

Balanced Budget, Patterns of Aggregate Fluctuations and an Overlapping Generations Model with Externalities

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Abstract

In an overlapping generations model, this paper picks up consumption taxes and labor income taxes as the means of eliminating the budget deficits. We theoretically compare how the different methods of expanding the budget revenue influence the probability of aggregate fluctuations due to agents' self-fulfilling beliefs. It will be shown that the analytical results in an OLG model completely overturn for the well-known results in the representative agent models such as the Ramsey optimal growth model and Woodford's finance constrained model (1986). This paper will identify what are the important factors affecting the patterns of growth and aggregate fluctuations.

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1 Introduction

In March 2012, European Union excluding England and Chechoslovak came to an arrangement over the constitution of new treaty aiming at the reinforcement of fiscal discipline. To avoid a deficit of annual expenditure and revenue, this new treaty obliges the government to balance the budget. This treaty is characterized as the pillar of preventing the recurrence of debt crisis. The current "Stability and Growth Pact" requests the budget deficits-GDP ratio to be less than 3% and thus it means that the more strict rule will be imposed on these European countries. In Europe, it is believed that there is the "regulating function" such as an increase in the value-added tax rates against the growing budget deficits.

For example, Greece strives to reduce the budget deficits-GDP ratio to 6.8% in 2012 and the value-added tax rate has been highly fixed at 23% since 2010. In September 2012, the Spanish value-added taxes were increased from 18% to 21% in order to accomplish the budget deficits-GDP ratio of 3.0% in 2014. In England, Conservative party announced the value-added tax increase by the degree of 2.5% in the emergency appropriations bill released in Jun 2010 and made a public commitment to cutback the national deficits rapidly to retain the best grade of government bonds.

To improve the budget revenue, the Japanese government also relies heavily on consumption taxes corresponding to the value-added taxes. The consumption tax rate will be increased to 8% in April 2014 and again to 10% in October 2015. The directional movement of consumption tax increases has been put into place for the first time in 17 years and this movement is evaluated as the first step toward the improvement of the Japanese fiscal health. It is estimated that the ratio of primary balance deficits to GDP will drop as much as 50 percent in 2015 relative to the ratio of 6.7% recorded in 2010. The Japanese government makes an international commitment for the primary balance to move into the black until 2020, but it is expected that the primary balance deficits-GDP ratio will remain at the 2.8% level that is converted into the size of 5.6% consumption tax rates. See Figures 1 and 2.

Endogenous tax theory is very appropriate framework for analyzing how the aggregate economy is dynamically affected if government taxes are actively utilized to eliminate the budget deficits. Let us put forward the representative literature concerning endogenous tax theory. Using the Ramsey model with constant returns in production, Schmitt-Grohe and Uribe (1997) showed that if the labor income tax rate is endogenously adjusted to balance the budget, government taxes can be the very strong factor generating aggregate fluctuations due to agents' self-fulfilling expectations. The emergence of indeterminacy is proved in the Ramsey growth model with endogenous labor income taxes. Based on Woodford's finance-constrained model (1986) with capital-labor substitution as studied in Grandmont *et al* (1998), Gokan (2006) also emphasizes the importance of endogenous labor income taxes for the occurrence of indeterminacy and endogenous cycles through local bifurcations. In the Ramsey model, however, Giannitsarou (2007) proved that as long as consumption taxes are actively used to balance the budget, the economic stability is always preserved and no indeterminacy arises.

Therefore, it is believed that agents' belief-driven aggregate fluctuations are less likely

if the government makes the most use of consumption taxes rather than labor income taxes to cut the budget deficits. We might consider that the existing literature provides the theoretical validity for tax policy implemented for the enhancement of government revenue in the many European countries and Japan.

Using an overlapping generations model (hereafter OLG model), this paper examines the robustness of the well-known analytical results in the Ramsey optimal growth model. Phrased differently, we compare how the probability of indeterminacy is influenced if government expenditure is financed through consumption and labor income taxes. We will obtain the analytical results that are completely reversed for the results obtained in the Ramsey optimal growth model. Using Woodford's finance constrained economy (1986) with externalities, Gokan (2008) compared how the patterns of growth are influenced by government expenditure under the alternative financing methods. This paper also shows that his analytical result completely overturns if this OLG model is used instead of Woodford's finance constrained model.

In the 2-period OLG model considered in the present paper, young agents do not care about their young period-consumption and thus completely save their labor income earned in youth. The agents cover their old period-consumption with the interest income. In Woodford's finance constrained economy (1986), in contrast, the representative worker behaves as the 2-period OLG model. That is, the worker saves all his labor income in youth in the form of money. In his old age, he makes a trade of the money for the final goods held by young agents to finance his old period-consumption. However, there is an important difference between these two models that the capital investment is financed only with the labor (capital) income in Woodford's finance constrained model (the OLG model).

The following will be understood by comparing the present paper with Gokan (2008). If endogenous taxes are *not* introduced in both the OLG model and Woodford's finance constrained model, the patterns of growth are mainly governed by the sizes of productive externalities as in the existing literature on "growth and indeterminacy". Moreover, the dynamic behaviors in the OLG model are almost the same as in Woodford's finance constrained model. Therefore, the financing ways of capital investment seem not to be so important in investigating the factors contributing to the differences of growth patterns. However, if endogenous taxes are introduced in this OLG model, how government expenditure financing is connected with the patterns of growth in the OLG model is completely different from in Woodford's finance constrained model. Unlike the growth models without government sector, therefore, we can identify the importance of the financing ways of capital investment as well as government expenditure financing.

As mentioned above, the present paper considers the two types of government expenditure financing, consumption tax and labor income tax financing. In the OLG model considered here, the agents work only in the young age and they retire when old. Thus, endogenous labor income taxes indicate the burdens of tax liability for the young agents. In contrast, endogenous consumption taxes means the liability to taxes for the old agents, because the agents consume only in the old age. In 2012, the ratio of elderly people in total population is 23% in Japan, 20% in Germany, 16% in England and 13% in USA. Many developed countries including Japan are rapidly aging. It is often argued that

as the population ages, the burdens of elderly people should be strengthened to lighten the burdens imposed on young workers. Therefore, this OLG model is very suitable for analyzing the dynamic consequences of the intergenerational burdens of tax liability.

It is also well understood that aggregate fluctuations can be endogenously explained if increasing returns resulting from externalities in production are incorporated in dynamic general equilibrium models. See Benhabib and Farmer (1994, 1996) and Perli (1998). Unlike the existing literature, the present paper simultaneously seizes endogenous taxes and productive externalities. By using an integrated framework, we can analytically understand the important factors influencing the patterns of growth and aggregate fluctuations.

There is several literature analyzing how the overlapping generations economy displays an indeterminate steady state. Cazzavillan and Pintus (2004, 2006) and Lloyd-Braga *et al* (2007) modified the OLG model as studied in this paper in that they abandon the assumption that agents consume only when old, and they investigate the theoretical relationship between the proportion of consumption to income and the likelihood of endogenous fluctuations. However, the government sector is completely ignored in the above works as the present paper.

The rest of the paper is organized as follows. Section 2 describes the framework of the model and section 3 analyzes the condition that steady states exist. In Sections 4 (5), the dynamic effects of endogenous consumption taxes (endogenous labor income taxes) are examined, depending on the levels of increasing returns. Section 6 compares the dynamic implications, while Section 7 describes the causes underlying the analytical consequences. Section 8 identifies what are the important factors affecting the patterns of growth and aggregate fluctuations by comparing the present paper with Gokan (2008). Section 8 provides concluding remarks.

2 Framework

2.1 Consumers

We consider a non-monetary, 2-period OLG model that are very similar to Cazzavillan (2001). There is a unique perishable good that can be consumed or invested. No population growth exists. Agents, who live for two periods, and are identical within each generation, care only about old period consumption, supply a variable quantity of labor hours, and save all their labor income by holding physical capital when young. During old age, they provide no labor, but rent physical capital to firms and consume all their income. Then we can specify the behavior of consumers as follows. They maximize at date t their lifetime utility

$$\frac{c_{t+1}^{1-\phi}}{1-\phi} - \frac{l_t^{1+\xi}}{1+\xi} \quad (1)$$

subject to the two budget constraints:

$$s_t = (1 - \tau_{wt}) \cdot w_t l_t, \quad (2)$$

$$(1 + \tau_{ct+1}) \cdot c_{t+1} = r_{t+1} \cdot s_t, \quad (3)$$

where l_t is the labor supply, c_{t+1} is the next period's consumption, which is taxed at the rate τ_{ct+1} , w_t is the real wage rate, which is taxed at the rate τ_{wt} , s_t is saving, and r_{t+1} is the real net interest rate at time $t + 1$.¹ The tax rates on consumption and labor income are specified below. For this optimization problem, the first order condition is

$$\frac{c_{t+1}^{-\phi}}{(1 + \tau_{ct+1})/r_{t+1}} = \frac{l_t^\xi}{(1 - \tau_{wt})w_t},$$

where the lefthand (righthand) side represents the marginal utility (disutility) of consumption (labor) per dollar. Combining this equation with the intertemporal budget constraint $(1 + \tau_{ct+1})c_{t+1}/r_{t+1} = (1 - \tau_{wt})w_t l_t$, we can obtain the unique solution characterized as the workers' offer curve,

$$\begin{aligned} l_t &= c_{t+1}^\Psi \\ &\equiv \Psi(c_{t+1}), \text{ where } \Psi \equiv \frac{1 - \phi}{1 + \xi}. \end{aligned} \quad (4)$$

Following the existing literature, we also focus on the gross substitutability between consumption and leisure and thus the following must be assumed:

Assumption 1 $0 < \Psi < 1$ ($\Leftrightarrow 0 < \phi < 1$ and $\xi > 0$)

Considering (2), (3) and (4), we can specify the wage-elasticity of labor supply as $\Psi/(1 - \Psi)$. Thus, A.1 ensures that labor supply elasticity is positive.

