional to their over(under) investment implements the optimal allocation through the efficient level of investment in renewable energy. While our analysis concentrates on the energy sector, the modeling of productivity improvements through the scrapping of old vintage capital stock might have applications in other areas.

Our paper is related to a number of papers in the literature. Parente (1994) studies a model in which firms choose to adopt new technologies as they gain firm-specific expertise through learning-by-doing. He identifies conditions under which equilibria in his model exhibit constant growth of per capita output. As in most of the literature on economic growth, Parente abstracts from issues related to energy. More recently, Golosov, Hassler, Krusell, and Tsyvinski (2011) built a macroeconomic model that incorporates the use of energy and the resulting environmental consequences. They derive a formula describing the optimal tax due to the externality from emissions and provide numerical values for the size of the tax in a calibrated version of their model. However, they abstract from endogenous technological progress. Van der Ploeg and Withagen (2011) investigates the possibility of a *green paradox* in the context of a growth model. Innovation in their model differs from ours.

Finally, Acemoglu et al. (2009) study a growth model that takes into consideration the environmental impact of operating “dirty” technologies. They examine the effects of policies that tax innovation and production in the dirty sectors. Their paper focuses on long run growth and sustainability and abstracts from the endogenous evolution of R&D expenditures and the corresponding cost reductions. They find that subsidizing research in the “clean” sectors can speed up environmentally friendly innovation without resorting to carbon taxes and the corresponding slowdown in economic growth. Consequently, optimal behavior in their model implies an immediate increase in clean energy R&D, followed by a complete switch toward the exclusive use of clean inputs in production.

2 The Environment

We assume discrete time and infinite horizon, $t = 0, 1, ...$There is a single consumption good per period and all markets are competitive. The economy is populated by a representative infinite-lived household. The household discounts the future at rate $\beta \in (0, 1)$ and values the period-t consumption good through a utility function $u(c_t)$. We assume that u is smooth, strictly increasing, strictly concave, and that the usual Inada conditions hold. There are three different kinds of firms, all owned by the household. In each period, the household chooses how much capital, k, to rent in the market at rate r_t and receives all profits resulting from firms' activities. All capital depreciates at the same rate, $\delta \in (0, 1)$.

The final good-producing firm uses capital, k, and energy, e, in order to produce output. The final good production function is given by $c_t \leq \mathcal{A}_t \cdot (k_t^g)^\theta (e_t)^{1-\theta}$, where $\mathcal{A}_t$ is a productivity parameter and $\theta \in (0, 1)$. Energy can be produced in two different ways by using a fossil or a renewable source. We assume that the two types of energy are perfect substitutes in the production of the final good. We let w_t denote the available stock of fossil fuel in period t, and f_t denote the fossil fuel used in energy production at t. Thus, $w_{t+1} \leq w_t - f_t$. The fossil-fuel-derived energy production function is given by $e_t^f \leq (f_t)^{1-\alpha_f} \left(k_t^f\right)^{\alpha_f}$, where $\alpha_f \in (0, 1)$. We assume a competitive sector of renewable energy producing firms. As these firms are heterogenous, we need to keep track of the identity of each individual firm. The renewable energy production function for firm j is given by $e_{j,t}^s \leq (A_{j,t})^{1-\alpha_s} \left(k_{j,t}^s\right)^{\alpha_s}$, where $A_{j,t}$ is a productivity parameter and $\alpha_s \in (0, 1)$. Total capital used in the economy cannot exceed the total supply; i.e., $k_t^g + k_t^f + \int_0^1 k_{j,t}^s dj \leq k_t$, all t.

Investment can boost productivity in the renewable energy sector. More precisely, we let $\iota_{j,t}$ denote the investment by renewable firm j in period t. We assume that the productivity of firm j evolves stochastically according to:

$$\ln A_{j,t+1} \leq \gamma + \ln A_{j,t} + \varepsilon_{j,t}\left(\left(\int_0^1 \iota_{j,t} k_{j,t}^s dj / \int_0^1 k_{j,t}^s dj\right), \sigma\right)$$

In other words, there is a spill-over effect, as aggregate investment affects the productivity of each individual firm. This creates an externality, implying a discrepancy between equilibrium and desirable levels of investment in renewable energy. We begin by characterizing desirable (efficient) allocations in this environment.

