Too Big to Cheat: Efficiency and Investment in Partnerships*

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Abstract

Many economic activities are organized as partnerships. These are ventures formed with capital contributions by partnership members that in exchange obtain a share of ownership. The design of the partnership rules dictates how much of the profits are distributed among the members and how much are reinvested. In this paper, we study the optimal design of partnerships under the assumption that one of the members (the founder) privately observes shocks to his liquidity needs. When the ownership share of the founder is large enough, private information is immaterial and the ownership structure remains constant over time. The founder has no incentives to misreport because a fraction of the increase in his payouts after reporting high liquidity needs is financed by disinvesting in the partnership. When the founder's ownership share is not big enough, the ownership structure of the partnership must vary over time to prevent misreporting. In the long run, only two possible outcomes have positive probability: Either (i) the ownership shares of the founder converge to zero (i.e., "immiserization," as in other models of private information), or (ii) the founder's share of the partnership becomes too big to cheat (i.e., his incentives to misreport vanish) and thus both partners own a constant positive share of the partnership forever.

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1 Introduction

Many economic activities are organized as partnerships. They are unincorporated organizations formed by two or more persons that join to carry on a trade or business. Each partner contributes money, property, labor, or skill, and each expects to share in the profits and losses.[1] They do not have access to the stock market and the ownership and control are vested to the partners.[2] The design of the partnership dictates how much of the profits are distributed among the members and how much are reinvested. Ideally, the profits that each member can take should depend on his willingness to consume or, more generally, on his liquidity needs. If these liquidity needs are private information, the design of the partnership must deal with incentive problems. Although there is a vast literature analyzing some aspects of the optimal design of a partnership, this literature is silent on the optimal arrangement in a setting in which liquidity needs are private information and a few partners must undertake investment decisions. Our paper fills this gap.

The model is as follows. Two risk-averse individuals contribute some initial capital to form a partnership. They use a neoclassical technology to produce goods. The partnership finances itself internally.[3] Partners decide which fraction of output is reinvested in capital that will be available for production the next period (*the investment plan*) and how to split the rest among the partners as payouts for consumption (the distribution plan). For simplicity, we assume that only the liquidity needs of one of the partners (hereafter *the founder* of the partnership) vary from period to period. We refer to the partner that does not face shocks to his liquidity needs as *the associate*. The environment is chosen so that in the absence of private information, the problem of analyzing the optimal design of the partnership is trivial: It simplifies to maximizing the lifetime utility of the owner of a sole proprietorship with a peculiar shock to his liquidity needs. Efficiency dictates that the ownership structure (i.e. the share of ownership held by each partner) is constant forever. In each period, as the liquidity needs of the founder increase, the efficient arrangement implies that his payout increases, the payout of the associate decreases, and investment in the partnership

[1]This definition is provided by the Internal Revenue Service in the US. They reported that in 2010 there were 3,248,481 partnerships that generated about 30 percent of the total net income of all US businesses.

[2]Corporations, on the other hand, represent the other set of firms in which the rest of economic activities are organized. They are distinguished from partnerships along two critical dimensions: (i) the financial sources to which they have access; i.e. they can issue issue shares in the stock market and can borrow and lend in the credit market; (ii) the ownership and control structure; i.e. it is owned by shareholders, while it is typically run by a separate group of managers. See Magill and Quinzii (1996), Chapter 6, Section 31, for a thorough discussion.

[3]This assumption is in line with a broad literature, surveyed by Hubbard (1998), that documents credit constraints, especially for small firms.

decreases. Thus, a liquidity shock triggers an increase in the founder's payout that is financed from two sources: *redistribution* (as the payout of the associate decreases) and *disinvestment* (as investment also decreases).

Under private information, the ownership structure becomes critical. In particular, there is a threshold for the share of the ownership held by the founder such that private information does not matter for levels above that threshold. To understand this result, recall that a part of the increase in the founder's payout after a high liquidity shock comes from reducing the associate's payout (redistribution), and the other part comes from decreasing investment (disinvestment). Interestingly, under full information, the fraction of the increase in the founder's payout that is financed through disinvestment is increasing in his ownership share. If the founder chooses to misreport, he will consume more this period at the cost of inducing underinvestment. As his share in the partnership increases, eventually the cost of misreporting (underinvestment) is larger than the benefits (a higher current payout). Disinvestment more heavily affects his future payouts and thus provides incentives for truthful revelation.

Our theory also has implications for the dynamics of the ownership structure: it is optimal to make ownership structure fluctuate to alleviate information problems. If the founder's share is below the threshold mentioned above, whenever he reports high liquidity needs, he receives a larger share of current payouts in exchange for a lower future ownership share. Thus, under private information the optimal ownership structure fluctuates over time to provide the right incentives, provided ownership is not sufficiently concentrated. Importantly, the long-run ownership structure of partnerships is such that private information does not matter. Two possible extreme structures achieve this outcome. On the one hand, for a sufficiently long sequence of low liquidity shocks, the founder's ownership share becomes sufficiently large to make the incentive problem irrelevant (i.e., incentives to misreport disappear). When that level of concentration is reached, the ownership structure remains unchanged forever and private information does not matter. On the other hand, if the founder is hit by a sufficiently long sequence of high shocks, his share in the partnership converges to zero; that is, the ownership structure is concentrated in the associate, and private information does not matter. Notice that as a result of the dynamics under private information the ownership structure in the long run is concentrated in both cases.

Despite its simplicity, the model seems to match some recently documented properties of firm dynamics. The sensitivity of investment to liquidity shocks decreases with age. Since disinvestment plays a key role in providing incentives, investment reacts more to liquidity shocks in the first years of the partnership, when private information does matter. In the long run, our theory predicts that the ownership structure is concentrated.[4] The model also yields prescriptions for the optimal design of partnerships. In particular, it prescribes that the ownership shares should be used to provide incentives for truthful revelation. Additionally, it predicts that partnerships with many members with small ownership shares are more likely to suffer from incentive problems than partnerships with fewer members with larger shares.

From the technical point of view, capital accumulation further complicates the analysis in this setting with private information. Solving this problem with the method developed by Abreu, Pearce, and Stacchetti (1990), APS hereafter, would imply iterating on utility possibility correspondences instead of functions. The value function (and the corresponding policy functions) would be recovered from the frontier of the fixed point of that operator on correspondences. However, while this is theoretically feasible,[5] a method based on iterations of correspondences makes the computation very demanding. This may explain why there are so few cases where the APS approach is numerically implemented.[6] We propose an alternative method to overcome these difficulties that characterizes the efficient frontier of a convex utility possibility correspondence by means of weights attached to each partner, which can be interpreted as that partner's ownership shares. The idea of substituting utility levels with ownership shares is borrowed from Lucas and Stokey (1984)'s analysis of optimal growth with many consumers. These shares become endogenous state variables that summarize the history.[7] This approach complements the methods pioneered by the seminal work of Spear and Srivastava (1987) and Abreu, Pearce, and Stacchetti (1990).

Related Literature. Pioneering contributions in the literature on constrained efficient allocations with private information abstracted from capital accumulation, as their main goal has been to study wealth distribution. In pure exchange economy settings, Green (1987), Spear and Srivastava (1987), Thomas and Worral (1990), and Atkeson and Lucas (1992) show that constrained efficiency dictates extreme levels of "immiserization," that is, the utility of (almost) every agent in the economy converges to the lower bound with probability 1. This result is quite robust since it

[4]This is in line with the description of ownership around the world in Porta, Lopez-De-Silanes, and Shleifer (1999).

[5]Espino (2005) adapted APS's method to characterize the limiting distribution in a framework with capital accumulation.

[6]A notable exception is Abraham and Pavoni (2008) who used APS's approach to solve for the constrained efficient allocation of a model with hidden actions and hidden savings.

[7]A related approach is developed by Beker and Espino (2013) to analyze endowment economies with limited commitment and belief heterogeneity.

is independent of the initial distribution of wealth. Our long-run analysis shows that this result does not hold if investment opportunities are available and there are only a few agents such that the disinvestment effect plays the critical role described above.

Some contributions include capital. Clementi and Hopenhayn (2006) study the interaction of limited liability and private information in a setting where a risk-neutral lender finances a project run by a risk-neutral entrepreneur who privately observes productivity shocks. In that setting, limited liability is the only impediment to implementing the first best. Once the level of promised utility of the entrepreneur reaches some threshold, limited liability does not bind and the firm attains its efficient size forever. On the other hand, liquidation of the firm is also an absorbing state and the entrepreneur attains its reservation value. While the interaction of two frictions—limited liability and private information—is the key to attain this limiting result, investment plays no role. Our theory does not appeal to limited liability and thus it isolates the interaction of private information and investment.[8]

Marcet and Marimon (1992) assume that output is subject to privately observed productivity shocks to show that a *risk-neutral* investor with *unlimited resources* can make a risk-averse entrepreneur follow the first-best investment plan. Thus, growth rates are unaffected since the shadow price of capital is exogenous as one agent is not only risk neutral but also has access to unlimited resources. This makes private information not affect capital accumulation as the risk-neutral agent can absorb all possible fluctuations at not extra cost. Khan and Ravikumar (2001) assume that household AK technologies are subject to privately observed idiosyncratic productivity shocks and show that growth rates are smaller relative to the complete risk-sharing benchmark, but the quantitative effect is small.[9] Clementi, Cooley, and Giannatale (2010) study a venture in which a risk neutral investor cannot monitor the entrepreneur's effort. They provide a rationale for a firm's decline since the incentive provision becomes more costly as the entrepreneur's wealth increases which leads to a drop in the return on investment. This sort of immiserization differs from our results as their source of the informational friction is different and the investor is assumed to be risk neutral.

Sleet and Yeltekin (2010) study constrained efficiency under private information in a model in which the shadow price of capital is endogenous. Interestingly, they also find that a fraction of

[8] There are two other related papers in the spirit of Clementi and Hopenhayn (2006). In a model with moral hazard, Quadrini (2004) studies the design of renegotiation-proof contracts. In a more recent paper, Cole, Greenwood, and Sanchez (2012) study the decision to adopt new technologies in a model with private information and costly monitoring.

[9] Also, the "shadow price of capital" here is exogenous since the production function is linear in the stock of capital.

agents is absorbed into "misery" with the rest "retiring into luxury."

The paper is organized as follows. Section 2 describes the physical environment of the model, defines feasibility and incentive compatibility, and shows how constrained efficient plans can be represented with ownership shares. Next, Section 3 characterizes the optimal design of partnerships. Section 4 numerically implements our approach and provides examples that examine, for example, how the importance of private information varies with the ownership structure of the partnership. Section 5 evaluates the role of the main assumptions in our analysis. Section 6 provides conclusions. Proofs are gathered in the appendix.