2.2 Firms and government

As for the producers' side, there are an infinite number of firms facing the same technology. Identical firms produce final good y_t by combining labor l_t and capital stock k_t . The production function exhibits Cobb-Douglas technology by

$$y_t = Ak_t^a l_t^b \bar{k}_t^{\alpha-a} \bar{l}_t^{\beta-b}, \text{ where } a + b = 1, \alpha > a \text{ and } \beta > b. \quad (5)$$

A is a scaling parameter. \bar{k}_t and \bar{l}_t denote the average economy-wide use of capital and labor, which are taken as parametric by individual capitalists. $\alpha - a$ and $\beta - b$ represent the degrees of the external effects derived from the average economy-wide use of capital and labor, respectively. Capital externality can be explained by Arrow's learning-by-doing, while the thick market hypothesis modeled by Diamond (1982) is a sufficient justification of the labor externality. We focus on the symmetric equilibrium in which $k_t = \bar{k}_t$ and

¹Defining the capital depreciation rate as δ , equation (3) must be written as $(1 + \tau_{ct+1}) \cdot c_{t+1} = (r_{t+1} + 1 - \delta) \cdot s_t$. However, this paper assumes the depreciation rate of capital equal to one, i.e., $\delta = 1$. Because we consider an OLG model, this assumption is regarded as realistically plausible.

$l_t = \bar{l}_t$ are satisfied. We can obtain

$$r_t = aAk_t^{\alpha-1}l_t^\beta \equiv r(k_t, l_t) \text{ and } w_t = bAk_t^\alpha l_t^{\beta-1} \equiv w(k_t, l_t). \quad (6)$$

Let us state the government behavior. The present paper will compare how the local dynamics are affected when the tax rates on consumption and labor income endogenously adjust to remove the budget deficits. Thus, we focus on the following budget constraint:

$$g = \tau_{wt} \cdot w_t l_t + \tau_{ct} \cdot c_t, \quad (7)$$

where g denotes constant government expenditure. (7) determines the tax rate on labor income (consumption) τ_{wt} (τ_{ct}) required to obtain the necessary revenue g if the consumption (labor income) tax rate is fixed at $\tau_{ct} = \tau_c$ ($\tau_{wt} = \tau_w$).

2.3 Market equilibrium

For each of the two government financing methods, we now derive the equilibrium conditions determining the intertemporal competitive equilibrium with perfect foresight with the sequence $(c_t, k_t) > 0$ for $t \geq 0$.

First, we think of the case that consumption taxes endogenously adjust to eliminate the budget deficits for a fixed level of government expenditure. From (2), (3), (4), (6) and (7), the goods market is in equilibrium, when saving at this period is equal to the new investment of physical capital, i.e., $s_t = k_{t+1}$. To facilitate the comparison, we set the fixed labor income tax rate equal to zero, i.e., $\tau_w = 0$. Then, we can express the goods market equilibrium condition as

$$w(k_t, \Psi(c_{t+1})) \cdot \Psi(c_{t+1}) = k_{t+1}. \quad (8)$$

Using (3), (4), (6), (7) and $s_t = k_{t+1}$, we can get

$$r(k_t, \Psi(c_{t+1})) \cdot k_t = c_t + g. \quad (9)$$

(9)² characterizes the situation where public spending is financed through consumption taxes. This case is generically called endogenous consumption taxes and (8) and (9) determine the intertemporal competitive equilibrium with perfect foresight with the sequence $(c_t, k_t) > 0$ for $t \geq 0$.

Finally, let us consider that labor income taxes endogenously adjust to balance the government budget. For the same reason above, the tax rate on consumption is set to zero, i.e., $\tau_c = 0$. As for the good market equilibrium, then, substituting $g = \tau_{wt} \cdot w_t l_t$, $s_t = k_{t+1}$, (4) and (6) into (2) leads to

$$w(k_t, \Psi(c_{t+1})) \cdot \Psi(c_{t+1}) - g = k_{t+1}. \quad (10)$$

²If we substitute the government budget $g = \tau_{ct+1} \cdot c_{t+1}$ into $(1 + \tau_{ct+1}) \cdot c_{t+1} = r_{t+1} k_{t+1}$ obtained from (2) and replace $t + 1$ by t , Equation (9) can be obtained.

Combining (3), (4), (6), $\tau_c = 0$ and $s_t = k_{t+1}$ yields

$$r(k_t, \Psi(c_{t+1})) \cdot k_t = c_t. \quad (11)$$

Equation (10) expresses the situation where public spending is financed through labor income taxes. This case is generally referred to as endogenous labor income taxes, and the intertemporal competitive equilibrium with perfect foresight is determined from (10) and (11).

3 Steady state analysis

Here, the existence of steady states is examined and it will be shown that the number of positive steady states is determined by the size of externalities, independently of how public spending is financed. Phrased differently, the conditions that the uniqueness and multiplicity of steady states appear are indifferent between endogenous consumption and income taxes. Needless to say, the economy is in steady state equilibrium when the economic variables are constant over time, i.e., $c_t = c_{t+1} = c^*$ and $k_t = k_{t+1} = k^*$ are satisfied. (x^* denotes the steady state value of x .)

First, we study the case that the tax rate on consumption endogenously adjusts to finance the preset level of government expenditure. Using (8) and (9), the steady state equilibria are defined as the solutions (k^*, c^*) in R_{++}^2 of

$$bA(k^*)^{\alpha-1}(c^*)^{\beta\Psi} = 1, \quad (12)$$

$$\frac{a}{b} \cdot k^* - g = c^* \quad (13)$$

From (12) and (13), the degree of externalities α and $\beta\Psi$ determines the number of non-zero steady states as follows.

Proposition 1 *Consumption taxes*

The steady state is uniquely determined, if $\alpha > 1$ (Figure 3) or $\alpha < 1$ and $\alpha + \beta\Psi > 1$ (Figure 4). When $\alpha + \beta\Psi < 1$ (Figure 5), there are the multiple steady states for $0 < g < g_{cS}$, and no steady state exists for $g > g_{cS}$.³ (Note that the two steady states coalesce together in Figure 5 when $g = g_{cS}$.)

When $\tau_w = 0$, equation (7) pins down the consumption tax rate τ_c^* required to obtain the public revenue g . Use of (7), (12) and (13) yields the steady state consumption tax rate,

$$\tau_{ci}^* = \left(\frac{a}{b}\right) \left(\frac{k_i^*}{c_i^*}\right) - 1. \quad (14)$$

³Gokan (2008) utilizes Woodford's finance-constrained model different from this OLG model and compares the welfare across two steady states. However, the analytical arguments are equally true of this paper in that the high steady state (k_2^*, c_2^*) strongly dominates the low steady state (k_1^*, c_1^*) .

If $\alpha + \beta\Psi > 1$, Figures 3 and 4 verify $d(k_i^*/c_i^*)/dg > 0$. Equation (14) means that the consumption tax rate must be increased to raise the budget revenue. We can conclude that no Laffer curve-relation exists. When $\alpha + \beta\Psi < 1$, in contrast, Figure 5 shows the existence of the two non-zero steady states for $0 < g < g_{cS}$ and an increase in g reduces the level of (k_1^*/c_1^*) , but raises the level of (k_2^*/c_2^*) . If the necessary revenue changes from 0 to g_{cS} , the low-(high-)steady state consumption tax rate must decrease (increase) from ∞ (0) to $\frac{1-\alpha}{\beta\Psi} - 1$ ($\equiv \tau_{cS}$).⁴ Thus, we can observe the Laffer curve-relationship between the tax rate τ_c^* and the tax revenue $\tau_c^* \cdot c^*(= g)$ and Figures 11 and 12 can be drawn.

Next, we consider the case where the tax rate on labor income is endogenously determined to satisfy the budget for a given value of government expenditure. For the same reason above, we set the consumption tax rate equal to zero, i.e., $\tau_c = 0$. From (10) and (11), we can obtain the steady state equilibria by solving

$$c^* = \frac{a}{b} \cdot (k^* + g), \quad (15)$$

$$aA(k^*)^\alpha (c^*)^{\beta\Psi-1} = 1. \quad (16)$$

(15) and (16) imply the following.

Proposition 2 *Labor income taxes:*

The steady state is uniquely determined, if $\alpha + \beta\Psi > 1$ (Figure 6). If g is not extremely large, the multiple steady states can be obtained, when $\alpha + \beta\Psi < 1$ (Figure 7).

Proof. See Figures 6 and 7. ■

Comparing Propositions 1 with 2, the number of steady states is determined only by the sizes of increasing returns and the elasticity of workers' offer curve, (i.e., $\alpha + \beta \cdot \Psi$) and it is independent of the ways of government expenditure financing.

Let us derive the steady state value of labor income tax rate τ_w^* required to obtain the revenue g . When $\tau_c = 0$, using (7), (15) and (16) yields the labor income tax rate,

$$\tau_{wi}^* = 1 - \left(\frac{a}{b}\right) \left(\frac{k_i^*}{c_i^*}\right). \quad (17)$$

When $\alpha + \beta\Psi > 1$, Figure 6 shows $d(k_i^*/c_i^*)/dg < 0$ and there exists a positive relations between the tax rate on labor income and the public revenue. In contrast, when $\alpha + \beta\Psi < 1$, Figure 7 illustrates that an increase in g reduces (k_1^*/c_1^*) , but raises (k_2^*/c_2^*) . Defining that g_{wS} is public spending such that the two steady states coalesce together in Figure 7, let us consider what happens if the necessary revenue changes from 0 to g_{wS} . Equation (17) means that the low-steady state tax rate on labor income must be increased from 0 to the revenue-maximizing tax rate $1 - \frac{\alpha}{1-\beta\Psi}$ ($\equiv \tau_{wS}$), while the high-steady state tax

⁴When $g = g_{cS}$, Fig.5 shows that eqs. (11) and (12) have the identical slopes and thus $\frac{c_i^*}{k_i^*} = \left(\frac{a}{b}\right) \left(\frac{\beta\Psi}{\alpha}\right)$ is satisfied. Substituting this equation in $\tau_{ci}^* = \left(\frac{a}{b}\right) \left(\frac{k_i^*}{c_i^*}\right) - 1$ yields the value of τ_{cS}^* . When $g \rightarrow 0$, Fig.5 indicates that $c_1^*/k_1^* \rightarrow 0$ and $c_2^*/k_2^* \rightarrow a/b$ are satisfied and thus we can verify $\tau_{c1}^* \rightarrow \infty$ and $\tau_{c2}^* \rightarrow 0$.

rate must be reduced from 1 to τ_{wS} .⁵ When increasing returns are relatively mild, we can verify the non-monotonic relations between the labor income tax rate τ_w^* and the tax revenue $\tau_w^* \cdot w^* l^*$ ($= g$). Only if $\alpha + \beta\Psi < 1$, we can draw the Laffer curve such as Figures 17 and 18, also in endogenous consumption taxes.

