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The Sequential Equal Surplus Division for Sharing International Rivers with Bifurcations

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Abstract We consider the problem of sharing water from a river among the group of countries located along it. The benefit of each country depends on the amount of water it consumes. An allocation of the water is efficient when it maximizes the total benefits of the countries. The problem of finding a fair welfare distribution can be modeled as a cooperative game. We introduce a new allocation rule, called the sequential equal surplus division, for sharing the total welfare resulting form the cooperation of countries along a river with bifurcations. This rule obeys the so-called Territorial Integration of all Basin States doctrine, which emphasizes compromise and fairness among countries. We provide two axiomatic characterizations of this rule.

Keywords River TU-game · Sequential Equal Surplus Division · Water allocation · Standard solution · Consistency · Fairness · Amalgamation

1 Introduction

An international river is one either flowing through the territory of two or more countries, or one separating the territory of two countries from one another. The fact that water is a scarce resource, characterized by its spatial and seasonal variations, with no substitute, and over which there is total dependency, has heightened both conflict and cooperation over a large number of international rivers. The trend in recent years with respect to international watercourses has been one of managing conflict and enhancing cooperation. Recognition by the international community of the virtues of cooperation at the bilateral, regional and multilateral levels has resulted in the recent conclusion of a number of treaties, protocols and conventions on international watercourses (see, e.g., Salman *et al.*, 2003).

The aim of this article is to introduce and study the fair distribution of welfare resulting from a socially optimal consumption plan for the set of countries located on an international river from upstream to downstream. We consider the case where there exists an additional flow of water that flows from upstream to downstream between each pair of neighboring countries. Due to the one-directionality of the river's flow, water inflow entering the territory of downstream countries cannot be consumed by

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upstream countries. The benefit of each country depends on the amount of water it consumes on its territory. In order to reach a socially optimal consumption plan, upstream countries may refrain from consuming all their water and let it pass to their downstream countries who derive a higher benefit from consuming water. Since every country benefits from consuming water, monetary compensations are necessary to encourage upstream countries to give up water. The difficulty is to find a fair agreement on the allocation of the social welfare resulting from an optimal consumption plan, which in turn determines monetary compensations.

Historically, distributing the optimal welfare that arises from the river water consumption has been at the center of international tensions and conflicts. The international law suggests several equity notions as guidelines for resolving international water disputes. Unfortunately, as explained in Kilgour and Dinar (1995) and Wolf (1999), these principles are often difficult to apply and even contradictory, especially for rivers, which require transfers because of the direction of the flow.

At a theoretical level, cooperative game theory provides useful insights into the way different parties that share a scarce resource may reach an agreement. The main characteristic of this branch of game theory is to focus on the possible distributions of the total gains generated by the cooperation of the different parties in order to achieve a stable and fair agreement. International water disputes are first studied from this point of view by Ambec and Sprumont (2002) and van den Brink *et al.* (2007), who consider rivers with one spring and without bifurcations. Khmelnitskaya (2010) extends the analysis to rivers with multiple springs or with bifurcations, while van den Brink *et al.* (2012) focus exclusively on the case where the river has multiple springs and no bifurcations. In this article, we consider international rivers with bifurcations. For a survey on this literature, we refer to Béal *et al.* (2012).

Different allocation rules for distributing the social welfare resulting from the optimal water allocation are discussed by these authors. These rules are characterized by translating into axioms the above-mentioned principles of international law. The views expressed in the three main doctrines advocated in international disputes are the efficiency of water use, the Absolute Territorial Sovereignty (ATS for short) and Territorial Integration of all Basin States (TIBS for short). Efficiency of the water use means that the optimal social welfare is fully shared among the countries. The ATI doctrine states that a country has absolute sovereignty over the area of any river basin on its territory. The TIBS doctrine attributes an equal use of water to all countries whatever their contribution to the flow. Within the cooperative game theoretical framework, efficiency of the water use is obviously the usual property of efficiency of the allocation rule. The combination of efficiency and the ATS doctrine has been very often translated into the property that the welfare distribution should be core-stable, *i.e.* that each coalition of countries should get a welfare at least equal to the welfare it can secure without the cooperation of any other country. The various game-theoretical interpretations of TIBS accord different shares of the surplus resulting from the cooperation of a coalition of countries.

We introduce a new allocation rule, the sequential equal surplus division as we call it, which relies on a weaker interpretation of the ATS doctrine and a more flexible interpretation of the TIBS doctrine. In particular, we would like to insist on four features of the sharing river problem.

Firstly, although core-stability is a nice requirement for an allocation rule, it is too strong if one sticks to the original statement of the ATS doctrine. In fact, the ATS doctrine only requires the stability condition for all singleton coalitions. The sequential equal surplus division satisfies this weaker interpretation of the ATS doctrine. Even if it is not core-stable, it also satisfies the intuitive requirement that the stability is satisfied for any coalition containing a country and all the countries located downstream.

Secondly, we believe that a minimal requirement that an allocation rule should satisfy is to coincide with the standard solution in the two-country case. This means that if the river flows through two countries, then each of them should get the welfare resulting from consuming the inflow entering its territory plus one half of the welfare surplus resulting from their cooperation. The reason is that the participation of each country is equally important in the achievement of the optimal water consumption whatever its location on the river. The allocation rules proposed so far in the literature, namely the downstream incremental distribution introduced in Ambec and Sprumont (2002), the upstream incremental distribution studied in van den Brink *et al.* (2007) for rivers without bifurcations, their generalization to rivers with bifurcations or multiple springs examined by Khmelnitskaya (2010), and the weighted hierarchical solutions studied in van den Brink *et al.* (2012), violate this very basic and intuitive requirement. On the contrary, it is satisfied by the sequential equal surplus division introduced in this article.

Thirdly, we aim at generalizing the well-accepted principle of the standard solution to n-country sharing river situations. The construction of the sequential equal surplus division is done sequentially by following the natural direction of the river's flow. At the spring of the river, the water can possibly flow through several branches. Together with the groups of countries located on each of these branches, the country located at the spring of the river achieves some surplus compared to the situation in which each branch would behave selfishly. This surplus is precisely the difference between the total welfare and the sum of the welfare achieved separately by the country located at the spring of the river and each of the downstream branches. In order to distribute this surplus, the country located at the spring of the river as well as the cooperating coalitions corresponding to the countries located on the downstream branches can be considered as single entities. As in the two-country case, the participation of each entity is necessary for the production of this surplus. Therefore, it seems natural to give to each entity an equal share of the surplus in addition to the welfare it can secure in the absence of cooperation. This is equivalent to reward each entity by the well-known equal surplus division for TU-games. For each remaining branch of the river, the obtained total payoff is what remains to be shared among its members. The sequential equal surplus division then consists of applying repeatedly the above step to all countries located at the "springs" of the remaining branches considered as single entities. The distribution of the surplus of cooperation between two or more coalitions of countries is also at the heart of the construction of many of the solutions proposed in the literature. Either this surplus is fully allocated to only one country as in the downstream incremental distribution, the upstream incremental distribution and their generalizations or it is shared on a more egalitarian basis as in the equal loss property solution studied in van den Brink *et al.* (2007). In any of these solutions, only the most upstream country of the downstream coalition and the most downstream country of the upstream coalition get a share of the surplus. Contrary to the sequential equal surplus division, these solutions somehow ignore the crucial role of the other participating countries in the coalitions located on the downstream branches.

Fourthly, since the sharing river problem is represented by a TU-game, we think that it is desirable to adapt several well-known principles used in cooperative TU-games to account for the one-directionality of the river's flow. Such desirable principles include consistency, fairness and amalgamation/collusion considerations. The first and the third principles are left out of the existing literature.[1] In this article, they are adapted to river TU-games and employed to provide two axiomatic characterizations of the sequential equal surplus division.

The rest of the article is organized as follows. Section 2 puts the international river basin problem within the context of cooperative game theory. Section 3 introduces the sequential equal surplus division and states a first result which describes some of its features. Several properties for an allocation rule are presented in section 4. Section 5 contains the two axiomatic characterizations. We conclude by considering a natural extension of the sequential equal surplus division.

2 The river sharing problem

2.1 International water conflicts

International river basins cover more than half of the land's surface. A total of 145 countries are riparian of one or more of the world's 263 international river basins. Because water is a finite and vulnerable resource, essential to sustain life, development and the environment, riparian countries may easily get into conflicts over shared water resources. The most common scenario is where a new or increased use by one or more countries results in the available water resources being inadequate to meet the needs of all users in a quantitative or even qualitative sense. International law offers a range of diplomatic means to resolve international disputes. Generally, water conflicts are settled through negotiations with an agreement as the final outcome.

We consider international rivers basins with bifurcations. A river bifurcation occurs when a river flowing in a single stream separates into two or more separate streams, called distributaries, which continue downstream. Such basins include international rivers with delta bifurcations, where the rivers split into many different slow-flowing channels that have muddy banks. Risk of conflicts over the distribution of water may emerge when the abstraction of large volumes of upstream water leaves insufficient flow to meet the needs of downstream ecosystems. Upstream and in-delta water diversions, water operations, and land use changes may affect the delta's water flows.

[1] Ansink and Weikard (2012) is an exception, but they deal with the sharing river problem from a bankruptcy point of view. Furthermore, they only consider rivers with one spring and without bifurcations.

