Equilibrium Matching and Termination

Cheng Wang and Youzhi Yang*

Earlier versions: 2007, 2008, 2012. This version: May 2013

Abstract

In an equilibrium model of the labor market with moral hazard, jobs are dynamic contracts, job separations are terminations of optimal dynamic contracts, and terminations are used both as an incentive device and as a means for minimizing the cost of compensation. Transitions from unemployment to new jobs are modeled as a process of random matching and Nash bargaining. Non-employed workers make consumption and saving decisions as in a standard growth model, but they must also decide whether or not to participate in the labor market. The stationary equilibrium of the model is characterized. We then calibrate the model to the U.S. labor market to study quantitatively the worker flows and distributions, and the compensation dynamics.

*Wang: School of Economics, Fudan University and Department of Economics, Iowa State University; chewang@iastate.edu. Yang: Department of Economics, Shanghai University of Finance and Economics; yang.youzhi@mail.shufe.edu.cn. Yang thanks the financial support from the Leading Academic Discipline Program, 211 Project for Shanghai University of Finance and Economics (the 4th phase). We thank the participants of various seminars and meetings for discussions and comments.

1 Introduction

This paper constructs an equilibrium model of the labor market where workers are risk averse and employment relationships are subject to moral hazard. Jobs are dynamic contracts with endogenous termination. Vacant firms and unemployed workers are randomly matched in the labor market to bargain over the values of the dynamic contract for each party. Following termination, firms go back to the labor market to seek new matches. Non-employed workers make optimal consumption and saving decisions and must also decide whether or not to participate in the labor market. Finally, firms freely enter and exit the labor market to endogenously determine the total number of jobs in the economy.

Essentially, what we do is to put an optimal dynamic contract with endogenous termination in the Mortensen and Pissarides (1994) model with risk averse workers. This allows us to combine, in a unified framework, three essential components of the labor market: the interaction between unemployed workers and vacant firms, the endogenous dynamics within the employment contract which leads to job separation, and the non-employed worker's decisions pertaining to labor force participation. Such a framework, described above, incorporates three existing classes of labor market models. The first is the equilibrium search-matching models (Mortensen and Pissarides, 1994) that emphasize matching and bargaining in the labor market and free entry and exit of firms. The second is the models of the labor market with private information, starting at least from the efficiency wage models (e.g., Shapiro and Stiglitz, 1984). These models emphasize the role of moral hazard and other types of information asymmetry in individual employment contracting, and in the determination of aggregate outcomes. The third is the standard neoclassical growth model for labor market analysis where risk averse agents make optimal decisions on intertemporal consumption and saving choices, as well as on labor force participation. Our model captures all the important aspects of the labor market that the above discussed models emphasize, individually but not simultaneously. More importantly, relative to the existing benchmark models, our model, with a single source of information asymmetry and no other exogenously imposed heterogeneities, goes farther on several diomensions in accounting for the observed labor market dynamics and distributions.

Relative to Mortensen and Pissarides (1994) where an exogenous stochastic process governs job separation, by treating jobs as an optimal dynamic contract with endogenous termination, separation is an endogenous process in our model, motivated by the efficient provision of incentives and risk sharing. In addition, instead of assuming that workers and firms cannot commit to long-term relationships and wages are bargained sequentially period by period, we postulate that firms can commit to any long-term contract they agree to enter and that each pair of matched firm and worker bargain only once over the course of the employment relationship, before it begins.[1]

The labor market literature has long recognized the importance of moral hazard for employment contracting. Moral hazard plays a central role in the models of efficiency wages (e.g., Shapiro and Stiglitz, 1984; MacLeod and Malcomson, 1998; Rocheteau, 2001) in generating equilibrium involuntary unemployment. In Den Haan, Ramey and Watson (1999), moral hazard gives rise to inefficient job destruction and helps propagate aggregate shocks. Another development of the literature studies the role of optimal termination in long-term employment contracting under moral hazard (Stiglitz and Weiss, 1983; Spear and Wang, 2005; Sannikov, 2008; Wang, 2011, 2013). Wang (2013), for example, studies the effects of firing costs in an equilibrium model of the labor market with moral hazard, long-term contracts and endogenous termination.[2] Our paper is the first to bring a fully optimal dynamic contract into the Mortensen-Pissarides framework to study the role of moral hazard for employment dynamics, worker flows and distributions. Moral hazard and dynamic contracting are essential to our theory in the sense that, given risk aversion, all the dynamics and flows and distributions in the model are ultimately motivated by moral hazard and generated by the optimal dynamic contract.[3]

A key feature of our model is that severance compensation plays a critical role in the individual worker' decision on labor market participation. Severance compensation that the worker receives upon termination is part of the optimal contract and its size depends on the worker's history with the firm. After termination the severance compensation becomes the (non-employed) worker's initial asset holding with which he then chooses optimally whether or not to return to the labor market to look for a new job. A newly terminated worker would stay in the labor market if the size of his severance compensation is sufficiently small. He would quit the labor force permanently if the size of his severance compensation is sufficiently large. And in equilibrium there are newly terminated workers whose severance compensation is neither sufficiently small to justify immediate returning back to the labor market, nor sufficiently large to justify staying permanently out of the labor force. These workers would quit the labor force temporarily, dissave, and eventually go back to the labor market once their asset holding is reduced to a sufficiently

[1]The assumption of sequential wage bargaining, although helpful in generating an ergodic wage distribution in Mortensen and Pissarides (1994), implies that wages are not sufficiently rigid for the model to match business cycle movements in the data (Hall, 2005; Shimer, 2005a). Rudanko (2009) models long-term wage contracts with limited commitment in a search-matching model of the labor market to generate the observed wage rigidity/volatility. Obviously, our model offers a potential alternative for accounting for the observed wage rigidity/volatility that is based on an optimal trade off between incentives and risk sharing.