4 Consumption taxes

This section considers the case of endogenous consumption taxes such that the tax rate on consumption τ_{ct} is endogenously determined to satisfy the budget (7) for a fixed level of government expenditure g . This tax policy implies that consumption tax rates are willingly used to obtain the preset level of budget revenue. To assess the dynamic consequences of this tax policy for the real economy, we analytically explore how the local dynamics are affected by changes in public spending financed through consumption taxes. Defining $dx_t \equiv x_t - x_i^*$, the linear approximation of (8) and (9) near the steady states (k_i^*, c_i^*) is expressed as

$$\begin{bmatrix} dk_{t+1} \\ dc_{t+1} \end{bmatrix} = \begin{bmatrix} \frac{1}{\beta(a/b)\Psi \cdot (k_i^*/c_i^*)} & -\frac{\alpha(a/b)}{\beta(a/b)\Psi \cdot (k_i^*/c_i^*)} \\ b/a & 0 \end{bmatrix} \begin{bmatrix} dk_t \\ dc_t \end{bmatrix}. \quad (18)$$

From (18), we can compute the determinant D and the trace T of the Jacobian matrix as

$$D = \left(\frac{b}{a}\right) \left(\frac{\alpha}{\beta\Psi}\right) \left(\frac{c_i^*}{k_i^*}\right) > 0, \quad (19)$$

$$T = \frac{1}{(a/b)\beta\Psi \cdot (k_i^*/c_i^*)}. \quad (20)$$

D denotes the determinant, which equals the product of the two characteristic roots in the matrix and T represents the trace, which is equal to the sum of the two characteristic roots.

If government expenditure is fixed at $g = 0$, this economy is identical to the framework with no government. Then, (13) generates $c_2^*/k_2^* = a/b$ and the uniqueness of steady state can be easily proved. If we substitute $c_2^*/k_2^* = a/b$ in (19) and (20), Figure 8 characterizes the patterns of local dynamics in the economy with no government sector.⁶ We can see that the local dynamics in this case completely coincide with in Woodford's finance constrained model with no government.⁷ Compare Figure 8 in the present paper

⁵When $g \rightarrow g_{wS}$, $\left(\frac{a}{b}\right) \left(\frac{k_i^*}{c_i^*}\right) \rightarrow \frac{\alpha}{1-\beta\Psi}$ is satisfied. Substituting this equation in (17) yields $\tau_{wS} = 1 - \frac{\alpha}{1-\beta\Psi}$. When $g \rightarrow 0$, Fig.7 indicates that $c_1^*/k_1^* \rightarrow \infty$ and $c_2^*/k_2^* \rightarrow a/b$ are satisfied. Substituting these relations in (17), we can get $\tau_{w1}^* \rightarrow \infty$ and $\tau_{w2}^* \rightarrow 0$.

⁶In the economy with no government, the trace and the determinant can be respectively expressed as $D = \alpha/(\beta\Psi)$ and $T = 1/(\beta\Psi)$. We can verify $D - T + 1 = \frac{\alpha+\beta\Psi-1}{\beta\Psi}$ and $D - 1 = \alpha/(\beta\Psi) - 1$. Thus, the stability properties in the plane $(\alpha, \beta\Psi)$ can be characterized as illustrated in Fig. 8.

⁷To be accurate, if the 100 % depreciation rate of capital is assumed in Woodford's model, Figure 8 in the present paper are quite the same as Figure 18 in Gokan (2008).

with Figure 18 in Gokan (2008) analyzing Woodford's model.⁸ It is very important to understand the difference between these models as follows. Private investment k_{t+1} is funded with labor (capital) income in this OLG model (Woodford's finance constrained economy). Therefore, the following can be obtained:

Proposition 3 *How private investment is funded does not matter to the patterns of growth and aggregate fluctuations in these growth models with no government sector.*

From here, let us think of the economy with the government sector, i.e., $g \neq 0$. Equations (15) and (16) show that public spending g influences the two characteristic roots through its influence on the steady state capital-consumption ratio (k_i^*/c_i^*). We can verify the following:

Lemma 1 *Irrespective of the values of public spending, point (T, D) is located on the line Δ , $D = \alpha \cdot T$.*

Proof. *The line Δ can be easily obtained if we combine (19) and (20) to eliminate (k_i^*/c_i^*) .*

■

Considering lemma 1, let us investigate the dynamic effects of public spending, depending on the sizes of external effects α and $\beta\Psi$. In other words, we will clarify how the aggregate economy fluctuates near the steady states in the case of this tax policy, when increasing returns are relatively mild and strong.

4.1 $\alpha + \beta\Psi < 1$

Let us consider that the degree of increasing returns is moderately low, that is, $\alpha + \beta\Psi < 1$ is satisfied. See Cases 1 and 2 in Figure 9. As proved in Proposition 1, if government expenditure g is not very large, the low steady state (k_1^*, c_1^*) and the high steady state (k_2^*, c_2^*) coexist and thus the Laffer curve-relation exists between the steady state consumption tax rate τ_c^* and the tax revenue $\tau_c^*c^* (= g)$. This section will clarify the local stability of the two steady states and the consumption tax rates on the Laffer curve τ_c^* . For the purpose, it is convenient to understand the next properties obtained in Figure 5:

Lemma 2 *As $g \rightarrow 0$, we can get $(c_1^*/k_1^*) \rightarrow 0$ and thus $D_1 \rightarrow 0$. When $0 < g < g_{cS}$, we can clarify $D_1 > T_1 - 1$ for the low steady state, while $D_2 < T_2 - 1$ is satisfied for the high steady state. Moreover, $D_i = T_i - 1$ is obtained at $g = g_{cS}$. ($i = 1, 2$).*

Proof. *Figure 5 shows that $(c_1^*/k_1^*) \rightarrow 0$, as $g \rightarrow 0$. Then, from (19), $D_1 \rightarrow 0$ is satisfied. For the rest, see Appendix A.* ■

⁸Using (2), (3) and (4), the labor supply curve is $\ln w_t = -\ln r_{t+1} + \left(\frac{1-\Psi}{\Psi}\right) \ln l_t$. From (6), the labor demand curve can be derived as $\ln w_t = \ln Ab + \alpha \ln k_t + (\beta - 1) \ln l_t$. Noting $\frac{1-\Psi}{\Psi} > \beta - 1$ ($\Leftrightarrow \beta\Psi < 1$), $\beta\Psi < 1$ ($\beta\Psi > 1$) means that the labor demand and supply curves cross with standard (wrong) slopes.

Because Figure 5 shows $d(c_1^*/k_1^*)/dg > 0$ [$d(c_2^*/k_2^*)/dg < 0$], Equation (19) means $dD_1/dg > 0$ [$dD_2/dg < 0$]. When g is increased from 0, (T_1, D_1) moves up from the horizontal axis, while (T_2, D_2) moves down from the point below the line (AB). The points (T_i, D_i) evaluated at the two steady states move on Δ toward the line (AB) and disappear, when g moves up through g_{cS} . Then, the system undergoes a saddle-node bifurcation such that the two steady states coalesce together and disappear. See Figure 10. To provide the complete characterization of the dynamic effects of g , let us see the following:

Lemma 3 *The line Δ crosses the point B at $\alpha = 1/2$.*

Proof. *As shown in Lemma 1, the line Δ is $D = \alpha \cdot T$. Thus, when $\alpha = 1/2$, this line crosses the point B. ■*

We define the value of g at which (T_1, D_1) crosses the interior segment (BC) as g_{cH} , and can easily derive $\tau_{c1}^* = \alpha/(\beta\Psi) - 1$ ($\equiv \tau_{cH}$) at $g = g_{cH}$. Now, let us summarize the dynamic effects of government expenditure.

Proposition 4 *Consumption taxes, $\alpha + \beta\Psi < 1$*

1) $0 < \alpha < 1/2$: *The low steady state is always a sink (locally indeterminate) for $0 < g < g_{cS}$ ($\infty > \tau_{c1}^* > \tau_{cS}$), while the high steady state (k_2^*, c_2^*) is always a saddle (locally determinate) for $0 < g < g_{cS}$ ($0 < \tau_{c2}^* < \tau_{cS}$). See Case 1 in Figures 9-10 and Figure 11.*

2) $1/2 < \alpha < 1$: *The low steady state is a sink (locally indeterminate) for $0 < g < g_{cH}$ ($\infty > \tau_{c1}^* > \tau_{cH}$), displays a Hopf bifurcation at $g = g_{cH}$ ($\tau_{c1}^* = \tau_{cH}$) and is a source for $g_{cH} < g < g_{cS}$ ($\tau_{cH} > \tau_{c1}^* > \tau_{cS}$), while the high steady state (k_2^*, c_2^*) is always a saddle (locally determinate) for $0 < g < g_{cS}$ ($0 < \tau_{c2}^* < \tau_{cS}$). See Case 2 in Figures 9-10 and Figure 12.*

Proof. *See Figure 10. ■*

For any sizes of capital externalities,⁹ the high steady state is locally determinate, and the economic variables monotonically approach this steady state. Thus, the economic stability is always preserved near the high steady state. This analytical result is consistent with in Giannitsarou (2007). She showed that no indeterminacy arises, and no Laffer curve relation exists in endogenous consumption taxes. Unlike her result, however, indeterminacy can emerge near the low steady state.