The Indus basin is an example of such international rivers with bifurcations for which international and subnational water disputes require the development of institutions for the management of water resources. On the one hand, soon after the partition of the Indian subcontinent between the two independent states of India and Pakistan, the provincial government of Indian Punjab discontinues the delivery of water to Pakistan. The delivery of water is restored a couple of weeks later. The two countries conclude the Inter-Dominion Agreement, and then sign the Indus Water Treaties in 1960, which divides the basin between the two countries and puts strong limitations on Indian projects that could manipulate the storage upstream of Pakistan. On the other hand, the Indus river travels southwards across Punjab and Sindh provinces in Pakistan before entering into the Arabian Sea through a delta close to the border with India. Over the last sixty years, a series of dams and irrigation schemes have been built in upstream parts of the river. Recurrent disputes over the competing use of water resources leads the government to set in place the Indus Water Accord in 1991, which apportions the use of the river's flow between the different provinces. Nevertheless, Pakistan's population has increased dramatically. As a result, the water initially stored behind dams intended for irrigation is now used in part for municipal supplies. The expanding upstream water diversions have greatly reduced inflows to the delta and have brought major intrusions of saline water from the ocean far into the central delta. Land in the area has become unsuitable for agriculture, and fishermen have been forced to fish near the Indian border as a result of destruction of ecosystems elsewhere in the delta, causing new water conflicts between these two countries (see, e.g., Seligman, 2008).

There exist many similar cases of water conflicts between several countries sharing an international river basin. Among them, we can mention the dispute on the Farakka barrage between Indian and Bangladesh. It started in 1951 when India decided to construct the Farakka barrage in order to divert waters from the Gange river water into a canal for irrigation and then into a channel of the Hooghly/Hugly River, which flows into Calcutta, India. The Ganges is a river delta, which takes its source in the Himalayas, and flows through China, India, Nepal and Bangladesh. It possesses the world's largest delta, and empties into the bay of Bengal. A number of large rivers flow through the Ganges delta, including the Hooghly river, which splits from the Ganges as a canal in Murshidabad District at the Farakka barrage. The Farakka barrage, even more than thirties years after its completion, is still a source of tension between India and Bangladesh. After its construction, the salinity of water and soil increased markedly. Bangladesh claims that the Farakka barrage deprives the country of a valuable source of water on which it depends because the Ganges waters are vital to irrigation, navigation and prevention of saline incursions in the Bangladesh Ganges delta region. Bangladesh holds that there should be joint control between India and Bangladesh over the waters of the Ganges as an international river. In 1996, India and Bangladesh signed the Farakka Dam Treaty. It stipulates the exact allocated amounts of water to be distributed between the two countries (see, e.g., Rahaman, 2006).

These examples show the importance of studying fair agreements between countries sharing an international river with bifurcations. The difference with a river having multiple springs and no bifurcations as analyzed by van den Brink *et al.* (2012) is easily seen if the the river is represented by a graph shaped like a tree along which each country has exactly one downstream neighbor. Nevertheless, a country may possess several upstream neighbors. A river with bifurcations is also represented by a graph shaped like a tree. In this case, each country may have several downstream neighbors whereas it possesses a unique upstream neighbor. The two types of rivers lead to opposite balances of negotiating power among the riparian countries. In a river with multiple springs, the balance of power is in favor of a downstream country who negotiates the amount of water to be released with possibly several upstream countries. Therefore, this downstream country is in a much better position than in the case of a river with bifurcations where it has a unique upstream neighbor. On the contrary, in a river with bifurcations, the balance of power is in favor of an upstream country who negotiates the amount of water to be released with possibly several downstream countries. It follows that an upstream country is in a much better position than in the case of a river with multiple springs where it has a unique downstream neighbor.

2.2 Sharing a river with bifurcations

To describe a river basin with bifurcations, let N be a finite set of agents of size $n \in \mathbb{N}$, representing countries, located along an international river. At the location of each country $i \in N$, rainfall and inflow from tributaries increase the total river flow by $e_i \in \mathbb{R}_+$. Water inflow at the territory of downstream countries cannot be consumed by upstream countries. The most upstream country along the river is called its spring or its root. We assume that the river may have many bifurcations. Such rivers can form complex networks of distributaries, especially in their deltas. Under these assumptions, the river is shaped like a rooted tree in which all edges are directed away from a selected country called the root. A river with bifurcations featuring the inflow of water on each country's location is shown in Figure 1, where the root 1 is the spring of the river.

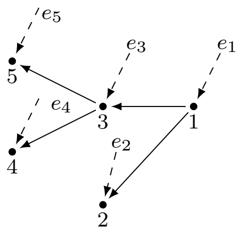

Fig. 1

Given a tree rooted at $r \in N$, for each country $i \in N \backslash \{r\}$, there is exactly one directed link $j \to i$; country j is the unique upstream neighbor of i and i is a downstream neighbor of j. Denote by $u(i)$ the unique upstream neighbor of $i \neq r$ and by $d(i)$ the possibly empty set of downstream neighbors of i. The notation $d[i]$ will denote the union of $d(i)$ and $\{i\}$. All countries having the same upstream neighbor are called brothers. We denote by $B_r(i)$ the set of all brothers of i. Country j is a *downstream agent* of i if there is a directed path from i to j, *i.e.* if there is a sequence of distinct countries $(i_0, i_1, \ldots, i_l)$ such that $i_0 = i$, $i_l = j$ and for each $q \in \{0, 1, \ldots, l-1\}$, $i_{q+1} \in d(i_q)$. The set $D[i]$ denotes the union of the set of all downstream countries of i and $\{i\}$. So, we have $d(i) \subseteq D[i] \backslash \{i\}$. If j is a downstream country of i, then we say that i is an *upstream country* of j. The set $U[i]$ denotes the union of the set of all upstream countries of i and $\{i\}$. So, we have $u(i) \in U[i] \backslash \{i\}$ for each $i \neq r$. The *depth* of a country $i \in N$ is the length of the unique directed path from r to i. The *depth of the tree* is the depth of its deepest countries. Note that the collection $\{d(i) : i \in N\}$ fully describes the river structure.

Example 1 *Let us illustrate these notions with the river in Figure 1. The set of downstream neighbors of country 1, the root, is* $d_1(1) = \{2, 3\}$*, the set of its downstream countries including itself is* $D_1[1] = \{1, 2, 3, 4, 5\}$*, whereas the set of downstream neighbors of country 3 is* $d_1(3) = \{4, 5\}$ *and* $D_1[3] = d_1[3]$*. Country 1 has no upstream neighbor and the upstream neighbor of country 3 is country 1:* $u_1(3) = 1$*. The set of upstream countries of country 4 including itself is* $U_1[4] = \{1, 3, 4\}$*, and the set of upstream countries of country 5 including itself is* $U_1[5] = \{1, 3, 5\}$*. Countries 4 and 5 are brothers since they have the same upstream neighbor:* $u_1(4) = u_1(5) = 3$*. We thus have* $B_1(4) = \{5\}$ *and* $B_1(5) = \{4\}$*. The deepest countries 4 and 5 have a depth equal to 2.* □

Each country $i \in N$ is endowed with a benefit function $b_i : \mathbb{R}_+ \longrightarrow \mathbb{R}$ that assigns to each amount $x_i \in \mathbb{R}_+$ of water consumed the benefit $b_i(x_i) \in \mathbb{R}$.

Assumption 1 (A1). The water inflow e_1 is strictly positive. Each benefit function b_i is strictly increasing, strictly concave, and differentiable on $\mathbb{R}_{++}$, $b_i(0) = 0$, and the first derivative $b_i^{(1)}(x_i)$ tends to infinity as x_i vanishes.

A consumption plan $x = (x_i)_{i \in N} \in \mathbb{R}^n_+$ satisfies the following constraints:

$$\forall k \in N, \sum_{j \in U[k]} x_j \leq \sum_{j \in U[k]} e_j, \text{ and } \sum_{j \in U[k] \cup B_r(k)} x_j \leq \sum_{j \in U[k] \cup B_r(k)} e_j. \quad (1)$$

The first constraints in (1) indicate that each country $k \in N$ consumes at most the sum of the water inflow at its location and the water inflows not consumed by its upstream countries. The second constraints in (1) indicate that the set of brothers of any country $k \in N$ including itself does not consume more water than the total inflow of water available to them. A consumption plan x induces the social welfare $\sum_{i \in N} b_i(x_i)$; x is optimal if it maximizes the social welfare. Under (A1) there is a unique optimal consumption plan.

In order to achieve the optimal consumption plan, some countries may refrain from consuming water. In turn, monetary compensations can be set up for these countries, which allow Pareto improvements. More precisely, money is available in unbounded quantities to perform side-payments. A compensation for country $i \in N$ is an amount of money $t_i \in \mathbb{R}$. If $t_i > 0$, then country i is a net beneficiary of monetary transfers; if $t_i < 0$, then country i is a net contributor to monetary transfers. A compensation scheme is a profile $t = (t_i)_{i \in N} \in \mathbb{R}^n$ of monetary transfers satisfying the budget constraint $\sum_{i \in N} t_i \leq 0$. A budget constraint is balanced if transfers add to zero. Countries value both water and money and are endowed with a quasi-linear utility function $u_i : \mathbb{R}_+ \times \mathbb{R} \longrightarrow \mathbb{R}$ that assigns to each pair $(x_i, t_i) \in \mathbb{R}_+ \times \mathbb{R}$ the utility $u_i(x_i, t_i) = b_i(x_i) + t_i$. A welfare distribution is a pair (x, t) where x is a consumption plan and t is a compensation scheme. The welfare distribution (x^*, t) is Pareto optimal if and only if x^* is the optimal consumption plan and the budget constraint is balanced. Each country i receives the payoff $z_i = u_i(x_i^*, t_i)$, and the sum of these payoffs is equal to the optimal social welfare $\sum_{i \in N} b_i(x_i^*)$. The problem is to find a fair distribution of this social welfare.

2.3 The river TU-game

A *cooperative game* with transferable utility (henceforth called a TU-game) is a pair (N, v) consisting of a finite agent set N of size $n \in \mathbb{N}$ and a coalition function $v : 2^N \longrightarrow \mathbb{R}$ satisfying $v(\emptyset) = 0$. An element S of 2^N is a coalition, and $v(S)$ is the maximal worth that the members of S can obtain by cooperating. For each nonempty coalition $S \in 2^N$, the *subgame* of (N, v) induced by S is the TU-game $(S, v_{|S})$ such that for any $T \in 2^S$, $v_{|S}(T) = v(T)$. A TU-game (N, v) is *superadditive* if $v(S \cup T) \geq v(S) + v(T)$ for any pair of disjoint coalitions S and T.