2 Model

Time is discrete and the time horizon is infinite. At date 0, consider a partnership that is formed by two agents, indexed $i = 1, 2$.[10] They operate a technology to produce a good that can be either consumed or invested. This technology is represented by a constant-returns-to-scale production function $F(K, L)$, where K is the current stock of capital and L denotes units of labor. Assume the partnership starts with capital $k_0 = k_{0,1} + k_{0,2}$, contributed by agents 1 and 2, respectively. Capital depreciates at the rate $\delta \in (0, 1)$. Given technological assumptions, there exists some $\overline{K}$ such that $X = [0, \overline{K}]$ denotes the sustainable levels of capital. Each partner is endowed with 1/2 unit of time each period and does not value leisure. In exchange for their capital and labor contributions, each partner gets an initial share of the partnership.

We assume that partners have preferences à la Diamond and Dybvig (1983) as they face liquidity shocks.[11] At the beginning of date t, partner i privately observes his shock $s_{i,t} \in S_{i,t} = \{s_L, s_H\}$, where $s_H > s_L$ and define $S_t = S_{1,t} \times S_{2,t}$.[12] These shocks are assumed to be i.i.d. across time and partners, where $\pi(s_{i,t}) > 0$ is the probability of $s_{i,t}$. Let $s_t = (s_{1,t}, s_{2,t}) \in S_t$ be the joint shock at date t with probability $\pi(s_t) = \pi(s_{1,t}) \times \pi(s_{2,t})$, while $s^t = (s_0, ..., s_t) \in S^t = \times_{j=0}^{t} S_t$ denotes the history of events from date 0 to date t. The probability at date 0 of any particular history s^t is given by $\pi(s^t) = \pi(s_0)...\pi(s_t)$.

Given a consumption path $\{c_{i,t}\}_{t=0}^{\infty}$ such that $c_{i,t} : S^t \to \mathbb{R}_+$, partner i's preferences are repre-

[10] All the analysis regarding the solution method also applies to the general case in which there is an arbitrary number of partners I.

[11] This class of preferences is standard in the literature; see Tirole (2005), Chapter 12. Moreover, liquidity shocks are multiplicative as in Atkeson and Lucas (1992).

[12] Notice that in this case we allow both partners to have liquidity shocks, as opposed to our benchmark case in later sections in which the shock affects only one partner. We do this for expositional purposes since in Section 5.2 we discuss the case in which both partners face liquidity shocks.

sented by

$$E\left\{\sum_{t=0}^{\infty}\beta^t \; s_{i,t} \; u(c_{i,t})\right\}=\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi(s^t) \; s_{i,t} \; u(c_i(s^t)),$$

where $u:\mathbb{R}_+\to\mathbb{R}$ is strictly increasing, strictly concave, and twice differentiable; $\lim_{c_t\to 0}u'(c_t)=+\infty$; and $\beta\in(0,1)$. A higher value of the partners liquidity needs implies that he is willing to take more resources from the partnership to consume more today compared with the future.

Since the problem of choosing the best investment and distribution plans for the partnership is restricted to solutions that are incentive compatible (i.e., partners have no incentives to misreport liquidity shocks) here we describe the reporting strategies available to the partners.[13]

At date t, partner i has privately observed the partial history s_i^t. Let $z_i=\{z_i(s_i^t)\}_{t=0}^{\infty}$ represent his reporting strategy, where $z_{i,t}:\times_{h=0}^{t}S_{i,h}=S_i^t\to S_{i,t}$ for all t, i.e. $z_i(s_i^t)\in S_{i,t}$ for all t. Let $z=(z_1,z_2)$ denote the joint reporting strategy. Let z_i^* be the truth-telling reporting strategy for which $z_i^*(s_i^t)=s_{i,t}$ for all t and $s_i^t\in S_i^t$.

Let $K'=\{K_{t+1}\}_{t=0}^{\infty}$ be an *investment plan* that every period allocates next-period capital, given a history of joint reports (i.e. $K_{t+1}:S^t\to\mathbb{R}_+$ for all t). Similarly, let $C=\{C_t\}_{t=0}^{\infty}$ be a *distribution plan*, where $C_t:S^t\to\mathbb{R}_+^2$ for all t. To interpret this, consider any joint realization s^t up to date t and any joint reporting strategy z. Partner i's consumption at date t is given by $C_i(z^t(s^t))$. Similarly, the stock of capital at date $t+1$ will be given by $K(z^t(s^t))\geq 0$. Any (C,K') satisfying these properties is called a *sequential plan*.

Definition 1 *Given K_0, a sequential plan (C,K') is feasible for the partnership, if for all joint reporting strategies z, all t, all s^t and $K_0=K$,*

$$K(z^t(s^t))+\sum_{i=1}^{2}C_i(z^t(s^t))\leq F(K(z^{t-1}(s^{t-1})),1)+(1-\delta)K(z^{t-1}(s^{t-1})). \tag{1}$$

Intuitively, feasibility at the firm's level means that part of the output is reinvested in the partnership, $K(z^t(s^t))-(1-\delta)K(z^{t-1}(s^{t-1}))$, and part is paid out to the partners, $C_1(z^t(s^t))+C_2(z^t(s^t))$. Importantly, note that there is no external finance available to the partnership.[14] In what follows, we refer to K as the stock of capital as well as the size of the partnership indistinctly.

[13]This restriction is without loss of generality since it can be shown that the relevant version of the celebrated Revelation Principle holds.

[14]In Section 5.4 we argue that if instead partners could obtain unlimited resources at a given interest rate r, our main result would disappear.

Now we introduce the concept of incentive compatibility. Let s^{t-1} be any arbitrary joint partial history reported. Let $\widehat{z}$ be an aggregate continuation reporting strategy from period t onward. Given a sequential plan (C, K'), partner i's utility at date t is

$$\begin{aligned} & U_{i,t}(C, K', \widehat{z} \| s^{t-1}) \\ = \; & \sum_{j=0}^{\infty} \beta^j \sum_{s^{j+1}} \pi(s^{j+1}) \; s_{i,t+j} \; u(C_i(s^{t-1}, \widehat{z}^j(s^j))), \\ = \; & \sum_{s_t \in S_t} \pi(s_t) \; \left(s_{i,t} \; u(C_i(s^{t-1}, \widehat{z}_0(s_t))) + \beta \; U_{i,t+1}(C, K', \widehat{z}'(s_t) \| (s^{t-1}, \widehat{z}_0(s_t)) \right), \end{aligned}$$

where $\widehat{z}'(s)$ is the continuation reporting strategy from period $t+1$ onward induced by $\widehat{z}$ when the first element s is kept constant. When $t = 0$, we directly write $U_i(C, K', z) = U_{i,0}(C, K', z)$ for any z.

The following definition says that a sequential plan is *incentive compatible* if truth-telling is the best response for each partner. Let s_{-i} be a joint liquidity shock that excludes partner i's element (e.g., $s_{-1} = s_2$).

Definition 2 *Given $K \in X$, a sequential plan (C, K') is incentive compatible if, for each partner i, for all $t \geq 0$, all s^{t-1}, and any continuation z_i',*

$$\begin{aligned} & \sum_{s_{-i}} \pi(s_{-i}) \left(s_i \; u(C_i(s^{t-1}, s_i, s_{-i})) + \beta \; U_{i,t+1}(C, K', z_i^*, z_{-i}^* \| (s^{t-1}, s_i, s_{-i})) \right) \\ \geq & \sum_{s_{-i}} \pi(s_{-i}) \left(s_i \; u(C_i(s^{t-1}, \widetilde{s}_i, s_{-i})) + \beta \; U_{i,t+1}(C, K', z_i', z_{-i}^* \| (s^{t-1}, \widetilde{s}_i, s_{-i})) \right) \end{aligned} \tag{2}$$

for all $(s_i, \widetilde{s}_i)$.

Thus, incentive compatibility ensures that partners have no incentive to misreport their liquidity shocks. Note that Definition 2 takes into account that partners can choose a continuation reporting strategy every period after they have observed their own history of shocks.[15]

2.1 Constrained efficient partnerships

To solve for the constrained efficient arrangement, we consider a fictitious planner that chooses among plans that are incentive compatible and feasible. Let $\Psi(K)$ be the utility possibility

[15]This implementation concept can be interpreted as the natural extension of Bayesian implementation for this particular dynamic environment. See Espino (2005).

correspondence—that is, the level of utility of the partners that can be attained given a partnership of size K and plans that are incentive compatible and feasible,

$$\Psi(K) \equiv \left\{ w \in \mathbb{R}_+^2 : \exists\ (C, K') \text{ satisfying } (1) - (2) \right.$$
$$\left. \text{and } w_i \leq U_i(C, K', z^*)\ \forall\ i,\ K_0 = K \right\}.$$

Definition 3 *An incentive-compatible, feasible sequential plan (C^*, K'^*) is constrained efficient if there is no alternative incentive-compatible, feasible sequential plan (C, K') such that $U_i(C, K', z^*)$ $> U_i(C^*, K'^*, z^*)$ for all i.*

The intuition of the solution strategy that we propose below to characterize efficient partnerships is as follows. Imagine that the partnership evaluates alternative plans weighting the utility of partner i using a coefficient θ_i. These weights map into property rights on current as well as future payouts to the partners. Although the assignment is indirect, hereafter we refer to these coefficients (θ_1, θ_2) as the *ownership shares* of partner 1 and 2, respectively. So the best distribution and investment plan maximizes the weighted sum of the partners' utility given these ownership shares.[16] In order to implement this approach, we need the following result.

Remark 1 *The set of constrained efficient plans can be parameterized by ownership shares $(\theta_1, \theta_2) \in \mathbb{R}_+^2$. That is, (C^*, K'^*) is constrained efficient if and only if (C^*, K'^*) is the corresponding plan that solves*

$$h^*(K, \theta) = \sup_{w\, \in\, \Psi(K)} \theta_1 w_1 + \theta_2 w_2, \tag{3}$$

for some $(\theta_1, \theta_2) \in \mathbb{R}_+^2$.

Lemma 1 in the appendix shows that the utility possibility correspondence is convex valued, a result that is key for applying our alternative characterization. As in Atkeson and Lucas (1992), it is key that the set of sequential plans satisfying the constraints (1)-(2) is convex.[17] Remark 1 is a direct consequence of convexity.

[16] Our approach to define the objective of the partnership is in the spirit of the general equilibrium approach under incomplete markets; see Magill and Quinzii (1996), Chapter 6, Section 31.

[17] This follows from the assumption that the liquidity shocks affect partners' utility multiplicatively.

