

# DI–economies: Contracts, Incentive Compatibility and Learning under Maximin Preferences

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## Introduction

This is a purely theoretical project aimed to study economic models with asymmetrically distributed information. The construction and study of the models of this type is a problem of great theoretical significance and develops microeconomic theory and the theory of general economic equilibrium.

Our analysis is based on a contractual approach proposed and developed by Marakulin in a series of papers of the early 21st century. Studies have shown that this approach works very efficiently and has great potential for further development. Its main advantage is that it presents a new model of perfect competition, the simplest among known in the literature and expressed in a flexible form in the permitting of partial breaking of contracts (barter, production, etc.) and subsequent consideration of stable webs of contracts. In this way, not addressing to value categories, one can describe equilibrium allocation in many economic models: pure exchange, with the production of Arrow—Debreu type and public goods, incomplete markets and etc. Thus, the project is developing previously started investigations of economies with asymmetrically distributed information in the general context of the contractual approach.

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# 1 DI-economies: new features and specification

Economic modeling under differentiated information has the following important specific features significantly affecting their mathematical analysis:

- Information is modeled as a *partition* of the set  $\Omega$  of elementary states of the future. Each agent has its own partition and can distinguish only its elements (subsets of  $\Omega$ ), but is unable to distinguish states of any given element of the partition.

- Individuals consume “contingent” commodity bundles, in which consumption is specified for each of the future states of nature. This bundle (map) will be *information admissible* if it will have equal quantities in indistinguishable states (measurability regarding information partition).

- Information can be dynamically changed, which affects the stability of resource allocation. Typically it is assumed that the information is stored and this expands the space of information feasible contingent commodity bundles. There are theoretical structures where the change of information is described endogenously according to a *rule of information sharing*.

- Agents can extract information from the price distribution and through contractual interaction. In this way, classical concept of *REE-equilibrium* (rational expectations equilibrium) and a new contractual *Interim Equilibrium* are introduced.

- Along with the usual motives of personal gain (no other choices that benefit everyone member of the coalition), the stability of an allocation may be at risk of information cheating (fraud), when for a group of agents it is profitable to send other individuals false signal about the realized state of nature; this will only work if the agents themselves can not distinguish the true state from the false one. Allocations stable with respect to these threats are called *incentive compatible*.

Models with asymmetrically informed agents in framework of *General Equilibrium Theory* started to develop from the seminal Radner’s works late 60’s and 70’s of the last century, see ?. Here the standard notion of Walrasian equilibrium was generalized to the case of DI-model (WEE-equilibrium) and a fundamentally new concept of *Rational Expectations Equilibrium* was suggested. The first one is almost a standard definition of competitive equilibrium, written in terms of the expected utilities, but under the requirements of information permissibility (measurability) for consumption bundles. This is an equilibrium of “ex ante” type, *i.e.* it is assumed that all economic activities are planned today and tomorrow, after the resolution of uncertainty, all commitments will be certainly fulfilled. The second case is fundamentally different: it is assumed that getting the price signals agents can extract information from them and form finer partitions; then for each element of this new partition the optimal plan of actions (purchases and sales) is specified and this will be implemented if “tomorrow” individual realizes “the event from the element of informational partition”. Of course, the set of individual plans should be jointly implemented, that is plans must be balanced.

econ.\coal.	$\wedge_S k_S^i(\mathbb{P})$	$\wedge_S P_i$	$P_i$	$\{k_S^i(\mathbb{P})\}$	$P_i^{lim}$	$\vee_S P_i$	$\vee_S k_S^i(\mathbb{P})$	author
$\wedge_{\mathcal{I}} P_i$		strong coarse	$\alpha$			$\beta$		Yannelis
$P_i$		coarse	private			fine		Yannelis
$\vee_{\mathcal{I}} P_i$		$\gamma$	$\delta$			weak fine		Yannelis
$\forall f$ -allocations		coarse				fine		Wilson
$\wedge_{\mathcal{I}} k_{\mathcal{I}}^i(\mathbb{P})$	coarse							Allen
$\{k_{\mathcal{I}}^i(\mathbb{P})\}$							fine	Allen
$\{P_i^{max}\}$				k-core				Schwalbe
$\{P_i^{lim}\}$				k-limit	limit			Marakulin

