Relational Contracts in a Persistent Environment

Suehyun Kwon*

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Abstract

This paper studies relational contracts with partially persistent states, where the distribution of the state depends on the previous state. The optimal contracts have properties similar to those of stationary contracts in Levin (2003), but stationary contracts are no longer optimal. With endogenous states, the dynamic enforcement constraint and the second-best contract are different from the i.i.d. case. The paper then applies the results to study implications for markets where the principal and the agent can be matched with new partners. (JEL C73, D86, L14)

1 Introduction

Most literature assumes that in a repeated interaction, the states are independent and identically distributed over time. But the real-world interactions don't always take place in an i.i.d. environment. A shock to the cost of raw material is likely to persist for some time, and if it becomes costly to perform a task this year, a firm may not expect the cost of performing the task next

*Kwon: University College London, suehyun.kwon@ucl.ac.uk, Department of Economics, University College London, Gower Street, London, WC1E 6BT United Kingdom. I'm very grateful to Glenn Ellison. I thank Michael Powell, Juuso Toikka and Muhamet Yildiz for helpful comments and Samsung Scholarship for financial support.

year to be distributed in the same way as it would after a good year. The production technology this period can also depend on the past realization of the productivity. Anticipating the persistence of the states, the employers may not expect the same effectiveness of the compensation scheme every period, and the optimal compensation scheme may in fact depend on the state.

I study a relational contract model similar to that of Levin (2003) when the states are partially persistent and there is moral hazard. The principal and the agent trade every period over an infinite horizon, and both parties are risk-neutral with a common discount factor. At the beginning of each period, the payoff-relevant state is realized and becomes observable to both the principal and the agent. Under a relational contract, the principal offers a compensation scheme each period, and the agent decides whether or not to accept it and how much effort to exert if he accepts the offer. The principal doesn't observe the agent's effort, which leads to moral hazard, but he observes the outcome, which is a noisy signal of the agent's effort, and therefore can promise contingent payments on outcomes.

The main results of the paper are in two parts. I characterize the optimal relational contracts, both for exogenous states and endogenous states. The second part applies the results to study the markets for random matching.

There is a large literature on relational contracts, including Levin (2003) and Baker, Gibbons, and Murphy (2002). Earlier literature on relational contracts focused on symmetric information case. See for example, Shapiro and Stiglitz (1984), Bull (1987), MacLeod and Malcomson (1989), Kreps (1990). More recent papers consider environments with asymmetric information, and most of the literature assumes that the environment is either stationary or i.i.d. over time. My paper is most closely related to Levin (2003), where he shows that for i.i.d. states, the principal can focus on maximizing the joint surplus and the optimal contracts can be stationary. The necessary and sufficient condition to implement an effort schedule with stationary contracts is that it satisfies the IC constraint and one other constraint. The optimal contract either implements the first best or is a step function. Other related literature is discussed at the end of this section.

Section 3 considers the results that hold for any type of persistence when the states evolve exogenously and are observable before the agent exerts effort. As was the case with i.i.d. states, the distribution of the joint surplus between the principal and the agent can be separated from the problem of efficient-contracting, and in characterizing the Pareto-optimal contracts, it is sufficient to focus on the joint surplus from the relationship. When the states follow a first-order Markov chain, the realization of the state this period is a sufficient statistic for the distribution of the future states, and the principal can provide all incentives by the bonus payments at the end of this period. In particular, the principal can offer a history-independent contract. However, it may not be stationary as defined in Levin (2003), and the fixed wage has to depend on the state. Under a relational contract, there is a temptation to renege, and the self-enforcement leads to the dynamic enforcement constraint as in the i.i.d. case. The necessary and sufficient condition for an effort schedule to be implementable by a history-independent contract is that it satisfies the IC constraint and the dynamic enforcement constraint. I also show that the optimal contract either implements the first-best level of effort, or it takes the form of a step function.

In Section 4, I consider an alternative model in which the state is endogenous. From an applied perspective, there are often environments where the agent's effort affects the distribution of the state. Specifically, I consider the environment in which the productivity is the state variable. The distribution of productivity for the next period depends on the current productivity and the agent's effort, which implies that the agent's effort affects the distribution of states in all future periods. When the productivity is observable and the persistence is of first-order, however, some of the results in Section 3 generalize to this environment.[1] What changes with endogenous states are the dynamic enforcement constraint and the second-best contract. The bonus cap depends on the realization of the state for the next period, and we don't have a uniform bonus cap anymore. The second-best contract is no longer a step function.

[1]The principal can focus on maximizing the joint-surplus, and he can do so with history-independent contracts. Stationary contracts are suboptimal.

I also consider two mechanisms through which the persistence of the states affect relational contracts. When the states are persistent, the joint surplus in the first best can vary with the state, and incentive provision for given bonus cap can also vary with the state. I consider two mechanisms separately, holding the other constant. I find that in both cases, if the joint surplus in the first best increases with the state, or if the implementable level of effort for given bonus cap increases with the state, the difference in the joint surplus between the first best and the second best decreases with the state. The principal prefers relational contracts to full-commitment contracts if and only if the initial state is sufficiently high.

The next section discusses the implications for the markets for random matching where the principal and the agent can be randomly, anonymously, and costlessly matched with new partners. The nature of the state leads to starkly different implications for the market. The degree of cooperation varies with the nature of the state, and it also highlights the difference between the i.i.d. states and the persistent states. The key step is to consider the bonus cap from the dynamic enforcement constraint. When the states are i.i.d., or if the states are persistent but common to all principal-agent pairs, cooperation is impossible; the parties cannot credibly promise any bonus payment, and the principal cannot induce any level of effort from the agent. On the other hand, if the state is persistent and agent-specific, the market turns into the market for lemons, and there is no market. The principal and the agent stay in the same relationship forever. If the state is persistent and relationship-specific, there will be a market, and the principal and the agent leave the current relationship if and only if the expected joint surplus falls below some threshold.

There are some papers on relational contracts with persistent states. Thomas and Worrall (2010) consider a two-sided incentive problem where the states and the efforts are observable and the players have limited liability. McAdams (2011) considers joint-partnership games in which the states are persistent and both the states and efforts are observable. The players decide whether to stay in the relationship and how much effort to exert. The main difference from my model is that there is no asymmetric information in their models, and there is limited liability in Thomas and Worrall (2010).

In Kwon (2012), I consider moral hazard with persistent states and full commitment. States are unobservable in Kwon (2012). Garrett and Pavan (2011, 2012) have moral hazard and persistent private information. There are also papers on dynamic adverse selection with persistent private information. Athey and Bagwell (2008) study collusion with private cost shocks, and Battaglini (2005) considers consumers with Markovian types. Escobar and Toikka (2012) show folk theorem results with Markovian types and communication.

The market setting in my paper is related to literature on repeated games with rematching. Eeckhout (2006), Ghosh and Ray (1996), Kranton (1996) and Watson (1999) among others consider repeated interactions when the players can exit the relationship in any period. The stage game in these papers are similar to the prisoner's dilemma, and most of them don't have monetary transfers. The equilibrium strategy is often to start small, which contrasts with the stationary behavior in my model. The market setting is also related to MacLeod and Malcomson (1998).

Lastly, this paper is also related to literature on partnership games with persistent states. Rotemberg and Saloner (1996) and Haltiwanger and Harrington (1991) study collusion in nonstationary markets. In Rotemberg and Saloner, the potential gain from deviating is higher in a higher state, and the future surplus is not affected by the state. In my first model in Section 5, the gain from deviating is constant across the states, and it is the future surplus that varies with the state; my model is closer to Haltiwanger and Harrington.

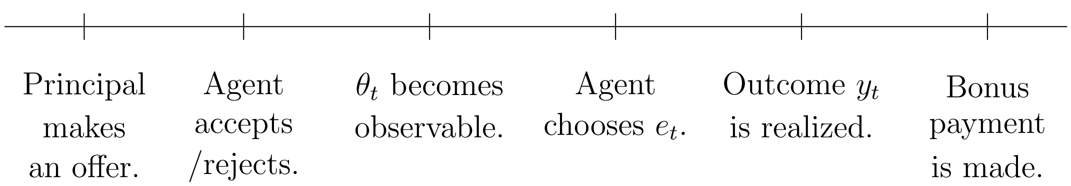
The rest of the paper is organized as follows. Section 2 describes the model, and the general results are presented in Section 3. I consider an alternative model in Section 4 in which the state variable is endogenous. Section 5 discusses the types of persistent states and their implications on the joint surplus in the second best. Section 6 applies the results to the markets for random matching. Section 7 concludes.

2 Model

The principal and the agent have the opportunity to trade over an infinite horizon, $t = 0, 1, 2, \cdots$. Both the principal and the agent are risk-neutral, and the common discount factor is $\delta < 1$.

The principal has limited commitment power and can only employ relational contracts. At the beginning of period t, the principal offers a compensation scheme to the agent, which consists of a fixed salary w_t and a contingent payment b_t. Both the fixed salary and the contingent payment can be functions of the history, which I will define momentarily. The agent decides whether to accept the offer, and a payoff-relevant parameter θ_t is realized. Both the principal and the agent observe the state. Note that the principal offers the compensation scheme before the realization of the state; he offers a function of the state as fixed salary, and the bonus payment is a function of the performance outcome.

Timing in Each Period



The state θ_t is drawn from the support $\Theta = [\underline{\theta}, \bar{\theta}]$. The distribution of the state θ_t depends only on the previous state θ_{t-1}. Denote the distribution of θ_t by $P(\theta_t|\theta_{t-1})$. The distribution of the state doesn't depend on the time index, and we have $P(\theta_t|\theta_{t-1}) = P(\theta_1|\theta_0)$ for all $t \geq 1$. In the initial period, the state θ_0 is distributed by $P_0(\theta_0)$. The distributions $P(\theta_t|\theta_{t-1})$ and $P_0(\theta_0)$ are common knowledge.