[2]There is also a large literature on unemployment insurance with moral hazard, including Shavell and Weiss (1979), Wang and Williamson (1996), Hopenhayn and Nicolini (1997).

[3]See Guerrieri (2008), Guerrieri, Shimer and Wright (2010), and Moen and Rosen (2011) for competitive search models with adverse selection.

low level. Thus our model generates not only flows into the state of nonparticipation from the state of employment, but also flows out from the state of nonparticipation and into the state of unemployment.

With the design described above, our model is able to produce outcomes that account, simultaneously, for four sets of observations that characterize modern labor markets. First, our model generates the stocks and composition of all three states of the labor market (employment, unemployment and nonparticipation), flows between employment and unemployment, and flows between participation (employment and unemployment) and nonparticipation. In particular, workers who produce a sequence of low outputs are laid off involuntarily because they have become too poor to motivate; workers who produce a sequence of high outputs retire from the firm because they are too expensive to compensate and motivate.[4] Moreover, retired workers whose severance compensation is not sufficiently large to justify permanent withdraw from the labor force will return to the labor market after being out of it for some periods of time. In addition, in each of the labor market states, there is a non-degenerate distribution of employed or non-employed workers who differ in their individual histories in the labor market.

Labor market data show significant flows of workers among all three states of the labor market.[5] Recent efforts to model explicitly the state of nonparticipation and the flows into and out of that state include Moscarini (2003) where two sources of worker heterogeneity and private information are introduced into the Mortensen-Pissarises framework to generate flows into the state of nonparticipation. Specifically, the match productivity depends on a match specific variable, as well as a non-match-specific variable that captures the ability of the worker. The values of both variables are learned during a match, workers whose non-match-specific variable is learned to be sufficiently low choose to withdraw from the labor market. Alvarez and Veracierto (1999) and Veracierto (2008) consider all three labor market states but they only put restrictions on the stocks of workers in the three states and do not model the flows into and out of the state of nonparticipation. Krusell, Mukoyama, Rogerson and Sahin (2011) put random matching in a neoclassical growth model with persistent idiosyncratic shocks to generate not only all three states of the labor market, but also flows among them. All the models discussed above rely either on exogenous idiosyncratic/aggregate shocks or on exogenous worker heterogeneity for generating the state of nonparticipation and related flows. In contrast, all workers and firms are identical in our model, and the model's only source of uncertainty - the output the worker produces - is an endogenous random variable that is determined by the worker's optimal effort choice. This aspect of our model is similar to Wang (2011) which nevertheless does not consider matching and

[4]This draws upon Spear and Wang (2005), Sannikov (2008), and Wang (2011).

[5]See Fallick and Fleischman (2004), Nagypal (2005), and Shimer (2005b).

bargaining in the labor market, neither does it model the flow out of the state of nonparticipation, and there is not a distinction between temporary and permanent nonparticipation.

Second, our model generates job stability and mobility dynamics that are consistent with empirical observations. Farber (1999) observes that three central facts describe job stability and mobility in modern labor markets: (1) long-term employment relationships are common; (2) most new jobs end early; and (3) the probability of a job ending declines with tenure. In our model, employment contracts are long-term because they offer the most efficient combination of incentives and risk sharing. New jobs end early because the expected utility for new hires starts low, and in order to enforce incentives this low expected utility must be made even lower in case the worker produces a low output, and this can only be achieved by way of terminating the worker.[6] On the other hand, older workers who on average have longer tenures and higher expected utilities are more likely to receive a pay cut rather than a layoff notice in case a low output is produced. Last, in our model, workers who have a longer tenure with the firm on average have a history of better performance with the firm which, in turn, implies that termination is less likely to be used for the purpose of enforcing incentives, although these workers are more likely to retire from that firm.

Third, our model, after being calibrated to the U.S. data, generates a monotonic average compensation-tenure relationship that is commonly observed in modern labor markets. Why do wages increase with tenure? There is a large literature that seeks to provide an explanation. One strand of this literature suggests that this is so because firm-specific productivity increases with tenure.[7] Another strand of the literature explains the same observation based on the idea of on-the-job-search.[8] Our explanation is based on the efficient provision of dynamic incentives and risk sharing. In our model, workers who have stayed longer are more likely to have stronger performance records, are promised higher expected utilities, and are thus given higher compensation.