If $0 < \alpha < 1/2$, the low steady state is always a sink (locally indeterminate) for all the feasible tax rates on consumption, $\infty > \tau_{c1}^* > \tau_{cS}$ (i.e., government expenditure, $0 \leq g < g_{cS}$). See Figure 11. Even if $1/2 < \alpha < 1$, the low steady state is a sink (locally indeterminate) for some interval of consumption tax rate, $\infty > \tau_{c1}^* > \tau_{cH}$ (i.e., government expenditure, $g_{cH} < g < \infty$). See Figure 12. Irrespective of the sizes of capital externalities, however, the steady state consumption tax rates generating indeterminacy exist in the downward-sloping side of Laffer curve.

⁹If capital stock is (is not) interpreted including human capital, the size of capital externalities α is above (below) $1/2$.

4.2 $\alpha + \beta\Psi > 1$

Finally, we consider that the degree of increasing returns is moderately high, that is, $\alpha + \beta\Psi > 1$.¹⁰ See Cases 3-5 in Figure 9. From Section 3, the uniqueness of the steady state can be always verified for $g \geq 0$. Unlike the case of $\alpha + \beta\Psi < 1$, we can obtain the monotonic relationship between the consumption tax rate τ_c^* and the tax revenue $\tau_c^*c^*(=g)$. The tax revenue always increases with the steady state tax rate on consumption. It is important to understand how the initial (final) point of determinant D at $g = 0$ ($g \rightarrow \infty$) is located, depending on the degree of capital and labor externalities α and $\beta\Psi$. Let us check the next lemma:

Lemma 4 1) We can see that $(c^*/k^*) \rightarrow 0$ and thus $D \rightarrow 0$, as $g \rightarrow \infty$.

2) For $0 < \alpha < 1/2$ or $1/2 < \alpha$ and $\alpha < \beta\Psi$, the steady state is a sink (locally indeterminate) at $g = 0$.

3) When $\alpha > \beta\Psi$, the steady state is a source at $g = 0$.

Proof. 1) See Figures 3 and 4.

2) and 3) See Figure 8. ■

Considering the last half in lemma 2, Appendix A, Figures 3 and 4, $D > T - 1$ is always satisfied for $g \geq 0$ if $\alpha + \beta\Psi > 1$. Figures 3 and 4 clarify $d(c^*/k^*)/dg < 0$ and thus Equation (19) means $dD/dg < 0$. Noting lemma 4, let us summarize how government expenditure influences the local dynamics around the steady state.

Proposition 5 Consumption taxes, $\alpha + \beta\Psi > 1$

1) $0 < \alpha < 1/2$ or $1/2 < \alpha$ and $\alpha < \beta\Psi$: The steady state is always a sink (locally indeterminate) for $0 \leq g < \infty$ ($0 \leq \tau_c^* < \infty$). See Cases 3 and 4 in Figures 9 and 13.

2) $\alpha > \beta\Psi$: The steady state is a source for $0 \leq g < g_{cH}$ ($0 < \tau_c^* < \tau_{cH}$), displays a Hopf bifurcation at $g = g_{cH}$ ($\tau_c^* = \tau_{cH}$) and is a sink (locally indeterminate) for $g_{cH} < g < \infty$ ($\tau_{cH} < \tau_c^* < \infty$). See Case 5 in Figures 9, 13 and Figure 14.

Proof. See Figure 13. ■

From Proposition 4, the following can be understood. If $(\alpha, \beta\Psi)$ exists in the Case 3 or 4 in Figure 9, indeterminacy can arise for all the range of government expenditure, $0 \leq g < \infty$ and no divergent equilibrium arises. In contrast, if $(\alpha, \beta\Psi)$ exists in the Case 5 in Figure 9, indeterminacy can emerge for extremely large range of government expenditure $g_{cH} < g < \infty$, but the steady state is a source (locally divergent) for relatively small range of government expenditure $0 \leq g < g_{cH}$.

5 Labor income taxes

In this section, we consider the case where labor income tax rates endogenously adjust to satisfy the budget (7) for a given value of government expenditure. This tax policy

¹⁰We should note that if the wage-elasticity of labor supply is set at infinity, i.e., $\Psi \rightarrow 1$, as in real business cycle literature, then this condition does not necessarily imply the strong sizes of increasing returns.

means that labor income taxes are exclusively used to remove the budget deficits. Section 3 proves that the conditions for the steady states to exist are almost the same as in endogenous consumption taxes. This section analyzes how the transitional dynamics are affected by changes in public spending financed through labor income taxes.

Defining $dk_t \equiv k_t - k^*$ and $dc_t \equiv c_t - c^*$, the dynamic motions determined by (10) and (11) are locally approximated near the steady states as

$$\begin{bmatrix} dk_{t+1} \\ dc_{t+1} \end{bmatrix} = \begin{bmatrix} 0 & \frac{b}{a} \\ -\frac{\alpha}{\beta} \frac{(c_i^*/k_i^*)}{\Psi} & \frac{1}{\beta\Psi} \end{bmatrix} \begin{bmatrix} dk_t \\ dc_t \end{bmatrix}, \quad (21)$$

From (21), the determinant D and the trace T of this matrix can be derived as

$$D = \left(\frac{\alpha}{\beta}\right) \left(\frac{b}{a}\right) \frac{(c_i^*/k_i^*)}{\Psi} \quad (22)$$

$$T = \frac{1}{\beta\Psi}. \quad (23)$$

As in the endogenous consumption taxes, (22) and (23) mean that government expenditure influences the local dynamics (i.e., the two characteristic roots) through its influence on the steady state capital-consumption ratio (k_i^*/c_i^*).

Using (22) and (23), Section 5-1 thinks that the degree of increasing returns is relatively low, i.e., $\alpha + \beta\Psi < 1$ and the relatively high sizes of increasing returns, i.e., $\alpha + \beta\Psi > 1$ are considered in Section 5-2.

5.1 $\alpha + \beta\Psi < 1$

If government expenditure g is below g_{wS} , we can verify that there coexist the low steady state (k_1^*, c_1^*) and the high steady state (k_2^*, c_2^*) and thus we can verify the Laffer curve-relation between the steady state tax rate τ_{wi}^* and the tax revenue $\tau_{wi} w_t l_t (= g)$. To characterize the dynamic effects of endogenous labor income taxes, it is very useful to understand the following properties in Figure 7.

Lemma 5 *As $g \rightarrow 0$, we can clarify $(c_1^*/k_1^*) \rightarrow 0$ and thus $D_1 \rightarrow 0$. When $0 < g < g_{wS}$, we can clarify $D_1 > T_1 - 1$ for the low steady state (k_1^*, c_1^*), while $D_2 < T_2 - 1$ is satisfied for the high steady state (k_2^*, c_2^*). Moreover, $D_i = T_i - 1$ is obtained at $g = g_{wS}$. ($i = 1, 2$).*

Proof. *Figure 7 shows that $(c_1^*/k_1^*) \rightarrow \infty$, as $g \rightarrow 0$. Then, from (22), $D_1 \rightarrow \infty$ is satisfied. For the rest, see Appendix B. ■*

From Figure 7, we can see $d(c_1^*/k_1^*)/dg < 0$ [$d(c_2^*/k_2^*)/dg > 0$] and Equation (22) means $dD_1/dg < 0$ [$dD_2/dg > 0$]. If g is increased from 0, (T_1, D_1) moves down from the point above the line (AB), while (T_2, D_2) moves up from the point between below the line (AC) and above the horizontal axis. If g moves up through g_{wS} , the points (T_i, D_i) evaluated at the two steady states move on the line $T = 1/(\beta\Psi)$ toward the line (AB) and disappear. Then, a saddle-node bifurcation occurs. See Figure 16.

Defining the value of g at which (T_1, D_1) crosses the interior segment (BC) as g_{wH} , we can easily get $\tau_{w1}^* = 1 - \alpha/(\beta\Psi) (\equiv \tau_{wH})$ at $g = g_{wH}$. Now, we can summarize the dynamic effects as follows:

Proposition 6 *Labor income taxes, $\alpha + \beta\Psi < 1$*

1) $0 < \beta\Psi < 1/2$: *The low steady state is always a source (locally divergent) for $0 < g < g_{wS}$ ($1 > \tau_{w1}^* > \tau_{wS}$), while the high steady state (k_2^*, c_2^*) is always a saddle (locally determinate) for $0 < g < g_{wS}$ ($0 < \tau_{w2}^* < \tau_{wS}$). See Case 6 in Figures 15, 16 and Figure 17.*

2) $1/2 < \beta\Psi < 1$: *The low steady state is a source for $0 < g < g_{wH}$ ($1 > \tau_{w1}^* > \tau_{wH}$), displays a Hopf bifurcation at $g = g_{wH}$ ($\tau_{w1}^* = \tau_{wH}$) and is a sink (locally indeterminate) for $g_{wH} < g < g_{wS}$ ($\tau_{wH} > \tau_{w1}^* > \tau_{wS}$), while the high steady state (k_2^*, c_2^*) is always a saddle (locally determinate) for $0 < g < g_{wS}$ ($0 < \tau_{w2}^* < \tau_{wS}$). See Case 7 in Figures 15, 16 and Figure 18.*

Proof. See Figure 16. ■

For any sizes of labor externalities, the high steady state is always a saddle (locally determinate) for any feasible tax rates on labor income $0 < \tau_{w1}^* < \tau_{wS}$ (i.e., government expenditure, $0 < g < g_{wS}$). If $0 < \beta\Psi < 1/2$, the low steady state is always a source (locally divergent) for all the feasible tax rates on consumption, $1 > \tau_{w1}^* > \tau_{wS}$ (i.e., government expenditure, $0 < g < g_{wS}$). See Figure 17. Unlike endogenous consumption taxes, no indeterminacy arises. If $1/2 < \beta\Psi < 1$, the low steady state is a source (locally divergent) for some interval of labor income tax rate, $1 > \tau_{w1}^* > \tau_{wH}$ (i.e., government expenditure, $0 < g < g_{wH}$) and indeterminacy can arise for $\tau_{wH} > \tau_{w1}^* > \tau_{wS}$ (i.e., $g_{wH} < g < g_{wS}$). The labor income tax rates generating indeterminacy exist only in the downward-sloping side of Laffer curve as in endogenous consumption taxes. See Figure 18.