Assumption 1. *The distribution of state θ_{t+1} when the previous state was θ_t is given by $P(\theta_{t+1}|\theta_t)$ and is identical for all $t \geq 0$.*

After the principal offers a compensation scheme, the agent decides whether or not to accept, $d_t \in \{0, 1\}$. If the agent accepts the compensation scheme, the agent chooses how much effort to exert, $e_t \in \mathcal{E} = [0, \bar{e}]$. The cost of effort, $c(e_t, \theta_t)$, increases with e with $c(e = 0, \theta) = 0$ for all θ and $c_{ee} > 0$. The agent's effort generates outcome y_t with the distribution $F(y|e, \theta)$ and the support $\mathcal{Y} = [\underline{y}, \bar{y}]$.[2] The expected per period joint surplus can be written as a function of θ and e, $S(e, \theta) = \mathbb{E}[y|e, \theta] - c(e, \theta)$. Throughout the paper, when capitalized, $S(e, \theta)$ denotes per-period joint surplus in state θ if the agent chooses effort e.

I allow the distribution of the outcome and the cost function to depend on the state. If neither of them depends on the state, we are back to i.i.d. environment, and in general, we can have one or the other to be state-dependent.

Each period, there are three pieces of payoff-relevant information: The cost-relevant parameter θ_t, the agent's effort e_t, and the outcome y_t. The agent observes all three parameters, but the principal observes only θ_t and y_t. The performance outcome is $\phi_t = \{\theta_t, y_t\}$, and the set of all performance outcomes is denoted by Φ.

At the end of each period, the principal is obliged to pay the fixed salary w_t, but the contingent payment is only promised. Denote the total payment to the agent by W_t; $W_t = w_t + b_t$ if the contingent payment is made, and it is $W_t = w_t$ if not.

If the agent rejects the principal's offer, the parties receive their outside option for the period. The agent's outside option is $\bar{u}$, and the principal's outside option is $\bar{\pi}$. The joint surplus from the outside option is denoted by $\bar{s} = \bar{u} + \bar{\pi}$.

Assumption 2 (Efficiency)**.** *The maximum joint surplus is strictly bigger than the outside option for any state, but the outside option is weakly better than no effort. For all* $\theta \in \Theta$, $\max_e S(e, \theta) > \bar{s} \geq S(0, \theta)$.

I assume that for any state θ, the maximum joint surplus is strictly bigger than the outside option, but the outside option is weakly better than no effort. I also assume that the outside options $\bar{u}, \bar{\pi}$ are independent of the state and

[2] Most results of Section 3 and 4 hold for any distribution of the outcome. The characterization of the second-best contract requires an additional assumption.

constant over time. In Section 6, I consider markets for random matching, and there will be endogenous outside options.

Given the distribution of the states, $P(\theta_{t+1}|\theta_t)$, we can define the distribution of $\theta_{t+\tau}$ given θ_t, $P(\theta_{t+\tau}|\theta_t)$. Let $p(\theta_{t+1}|\theta_t)$ be the pdf of θ_{t+1}, then we have

$$p(\theta_{t+\tau}|\theta_t) = \int \cdots \int p(\theta_{t+\tau}|\theta_{t+\tau-1}) \cdots p(\theta_{t+1}|\theta_t) d\theta_{t+\tau-1} d\theta_{t+1},$$

and $P(\theta_{t+\tau}|\theta_t)$ can be constructed from $p(\theta_{t+\tau}|\theta_t)$. The discounted payoffs to the parties from date t given θ_{t-1} are

$$u_t(\theta_{t-1}) = (1-\delta)\mathbb{E}[\sum_{\tau=t}^{\infty} \delta^{\tau-t}\{d_\tau(W_\tau - c(e_\tau, \theta_\tau)) + (1-d_\tau)\bar{u}\}|\theta_{t-1}],$$
$$\pi_t(\theta_{t-1}) = (1-\delta)\mathbb{E}[\sum_{\tau=t}^{\infty} \delta^{\tau-t}\{d_\tau(y_\tau - W_\tau) + (1-d_\tau)\bar{\pi}\}|\theta_{t-1}],$$

where the expectations are taken over $P(\theta_\tau|\theta_{t-1})$, (d_τ, W_τ, e_τ), $\tau \geq t$, and $F(\cdot|e, \theta)$. In period 0, the expectation is also taken over $P_0(\theta_0)$. At each period, the parties maximize their expected payoffs. I define the expected joint surplus from period t as

$$s_t(\theta_{t-1}) = u_t(\theta_{t-1}) + \pi_t(\theta_{t-1}).$$

Note that $s_t(\theta_{t-1})$ is the per period average expected joint surplus, as it is discounted by $1-\delta$. When capitalized, $S(e, \theta)$ is the expected joint surplus from the given period for e, θ.

Let $h^t = (w_0, d_0, \phi_0, W_0, \cdots, w_{t-1}, d_{t-1}, \phi_{t-1}, W_{t-1})$ be the history up to period t and $\mathcal{H}^t$ be the set of possible period t histories. Given any period t and history h^t, a relational contract specifies the compensation the principal offers, whether or not the agent accepts it, and if the agent accepts the offer, it also specifies the effort level. The compensation w_t, b_t are allowed to be functions of the history, and they are functions of the following form:

$$w_t : \mathcal{H}^t \times \Theta \to \mathbb{R},$$
$$b_t : \mathcal{H}^t \times \Phi \to \mathbb{R}.$$

A relational contract is self-enforcing if it forms a perfect public equilibrium of the repeated game.

3 Observable and Exogenous States

This section characterizes the optimal relational contracts when the states are observable and exogenously given. The optimal contract can be independent of history, but it may not be stationary as in Levin (2003), and the fixed wage depends on the state. The self-enforcement leads to the dynamic enforcement constraint as with i.i.d. states. The necessary and sufficient condition to implement an effort schedule with stationary contracts is strictly stronger than the dynamic enforcement constraint. An optimal contract either implements the first-best level of effort or takes the form of a step function.

A relational contract forms a perfect public equilibrium of the repeated game, and there is multiplicity of equilibria. Instead of characterizing all relational contracts, I focus on efficient contracting and focus on the Pareto Frontier of the payoffs. The first result is to note that the problem of efficient contracting can be separated from the problem of distribution even if the states are persistent. The intuition is same as in Levin (2003). The principal can always adjust the fixed salary to redistribute the surplus.

Proposition 1. *Suppose there exists a relational contract with expected joint surplus $s > \bar{s}$. Any expected payoff pair (u, π) with $u \geq \bar{u},\ \pi \geq \bar{\pi},\ u + \pi = s$ can be implemented with a relational contract.*

Any proof that is not presented in the text is in the appendix. As long as the expected payoff is greater than the outside option, the parties are willing to initiate the contract. The principal can adjust the distribution of the joint surplus by the fixed salary of the initial period, and the resulting contract is still self-enforcing because the incentives are not affected. Given Proposition 1, we can restrict attention to optimal relational contracts that maximize the joint surplus from the contract.

The next result is that despite the persistence of the states, the maximum joint surplus can be achieved with history-independent contracts. I define history-independent contracts as follows:

Definition 1. *A contract is independent of history if* $W_t = w(\theta_t) + b(\phi_t),\ e_t = e(\theta_t)$ *at every* t *on the equilibrium path for some* $w : \Theta \to \mathbb{R},\ b : \Phi \to \mathbb{R}$ *and* $e : \Theta \to \mathcal{E}$*.*

Compared to stationary contracts in Levin (2003), the fixed wage in a history-independent contract may depend on the realization of the state.

Definition 2. *A contract is stationary if* $W_t = w + b(\phi_t),\ e_t = e(\theta_t)$ *at every* t *on the equilibrium path for some* $w \in \mathbb{R},\ b : \Phi \to \mathbb{R}$ *and* $e : \Theta \to \mathcal{E}$*.*

Note that the contract is independent of history on the equilibrium path. Without loss of generality, we can assume that off the equilibrium path, the parties revert to the static equilibrium of taking the outside option every period. With a history-independent contract, the principal offers the identical compensation scheme every period. The compensation scheme is independent of the history, and it only depends on the performance outcome of the given period. The fixed salary may depend on the state, but given the same state, the fixed salary is constant over time.

Proposition 2. *The maximum joint surplus can be attained with a history-independent contract.*

The proof of Proposition 2 goes as follows. When the states follow a Markov process, the current state is a sufficient statistic for the distribution of future states. Since it is observable to both the principal and the agent, there is no information asymmetry regarding the distribution of the states or continuation values. Together with risk-neutrality, the principal can provide a fixed continuation value for each state and provide all incentives by the bonus payments in the given period. The incentive provision becomes myopic, and the principal can further isolate each state, since it is observable at the beginning of each period. Then the principal can provide optimal incentives in each state in each period.

It is crucial that the states are observable to both the principal and the agent and that they are both risk-neutral. If the state wasn't observable to the principal, the principal updates his belief about the state after observing the outcome. The principal and the agent can have different priors on the state after the agent deviates, and the agent's deviation payoff is different from what the principal believes he's providing the agent with. The agent's IC constraint has to take into account future periods, and we can no longer isolate the incentive provision by each period. The principal also cannot frontload all payments if the agent wasn't risk-neutral. The key to the proof is to recognize that the principal can provide a constant continuation value for each state, independent of the history, and all incentives can be provided by the bonus payments.

However, the fixed wage in the optimal history-independent contract may vary with the state. It only depends on the current state, but since the states are partially persistent, providing a constant fixed wage for all states may be suboptimal.

Theorem 1. *An effort schedule $e(\theta)$ with expected payoffs $u(\theta), \pi(\theta)$ can be implemented with a stationary contract if and only if there exists a payment schedule $W : \Phi \to \mathbb{R}$ such that for all $\theta \in \Theta$,*

$$e(\theta) \in \arg\max_e \mathbb{E}_y[W(\phi)|e,\theta] - c(e,\theta),$$

$$\inf_\theta [\frac{\delta}{1-\delta}(u(\theta)-\bar{u}) + \inf_y W(\phi)] + \inf_\theta [\frac{\delta}{1-\delta}(\pi(\theta)-\bar{\pi}) - \sup_y W(\phi)] \geq 0,$$

$$u(\theta) \geq \bar{u},\ \pi(\theta) \geq \bar{\pi}.$$

Before proving Theorem 1, I will show the necessary and sufficient condition to implement an effort schedule with history-independent contracts.

Theorem 1 shows that the necessary and sufficient condition for a stationary contract is strictly stronger than the dynamic enforcement constraint.

With relational contracts, neither the principal or the agent commits to the contingent payment, and there exists a temptation to renege on the promised payment. The contract is self-enforcing if the principal and the agent have no incentives to renege. Since we are interested in the maximum joint surplus, there is no loss of generality in assuming that a deviation leads to the static equilibrium behavior. If the principal offers an unexpected compensation scheme, the agent either rejects the offer, or he accepts the offer but exerts zero effort. Following a deviation, the parties receive their outside options $\bar{\pi}$ and $\bar{u}$.