Table 1: *Known in literature variety of cores for DI-economies*

Along with the competitive equilibrium the theory of DI-economy introduces and analyzes original core concepts—specific form of *cooperative equilibrium*, where the final allocation is stable with respect to any intention of the coalition (the group) individuals to form an autonomous subeconomy. However, the existing literature is extremely diversified—there is a huge amount of reasonable alternative definitions of the core. Due to the specific information and the ability of individuals to distinguish between states of world these variants are differentiated, mainly in two dimensions. First of all one needs to determine which allocations are information admissible (measurability regarding the information partition) and which are not. Often these requirements for the final allocation of the economy as a whole differ from the similar requirements applied for dominating via coalition allocations. Typical choices are:

- Common knowledge—each individual is able to understand the consumption of any other individual (infimum of individual partitions).
- Private knowledge—individuals are able to understand their own consumption.
- Perfect knowledge—information is pulled, that is each individual is endowed with the ability to distinguish everything that distinguishes any other agent (supremum of individual partitions).
- It is also applied intermediate solutions produced via the rules of information sharing.

Next, for each type of core the measurability of the corresponding allocations is specified. For instance, if we use a private knowledge as for the dominating via coalitions and for the final allocation, then we come to the concept of *Private Core* by Yannelis; *Coarse Core* is obtained when one uses common knowledge for the final and dominating allocations, etc. Available in literature (not all!) and other possible variants of core definitions are presented in the Table ???. Here  $P_i$  is an information partition of agent  $i$ ,  $\wedge$  and  $\vee$  denote infimum and supremum,  $k_S^i(\mathbb{P})$  is a partition which agent  $i$  can apply according to the rule  $k_S$  of information sharing ( $S \subseteq \mathcal{I}$  is a coalition,  $\mathcal{I} = \{1, \dots, n\}$  is a set of all individuals). The columns of the table correspond to the requirements for coalition dominance, similarly rows for the final allocations.

The non-emptiness of the core of any type, and thus, the validness of the proposed solutions is a matter of the theorems, which generalizes the known Scarf's theorem. From the information point of view, the answer is the following: allocations under consideration would be measurable with respect to information structure finer than the information used to coalitional domination. Now the critical question is the following.

*What the core concept (available in the literature or new) is the most theoretically adequate for DI-economy? This should be the incentive compatible concept (stability with respect to the information of fraud)! Which concept of equilibrium does it meet?*

Contractual approach, which we commonly use in our study allows significantly progress in the theoretical analysis of the problem.

## 2 DI-economies: preliminary results

From the information point of view DI-economy potentially can have two types of solutions: *ex ante* (all ahead of time with the full responsibility of individuals to the obligations assumed), and *interim* (optional agreement, everything can be replayed at the last moment). In these cases the application of contractual analysis leads us to the following conclusions.

### 2.1 Ex ante

The situation is similar to classical markets where informational specification is almost not taken into account. If the information exchange is not possible, then the most appropriate are mutually complementary concepts of *Private Core* and *(quasi-)Equilibrium* by Yannelis. The main argument in favor of this stands the coincidence of core and equilibria under perfect competition conditions, that can also be established applying the contractual approach.

If information exchange is allowed, then endogenously given a *rule of information sharing* by Allen is its adequate model: it can be used to define almost all known in literature versus of the core. Many of these concepts can be represented in terms of *limit information* which individuals are able to reach as a result of the information exchange, simultaneously going on (iteratively) with the contractual interaction.