Fourth, relative to standard search-matching models, our model provides a better account for the observed large wage dispersion among individual workers. Hornstein, Krusell and Violante (2011) observes that standard equilibrium search-matching models can generate only a very small, about 3.6%, differential between the average and the lowest wages paid in the U.S. labor market, and the observed Mm ratio - the ratio between the average wage and lowest wage paid - is at least

[6] The logic of the efficiency wage models applies. Under the assumption of non-negative compensation, retaining a worker while giving him an expected utility that is too low would not be consistent with incentives. The difference though is that termination never actually occurs in the efficiency wage models.

[7] See for example Brown (1989).

[8] That is, as the firm's optimal response to the worker's outside offers, compensation is backloaded to reduce worker quit. See for example Burdett and Coles (2003).

twenty times larger than what the model is able to generate.[9] Our model generates a much larger wage dispersion which is compatible with the short average unemployment duration in the data. In our model, over time identical workers who start employment with equal expected utility fan out in expected utility and compensation, as the outputs they produce follow a stochastic process and the compensation, both current and future, they receive are tied to performance through the optimal dynamic contract.

Here a reference must be made particularly to Moscarini (2005) for endogenizing the employment and termination dynamics in Mortensen and Passarides (1994), and for generating equilibrium wage dispersion. Instead of viewing employment relationships as a dynamic contract, he uses the model of Jovanovic (1979) to describe the process of job dynamics and separation. There, the firm learns efficiently about the worker's productivity over time and compensates him optimally, and separation occurs once the perceived productivity of the worker becomes sufficiently low.

The model is presented in the next section. In Sections 3 and 4 we formulate and analyze a stationary equilibrium of the model. We then calibrate the model to the U.S. labor market in Section 5 to study the structure of the equilibrium dynamic contract, the stocks and flows of the labor market, and worker compensation dynamics. Section 6 concludes the paper.

2 Model

Let time be denoted $t = 1, 2, \cdots$ The model economy has one consumption good, and is populated by one unit of homogeneous workers. Workers survive into the next period with a constant probability $\Delta \in (0, 1)$. At the beginning of each period, $1 - \Delta$ units of workers are born so that the measure of workers in each period is constant at one. Workers who are born in period $\tau (\geq 1)$ have the following preferences:

$$\mathbb{E}_\tau \left[\sum_{t=\tau}^{\infty} (\beta\Delta)^{t-\tau} (u(c_t) - \phi(a_t)) \right].$$

Here $\beta \in (0, 1)$ denotes the worker's discount factor. $u : \mathbb{R}_+ \to \mathbb{R}$ is the worker's period utility function, c_t his consumption in period t. $\phi : 0 \cup [\underline{a}, \overline{a}] \to \mathbb{R}$, where $\overline{a} > \underline{a} \geq 0$, denotes the worker's disutility function, a_t his effort. Assume if the worker is not employed in any period t,

[9] As the paper explains, "The short unemployment durations, as in the U.S. data, reveal that agents in the model do not find it worthwhile to wait because frictional wage inequality is tiny. The message of search theory is that 'good things come to those who wait", so if the wait is short, it must be that good things are not likely to happen.' (page 9.) The paper further shows that the extensions of the standard search and matching models can only modestly improve their performance on accounting for the observed Mm ratio.

then $a_t = 0$; if he is employed, then $a_t \in [\underline{a}, \overline{a}]$. We make the following assumptions on u and ϕ: u is bounded, strictly increasing, strictly concave, and twice differentiable; ϕ is strictly increasing, strictly convex, twice differentiable on $[\underline{a}, \overline{a}]$, and $\phi(\underline{a}) \geq \phi(0)$.

The model economy is also populated with a large measure of homogeneous firms. Firms maximize expected discounted profits, and they discount future profits using a constant discount factor $1/(1+r)$, where $r > 0$ denotes the interest rate firms face. In any given period, some firms are in the market, the rest not. Firms are allowed to freely enter or exit the market and so the measure of the firms that are in the market, denoted $\gamma(\geq 0)$, is an endogenous variable. Firms in the market must be matched with a worker in order to produce. A matched pair of firm and worker creates a job.

In any given period, the total measure of matches formed in the labor market is equal to $M(\eta_U, \gamma - \eta_E)$, where η_U is the measure of unemployed workers (non-employed and actively looking for a job) in the labor market, and η_E is the measure of employed workers when the labor market opens, and hence $\gamma - \eta_E$ is the measure of vacant (recruiting) firms in the labor market. Throughout the paper, we assume the matching function is such that

$$0 \leq M(\eta_U, \gamma - \eta_E) < \min\{\eta_U, \gamma - \eta_E\},$$

so there is always a positive measure of workers and firms that are not matched.

A firm that fails to find a match could either exit the market or operate as a dormant firm in the remainder of the period, while waiting for the labor market to open next period. We follow the literature to assume that a vacant firm must incur a fixed cost $c_0(\geq 0)$ in order to stay open to job applications.

The matched firm and worker Nash bargain over a dynamic employment contract. This dynamic contract specifies a history contingent rule for compensating and terminating the worker. Once they agree on a specific contract, this contract cannot be renegotiated in any future periods.