2.2 Recursive representation

Here we provide a recursive representation of the dynamic problem discussed above. Hereafter we restrict the ownership shares of the partners to add up to 1; that is, $\Delta \equiv \{\theta \in \mathbb{R}^2_+ : \theta_1 + \theta_2 = 1\}$.[18] In the appendix, we show that h^* solves the following operator:

$$(Th)(k,\theta) = \max_{(c,w',k')} \sum_{i=1}^{2} \theta_i \left\{ \sum_s \pi(s) \left[s_i \; u(c_i(s)) + \beta \; w_i'(s) \right] \right\}, \tag{4}$$

subject to

$$k'(s) + \sum_{i=1}^{2} c_i(s) = f(k) + (1-\delta)k \tag{5}$$

$$\begin{aligned} &\sum_{s_{-i}} \pi(s_{-i}) \left(s_i u(c_i(s_i, s_{-i})) + \beta \; w_i'(s_i, s_{-i}) \right) \\ &\geq \sum_{s_{-i}} \pi(s_{-i}) \left(s_i u(c_i(\widetilde{s}_i, s_{-i})) + \beta \; w_i'(\widetilde{s}_i, s_{-i}) \right) \end{aligned} \tag{6}$$

for all $(s_i, \widetilde{s}_i)$ and

$$c_i(s) \geq 0, \qquad w_i'(s) \geq 0 \qquad \text{for all } s \text{ and all } i, \tag{7}$$

$$\min_{\theta' \in \Delta} \left[h(k'(s), \theta') - \sum_{i=1}^{2} \theta_i'(s) \; w_i'(s) \right] \geq 0 \qquad \text{for all } \theta' \text{and } s. \tag{8}$$

Notice that the optimization problem takes as given the size of the firm k and the share of ownerships θ, distributes output between current payouts (consumption of the partners) and investment, and assigns continuation utility levels to the partners. The optimization problem defined in condition (8) characterizes the set of continuation utility levels attainable at the future stock of capital k' and ownership shares θ' (see appendix). The value of θ' that attains the minimum in (8) are the next-period ownership shares that are consistent with the entitlement of continuation utilities.

Let $\left(\widehat{c}, \widehat{k}', \widehat{w}'\right)$ denote the set of policy functions solving (4) - (8), while $\widehat{\theta}'$ denotes the corresponding next period ownership shares solving (8). Given (k_0, θ_0), we say that a set of policy functions $\left(\widehat{c}, \widehat{k}', \widehat{w}'\right)$ generates a sequential plan $(\widehat{C}, \widehat{K}')$ if

$$\begin{aligned} \widehat{C}_i(s^t) &= \widehat{c}_i(\widehat{K}(s^{t-1}), \theta(s^{t-1}))(s_t), \\ \theta(s^{t-1}, s_t) &= \widehat{\theta}'(\widehat{K}(s^{t-1}), \theta(s^{t-1}))(s_t), \\ \widehat{K}(s^{t-1}, s_t) &= \widehat{k}'(\widehat{K}(s^{t-1}), \theta(s^{t-1}))(s_t), \end{aligned} \tag{9}$$

[18] This restriction is innocuous because solutions are homogenous of degree 0 with respect to (θ_1, θ_2).

for all i and all (t, s^t), where K_0 is given.

Proposition 1 *h^* is a fixed point of T. Moreover, a plan (C^*, K'^*) is constrained efficient at K_0 if and only if it is generated by the set of policy functions at (9).*

Thus, the value of any plan that can be attained with a feasible incentive-compatible sequential plan (C, K') can also be attained by splitting output between total current payouts and investment and then delivering current payouts and contingent future ownership shares to each partner.

Now we provide an algorithm capable of finding the value function h^* and its corresponding policy functions. Let h^{**} be the value function solving the recursive problem for which the incentive compatibility constraints are ignored (i.e., the full information case). Evidently, $h^*(k, \theta) \leq h^{**}(k, \theta)$ for all (k, θ).

Proposition 2 *Let $h_0 = h^{**}$ and denote $h_n = T^n(h^{**})$. Then, $\{h_n\}$ is a monotone decreasing sequence and $\lim_{n\to\infty} h_n = h^*$.*

The main gain of this method with respect to the traditional APS approach is in terms of applicability and tractability. Our approach identifies attainable levels of next-period utility by iterating directly on the utility possibility frontier with no need to know the utility possibility correspondence a priori. This greatly simplifies the computational burden. Of course, the APS approach outperforms ours if the problem is one in which the utility possibility correspondence is not convex valued. Indeed, our approach (at least as stated in this paper) is not appropriate to handle such problems.

3 On the design of partnerships

This section provides the main insights about the design of partnership under two alternative assumptions on the observability of liquidity shocks: full information and private information. For expositional purposes, in this section we assume that only agent 1 (hereafter the founder) faces liquidity shocks, while agent 2 (the associate) does not.[19] Most of the economic insights, however, carry over to the case of two partners facing privately observed liquidity shocks. Section 5 shows how the long-run results are modified in that case. Since $\theta_1 = 1 - \theta_2$, to simplify notation hereafter we refer to the founder's ownership share of the partnership directly as θ.

[19]Importantly, the associate is risk averse and has no access to unlimited resources. These two features are critical to results; see Section 5 for a discussion.

3.1 Partnerships under full information

The full information case reduces to the problem of a sole proprietorship with an investor with preferences featuring peculiar shocks to liquidity needs. That is, this shock will in general be a weighted average of the liquidity shocks of both partners, although here the second agent's shock is constant at 1.

To see this more clearly assume that the partners have CRRA preferences with relative risk aversion coefficient σ. The necessary and sufficient first-order conditions of the full information problem imply

$$c_1(s) = \frac{(\theta\ s)^{1/\sigma}}{(\theta\ s)^{1/\sigma} + (1-\theta)^{1/\sigma}}(F(k,1) + k(1-\delta) - k'(s)) \tag{10}$$

$$c_2(s) = \frac{(1-\theta)^{1/\sigma}}{(\theta\ s)^{1/\sigma} + (1-\theta)^{1/\sigma}}(F(k,1) + k(1-\delta) - k'(s)), \tag{11}$$

where $(F(k,1) + k(1-\delta) - k'(s))$ represents the total amount paid out to the partners and

$$\theta'(s) = \theta \tag{12}$$

for all s. Notice that (12) follows from the first-order conditions with respect to w'. Importantly, this means that efficiency implies that the ownership shares of the partnership are kept constant. Indeed, this result holds more generally for full information plans and, in particular, it does not depend on CRRA preferences. The underlying intuition for keeping these ownership shares optimally constant can be grasped as follows. The planner's problem is an artificial device to characterize a particular set of plans in which ownership shares, $(\theta, 1-\theta)$, are the ex ante planner's valuation of delivering one more unit of expected discounted utility to the partners. Thus, the ex ante relative valuation is $\theta/(1-\theta)$. Similarly, $(\theta'(s), 1-\theta'(s))$ denotes the valuation of delivering one more unit of expected discounted utility to the partners next period if today's shock is s. Consider now the relative valuation of delivering one unit of expected discounted utility next period after a realization of the shock s. In the full information case, this will be given by the ratio $\frac{\theta'(s)}{(1-\theta'(s))} = \frac{\theta\beta\pi(s)}{(1-\theta)\beta\pi(s)}$ and thus the relative valuation remains unchanged since agents discount the future equally. On the other hand, when there is private information future expected discounted utility plays an additional role since it provides incentives for truthful reporting. Its relative valuation can vary as time and uncertainty unfold.

Observe that (10)-(12) imply that the problem reduces to choosing total payouts and investment in a sole proprietorship with an "aggregate" investor with preferences at date t,

$$S(\theta, s_t)\frac{(C_t)^{1-\sigma}}{1-\sigma}$$

where $C_t = c_{1,t} + c_{2,t}$, denotes total payouts and

$$S(\theta, s_t) = \left((\theta\ s_t)^{1/\sigma} + (1-\theta)^{1/\sigma}\right)^{\sigma}$$

is an "aggregate" liquidity shock. An important observation is that this shock depends on the ownership structure through θ. It follows by standard arguments that investment is decreasing in S_t, and so the fraction of the increase in the founder's payout that is financed through disinvestment is increasing in his ownership share.

3.2 Partnerships under private information

Here we argue that the best arrangement between partners under full information and under private information coincide under a certain ownership structure. In what follows, we refer to (θ_t, k_t) as the ownership structure and the size of the partnership at date t, respectively. The following result establishes that, for a given size of firm k, the founder's incentive to misreport vanish as his share of the partnership becomes big enough.

Proposition 3 (Too big to cheat) *For each partnership size k, there exists some value of the founder's share of ownership $\overline{\theta}(k) \in (0,1)$ such that for all (θ, k) with $\theta \in \left[\overline{\theta}(k), 1\right]$ the full information plan satisfies the incentive compatibility constraints.*

The underlying intuition for this result can be grasped as follows. As mentioned previously, as the founder reports high liquidity needs, he receives more funds. These resources come from two sources: (i) decreasing the funds received by the associate and (ii) reducing investment or disinvestment. If the founder's ownership share is large enough, most of the extra funds he receives are financed by disinvestment. This occurs because of two main reasons. First, there is not much left to redistribute from the associate as his payout approaches zero. And second, the associate's allowance is small; therefore, the cost of taking away an extra unit of consumption is large. Since disinvestment implies that misreporting can heavily decrease the future utility of the founder himself, as the founder's participation in the partnership passes a threshold, cheating becomes undesirable.

Now we move a step forward to argue that private information does not matter under a certain ownership structure of the partnership. That is, we show that one can always find a subset of ownership shares such that the investment and distribution plans under private information actually coincide with the full information plans.

Consider the full information policy function for capital accumulation $k'_{FI}(\theta,k)(s_L)$ and $k'_{FI}(\theta,k)(s_H)$ and define the bounds of sustainable levels of capital as

$$\begin{aligned} k'_{FI}(\theta,k_{\min}(\theta))(s_H) &= k_{\min}(\theta), \\ k'_{FI}(\theta,k_{\max}(\theta))(s_L) &= k_{\max}(\theta). \end{aligned}$$

Since policy functions are continuous, these bounds, $k_{\min}(\theta)$ and $k_{\max}(\theta)$, are both continuous functions with respect to θ. Let $\Gamma \equiv \{(\theta,k) : \theta \in [0,1],\ k \in [k_{\min}(\theta), k_{\max}(\theta)]\}$ and define $\theta^* \equiv \sup\{\overline{\theta}(k) : \left(\overline{\theta}(k),k\right) \in \Gamma\} \in (0,1)$.

Proposition 4 (Too big to cheat, forever) *Suppose that $(\theta_t,k_t) \in [\theta^*,1] \times [k_{\min}(\theta_t),k_{\max}(\theta_t)]$ at some t. Then the constrained efficient and the full information plans coincide, and $\theta_{t+n} = \theta_t$ for all $n \geq 0$.*

This is our main result which argues that the ownership structure remains unchanged and private information becomes irrelevant once the ownership structure and the size of the partnership reach a region in which (i) the founder's ownership share becomes big enough and (ii) the size of the partnership converges to an interval with bounds determined in the full information case. This aforementioned region (defined in Proposition 4) can be reached either immediately (as the initial ownership structure and the initial size of the partnership starts there) or in the long run (as the ownership structure and the size of the partnership converge as time and uncertainty unfold). We discuss this latter possibility in what follows.

We now consider the evolution of the ownership structure of the partnership. Let S^∞ be the set of infinite sequences of liquidity shocks, with typical element $s^\infty = \{s_0,...,s_t,...\}$, where $\mathcal{B}(S^\infty)$ are the corresponding Borel sets. Let $\{\theta_t\}_{t=0}^\infty$ be the stochastic process for ownership shares generated by the set of policy functions solving (4). That is, $\theta_t : S^\infty \to [0,1]$, where $\theta_t(s^\infty)$ denotes a particular realization at date t. It will be shown that the stochastic process of the ratio of ownership shares is a nonnegative martingale.