Recall that when the states are persistent, the discounted payoffs at period t should be conditional on state θ_{t-1}:

$$u_t(\theta_{t-1}) = (1-\delta)\mathbb{E}[\sum_{\tau=t} \delta^{\tau-t}\{d_\tau(W_\tau - c(e_\tau, \theta_\tau)) + (1-d_\tau)\bar{u}\}|\theta_{t-1}],$$
$$\pi_t(\theta_{t-1}) = (1-\delta)\mathbb{E}[\sum_{\tau=t} \delta^{\tau-t}\{d_\tau(y_\tau - W_\tau) + (1-d_\tau)\bar{\pi}\}|\theta_{t-1}],$$

and the expected joint surplus from $t+1$ is $s_{t+1}(\theta_t) = u_{t+1}(\theta_t) + \pi_{t+1}(\theta_t)$.

The principal makes the promised payment if and only if

$$\frac{\delta}{1-\delta}(\pi_{t+1}(\theta_t) - \bar{\pi}) \geq \sup_y b(\theta_t, y), \forall \theta_t,$$

and for the agent to make the promised payment, we need

$$\frac{\delta}{1-\delta}(u_{t+1}(\theta_t) - \bar{u}) \geq -\inf_y b(\theta_t, y), \forall \theta_t.$$

From Proposition 1, the principal can redistribute the surplus by adjusting the fixed wage, and the above inequalities can be combined in the dynamic enforcement constraint:

$$(DE) \quad \frac{\delta}{1-\delta}(s_{t+1}(\theta_t) - \bar{s}) \geq \sup_y W(\theta_t, y) - \inf_y W(\theta_t, y).$$

The enforceable effort schedules are characterized by the agent's IC constraint and the dynamic enforcement constraint.

Propositions 3 and 4 generalize the results for stationary contracts in Levin (2003) to history-independent contracts. The main intuition is that the states are observable and Markov.

Proposition 3. *An effort schedule $e(\theta)$ with expected joint surplus $s(\theta)$ can be implemented with a history-independent contract if and only if there exists a payment schedule $W : \Phi \to \mathbb{R}$ such that for all $\theta \in \Theta$,*

$$\begin{aligned} (IC) \quad & e(\theta) \in \arg\max_e \mathbb{E}_y[W(\phi)|e,\theta] - c(e,\theta), \\ (DE) \quad & \frac{\delta}{1-\delta}(s(\theta) - \bar{s}) \geq \sup_y W(\theta, y) - \inf_y W(\theta, y). \end{aligned}$$

Note that the continuation payoffs from period $t+1$ matter for the dynamic enforcement constraint, but they don't enter the agent's IC constraint. Since the states are persistent, the continuation payoffs $u_{t+1}(\theta_t)$ and $\pi_{t+1}(\theta_t)$ depend on the state θ_t. But the principal also observes θ_t, and by Proposition 2, the principal can offer a history-independent continuation contract, and the continuation value is independent of the outcome y_t. Therefore, even though the agent's expected payoff from period t is $W(\phi_t) + \delta u_{t+1}(\theta_t)$, $u_{t+1}(\theta_t)$ doesn't matter for the agent's IC constraint.

Now, we can prove Theorem 1.

Proof of Theorem 1. ($\Rightarrow$) Suppose $e(\theta)$ can be implemented with a stationary contract $w \in \mathbb{R}$, $b : \Phi \to \mathbb{R}$. We need both parties to make the bonus payment:

$$\begin{aligned} \frac{\delta}{1-\delta}(\pi(\theta) - \bar{\pi}) &\geq \sup_y b(\phi) = \sup_y (W(\phi) - w), \\ \frac{\delta}{1-\delta}(u(\theta) - \bar{u}) &\geq -\inf_y b(\phi) = -\inf_y (W(\phi) - w). \end{aligned}$$

We can combine the inequalities to

$$\frac{\delta}{1-\delta}(u(\theta) - \bar{u}) + \inf_y W(\phi) \geq w \geq -\frac{\delta}{1-\delta}(\pi(\theta) - \bar{\pi}) + \sup_y W(\phi).$$

Since the inequality holds for all θ, we get the second condition,

$$\inf_{\theta}[\frac{\delta}{1-\delta}(u(\theta)-\bar{u})+\inf_{y} W(\phi)]+\inf_{\theta}[\frac{\delta}{1-\delta}(\pi(\theta)-\bar{\pi})-\sup_{y} W(\phi)] \geq 0.$$

The agent's IC constraint has to be satisfied, and the continuation values for both parties are weakly greater than the outside options.

($\Leftarrow$) When the conditions are satisfied, the agent chooses $e(\theta)$ in each θ, and the parties are willing to initiate the relationship. When the second condition is satisfied, we can pick w such that

$$\inf_{\theta}[\frac{\delta}{1-\delta}(u(\theta)-\bar{u})+\inf_{y} W(\phi)] \geq w \geq -\inf_{\theta}[\frac{\delta}{1-\delta}(\pi(\theta)-\bar{\pi})-\sup_{y} W(\phi)],$$

and the parties will make the bonus payment. By construction, the contract is self-enforcing in every period. □

Theorem 1 shows that stationary contracts may be suboptimal; the second condition in Theorem 1 is strictly stronger than the dynamic enforcement constraint. Specifically, we know that providing a constant fixed wage is no longer optimal when the states are persistent. Also note that when a contract is stationary, the continuation value for the agent may vary with the state. The principal cannot frontload all the payments and provide the constant continuation value.

We also know from the dynamic enforcement constraint that both the joint surplus per period and the expected discounted joint surplus decrease with the outside option $\bar{s}$.

Corollary 1. *The per period joint surplus and the expected joint surplus weakly decrease with the outside option* $\bar{s}$.

Lastly, from Proposition 3, we obtain the following characterization of optimal contracts. When the distribution of the outcome satisfies the Mirrlees-Rogerson constraints, together with risk-neutrality of both parties, the principal wants to use the strongest incentives possible. If an optimal contract cannot induce the first-best effort $e^{FB}(\theta_t)$ in state θ_t, the DE constraint binds, and the compensation scheme is a step function.

Assumption 3. *The distribution of the outcome $F(y|e,\theta)$ satisfies the Mirrlees-Rogerson constraints: $F(y|e,\theta)$ has the monotone likelihood ratio property, (f_e/f increases with y) and $F(y|e,\theta)$ is convex in e for any θ.*

Proposition 4. *Suppose Assumption 3 holds. An optimal contract either (i) implements $e^{FB}(\theta_t)$ or (ii) takes the form of a step function at each θ_t. When $e(\theta_t) < e^{FB}(\theta_t)$, there exists $y(\theta_t)$ such that $W(\theta_t, y) = \bar{W}(\theta_t)$ for $y \geq y(\theta_t)$ and $W(\theta_t, y) = \underline{W}(\theta_t)$ for $y < y(\theta_t)$. $\bar{W}(\theta_t) = \underline{W}(\theta_t) + \frac{\delta}{1-\delta}(s_{t+1}(\theta_t) - \bar{s})$, and the likelihood ratio $f_e/f(y|e(\theta_t))$ changes the sign at $y(\theta_t)$.*

When the states are observable and exogenously given, history-independent contracts generalize the stationary contracts for i.i.d. states in Levin (2003). However, stationary contracts are suboptimal with persistent states, and the fixed wage under the optimal contract depends on the state.

4 Endogenous States

In practice, it is often natural to assume that the state variable is endogenous. Human capital is likely to be developed by the agent's effort over time, and the productivity is also often endogenous. If the outcome this period determines the productivity for the next period, the outcome itself is the state variable and is endogenous.

This section characterizes optimal relational contracts with endogenous states when the state is observable to both the principal and the agent. The agent's effort and the productivity this period determine the distribution of the productivity next period, and the outcome is a function of the productivity. Since the agent's effort affects the distribution of the productivity, it is an endogenous state variable. However, when the productivity is observable to both the principal and the agent, some of the results in the previous section generalize to this model. I show that the problem of efficient contracting can be separated from the distribution of joint surplus, and the maximum joint surplus can be attained with history-independent contracts. What is different with endogenous states is the form of the dynamic enforcement constraint and the second-best contract. There is no uniform bonus cap for the given state, and the second-best contract is not a step function any more.

The productivity θ_t is drawn from $\Theta = [\underline{\theta}, \bar{\theta}]$. The distribution of θ_t depends on θ_{t-1} and e_{t-1} and is time-homogeneous. Denote the distribution by $P(\theta_t|\theta_{t-1}, e_{t-1})$. The distribution of θ_0 is given by $P_0(\cdot)$. Given θ_t, the principal gets the outcome $y_t = y(\theta_t)$ as a deterministic function of the productivity. A performance outcome is $\phi_t = (\theta_t, y_t, \theta_{t+1})$. Note that the outcome need not be deterministic. I assume it to be deterministic to simplify the analysis, but the same argument works if it is stochastic and the expected outcome is a function of the state.

The timing of the model is as follows. At the beginning of period t, the principal offers a contract to the agent, and the agent decides whether to accept it. The agent decides how much effort to exert, and the outcome is realized as a function of the productivity, which is known from previous period. The productivity for the next period is realized. The principal and the agent make the payments.

First set of results are analogous to Propositions 1-3.

Proposition 5. *Suppose there exists a relational contract with expected joint surplus $s > \bar{s}$. Any expected payoff pair (u, π) with $u \geq \bar{u}$, $\pi \geq \bar{\pi}$, $u + \pi = s$ can be implemented with a relational contract.*

The proof of Proposition 5 is the same as the proof of Proposition 1 verbatim. The agent accepts the contract as long as the expected payoff is greater than his outside option, and the principal can always redistribute the surplus by the fixed wage.

Proposition 6. *The maximum joint surplus can be attained with a history-independent contract. Furthermore, it is optimal to provide a constant continuation value to the agent for every state.*

The key to the proof of Proposition 6 is that θ' is a sufficient statistic for the outcome and the states in the future. Since the principal and the agent are risk-neutral and the productivity is observed before they make the payments, the principal can provide all incentives by the present compensation and provide a constant expected payoff to the agent in every state. Under an optimal contract, the principal chooses the bonus payments to maximize the expected joint surplus.