In the TU-game (N, v), each agent $i \in N$ may receive a payoff $z_i \in \mathbb{R}$. A *payoff vector* $z = (z_1, \ldots, z_n) \in \mathbb{R}^n$ lists a payoff z_i for each $i \in N$. For any nonempty coalition $S \in 2^N$ the notation z_S stands for $\sum_{i \in S} z_i$. An *allocation rule* Φ on a class of TU-games $\mathcal{C}$ is a map that assigns to each TU-game $(N, v) \in \mathcal{C}$ a payoff vector $\Phi(N, v) \in \mathbb{R}^n$.

One way to solve a river sharing problem is by treating it as a TU-game. We follow the presentation by Khmelnitskaya (2010) who constructs the following TU-game from a river problem where the river admits bifurcations. The player set N is the set of riparian countries located on a rooted tree described by the collection of downstream neighbors $\{d(i) : i \in N\}$. Due to the direction of the river's flow, each arc $i \to j$ is controlled by the tail i. Indeed, country i does not cooperate with country j if it refuses to pass water to j. The definition of the coalition function v takes into account this fact. Since the total worth to distribute is equal to the optimal social welfare, the worth of the grand coalition N is given by $v(N) = \sum_{i \in N} b_i(x_i^*)$. Further, a group of countries S on the river can cooperate if there exists $k \in S$ such that for any pair $\{i, j\} \subseteq S$, $i \in D[k]$ and $j \in D[k]$, and $u(i) \in D[k]$ for each $i \in S \backslash \{k\}$. This condition means that the two countries either are located on the same branch/distributary of the river, that is $k \in \{i, j\}$, which in turn implies that each other country between them also cooperates in S, or belong two different branches of the river but have a common upstream country in $S \backslash \{i, j\}$, which coordinates the allocation of water between them. If coalition S satisfies this condition, S is said to be *connected*. By definition, N is connected, and each singleton $\{i\}$ is viewed as a connected coalition.

Example 2 *Consider again the rooted tree of Figure 1. For each $i \in N$, coalition $D[i]$ formed by the union of $\{i\}$ and all its downstream countries $D(i)$ is a connected coalition. Coalition $\{1, 2, 3\}$ is also a connected coalition, whereas coalition $\{2, 3\}$, formed by the two brothers 2 and 3, is not connected because these two members trivially don't have a common upstream neighbor in $\{2, 3\}$.* □

The worth of each connected coalition S is given by the social welfare that it can secure for itself without the cooperation of the other countries. Therefore, for any connected coalition S, the worth $v(S)$ is given by:

$$v(S) = \sum_{k \in S} b_k(x_k^S),$$

where $x^S = (x_i^S)_{i \in S}$ solves

$$\max \sum_{k \in S} b_k(x_k) \text{ s.t.}$$

$$\forall k \in S, \quad \sum_{j \in U[k] \cap S} x_j \leq \sum_{j \in U[k] \cap S} e_j, \qquad \sum_{j \in U[k] \cup B_r(k) \cap S} x_j \leq \sum_{j \in U[k] \cup B_r(k) \cap S} e_j. \tag{2}$$

Assumption (A1) ensures that there is a unique solution to this program. A coalition S which is not connected admits a unique partition into maximally (with respect to set inclusion) connected parts, called its *components*. By definition of a TU-game, countries belonging to $N \backslash S$ do not cooperate with S and act non-cooperatively. Since the benefit functions are strictly increasing in the water consumed, each country located between two components of S will consume all the water inflow entering its location. Therefore, a component of S will never receive the water left over by another component located upstream. It follows that the worth of a non-connected coalition S is the sum of the worths of its components. Note also that a river TU-game is superadditive.

By allocating a payoff z_i to each $i \in N$, we determine a compensation scheme t as follows: for each country $i \in N$, $t_i = z_i - b_i(x_i^*)$. If $z_N = v(N)$, then the optimal social welfare is integrally allocated among the countries and t is budget balanced. The difficulty is thus to find a fair agreement on the allocation of the social welfare resulting from an optimal consumption plan, which in turn determines monetary compensations.

3 The sequential equal surplus division

3.1 Customary international law practice

Sharing river water has often been at the center of upstream-downstream tensions and conflicts. Many principles of international law have been developed to prevent or resolve international water disputes. Unfortunately, they are not easy to apply and often are contradictory because the effects are one-way and the property rights over water are not well defined (Wolf, 1999, Kilgour and Dinar, 1995, Kliot *et al.*, 2001). The first principle is the efficiency of water use. It indicates that the total welfare resulting from an implementation of the optimal consumption plan should be fully redistributed among the riparian countries. The second principle is the doctrine of “Absolute Territorial Sovereignty” (ATS for short), often initially claimed by upstream countries. This doctrine argues that a country has absolute rights to water flowing through its territory. If we extend this principle to each group of countries, this doctrine implies that each group of countries receive a total payoff at least equal to the worth they can obtain by agreeing to cooperate. Translated into game theoretical terms, the combination of water use efficiency and the extended interpretation of the ATS doctrine implies core stability.[2] The third principle is the doctrine of “Territorial Integration of all Basin States” (TIBS). Symmetrically, this principle favors downstream countries to which it accords “equal” use, without regard to their contribution to the flow. Ambec and Sprumont (2002) apply an extreme case of the TIBS doctrine, called the “Absolute Territorial Integrity” (ATI). It aims at protecting downstream country by stating that a downstream country has an absolute right to as much water as it can use. Under this principle, the upstream country has a legal obligation to leave as much water in an international river as the downstream countries require. Translated into game theoretical terms, this doctrine indicates that each coalition of countries should get a total payoff as least equal to the worth that it can secure for itself when the upstream countries let pass all the water flow. In the context of a river with one spring and without bifurcations – the river is shaped like a line – Ambec and Sprumont (2002) show that there is a unique efficient allocation rule that satisfies both the ATS and ATI doctrines. van den Brink *et al.* (2007) provide an axiomatic characterization of this solution. They also discuss this solution and note that it favors too much the downstream countries. Indeed, upstream countries can stop the cooperation with the downstream countries by consuming their total water inflow and so can claim much more than they get under the solution designed by Ambec and Sprumont (2002). As underlined by Kliot *et al.* (2001), the ATI doctrine is too restrictive, and therefore rarely used in practice.

[2] Recall that a payoff vector z belongs to the core of the TU-game (N, v) if $z_N = v(N)$ and $z_S \geq v(S)$ for each other coalition S.

3.2 ATS and TIBS doctrines and the division of the surplus

In this article we look for solutions that can be motivated by the ATS and TIBS doctrines, but with a more flexible interpretation of the TIBS doctrine. The general TIBS doctrine requires that the interests of all riparian countries has to be taken into account when allocating and using the waters of international rivers courses. This doctrine has been applied by international courts and also by national courts of federal countries. For instance, as mentioned above, allocations of the Indus river's flow within Pakistan have proved to be problematic. The creation of the Indus River System Authority in 1993 allowed representatives from both the federal government and the provinces within Pakistan to agree on an equitable intra-country allocation. The TIBS doctrine is also endorsed by the Helsinki Rules. The Helsinki Rules (1966) were the first attempt to create global standards for countries regulating how rivers and their connected groundwaters that cross national boundaries may be used.

We first notice that the so-called standard solution, a very popular allocation rule for the two-agent case, satisfies these two principles in a broader sense. It assigns first to a country its stand-alone worth and then distributes an equal share of the left surplus created by the cooperation. Rephrased in terms of the river sharing problem, the standard solution assigns first to each country the benefit of consuming the water inflow entering its own territory and then allocates half of the surplus that can be created by the two countries when they coordinate their water consumption. That is, country $i \in \{1, 2\}$ gets:

$$v(\{i\}) + \frac{v(\{1,2\}) - v(\{2\}) - v(\{1\})}{2}.$$

Since the river TU-game is superadditive, $v(\{1,2\}) - v(\{2\}) - v(\{1\}) \geq 0$, which implies that each country gets at least its stand-alone payoff: the ATS principle is well satisfied. On the other hand, since the surplus resulting from cooperation is equally distributed among them, the TIBS principle is also satisfied. This is the reason why we think that a desirable property for a solution on the class of river TU-games is that it implements the standard solution in the two-agent situation. Below, we will translate this requirement into a property.

We would like to generalize this principle from the two-country situation to the situation where n countries are located along a river with bifurcations. So, assume that countries in set N of size $n > 2$ are located along a river with bifurcations represented by the collection $\{d(i) : i \in N\}$. Pick any country $i \in N$ and focus on the welfare produced by this country and all its downstream countries. The worth $v(D[i])$ is jointly created by country i and the existing coalitions $D[j]$ for each $j \in d(i)$. The reason is obvious: on the one hand, country i can threaten to consume all the water inflow available to it instead of letting some water flowing out toward its downstream countries; but on the other hand, the optimal consumption plan might require its downstream countries to consume extra water from i. In other words, country i and each coalition of countries $D[j]$ for each $j \in d(i)$ have to cooperate in order to produce the total worth $v(D[i])$. In the absence of cooperation, the total welfare achieved by these countries reduces to:

$$v(\{i\}) + \sum_{j \in d(i)} v(D[j]).$$

If one considers each coalition $D[j]$ as a single entity in the problem of distributing the total welfare $v(D[i])$, then it is natural to apply the well-known equal surplus division to such a $|d[i]|$-agent situation.[3] We have:

$$v(\{i\}) + \frac{1}{|d[i]|}\Big(v(D[i]) - v(\{i\}) - \sum_{j \in d(i)} v(D[j])\Big)$$

to country i and, for each coalition of downstream countries $D[j]$, $j \in d(i)$,

$$v(D[j]) + \frac{1}{|d[i]|}\Big(v(D[i]) - v(\{i\}) - \sum_{j \in d(i)} v(D[j])\Big).$$

The allocation rule that we construct consists in a sequential application of the equal surplus division principle from upstream to downstream, which means that for country i and its downstream countries, ~~it might remain more than~~ $v(D[i])$ to share. Before defining formally this allocation rule, we provide an example.