Proposition 5 (Ownership share dynamics) *The ratio of ownership shares satisfies the following properties:*

1. It is a nonnegative martingale; i.e., for all t and all s^t,

$$\mathbf{E}\left[\frac{\theta_{t+1}}{(1-\theta_{t+1})} \parallel s^t\right] = \frac{\theta_t(s^\infty)}{(1-\theta_t(s^\infty))} \qquad s^\infty - a.s.$$

2. It follows by the martingale convergence theorem that

$$\frac{\theta_t(s^\infty)}{(1-\theta_t(s^\infty))} \to \frac{\widehat{\theta}(s^\infty)}{\left(1-\widehat{\theta}(s^\infty)\right)} \qquad s^\infty - a.s.$$

for some random variable $\widehat{\theta}$ on $(S^\infty, \mathcal{B}(S^\infty))$.

The next result shows that private information does not matter in the long run; that is, the limiting allocation is incentive compatible and so coincides with the full information allocation.

Proposition 6 (Long-run convergence to full information) *The partnership's ownership structure and size, (θ_t, k_t), reach the region in which the constrained efficient and the full information plans coincide with probability 1 in which $\theta_t \to \{0, [\theta^*, 1]\}$ a.s.*

This proposition shows that incentives to misreport vanish in the long run. This occurs because in the long run either the associate owns the entire partnership or the founder's ownership share is sufficiently large. In the former case, private information does not matter because there is only one active partner. That case actually resembles the immiserization result found in other settings with private information. In the latter case, the founder's share is sufficiently large that the costs of lying that he internalizes, as opposed to the total costs to the partnership, dominate the benefits of misreporting. The initial distribution of shares is key to determining which of these two possibilities will be the long-run outcome. For instance, if the initial shares of the founder are big enough, implying $\theta_0 \geq \theta^*$, the best arrangement under private information coincides with the one of full information from the day of the partnership formation. On the other hand, if the initial shares of the founder are not big enough (i.e. $\theta_0 < \theta^*$) the long-run outcome could reach both outcomes with positive probability.

4 Numerical example

In this section, we present the results of solving for the optimal plans using the approach discussed previously. The main motivation for these exercises is to enrich our understanding of the design of the partnership under private information. In addition to our benchmark case, several economies are computed to disentangle the role of specific assumptions: (i) the benchmark case with full information, (ii) private information without investment, and (iii) private information with investment and two partners with liquidity shocks. The parameters used to solve the model are described in Table 1.[20]

Table 1: Parameter values

Variable	Value	Description
$f(K)$	$K^{\alpha}L^{1-\alpha}$	Production function
$u(c)$	$\frac{c^{(1-\sigma)}}{1-\sigma}$	Utility function
δ	0.07	Depreciation rate
α	0.3	Capital share
β	0.9	Discount factor
σ	0.5	CRRA
$\pi(L)$	0.5	Probability state Low
$s(L)$	0.8	State low
$s(H)$	1.2	State high

4.1 Value and policy functions

Figure 1 shows the solution for function h^* . It has the properties described in the theoretical characterization. The function h^* is increasing and concave in the size of the partnership. Additionally, this function is also convex in the founder's share of ownership of the partnership, θ.

There is a more standard way of looking at this function. The utility possibility frontiers, shown in Figure 2, display the combination of lifetime utility levels that the founder and the associate can achieve given the current size of the partnership. The solid blue line corresponds to a small partnership, while the dashed red line corresponds to a large partnership. As expected, the frontier is concave and increasing in k.

To analyze how the allocations of consumptions are shaped by private information, we focus on Figure 3. It plots the founder's payout under private information, relative to his payout under full

[20]The values for the exponent in the production function are in line with literature, as it is the value of the depreciation rate. The utility function is CRRA with parameter $\sigma = 1/2$. This value is lower than what it is usual in macroeconomics, but it is in line with other studies of private information as those by Hopenhayn and Nicolini (1997) and Pavoni (2007). Setting $\sigma = 1/2$ is useful because $u(0)$ is finite and the results in previous sections assumed the utility function is bounded. We also computed the model with $\sigma = 0.99$ and obtained very similar results.

Figure 1: The solution for h^*

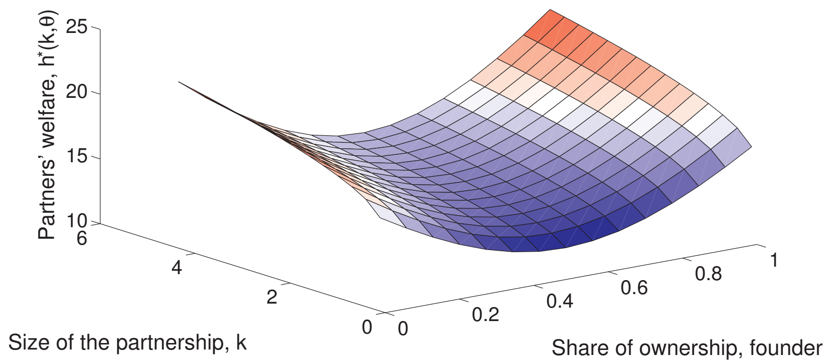
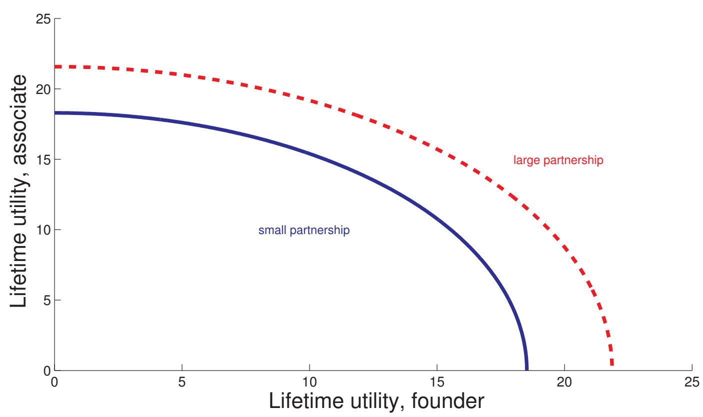
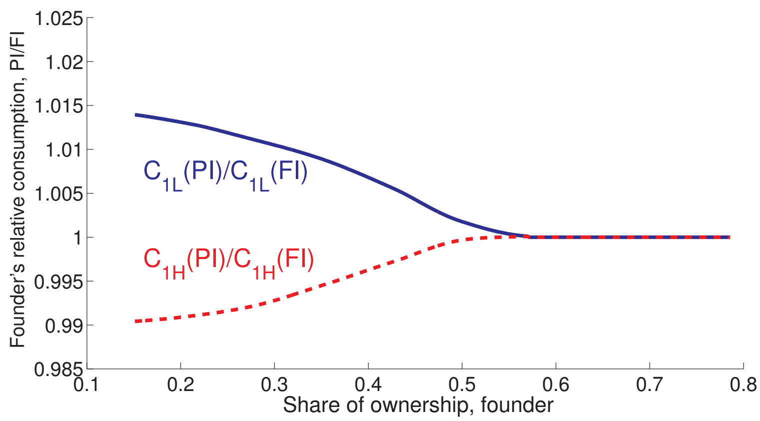


Figure 2: The utility possibility frontier



information, as a function of his ownership share of the partnership. First consider the solid blue line: the founder's payout when he reports a low liquidity shock. Notice that this line is above 1 for low values of the founder's ownership share of the partnership and is equal to 1 for shares larger than 0.6, when the founder's ownership share reaches the threshold after which he is "too big to cheat." The fact that the ratio is greater than 1 should be interpreted together with the payout of the founder after he reported a high liquidity shock (dashed red line). This ratio is smaller than 1 for low values of the founder's ownership share of the partnership and equal to 1 for shares larger than 0.6. Together, these patterns show that under private information the founder's payout does not react to liquidity needs as much as under full information. This occurs because, to make his report compatible with incentives, the founder receives a higher payout when his report is low and a lower payout when it is high, compared with the full information case. Hence, under private information, the founder cannot fully insure his liquidity needs.

Figure 3: Information and the founder's payout

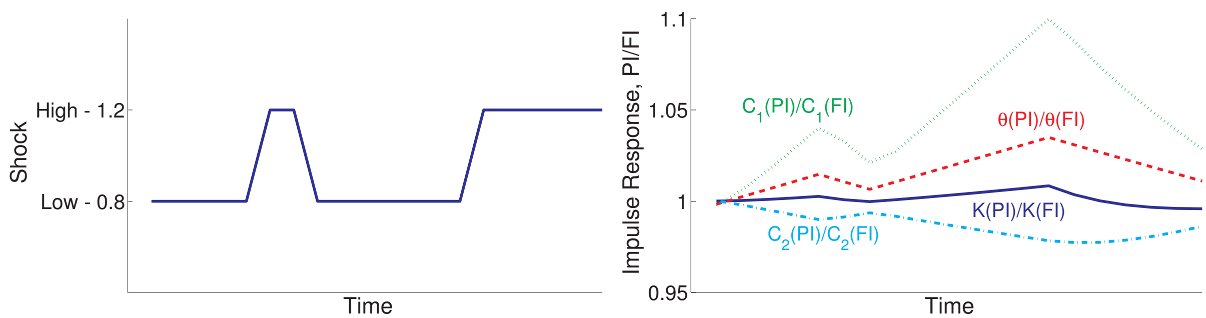


Note: PI/FI, Private information / private infomration.

We generate a sequence of shocks and plot the optimal reaction of several variables under full information and private information to further illustrate some properties of the model. At period 0, we initialize partnership (θ, k) at the same values in both cases (full and private information). The left panel of Figure 4 displays the sequence of shocks and the right panel the variables of interest.

Notice the sequence of shocks chosen consists predominantly of low shocks. Under private information, the founder must be rewarded for truthfully reporting this shock, which is shown in the right panel of Figure 4. Notice that during all the periods in this figure, the founder's payouts under private information are larger than under full information (i.e. the dotted green line is above 1). This occurs because under private information there is history dependence, shown by the fact that the ratio of the founder's ownership share of the partnership (dashed red line) is above 1. Notice that the higher payout to the founder under private information relative to full information

Figure 4: Dynamics of the main variables with private and full information



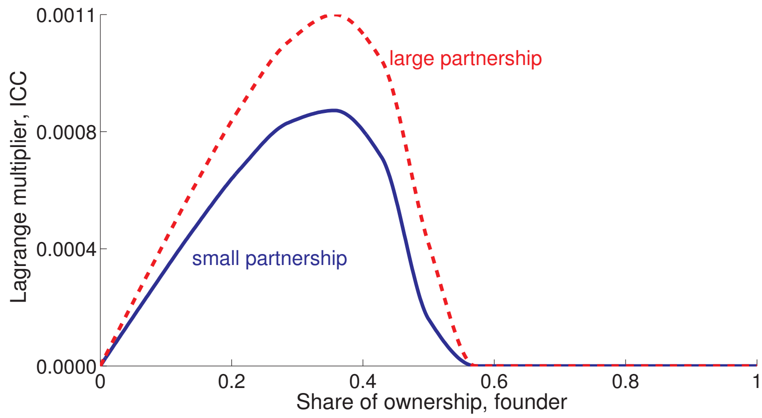
is compensated by the opposite behavior in the associate's payout. However, not only consumption, but also capital accumulation, is distorted under private information. In particular, the size of the partnership k increases more under private information than under full information as the founder reports low liquidity shocks. As we see more clearly below, this is a direct consequence of the fact that capital accumulation—and so the size of the partnership—is manipulated to provide incentives.