3 The Planner´s Problem

Efficient allocations are identical to those solving a social planning problem. The social planner's problem in period t for our economy is as follows:

$$\max_{\{c_t, w_{t+1}^f, \{\iota_{j,t+1}\}_{j=0}^1, \{k_{j,t+1}^s\}_{j=0}^1, k_{t+1}^f, k_{t+1}^g, e_t, \{e_{j,t+1}^s\}_{j=0}^1, e_t^f, \{A_{j,t+1}\}_{j=0}^1\}_{t=0}^\infty} \sum_{t=0}^\infty \beta^t u(c_t)$$

$$\text{s.t. } c_t + k_{t+1} + \int_0^1 \Psi(\iota_{j,t}) k_{j,t}^s dj \leq \mathcal{A}_t \cdot (k_t^g)^\theta (e_t)^{1-\theta} + (1-\delta) k_t : \mu_{R,t}$$

$$w_{t+1} \leq w_t - f_t : \mu_{W,t}$$

$$e_{j,t}^s \leq (A_{j,t})^{1-\alpha_s} \left(k_{j,t}^s\right)^{\alpha_s} : \mu_{S,t}^j$$

$$e_t^f \leq (f_t)^{1-\alpha_f} \left(k_t^f\right)^{\alpha_f} : \mu_{F,t}$$

$$k_t \geq k_t^g + k_t^f + \int_0^1 k_{j,t}^s dj : \mu_{K,t}$$

$$\ln A_{j,t+1} \leq \gamma + \ln A_{j,t} + \varepsilon_{j,t}\left(\left(\int_0^1 \iota_{j,t} k_{j,t}^s dj / \int_0^1 k_{j,t}^s dj\right), \sigma\right) : \mu_{A,t}^j$$

$$e_t \leq e_t^f + \int_0^1 e_{j,t}^s k_{j,t}^s dj : \mu_{E,t}$$

$$k_{t+1} \geq 0, w_{t+1} \geq 0, \text{ all } t$$

$$k_0 > 0, A_{j,t} > 0, w_0 > 0, \text{ given}$$

The FOCs for the planner´s problem, which are also sufficient in this model, are:

$$\partial c_t : \beta^t u\'(c_t) = \mu_{R,t} \tag{1}$$

$$\partial k_{t+1} : -\mu_{R,t} + (1-\delta)\mu_{R,t+1} + \mu_{K,t+1} = 0 \tag{2}$$

$$\partial w_{t+1} : \mu_{W,t} = \mu_{W,t+1} \tag{3}$$

$$\partial f_t : -\mu_{W,t} + \mu_{F,t}(1-\alpha)\left(\frac{k_t^f}{f_t}\right)^{\alpha_f} = 0 \tag{4}$$

$$\partial e^s_{jt} : \int_0^1 \mu^j_{S,t} dj = \mu_{E,t} \text{ and } \mu^j_{S,t} = \mu^i_{S,t} \text{ for almost all } j \text{ and } i$$

Notice that the above implies that

$$\mu^j_{S,t} = \mu_{E,t} \text{ for almost all } j \tag{5}$$

Also note that the marginal utility of having firm i producing an extra infinitesimal amount of energy should be equal to the marginal utility of having firm j producing an extra infinitesimal amount of energy, i.e. $\mu^j_{S,t} = \mu^i_{S,t}$, for almost all j and i.

$$\partial e^f_t : -\mu_{F,t} + \mu_{E,t} = 0 \tag{6}$$

$$\partial e_t : \mu_{R,t} (1-\theta) \mathcal{A}_t \left(\frac{k^g_t}{e_t} \right)^{\theta} = \mu_{E,t} \tag{7}$$

$$\partial k^g_t : \mu_{R,t} \theta \mathcal{A}_t \left(\frac{e_t}{k^g_t} \right)^{1-\theta} = \mu_{K,t} \tag{8}$$

$$\partial k^f_t : \mu_{F,t} \alpha_f \left(\frac{f_t}{k^f_t} \right)^{1-\alpha_f} = \mu_{K,t} \tag{9}$$