Proposition 7. *An effort schedule $e(\theta)$ with expected joint surplus $s(\theta)$ can be implemented with a history-independent contract if and only if there exists a payment schedule $b : \Phi \to \mathbb{R}$ such that for all $\theta, \theta' \in \Theta, y \in \mathcal{Y}$,*

$$(IC) \quad e(\theta) \in \arg\max_{e} \mathbb{E}_{y,\theta'}[b(\phi)|e, \theta] - c(e, \theta),$$

$$(DE) \quad 0 \leq b(\theta, y, \theta') \leq \frac{\delta}{1-\delta}(s(\theta') - \bar{s}).$$

In Proposition 7, the bonus cap now depends on the realization of the productivity for the next period. This is because the continuation values depend on the productivity for the next period. Note also that the bounds depend on θ', and we don't have a uniform bonus cap when the current state is θ. The rest of the argument is similar to the proof of Proposition 3.

Remark 1. *When the states are observable to both the principal and the agent, even if the states are endogenous, the optimal relational contracts are independent of history. There is no information asymmetry about the distribution of future states, and together with risk-neutrality, we know that the optimal contracts are independent of history and the necessary and sufficient condition to implement an effort schedule is the IC constraint and the DE constraint.*

Remark 2. *The difference between the endogenous states and the exogenous states is that instead of having a uniform bonus cap for the given state, the bonus cap now depends on the realization of the productivity for the next period.*

We also obtain the analogue of Theorem 1 for endogenous states.

Theorem 2. *An effort schedule $e(\theta)$ with expected payoffs $u(\theta), \pi(\theta)$ can be implemented with a stationary contract if and only if there exists a payment* *schedule* $W : \Phi \to \mathbb{R}$ *such that for all* $\theta \in \Theta$,

$$e(\theta) \in \arg\max_e \mathbb{E}_{y,\theta'}[W(\phi) + \delta u(\theta')|e,\theta] - c(e,\theta),$$
$$\inf_\phi [\frac{\delta}{1-\delta}(u(\theta') - \bar{u}) + W(\phi)] + \inf_\phi [\frac{\delta}{1-\delta}(\pi(\theta') - \bar{\pi}) - W(\phi)] \geq 0,$$
$$u(\theta) \geq \bar{u},\ \ \pi(\theta) \geq \bar{\pi}.$$

The second condition is strictly stronger than the dynamic enforcement constraint, and stationary contracts may be suboptimal. With endogenous states, the agent's IC constraint now has to take into account the continuation values.

Lastly, the second-best contract with endogenous states is no longer a step function. When the Mirrlees-Rogerson constraints are satisfied, we can replace the agent's IC constraint with the first-order condition, and the principal wants to maximize the agent's effort when $e(\theta) < e^{FB}(\theta)$. From the monotone likelihood ratio property, the principal provides the maximum bonus when the likelihood ratio is positive and the minimum bonus when the likelihood ratio is negative. However, the bonus cap depends on the realization of the productivity for the next period, and the second-best contract is not a step function.

Assumption 4. *The distribution of the outcome* $F(\theta'|e,\theta)$ *satisfies the Mirrlees-Rogerson constraints:* $F(\theta'|e,\theta)$ *has the monotone likelihood ratio property, (*f_e/f *increases with* θ'*) and* $F(\theta'|e,\theta)$ *is convex in* e *for any* θ.

Theorem 3. *Suppose Assumption 4 holds. An optimal history-independent contract either (i) implements* $e^{FB}(\theta)$ *or (ii) takes the following form: When* $e(\theta) < e^{FB}(\theta)$*, there exists* $g(\theta)$ *such that* $W(\theta, y, \theta') = \bar{W}(\theta')$ *for* $\theta' \geq g(\theta)$ *and* $W(\theta, y, \theta') = \underline{W}(\theta')$ *for* $\theta' < g(\theta)$. $\bar{W}(\theta') = \frac{\delta}{1-\delta}(s(\theta') - \bar{s})$, $\underline{W}(\theta') = 0$*, and the likelihood ratio* $f_e/f(\theta'|e(\theta))$ *changes the sign at* $g(\theta)$.

5 Joint Surplus in the Second Best

I consider the joint surplus in the second best for two types of persistence in this section. The states are exogenous. The first case is in which the joint surplus in the first best increases with the state. When the cost function is separable and strictly decreases with the state, incentive provision is identical in each state, and in particular, given a bonus cap, the principal can implement the same level of effort in every state. The second type of persistence I consider is when the incentive provision becomes easier in a higher state. The joint surplus in the first best is identical in all states. In both cases, the difference in joint surplus between the first best and the second best decreases with the state. The principal prefers relational contracts only if the initial state is sufficiently high. The first case only exists with persistent states, but the second case has similar effect with i.i.d. states.

5.1 Joint Surplus Varies with the State

In this section, I consider the case in which the joint surplus varies with the state and the incentive provision is constant across the states. Specifically, I assume the following.

Assumption 5. *The cost of effort is separable and strictly decreases with the state: There exist* $c_1 : \mathcal{E} \to \mathbb{R},\ c_2 : \Theta \to \mathbb{R}$ *such that*

$$c(e, \theta) = c_1(e) + c_2(\theta), \forall e \in \mathcal{E}, \theta \in \Theta$$

and $c_2' < 0$ *for all* $\theta \in \Theta$.

Assumption 6. $F(\cdot|e, \theta)$ *is independent of* θ.

Assumption 7. $\theta_t > \theta_t'$ *implies* $P(\cdot|\theta_t)$ *FOSD* $P(\cdot|\theta_t')$.

I also define $\Delta W(\theta)$ as the minimum bonus cap to be able to induce the first-best level of effort in state θ. Given a state θ, $e^{FB}(\theta)$ can be a solution to

$$\begin{aligned} e(\theta) &\in \arg\max_e \mathbb{E}_y[W(\phi)|e] - c(e,\theta), \\ \Delta W &\geq \sup_y W(\theta,y) - \inf_y W(\theta,y) \end{aligned}$$

if and only if $\Delta W \geq \Delta W(\theta)$.

As a benchmark, I first show the implications of Assumptions 5-7 in the first best and in the case the principle has a within-period commitment power.

Proposition 8. *Suppose Assumptions 5-7 hold. Both the per period expected joint surplus and the future discounted joint surplus in the first best strictly increases with the state. The first-best level of effort is constant across all states $\theta \in \Theta$. The minimum bonus cap to implement the first-best level of effort, $\Delta W(\theta)$, is also constant across the state. If the principal can credibly promise $W(\phi)$, the principal implements the same level of effort, $e^* = e^{FB}$ in all states.*

Proof. The expected joint surplus in state θ is given by

$$\mathbb{E}_y[y|e] - c(e,\theta) = \mathbb{E}_y[y|e] - c_1(e) - c_2(\theta),$$

and the first-best level of effort is the maximand of

$$\int_y yf(y|e)dy - c_1(e) - c_2(\theta).$$

Since the cost of effort is separable, the maximization problems for any two states are constant transformations of each other, and the first-best level of effort is constant across the states. The cost strictly decreases with the state, and the expected per period joint surplus in the first best in state θ strictly increases with the state. By the persistence of states, the future discounted joint surplus also increases with the state. Since the maximization problems are a constant transformation of each other, $\Delta W(\theta)$ is constant across the states.

If the principal can commit to bonus payments, the only constraint is the agent's IC constraint. By the efficiency assumption, it is efficient to induce the first-best level of effort than to take the outside option in all states θ, and the principal induces the first-best level of effort in all θ. $\square$

Now consider relational contracts under Assumption 5. Define $s^{FB}(\theta)$ as the discounted future joint surplus when the previous state is θ. We know from Proposition 8 that $\Delta W(\theta)$ is constant over θ. Denote $\Delta W(\theta) = \Delta W^*$. If $s^{FB}(\underline{\theta}) \geq \Delta W^*$, the principal can implement the first-best level of effort in all states with relational contracts, and the problem becomes trivial. I will make the following assumption:

Assumption 8. *The principal cannot induce the first-best level of effort in the lowest state:*

$$s^{FB}(\underline{\theta}) < \Delta W^*.$$

Define $e(\theta|\Delta W)$ to be the solution to the optimization problem

$$\begin{aligned} max_e \mathbb{E}_y[y - c|e, \theta] \quad \text{s.t. } & e(\theta) \in \arg\max_e \mathbb{E}_y[W(\phi)|e] - c(e, \theta), \\ & \Delta W \geq \sup_y W(\theta, y) - \inf_y W(\theta, y). \end{aligned}$$

$e(\theta|\Delta W)$ is the level of effort that maximizes the per period joint surplus in state θ when the bonus cap is ΔW. If $\Delta W \leq \Delta W(\theta)$, the principal cannot implement the first-best level of effort, and $e(\theta|\Delta W) < e^{FB}(\theta)$. Since the principal can always mimic the payments with $\Delta W'$ if $\Delta W \geq \Delta W'$, the implementable level of effort weakly increases with the bonus cap, and we have $e(\theta|\Delta W) \geq e(\theta|\Delta W'), \forall \theta$.

Proposition 9. *The implementable level of effort $e(\theta|\Delta W)$ weakly increases with ΔW for all θ.*

Proof. The proof follows from the fact that the principal can always mimic the compensation scheme with $\Delta W'$ if $\Delta W \geq \Delta W'$. $\square$

Under relational contracts, the expected joint surplus from the following period limits the principal's ability to induce effort, and Assumption 5 states that the joint surplus in the first best strictly increases with the state. The implementable level of effort is lower in a worse state, and the difference in the expected joint surplus is reinforced by the implementable effort. Under Assumption 5, the joint surplus under relational contracts increases with the state, and the difference in the joint surplus between the first best and the second best decreases with the state.

Proposition 10. *Suppose Assumptions 5-8 hold. Let $s^{SB}(\theta)$ be the expected joint surplus under an optimal relational contract. $s^{SB}(\theta)$ strictly increases with θ, and $\frac{\partial s^{SB}}{\partial \theta} \geq \frac{\partial s^{FB}}{\partial \theta} > 0$. The difference in the joint surplus between the first best and the second best, $s^{FB}(\theta) - s^{SB}(\theta)$, weakly decreases with the state. The difference is strictly positive at $\underline{\theta}$, and it is weakly bigger than zero at all θ.*

Proof. We know from Proposition 9 that the implementable level of effort, $e(\theta|\Delta W)$, weakly increases with ΔW. From Assumption 5, the expected joint surplus in the first best increases with the state, and Assumption 8 says that the expected joint surplus in the state $\underline{\theta}$ is less than the minimum bonus cap to induce the first-best level of effort. Since the distribution of the states increases with the state in the sense of first-order stochastic dominance, the implementable level of effort under an optimal relational contract increases with the state, and the expected joint surplus in the second best also increases with the state.