3 For any finite set A, the notation $|A|$ stands for the cardinality of A.

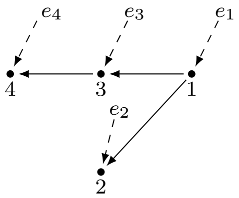

Fig. 2

Example 3 *Consider the river with bifurcations represented in Figure 2. Initially the four countries cooperate with each other and the welfare $v(\{1,2,3,4\})$ is to be distributed. The countries receive their payoff sequentially according to their distance to the spring of the river. We compute the payoff of country 1 first on the basis of what would happen if it refuses to cooperate. In such a case, the grand coalition would partition into three coalitions: $\{1\}$, $\{2\}$ and $\{3,4\}$, where $\{3,4\}$ acts as a single entity. How much should country 1 get from the worth $v(\{1,2,3,4\})$ as it threatens to refuse the cooperation? No doubt these three entities should first get the total worth they can secure without the cooperation of the other countries, i.e. $v(\{1\})$, $v(\{2\})$ and $v(\{3,4\})$, respectively. As for the surplus $v(\{1,2,3,4\}) - v(\{1\}) - v(\{2\}) - v(\{3,4\})$, one can argue that since country 1 is now negotiating with coalition $\{2\}$ and with coalition $\{3,4\}$ as a whole and the surplus is jointly created by these three parties, the equal surplus division for 3-agent TU-games can be applied so that each party should get one third of the joint surplus. Consequently, country 1 obtains its individual worth plus one third of the joint surplus:*

$$v(\{1\}) + \frac{1}{3}\Big(v(\{1,2,3,4\}) - v(\{1\}) - v(\{2\}) - v(\{3,4\})\Big).$$

The welfare shares remaining for coalitions $\{2\}$ and $\{3,4\}$, which we call the egalitarian remainders for $\{2\}$ and $\{3,4\}$, are given respectively by:

$$v(\{2\}) + \frac{1}{3}\Big(v(\{1,2,3,4\}) - v(\{1\}) - v(\{2\}) - v(\{3,4\})\Big) \tag{3}$$

and

$$v(\{3,4\}) + \frac{1}{3}\Big(v(\{1,2,3,4\}) - v(\{1\}) - v(\{2\}) - v(\{3,4\})\Big).$$

Then, the above procedure has to be repeated on each branch of the river with respect to the direction of the river's flow. Thus, both remaining coalitions $\{2\}$ and $\{3,4\}$ behave independently on their own branch of the river in order to negotiate the allocation of their respective remainder. On the one hand, country 2 is the unique country on its branch of the river so that the final payoff of 2 is given by (3). On the other hand, country 3 can now be considered as the spring of the branch of the river on which it is located. As above, the egalitarian remainder for country 4 will be:

$$v(\{4\}) + \frac{1}{2}\bigg(v(\{3,4\}) + \frac{1}{3}\Big(v(\{1,2,3,4\}) - v(\{1\}) - v(\{2\}) - v(\{3,4\})\Big) - v(\{3\}) - v(\{4\})\bigg),$$

while agent 3 will get:

$$v(\{3\}) + \frac{1}{2}\bigg(v(\{3,4\}) + \frac{1}{3}\Big(v(\{1,2,3,4\}) - v(\{1\}) - v(\{2\}) - v(\{3,4\})\Big) - v(\{3\}) - v(\{4\})\bigg).$$

□

Extending this argument to each n-country river TU-game, we obtain a solution on $\mathcal{C}_0$, called the *sequential equal surplus division*. The sequential equal surplus division on $\mathcal{C}_0$, denoted by Φ^e, is defined as follows. Pick any $(N,v) \in \mathcal{C}_0$, any $i \in N$, and define the *egalitarian remainder* α for coalition $D[i]$ as:

$$\alpha(D[i]) = \begin{cases} v(N) & \text{if } i = r, \\ v(D[i]) + \dfrac{\alpha(D[u(i)]) - \sum\limits_{j \in d(u(i))} v(D[j]) - v(\{u(i)\})}{|d[u(i)]|} & \text{otherwise.} \end{cases} \tag{4}$$

Note that by definition $i \in d(u(i))$ for each $i \neq r$. The sequential equal surplus division assigns to each $i \in N$, the payoff given by[4]:

$$\Phi^e_i(N,v) = v(\{i\}) + \frac{1}{|d[i]|}\Big(\alpha(D[i]) - \sum_{j \in d(i)} v(D[j]) - v(\{i\})\Big). \quad (5)$$

Let us verify that the sequential equal surplus division is an efficient allocation rule and provides a compromise between the ATS and TIBS doctrines.

We show below that for each coalition formed by a country and its downstream countries, the egalitarian remainder is fully redistributed among its members, *i.e.* for each $i \in N$, it holds that $\Phi^e_{D[i]}(N,v) = \alpha(D[i])$. We have $D[r] = N$, $\alpha(N) = v(N)$ by (4) and so $\Phi^e_N(N,v) = v(N)$, which proves efficiency.

The ATS doctrine prescribes that each country i has the right to all water on its territory, *i.e.* $\Phi^e_i(N,v) \geq v(\{i\})$. This doctrine can be extended to all coalitions. In our case, this doctrine is satisfied only for coalitions formed by a country and all its downstream countries, *i.e.* for each $i \in N$, it holds that $\Phi^e_{D[i]}(N,v) \geq v(D[i])$. Therefore, a coalition consisting of any country and all its downstream countries obtains a share of the total welfare that is at least as large as the welfare they can achieve without the cooperation of their upstream countries in the river.[5]

The TIBS doctrine states that each country has the right to all water flowing in its territory, no matter where the river's flow enters. This means that each country has the right to a certain share of the total welfare resulting from the cooperation of all members living along the river. This share is measured by the non-negative difference $\Phi_i(N,v) - v(\{i\}) \geq 0$ given by (5).

These properties are summarized in the following proposition and stated for all river TU-games.

Proposition 1 *For each river TU-game (N,v), and each country $i \in N$, it holds that:*
(i) $\Phi^e_{D[i]}(N,v) = \alpha(D[i])$.
(ii) $\alpha(D[i]) \geq v(D[i])$;
(iii) $\Phi_i(N,v) \geq v(\{i\})$.

Proof. Pick any river TU-game (N,v) and any $i \in N$. The proof of part (i) is by induction on the number of downstream countries of i.
Initial step: Assume that i has no downstream countries, *i.e.* $D[i] = \{i\}$. Thus, $d(i) = \emptyset$ and (5) imply:

$$\Phi^e_{D[i]}(N,v) = \Phi^e_i(N,v) = \alpha(\{i\}).$$

Induction hypothesis: Assume that the assertion holds when $D[i]$ contains at most $q < n$ elements.
Induction step: Assume that $D[i]$ contains $q+1$ elements. Then:

$$\Phi^e_{D[i]}(N,v) = \Phi^e_i(N,v) + \sum_{j \in d(i)} \Phi^e_{D[j]}(N,v).$$

Since $j \in d(i)$, each $D[j]$ contains at most q elements. By the induction hypothesis, the right-hand side of the above equality is equivalent to:

$$\Phi^e_i(N,v) + \sum_{j \in d(i)} \alpha(D[j]).$$

Using the definition (4) of the egalitarian remainder and the fact that for each $j \in d(i)$, $u(j) = i$, the previous expression can be rewritten as follows:

[4] Φ^e generalizes the individual standardized remainder vectors proposed by Ju, Borm and Ruys (2007) for the class of all TU-games. Each individual standardized remainder vector is constructed from a bijection over the player set. Because a bijection induces a total order on the player set, it is isomorphic to a line. In this special case, the resulting individual standardized remainder vector coincides with (4). The major difference with our allocation rule is that more than two coalitions may negotiate to share a surplus when the river continues on different branches.

[5] This property is similar to the subsidy-free property used in Aadland and Kolpin (1998) for the related problem of sharing the cleaning costs of irrigation ditches.

$$
\begin{aligned}
& v(\{i\}) + \frac{1}{|d[i]|}\Big(\alpha(D[i]) - \sum_{j\in d(i)} v(D[j]) - v(\{i\})\Big) \\
& + \sum_{j\in d(i)} \Big(v(D[j]) + \frac{1}{|d[i]|}\Big(\alpha(D[i]) - \sum_{k\in d(i)} v(D[k]) - v(\{i\})\Big)\Big) \\
= \; & v(\{i\}) + \frac{1}{|d[i]|}\Big(\alpha(D[i]) - \sum_{j\in d(i)} v(D[j]) - v(\{i\})\Big) \\
& + \sum_{j\in d(i)} v(D[j]) + \frac{|d(i)|}{|d[i]|}\Big(\alpha(D[i]) - \sum_{k\in d(i)} v(D[k]) - v(\{i\})\Big) \\
= \; & \alpha(D[i]).
\end{aligned}
$$

Therefore, we get:

$$
\begin{aligned}
\Phi^e_{D[i]}(N,v) &= \Phi^e_i(N,v) + \sum_{j\in d(i)} \alpha(D[j]) \\
&= \alpha(D[i]),
\end{aligned}
$$

as desired.

The proof of part (ii) is by induction on the depth q of a country i in the rooted tree describing the river structure.

Initial step: Assume that $q = 0$ so that i is the root r of the rooted tree on N, *i.e.* $D(r) = N$. By (4), we get $\alpha(N) = v(N)$, as desired.

Induction hypothesis: Assume that the assertion is true for each country $i \in N$ such that its depth is inferior or equal to $q < q_N$ where q_N is the depth of the rooted tree.