If this rule of information sharing is monotonic on the set of information structures, then the limit information does not depend on the order of agents' contractual interactions (?) and is uniquely determined. In this case, the concepts of private core and (quasi-)equilibrium are again the most appropriate types of stable *ex ante* agreement, but now they are specified for a limit information.

## 2.2 Interim notions

The only classical concept of this type is known as *Rational Expectations Equilibrium*. This concept qualifies the possibility of individuals to extract information from a price distribution and, therefore, exchange of information is going on and is involved in reaching of equilibrium. However, what type of core is it correspond? How is the exchange of information going and what is the incentive for it? These are the most important theoretical questions that needed to be resolved. The dynamic views involved into the contract-based approach help us in their analysis.

The critical property is that, unlike concepts like “ex ante”, now the contractual process can continue up until complete resolution of uncertainty. Specifically, this moment comes when nature has already implemented some state  $\omega$ , but agents are still able to understand it only as an element of their information partition (they can not distinguish it from other  $\omega' \in P_i(\omega)$ ). What a stable agreement is now as a result of contractual process? Here all individuals are aware that the contractual process can only end in a moment when uncertainty is resolved, and even before that. They will be interested in different consumption bundles specified their information partitions. Thus agent’s interests vary depending on the implementation of the different elements of information partition, and thus individual receives several realizations of his/her future consumption and utility evaluation. Moreover, each of these options is important for the agent. Thus, instead of one individual in the contractual process are involved different types of its “informational implementation” (faces), one can say that the agents multiply. Formally it means that the economy is transformed to form in which the set of agents is described as a set

$$\mathfrak{S} = \{(i, E) \mid i \in \mathcal{I}, E \in P_i\}.$$

Moreover, these “multiplied” agents are able to consume only in those states of nature, that correspond to their information type, *i.e.*  $(i, E) \in \mathfrak{S}$  consumes a bundle  $x_i^E \in \mathbb{R}_+^l$  in every of possible states  $\omega \in E$  and can not consume anything in states outside of  $E$ . What does it mean for contractual process and which contracts these agents are able to conclude?

As soon as  $(i, E)$  exists only if the event  $E \in P_i$  is implemented, then it is should be reflected in the contracts. We set

$$v_i^E : \Omega \rightarrow \mathbb{R}^l : v_i^E(\omega) = v_i^E(\omega'), \forall \omega, \omega' \in E \ \& \ v_i^E(\omega) = 0, \forall \omega'' \in \Omega \setminus E.$$

Now collection  $(v_i^E)_{\mathfrak{S}}$  can be treated as a contract in a new economy with informational types of agents subject to  $\sum_{\mathfrak{S}} v_i^E = 0$ .

Applying contracts of this type in contractual process, the economy should arrive in at least steady state defined as the allocation of the core of the economy with differentiated agents  $\mathfrak{S}$ . These allocations have the following type of stability: there is no coalition  $S \subseteq \mathcal{I}$  and event  $A \subseteq \Omega$  such that all agents  $(i, E)$ , satisfying  $i \in S$ ,

$E \subseteq A$  (these are the “informational faces” of individuals from  $S$  at event  $A$ ), which are able breaking if necessary old and entering into new contracts, to become better off. If this is not fulfilled, the contractual process can not stop because there is always a group of individuals which are interested in improving their situation and allocation will be changed. However, will it be the final point? Most probably not.

Indeed, at the time when tomorrow is already came and the nature is realized an event  $\omega \in \Omega$ , but the agents are still able to understand it only in the form of events  $P_i(\omega)$  (an element of partition including  $\omega$ ), they do not need to worry about consumption in states outside of  $P_i(\omega)$  and the utility level of individuals  $(i, E)$  realized for  $E \subseteq \Omega \setminus P_i(\omega)$  should not affect the domination. This type of stability corresponds to the concept of *interim core*. However, do allocations of this type exist and is economy capable of to achieve them?

The answer to this question is hid in the dynamic nature of the contractual process, which allows the sharing of information among economic agents. However, it is an act purely voluntary and one can reasonable assume that agents do it only based on the personal gain. However, that can serve as an incentive to disclose information? Only the opportunity to sign a new mutually beneficial contract (or break an old one)!