4.2 Too big to cheat

As we have shown in the theoretical characterization, if the founder's ownership share of the partnership is too large, he does not have an incentive to cheat. Figure 5 shows the Lagrange multiplier of the incentive compatibility constraint as a function of the founder's ownership share of the partnership. As his share starts increasing from 0, the Lagrange multiplier of the incentive compatibility constraint also increases, indicating that relaxing this constraint would increase the partners' welfare, h^*. However, when the value of the founder's ownership share is around 0.38, the multiplier peaks and then starts to decrease, reaching 0 when his share is around 0.56. Thereafter, relaxing the incentive compatibility constraint does not increase the partners' welfare: *private information does not matter.*

To understand this result, it is important to notice that the increase in the founder's payout after he reports high liquidity needs is financed not only by redistribution (i.e., a reduction in the associate's payout), but also by *disinvestment* (i.e., a reduction in the funds reinvested in the partnership). Define the share financed with disinvestment as

Figure 5: Too big to cheat



Note: ICC, incentive compatibility constraint.

$$\text{Disinvestment share} = \frac{k'_L - k'_H}{c_{1H} - c_{1L}}; \tag{13}$$

that is, the fraction of the higher payout financed by of investing less.

Figure 6 shows the disinvestment share defined in (13) and illustrates that it is increasing in the founder's ownership share. Notice that the founder's incentives for misreporting come from the fact that he can take resources from the associate. Thus, the reduction in the future size of the partnership after a report of a high shock provides an incentive for truthful revelation since this lowers his payouts from tomorrow onward.

Now, if disinvestment is useful to provide incentives, we should see more disinvestment under private information. The share financed with disinvestment under private information is actually higher than under full information (Figure 7). This implies that investment is distorted, and thus the size of the partnership is altered, by the presence of private information. Every time the founder reports a high shock, investment decreases more under private information than under full information because this helps to provide incentives for truthful revelation.[21]

[21]This is the key difference to the work of Marcet and Marimon (1992), as discussed below.

Figure 6: Disinvestment share

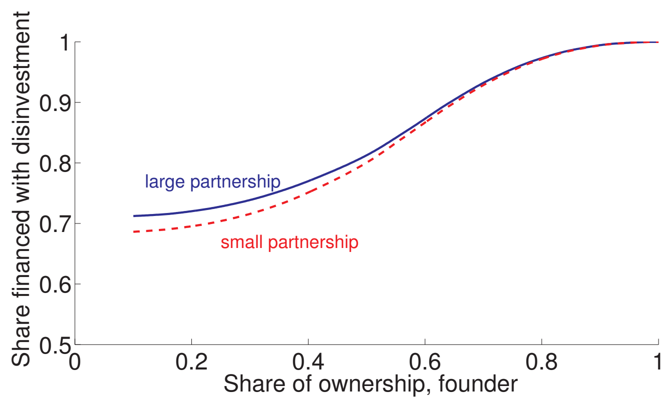
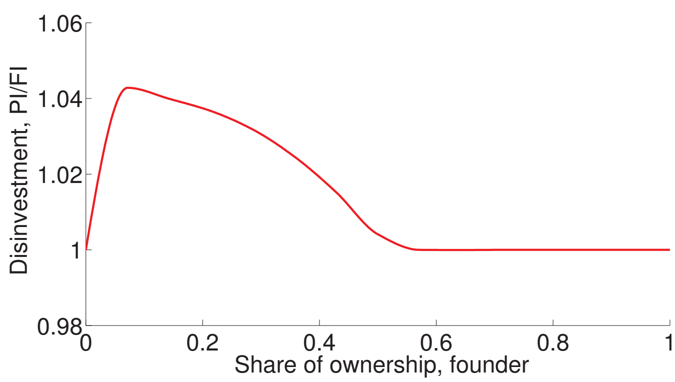


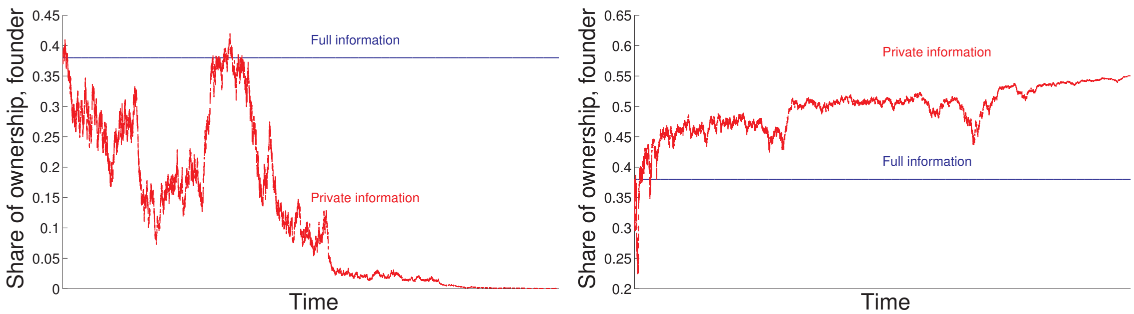
Figure 7: Disinvestment, private information and full information



4.3 The ownership structure in the long run

The theoretical section shows that ownership shares vary over time under private information. This raises the following question: What happens to the ownership structure in the long run? Figure 8 shows the two opposite cases described in Proposition 6. In the left panel, the share of the founder converges to 0. This implies that the ownership is concentrated in the associate. In the right panel, the founder's share of ownership reaches 0.56. At that point, the founder's ownership share of the partnership is big enough that the incentive compatibility constraint does not bind. From this point and thereafter, his share remains constant and so efficiency dictates that the ownership structure no longer needs to change.

Figure 8: Long-run convergence



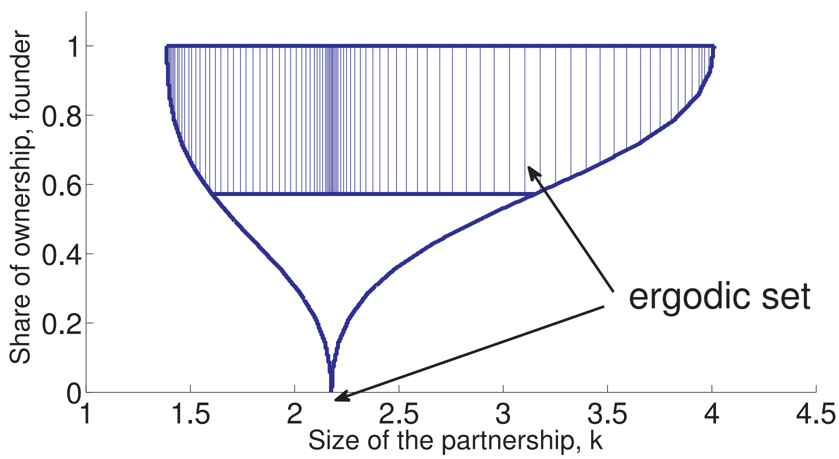
Which case is realized depends on the realization of shocks and the initial conditions. To characterize the role of these factors, we analyze the convergence of 1,000 simulations for different starting conditions. Table 2 shows that the initial size of the partnership matters for the ownership structure in the long run. When the founder's initial share of ownership is 28 percent, the partnership converges to the too-big-to-cheat' region 31 percent of the time if the initial partnership size is small and 25 percent of the time if the initial size is large. The initial ownership structure matters even more the size of the partnership. For instance, as the founder's initial share is increased to 48 percent, the partnership structure converges to the too-big-to-cheat region more frequently for different initial sizes, as shown in the second column of Table 2.

Table 2: Convergence to the too-big-to-cheat region

	Initial θ	
Initial k	0.28	0.48
1.32	31%	66%
1.79	25%	65%

Figure 9 characterizes the ergodic set in the space of the state variables: the size of the partnership and the founder's ownership share. Once the state variables take values in this region, they stay there forever. As in Figure 8, there are actually two regions. The clearest is the striped region with the founder's ownership share of the partnership between 0.56 and 1: the too-big-to-cheat-region. The founder's share remains constant anywhere within that region but the capital stock that defines the size of the partnership fluctuates between the boundaries. There is also another region that contains exactly one point in the state space: where the founder's ownership share of the partnership is 0 and capital is 2.15. In this second region, the capital stock—and thus the size of the partnership—is constant because only the founder has a liquidity shock but his ownership share is 0.

Figure 9: Ergodic set



5 Discussion of main assumptions

This section discusses the role of the main assumptions in our analysis above.

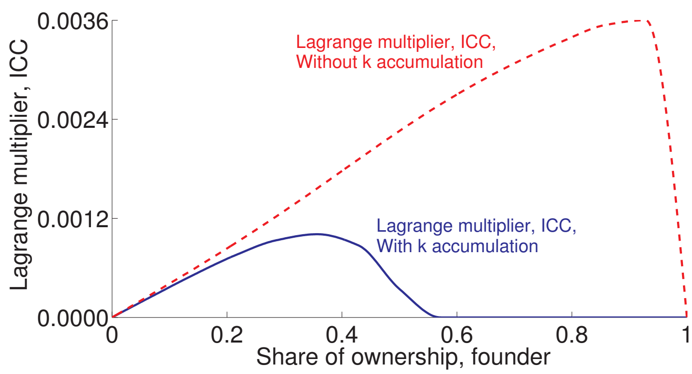
5.1 The role of investment

What is the role of investment and capital accumulation? To answer this question, we study the same problem as above but assume that the resources available cannot be affected by investment and thus are given every period. To facilitate the comparison, the level of resources fixed in the case without capital accumulation is set at the output produced at the mean of the steady-state level of capital in the case with capital accumulation.

Figure 10 shows the value of the Lagrange multiplier of the incentive compatibility constraint in both the case with and without capital accumulation. The solid blue line is the Lagrange multiplier in the case with capital accumulation. As shown in Figure 5, the multiplier starts to decrease when the founder's ownership share of the partnership increases and eventually reaches 0. This means that the full information allocation is actually implementable under private information. The dashed red line represents the value of the Lagrange multiplier of the incentive compatibility constraint in the case without capital accumulation. Remarkably, it is increasing in the founder's ownership share of the partnership in the interval $[0, 1)$ and drops discontinuously to 0 as this share is 1.