Consider the difference in per period joint surplus between the first best and the second best.

$$
\begin{aligned}
&S(e^{FB}, \theta) - S(e(\theta|\Delta W), \theta) \\
=&(\mathbb{E}[y|e^{FB}] - c(e^{FB}, \theta)) - (\mathbb{E}[y|e(\theta|\Delta W)] - c(e(\theta|\Delta W), \theta)) \\
=&(\mathbb{E}[y|e^{FB}] - c_1(e^{FB})) - (\mathbb{E}[y|e(\theta|\Delta W)] - c_1(e(\theta|\Delta W))).
\end{aligned}
$$

Given ΔW, $e(\theta|\Delta W)$ is constant across the states, and we also know that

$$\mathbb{E}[y|e(\theta|\Delta W)] - c_1(e(\theta|\Delta W))$$

increases with ΔW. Therefore, the difference in the per period joint surplus,

$$S(e^{FB}, \theta) - S(e(\theta|\Delta W), \theta),$$

decreases with the state, and by the persistence of the states, the difference in the expected joint surplus also decreases with the state. From Assumption 8, the difference is strictly positive at $\underline{\theta}$. □

When the per period joint surplus in the first best increases with the state, the persistence of the states enter the optimization problem through the bonus cap, and the expected joint surplus under an optimal relational contract also increases with the state. The dynamic enforcement constraint magnifies the impact of persistent states, and the expected joint surplus varies more in the second best than in the first best.

5.2 Incentive Provision Varies with the State

This section considers the alternative case in which the joint surplus in the first best is constant across the state but the incentive provision varies with the state.

I assume that the first-best level of effort is constant across the states. This is without loss of generality for any interior solution e^{FB}. I also assume that for given bonus cap, the maximum per period joint surplus strictly increases with the state, and the principal cannot implement the first-best level of effort in the worst state, even with the expected joint surplus in the first best.

Assumption 9. *The first-best level of effort is constant in all states. The per period joint surplus in the first best is constant across the states:* $S(e^{FB}, \theta) = S^*$ *for all* θ*.*

Assumption 10. *For given bonus cap ΔW, if the principal cannot induce the first-best level of effort, the maximum per period joint surplus strictly increases with the state. i.e., $S(e(\theta|\Delta W), \theta)$ strictly increases with θ for all $e(\theta|\Delta W) < e^{FB}$.*

Assumption 11. *The principal cannot implement the first-best level of effort in the lowest state, that is $e(\underline{\theta}|s^{FB}) < e^{FB}$.*

Under the second set of assumptions, the expected joint surplus in the second best strictly increases with the state, and the difference in the expected joint surplus between the first best and the second best decreases with the state. We have the following proposition which is an analogue of Proposition 10.

Proposition 11. *Suppose Assumptions 7, 9, 10 and 11 hold. There exists $\theta^* \in \Theta$ such that $s^{SB}(\theta)$ strictly increases with θ for $\theta < \theta^*$, and $s^{SB}(\theta) = s^{FB}$ for $\theta \geq \theta^*$. The difference in the joint surplus between the first best and the second best, $s^{FB} - s^{SB}(\theta)$, decreases with the state. The difference is strictly positive at $\underline{\theta}$, and it is weakly bigger than zero at all θ.*

Proof. By Assumptions 10, 11 and the persistence of the states, the per period joint surplus in the second best weakly increases with θ, and it increases strictly for all θ such that $e(\theta|s^{SB}(\theta)) < e^{FB}$. Therefore, the expected joint surplus in the second best also increases with the state. Since the first-best joint surplus is constant across the states, the difference between the first best and the second best decreases with the state. □

I have considered two types of persistent states. In both environments, the difference in the expected joint surplus between the first best and the second best decreases with the state. If the two factors, the level of joint surplus in the first best and the difficulty of incentive provision, move in the same direction, the effect will be magnified. If they move in the opposite directions, the difference in the joint surplus will be determined by which effect dominates.

5.3 Benefits from Relational Contracts

Suppose there exists a positive benefit from relational contracts. I define full-commitment contracts as contracts under which the principal specifies the compensation scheme as functions of history and commit to both the fixed wage and the bonus payments. In my model, the only constraint under full-commitment contracts is the agent's IC constraints, and the principal can implement the first best under full-commitment contracts.

There could be gains from relational contracts as it is often impractical to write complete contracts. Performance measures can be hard to describe, and often, the best performance measure is a subjective measurement. When there is positive benefit $x > 0$ from relational contracts, the principal prefers the relational contracts over full-commitment contracts if and only if the benefit is bigger than the difference in the expected joint surplus.

Proposition 12. *Suppose Assumptions 5-8 hold. Let $x > 0$ be the benefit from relational contracts. The principal prefers relational contracts if and only if the prior on the states is sufficiently high:*

$$\int_{\theta_0} s^{SB}(\theta_0)dP_0(\theta_0) + x \geq \int_{\theta_0} s^{FB}(\theta_0)dP_0(\theta_0).$$

Proof. The principal can implement the first best with full-commitment contracts. Given prior P_0 on the state, the difference in the expected joint surplus between the full-commitment contract and the optimal relational contract is given by

$$\int_{\theta_0} (s^{FB}(\theta_0) - s^{SB}(\theta_0))dP_0(\theta_0) - x.$$

□

Proposition 13. *Suppose Assumptions 7, 9, 10 and 11 hold. Let $x > 0$ be the benefit from relational contracts. The principal prefers relational contracts if and only if the prior on the states is sufficiently high:*

$$\int_{\theta_0} s^{SB}(\theta_0)dP_0(\theta_0) + x \geq s^{FB}.$$

Proof. The principal can implement the first best with full-commitment contracts, and the joint surplus in the first best is constant. □

6 Market for Matching

This section considers a market for matching when there is a continuum of principal-agent pairs. In any given period, the principal and the agent have an option to exit the current relationship. If they exit, they will be randomly, anonymously and costlessly rematched with new partners. The main result of this section is that the nature of the underlying state leads to different implications for the market. If the state is agent-specific, the principal-agent pairs remain in the current relationships regardless of the realization of the state or the past history, and there will be no market for matching. If the state is relationship-specific, there will be a market, and the pair leaves the relationship if and only if the expected joint surplus falls below some threshold. If the state is a macro shock, common to all principal-agent pairs, then cooperation is impossible, and the principal cannot induce the agent to put in any effort. Cooperation is also impossible if the states are i.i.d..

The literature on relational contracts take the outside options as exogenous. The goal of this section is to consider the market and to endogenize the outside options. If a continuum of principal-agent pairs in the same contractual environment have options to be matched with new partners, the market forms endogenous outside options for the principal-agent pairs. The implications highlight the difference between the i.i.d. states and persistent states, and also the difference among the types of persistent states.

The timing of the game is as follows. In each period, the principal offers a compensation scheme, and the agent decides whether or not to accept it. After the agent decides, the state is realized and becomes observable to both the principal and the agent. If the agent accepted, he decides how much effort to put in, and the outcome is realized. The principal and the agent make the contingent bonus payment and decide whether or not to stay in the relationship. If they both decide to stay, they move on to the next period. If one of them exits, both the principal and the agent will be matched with new partners and start in the next period. If the agent rejected the offer, both receive their exogenous outside options and decide whether to stay or exit.

With a market for matching, the outside options for the principal and the agent are endogenously determined in an equilibrium. However, given a continuum of principal-agent pairs, each pair takes the outside options as given, and we can apply the analysis from Section 3. I allow for exogenous outside options as well, but this doesn't affect the analysis, and we can restrict attention to endogenous outside options.[3] In this section, $\bar{s}$ refers to the endogenous outside option through matching.

The equilibria of the game depend on the strategies when the principal and the agent are matched with new partners. For most part of this section, I focus on equilibria in which the principal and the agent always maximize the joint surplus, when they are matched with new partners. I will discuss at the end of the section what happens if they don't maximize the joint surplus. Also, the analysis in this section doesn't rely on the stationarity or symmetry of the strategies. The principal-agent pairs are allowed to use non-stationary contracts, and each pair can use different contracts.

6.1 When the States are i.i.d.

This section considers the i.i.d. states as a benchmark. Cooperation is impossible if there is frictionless market for matching and if the principal-agent pairs maximize the joint surplus in a new relationship.

Proposition 14. *Suppose the states are i.i.d., and the principal and the agent can be randomly, anonymously, and costlessly matched with new partners. If the principal-agent pairs maximize the joint surplus in a new relationship, the principal cannot induce any level of effort from the agent.*

Proof. After the outcome is realized, the principal makes the bonus payment

[3] Without loss of generality, we can assume that if the principal anticipates that the agent will reject the offer in the next period, he chooses to exit the relationship.

if and only if

$$\frac{\delta}{1-\delta}(\pi - \bar{\pi}) \geq \sup_{y} b(\theta, y), \forall \theta,$$

and the agent makes the bonus payment if and only if

$$\frac{\delta}{1-\delta}(u - \bar{u}) \geq -\inf_{y} b(\theta, y), \forall \theta.$$

Together, we have

$$\frac{\delta}{1-\delta}(s - \bar{s}) \geq \sup_{y} b(\theta, y) - \inf_{y} b(\theta, y).$$

However, if they maximize the joint surplus when matched with new partners and the states are i.i.d., we get $s = \bar{s}$, and the bonus payment has to be the same for all outcomes. The agent has no incentive to put in any effort. □

Note that I don't require the strategies to be independent of history or symmetric. The only requirement is that the principal and the agent maximize the joint surplus when they are matched with new partners, but when they do, they can't have any level of cooperation. In order to have any cooperation, they cannot maximize the joint surplus when they are matched with new partners; this case is considered in Section 6.5. Also note that, in order to have cooperation, they have to suppress the joint surplus. Delaying the payment doesn't help.