Assume that the process has reached a quasi-equilibrium in the model with differentiated agents—this is an element of the core with the additional possibility to partially break contracts. Now there are two possibilities:

(i) There is a couple of individuals that can enter into a new mutually beneficial contract, provided that one of them to share information with another (learn him/her to distinguish an event). Then contractual process (for differentiated agents) is further developed under a new information knowledges.

(ii) Even after the sharing of information agents are not able to enter into a new mutually beneficial contract. Now that’s a real final agreement and it is remarkable that in this way the economy arrives to the interim (quasi-)equilibrium, which is an element of the interim contractual core and meets REE-equilibrium.

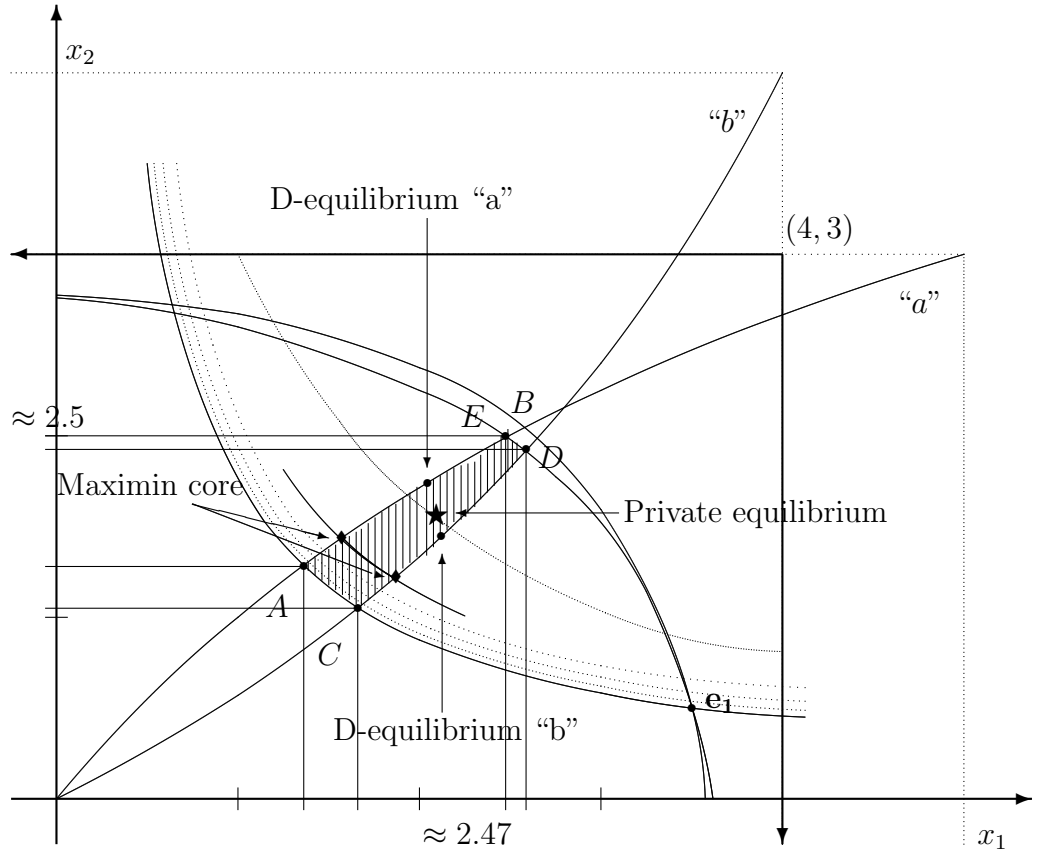
### 2.3 Example $2 \times 2 \times 2$ —economy

The basic ideas of the approach and the results of analysis of economy with differentiated agents can be illustrated by the following example.