Induction step: Pick any country $i \in N$ such that its depth is $q + 1$. It follows that the number of links on the path from the root r to $u(i)$ is equal to q. By the induction hypothesis and superadditivity of v, we get:

$$
\begin{aligned}
\alpha(D(i)) &= v(D[i]) + \frac{1}{|d_r[u(i)]|}\Big(\alpha(D[u(i)]) - \sum_{j\in d(u(i))} v(D[j]) - v(\{u(i)\})\Big) \\
&\geq v(D[i]) + \frac{1}{|d_r[u(i)]|}\Big(v(D[u(i)]) - \sum_{j\in d(u(i))} v(D[j]) - v(\{u(i)\})\Big) \\
&\geq v(D[i]),
\end{aligned}
$$

as desired.

The proof of part (iii) follows from (ii), the definition (5) of Φ^e and the superadditivity of the river TU-game (N, v). It suffices to note that the collection of coalitions $D[j]$, $j \in d(i)$, and the singleton $\{i\}$ constitute a partition of $D[i]$. ■

It is interesting to compare our solution to the solutions provided by Ambec and Sprumont (2002), van den Brink *et al.* (2007) for a river with one spring and without bifurcations and their extension to a river with one spring and bifurcations provided by Khmelnitskaya (2010). Assume first that the river has one spring and possibly many bifurcations, then the solution suggested by Khmelnitskaya (2010) is the so-called hierarchical outcome introduced by Demange (2004) in another context and defined by:

$$
\forall i \in N, \quad h_i(N,v) = v(D[i]) - \sum_{j\in d(i)} v(D[j]). \tag{6}
$$

Each country's payoff is equal to its marginal contribution to all its downstream countries when it cooperates with them. In case the river has no bifurcation — the river is the line $\{i \to i+1 : i \in N\backslash\{n\}\}$ — each country other than the country n located at the sink of the river has exactly one downstream neighbor $i+1$, so that $D[i] = \{i, i+1, \ldots, n\}$ and $D[i+1] = \{i+1, i+2, \ldots, n\}$ for $\{i+1\} = d(i)$. The hierarchical outcome reduces to:

$$h_n(N, v) = v(\{n\}) \text{ and for } i \in N \backslash \{n\},\ h_i(N, v) = v(\{i, \ldots, n\}) - v(\{i+1, \ldots, n\}).$$

As shown by Demange (2004), van den Brink *et al.* (2007) and Khmelnitskaya (2010), this solution possesses several advantages. Firstly, it is efficient and satisfies also point (ii) in Proposition 1 not only for coalitions formed by a country and all its downstream countries but for all coalitions, which means that this solution is core-stable. Secondly, it also gives an incentive for a country to cooperate with its downstream countries in the sense that all the surplus resulting from this cooperation is allocated to this country. Indeed, this surplus is given by:

$$v(D[i]) - \sum_{j \in d(i)} v(D[j]) - v(\{i\})$$

and i's payoff in (6) is exactly the same as:

$$v(\{i\}) + v(D[i]) - \sum_{j \in d(i)} v(D[j]) - v(\{i\}).$$

This solution is the opposite of the solution suggested by Ambec and Sprumont (2002) for a river with no bifurcations, which distributes all the surplus resulting from the cooperation of a country with its upstream countries to this country:

$$\text{the root } r = 1 \text{ gets } v(\{1\}) \text{ and each other country } i \text{ gets } v(\{1, 2, \ldots, i\}) - v(\{1, 2, \ldots, i-1\}).$$

One main drawback of these solutions is that they rely on two extreme interpretations of the TIBS doctrine. As a consequence, these solutions do not coincide with the standard solution for the two-agent case. On the contrary, our procedure is a comprise between the ATS and TIBS doctrines, without relying on an extreme interpretation of the latter. Of course, it is still possible to average between several solutions that do not violate the TIBS doctrine. This is the option taken by van den Brink *et al.* (2012) for river TU-games with multiple springs. Their solution assigns to every river TU-game a weighted average of different hierarchical outcomes, computed from various other directed trees than the natural one describing the river structure. This type of solutions has been introduced and axiomatically characterized by Béal *et al.* (2010, 2012) on the class of cycle-free graph TU-games. It generalizes the so-called average tree solution introduced and axiomatically characterized by Herings *et. al.* (2008) on the same class of games. Despite the interest of this type of solutions, we are inclined to think that averaging between different hierarchical outcomes makes the procedure of allocation often unclear and artificial for practitioners, especially in the context of the river sharing problem where the direction of river's flow is given once and for all. On the contrary, our solution is constructed recursively from the spring of the river and remains consistent with the direction of the river's flow.

Another way to compare (6) with our solution is to use point (i) of Proposition 1. Indeed, for each $i \in N$, we have:

$$\begin{aligned} \Phi^e_i(N, v) &= \Phi^e_{D[i]}(N, v) - \sum_{j \in d(i)} \Phi^e_{D[j]}(N, v) \\ &= \alpha(D[i]) - \sum_{j \in d(i)} \alpha(D[j]). \end{aligned}$$

Thus, each country's payoff is equal to its contribution to the egalitarian remainder when it cooperates with all its downstream countries.

As we will see in the next sections, our solution satisfies several natural principles of efficiency, fairness, consistency, and collusion.

4 Properties for resolving water disputes

Each river TU-game belongs to a larger class of TU-games, denoted by $\mathcal{C}_0$, and defined as follows:

(i) The countries of set N are located along a river with bifurcations represented by a rooted-tree.

(ii) The collection of possible coalitions of countries is restricted by the shape of the underlying river. Precisely, the worth of each nonempty coalition of countries is the sum of the worths of its connected components.

(iii) The resulting TU-game (N, v) is superadditive.

The only difference with a river TU-game is that the worth of a coalition is not necessarily defined by the program given in (2). In this section, we present some properties of allocation rules Φ on the class $\mathcal{C}_0$. The reason for this change is that among the six properties we will propose, two of them need the construction of a new TU-game from a given river TU-game. This new TU-game is not strictly speaking a river TU-game but belongs to $\mathcal{C}_0$ and represents a new situation that we derive from a river TU-game. For instance, we consider a river TU-game in which some countries may collude to act as a single entity. The other countries face a new situation that we represent by a companion TU-game constructed from the original river TU-game.

In the next section, we state two axiomatic characterizations of the sequential equal surplus division Φ^e. The first one is valid on the class of all river TU-games whereas the second one is valid on $\mathcal{C}_0$. For convenience, we present all properties on the class $\mathcal{C}_0$. The first property states that the allocation rule should coincide with the standard solution if the TU-game involves only two countries. As asserted above, this property combines the principles incorporated in the ATS and TIBS doctrines.

Standardness For each $(N, v) \in \mathcal{C}_0$ such that $N = \{1, 2\}$, it holds that:

$$\forall i \in \{1, 2\}, \quad \Phi_i(N, v) = v(\{i\}) + \frac{v(\{1, 2\}) - v(\{1\}) - v(\{2\})}{2}.$$

The principle of efficiency of water use is translated into the following property.

Efficiency For each $(N, v) \in \mathcal{C}_0$, it holds that:

$$\Phi_N(N, v) = v(N).$$

The TIBS doctrine emphasizes compromise and fairness. This being said, one easily admits that the principle of "equitable and reasonable utilization of the water resources" is an elusive concept. An upstream country may believe that its diversion is equitable and a downstream country may still object on the grounds that this action is not equitable. The TIBS doctrine is an intentionally loose approach to the sharing of resources; it allows countries and courts to evaluate each case with respect to different criteria such as the geography of the river, the projects elaborated by countries and their consequences on downstream countries. In order to elaborate on the TIBS requirement, we propose two fairness properties for two different situations. Each of these properties measures changes in payoffs as a result of individual change of strategy. Typically, we consider the case where a country stops to cooperate with one or all its neighboring countries by refusing to trade water in exchange of money, by building a barrage or by wasting resources.

Firstly, take a river TU-game (N, v), and consider the following situation. All countries cooperate, but country i envisages to cease the cooperation with all its downstream neighbors. At the same time, its upstream neighbor threats it to discontinue the delivery of water if it ceases to cooperate with its downstream neighbors. If both the project of i and the threat of its upstream neighbor are implemented, country i will become an isolated country along the river. Further, on each branch of the river from i, each group of downstream countries $D[j]$, $j \in d(i)$, will form a new coalition whose members will no longer cooperate with i. It follows that each $D[j]$ will induce a subriver with bifurcations of the international river, with root j and members $D[j]$. The corresponding river TU-game associated to its subriver is the subgame $(D[j], v_{|D[j]})$ of (N, v) on $D[j]$. Similarly, the corresponding river TU-game associated with the autarkic country i is the subgame $(\{i\}, v_{|\{i\}})$ of (N, v) on $\{i\}$. Note that these subriver TU-games are themselves river TU-games. The natural question is how should the total payoffs of these coalitions be affected by these changes? We interpreted the TIBS doctrine by requiring the same loss of welfare.

Downstream Fairness For each $(N, v) \in \mathcal{C}_0$ with at least two countries and each $i \in N$ such that $d(i) \neq \emptyset$, it holds that:

$$\forall j \in d(i), \quad \Phi_i(N, v) - \Phi_i(\{i\}, v_{|\{i\}}) = \Phi_{D[j]}(N, v) - \Phi_{D[j]}(D[j], v_{|D[j]}).$$

In particular, this property implies that this evolution of the cooperation structure yields the same total change in payoff for each coalition consisting of a downstream neighbor of i and all its downstream countries, *i.e.*

$$\forall j, k \in d(i), \quad \Phi_{D[j]}(N, v) - \Phi_{D[j]}(D[j], v_{|D[j]}) = \Phi_{D[k]}(N, v) - \Phi_{D[k]}(D[k], v_{|D[k]}).$$

In order to define the second fairness property, we consider a specific situation in which the distributaries inflow into different drainage basins in a short distance. This hydrological phenomenon happens when the river stream reaches watershed areas with flattened topography. The rivers flowing on the flat clay floor in these areas can easily change their river beds and in cases of big rainfall they can overflow and create short bifurcations. Because this type of international river structure is a rather rare natural phenomenon, we can alternatively view this situation as the subriver of a larger river. In the sequel, the whole river in such a situation is a called a *bifurcation*. In a bifurcation whose root is $r \in N$, each distributary of the river has r for tail. This implies that the depth of the associated rooted-tree is 1. Figure 3 depicts a bifurcation whose root is country 1.