Production then takes place immediately after a contract is agreed upon. In each period, the employed worker produces a random output θ from the set $\{\theta_1, \cdots, \theta_n\}$, with $\theta_1 < \theta_2 < \cdots < \theta_n$. The probability that each output level is produced depends on the effort a that the worker exerts in that period. Specifically, for each $i \in \{1, \cdots, n\}$ and each $a \in A \equiv [\underline{a}, \overline{a}]$, output θ_i is produced with probability $\pi_i(a) \geq 0$. Of course $\Sigma_i \pi_i(a) = 1$.

The model's information structure is the same as that in the standard model of moral hazard. Specifically, the worker's effort is not observed to the firm, but the output he produces is publicly observable and verifiable. Other parameters of the model are common knowledge to all agents in the model.

There is a risk free asset in the model: for each unit of the good invested in this asset, it returns $1 + r$ units of the good next period. The market for this asset is incomplete in that workers can save through this market by holding a nonnegative amout of the asset, but cannot borrow through this market by holding a negative amount of the asset. To avoid additional information asymmetry, we assume that all investments in this asset are public information, and can be transferred between workers and firms at zero costs.

In addition to the risk free asset described above, workers also have access to a competitive insurance market where one unit of consumption in the current period can be exchanged for $1/\Delta$ units of consumption in the next period conditional on the worker's survival.

As part of the model's physical environment, we make four assumptions about what contracts are feasible between the worker and the firm. First, contracts are subject to a non-negativity constraint that requires that compensation to the worker be non-negative. Second, once the worker and the firm agree on a contract, they can commit to not renegotiating the continuations of the contract in all future periods. Third, firms can fully commit to the terms of any long-term contract it agrees to enter, whereas workers' commitment to a long-term contract is limited: they are free to leave an ongoing long-term contract anytime they have a better outside opportunity. Fourth, severance compensation must be paid in lump-sum amounts to the worker immediately upon termination. After the ending of the employment relationship, no further interactions between the firm and the worker are feasible.

3 Equilibrium

In this section, we formulate the economy's stationary equilibrium. We first describe the economy's aggregate state variables. We then describe the problem of optimal contracting and the problems for the vacant firms and non-employed workers, taking the aggregate states as given. We then describe the process of bargaining between the matched worker and firm. The equilibrium is then defined by requiring that the aggregate states and individual optimizations be consistent with each other, the economy's stocks and flows be consistent with each other, and that the firms in the market be making zero profits.

3.1 The aggregate states

At the beginning of each period, the state of the economy is characterized by the set of aggregate state variables

$$\Sigma = \{\gamma, (\mu_E, \eta_E),\ (\mu_U, \eta_U),\ (\mu_N, \eta_N)\}.$$

Here, $\gamma (> 0)$ denotes the measure of firms in the market. The scalar $\eta_E \in [0, 1]$ denotes the measure of employed workers. The employed workers are distributed according to the distribution function $\mu_E : X \to [0, 1]$ where

$$X \equiv \left[\frac{u(0) - \phi(\bar{a})}{1 - \beta\Delta}, \frac{u(\infty) - \phi(0)}{1 - \beta\Delta}\right) \equiv [V_{\min}, V_{\max}).$$ [10]

The scalar $\eta_U \in [0, 1]$ is the measure of unemployed workers (non-employed and actively looking for a job). These workers are distributed in the amount of assets they hold according to the distribution function $\mu_U : \mathbb{R}_+ \to [0, 1]$. The scalar $\eta_N \in [0, 1]$ is the measure of non-employed workers who do not participate in the labor force. These workers are distributed, again in asset holding, according to the distribution function $\mu_N : \mathbb{R}_+ \to [0, 1]$. Clearly, the measures of the workers in the different states of the labor market, η_E, η_U, and η_N, must satisfy

$$\eta_E + \eta_U + \eta_N = 1. \tag{1}$$

Given the above, let λ denote the fraction of vacant firms to obtain a match in each period:

$$\lambda = \frac{M(\eta_U, \gamma - \eta_E)}{\gamma - \eta_E}. \tag{2}$$

Let ρ denote the fraction of unemployed workers to be matched with a vacant firm in each period:

$$\rho = \frac{M(\eta_U, \gamma - \eta_E)}{\eta_U}. \tag{3}$$

The restriction we put on the matching function M in Section 2 ensures $0 < \lambda, \rho < 1$.

In the next four subsections, we formulate a system of Bellman equations for the values of the firms and the workers, along with their optimal strategies, taking as given Σ, λ, and ρ.

3.2 The Optimal Contract

Once an unemployed worker is matched with a vacant firm, they bargain over an optimal contract which promises to deliver, over the course of the employment relationship, a sequence of state

[10] Note that since at this stage we are not putting any restrictions on the distribution function μ_E, the set X essentially includes all possible levels of expected utility for any worker in the model.