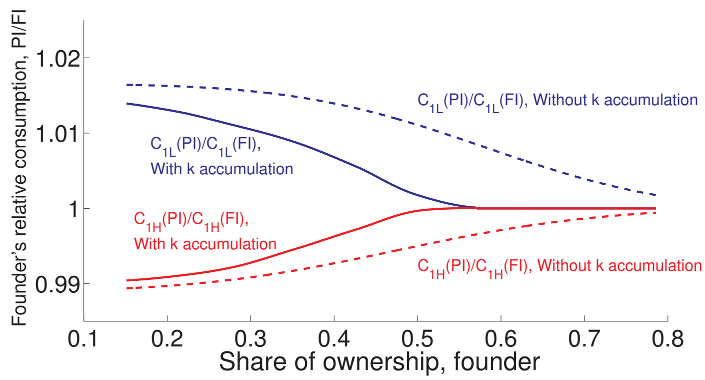
Because the behavior of these multipliers of the incentive-compatibility constraint differs so remarkably for the settings with and without capital accumulation, the allocation of consumption might also be expected to change. Figure 11 shows the payouts under private information relative to the consumption under full information for the cases with and without capital accumulation. The solid lines (blue and red) correspond to the case with capital accumulation. The blue (red) line is the relative payout under private information vs. full information as liquidity needs are low (high). As shown in Figure 3, as the founder's ownership share becomes sufficiently large, the payouts under private and full information coincide. This is not true in the case without capital accumulation, as shown by the dashed blue and red lines. These lines, which are the corresponding counterpart of the previous ones, show that payouts are more distorted in this case. This illustrates our argument that investment plays a key role as an additional instrument to provide incentives.

Figure 10: Lagrange multiplier of the incentive-compatibility constraint



Note: ICC, incentive compatibility constraint.

Figure 11: Relative consumption



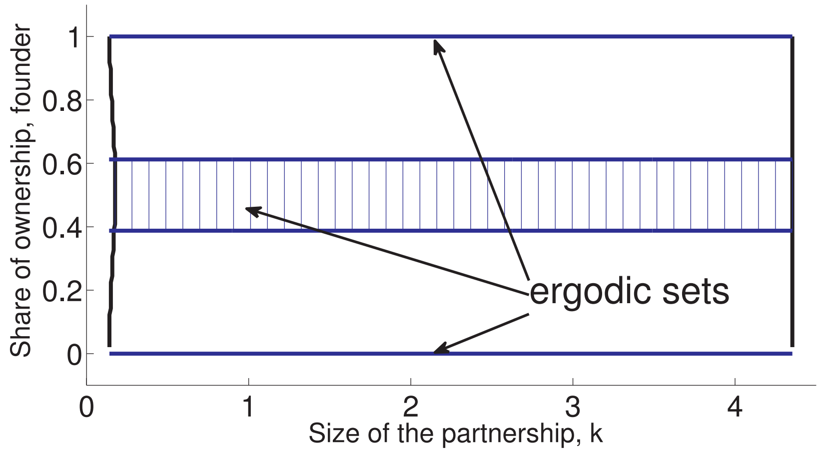
Note: PI/FI, Private information / private infomration.

5.2 Founder and associate with liquidity shocks

How would the long-run predictions change if both partners face liquidity shocks? To answer this question, we solved the case in which both partners face (uncorrelated) liquidity shocks under full information. Then, we constructed the regions in which both incentive compatibility constraints would not bind for that allocation. This allows us to replicate the analysis in Figure 9 for the case in which both partners face shocks. As shown in Figure 12, in this case there are three ergodic sets: (1) the founder's ownership share of the partnership is 0 and the associate's ownership share is too big to cheat, (2) the associate's ownership share is 0 and the founder's ownership share of the partnership is too big to cheat, and (3) both partners' shares are too big to cheat, which occurs when the ownership is close to equally distributed between the two partners (the founder's ownership share of the partnership is about one half).

This result is intuitive. When both partners face privately observed liquidity shocks, private information does not matter since both shareholders are too big to cheat such that they both internalize the negative impact of their own misreports on investment. The case in which both partners are big enough requires that the distribution of ownership to be (close) to equal between both partners. Notice that as more partners are added, the impact of the too-big-to-cheat effect driving the results obtained in region (3) will be weaker. This holds true even if ownership rights

Figure 12: Ergodic set, both partners with shocks



are equally distributed because the impact of misreports becomes diluted as property rights are more diffuse. Thus, our result is also related to that of Bolton and Von Thadden (1998) who show that a dispersed ownership structure helps in terms of liquidity provision but also creates incentive problems.

5.3 Small lies

The other important assumption in our framework is that the shock is discrete; in our particular setting, the support of the shock is $\{s_L, s_H\}$. The main mechanism at work here is the fact that with a few agents and capital accumulation, as the ownership share of one partner in the venture increases, a force (which we refer to as disinvestment) causes his incentives to misreport to decline. Here we argue that the same mechanism would be at work even if the shock's support was continuous such that the founder could tell a "marginal lie."

To see this argument, it is useful to consider the case in which the shock s has a continuous support between s_L and s_H in alternative models with and without capital accumulation. First, as we showed in the case with capital accumulation, reporting s_H when the realized shock is actually s_L is eventually not beneficial as the founder's ownership share of the partnership increases. The founder may have incentives to tell a smaller lie at that point if possible. Suppose it is some $\widetilde{s} \in (s_L, s_H)$. Again, there will be a value of the founder's ownership share of the partnership at which reporting $\widetilde{s}$ when s_L is realized is not beneficial. In fact, our theoretical result implies that for any $\widetilde{s} > s_L$, there is a value of the founder's ownership share of the partnership smaller than

1 at which the multiplier in the incentive compatibility constraint is 0. Thus, when the founder's ownership share of the partnership is in the neighborhood of 1, it must be the case that the only lie an agent will consider is marginal.[22]

This contrasts with the case without capital accumulation. Recall that full information implies that the ownership structure next period is independent of current reports and payout is increasing in current reports. Thus, without capital accumulation, the partner who privately observes a shock s_L would always want to report the largest possible lie, s_H. The contrast between incentives in settings with and without capital accumulation is the emphasis here, and this is clearly unchanged if the support of the shock is continuous instead of discrete.

5.4 No financial frictions

Here we discuss the role of risk aversion and internal financing, as opposed to the assumption of unlimited access to funds (i.e., deep pockets). This is important to contrast our results with the work of Marcet and Marimon (1992) who find that private information does not distort optimal investment.

Suppose that the associate is risk neutral. Notice that this is the case if the associate has both linear preferences and deep pockets.[23] In this case, it is simple to show that, as a direct consequence of evaluating the investment decision with the intertemporal marginal rate of substitution of the risk-neutral agent, the first-order condition that characterizes optimal investment is

$$1 = \beta\left(f'(k^*) + (1-\delta)\right). \tag{14}$$

This implies that the stock of capital that defines the size of the partnership will jump directly from k_0 to k^* to stay at that level forever. Consequently, since one partner is risk neutral in this setting, the stationary size of the partnership under private information coincides with the size under full information. Therefore, the reports of the partner with private information do not distort the optimal investment plan.

Why has the too-big-to-cheat result disappeared? With a risk-neutral agent with deep pockets, the Euler equation described above implies that the stock of capital is constant and as the level

[22]Even if the multiplier is no longer 0 in a certain region (i.e., the founder would like to cheat marginally), the mechanism identified in the case with capital accumulation will make the agent less willing to cheat. This would imply that the multiplier in the ICC will be smaller and the partnership's size and ownership structure will spend a substantial amount of time in that region (recall that changes in the ownership shares are proportional to the value of the multiplier).

[23]The latter implies that the associate can have negative consumption.

of output. Therefore, this case resembles the case without investment in which output is fixed at $f(k^*) - \delta k^*$. All the reasonings mentioned for that case also apply here.

Now, why is capital accumulation not used to provide incentives? As in the full information case with a risk-neutral associate, output is independent of liquidity shocks and thus it is optimal to finance all changes in the founder's payout by redistribution from the risk-neutral associate. With private information, the best arrangement also needs to provide incentives to report truthfully. This is optimally achieved only by redistribution since manipulating the associate's payout does not imply extra costs as a consequence of risk neutrality.

6 Conclusion

This paper studies a venture with two partners who share the stock of capital and must allocate resources to consumption of each partner and investment. One of the agents (the founder) is subject to shocks to liquidity needs that are privately observed. The other agent (the associate) does not face shocks and thus can potentially alleviate fluctuations in the founder's needs of liquidity. Constrained efficiency dictates that the ownership structure and its dynamic are fundamental to lessen the difficulties of private information. On the one hand, we show that if the ownership share of the founder becomes large enough, the full information arrangement can be implemented, private information becomes immaterial, and the ownership structure remains constant over time. The founder's incentives to misreport vanish because a fraction of the increase in his payout after reporting a high liquidity shock is financed by disinvesting in the partnership. Thus, allowing for capital accumulation is ultimately crucial in implementing the full information arrangement. On the other hand, if the founder's ownership share is not big enough, the occasionally binding incentive compatibility constraint makes the founder's payout increase less under private information than under full information after a high liquidity shock. More importantly, the history dependence caused by private information makes the ownership structure of the partnership vary over time. Consequently, in the lung run only two possible outcomes have positive probability: Either the founder's ownership share converges to 0 or it becomes large enough that the founder's incentives to cheat vanish.

Overall, the analysis suggests that capital accumulation, the number of partners, and the ownership structure in a venture are important for determining the extent to which private information matters. When there are fewer partners with large ownership shares in the partnership, they partially internalize the cost of misreporting liquidity needs in terms of distorting capital accumulation, which helps in overcoming the problem of private information.

Our analysis can be applied to different settings. For instance, the partnership could be reinterpret as an economic union among several countries. Then, the size of the countries (in terms of how much wealth they have relative to the union) would be important to determine the extent to which misreporting must be prevented by the design of the union's structure and regulations. In an economic union between a large and a small country, our results suggest that the small country would have incentives to misreport if the union regulations are not carefully designed. Moreover, our theory predicts that adding more countries to the union—and thereby reducing the share of the union of each member—would exacerbate information problems.

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7 Appendix

This appendix provides all the proofs of our results in the main text.

7.1 Recursive formulation

In this section, we provide the proofs for Propositions 1 and 2 in Section 2.2. Our analysis here generalizes to I partners with privately observed shocks to liquidity needs since our alternative recursive approach does not depend on our two-partner assumption. In order to do that, we first need to show that the utility possibility correspondence is convex valued.

Lemma 1 *$\Psi(K)$ is convex for all $K \in X$.*

Proof. Let w and $\widetilde{w} \in \Psi(K)$ and take any $\lambda \in [0,1]$. We need to show that $\lambda w + (1-\lambda)\widetilde{w} \in \Psi(K)$. Let $\left(\widehat{C}, \widehat{K}'\right)$ and $\left(\widetilde{C}, \widetilde{K}'\right)$ be the corresponding incentive compatible, feasible sequential plan such that

$$w \leq U\left(\widehat{C}, \widehat{K}', z^*\right) \text{ and } \widetilde{w} \leq U\left(\widetilde{C}, \widetilde{K}', z^*\right).$$

Define $\left(C^\lambda, K'^\lambda\right) = \lambda\left(\widehat{C}, \widehat{K}'\right) + (1-\lambda)\left(\widetilde{C}, \widetilde{K}'\right)$ and note that the concavity of $U(., z^*)$ with respect to (C, K') implies that

$$\lambda U\left(\widehat{C}, \widehat{K}', z^*\right) + (1-\lambda)U\left(\widetilde{C}, \widetilde{K}', z^*\right) \geq U\left(C^\lambda, K'^\lambda, z^*\right) \geq \lambda w + (1-\lambda)\widetilde{w}.$$

As the liquidity shocks are multiplicative, the same arguments in Atkeson and Lucas (1992) show that $\left(C^\lambda, K'^\lambda\right)$ satisfy (2).