$$\partial k^s_{j,t} : \left(\frac{\iota_{j,t} - \overline{\iota_t}}{\overline{k^s_t}} \right) \cdot \int_0^1 \mu^i_{A,t} di + \alpha_s \mu^j_{S,t} \left(\frac{A_{jt}}{k^s_{jt}} \right)^{1-\alpha_s} - \mu_{R,t} \Psi\left(\iota_{j,t}\right) = \mu_{K,t} \tag{10}$$

where $\overline{k^s_t} = \int k^s_{t,i} di$, and $\overline{\iota_t} = \int_0^1 \iota_{j,t} k^s_{j,t} dj / \int_0^1 k^s_{j,t} dj$. From (5) and (10), $\iota_{j,t}$ is a function of $\frac{A_{jt}}{k^s_{jt}}$ only.

$$\partial \iota_{j,t} : \mu_{R,t} \Psi'(\iota_{j,t}) k^s_{j,t} dj = \frac{k^s_{j,t}}{\overline{k^s_t}} \int_0^1 \mu^i_{A,t} di dj$$

$$\text{which implies } \mu_{R,t} \Psi'(\iota_{j,t}) = \left(\overline{k^s_t}\right)^{-1} \int_0^1 \mu^i_{A,t} di \text{ for almost all } j \tag{11}$$

Thus, all firms choose the same ι.

$$\partial A_{j,t+1} : \mu^j_{A,t} = \mu^j_{A,t+1} + \mu^j_{S,t+1} (1-\alpha_s) \left(A_{jt+1}\right)^{1-\alpha_s} \left(k^s_{jt+1}\right)^{\alpha_s} \tag{12}$$

Taking together (1), (2) and (4) we have:

$$-\beta^t u'(c_t) + (1-\delta)\beta^{t+1} u'(c_{t+1}) + \beta^{t+1} u'(c_{t+1}) \theta \mathcal{A}_{t+1} \left(\frac{e_{t+1}}{k^g_{t+1}} \right)^{1-\theta} = 0$$

or

$$\frac{u'(c_t)}{\beta u'(c_{t+1})} = (1-\delta) + \theta \mathcal{A}_{t+1} \left(\frac{e_{t+1}}{k^g_{t+1}} \right)^{1-\theta} \tag{13}$$

From (5) and (6),

$$\mu_{F,t} = \mu^j_{S,t} = \mu_{E,t} \tag{14}$$

From (7),

$$\frac{\mu_{R,t}}{\mu_{E,t}} = \left[(1-\theta)\,\mathcal{A}_t \left(\frac{k^g_t}{e_t} \right)^{\theta} \right]^{-1} \tag{15}$$

From (9) and (10)

$$\left(\int_0^1 \mu^i_{A,t} di \right) \left(\frac{\iota_{j,t} - \overline{\iota_t}}{\overline{k^s_t}} \right) + \mu^j_{S,t} \alpha_s \left(\frac{A_{jt}}{k^s_{jt}} \right)^{1-\alpha_s} - \mu_{R,t} \Psi\left(\iota_{j,t}\right) = \mu_{F,t} \alpha_f \left(\frac{f_t}{k^f_t} \right)^{1-\alpha_f}$$

or

$$\frac{\left(\int_0^1 \mu^i_{A,t} di \right)}{\mu_{F,t}} \left(\frac{\iota_{j,t} - \overline{\iota_t}}{\overline{k^s_t}} \right) + \frac{\mu^j_{S,t}}{\mu_{F,t}} \alpha_s \left(\frac{A_{jt}}{k^s_{jt}} \right)^{1-\alpha_s} - \frac{\mu_{R,t}}{\mu_{F,t}} \Psi\left(\iota_{j,t}\right) = \alpha_f \left(\frac{f_t}{k^f_t} \right)^{1-\alpha_f} \tag{16}$$