Suppose we have two agents  $i = 1, 2$ , two states of nature  $\Omega = \{a, b\}$  and two physically different goods; here  $\mathbb{R}^2 \times \mathbb{R}^2 = \mathbb{R}^4$  is the space of contingent goods. Suppose the 1st individual is not able to distinguish anything, and 2nd in contrast, understands all events, *i.e.*

$$P_1 = \{\Omega\} \text{ and } P_2 = \{\{a\}, \{b\}\}.$$

Agents can consume the products in any non-negative quantities and their preferences are described via the following (logarithmic) utility functions.



*D-core: shaded curved area ACDE*

$$u_x = \ln(x_1(a)) + \ln(x_2(a)) + \ln(x_1(b)) + \ln(x_2(b)),$$

$$u_y = 2 \ln(y_1(a)) + \ln(y_2(a)) + \ln(y_1(b)) + 2 \ln(y_2(b)),$$

where “ $x$ ” corresponds to 1st agent consumption and “ $y$ ” denotes 2nd. Similarly, for the initial endowments:

$$\mathbf{e}_x = \left( \left( 3\frac{1}{2}, \frac{1}{2} \right)_a, \left( 3\frac{1}{2}, \frac{1}{2} \right)_b \right),$$

$$\mathbf{e}_y = \left( \left( 1\frac{1}{2}, 2\frac{1}{2} \right)_a, \left( \frac{1}{2}, 3\frac{1}{2} \right)_b \right).$$

Hence one can find the aggregate endowments of products in the different states of the world.

$$\bar{\mathbf{e}}(a) = (5, 3), \quad \bar{\mathbf{e}}(b) = (4, 4).$$

Next, consider a graphical representation of the core and equilibria in the framework of a generalized “Edgeworth box”. Here the coordinate system with the origin at the lower left corner and the axes are reaching the right and upward corresponds to the consumption of 1st agent, the figure shows its initial endowments and indifference curves (points with the same utility)—curves convex downward.

The upper right corner at the point (4, 3) is the origin of the coordinate system used for 2nd agent. The coordinate axes are going down left and right. Any point of the rectangle obtained as the intersection of the two axes of the coordinate system

corresponding to the consumption of individuals (in their system), and allocation as a whole (by balance constraints) and taking into account information measurability of 1st individual consumption. Thin strokes are also applied to denote two coordinate systems corresponding to states “ $a$ ” and “ $b$ ”, their intersection implements coordinates of 2nd individual. These systems correspond to agents  $(2, a)$  and  $(2, b)$ , their indifference curves are convex upward in the figure.

The two curves that connect the origin of the 1st to the origins of “ $a$ ” and “ $b$ ” represent variants of the Pareto frontier as if there is no other state of nature. The area between these two curves, limited also by indifference curves emanating from the point of initial endowments  $e_1$ , a shaded area with extremities  $ACDE$ , presents core allocations of model with differentiated agents. Two black fat points are D-(quasi)equilibrium allocations, but just one of them, corresponding to the state “ $b$ ”, is stable relative to the partial breaking of contracts (has a chance of continued existence). The asterisk in the center denotes the private equilibrium.

So, if the information exchange does not occur, from a theoretical point of view the economy could realized two equilibrium outcomes:

(i) private equilibrium (ex ante)  $x \approx (\frac{25}{12}, \frac{25}{16})$ —if individuals trust each other and always perform their commitments;

(ii) D-(quasi)equilibrium “ $b$ ” (interim)  $x = (\frac{36}{17}, \frac{36}{25})$ —if individuals do not incur obligations and are always ready to change the arrangements.

However, our contractual process may not be completed now, it will be so if for the D-quasi-equilibrium 2nd agent will think it pays him to share the information with the first agent: if the 1st will learn to distinguish between “ $a$ ” and “ $b$ ” (market falls), then it will allow him to enter into a new mutually beneficial contract. In order to justify this for the market “ $a$ ” it is sufficient to calculate the gradients of utilities  $\nabla_a u_1(x(a), x(b))$  and  $\nabla_a u_2(y(a), y(b))$  and make sure that they are not collinear, and similarly for the market “ $b$ ”. As a result of this second stage of the contracting process economy will come to an allocation wherein each of markets will be realized usual Walrasian equilibrium.