It follows by standard arguments that $\left(C^\lambda, K'^\lambda\right)$ is feasible as F is concave. ■

The next result is one of the key steps to make our alternative approach computationally simpler and it follows from Lucas and Stokey (1984).

Lemma 2 $w \in \Psi(K)$ if and only if $w \geq 0$ and

$$\min_{\widetilde{\theta} \in \Delta}\left[h^*(K, \widetilde{\theta}) - \sum_{i=1}^{I} \widetilde{\theta}_i \ w_i\right] \geq 0.$$

Let $\Delta^I \equiv \{\theta \in \mathbb{R}_+^I : \sum_{i=1}^I \theta_i = 1\}$ and $\|h\| = \sup_{(k,\theta)}\left\{\mid h(k,\theta) \mid : \theta \in \Delta^I\right\}$ and define

$$F \equiv \{h : X \times \mathbb{R}_+^I \to \mathbb{R}_+ : \ h \text{ is continuous and } \ \|h\| < \infty\}$$

be the set of continuous and bounded function mapping $X \times \mathbb{R}_+^I$ into $\mathbb{R}_+$,while

$$F_H \equiv \{h \in F : f \text{ is HOD 1 in } \theta\}$$

denotes its subset of functions, which are homogeneous of degree 1 (HOD 1) with respect to θ. Consider the metric induced by $\|.\|$ and observe that $(F_H, \|.\|)$ is a closed subset of the Banach space $(F, \|.\|)$ and, consequently, a Banach space itself.

Let $\mathcal{W}(k)$ denote the constraint correspondence defined by (5) - (8) at any initial stock of capital k. Any $(c, w', k') \in \mathcal{W}(k)$ will be referred as a *feasible incentive-compatible recursive plan.* We say that $h \in F_H$ is *preserved under* T if $h(k,\theta) \leq (Th)(k,\theta)$ for all (k,θ).

Lemma 3 (Self-generating) *If $h \in F_H$ is preserved under T, then*

$$(Th)(k,\theta) \leq h^*(k,\theta)$$

for all (k,θ).

Proof. Take any arbitrary $\left(\widehat{c}_0(s_0), \widehat{k}'_1(s_0), \widehat{w}'_0(s_0)\right)_{s_0 \in S} \in \mathcal{W}(k_0)$ while $\theta'(s_0)$ denotes the corresponding vector of ownership shares such that

$$\widehat{\theta}'(s_0) =_{\theta' \in \Delta^I} \left[h(\widehat{k}'_1(s_0), \theta') - \sum_{i=1}^{I} \theta'_i \ w'_{i,0}(s_0) \right]$$

Since h is preserved under T, we have that

$$\widehat{\theta}'(s_0)\widehat{w}'_0(s_0) \leq h(\widehat{k}'_1(s_0), \widehat{\theta}'(s_0)) \leq (Th)(\widehat{k}'_1(s_0), \widehat{\theta}'(s_0)),$$

and therefore $\widehat{w}'_0(s_0)$ is a vector of utility levels that are attainable given $\widehat{\theta}'(s_0)$. This means that, for each $s_0 \in S$, there exists some $\left(\widehat{c}_1(s_0,s_1), \widehat{k}'_2(s_0,s_1), \widehat{w}'_1(s_0,s_1)\right)_{s_1 \in S} \in \mathcal{W}(\widehat{k}'_1(s_0))$ such that

$$\widehat{w}'_0(s_0) = \sum_{s_1} \pi(s_1) \left[s_i \ u(\widehat{c}_{i,1}(s_0,s_1)) + \beta \ \widehat{w}'_{i,1}(s_0,s_1) \right].$$

Following this strategy repeatedly T times, we can conclude that for any arbitrary $\theta_0 \in \Delta^I$

$$\begin{aligned}
& \sum_{i=1}^{I} \theta_{i,0} \left\{ \sum_{s_0} \pi(s_0) \left[s_i \ u(\widehat{c}_{i,0}(s_0)) + \beta \ \widehat{w}'_{i,0}(s_0) \right] \right\} \\
= & \sum_{i=1}^{I} \theta_{i,0} \left\{ \sum_{s_0} \pi(s_0) s_i \ u(\widehat{c}_{i,0}(s_0)) \right. \\
& \left. + \beta \sum_{s_0} \pi(s_0) \ \sum_{s_1} \pi(s_1) \left[s_i \ u(\widehat{c}_{i,1}(s_0,s_1)) + \beta \ \widehat{w}'_{i,1}(s_0,s_1) \right] \right\} \\
= & \sum_{i=1}^{I} \theta_{i,0} \ E \left(\sum_{t=0}^{T} \beta^t \ s_{i,t} u(\widehat{c}_{i,t}) \right) + \beta^{T+1} \sum_{i=1}^{I} \theta_{i,0} \ E \left(\widehat{w}'_{i,T+1} \right) \\
\leq & \sum_{i=1}^{I} \theta_{i,0} \ E \left(\sum_{t=0}^{T} \beta^t \ s_{i,t} u(\widehat{c}_{i,t}) \right) + \beta^{T+1} \left\| h \right\| \\
\leq & \sum_{i=1}^{I} \theta_{i,0} \ E \left(\sum_{t=0}^{\infty} \beta^t \ s_{i,t} u(\widehat{c}_{i,t}) \right)
\end{aligned}$$

where the first inequality holds since $h \in F_H$ and (8) while the last one follows from the Dominated Convergence Theorem and $\beta \in (0,1)$.

Notice that $(\widehat{c}, \widehat{k}')$ is sequentially feasible by construction. Now we argue that it is incentive compatible as well. To see this, denote recursively $W_{i,t}(s^t) = \widehat{w}'_{i,t}(s_0, ..., s_t)$ and observe that by construction

$$\begin{aligned}
& \left| U_{i,t}(\widehat{c}, \widehat{k}')(s^t) - W_{i,t}(s^t) \right| \\
= \;& \beta \left| \sum_{s_{t+1}} \pi(s_{t+1}) \left(U_{i,t}(\widehat{c}, \widehat{k}')(s^t, s_{t+1}) - W_{i,t+1}(s^t, s_{t+1}) \right) \right| \\
\leq \;& \beta \sup_{s_{t+1}} \left| U_{i,t}(\widehat{c}, \widehat{k}')(s^t, s_{t+1}) - W_{i,t+1}(s^t, s_{t+1}) \right| \\
\leq \;& \beta^k \sup_{(s_{t+1}, \ldots s_{t+k})} \left| U_{i,t}(\widehat{c}, \widehat{k}')(s^t, s_{t+1}, ..., s_{t+k}) - W_{i,t+k}(s^t, s_{t+1}, ..., s_{t+k}) \right|
\end{aligned}$$

Observe that $0 \leq W_{i,t}(s^t) \leq \|h\| < \infty$ for all i and all s^t while $\widehat{c}$ is uniformly bounded by construction. Taking the $\limsup$ as $k \to \infty$ for this last expression, we can conclude that $U_{i,t}(\widehat{c}, \widehat{k}')(s^t) = W_{i,t}(s^t)$ for all i and all s^t and, then, incentive compatibility follows immediately.

Since both $\left(\widehat{c}_0(s_0), \widehat{k}'_1(s_0), \widehat{w}'_0(s_0)\right)_{s_0} \in \mathcal{W}(k_0)$ and the corresponding sequential plan $(\widehat{c}, \widehat{k})$ is arbitrary, we can conclude that

$$\begin{aligned}
& \sum_{i=1}^{I} \theta_i \left\{ \sum_{s_0} \pi(s_0) \left[s_i \; u(\widehat{c}_{i,0}(s_0)) + \beta \; \widehat{w}'_{i,0}(s_0) \right] \right\} \\
\leq \;& \sum_{i=1}^{I} \theta_i \; E \left(\sum_{t=0}^{\infty} \beta^t \; s_{i,t} u(\widehat{c}_{i,t}) \right) \\
\leq \;& h^*(k, \theta)
\end{aligned}$$

and therefore, since weak inequalities are preserved in the limit, we have that

$$\begin{aligned}
Th(k, \theta) \;&=\; \max_{\left(\widehat{c}, \widehat{k}', \widehat{w}'\right) \in \mathcal{W}(k_0)} \sum_{i=1}^{I} \theta_{i,0} \left\{ \sum_{s_0} \pi(s_0) \left[s_i \; u(\widehat{c}_i) + \beta \; \widehat{w}'_i \right] \right\} \\
&\leq\; h^*(k, \theta).
\end{aligned}$$

and this completes the proof. ■

Now we are prepared to prove our two main results in Section 2.2.

Proof of Proposition 1. Given (K, θ), take any $w \in \Psi(K)$ for which (C, K') denotes the corresponding feasible incentive-compatible plan. Observe that

$$\sum_{i=1}^{I} \theta_i \; U_i(C, K') = \sum_{i=1}^{I} \theta_i \sum_{s_0 \in S_0} \pi(s_0) \; \left[s_{i,0} \; u(C_i(s_0)) + \beta \; U_{i,1}(C, K' \| (s_0)) \right]$$

Notice that $(U_{i,1}(C, K' \| (s_0)))_{i=1}^{I} \in \Psi(K(s_0))$ for all s_0. It follows by Lemma 2 that

$$h^*(K'(s_0), \theta') \geq \sum_{i=1}^{I} \theta'_i \; U_{i,1}(C, K' \| (s_0))$$

for all $\theta' \in \Delta^I$ and all s_0. Therefore, $(C_i, K', U_{i,1}(C, K'))_{i=1}^I \in \mathcal{W}(K_0)$ and then

$$\sum_{i=1}^{I} \theta_i \; U_i(C, K') \leq (Th^*)(K, \theta)$$

Since weak inequalities are preserved in the limit, we can conclude that

$$h^*(K, \theta) = \sup_{(C,K')} \sum_{i=1}^{I} \theta_i U_i(C, K') \leq (Th^*)(K, \theta),$$

for all (K, θ) (i.e. h^* is preserved under T) and thus $h^*(K, \theta) = (Th^*)(K, \theta)$ for all (K, θ). ■

Proof of Proposition 2. It is a routine exercise to show that T is a monotone operator (i.e., if $f \geq g$ then $Tf \geq Tg$). Also, observe that $h^* = Th^* \leq \widehat{T}h^* \leq \widehat{T}h^{**} = h^{**}$ by Proposition 1. Monotonicity implies that $T^n(h^{**}) \leq T^{n-1}(h^{**})$ and thus $h_n \geq h_{n+1} \geq h^*$. Since this implies that $\{h_n\}$ is a monotone decreasing sequence of uniformly bounded functions, then there exists a function $h_\infty \geq h^*$ such that $\lim_{n\to\infty} h_n = h_\infty$. It remains to show that $h_\infty \leq h^*$, for which it is sufficient that h_∞ is preserved under T due to Proposition 3.