5.2 $\alpha + \beta\Psi > 1$

We have already known that the unique steady state always exists for $g \geq 0$, when $\alpha + \beta\Psi > 1$. Then, the tax revenue $\tau_w w_i l_t (= g)$ always increases with the steady state tax rate τ_w^* . To analyze the transitional effects, it is necessary to understand the following.

Lemma 6 1) *We can see that $(c^*/k^*) \rightarrow \infty$ and thus $D \rightarrow \infty$, as $g \rightarrow \infty$.*

2) *Suppose that government expenditure is set to zero, i.e., $g = 0$. For $\alpha < \beta\Psi$, the steady state is a sink (locally indeterminate), while the steady state is a source for $0 < \beta\Psi < 1/2$ or $1/2 < \beta\Psi$ and $\alpha > \beta\Psi$.*

Proof. 1) See Figure 6.

2) See Figure 8. ■

The last half in lemma 5 and Figure 6 imply that $D > T - 1$ is satisfied for $g \geq 0$ if $\alpha + \beta\Psi > 1$. Figure 6 shows $d(c^*/k^*)/dg > 0$ and Equation (22) means $dD/dg > 0$. Noting lemma 6, therefore, the following can be described.

Proposition 7 *Labor income taxes, $\alpha + \beta\Psi > 1$*

1) $0 < \beta\Psi < 1/2$ or $1/2 < \beta\Psi$ and $\alpha > \beta\Psi$: *The steady state is always a source (locally divergent) for $0 \leq g < \infty$ ($0 \leq \tau_w^* < 1$). See Cases 8 and 9 in Figures 15 and 19.*

2) $\alpha < \beta\Psi$: *The steady state is a sink (locally indeterminate) for $0 \leq g < g_{wH}$ ($0 < \tau_w^* < \tau_{wH}$), displays a Hopf bifurcation at $g = g_{wH}$ ($\tau_w^* = \tau_{wH}$) and is a source for $g_{wH} < g < \infty$ ($\tau_{wH} < \tau_w^* < 1$). See Case 10 in Figures 15, 19 and Figure 20.*

Proof. See Figure 19. ■

From Proposition 6, the following property can be also understood. If $(\alpha, \beta\Psi)$ exists in Case 8 or 9 in Figure 15, the steady state is always a source (locally divergent) for all the range of government expenditure, $0 \leq g < \infty$ and no convergent equilibrium path exists. In contrast, if $(\alpha, \beta\Psi)$ exists in Case 10 in Figure 15, the steady state is a source (locally divergent) for extremely wide range of government expenditure $g_{wH} < g < \infty$ and indeterminacy can emerge only for relatively small range of government expenditure $0 < g < g_{wH}$.

6 Comparison

The previous sections analyze how the local dynamics are influenced by increasing public spending financed through labor income and consumption for given sizes of increasing returns in production. Using the analytical results obtained in the previous sections, this section compares how the alternative government expenditure financing influences the patterns of economic growth, and particularly, the probability of indeterminacy. We establish that the analytical outcomes in this OLG model completely overturn for the well known results in the Ramsey growth model. In the Ramsey growth model, the following is well understood.

Remark 1 *If government expenditure is financed through labor income taxes, indeterminacy can arise in the upward-sloping side of the Laffer curve. See Schmitt-Grohe and Uribe (1997). However, if government expenditure is financed by imposing consumption taxes, no indeterminacy arises and no Laffer curve-relation exists. See Giannitsarou (2007).*

From the viewpoint of economic stability, Remark 1 recommends that consumption taxes should be used as a means of removing the budget deficits rather than labor income taxes. Therefore, this well-known result analytically suggests the validity of tax policy designed to shrink the fiscal deficits in many European countries and Japan. Based on the OLG model proposed by Reichlin (1986), let us check the robustness of the well-known result in the Ramsey model.

Suppose that increasing returns are relatively mild, i.e., $\alpha + \beta\Psi < 1$. Then, the high steady state is locally determinate for any methods of government financing. If $(\alpha, \beta\Psi)$ exists in Case 1 (6) of Figure 9 (15), in contrast, the low steady state is locally indeterminate (divergent) for any feasible values of government expenditure financed through consumption (labor income) taxes. If the parameter $(\alpha, \beta\Psi)$ exists in the Case 2 (7) of Figure 9 (15), the low steady state is locally divergent (indeterminate) for some range of

government expenditure $0 < g < g_{cH}$ [$0 < g < g_{wH}$],, but it is locally indeterminate (divergent) for the other range of government expenditure $g_{cH} < g < g_{cS}$ ($g_{wH} < g < g_{wS}$). If $\alpha + \beta\Psi < 1$, therefore, an indeterminacy (a divergency) of equilibria is more likely to arise near the low steady state if consumption (labor income) taxes are chosen to eliminate the budget deficits.

Next, suppose that increasing returns are relatively high, i.e., $\alpha + \beta\Psi > 1$. Then, the uniqueness of steady state can be always clarified. If the economy exists in Case 3 or 4 (8 or 9) of Figure 9 (15), the steady state is locally indeterminate (divergent) for all the values of government expenditure $0 \leq g < \infty$ in endogenous consumption [labor income] taxes. For Case 5 (10) of Figure 9 (15), the steady state is locally indeterminate (divergent) for extremely wide range of government expenditure $g_{cH} < g < \infty$ ($g_{wH} < g < \infty$), while it is locally divergent (indeterminate) for relatively small range of government expenditure $0 < g < g_{cH}$ ($0 < g < g_{wH}$). If $\alpha + \beta\Psi > 1$, therefore, endogenous consumption (labor income) taxes are more likely to drive the economy to an indeterminacy (a divergence) of equilibria.

Unlike the Ramsey growth model, therefore, we can conclude the following.

Proposition 8 *The probability of indeterminacy is much higher if the government willingly utilizes consumption taxes rather than labor income taxes as a means of obtaining the necessary revenue.*

This analytical result is completely the opposite of the well-know results described in *Remark 1*. As consumption (labor supply) is done only by old (young) agents in this OLG model, consumption (labor income) tax-financing means that old (young) generations are forced to bear the burden. The intergenerational burden are frequently argued in many countries facing an aging economy. Therefore, this paper also suggests an example that agents' belief-driven aggregate fluctuations are more likely if the government lightens the load for the young people at the sacrifice of the old people.

7 Interpretation

This section clarifies the reasons behind the analytical results in Section 6. It is relatively easy to interpret the reasons if we focus on the empirically plausible sizes of labor externalities, i.e., $\beta < 1$. See Basu and Fernald (1995, 1997). For the purpose, let us restate the worker's F.O.C

$$\frac{c_{t+1}^{-\phi}}{(1 + \tau_{ct+1})/r_{t+1}} = \frac{l_t^\xi}{(1 - \tau_{wt})w_t} \quad (24)$$

Suppose that agents expect the rate of return on capital stock r_{t+1} to rise in the near future, when the economy is in its steady state. Footnote 8 shows that the optimistic expectation induces the labor supply curve to shift rightward. If the labor demand curve is downward sloping, i.e., $\beta < 1$, the hours of work l_t rise. (8) and (10) mean that the increase in l_t raises future capital k_{t+1} and thus the returns on capital r_{t+1} is reduced. Moreover, the substitutability between leisure and consumption generates a higher level of future consumption c_{t+1} . See Assumption 1.

Then, the righthand side in (24) increases, while the lefthand side in (24) decreases. Future labor l_{t+1} in the real interest rate r_{t+1} must appropriately adjust to satisfy (24). If the increase in l_{t+1} is not (is) large relative to the increase in l_t , the steady state is convergent (divergent) and thus an indeterminacy (a divergence) of equilibria actually happens.

In endogenous consumption taxes, the tax rates are determined by $\tau_{ct+t} = g/l_{t+1}^{1/\Psi}$. Noting the substitutability between leisure and consumption, i.e., $0 < \Psi < 1$, the tax rate τ_{ct+1} decrease easily when the future labor l_{t+1} rises to be consistent with (24). Then, the future labor l_{t+1} must increase less greatly to satisfy (24), because the righthand side in (24) increases more largely for a given increase in l_{t+1} . Therefore, a convergence of equilibria is more likely.

In endogenous labor income taxes, the labor income tax rate is specified as $\tau_{wt} = g/(bAk_t^\alpha l_t^\beta)$. Noting that $\beta < 1$ is assumed in this section, the decrease in the tax rate τ_{wt} is not so large, when l_t rises. Then, the increase in the righthand side is still high and the future labor l_{t+1} need to increase greatly to satisfy (24). Thus, a divergence of equilibria is more likely in this financing method.

8 Comparison 2

To associate the results in this OLG model with in Woodford's finance constrained economy, let us check the agents' consumption behaviors arising in these models. As agents do not care about their young period-consumption in this OLG model [Woodford's finance constrained economy (1986)], all their labor income earned in youth are completely saved in the form of capital [money].¹¹

We can obtain the same dynamic equations as (8) and (9) in endogenous capital income taxes even if we replace the budget in old period (3) and the government budget (7) by $c_{t+1} = (1 - \tau_{rt+1}) \cdot r_{t+1}s_t$ and $g = \tau_{rt} \cdot r_t k_t$, respectively, where τ_{rt} denotes the capital income tax rate. In this OLG model, therefore, consumption tax-financing is analytically equivalent to capital income tax-financing and the arguments in endogenous consumption taxes are equally true of the ones in endogenous capital income taxes.¹²

Using Woodford's finance constrained economy [the OLG model], Gokan (2008) [the present paper] proved the following.