contingent compensation/consumption to the worker. As part of the employment agreement, the worker gives the firm all the assets he holds before the job starts and promises not to hold any assets before the job ends.[11] Given this qualification, any contract, denoted σ, takes the following recursive form:

$$\{a(V), \Omega(V), \{c_i(V), s_i(V), V_i(V)\}_{i=1}^n, V \in \Phi\}.$$

Here of course we follow Green (1987) and Spear and Srivastava (1987) to use the worker's beginning of period expected utility $V \in \Phi$ as a state variable to summarize the worker's history at the firm. The set $\Phi \subseteq X$ contains all expected utilities of the employed worker that can be delivered by a feasible and incentive compatible contract (to be defined shortly). Note Φ is an endogenous variable of the model. Then, for all $V \in \Phi$, $a(V) \in A$ denotes the worker's effort in the current period that the contract recommends, and $\Omega(V) \subseteq \{1, \cdots, n\}$ denotes the set of the worker's current output realizations in which the worker is retained and outside which the worker is terminated. Finally, $c_i(V) \in \mathbb{R}_+$, $s_i(V) \in \mathbb{R}_+$ and $V_i(V) \in \Phi$ are, respectively, the worker's current period compensation (consumption), the severance compensation he receives if he is terminated (i.e., $i \notin \Omega(V)$), and the continuation expected utility he is promised to receive if he is retained (i.e., $i \in \Omega(V)$), all conditional on his current period output being θ_i.

The optimal contract is then defined as a solution to a Bellman equation to be described below. For that, three value functions must be defined. First, for all $V \in \Phi$, let $U(V) \in \mathbb{R}$ denote the maximum value of a firm who currently employs a worker with expected utility V. Second, let $\overline{U} \in \mathbb{R}$ denote the value of a vacant firm: a firm that is free to hire a new worker at the beginning of a period. Third, for all $s \in \mathbb{R}_+$, let $v(s) \in X$ denote the maximized beginning-of-period (before the labor market opens) expected utility of a non-employed worker holding assets s. Notice that $U(V)$ and V represent the firm and worker's values in the contract respectively, and $\overline{U} - s$ and $v(s)$ their values exiting the contract, respectively.

Taking $\overline{U}$ and v as given (they will be determined in the following two subsections), the optimal contract and the firm's value function U must satisfy the following Bellman equation:

[11] This assumption that the employed worker is not allowed to save is made without loss of generality, under the model's assumption that the worker's assets are publicly observable and freely *transferable* between the contracting parties. Specifically, it is straightforward to show that any contract that allows the worker to hold assets during employment is weakly dominated by a contract which instead dictates that the worker submit his assets to the firm before the job starts and consume all this compensation each period as the job lasts. To save space we leave this for the reader to consider.

For all $V \in \Phi$,

$$\begin{aligned} U(V) &= \max_{a,\Omega,\{c_i,s_i,V_i\}_{i=1}^n} \sum_{i \notin \Omega} \pi_i(a) \left[(\theta_i - c_i) + \frac{\Delta}{1+r}(\overline{U} - s_i) \right] \\ &+ \sum_{i \in \Omega} \pi_i(a) \left[(\theta_i - c_i) + \frac{\Delta}{1+r} U(V_i) \right] + \frac{1-\Delta}{1+r}\overline{U} \end{aligned} \tag{4}$$

subject to

$$\sum_{i \notin \Omega} \pi_i(a)(u(c_i) + \beta\Delta v(s_i)) + \sum_{i \in \Omega} \pi_i(a)(u(c_i) + \beta\Delta V_i) - \phi(a) = V, \tag{5}$$

$$a \in \arg\max_{a' \in [\underline{a},\overline{a}]} \left\{ \sum_{i \notin \Omega} \pi_i(a')(u(c_i) + \beta\Delta v(s_i)) + \sum_{i \in \Omega} \pi_i(a')(u(c_i) + \beta\Delta V_i) - \phi(a') \right\}, \tag{6}$$

$$\Omega \subseteq \{1, \cdots, n\}, \tag{7}$$

$$c_i \geq 0, \ \forall i, \tag{8}$$

$$s_i \geq 0, \forall i \notin \Omega, \tag{9}$$

$$V_i \in \Phi, \forall i \in \Omega, \tag{10}$$

$$V_i \geq v(0), \ \forall i \in \Omega, \tag{11}$$

where Φ, the set of all expected utilities for an employed worker that can be delivered by a feasible and incentive compatible contract, is the largest self-generating set of the mapping $B : 2^X \to 2^X$ defined as follows: for all $\Phi' \subseteq X$,

$$B(\Phi') \equiv \{V \in X | \exists \{a, \Omega, \{c_i, s_i, V_i\}_{i=1}^n\} \ s.t. \ (5) - (9), \ (11), \text{ and } V_i \in \Phi', \ \forall i \in \Omega\}. \tag{12}$$

Note that since each worker dies with probability $(1 - \Delta)$, the firm faces in each period a constant probability of $(1-\Delta)$ to become vacant next period. Constraint (5) is a promise-keeping constraint, (6) is an incentive-compatibility constraint. Constraint (8) requires the worker's compensation be non-negative: a limited liability constraint. Constraint (10) requires that if the worker is retained, then his continuation expected utility V_i must be deliverable by a feasible and incentive compatible contract. Constraint (11) is a self-enforcing constraint. Under (11), the worker will not have incentives to leave the contract in any state of the world in which the contract prescribes retention. [12] Last, (12) provides a Bellman equation for the state space of the dynamic contract.[13]