6.2 Relationship-Specific State

Suppose that the states are persistent and the state is specific to the pair of principal and agent. If they exit the current relationship, the initial state in a new relationship is drawn from a known distribution G and is i.i.d. across the new pairs of principals and agents. Then there is endogenous threshold for the joint surplus such that the principal and the agent exit the relationship if and only if the expected joint surplus falls below the threshold.

Proposition 15. *Suppose the initial state is i.i.d. across the new pairs of* *principals and agents and is drawn from a known distribution G. The principal and the agent exit the current relationship if and only if the expected joint surplus falls below some threshold. When the state is such that they will exit, the agent doesn't put in any effort.*

Proof. With persistent states, the maximum joint surplus differs across the pairs and depends on the state θ. The principal and the agent stay in the relationship if and only if $s(\theta) \geq \bar{s}$, where $\bar{s}$ is the expected joint surplus from being matched with a new partner. If $s(\theta) < \bar{s}$, the bonus payment is the same for all outcomes, and the agent doesn't put in any effort. □

When the state is specific to the principal-agent pair, they remain in the relationship if and only if the expected joint surplus is above the threshold. Since the states are observable and the principal and the agent maximize the joint surplus, the state in this period completely summarizes the expected joint surplus from the next period and on, and the exit behavior is determined by the realization of the state.

Contrary to the i.i.d. case, there is some degree of cooperation in any equilibrium, except for the degenerate case in which the expected joint surplus from the new distribution G dominates the expected joint surplus after any state. Even if the principal and the agent maximize the joint surplus in every relationship, the principal can induce the agent to exert effort in a good state.

6.3 Agent-Specific State

Next, consider the case in which the state is the type of the agent. It can be interpreted as the productivity of the agent. When the agent is matched with a new principal, the distribution of the state is determined by his type in the last period, which is the last realization of the state in the agent's previous relationship. In this case, I show the market for matching turns into the market for lemons; there cannot be a market for matching, and all principal-agent pairs stay in their relationship forever.

Proposition 16. *Suppose when a principal and an agent is matched, the initial state is drawn from the distribution $P(\cdot|\theta)$ where θ is the last realization of the state of the agent. The principal and the agent never exit the current relationship, and there is no market for rematching.*

Proof. From Proposition 1, we can focus on the joint surplus from the relationship, and the principal and the agent remain in the current relationship if and only if $s(\theta) \geq \bar{s}$. Let $\hat{\Theta} = \{\theta | s(\theta) < \bar{s}\} \subset \Theta$ be the set of states after which the principal and the agent exit the relationship. Let F be the distribution of the states in the given period. When the principal and the agent decide whether to stay in the relationship, the outside option must satisfy

$$\bar{s} = \int_{s \in \hat{\Theta}} s(\theta) dF,$$

which is a contradiction to the definition of $\hat{\Theta}$. Therefore, $\hat{\Theta}$ is degenerate and can only be $\emptyset$.

If the principal and the agent maximize the joint surplus when they are matched with new partners, the lowest type is indifferent between staying in and exiting the relationship. When the realized state is such, the parties cannot make any bonus payment, and the agent puts in no effort. If they don't maximize the joint surplus, even the lowest type prefers to stay in the current relationship. The principal and the agent never exit the relationship, and there is no market for rematching. □

Proposition 16 shows that if the underlying state is the type of the agent, the market for matching turns into a market for lemons, and there will not be a market. Only the lowest type can exist in the market, and all principal-agent pairs stay in the current relationship.

Note that the result doesn't depend on the strategies of the principal-agent pairs when matched with new partners. Proposition 16 holds even if the principal and the agent don't maximize the joint surplus in new relationships.

6.4 Macro Shock

This section considers a macro shock. The state is common to all principal-agent pairs. In this case, the principal cannot induce the agent to put in any effort, and cooperation is impossible.

Proposition 17. *Suppose the state is common to all principal-agent pairs. If the principal and the agent maximize the joint surplus in every relationship, the principal cannot induce the agent to put in any effort.*

Proof. The proof is the same as in the i.i.d. case. If the state is common to all principal-agent pairs, $s = \bar{s}$ in the dynamic enforcement constraint, and the principal pays the same bonus for all payments. The agent has no incentive to put in any effort. □

If the state is common to all principal-agent pairs, the expected joint surplus from the next period and on is the same whether they remain in the current relationship or are matched with new partners. Then, the principal and the agent have no incentive to pay the bonus payment, and without the bonus payments, cooperation is impossible.

6.5 Not Maximizing the Joint Surplus

This section discusses what happens when the principal and the agent don't maximize the joint surplus when they are matched with new partners. The degree of cooperation depends on the endogenous outside option determined in a market, and the maximum cooperation the principal and the agent can have weakly decreases with the outside option. The maximum cooperation is possible if they revert to the static equilibrium when matched with new partners. However, any equilibrium in which the pairs don't maximize the joint surplus is not renegotiation-proof.

Proposition 18. *The maximum joint surplus weakly decreases with the outside option $\bar{s}$ determined in an equilibrium. The maximum joint surplus is the largest if the principal and the agent revert to the static equilibrium when* *matched with new partners. Ex ante maximum joint surplus also weakly decreases with the outside option.*

Proof. We know from Corollary 1 that without a market, the joint surplus decreases with the outside option $\bar{s}$. When there is a market, if $s(\theta) < \bar{s}$, the principal and the agent cannot make any bonus payment in that period, the agent puts in no effort, and the parties exit the relationship at the end of the period. When $\bar{s}$ increases, the set of states after which the relationship ends increases. Increase in $\bar{s}$ also means that the bonus cap in a given state is lower, and the per period joint surplus the principal can induce is weakly lower. Together, the maximum joint surplus in every state weakly decreases with $\bar{s}$. Since the endogenous outside option is the minimum when the parties revert to the static equilibrium, the parties can attain the maximum joint surplus if they revert to the static equilibrium in new relationships. When the maximum joint surplus in each state weakly decreases with the outside option, ex ante maximum joint surplus also weakly decreases. □

We know from Proposition 1 that the distribution of joint surplus doesn't affect efficient contracting. If the parties are not maximizing the joint surplus, they can always renegotiate and redistribute the surplus. The only contracts that are renegotiation-proof are the ones that maximize the joint surplus in every relationship, and for such contracts, I have shown the following.

Remark 3. *When there is a market for matching, the persistence of states leads to very different outcomes from the i.i.d. states. If the principal and the agent always maximize the joint surplus, and if there is a frictionless market for matching, cooperation is impossible with i.i.d. states or macro shocks. On the contrary, there is always some degree of cooperation with relationship-specific states, and there is no market if the states are specific to the agent.*

Remark 4. *Ex ante joint surplus is strictly lower with the market, if the states are i.i.d. or common to all principal-agent pairs and if the principal and the agent always maximize the joint surplus. The market doesn't affect the ex ante joint surplus with agent-specific states. The effect of market on ex ante joint surplus is ambiguous for relationship-specific states.*

When the states are specific to the relationship, there are two effects of a market on ex ante joint surplus. There are some states in which the agent puts in no effort because the principal and the agent will exit the relationship, but they can also have a new draw instead of staying in the low states. The overall effect depends on which effect dominates.

7 Conclusion

I study relational contracts in a persistent environment in this paper. I show that when there is no asymmetric information about the state, history-independent contracts are optimal, and I characterize the necessary and sufficient condition to implement an effort schedule with history-independent contracts. These properties show that history-independent contracts are appropriate generalization of stationary contracts. However, stationary contracts are no longer optimal when the states are partially persistent. The necessary and sufficient condition to implement an effort schedule with stationary contracts is strictly stronger than the dynamic enforcement constraint. I also show that with endogenous states, the bonus cap depends on the realization of the state for the next period, and the second-best contract is not a step function anymore.

Suboptimality of stationary contracts means that the persistence of the underlying environment changes the optimal contract qualitatively. If the environment is persistent, providing bonus payments may not be sufficient, and the fixed wage may also have to vary with the state. Given that many compensation schemes have a constant fixed wage in every state, the firms will be strictly better off with state-dependent wages.

When the states are observable and follow a first-order Markov chain, the state in any given period is a sufficient statistic for the distribution of future states. In particular, the outcome doesn't have any information about the distribution of future states, and the principal can provide the incentives by the bonus payments in the given period. It is optimal to provide the same expected per period payoff in every state. If the continuation contract for a given state in some period provides the maximum joint surplus for the given state, the principal can provide the same continuation contract in every period for the given state. Since the agent gets the same expected payoff in all states, the agent's IC constraints are still satisfied when the principal replaces the continuation contract, and the optimal contract can be independent of history. The principal can also redistribute the surplus through the fixed wage, and we get the dynamic enforcement constraint as with i.i.d. states. An effort schedule can be implemented with history-independent contracts if and only if it satisfies the IC constraint and the dynamic enforcement constraint.

If the states are endogenous, the bonus cap also depends on the realization of the state for the next period, and there is no uniform bonus cap for the given state. The second-best contract reflects this bonus cap and is no longer a step function.

Persistent states can affect the relational contracts through two mechanisms. The persistence of the states implies that if the joint surplus depends on the state, the bonus cap also varies with the state, and the implementable level of effort depends on the state, even if the incentive provision for the given bonus cap is identical in each state. On the other hand, the incentive provision for the given bonus cap can also change with the state. If the joint surplus in the first best increases with the state, or if the implementable level of effort for given bonus cap increases with the state, the difference in the joint surplus between the first best and the second best decreases with the state. The principal prefers the relational contracts to full-commitment contracts only if the initial state is sufficiently high.

I show that the nature of the state has starkly different implications for the market when the principal and the agent can be randomly, anonymously, and costlessly matched with new partners. Cooperation is impossible if the states are i.i.d. and the parties maximize the joint surplus in every relationship. If the states are persistent, we get varying degree of cooperation depending on the type of the state. If it's agent-specific, the principal and the agent stay in the relationship forever, and there is no market. If it's relationship-specific, they exit the current relationship if and only if the expected joint surplus falls below some threshold. With macro shocks, cooperation is impossible, and the principal cannot induce any level of effort.

I have considered partially persistent environments where the states are observable and the persistence is of first-order. If the states are observable, both with exogenous and endogenous states, the optimal contract can be independent of history. However, if the states are unobservable, there can be information asymmetry between the principal and the agent about the future states. The belief about the agent's effort matters for the future, and the relational contract will likely have to take into account the private information. It will be interesting to study relational contracts and their implications for the market when the information about the future states is no longer symmetric.