Now from (11),

$$\begin{aligned} \frac{\left(\int_0^1 \mu^i_{A,t} di \right)}{\mu_{F,t}} &= \frac{\mu_{R,t}}{\mu_{F,t}} \Psi'\left(\iota_{j,t}\right) \overline{k^s_t} \Longrightarrow \\ \frac{\left(\int_0^1 \mu^i_{A,t} di \right)}{\mu_{F,t}} \left(\frac{\iota_{j,t} - \overline{\iota_t}}{\overline{k^s_t}} \right) - \frac{\mu_{R,t}}{\mu_{F,t}} \Psi'\left(\iota_{j,t}\right) &= \left[\Psi'\left(\iota_{j,t}\right) \left(\iota_{j,t} - \overline{\iota_t}\right) - \Psi\left(\iota_{j,t}\right) \right] \frac{\mu_{R,t}}{\mu_{F,t}} \end{aligned}$$

Using (14) and (15) in (16) we obtain

$$\alpha_s \left(\frac{A_{jt}}{k^s_{jt}} \right)^{1-\alpha_s} + \left[\Psi'\left(\iota_{j,t}\right) \left(\iota_{j,t} - \overline{\iota_t}\right) - \Psi\left(\iota_{j,t}\right) \right] \left[(1-\theta)\,\mathcal{A}_t \left(\frac{k^g_t}{e_t} \right)^{\theta} \right]^{-1} = \alpha_f \left(\frac{f_t}{k^f_t} \right)^{1-\alpha_f} \tag{17}$$

The above equation characterizes the optimal ι in our model. Equations (7) and (8) imply

$$\frac{\mu_{R,t} \theta \mathcal{A}_t \left(\frac{e_t}{k^g_t} \right)^{1-\theta}}{\mu_{R,t} (1-\theta)\,\mathcal{A}_t \left(\frac{k^g_t}{e_t} \right)^{\theta}} = \frac{\mu_{K,t}}{\mu_{E,t}}$$

Using (14) and (9) in the equation above

$$\frac{\theta}{1-\theta} \left(\frac{e_t}{k^g_t} \right) = \alpha_f \left(\frac{f_t}{k^f_t} \right)^{1-\alpha_f}$$

next, equations (3) and (4) imply

$$\mu_{F,t}\left(1-\alpha\right)\left(\frac{k_t^f}{f_t}\right)^{\alpha_f}=\mu_{F,t+1}\left(1-\alpha\right)\left(\frac{k_{t+1}^f}{f_{t+1}}\right)^{\alpha_f}$$

Using (1), (6) and (7) in the equation above, we obtain:

$$u\acute{}(c_t)\left[\left(1-\theta\right)\mathcal{A}_t\left(\frac{k_t^g}{e_t}\right)^{\theta}\right]\left[\left(1-\alpha\right)\left(\frac{k_t^f}{f_t}\right)^{\alpha_f}\right]=\beta u\acute{}(c_{t+1})\left[\left(1-\theta\right)\mathcal{A}_{t+1}\left(\frac{k_{t+1}^g}{e_{t+1}}\right)^{\theta}\right]\left[\left(1-\alpha\right)\left(\frac{k_{t+1}^f}{f_{t+1}}\right)^{\alpha_f}\right]$$

The above equation has a natural interpretation. The term $\left[\left(1-\theta\right)\mathcal{A}_t\left(\frac{k_t^g}{e_t}\right)^{\theta}\right]$ is the marginal productivity of energy in the production of the good, which in a competitive equilibrium is equal to the price of energy. The term $\left[\alpha_f\left(\frac{f_t}{k_t^f}\right)^{1-\alpha_f}\right]$ is the marginal productivity of capital in the production of energy in the fuel sector. Finally, the term $\left[\left(1-\theta\right)\mathcal{A}_t\left(\frac{k_t^g}{e_t}\right)^{\theta}\right]\left[\alpha_f\left(\frac{f_t}{k_t^f}\right)^{1-\alpha_f}\right]$ equals $\theta\mathcal{A}_t\left(\frac{e_t}{k_t^g}\right)^{1-\theta}$, which is the marginal productivity of capital in the production of the final good.