3.3 The Vacant Firm

Consider the value of a firm that is vacant at the beginning of a period. With probability λ this firm is matched with an unemployed worker in the current labor market. This unemployed worker's asset holding s is drawn randomly from the distribution μ_U. Once matched, the worker exchanges his assets s for an employment contract that promises expected utility $V_m(s)$ for him. The value of $V_m(s)$, which is the outcome of the Nash bargaining between the two parties, will be given in section 3.5. where the bargaining process is described. The value of a firm that is vacant at the beginning of a period, $\overline{U}$, thus must solve the following Bellman equation:

$$\overline{U} = -c_0 + \left[\lambda \int_{\mathbb{R}_+} (U(V_m(s)) + s) d\mu_U(s) + (1-\lambda)\frac{\overline{U}}{1+r}\right]. \tag{13}$$

Remember c_0 is the fixed cost any firm in the labor market must incur for posting a vacancy.[14]

3.4 The Non-Employed Worker

Consider the problem of a worker who is not employed at the beginning of a period. This worker must decide how much to consume in the current period and, more importantly, whether or not to participate in the labor market. The state variable for these decisions is s, the assets he holds at the beginning of the period. The key part of the problem is to determine $S_U \subseteq \mathbb{R}_+$, the set of assets with which the non-employed worker would choose to participate in the current labor market. This S_U should contain all s with which there is a joint surplus for the worker and the firm he is matched with, should a match occur. In other words, in order for the worker to be willing to participate in the labor market, there must exist a feasible and incentive compatible contract to make the worker and the firm both better off should they form a match. Thus S_U, and its complement S_N, the set of all s with which the worker would not participate in the labor market, must satisfy the following Bellman equation:

$$S_U \equiv \left\{ s \in \mathbb{R}_+ \middle| \exists V \in \Phi \ s.t. \ U(V) + s \geq \frac{1}{1+r}\overline{U} \text{ and } V \geq V_n(s) \right\}, \tag{14}$$

[12] If the contract prescribes retention but the worker quits, then the worker receives no severance compensation from the firm.

[13] This follows Abreu, Pearce and Stacchetti (1990) and Wang (1995).

[14] Implicit in equation (13) is our assumption that assets are freely transferable between the worker and the firm. Suppose assets are not freely transferable. Suppose for example assets could not be transferred at all between the worker and the firm. Then an additional state variable, namely the worker's asset holding, must be defined for recursively formulating the dynamic contract. We leave this possibility for future work.

$$S_N \equiv \mathbb{R}_+ \setminus S_U, \tag{15}$$

where

$$V_n(s) = \max_{c \in [0,s]} \left\{ (u(c) - \phi(0)) + \beta \Delta v \left(\frac{1+r}{\Delta}(s-c) \right) \right\}, \ \forall s \in \mathbb{R}_+, \tag{16}$$

$$v(s) = \begin{cases} \rho V_m(s) + (1-\rho) V_n(s), \text{ if } s \in S_U \\ V_n(s), \text{ if } s \in S_N \end{cases}, \forall s \in \mathbb{R}_+. \tag{17}$$

In the above, $v(s) \in X$ is the maximized beginning-of-period (before the labor market opens) expected utility of a non-employed worker with assets s; $V_n(s)$ denotes the ex-post expected utility of this worker conditional on he not being matched with a firm (either he chose not to participate in the market ($s \in S_N$), or he went to the market ($s \in S_U$) but failed to find a match); $V_m(s)$ denotes the bargained expected utility of this worker conditional on he being matched with a firm (that is, he chose to participate in the market ($s \in S_U$) and found a match).

Equation (16) describes the optimization problem for the non-employed worker who is not matched with a firm, either he was in the market but failed to find a match, or he chose not to participate in the labor market. The problem for this worker, which is the same as that for the consumer in a typical growth model, is one of finding the optimal consumption and saving scheme. Equation (17) holds by definition.

3.5 The Nash Bargaining

The values of the optimal contract for the matched vacant firm and the unemployed worker are determined through Nash bargaining as follows: $\forall s \in S_U$,

$$V_m(s) = \arg\max_V \left(U(V) + s - \frac{1}{1+r}\overline{U} \right)^{\omega} (V - V_n(s))^{1-\omega} \tag{18}$$

subject to

$$V \in \Phi, \ U(V) + s \geq \frac{1}{1+r}\overline{U}, \ V \geq V_n(s). \tag{19}$$

Here the parameter $\omega \in (0,1)$ is the exogenously given bargaining weight for the firm. Since in each period each firm and each worker can find at most one match, $\overline{U}/(1+r)$ is the firm's reservation utility, and $V_n(s)$ is the worker's reservation utility. The bargaining thus involves choosing a level of expected utility V in the set Φ of deliverable expected utilities for the worker, and this expected utility must exceed the worker's reservation utility, must make the firm better off than its reservation utility, and must maximize the Nash product of surpluses.

Note that implicit in Equation (18) is the assumption that (a) the Nash bargaining problem has a solution and (b) the solution is unique. Proposition 1 in the next section will confirm this.

3.6 Equilibrium

A stationary equilibrium of the model is now defined.