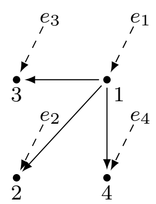

Fig. 3

Now consider a river TU-game (N, v) where the river is a bifurcation. Assume that country r envisages to no longer pass water to the distributary going from r to country i. Instead, country r can either decide to consume this amount of water or to distribute it to its other downstream neighbors. A natural question arises: under the TIBS doctrine, how should the payoff of each of the remaining downstream neighbors of r be affected if r decides to implement this strategy? Here, we interpret the TIBS doctrine by requiring the same payoff variation between any pair of remaining countries. Note that this situation is different from the previous one since country r only stop cooperating with one downstream neighbor.

Bifurcation Fairness For each $(N, v) \in \mathcal{C}_0$ such that the underlying river is a bifurcation with root $r \in N$ and $|N| \geq 3$, the following holds. For each downstream neighbor i of r we have:

$$\forall j, k \in N \backslash \{i\}, \quad \Phi_j(N, v) - \Phi_j(N \backslash \{i\}, v_{|N \backslash \{i\}}) = \Phi_k(N, v) - \Phi_k(N, \backslash \{i\}, v_{|N \backslash \{i\}}).$$

Note that the property above applies to bifurcations with at least three members. Otherwise, we can apply standardness if the bifurcation contains two members and efficiency if it is a singleton.

The next property incorporates a consistency principle. Informally, a consistency principle states the following. Fix a solution for a class of TU-games. Assume that some agents leave the game with their payoffs, and examine the reduced problem that the remaining agents face. The solution is consistent if for this reduced game, there is no need to re-evaluate the payoffs of the remaining agents. As noted by Aumann (2008) and Thomson (2011), the consistency principle has been examined in the context of a great variety of concrete problems of resource allocation. In one form or another, it is common to almost all solutions and often plays a key role in axiomatic characterization of the solutions. In the context of river TU-games, we design a consistency property which takes into account the direction of the river's flow.

Pick any river TU-game (Nv) with root $r \in N$ and $i \in N$. Assume that the payoffs have been distributed according to the payoff vector $z \in \mathbb{R}^n$ and that the countries $N \backslash D[i]$ leave the game with their component of the vector z. Let us re-evaluate the situation of $D[i]$ at this point. To do this, we define the reduced game they face. The worths of coalitions in the reduced games can depend on (i) the worths that these coalitions could earn on their own in the original game, (ii) what these coalitions could earn with the leaving countries and (iii) the payoff with which the leaving countries left the game. Our *reduced TU-game* $(D[i], v_{z,i})$ induced by $D[i]$ and z is defined as follows: $v_{z,i}(D[i]) = v(N) - z_{N\backslash D[i]}$ and for each other coalition $S \subset D[i]$, $v_{z,i}(S) = v(S)$. Thus, $v_{z,i}(D[i])$ is the total worth left for the remaining countries who interact according to $(D[i], v_{z,i})$ on the induced subriver on $D[i]$.[6] Note that for a river TU-game, its reduction is not necessarily superadditive. This depends on the worth $v(N) - z_{N\backslash D[i]}$. For instance, each reduced game $(D[i], v_{\Phi^e,i})$ constructed from Φ^e remains superadditive by point (ii) of Proposition 1. More generally, if (N, v) belongs to $\mathcal{C}_0$, then $(D[i], v_{\Phi^e,i})$ belongs to $\mathcal{C}_0$ as well. Consequently, we can restrict the analysis to allocation rules that preserve this superadditivity property. In this way, the following consistency property can be applied to the class $\mathcal{C}_0$.

Downstream Consistency For each $(N, v) \in \mathcal{C}_0$ and each $i \in N$, it holds that:

$$(D[i], v_{\Phi,i}) \in \mathcal{C}_0 \text{ and } \forall j \in D[i], \quad \Phi_j(N, v) = \Phi_j(D[i], v_{\Phi,i}).$$

Downstream consistency is a robustness requirement guaranteeing that a coalition of downstream countries respects the recommendations made by Φ when the other countries have already received their payoffs according to the solution Φ. In particular, downstream consistency is compatible with a solution constructed recursively from the spring of the river.

The last property incorporates an amalgamation principle. This principle says something about the changes in payoffs when two or more countries are amalgamated to act as if they were a single country. It states that if some countries are amalgamated into one entity, just because they have colluded, then the payoff of these countries in the new game coincides with the sum of the payoffs of the amalgamated country in the original game.[7]

Pick any river TU-game (N, v) with root $r \in N$, and any $i \in N$. For each $j \in d(i)$, the members of $D[j]$ collude and act as a single entity so that they are amalgamated into a new single entity denoted by $\overline{D[j]}$. From this operation of amalgamation, we define a companion river TU-game as follows. The country set N^i is given by:

$$N^i = \Big[N \backslash \Big(\bigcup_{j \in d(i)} D[j] \Big) \Big] \bigcup_{j \in d(i)} \Big\{ \overline{D[j]} \Big\}.$$

The rooted tree on N^i contains the directed links $i \to \overline{D[j]}$ for each $j \in d(i)$ plus all the original directed links between pairs of countries that belong to $N \backslash \big(\bigcup_{j \in d(i)} D[j] \big)$.

Example 4 *The amalgamation process is illustrated by the river in Figure 4 in which the inflow entering each country is not specified for keep the picture simple enough. Part (b) represents the river obtained after amalgamation of the countries located on each branch downstream of country 3. Therefore, the coalitions* $\{5, 9\}$, $\{4, 7, 8\}$ *and* $\{6\}$ *have been transformed into single entities* $\overline{\{4, 7, 8\}}$, $\overline{\{5, 9\}}$ *and* $\overline{\{6\}}$ *respectively. Note that the amalgamated entity* $\overline{\{6\}}$ *is formally identical to the original one* $\{6\}$. *Country 3 and its upstream countries are not affected. Regarding the links of the new river, the links* $5 \to 9$, $4 \to 7$ *and* $4 \to 8$ *within the former branches completely disappear. The former links* $3 \to 4$, $3 \to 5$ *and* $3 \to 6$ *are somehow replaced by links* $3 \to \overline{\{4, 7, 8\}}$, $3 \to \overline{\{5, 9\}}$ *and* $3 \to \overline{\{6\}}$ *respectively. The other links are not affected.* □

Since the members of $D[j]$ behave as a single entity, the coalitions contained in $D[j]$ as well as their links are not taken into account in the description of the new coalition function. Therefore, we define (N^i, v^i) as follows: for each $S \in 2^{N^i}$,

$$v^i(S) = \begin{cases} v(S) & \text{if } S \cap D[j] = \emptyset \text{ and } j \in d(i), \\ v\Big(\big(S \backslash \{ \overline{D[j]} : j \in d(i) \} \big) \bigcup_{\{j \in d(i), \overline{D[j]} \in S\}} D[j] \Big) & \text{otherwise.} \end{cases}$$

[6] This reduced TU-game, also called the projected reduced game, is customary used in cooperative game theory to construct consistency axioms.

[7] This principle has been used by Lehrer (1988), Albizuri (2001) and Albizuri, Aurrekoetxea (2006), among others, in order to characterize power indexes in voting games.

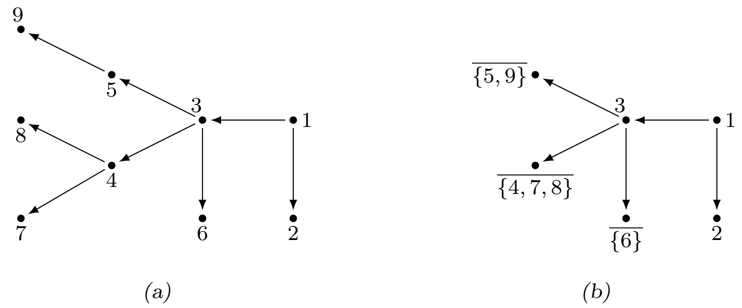

Fig. 4

In particular, for each player $\overline{D[j]}$, we have $v^i(\{\overline{D[j]}\}) = v(D[j])$. Of course, if $d(i)$ is empty, then (N^i, v^i) coincides with (N, v). Note that each (N^i, v^i) belongs to $\mathcal{C}_0$.

Downstream Amalgamation For each river TU-game $(N, v) \in \mathcal{C}_0$ and each $i \in N$ such that $d(i) \neq \emptyset$, it holds that:

$$\forall j \in d(i), \quad \Phi_{D[j]}(N, v) = \Phi_{\overline{D[j]}}(N^i, v^i).$$

A similar principle has been used in Ansink and Weikard (2012) in the context of a river with one spring and without bifurcations. But, these authors depart from the TU-game approach by assuming that each country along the river has a claim to the river's flow and that the sum of downstream claims exceeds the sum of downstream endowments at each location. They transform the full problem into an ordered list of two-agent interdependent river sharing problems, each of which is formally equivalent to a bankruptcy problem and is solved through a bankruptcy rule.

5 Two characterizations of Φ^e

This section contains the main results of this article. We start by showing that Φ^e satisfies all the properties listed in the previous section.