Remark 2 *Based on Woodford's finance constrained economy (1986) [the OLG model], Gokan (2008) [the present paper] clarified that an indeterminacy of equilibria is more [less] likely in endogenous labor income taxes, while a divergence of equilibria is more [less] likely in endogenous capital income taxes.*

¹¹In Woodford's finance constrained economy, the representative worker maximizes the utility $c_{t+1}^{1-\phi}/(1-\phi) - l_t^{1+\xi}/(1+\xi)$, subject to the budget constraint $p_{t+1}c_{t+1} = (1 - \tau_{wt})p_t w_t l_t = M_{t+1}$ where p_i is the i th period price of consumption, M_{t+1} is the nominal outside money held at the end of this period.

¹²In this OLG model (Woodford's finance constrained economy), consumption tax-financing is the same as capital [labor] income tax-financing. This is because the workers care only about the old period-consumption and save all their labor income earned in youth in the form of capital (money).

As emphasized in Section 4, private investment k_{t+1} is funded with labor (capital) income in the OLG model (Woodford's finance constrained economy).¹³ Proposition 3 shows that the patterns of growth are not influenced by how private investment is financed if the government sector is absent in these two models. However, Remark 2 shows that if how public spending is financed is analytically considered in these models, tax policy has the completely opposite implications for the patterns of growth and aggregate fluctuations, depending on how private investment is funded. Therefore, the financing methods of private investment as well as public spending can be regarded as the important factors affecting the dynamically economic behaviors.

Finally, let us consider the local dynamics if all the tax rates on labor and capital income and consumption are fixed at $\tau_{wt} = \tau_w$, $\tau_{rt} = \tau_r$ and $\tau_{ct} = \tau_c$. The existing literature can be summarized as follows:

Remark 3 *In the Ramsey optimal growth model [Woodford's finance constrained model], the fixed tax rates on labor and capital income have no impact on local dynamics if government expenditure endogenously adjusts to balance his budget for fixed income tax rates. See Guo and Lansing (1998) [Gokan (2008)]*

Then, we can get the two dynamic equations $(1 - \tau_w) w(k_t, \Psi(c_{t+1})) \cdot \Psi(c_{t+1}) = k_{t+1}$ and $(1 - \tau_r) r(k_t, \Psi(c_{t+1})) \cdot k_t = (1 + \tau_c) c_t$ instead of (8) and (9) or (10) and (11). Then, we can easily verify the uniqueness of steady state and derive the same Trace and Determinant as in Footnote 6. Unlike *Remarks 1 and 2*, therefore, *Remark 3* hold also in this OLG model.

9 Concluding remarks

Based on the OLG model proposed by Reichlin (1986) with production externalities, this paper examines how the patterns of growth are differently influenced by adjusting consumption or labor income tax rate as a means of removing the budget deficits. By comparing the present paper with Gokan (2008) analyzing Woodford's finance constrained model (1986), the following can be clarified.

If the government sector is absent in these growth models, the patterns of growth are entirely determined only by the degree of production externalities and are not influenced by how private investment is funded. However, if the government expenditure financing is introduced in these models, the analytical consequences of tax policy for the patterns of growth are completely reversed, depending on whether private investment is funded with labor or capital income.

As for the analytical results in the OLG model on which the present paper focuses, we can summarize as follows.

Suppose that increasing returns in production are moderately low. If public spending is not so large, the two non-zero steady states coexist. Irrespective of public financing

¹³In woodford's finance constrained model, the behavior of private investment is specified by $k_{t+1} = \gamma [(1 - \delta) k_t + (1 - \tau_{rt}) r_t k_t]$ that δ denotes the depreciation rate of capital and γ denotes the capitalists' discount rate on the future.

methods, the high steady state is locally determinate. However, when consumption (labor income) taxes are willingly chosen to cut the budget deficits, the low steady state is more likely to be locally indeterminate (locally divergent), but less likely to be locally divergent (locally indeterminate). The Laffer curve-relation exists between the tax rate and the tax revenue, and an indeterminacy (a divergency) of equilibria emerges only in the downward-side of the Laffer curve.

Next, suppose that increasing returns are moderately high. For any non-negative values of government expenditure financed through labor income and consumption taxes, the uniqueness of the steady state is always verified, and there exists the monotonically positive relation between the tax rate and the tax revenue. If consumption (labor income) tax rates endogenously adjust to eliminate the budget deficits, however, the steady state is locally indeterminate (locally divergent) for extremely wide range of values of public spending and is locally divergent (locally indeterminate) for relatively narrow range of public spending.

In this OLG model considered in the present paper, agents do not care about the young period-consumption. They save all the labor income earned when young and cover the old period-consumption with the interest income. In this economy, we can easily prove that consumption tax-financing is analytically equivalent to capital income tax-financing. That is why the above arguments are equally true of the comparison between endogenous capital and labor income taxes. Using Woodford's finance constrained model (1986), Gokan (2008) showed that an indeterminacy (a divergence) of equilibria is more likely to occur in endogenous labor (capital) income taxes. The analytical results in the present paper also overturns for the ones in Woodford's finance constrained model. Noting that private investment is funded with labor (capital) income in this OLG model (Woodford's finance constrained economy), the consequence of cutting the budget deficits for the real economy depends greatly on how the private investment is funded.

As for the Ramsey optimal growth model, Schmitt-Grohe and Uribe (1998) showed that indeterminacy can arise only in the upward-side of the Laffer curve if public spending is financed through labor income taxes. In contrast, Giannitsarou clarified that if consumption taxes are actively used to obtain the necessary revenue, no indeterminacy arises and the Laffer curve-relation disappears between steady state tax rate and tax revenue. Unlike the OLG model, therefore, an indeterminacy of equilibria is more likely in endogenous labor income taxes than in endogenous consumption taxes. Therefore, the analytical results in this paper are also the opposite of the ones in the Ramsey growth model.

Appendix A (Proof of lemma 2)

Let us prove the latter in lemma 2. From (11), we can obtain:

$$\frac{dc_i^*}{dk_i^*} = -\frac{\alpha - 1}{\beta\Psi} \left(\frac{c_i^*}{k_i^*} \right). \quad (i = 1, 2) \quad (\text{A-1})$$

Using (12), the following can be obtained.

$$\frac{dc_i^*}{dk_i^*} = \frac{a}{b}. \quad (i = 1, 2) \quad (\text{A-2})$$

Fig. 5 illustrates that at the low (high) steady state,

$$(\text{A-1}) > (<) (\text{A-2}) \quad (\text{A-3})$$

Use of (15) and (16) provides

$$\begin{aligned} & D_i - T_i + 1 \\ &= \left(\frac{b}{a}\right) \left[\frac{\alpha - 1}{\beta\Psi} \left(\frac{c_i^*}{k_i^*}\right) + \frac{a}{b} \right]. \quad (i = 1, 2) \end{aligned} \quad (\text{A-4})$$

Considering (A-3) and (A-4), we can verify

$$D_1 < T_1 - 1 \text{ and } D_2 > T_2 - 1.$$

Appendix B (Proof of lemma 5)

Let us prove the latter in lemma 5. From (16), we can obtain:

$$\frac{dc_i^*}{dk_i^*} = \frac{\alpha}{1 - \beta\Psi} \left(\frac{c_i^*}{k_i^*}\right). \quad (i = 1, 2) \quad (\text{B-1})$$

Noting (15), the following can be obtained.

$$\frac{dc_i^*}{dk_i^*} = \frac{a}{b}. \quad (i = 1, 2) \quad (\text{B-2})$$

Fig. 7 shows that at the low (high) steady state,

$$(\text{B-1}) > (<) (\text{B-2}) \quad (\text{B-3})$$

Considering (22) and (23) provides

$$\begin{aligned} & D_i - T_i + 1 \\ &= \left[\frac{a}{b} - \frac{\alpha}{1 - \beta\Psi} \left(\frac{c_i^*}{k_i^*}\right) \right]. \quad (i = 1, 2) \end{aligned} \quad (\text{B-4})$$

From (B-3) and (B-4), we can verify

$$D_1 < T_1 - 1 \text{ and } D_2 > T_2 - 1.$$

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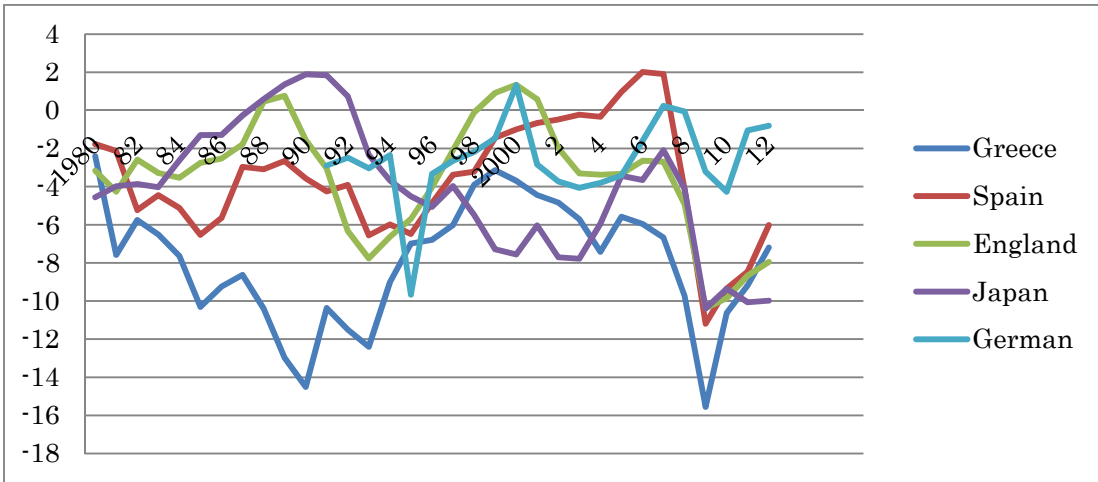


Fig.1: Financial Balance-GDP ratio

Source: IMF – World Economic Outlook Databases (April, 2012)



Fig.2: VAT (Value Added Tax) rates

Source: Ministry of Finance Japan (January 2012)