A Proofs

Proof of Proposition 1. Consider the relational contract that provides s. The principal offers in the initial period $w(\theta_0), b(\phi_0)$, and if the agent accepts, he exerts effort $e(\theta_0)$. The continuation payoffs under the contract are denoted by $u(\phi_0)$ and $\pi(\phi_0)$, and the expected payoffs from the contract are u_0 and π_0. Without loss of generality, we can assume that off the equilibrium path, the parties revert to the static equilibrium of $(\bar{u}, \bar{\pi})$. The first period payment W is a function of ϕ_0.

The contract is self-enforcing if and only if the following conditions hold:

$$
\begin{aligned}
&(i)\ u_0 \geq \bar{u},\ \ \pi_0 \geq \bar{\pi},\\
&(ii)\ e(\theta_0) \in \arg\max_e \mathbb{E}_y[(1-\delta)W(\phi_0) + \delta u(\phi_0)|e,\theta_0] - c(e,\theta_0),\\
&(iii)\ b(\phi_0) + \frac{\delta}{1-\delta}u(\phi_0) \geq \frac{\delta}{1-\delta}\bar{u},\\
&\quad -b(\phi_0) + \frac{\delta}{1-\delta}\pi(\phi_0) \geq \frac{\delta}{1-\delta}\bar{\pi},
\end{aligned}
$$

and (iv) each continuation contract is self-enforcing.

Given any (u, π) such that $u \geq \bar{u},\ \pi \geq \bar{\pi},\ u + \pi = s$, the principal can offer the same $b(\phi_0)$ and continuation contracts and adjust $w(\theta_0)$ to

$$\hat{w}(\theta_0) \equiv w(\theta_0) + \frac{\pi - \pi_0}{1-\delta}.$$

The conditions are satisfied with the new contract, and it provides (u, π) as the expected payoffs. □

Proof of Proposition 2. Suppose a contract that maximizes the joint surplus provides w_t, b_t and the agent chooses e_t. The first step is to construct an alternative contract $\hat{w}_t$, $\hat{b}_t$ under which the agent chooses the same level of effort e_t and his expected payoff is constant in every state.

When the states are observable and exogenously given, the distribution of the states from period $t+1$ only depends on θ_t, and the outcome y_t doesn't carry any information about the future states. The principal can adjust the contingent payment b_t and keep the expected payoff in each state constant. Specifically, consider the following contract. Let $u_t(h^t, \phi_t)$ be the continuation value of the agent under the given contract, and define $\hat{w}_t$, $\hat{b}_t$ as the following:

$$\hat{b}_t(h^t, \phi_t) \equiv b_t(h^t, \phi_t) + \frac{\delta}{1-\delta}(u_t(h^t, \phi_t) - \bar{u}),$$
$$\hat{w}_t(h^t, \theta_t) \equiv \bar{u} - \mathbb{E}_{y_t}[\hat{b}_t(h^t, \phi_t) | e_t(h^t, \theta_t)].$$

From

$$\hat{b}_t(h^t, \phi_t) + \frac{\delta}{1-\delta}\bar{u} = b_t(h^t, \phi_t) + \frac{\delta}{1-\delta}u_t(h^t, \phi_t),$$

the agent chooses the same level of effort e_t under the new contract. The agent's expected payoff is $\bar{u}$ for all t, h^t, θ_t.

The next step is to show that we can choose $\tilde{w} : \Theta \to \mathbb{R}$, $\tilde{b} : \Phi \to \mathbb{R}$ such that the principal offers $\tilde{w}$, $\tilde{b}$ in every period. Consider $\hat{w}_t$ and $\hat{b}_t$. The agent's expected payoff is constant over all t, h^t, and θ_t, which implies that the agent's IC constraint is determined by the within period compensation scheme. Specifically, the agent chooses e such that

$$e_t(h^t, \theta_t) \in \arg\max_e \mathbb{E}_y[\hat{b}_t(h^t, \phi_t) | e, \theta_t] - c(e, \theta_t).$$

When the agent's IC constraints are myopic, the principal can replace a compensation scheme for any given period with another compensation scheme without affecting the incentives. The principal can also treat each θ separately, because the state is observable before the agent chooses the effort. Specifically, let $\tilde{b}$ be the compensation scheme that maximizes the expected per period joint surplus for state θ:

$$\begin{aligned} \tilde{b}(\theta, \cdot) &\equiv \arg\max_{\hat{b}_t(h^t,\theta,\cdot),h^t} \mathbb{E}_y[y|e_t(h^t,\theta),\theta] - c(e_t(h^t,\theta),\theta) \\ \text{s.t. } e_t(h^t,\theta) &\in \arg\max_e \mathbb{E}_y[\hat{b}_t(h^t,\theta,\cdot)|e,\theta] - c(e,\theta). \end{aligned}$$

If there's multiplicity of the compensation schemes, we can pick one without loss of generality.

Given $\tilde{b} : \Phi \to \mathbb{R}$, the agent chooses $e : \Theta \to \mathcal{E}$ such that

$$e(\theta) \in \arg\max_e \mathbb{E}_y[\tilde{b}(\phi)|e,\theta] - c(e,\theta).$$

Define $\tilde{w}$ as

$$\tilde{w}(\theta) \equiv \bar{u} - \mathbb{E}_y[\tilde{b}(\phi)|e(\theta),\theta],$$

and we have a history-independent contract that maximizes the expected joint surplus. By construction, it is self-enforcing, and it provides the same expected payoff to the agent in all $t,\ h^t,\ \theta_t$. Let s^* be the minimum expected per period joint surplus over the states under $\tilde{b},\ \tilde{w}$:

$$s^* \equiv \min_\theta \{\mathbb{E}_y[y|e(\theta),\theta] - c(e(\theta),\theta)\}.$$

The principal can adjust the fixed salary and can provide any u such that $\bar{u} \le u \le s^* - \bar{\pi}$ to the agent as the constant expected payoff. $\square$

Proof of Proposition 3. ($\Rightarrow$) Suppose $e(\theta)$ is implementable. Let $u(\theta)$ and $\pi(\theta)$ be the continuation value for the agent and the principal when the previous state was θ. The IC constraint has to be satisfied, and we also know that

$$\frac{\delta}{1-\delta}(\pi(\theta)-\bar{\pi}) \geq \sup_y b(\theta, y), \forall\theta, \tag{1}$$

$$\frac{\delta}{1-\delta}(u(\theta)-\bar{u}) \geq -\inf_y b(\theta, y), \forall\theta \tag{2}$$

have to hold. Adding the two inequalities, we have the dynamic enforcement constraint.

($\Leftarrow$) Suppose $W(\phi)$ and $e(\theta)$ satisfy the IC constraint and the dynamic enforcement constraint. Define

$$b(\phi) = W(\phi) - \inf_\phi W(\phi),$$
$$w(\theta) = \bar{u} - \mathbb{E}_y[W(\phi)|e(\theta), \theta],$$

and consider the history-independent contract with $w(\theta)$, $b(\phi)$ and $e(\theta)$. The parties revert to the static equilibrium if a deviation occurs. The agent receives $\bar{u}$ as expected payoff in each state, and the principal receives $\pi(\theta) = s(\theta) - \bar{u}$ if the previous state was θ. By the dynamic enforcement constraint, $s(\theta) \geq \bar{s}$ and $\pi(\theta) \geq \bar{\pi}$ for all θ. From the IC constraint, the agent chooses $e(\theta)$ in each state θ, and it can be verified that Inequalities (1) and (2) are satisfied. By construction, the contract is self-enforcing in every period. □

Proof of Corollary 1. From the dynamic enforcement constraint, the bonus cap decreases with the outside option $\bar{s}$. The maximum per period joint surplus weakly decreases with $\bar{s}$, which further suppresses the bonus cap through the expected joint surplus. Therefore, both the per period joint surplus and the expected joint surplus decrease with $\bar{s}$. □

Proof of Proposition 4. We know from Proposition 1 that we can focus on maximizing the joint surplus, and Proposition 2 implies that we can focus on history-independent contracts. By the Mirrlees-Rogerson constraints, we can replace the agent's IC constraint with the first-order condition. The optimal history-independent contract solves

$$\max_{e(\cdot),W(\cdot,\cdot)} \mathbb{E}_{\theta,y}[y - c|e(\theta), \theta]$$

subject to

$$\frac{d}{de}\{\mathbb{E}_y[W(\theta, y) - c(e, \theta)|e = e(\theta), \theta]\} = 0, \ \forall\theta,$$
$$\frac{\delta}{1-\delta}(s(\theta) - \bar{s}) \geq \sup_{\theta,y} W(\theta, y) - \inf_{\theta,y} W(\theta, y),$$
$$s(\theta_0) = (1-\delta)\mathbb{E}[\sum_{t=0} \delta^t \{d_t(y_t - c(e_t, \theta_t)) + (1 - d_t)\bar{s}\}|\theta_0].$$

From the Mirrlees-Rogerson constraints, the principal wants to maximize e when $e(\theta_t) < e^{FB}(\theta_t)$. We get

$$W(\theta, y) = \begin{cases} \bar{W}(\theta) & \text{if } y \geq y(\theta) \\ \underline{W}(\theta) & \text{if } y < y(\theta). \end{cases},$$

and f_e changes the sign at $y(\theta)$, and $\bar{W}(\theta) = \underline{W}(\theta) + \frac{\delta}{1-\delta}(s(\theta) - \bar{s})$. □

Proof of Proposition 5. Consider the relational contract that provides s. The principal offers in the initial period $w(\theta_0)$, $b(\phi_0)$, and if the agent accepts, he exerts effort $e(\theta_0)$. The continuation payoffs under the contract are denoted by $u(\phi_0)$ and $\pi(\phi_0)$, and the expected payoffs from the contract are u_0 and π_0. Without loss of generality, we can assume that off the equilibrium path, the parties revert to the static equilibrium of $(\bar{u}, \bar{\pi})$. The first period payment W is a function of ϕ_0.