Proposition 2 *The allocation rule Φ^e satisfies standardness, efficiency, downstream fairness, bifurcation fairness, downstream consistency, and downstream amalgamation on $\mathcal{C}_0$.*

Proof. From (4) and (5), it is easily verified that Φ^e satisfies standardness. The fact that Φ^e satisfies efficiency follows directly from part (i) in Proposition 1 and (4). To check that Φ^e satisfies downstream fairness, pick any $(N, v) \in \mathcal{C}_0$ with root $r \in N$ and any $i \in N$ such that $d(i) \neq \emptyset$. For each $j \in d(i)$, we have:

$$\begin{aligned}
\Phi^e_{D[j]}(N, v) - \Phi^e_{D[j]}(D[j], v_{|D[j]}) &= \Phi^e_{D[j]}(N, v) - v(D[j]) \\
&= \alpha(D[j]) - v(D[j]) \\
&= \frac{1}{|d[i]|}\left(\alpha(D[i]) - \sum_{k \in d(i)} v(D[k]) - v(\{i\})\right) \\
&= \Phi^e_i(N, v) - v(\{i\}) \\
&= \Phi^e_{\{i\}}(N, v) - \Phi^e_{\{i\}}(\{i\}, v_{|\{i\}}),
\end{aligned}$$

where the first equality follows from the fact that Φ^e satisfies efficiency. The second equality follows from part (i) in Proposition 1. The third and four equalities come from $u(j) = i$ and (4)–(5) respectively. Therefore, Φ^e satisfies downstream fairness.

Next, consider any $(N, v) \in \mathcal{C}_0$ where the underlying river is a bifurcation containing at least three countries and whose root is $r \in N$. Pick any branch $r \to i$. In order to verify that Φ^e satisfies bifurcation fairness, we first compute $\Phi^e_j(N, v)$ for each $j \in N$. By construction and definition of the egalitarian remainder, we have: $D[r] = N$, $\alpha(N) = v(N)$ and, for each $j \in N\backslash\{r\}$, $u(j) = r$ and also $D[j] = \{j\}$. Consequently, from (4) and (5) we get:

$$\forall j \in N, \quad \Phi^e_j(N, v) = v(\{j\}) + \frac{1}{n}\left(v(N) - \sum_{k \in N} v(\{k\})\right). \tag{7}$$

By a similar computation on the subgame $(N\backslash\{i\}, v_{|N\backslash\{i\}}) \in \mathcal{C}_0$, we get that for each $j \in N\backslash\{i\}$:

$$\Phi^e_j(N\backslash\{i\}, v_{|N\backslash\{i\}}) = v_{|N\backslash\{i\}}(\{j\}) + \frac{1}{|N|-1}\Bigg(v_{|N\backslash\{i\}}(N\backslash\{i\}) - \sum_{k\in N\backslash\{i\}} v_{|N\backslash\{i\}}(\{k\})\Bigg). \qquad (8)$$

From (7)–(8) and the fact that $v(\{j\}) = v_{|N\backslash\{i\}}(\{j\})$ for each $j \in N\backslash\{i\}$, we get:

$$\forall j, k \in N\backslash\{i\}, \quad \Phi^e_j(N, v) - \Phi^e_j(N\backslash\{i\}, v_{|N\backslash\{i\}}) = \Phi^e_k(N, v) - \Phi^e_k(N\backslash\{i\}, v_{|N\backslash\{i\}}),$$

which proves that Φ^e satisfies bifurcation fairness.

In order to verify that Φ^e satisfies downstream consistency, we first compare the payoffs $\Phi^e_i(N, v)$ and $\Phi^e_i(D[i], v_{\Phi^e,i})$ for any $(N, v) \in \mathcal{C}_0$ and any $i \in N$. By definition of $(D[i], v_{\Phi^e,i})$, we have $v_{\Phi^e,i}(D[i]) = v(N) - \Phi^e_{N\backslash D[i]}(N, v)$. Because Φ^e satisfies efficiency and by part (i) of Proposition 1, we get:

$$v_{\Phi^e,i}(D[i]) = \Phi^e_{D[i]}(N, v) = \alpha(D[i]).$$

Using the definition of $(D[i], v_{\Phi,i})$, it follows that:

$$\begin{aligned}
\Phi^e_i(D[i], v_{\Phi^e,i}) &= v_{\Phi^e,i}(\{i\}) + \frac{1}{|d[i]|}\left(v_{\Phi^e,i}(D[i]) - \sum_{j\in d(i)} v_{\Phi^e,i}(D[j]) - v_{\Phi^e,i}(\{i\})\right)\\
&= v(\{i\}) + \frac{1}{|d[i]|}\left(\alpha(D[i]) - \sum_{j\in d(i)} v(D[j]) - v(\{i\})\right)\\
&= \Phi^e_i(N, v),
\end{aligned}$$

as desired for country i. It follows that the egalitarian remainder for each $j \in d(i)$ is the same in $(D[i], v_{\Phi^e,i})$ and (N, v). So, using the definition of $(D[i], v_{\Phi^e,i})$, we also have $\Phi^e_j(D[i], v_{\Phi^e,i}) = \Phi^e_j(N, v)$ for each $j \in d(i)$. Continuing in this fashion for each downstream country of i, we reach the desired conclusion.

Finally, consider any $(N, v) \in \mathcal{C}_0$ and any $i \in N$ such that $d(i) \neq \emptyset$. Pick any $j \in d(i)$. On the one hand, by part (i) of Proposition 1, we have $\Phi^e_{D[j]}(N, v) = \alpha(D[j])$. On the other hand, from (5) and definition of $(N^i, v^i) \in \mathcal{C}_0$, we deduce that $\Phi^e_k(N^i, v^i) = \Phi^e_k(N, v)$ for each k who does not belong to the set of i's downstream countries. To understand this equality, note that the egalitarian remainder for such a k in (N, v) and in (N^i, v^i) do not rely on proper coalitions of $D[j]$, $j \in d(i)$ so that the computations from v give the same results as the computations from v^i. Therefore, for each $j \in d(i)$, the egalitarian remainder for $D[j]$ in (N, v) coincides with the egalitarian remainder for $\overline{D[j]}$ in (N^i, v^i). Because each country $\overline{D[j]}$, $j \in d(i)$, has no downstream neighbor in the new tree induced by N^i, we easily conclude that $\Phi^e_{\overline{D[j]}}(N^i, v^i) = \alpha(D[j]) = \Phi^e_{D[j]}(N, v)$. This proves that Φ^e satisfies downstream amalgamation. ■

Combining efficiency and downstream fairness we obtain a characterization of the sequential equal surplus division on $\mathcal{C}_0$.

Proposition 3 *The sequential equal surplus division Φ^e is the unique allocation rule that satisfies efficiency and downstream fairness on $\mathcal{C}_0$.*

Proof. By Proposition 2, Φ^e satisfies efficiency and downstream fairness. Next, consider an allocation rule Φ that satisfies efficiency and downstream fairness on $\mathcal{C}_0$. To show: $\Phi = \Phi^e$. Pick any $(N, v) \in \mathcal{C}_0$. First note that, for each $i \in N$, we have:

$$\Phi_i(N, v) = \Phi_{D[i]}(N, v) - \sum_{j\in d(i)} \Phi_{D[j]}(N, v).$$

It remains to prove that, for each $i \in N$, $\Phi_{D[i]}(N, v)$ is uniquely determined by efficiency and downstream fairness. We proceed by induction on the depth q of the countries on the river.
Initial step: If $q = 0$, then $i = r$. By efficiency, we get:

$$\Phi_{D[r]}(N, v) = \Phi_N(N, v) = v(N).$$

Thus, $\Phi_{D[r]}(N,v)$ is uniquely determined.
Induction hypothesis: Assume that the assertion is true for each country $i \in N$ whose depth is inferior or equal to $q < q_N$, where q_N is the depth of the river on N.
Induction step: Pick any country $i \in N$ whose depth is equal to $q+1$. By downstream fairness and efficiency of Φ, we have:

$$\Phi_{u(i)}(N,v) - v(\{u(i)\}) = \Phi_{D[i]}(N,v) - v(D[i]).$$

Denote this quantity by δ. By downstream fairness and efficiency, we obtain:

$$\forall j \in d(u(i)), \quad \Phi_{D[j]}(N,v) = v(D[j]) + \delta.$$

We also have $\Phi_{u(i)}(N,v) = v(\{u(i)\}) + \delta$. Summing all these equalities, we get:

$$\Phi_{D[u(i)]}(N,v) = \sum_{j \in d[u(i)]} v(D[j]) + |d[u(i)]|\delta.$$

By the induction hypothesis, $\Phi_{D[u(i)]}(N,v)$ is uniquely determined. As a consequence, the parameter δ is uniquely determined. This gives the result for country i since $\Phi_{D[i]}(N,v) = v(D[i]) + \delta$. ■

Remark 1 Note that Proposition 3 remains valid on the subclass of all river TU-games. The reason is that each subgame of a river TU-game induced by a set of downstream countries also defines a river TU-game on the corresponding subriver. Because downstream fairness only uses a river TU-game and the subgames on the sets of downstream countries, the assertion follows.

The next proposition provides an alternative characterization of the sequential equal surplus division.

Proposition 4 *The sequential equal surplus division Φ^e is the unique allocation rule that satisfies efficiency, standardness, bifurcation fairness, downstream consistency and downstream amalgamation on $\mathcal{C}_0$.*

In order to prove the above statement, we need an intermediary result establishing that if an allocation rule satisfies efficiency, standardness and bifurcation fairness for the subclass of TU-games where the river is a bifurcation, then it coincides with Φ^e.