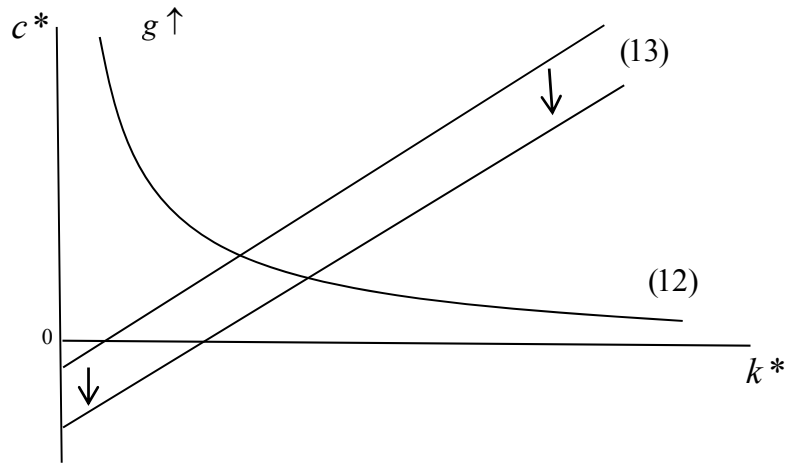


Fig. 3: Endogenous consumption taxes and $\alpha > 1$

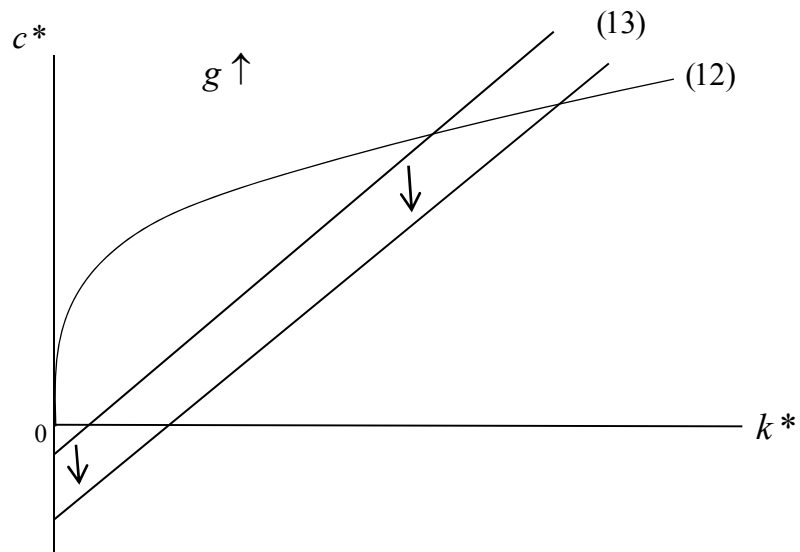


Fig. 4: Endogenous consumption taxes, $\alpha + \beta \cdot \Psi > 1$ and $\alpha < 1$

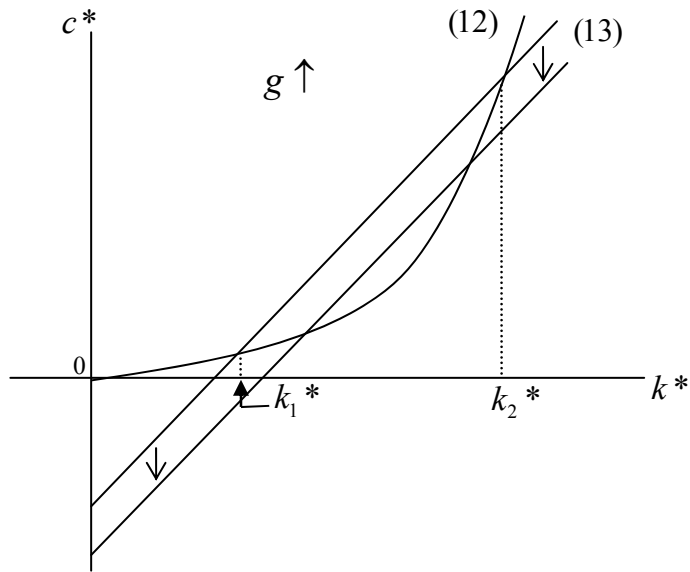


Fig. 5: Endogenous consumption taxes and $\alpha + \beta \cdot \Psi < 1$

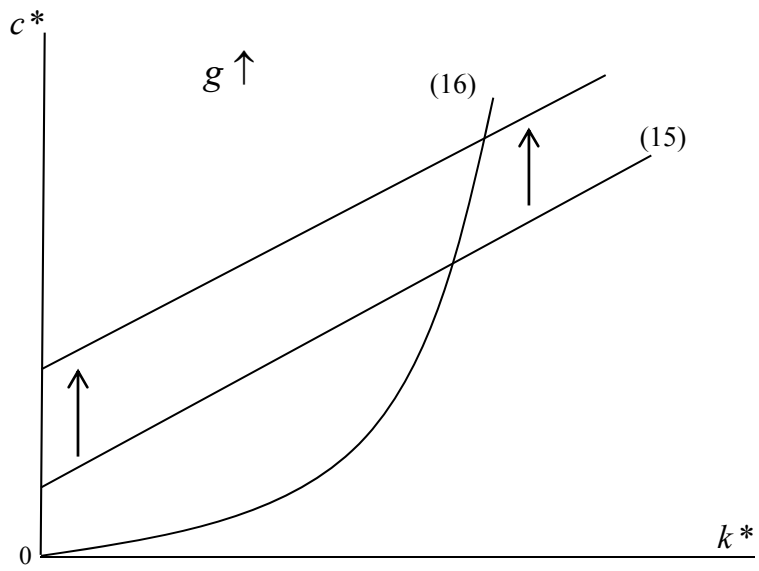


Fig. 6: Endogenous labor income taxes and $\alpha + \beta \cdot \Psi > 1$

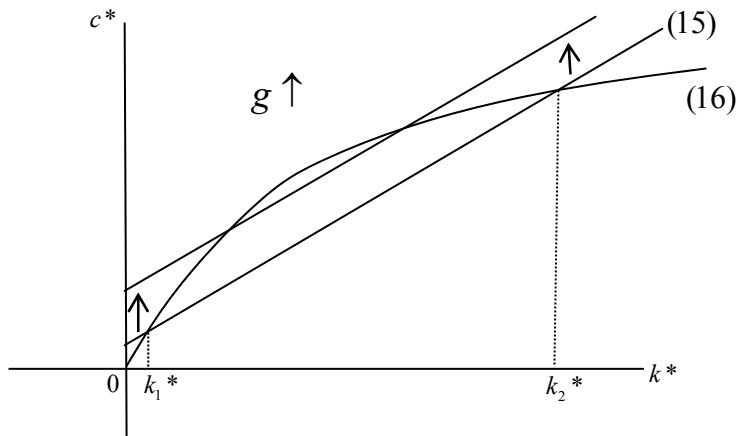


Fig. 7: Endogenous labor income taxes and $\alpha + \beta \cdot \Psi < 1$

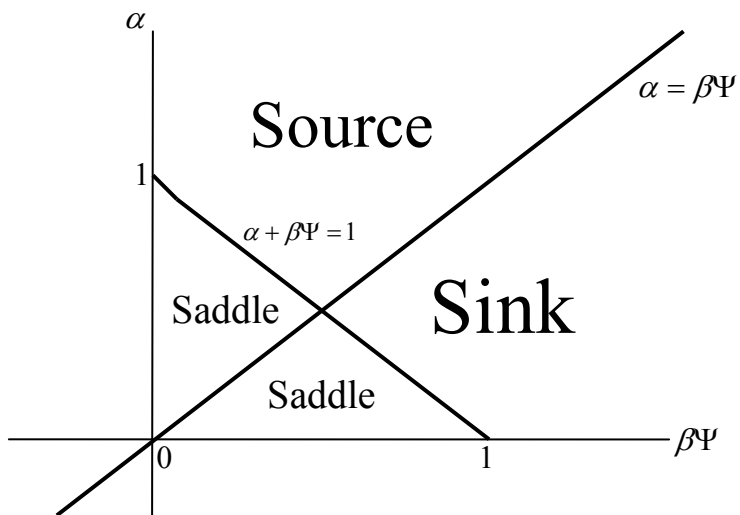


Fig. 8: Benchmark case

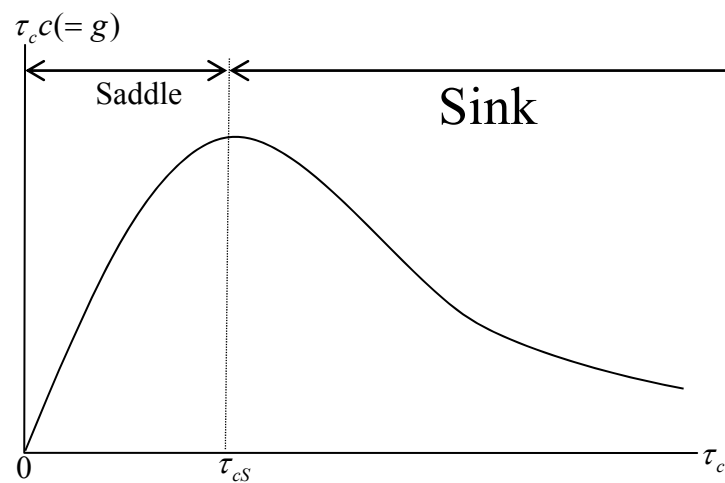


Fig.11: Consumption taxes, $0 < \alpha < \frac{1}{2}$ and $\alpha + \beta \cdot \Psi < 1$ (Case 1)