From equation (12),

$$\mu_{A,t}^j=\mu_{A,t+1}^j+\mu_{S,t+1}^j\left(1-\alpha_s\right)\left(A_{j,t+1}\right)^{1-\alpha_s}\left(k_{j,t+1}^s\right)^{\alpha_s}$$

and

$$\mu_{A,t+1}^j=\mu_{A,t+2}^j+\mu_{S,t+2}^j\left(1-\alpha_s\right)\left(A_{j,t+2}\right)^{1-\alpha_s}\left(k_{j,t+2}^s\right)^{\alpha_s}$$

Combining these equations, we obtain

$$\mu_{A,t}^j=\left(1-\alpha_s\right)\sum_{\tau=1}^{\infty}\mu_{S,t+\tau}^j\left(A_{j,t+\tau}\right)^{1-\alpha_s}\left(k_{j,t+\tau}^s\right)^{\alpha_s}$$

Using equation (7) and (14) in the equation above, we have

$$\mu_{A,t}^j=\left(1-\alpha_s\right)\sum_{\tau=1}^{\infty}\mu_{R,t+\tau}\left(1-\theta\right)\mathcal{A}_{t+\tau}\left(\frac{k_{t+\tau}^g}{e_{t+\tau}}\right)^{\theta}\left(A_{j,t+\tau}\right)^{1-\alpha_s}\left(k_{j,t+\tau}^s\right)^{\alpha_s}$$

Using equation (1) in the equation above

$$\mu_{A,t}^j=\left(1-\alpha_s\right)\sum_{\tau=1}^{\infty}\beta^{t+\tau}u\acute{}(c_{t+\tau})\left(1-\theta\right)\mathcal{A}_{t+\tau}\left(\frac{k_{t+\tau}^g}{e_{t+\tau}}\right)^{\theta}\left(A_{j,t+\tau}\right)^{1-\alpha_s}\left(k_{j,t+\tau}^s\right)^{\alpha_s}$$

By integrating both sides, we obtain

$$\int_0^1\mu_{A,t}^i di=\left(1-\alpha_s\right)\sum_{\tau=1}^{\infty}\beta^{t+\tau}u\acute{}(c_{t+\tau})\left(1-\theta\right)\mathcal{A}_{t+\tau}\left(\frac{k_{t+\tau}^g}{e_{t+\tau}}\right)^{\theta}\int_0^1\left(A_{i,t+\tau}\right)^{1-\alpha_s}\left(k_{i,t+\tau}^s\right)^{\alpha_s}di$$

Finally, using (11) in the equation above

$$\Psi'(\iota_{j,t}) = (1-\alpha_s)\sum_{\tau=1}^{\infty}\frac{\beta^{\tau}u'(c_{t+\tau})}{u'(c_t)}(1-\theta)\,\mathcal{A}_{t+\tau}\left(\frac{k^g_{t+\tau}}{e_{t+\tau}}\right)^{\theta}\int_0^1 A_{i,t+\tau}\left(\frac{k^s_{i,t+\tau}}{A_{i,t+\tau}}\right)^{\alpha_s}di \qquad (18)$$

Note that, the right hand side of the above equation is independent of j, hence the optimal allocation implies that all renewable firms choose the same adaptation level. Moreover, (17) implies that in an optimal allocation the ratio of capital to technology should be equal across all renewable energy firms. We summarize our findings in the following.

Proposition 1. *The optimal allocation implies* $\frac{k^s_{i,t}}{A_{i,t}} = \frac{k^s_t}{A_t}$ *and* $\iota_{i,t} = \iota_t$*, all* i*.*

The next section discusses a decentralized version of our model and characterizes a competitive equilibrium for the corresponding economy.

4 Competitive Equilibrium

Here we solve for a competitive equilibrium of our economy and demonstrate that, as argued earlier, there is discrepancy between equilibrium and optimal allocations. We also discuss the role of policy in restoring efficiency.

The household's problem is given by the following.

$$\max\sum_{t=0}^{\infty}\beta^t u(c_t)$$

$$\text{s.t.}\quad \sum_{t=0}^{\infty}p_t\left[c_t + k_{t+1} - (1-\delta)k_t\right] \le \sum_{t=0}^{\infty}p_t\left[r_t k_t + p^f_t f_t + \pi^g_t + \pi^f_t + \int_0^1 \pi^s_{j,t}dj\right] : \lambda$$

$$w_{t+1} \le w_t - f_t : \mu_t$$

where p_t is the Arrow-Debreu price of the period-t final good, r_t is the rental price of capital at t, p^f_t is the price of fossil fuel in period t, and π stand for the respective firms' profits. The FOC, which are also sufficient for a maximum, are