Definition 1 *A stationary equilibrium of the model is a tuple* $\{\Sigma, \lambda, \rho, \sigma\}$, *where*

$$\sigma \equiv \left\{ \begin{array}{c} a(V), \Omega(V), \{c_i(V), s_i(V), V_i(V)\}_{i=1}^n, V \in \Phi \\ \overline{U}; U(V), V \in \Phi \\ v(s), s \in \mathbb{R}_+; V_m(s), s \in S_U; V_n(s), s \in \mathbb{R}_+ \end{array} \right\},$$

that satisfies the following conditions:

(i) λ *and* ρ *are given by (2) and (3).*

(ii) Conditional on Σ, ρ *and* λ, σ *solves the system of (Bellman) equations (4)-(19).*

(iii) Σ *is generated by* σ*.*

(iv) Free entry of firms into the market ensures

$$\overline{U} = 0. \tag{20}$$

4 Analysis

Three assumptions are made to facilitate the theoretical analysis of the model.

Assumption 1 $u(\infty) - u(0) \geq \beta\Delta(\phi(\underline{a}) - \phi(0))$.

Assumption 2 *There exists a feasible and incentive compatible one-period contract* σ_0 *that offers the worker expected utility* $V_0 \geq u(0) - \phi(0)$ *and the firm expected profit* $\Pi(V_0) > 0$*.*

Assumption 3 *The value function* $U : \Phi \rightarrow \mathbb{R}$ *is continuous and concave.*

Assumption 1 is easy to satisfy, a sufficient condition for which being $u(\infty) - \phi(\underline{a}) \geq u(0) - \phi(0)$. That is, the worker is better off working with the minimum level of effort and the highest amount of consumption than not working and not consuming. This assumption will be needed for establishing that the set Φ is an interval. Assumption 2 puts a positive lower bound on the surplus from contracting. Assumption 3 is made to help reduce the technical burden on the analysis. Note that the continuity and concavity of U could always be obtained by way of randomizing over a set of employment contracts, but that would make the problem even larger.[15]

To start of the analysis, observe first that

$$V_m(s) \geq v(s) \geq V_n(s), \ \forall s \in S_U. \tag{21}$$

This holds because the definitions of S_U and $V_m(s)$ imply $V_m(s) \geq V_n(s)$ and that $v(s)$ is a convex combination of $V_m(s)$ and $V_n(s)$ for all $s \in S_U$. Notice next that $V_n(0) = u(0) - \phi(0) + \beta\Delta v(0)$. This holds because, with $s = 0$, the non-employed worker's current consumption must be zero (as he is not able to borrow) and he will enter the next period with zero assets. Notice also that

$$v(0) \geq \frac{u(0) - \phi(0)}{1 - \beta\Delta} \geq V_{\min}.$$

This holds because the non-employed worker could always choose to stay out of the labor market permanently. We therefore have

$$v(0) \geq V_n(0) = u(0) - \phi(0) + \beta\Delta v(0) \geq V_{\min}. \tag{22}$$

The main body of the following analysis consists of four major results that characterize the model's equilibrium. Proposition 1 provides a foundation for much of the analysis to follow, both theoretical and quantitative. Proposition 2 establishes a connection between the firm's decision on worker termination and the worker's decision on whether to stay in the labor market after termination. Proposition 3 extends the result of Proposition 2, stating that the firm would not keep a worker whose expected utility is sufficiently high and the non-employed worker would stay out of the labor market at least temporarily if his asset holding is sufficiently large. Proposition 4 states that workers who are terminated with a large enough severance compensation would quit the labor market permanently.

Proposition 1 *(i)* $\Phi = [u(0) - \phi(\underline{a}) + \beta\Delta v(0),\ u(\infty) - \phi(\underline{a}) + \beta\Delta V_{\max})$. *(ii) The value function $v(\cdot)$ is well defined, continuous, and strictly increasing on $\mathbb{R}_+$. (iii) The value function $V_n(\cdot)$ is well defined, continuous, and strictly increasing on $\mathbb{R}_+$; (iv) The bargaining problem in equation (17) has a unique solution for all $s \in S_U$ and the value function $V_m(\cdot)$ is well defined, continuous, and strictly increasing on S_U.*

Thus the set of expected utilities that are feasible for the firm to deliver, Φ, is an interval. Since any promised utility must be weakly greater than $v(0)$ (the employed worker is free to quit

[15] See Athey and Bagwell (2001). In the calibrated version of the model, the numerically simulated value function U does appear smooth and concave.

the job any time to obtain $v(0)$) the set Φ becomes smaller as $v(0)$ increases. Note, however, that $v(0)$ is an endogenous variable of the model. The size of the interval Φ is thus also an endogenous variable of model which must be determined jointly with the rest of the optimal contract in equilibrium.