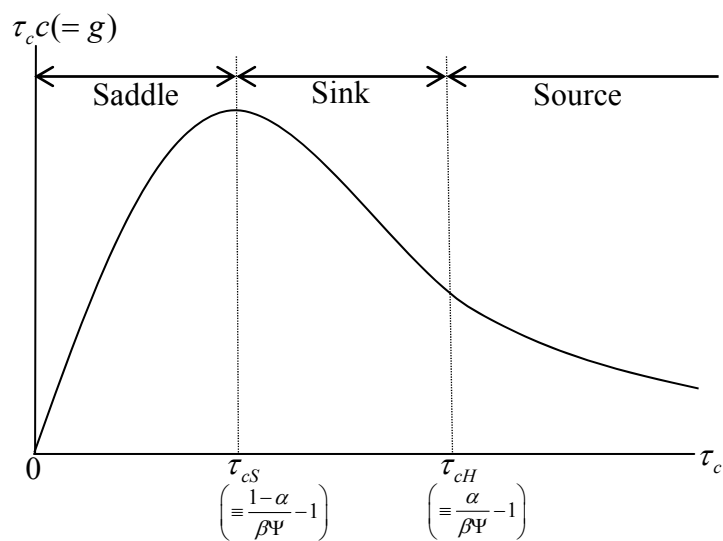


Fig.12: Consumption taxes, $\frac{1}{2} < \alpha < 1$ and $\alpha + \beta \cdot \Psi < 1$ (Case 2)

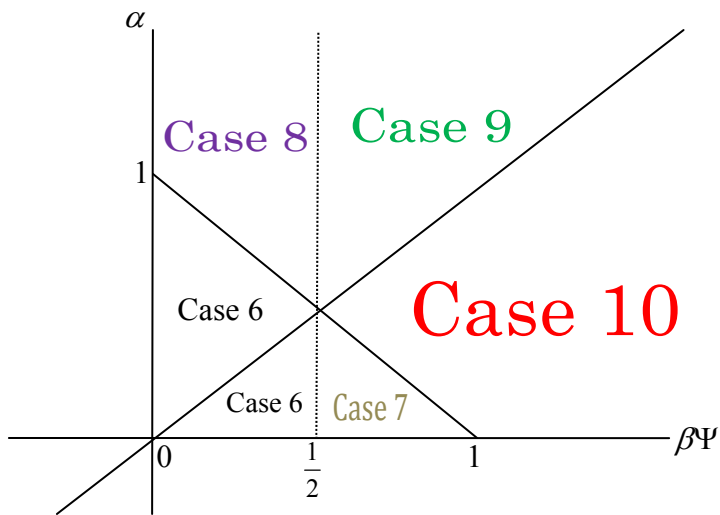


Fig.15: Labor income taxes

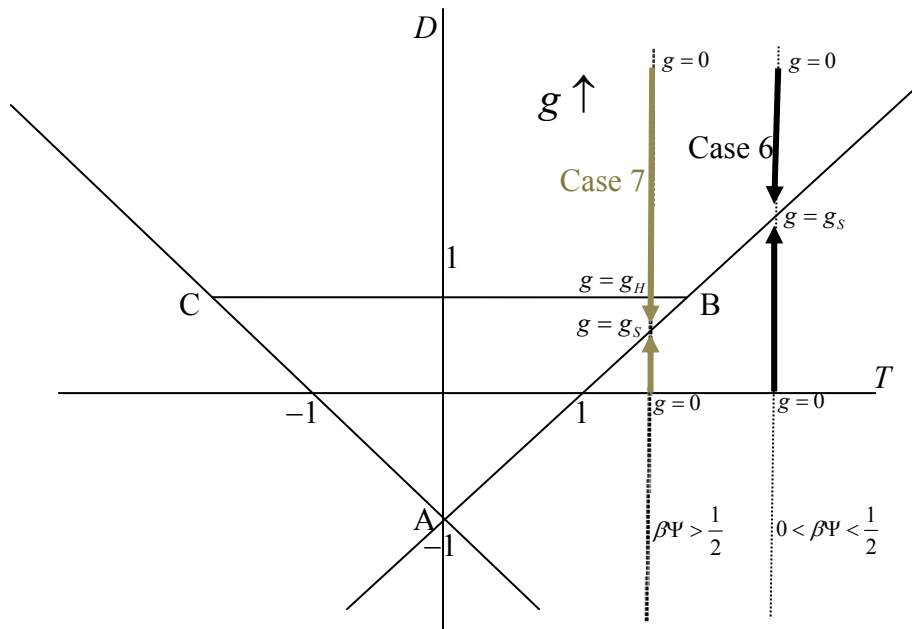


Fig. 16: Endogenous labor income taxes and $\alpha + \beta \cdot \Psi < 1$

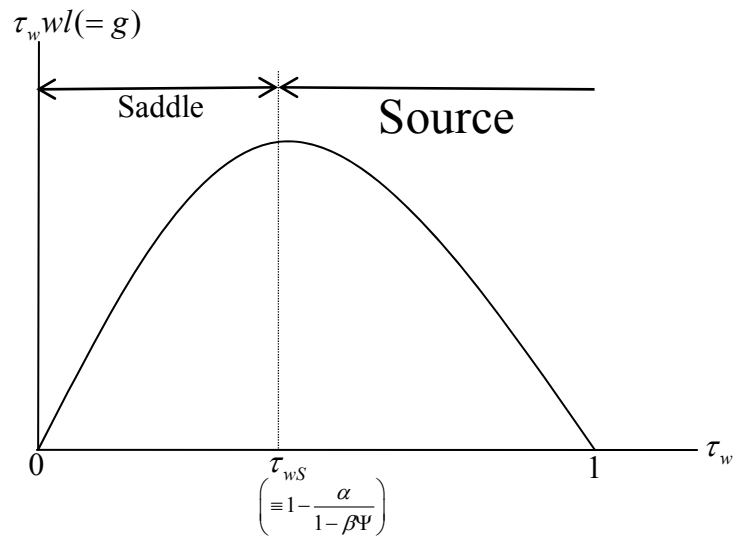


Fig.17: Labor income taxes, $0 < \beta\Psi < \frac{1}{2}$ and $\alpha + \beta \cdot \Psi < 1$ (Case 6)

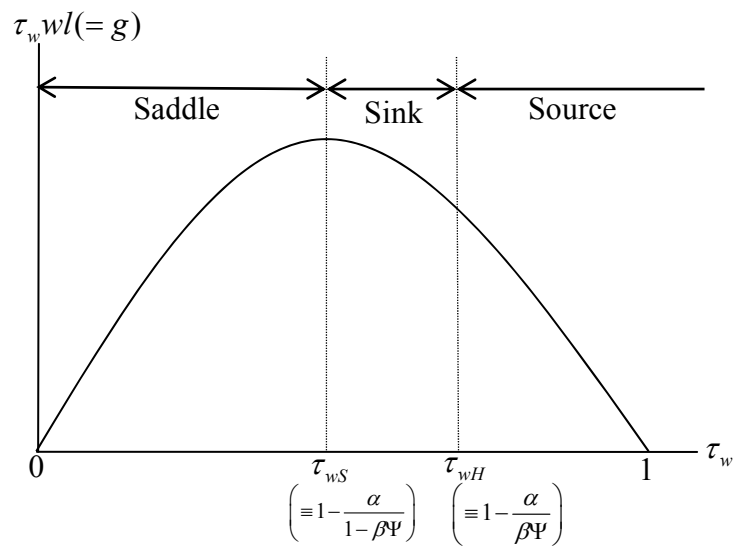


Fig.18: Labor income taxes, $\frac{1}{2} < \beta\Psi < 1$ and $\alpha + \beta \cdot \Psi < 1$ (Case 7)

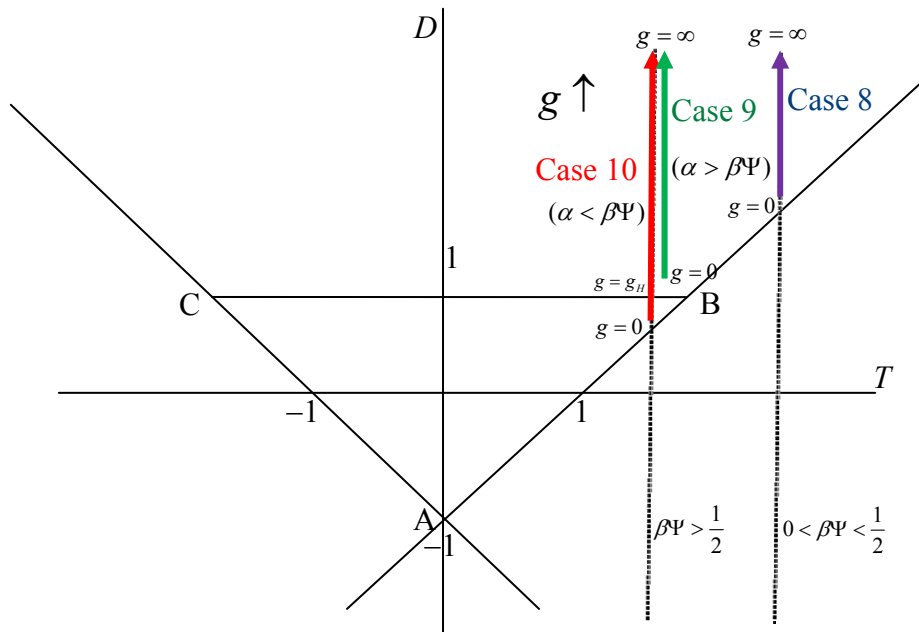


Fig.19: Endogenous labor income taxes and $\alpha + \beta \cdot \Psi > 1$

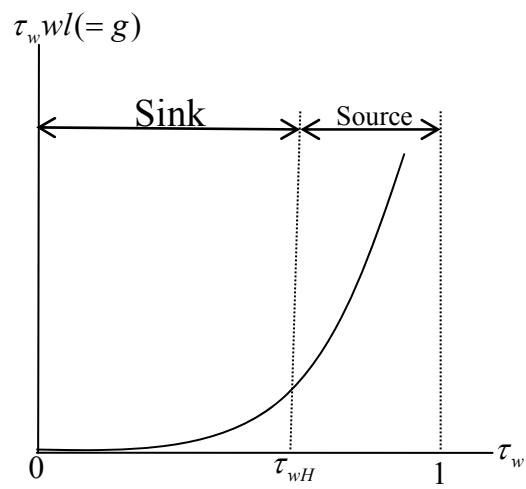


Fig.20: Labor income taxes, $\beta\Psi > \frac{1}{2}$, $\alpha < \beta\Psi$ and $\alpha + \beta \cdot \Psi > 1$ (Case 10)