### 3 Incentive compatibility: new results

The term “incentive compatibility” in DI-economics context is treated as a solution property such that acting individually or together (in a coalition), the agents are not able to get a profit from cheating with information (lie). In modern literature, this requirement is seen as critical. However, the available in the literature definitions do not take into account the fact that no lie is bad in itself, but the damage that it causes members of the society which are not involved in the fraudulent conspiracy. The effect of utility improving for the members of a coalition associated with the information sharing which expands the opportunities to find a mutually beneficial



arrangement, is often interpreted as fraud—despite the fact that members of the complementary coalition suffered no damage. Correct definition may be represented as follows. A solution concept is incentive compatible if the following is impossible.

(i) There are states  $a, b \in \Omega$  such that for a coalition  $S \subseteq \mathcal{I}$ , which members can understand that state  $a$  is realized (*i.e.*,  $\{a\} = \cap_S P_i(a)$ ), but complementing coalition  $T = \mathcal{I} \setminus S$  members are not able to distinguish between “ $a$ ” and “ $b$ ”. It is profitable for every member of  $S$  to send the false signal “ $b$ ” instead of true “ $a$ ” (possible after a new contract for  $S$  is concluded).

(ii) If (i) is implemented, at least one member of the complementary coalition  $T$  is worse off, *i.e.*, his/her utility in true state “ $a$ ” is lower as a result of contract implementation for the false state “ $b$ ”.

In known literature definitions requirement (ii) is not applied, but its addition is of theoretical significance: if the efficiency is treated in the limited framework of measurable contracts, then there is no its conflict with incentives—each member of the complementing coalition is not able to distinguish false signal from the true one, but it does not affect the performance of the contract and the final result—after final authorization of uncertainty the result for the agent will be the same. Thus, in presented treatment *private* contractual core and equilibrium, solutions for a model with differentiated agents and interim concepts of the previous section are transfer coalitionally incentive compatible. According to presented in the literature definition it is not so and, for example, private core and equilibrium are just *individually* incentive compatible (used for singleton coalitions), or in a slightly more general case, when all members of the coalition well understand the state implemented ( $P_i(a) = \{a\} \forall i \in S$ ), but these are the cases when the coalition does not need to exchange information. Note that the measurability of contracts is a sufficient but not a necessary property for incentive compatibility. In our opinion, the two effects—fraud with information and effective information sharing—to be split and analyze, respectively.

Let  $\Omega = \{a, b, c\}$  and there are three agents with the information:

$$P_1 = \{\{a, b, c\}\}, \quad P_2 = \{\{a\}, \{b, c\}\}, \quad P_3 = \{\{a\}, \{b\}, \{c\}\}.$$

Suppose endowments and a posteriori utility 2nd individual in the states “ $a$ ” and “ $b$ ” are equal and the same holds for the third agent. Suppose that preferences and endowments in the state “ $c$ ” are such that the Pareto optimal contracts in the event  $\{b, c\}$  between 2nd and 3rd agents differ from Pareto border in the state “ $a$ ”. Then if the state “ $b$ ” is implemented and after the false signal “ $a$ ” from the coalition  $S = \{2, 3\}$ , members of  $S$  can win by definition (i)—without damage to the 1st agent ((ii) is violated). It can happen, for example, in a private equilibrium...

## 4 Ambiguity and maximin expected utilities

Here the term “ambiguity” means that individuals not only know what state will happen tomorrow, but also they do not know the likelihood of its implementation: they can only estimate the probability of some events which are the elements of agent’s informational partition. Thus, this is an uncertainty of a higher level and now even formally one can not apply the expected utilities which one can find in most of current literature.