Given (k, θ), $h_\infty(k, \theta) \leq h_n(k, \theta)$ implies that, for all n, there exists $(c^n, k'^n, w'^n) \in \mathcal{W}^n(k)$ that attains $h_\infty(k, \theta)$. Observe that (c^n, k'^n, w'^n) lies in a compact set and, thus, it has a convergent subsequence with limit point $\left(\widehat{c}, \widehat{k}', \widehat{w}'\right)$. Notice that incentive compatibility is preserved in the limit; also, we have that for all n and all s_0

$$h_n(\widehat{k}'(s_0), \theta') \geq \sum_{i=1}^{I} \theta'_i \; \widehat{w}_i'^n(s_0), \quad \text{for all } \theta'$$

and then

$$h_\infty(\widehat{k}'(s_0), \theta') \geq \sum_{i=1}^{I} \theta'_i \; \widehat{w}'_i(s_0),$$

Therefore, $\left(\widehat{c}, \widehat{k}', \widehat{w}'\right) \in \mathcal{W}^\infty(k)$ for which

$$h_\infty(k, \theta) = \sum_{i=1}^{I} \theta_i \sum_{s_0} \pi(s_0) \left[u(\widehat{c}_i(s_0)) + \beta \; \widehat{w}'_i(s_0)\right].$$

Finally, since $\left(\widehat{c}, \widehat{k}', \widehat{w}'\right) \in \mathcal{W}^\infty(k)$ is arbitrary, we can conclude that

$$\begin{aligned} (Th_\infty)(k, \theta) \quad &\geq \quad \sum_{i=1}^{I} \theta_i \sum_{s_0} \pi(s_0) \left[u(\widehat{v}_i(s_0)) + \beta \; \widehat{w}'_i(s_0)\right] \\ &= \quad h_\infty(k, \theta), \end{aligned}$$

for all (k, θ) and thus h_∞ is preserved under T by definition. ■

7.2 Efficiency and investment

In this section we provide the proofs of our main results in Section 3. The analysis is restricted to the case where I=2 and only agent 1 faces shocks to liquidity needs.

Proof of Proposition 3. First, we show that the full information plan is incentive compatible as $\theta = 1$.

Consider the recursive problem (4)-(8) for the case in which the incentive compatibility constraints are absent. This problem reduces to the well-known neoclassical growth model

$$h(\theta, k) = \max \ \theta \left\{ \sum_s \pi(s) \left[s \ u(c_1(s)) + \beta \ w_1'(s) \right] \right\} + (1-\theta) \left\{ \sum_s \pi(s) \left[u(c_2(s)) + \beta \ w_2'(s) \right] \right\}$$

subject to

$$c_1(s) + c_2(s) + k'(s) = f(k) + (1-\delta)k$$

$$\min_{\theta' \in \Delta} \left[h(\theta', k'(s)) - (\theta' \ w_1'(s) + (1-\theta') \ w_2'(s)) \right] \geq 0$$

Let $(c(s)(\theta, k), c_2(s)(\theta, k), k'(s)(\theta, k), \theta'(s)(\theta, k), w_1'(s)(\theta, k), w_2'(s)(\theta, k))$ be the set of continuous policy functions solving the problem above. Notice that in this case the law of motion of ownership shares satisfies

$$\theta'(s)(\theta, k) = \theta$$

for $s = s_L, s_H$ and all (θ, k).

Since $\theta_s'(\theta, k) = \theta = 1$, then $h(1, k'(s)) = w_1'(s)$ for all (s, θ, k) and the value function reduces to

$$h(1, k) = \pi(s_L) \left[s_L u(c_1(s_L)(1, k)) + \beta w_1'(s_L)(1, k) \right] + \pi(s_H) \left[s_H u(c_1(s_H)(1, k)) + \beta w_1'(s_H)(1, k) \right].$$

Suppose that the corresponding full information plan, characterized by $(c_{1s}(\theta, k), 0, k_s'(\theta, k), \theta)$, is not incentive compatible; i.e.,

$$s_L \ u(c_1(s_L)(1, k)) + \beta \ h(1, k'(s_L)(1, k)) < s_L \ u(c_1(s_H)(1, k)) + \beta \ h(1, k'(s_H)(1, k))$$

while

$$c_1(s_L)(1, k) + k'(s_L)(1, k) = f(k) + (1-\delta)k$$

$$c_1(s_H)(1, k) + k'(s_H)(1, k) = f(k) + (1-\delta)k$$

This implies that $(c_1(s_H)(1, k), k'(s_H)(1, k))$ is feasible at $s = s_L$ and

$$w_1'(s_H)(1, k) = h(1, k'(s_H)(1, k)).$$

This contradicts that $(c_1(s_L)(1, k), k'(s_L)(1, k), w_1'(s_L)(1, k))$ is part of the unique solution at $(1, k)$.

To complete the proof we need to argue that, conditional upon k, we can find some $\overline{\theta}(k) < 1$ such that the incentive compatibility constraint is also satisfied. Since the solution is unique, the incentive compatibility constraint must hold with strict inequality when $\theta = 1$. Now, since the policy functions in the full information case are continuous, it must be the case that there exists some $\overline{\theta}(k) < 1$ such that the incentive compatibility constraint is also satisfied for $\theta \in [\overline{\theta}(k), 1]$. ■

Proof of Proposition 4. First notice that Γ is compact set since $k_{\min}(\theta)$ and $k_{\max}(\theta)$ are both continuous functions. Then, for all $\theta \in [\theta^*, 1]$ and $k \in [k_{\min}(\theta), k_{\max}(\theta)]$ the incentive compatibility constraints are not binding. Hence, $\theta'(s)(\theta, k) = \theta$ for all (s, θ, k) and $k' \in [k_{\min}(\theta), k_{\max}(\theta)] = [k_{\min}(\theta'), k_{\max}(\theta')]$. The stock of capital remains in this ergodic set and, more in general, the corresponding plan coincides with the full information plan. ■

Proof of Proposition 5. From the ratio of FOC we obtain

$$\frac{\theta'(L)}{(1-\theta'(L))} \quad = \quad \frac{\theta\ \pi(L)+\eta_1(L)}{(1-\theta)\ \ \pi(L)} = \frac{\theta}{(1-\theta)} + \frac{\eta_1(L)}{(1-\theta)\ \ \pi(L)},$$

where η is the multiplier of the incentive compatibility constraint and $\pi(L)$ the probability of the realization of the low shocks.

$$\frac{\theta'(H)}{(1-\theta'(H))} \quad = \quad \frac{\theta\ \pi(H)-\eta_1(L)}{(1-\theta)\ \ \pi(H)} = \frac{\theta}{(1-\theta)} - \frac{\eta_1(L)}{(1-\theta)\ \ \pi(H)}$$

and then

$$\mathbf{E}\left[\frac{\theta'(s_1)}{(1-\theta'(s_1))}\right] = \frac{\theta}{(1-\theta)}.$$

■

Proof of Proposition 6. Let $\Omega = \{s^\infty \in S^\infty : \theta_t(s^\infty) \to \widehat{\theta}(s^\infty)\}$ and take any $s^\infty \in \Omega$. If $\widehat{\theta}(s^\infty) = 0$ the limiting plan is trivially first best.

Now if $\widehat{\theta}(s^\infty) \geq \theta^*$, it follows by proposition (4) that the limiting plan is first best.

Finally, we need to show that $\widehat{\theta}(s^\infty) \notin (0, \theta^*)$; i.e. the limiting plan can converge to an plan where the ICC is binding only for zero-probability sequences.

Case 1: $\overline{\theta}(k) = \theta^*$ for all k

Suppose that the state s_L occurs infinitely often and consider that infinite subsequence $\{s_{t_n}\}_{n=0}^\infty$ in which $s_{t_n} = s_L$ for all n. Since $\{k_{t_n}\}_{n=0}^\infty$ is a sequence in a compact set, it must have a convergent subsequence with limit $\widehat{k}(s^\infty) \in \times \left[k_{\min}(\widehat{\theta}(s^\infty)), k_{\max}(\widehat{\theta}(s^\infty))\right]$; to simplify notation suppose that it is the sequence $\{k_{t_n}\}_{n=0}^\infty$ itself. Since $\theta_{t_n+1} = \theta'(\theta_{t_n+1}, k_{t_n})(s_L)$, it follows by continuity that taking the limit

$$\widehat{\theta}(s^\infty) = \theta'(\widehat{\theta}(s^\infty), \widehat{k}(s^\infty))(s_L)$$

i.e. the ICC does not bind. But this contradicts that $\widehat{\theta}(s^\infty) \notin (0, \theta^*)$ and consequently, as in Thomas and Worral (1990), $\{\theta_t\}_{t=0}^\infty$ can converge to some number in the interval $(0, \theta^*)$ only for sequences where s_L occurs only finitely often. Those events occur with zero probability.

Case 2: $\overline{\theta}(k)$ can vary with k.

First if $\widehat{\theta}(s^\infty) < \overline{\theta}(k)$ for all k, then the argument follows as in Case 1.

So, suppose that $\theta^* > \widehat{\theta}(s^\infty) > \overline{\theta}(k)$ for some k and let $k(s^\infty)$ be defined such that $\widehat{\theta}(s^\infty) = \overline{\theta}(k(s^\infty))$. Notice that $k(s^\infty) \in (k_{min}(\widehat{\theta}(s^\infty)), k_{max}(\widehat{\theta}(s^\infty)))$ and assume, without lost of generality, that $k(s^\infty) > k$. As long as $\widehat{\theta}(s^\infty) \geq \overline{\theta}(k_t(s^\infty))$, $\theta'_{s_t}\left(\widehat{\theta}(s^\infty), k_t(s^\infty)\right) = \widehat{\theta}(s^\infty)$ for all s_t.

Since s^∞ belongs to a set with positive probability, there exists some finite T such that $k_T(s^\infty) \geq k(s^\infty)$.

If $\widehat{\theta}(s^\infty) < \overline{\theta}(k_T(s^\infty))$, the full information plan does not satisfy the incentive compatibility constraint at $(\widehat{\theta}(s^\infty), k_T(s^\infty))$ and so the argument follows as in Case 1.

If $\widehat{\theta}(s^\infty) = \overline{\theta}(k_T(s^\infty))$ the full information plan satisfies the incentive compatibility constraint at T, θ' remains unchanged and k' moves up and with s. Then, the full information constraint does not satisfy the incentive compatibility constraint at $(\widehat{\theta}(s^\infty), k'_L(\widehat{\theta}(s^\infty), k_T(s^\infty)))$ since $k'_L(\widehat{\theta}(s^\infty), k_T(s^\infty)) > k_T(s^\infty)$. Again, the argument follows as in Case 1. ■