Lemma 1 *If an allocation rule Φ satisfies efficiency, standardness and bifurcation fairness on $\mathcal{C}_0$, then for $(N,v) \in \mathcal{C}_0$ such that the underlying river is a bifurcation, it holds that $\Phi(N,v) = \Phi^e(N,v)$.*

Proof. By Proposition 2, Φ^e satisfies efficiency, standardness and bifurcation fairness on $\mathcal{C}_0$. Consider any allocation rule Φ that satisfies efficiency, standardness and bifurcation fairness on $\mathcal{C}_0$ and pick any $(N,v) \in \mathcal{C}_0$ such that the underlying river is a bifurcation. We proceed by induction on the number of elements of N.
Initial step: If $N = \{j\}$ for some $j \in N$, then by efficiency, $\Phi_j(N,v) = \Phi^e_j(N,v)$. If $N = \{i,j\}$, then by standardness, $\Phi_i(N,v) = \Phi^e_i(N,v)$ and $\Phi_j(N,v) = \Phi^e_j(N,v)$.
Induction hypothesis: Assume that the statement is true for N with at most $q \in \mathbb{N}$ countries.
Induction step: Assume that N contains $q+1$ countries, that $r \in N$ is the root, and consider the branch $r \to i$ for some $i \in N\backslash\{r\}$. By bifurcation fairness and the induction hypothesis we have:

$$\begin{aligned}\forall j,k \in N\backslash\{i\}, \quad \Phi_j(N,v) - \Phi_k(N,v) &= \Phi_j(N\backslash\{i\}, v_{|N\backslash\{i\}}) - \Phi_k(N,\backslash\{i\}, v_{|N\backslash\{i\}}) \\ &= \Phi^e_j(N\backslash\{i\}, v_{|N\backslash\{i\}}) - \Phi^e_k(N,\backslash\{i\}, v_{|N\backslash\{i\}}) \\ &= \Phi^e_j(N,v) - \Phi^e_k(N,v).\end{aligned}$$

Thus there is a constant $\delta \in \mathbb{R}$ such that:

$$\forall j \in N\backslash\{i\}, \quad \Phi_j(N,v) - \Phi^e_j(N,v) = \delta.$$

Because these equalities remain valid whatever the chosen branch, we have:

$$\forall j \in N, \quad \Phi_j(N,v) - \Phi^e_j(N,v) = \delta.$$

By efficiency, $\Phi_N(N,v) = \Phi^e_N(N,v) = v(N)$ and so $\delta = 0$. This shows that, for each $j \in N$, $\Phi_j(N,v) = \Phi^e_j(N,v)$. ■

Proof. (of Proposition 4). By Proposition 2, Φ^e satisfies efficiency, standardness, bifurcation fairness, downstream consistency and downstream amalgamation on $\mathcal{C}_0$. Next, pick any allocation rule Φ that satisfies standardness, bifurcation fairness, downstream consistency and downstream amalgamation on $\mathcal{C}_0$ and consider any $(N, v) \in \mathcal{C}_0$. To show: for each $i \in N$, $\Phi_i(N, v)$ is uniquely determined. We proceed by induction on the depth of the river.
Initial step: Assume that the depth of the river is equal to zero or one. In such case, the river is a bifurcation. By Lemma 1, the payoffs $(\Phi_i(N, v))_{i\in N}$ are uniquely determined.
Induction hypothesis: Assume that the payoffs $(\Phi_i(N, v))_{i\in N}$ are uniquely determined for each river whose depth is $q_N < n - 1$.
Induction step: Assume that the depth of the river, rooted at $r \in N$, is $q_N + 1 > 1$. From (N, v) and r, construct the companion TU-game (N^r, v^r) where r's downstream countries are amalgamated. By construction, r is now the root of a bifurcation, *i.e.* we have $r \to \overline{D[j]}$ for each $j \in d(r)$. Since by assumption there is at least one country with no downstream neighbor whose depth is strictly greater than one, (N^r, v^r) does not coincide with (N, v). By Lemma 1, for each $j \in d(r)$, the payoff $\Phi_{\overline{D[j]}}(N^r, v^r)$ is uniquely determined. By downstream amalgamation, we get:

$$\forall j \in d(r), \quad \Phi_{\overline{D[j]}}(N^r, v^r) = \Phi_{D[j]}(N, v),$$

which means that the total payoff of each coalition $D[j]$, $j \in d(r)$, is uniquely determined in the original game (N, v). Using efficiency, we obtain that country r's payoff is uniquely determined in (N, v):

$$\Phi_r(N, v) = v(N) - \sum_{j\in d(r)} \Phi_{D[j]}(N, v).$$

It remains to show that the payoff of each other country is uniquely determined. Consider any $j \in d(r)$ and construct the reduced TU-game $(D[j], v_{\Phi,j}) \in \mathcal{C}_0$. By definition of the reduced TU-game, we have:

$$\forall S \subset D[j], \quad v_{\Phi,j}(S) = v(S) \text{ and } v_{\Phi,j}(D[j]) = v(N) - \Phi_{N\setminus D[j]}(N, v).$$

Note that $\Phi_{N\setminus D[j]}(N, v)$ is uniquely determined by the previous step so that $(D[j], v_{\Phi,j})$ is well defined. By construction, the depth of each subriver on $D[j]$, $j \in d(r)$, is at most q_N. By the induction hypothesis, for each $j \in d(r)$ and $i \in D[j]$, $\Phi_i(D[j], v_{\Phi,j})$ is uniquely determined. By downstream consistency we get:

$$\forall j \in d(r), \forall i \in d(j), \quad \Phi_i(D[j], v_{\Phi,j}) = \Phi_i(N, v),$$

which means that the payoff of each downstream country of r is uniquely determined in (N, v). This completes the proof. ■

Remark 2. Note that on the subclass of TU-games where the underlying river has no bifurcation, Φ^e is characterized by standardness, efficiency, downstream amalgamation, and downstream consistency.

6 Concluding remarks

We conclude this article by considering a natural extension of the sequential equal surplus division. It is perhaps possible to elaborate a more realistic distribution of the surplus generated by the cooperation of several countries. Consider again the river in Figure 2 and the distribution of the surplus

$$v(\{1, 2, 3, 4\}) - v(\{1\}) - v(\{2\}) - v(\{3, 4\})$$

created by the three negotiating coalitions $\{1\}$, $\{2\}$ and $\{3, 4\}$. Further, suppose that the benefit functions yield an optimal consumption plan x^* such that $x^*_3 + x^*_4 = e_3 + e_4$. This extreme situation raises the question of which (asymmetrical) shares of the surplus should be distributed. In fact, the equality $x^*_3 + x^*_4 = e_3 + e_4$ means that the optimal total water consumption of the branch of the river on which countries 3 and 4 are located coincides with its total inflow. In other words, the achievement of the optimal welfare does not require the spring of the river to let pass any water to its downstream country 3. As such, the branch of the river on which countries 3 and 4 are located does not contribute any unit to the surplus. Therefore, it makes sense to deprive coalition $\{3, 4\}$ of any share of the surplus, leaving this coalition with a remainder equal to $v(\{3, 4\})$. This example is rather extreme but can be easily generalized to any situation. It is enough to require, at each step, that each negotiating coalition gets a share of the surplus which is proportional to the extra amount of water it consumes in order to reach the optimal consumption plan. This intuitive aspect of the river sharing problem can be formalized as follows.

In the definition (5) of Φ^e_i for country i, recall that the surplus

$$\alpha(D[i]) - \sum_{j\in d(i)} v(D[j]) - v(\{i\})$$

is equally redistributed among i and all coalitions $D[j]$, $j \in d(i)$, formed by i's downstream countries, *i.e.* is equally redistributed among $|d[i]|$ negotiating coalitions. Now, for each downstream neighbor $j \in d(i)$, denote by $e^*_{i,j}$ the amount of water that i lets pass to country j with respect to the optimal consumption plan $x^* = (x^*_i)_{i\in N}$. Furthermore, define $e^*_i \geq e_i \geq 0$ the total amount of water which is available at i's location with respect to x^*. More specifically:

$$e^*_i = x^*_i + \sum_{j\in d(i)} e^*_{i,j} > 0,$$

where the strict inequality comes from the fact that x^*_i is strictly positive under (A1). If each branch of the river $D[j]$, $j \in d(i)$, obtains a share of the surplus proportional to the quantity $e^*_{i,j}$, then its weighted remainder becomes:

$$\alpha^w(D[j]) = v(D[j]) + \frac{e^*_{i,j}}{e^*_i}\Big(\alpha^w(D[i]) - \sum_{k\in d(i)} v(D[k]) - v(\{i\})\Big)$$

so that the share of the surplus which goes to country i is

$$\frac{x^*_i}{e^*_i}\Big(\alpha^w(D[i]) - \sum_{k\in d(i)} v(D[k]) - v(\{i\})\Big).$$

The remainder of $D[r]$ remains $v(N)$ for the spring r of the river. The new allocation rule $\Phi^{w,e}$ constructed from these weighted remainders assigns to every $i \in N$ in each river TU-game (N, v) the payoff:

$$\Phi^{w,e}_i(N, v) = v(\{i\}) + \frac{x^*_i}{e^*_i}\Big(\alpha^w(D[i]) - \sum_{j\in d(i)} v(D[j]) - v(\{i\})\Big).$$

Proceeding as in the proof of Proposition 1, we conclude that $\Phi^{w,e}$ satisfies points (i), (ii) and (iii) of Proposition 1 and so is efficient on the class of all river TU-games. The allocation rule $\Phi^{w,e}$ does not satisfy downstream fairness anymore. Nevertheless, it satisfies the following weighted version of downstream fairness: for each river TU-game (N, v) with at least two countries and each $i \in N$ such that $d(i) \neq \emptyset$, it holds that:

$$\forall j \in d(i), \quad e^*_{i,j}\Big(\Phi_i(N, v) - \Phi_i(\{i\}, v_{|\{i\}})\Big) = x^*_i\Big(\Phi_{D[j]}(N, v) - \Phi_{D[j]}(D[j], v_{|D[j]})\Big).$$

In particular, if $e^*_{i,j} = 0$, then this property implies that $\Phi_{D[j]}(N, v) = \Phi_{D[j]}(D[j], v_{|D[j]})$ because $x^*_i > 0$. It is indeed true for $\Phi^{w,e}$ since, in such a case, $\alpha^w(D[j]) = v(D[j])$ so that $\Phi^{w,e}_{D[j]}(D[j], v_{|D[j]}) = v(D[j]) = \Phi^{w,e}_{D[j]}(N, v)$ by point (i) of Proposition 1. This is consistent with the extreme example considered at the beginning of the section. Remark 1 and straightforward modifications of the proof of Proposition 3 yield that $\Phi^{w,e}$ is the unique allocation rule which satisfies efficiency and the weighted version of downstream fairness on the class of all river TU-games. It is however not possible to adapt similarly Proposition 4 to characterize $\Phi^{w,e}$.

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