This can be achieved, for example, for *maximin* preferences. Preferences of this type were introduced and axiomatized in ?, and then in (?) they were reformulated as presented below and applied for DI-economies.

• *Let the agents of DI-economy have a posteriori utility functions  $u_i : \Omega \times \mathbb{R}^l \rightarrow \mathbb{R}$  and let individual probability distributions  $\mu_i(\cdot)$  be associated with the information partitions  $P_i$ ,  $i \in \mathcal{I}$ .*

**Maximin expected preferences** are defined as follows

$$U_i^{min}(x_i) = \sum_{E \in P_i} \left( \min_{\omega \in E} u_i(\omega, x_i(\omega)) \right) \mu_i(E), \quad x_i : \Omega \rightarrow \mathbb{R}_+^l.$$

In substantial terms maximin preferences correspond to the cautious behavior of economic agents, *i.e.*, for a pessimistic perception of future events that is valued as a guaranteed minimum: it seems to be a plausible scenario.

From the modeling point of view a most relevant option of maximin preferences is a special case of a posteriori preferences when they are identical for agent’s indistinguishable states of the world. This is a case where a posteriori utility function  $u_i(\cdot, y)$  is measurable regarding the agent’s information partition for any fixed consumption  $y \in \mathbb{R}_+^l$  (*i.e.* it is a constant for the elements of partition). Indeed, the minimum in the latter formula can be correctly interpreted only in the case of comparable values, for example, it can be money. However even if it is so it is not clear why for the same consumption in indistinguishable for the individual states one attaches the different (utility) value—no reason for this, for beginning an agent needs at least to understand what the difference is...

The main advantage of these preferences and a subject of theoretical interest is presented in the Theorem below.

• *Core allocations of economy  $\mathcal{E}^{di}$ , in which all agents are equipped with maximin preferences, form **maximin core**  $\mathcal{C}^{min}(\mathcal{E}^{di})$ ; these allocations are not assumed to be measurable.*

**Theorem.** *Let a posteriori utilities  $u_i : \Omega \times \mathbb{R}_+^l \rightarrow \mathbb{R}$  of all individuals obey*

- (i)  $\forall \omega \in \Omega$   $u_i(\omega, \cdot)$  *is strictly monotonic for each commodity,*
- (ii)  $\forall y \in \mathbb{R}_+^l$   $u_i(\cdot, y)$  *and initial endowments  $\mathbf{e}_i(\cdot)$  are measurable subject to individual information.*

Then every allocation of maximin core  $\mathcal{C}^{min}(\mathcal{E}^{di})$  is **transfer coalitionally incentive compatible**.

An interesting property of maximin core elements is that they also have specific measurability relative to the individual information, but now it is the measurability of utilities.

**Proposition.** *In theorem's conditions assume in addition that for each state of nature there is its distinguishing individual, i.e.  $\forall \omega \in \Omega \exists i \in \mathcal{I} \mid P_i(\omega) = \{\omega\}$ .*

*Then for  $x = (x_i)_{i \in \mathcal{I}} \in \mathcal{C}^{min}(\mathcal{E}^{di})$ ,  $\forall i \in \mathcal{I}$  function  $u_i(\omega, x_i(\omega))$  is measurable relative to  $P_i$ , i.e.  $u_i(\omega, x_i(\omega)) = u_i(\omega', x_i(\omega'))$ ,  $\omega' \in P_i(\omega)$ ,  $\forall \omega, \omega' \in \Omega$ .*

This observation has theoretical significance and will help us to specify how contractual process is going on for maximin preferences. Let us examine this problem in more detail, but first notice that now we have got the possibility to demonstrate the elements of maximin core in the previous Figure—in spite of the fact that contingent commodity space is 4-dimensional. One can observe that 1st agent consumption can be presented via two points which have to belong to Pareto boundaries for “a” and “b” and, simultaneously, to a common indifference curve of his/her a posteriori utility.