$$\partial c_t : \beta^t u'(c_t) = \lambda p_t$$

$$\partial k_{t+1} : \lambda\left[-p_t + (1-\delta+r_{t+1})p_{t+1}\right] = 0$$

$$\partial w_{t+1} : \mu_t = \mu_{t+1}$$

$$\partial f_t : -\mu_t + \lambda p_t p^f_t = 0$$

These equations can be rewritten as

$$\frac{p_{t+1}}{p_t} = \frac{p^f_t}{p^f_{t+1}}$$

$$\beta^t \frac{u\'(c_t)}{p_t} = \lambda$$

$$1 - \delta + r_{t+1} = \frac{p_t}{p_{t+1}}$$

$$\beta^t u\'(c_t) p_t^f = \mu_t = \mu$$

The Final-Good Firm's Problem is

$$\max \left[\mathcal{A}_t \cdot (k_t^g)^\theta (e_t)^{1-\theta} - r_t k_t^g - p_t^e e_t\right]$$

The first order conditions are

$$\partial k_t^g : \theta \mathcal{A}_t \left(\frac{e_t}{k_t^g}\right)^{1-\theta} = r_t$$

$$\partial e_t : (1-\theta)\, \mathcal{A}_t \left(\frac{k_t^g}{e_t}\right)^{\theta} = p_t^e \tag{19}$$

The Fossil-Fuel Firm's Problem is:

$$\max \left[p_t^e \left(f_t\right)^{1-\alpha_f} \left(k_t^f\right)^{\alpha_f} - r_t k_t^f - p_t^f f_t\right]$$

$$\partial k_t^f : p_t^e \alpha_f \left(\frac{f_t}{k_t^f}\right)^{1-\alpha_f} = r_t \tag{20}$$

$$\partial f_t : p_t^e \left(1-\alpha_f\right) \left(\frac{k_t^f}{f_t}\right)^{\alpha_f} = p_t^f$$

The Renewable Firm j's problem is:

$$\max_{\{\iota_{j,t}\}_{t=0}^{\infty}} \sum_{\tau=0}^{\infty} \beta^\tau \frac{u\'(c_{t+\tau})}{u\'(c_t)} \left[p_{t+\tau}^e \left(A_{j,t+\tau}\right)^{1-\alpha_s} \left(k_{j,t+\tau}^s\right)^{\alpha_s} - r_{t+\tau} k_{j,t+\tau}^s - \Psi(\iota_{j,t+\tau}) k_{j,t+\tau}^s\right]$$

$$\begin{aligned} \text{s.t. } \ln A_{t+1}^j &\leq \gamma + \ln A_t^j + \varepsilon_{j,t}\left(\left(\int_0^1 \iota_{j,t} k_{j,t}^s dj / \int_0^1 k_{j,t}^s dj\right), \sigma\right) : \lambda_{A,t}^j \\ \Psi(\iota_{j,t+\tau}) k_{j,t+\tau}^s &\leq k_{j,t+\tau}^s \\ \iota_{j,t} &\geq 0, \text{ and } A_0 \text{ given} \end{aligned}$$

Here, $\Psi(\iota_{j,t})$ is a convex function, with $\Psi(0) = 0$, $\Psi' > 0$, $\Psi'' > 0$, $\lim_{x \to 0} \Psi'(x) = 0$. The FOC are

$$\partial k_{j,t}^s : p_t^e \alpha_s \left(\frac{A_{j,t}}{k_{j,t}^s}\right)^{1-\alpha_s} = r_t + \Psi(\iota_{j,t})$$

$$\partial \iota_t^i : \Psi'(\iota_{j,t}) k_{j,t}^s = o \lambda_{A,t}^j = 0 \tag{21}$$

Condition (21) implies that in a competitive equilibrium without policy intervention the first best is not achievable. We summarize this in the following.

Proposition 2. *In any competitive equilibrium, $\iota_{j,t} = 0$, all j, t.*

5 Policy

In this section we will discuss the implementation of the optimal allocation through policy. Our main finding, summarized in the following Proposition, asserts that the optimum can be supported via the use of a simple tax/subsidy policy.