The contract is self-enforcing if and only if the following conditions hold:

$$(i)\ u_0 \geq \bar{u},\ \ \pi_0 \geq \bar{\pi},$$

$$(ii)\ e(\theta_0) \in \arg\max_{e} \mathbb{E}_{\theta_1}[(1-\delta)W(\phi_0) + \delta u(\phi_0)|e,\theta_0] - c(e,\theta_0),$$

$$(iii)\ b(\phi_0) + \frac{\delta}{1-\delta}u(\phi_0) \geq \frac{\delta}{1-\delta}\bar{u},$$

$$-b(\phi_0) + \frac{\delta}{1-\delta}\pi(\phi_0) \geq \frac{\delta}{1-\delta}\bar{\pi},$$

and (iv) each continuation contract is self-enforcing.

Given any (u,π) such that $u \geq \bar{u},\ \pi \geq \bar{\pi},\ u+\pi = s$, the principal can offer the same $b(\phi_0)$ and continuation contracts and adjust $w(\theta_0)$ to

$$\hat{w}(\theta_0) \equiv w(\theta_0) + \frac{\pi - \pi_0}{1-\delta}.$$

The conditions are satisfied with the new contract, and it provides (u,π) as the expected payoffs. □

Proof of Proposition 6. Suppose a contract that maximizes the joint surplus provides $w_t,\ b_t$ and the agent chooses e_t. The first step is to construct an alternative contract $\hat{w}_t,\ \hat{b}_t$ under which the agent chooses the same level of effort e_t and his expected payoff is constant in every state.

When the states are observable, the distribution of the states from period $t+1$ only depends on θ_{t+1}, which is observed before the principal makes payments in period t. The principal can adjust the contingent payment b_t and keep the expected payoff in each state constant. Specifically, consider the following contract. Let $u_t(h^t,\phi_t)$ be the continuation value of the agent under the given contract, and define $\hat{w}_t,\ \hat{b}_t$ as the following:

$$\hat{b}_t(h^t,\phi_t) \equiv b_t(h^t,\phi_t) + \frac{\delta}{1-\delta}(u_t(h^t,\phi_t) - \bar{u}),$$

$$\hat{w}_t(h^t,\theta_t) \equiv \bar{u} - \mathbb{E}_{\theta_{t+1}}[\hat{b}_t(h^t,\phi_t)|e_t(h^t,\theta_t)].$$

From

$$\hat{b}_t(h^t, \phi_t) + \frac{\delta}{1-\delta}\bar{u} = b_t(h^t, \phi_t) + \frac{\delta}{1-\delta}u_t(h^t, \phi_t),$$

the agent chooses the same level of effort e_t under the new contract. The agent's expected payoff is $\bar{u}$ for all $t,\ h^t,\ \theta_t$.

The next step is to show that we can choose $\tilde{w} : \Theta \to \mathbb{R},\ \tilde{b} : \Phi \to \mathbb{R}$ such that the principal offers $\tilde{w},\ \tilde{b}$ in every period. Consider $\hat{w}_t$ and $\hat{b}_t$. The agent's expected payoff is constant over all $t,\ h^t$, and θ_t, which implies that the agent's IC constraint is determined by the within period compensation scheme. Specifically, the agent chooses e such that

$$e_t(h^t, \theta_t) \in \arg\max_e \mathbb{E}_{\theta_{t+1}}[\hat{b}_t(h^t, \phi_t)|e, \theta_t] - c(e, \theta_t).$$

When the agent's IC constraints are myopic, the principal can replace a compensation scheme for any given period with another compensation scheme without affecting the incentives. Let $\tilde{b}$ be the compensation scheme that maximizes the expected per period joint surplus for state θ:

$$\begin{aligned}
\tilde{b}(\theta, \cdot) &\equiv \arg\max\nolimits_{\hat{b}_t(h^t,\theta,\cdot),h^t} \mathbb{E}_{y,\theta'}[\delta y(\theta')|e_t(h^t, \theta), \theta] - c(e_t(h^t, \theta), \theta) \\
\text{s.t. }\ & e_t(h^t, \theta) \in \arg\max_e \mathbb{E}_{y,\theta'}[\hat{b}_t(h^t, \theta, \cdot)|e, \theta] - c(e, \theta).
\end{aligned}$$

If there's multiplicity of the compensation schemes, we can pick one without loss of generality.

Given $\tilde{b} : \Phi \to \mathbb{R}$, the agent chooses $e : \Theta \to \mathcal{E}$ such that

$$e(\theta) \in \arg\max_e \mathbb{E}_{y,\theta'}[\tilde{b}(\phi)|e, \theta] - c(e, \theta).$$

Define $\tilde{w}$ as

$$\tilde{w}(\theta) \equiv \bar{u} - \mathbb{E}_{y,\theta'}[\tilde{b}(\phi)|e(\theta), \theta],$$

and we have a history-independent contract that maximizes the expected joint surplus. By construction, it is self-enforcing, and it provides the same expected payoff to the agent in all $t,\ h^t,\ \theta_t$. □

Proof of Proposition 7. ($\Rightarrow$) Suppose $e(\theta)$ is implementable with a history-independent contract. If $u(\theta)$ is not constant across the states, we can define another history-independent contract as follows:

$$\begin{aligned}\hat{w}(\theta) &= w(\theta),\\ \hat{b}(\phi) &= b(\theta) + \frac{\delta}{1-\delta}(u(\theta) - \bar{u}).\end{aligned}$$

The new contract is history-independent, and it implements $e(\theta)$ since the agent's incentives are not affected. The continuation value for the agent is $\bar{u}$ in every state. The IC constraint has to be satisfied, and we know that

$$\begin{aligned}\frac{\delta}{1-\delta}(s(\theta') - \bar{s}) = \frac{\delta}{1-\delta}(\hat{\pi}(\theta') - \bar{\pi}) &\geq b(\theta, y, \theta'), \forall \theta, \theta', y,\\ 0 = \frac{\delta}{1-\delta}(\hat{u}(\theta') - \bar{u}) &\geq -b(\theta, y, \theta'), \forall \theta, \theta', y\end{aligned}$$

have to hold, where $\hat{u}(\theta)$ and $\hat{\pi}(\theta)$ are the continuation values for the agent and the principal given θ. Combining the two inequalities, we get the dynamic enforcement constraint.

($\Leftarrow$) Suppose $b(\phi)$ and $e(\theta)$ satisfy the conditions. Let w be defined as follows:

$$w(\theta) = \bar{u} - \mathbb{E}_{y,\theta'}[b(\phi)].$$

The agent's continuation value is $\bar{u}$ in every state, and the principal receives $s(\theta) - \bar{u} > \bar{\pi}$; the parties are willing to initiate the relationship. From the IC constraint, the agent chooses $e(\theta)$. The parties revert to the static equilibrium if a deviation occurs, and from the dynamic enforcement constraint, we have the following:

$$\begin{aligned}\frac{\delta}{1-\delta}(s(\theta') - \bar{s}) = \frac{\delta}{1-\delta}(\pi(\theta') - \bar{\pi}) &\geq b(\theta, y, \theta'), \forall \theta, \theta', y,\\ 0 = \frac{\delta}{1-\delta}(u(\theta') - \bar{u}) &\geq -b(\theta, y, \theta'), \forall \theta, \theta', y\end{aligned}$$

The parties make the bonus payments. By construction, the contract is self-enforcing in every period. $\square$

Proof of Theorem 2. ($\Rightarrow$) Suppose $e(\theta)$ can be implemented with a stationary contract $w \in \mathbb{R}$, $b : \Phi \to \mathbb{R}$. We need both parties to make the bonus payment:

$$\frac{\delta}{1-\delta}(\pi(\theta') - \bar{\pi}) \geq b(\phi) = W(\phi) - w,$$
$$\frac{\delta}{1-\delta}(u(\theta') - \bar{u}) \geq -b(\phi) = -(W(\phi) - w).$$

We can combine the inequalities to

$$\frac{\delta}{1-\delta}(u(\theta') - \bar{u}) + W(\phi) \geq w \geq -\frac{\delta}{1-\delta}(\pi(\theta') - \bar{\pi}) + W(\phi).$$

Since the inequality holds for all ϕ, we get

$$\inf_{\phi}[\frac{\delta}{1-\delta}(u(\theta') - \bar{u}) + W(\phi)] + \inf_{\phi}[\frac{\delta}{1-\delta}(\pi(\theta') - \bar{\pi}) - W(\phi)] \geq 0.$$

The agent's IC constraint has to be satisfied, and the continuation values for both parties are weakly greater than the outside options.

($\Leftarrow$) When the conditions are satisfied, the agent chooses $e(\theta)$ in each θ, and the parties are willing to initiate the relationship. When the second condition is satisfied, we can pick w such that

$$\inf_{\phi}[\frac{\delta}{1-\delta}(u(\theta') - \bar{u}) + W(\phi)] \geq w \geq -\inf_{\phi}[\frac{\delta}{1-\delta}(\pi(\theta') - \bar{\pi}) - W(\phi)],$$

and the parties will make the bonus payment. By construction, the contract is self-enforcing in every period. $\square$

Proof of Theorem 3. We know from Proposition 5 that we can focus on maximizing the joint surplus, and Proposition 6 implies that we can focus on history-independent contracts. By the Mirrlees-Rogerson constraints, we can replace the agent's IC constraint with the first-order condition. The optimal history-independent contract solves

$$\max_{e(\cdot),W(\cdot)} \mathbb{E}_{\theta,y,\theta'}[\delta y(\theta') - c(e,\theta)|e(\theta),\theta]$$

subject to

$$\begin{aligned}
&\frac{d}{de}\{\mathbb{E}_{y,\theta'}[W(\theta,y,\theta') - c(e,\theta)|e = e(\theta),\theta]\} = 0,\ \forall\theta, \\
&0 \leq b(\theta,y,\theta') \leq \frac{\delta}{1-\delta}(s(\theta') - \bar{s}), \\
&s(\theta_0) = (1-\delta)\mathbb{E}[\sum_{t=0} \delta^t\{d_t(y_t - c(e_t,\theta_t)) + (1-d_t)\bar{s}\}|\theta_0].
\end{aligned}$$

From the Mirrlees-Rogerson constraints, the principal wants to maximize e when $e(\theta_t) < e^{FB}(\theta_t)$. We get

$$W(\theta,y,\theta') = \begin{cases} \frac{\delta}{1-\delta}(s(\theta') - \bar{s}) & \text{if } \theta' \geq g(\theta) \\ 0 & \text{if } \theta' < g(\theta). \end{cases},$$

and f_e changes the sign at $g(\theta)$. □

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