What will happen with the maximin utility of individuals after informational sharing? For instance, suppose there are three states of nature  $\Omega = \{a, b, c\}$ , that the individual is unable to differentiate. Now his/her maximin utility takes the form  $U^{min}(x(a), x(b), x(c)) = \min\{u(a, x(a)), u(b, x(b)), u(c, x(c))\}$ .

Suppose now that the agent has learned to distinguish “a”: what is happened with his/her utility, is it changed or not? For Bayesian utility<sup>1</sup> nothing happens, but it is not so for maximin. Now one can not just write down the amount of utility in the state “a” plus utility minimum for two other states: under such decomposition the utility is unreasonable changed (increased?). No objections raise the representation

$$u(a, x(a))\mu(a) + \min\{u(b, x(b)), u(c, x(c))\}\mu(\{b, c\}),$$

where  $\mu(E)$  presents a probability for a new information partition. After one more information sharing utility is transformed to the form

$$u(a, x(a))\mu'(a) + u(b, x(b))\mu'(b) + u(c, x(c))\mu'(c),$$

where  $\mu'(\cdot)$  is a new probability distribution such that

$$\mu'(a) = \mu(a), \mu'(b) + \mu'(c) = \mu(\{b, c\}).$$

Thus during the information sharing process maximin utility is transformed into Bayesian (one can assume that individual was endowed with a shadow Bayesian utility). But what are the incentives for this process and at what is the economic system new state?

Similarly to the previous section, we can assume that the contractual process with maximin preferences is going on the model with differentiated agents, but now

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<sup>1</sup>This is expected a posteriori utility, presented in a form  $U(x) = \sum_{\omega \in \Omega} u(\omega, x(\omega))\mu(\omega)$ , where  $\mu(\cdot)$  is known for individual probability distribution for future states of nature (prior).

each new agent can consume any contingent goods bundles within an individual presenting event (no measurability in consumption). After the system reaches a stable web of contract (stability relative to simultaneous signing of new contracts and partial breaking of old ones), the process stops and, as before, it will continue its development after an information sharing will be realized. Now, however, uninformed agent is more interested in the sharing of information, because his/her incentive is to find a new opportunity for a partial breaking of contracts. Opportunities to enter into new mutually beneficial agreements without breaking old ones are totally exhausted. This is so because when the utilities are of maximin type no restrictions on the measurability of the new contract are imposed (all is valid), and also by the last statement on measurable utility: mutually beneficial exchange in any of the states of nature is impossible.

The last quest: What the state of the economy has been reached via presented process? The answer is—the analogue of contractual interim equilibrium, which corresponds to the concept of Rational Expectations Equilibrium. So we proposed a new model of how economy may arrive to REE-equilibrium.

## References

- Aliprantis, C. ed.** (2001). *Economic Theory*, Vol. 18. Berlin/Heidelberg: Springer–Verlag (special issue)
- de Castro, L. I. and N. C. Yannelis** (2010). Ambiguity aversion solves the conflict between efficiency and incentive compatibility// Working paper
- Glycopantis, D. and N. C. Yannelis** (2005). *Differential information economies (collected papers)*. Berlin/Heidelberg: Springer–Verlag
- Gilboa, I. and D. Schmeidler** (1989). Maxmin expected utility with non-unique prior// *Journal of Mathematical Economics*, **18**(2), 141–153
- Marakulin, V. M.** (2009). Economies with asymmetrically informed agents: the concept of limit information// *Journal of the New Economic Association*, **1–2**, 62–85 (in Russian and English transl.)
- Marakulin, V. M.** (2011). Contracts and domination in competitive economies// *Journal of the New Economic Association*, **9**, 10–32 (in Russian)
- Radner, R.** (1982). Equilibrium under uncertainty// *in*: K. J. Arrow and M. D. Intriligator, eds., *Handbook of Mathematical Economics*, Vol. II (Amsterdam: North-Holland), 923–1006