Proposition 3. *The revenue-neutral policy function $\Phi_t(k,\iota_{j,t}) = \Psi'(\iota_t^*)(\iota_{j,t}-\iota_t^*)k_{j,t}^s$ supports the social planner's solution, where ι_t^* is the solution to the planner's problem.*

To see this, consider a general sequence of policy functions, $\{\Phi_t\}_t$. We think of Φ_t as a potential tax or subsidy schedule imposed on renewable energy-producing firms and we assume that Φ_t is a function of the variables $\left(e_{j,t}^s, k_{j,t}^s, \Psi(\iota_{j,t})k_{j,t}^s\right)$, all t. In this case the firm j's problem becomes

$$\max_{\{\iota_{j,t}\}_{t=0}^{\infty}} \sum_{\tau=0}^{\infty} \beta^{\tau} \frac{u\'(c_{t+\tau})}{u\'(c_t)} \left[p_{t+\tau}^e \left(A_{j,t+\tau}\right)^{1-\alpha_s} \left(k_{j,t+\tau}^s\right)^{\alpha_s} - r_{t+\tau}k_{j,t+\tau}^s - \Psi(\iota_{j,t+\tau})k_{j,t+\tau}^s + \Phi_{t+\tau}^j\right]$$

$$\begin{aligned} \text{s.t. } \ln A_{t+1}^j &\leq \gamma + \ln A_t^j + \varepsilon_{j,t}\left(\left(\int_0^1 \iota_{j,t}k_{j,t}^s dj \Big/ \int_0^1 k_{j,t}^s dj\right), \sigma\right) : \lambda_{A,t}^j \\ \iota_{j,t} &\geq 0, \text{ and } A_0 \text{ given} \end{aligned}$$

The FOC are

$$\partial k_{j,t}^s : p_t^e \cdot \alpha_s \left(\frac{A_{j,t}}{k_{j,t}^s}\right)^{1-\alpha_s} + \Psi'(\iota_t^*)(\iota_{j,t}-\iota_t^*) = r_t + \Psi(\iota_{j,t}) \tag{22}$$

$$\partial \iota_t^i : \Psi'(\iota_{j,t}) = \Psi'(\iota_t^*) \tag{23}$$

where (23) sets $\iota_{j,t} = \iota_t^*$ and (22) implies:

$$\begin{aligned} & p_t^e \cdot \alpha_s \left(\frac{A_{j,t}}{k_{j,t}^s}\right)^{1-\alpha_s} + \Psi'(\iota_t^*)(\iota_{j,t}-\iota_t^*) = r_t + \Psi(\iota_{j,t}) \\ \iff & (1-\theta)\mathcal{A}_t\left(\frac{k_t^g}{e_t}\right)^{\theta} \cdot \alpha_s \left(\frac{A_{j,t}}{k_{j,t}^s}\right)^{1-\alpha_s} + \Psi'(\iota_t^*)(\iota_{j,t}-\iota_t^*) \\ = & \Psi(\iota_{j,t}) + (1-\theta)\mathcal{A}_t\left(\frac{k_t^g}{e_t}\right)^{\theta} \alpha_f \left(\frac{f_t}{k_t^f}\right)^{1-\alpha_f} \\ \iff & \alpha_s\left(\frac{A_{jt}}{k_{jt}^s}\right)^{1-\alpha_s} + \left[\Psi'(\iota_t^*)(\iota_{j,t}-\iota_t^*) - \Psi(\iota_{j,t})\right]\left[(1-\theta)\mathcal{A}_t\left(\frac{k_t^g}{e_t}\right)^{\theta}\right]^{-1} = \alpha_f\left(\frac{f_t}{k_t^f}\right)^{1-\alpha_f} \end{aligned}$$

Note that the last expression is identical to (16). Thus, by choosing the optimal allocation ratio of capital to technology and the optimal level of ι, the optimal allocation is implemented.

6 Extensions - Discussion

We are currently working on a number of extensions. First, we are further characterizing the efficient allocation, including the implied fossil fuel use, both at steady state and during the transition. A second step involves introducing an environmental externality and investigating whether our model gives rise to a "Green Paradox." Lastly, we intend to calibrate the model to world data. Quantitative questions involve pinning down the magnitude of the cross-country spillover effect (as well as γ, α_s, α_f, Ψ), and characterizing the distribution across productivities for renewable energy firms.

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