Proving parts (ii)-(iv) of Proposition 1 is a demanding task. The difficulty is that the three value functions $v(\cdot)$, $V_n(\cdot)$ and $V_m(\cdot)$ are jointly defined and thus must be jointly characterized. Observe, however, that $v(\cdot)$ plays a more central role here. Specifically, $v(\cdot)$ provides a link between the firm's problem of optimal contracting and the non-employed worker's optimization problem. As we show in Appendix (the proof of Proposition 1), if $v(\cdot)$ is well defined and continuous, then $V_n(\cdot)$ is well defined and continuous, then the bargaining problem (18) has a unique solution and $V_m(\cdot)$ is well defined and continuous. Given these, our strategy is to formulate the function $v(\cdot)$ as a fixed point of a contraction mapping on a space of bounded and continuous functions, and then apply the contraction mapping theorem to show that $v(\cdot)$ is uniquely defined and continuous, and then to obtain the desired properties for $V_n(\cdot)$ and $V_m(\cdot)$.[16]

By Proposition 1, we also have $v(\mathbb{R}_+) = [v(0), V_{\max})$. With this, and given (22), constraints (10)-(11) can be combined to read as

$$V_i \in [v(0), u(\infty) - \phi(\underline{a}) + V_{\max}),\ \forall i \in \Omega. \tag{23}$$

Corollary 1 *With the optimal contract, it holds that for all $V \in \Phi$, (i) $U(v(s_i)) < \overline{U} - s_i,\ \forall i \notin \Omega$; (ii) $U(V_i) \geq \overline{U} - v^{-1}(V_i),\ \forall i \in \Omega$.*

Note that since the function $v(\cdot)$ is strictly increasing, $v^{-1}(V_i)$, which is the lump-sum compensation the firm would have to give to the worker if it had chosen to terminate him, not to retain him, is well defined. The corollary thus simply says that in any state of retention, for the given expected utility the worker is retained at, the value of retention is higher than the value of termination; and in any state of termination, the value of termination is higher than that of retention. In other words, the optimal termination policy is ex post efficient. Notice also that since in equilibrium $\overline{U} = 0$, the corollary implies that it may be rational for the firm to retain a worker that has a negative value to it, as long as the value of terminating him is even lower.

Observe that $U(V) \to -\infty$ as $V \to u(\infty) - \phi(\underline{a}) + \beta\Delta V_{\max}$. This holds because, independent of the contract used, the expected profits of a firm are bounded from above while the cost of

[16] A full analysis of the firm and worker's optimization problems would require characterizing the value functions $\overline{U}$, $U(\cdot)$ and $v(\cdot)$ simultaneously in a unified fixed point argument. A difficulty is that the function $U(\cdot)$ is not bounded so the contraction mapping theorem could not be applied for the proof of existence.

delivering V to the worker goes to infinity as V goes to $u(\infty) - \phi(\underline{a}) + \beta \Delta V_{\max}$. With this and Assumption 3, we let

$$V^* = \max \left\{ V : \; V \in \arg\max_{V' \in \Phi} U(V') \right\}. \tag{24}$$

This V^* exists, is unique and has the following interpretation: if the firm were free to offer any expected utility from Φ to a newly matched worker with assets $s = 0$ (i.e., suppose all the bargaining power were given to the firm), then V^* is the starting expected utility for the worker with which the firm achieves its maximum value, $U(V^*)$.

Our second main result concerns how the flow out of employment is divided between the flow into unemployment and that into the state of nonparticipation. Under what conditions would the worker, upon termination, stay in the labor market to look for a new job; under what conditions would he choose to quit the labor force, temporarily or permanently? The answer is that it depends on the size of the severance compensation the worker receives upon termination which, of course, depends in turn on the worker's history with the firm. The worker would stay in the labor market after termination (flowing from employment to unemployment) if his expected utility upon termination is sufficiently low and thus the size of his severance compensation is sufficiently small. The worker would quit the labor force (flowing from employment to nonparticipation) if his expected utility upon termination is sufficiently high and thus his severance compensation is sufficiently large.

To formally describe and derive this result, let

$$\widehat{\Psi} \equiv \{ s \in \mathbb{R}_+ : \; U(v(s)) \geq \overline{U} - s \}.$$

By Corollary 1, any $s \in \widehat{\Psi}$ would not be given by the firm to a worker as severance compensation in any state of termination. Next, let

$$\widehat{\Psi}_U \equiv \{ s \in \mathbb{R}_+ : \; v(s) \leq V^* \; and \; U(v(s)) < \overline{U} - s \},$$

$$\widehat{\Psi}_N \equiv \{ s \in \mathbb{R}_+ : \; v(s) > V^* \; and \; U(v(s)) < \overline{U} - s \}.$$

For each $s \in \widehat{\Psi}_U \bigcup \widehat{\Psi}_N$, the firm may terminate the worker with s being his severance compensation. The difference between $\widehat{\Psi}_U$ and $\widehat{\Psi}_N$ is: if $s \in \widehat{\Psi}_U$, then the worker is terminated from the left hand side of the firm's value function (*i.e.*, $v(s) \leq V^*$); if $s \in \widehat{\Psi}_N$, then the worker is terminated from the right hand side of the firm's value function (*i.e.*, $v(s) > V^*$).

Proposition 2 *In equilibrium,* $S_U = \widehat{\Psi}_U \bigcup \widehat{\Psi}$ *and* $S_N = \widehat{\Psi}_N$.

That is, in equilibrium, any level of asset holding s that makes the worker willing to stay in the labor market is a level of severance compensation with which either the firm does not have an incentive to terminate the worker or the worker is terminated from the left hand side of the firm's value function. Moreover, in equilibrium any level of asset holding s with which the worker would quit the labor market is a level of severance compensation with which the worker is terminated from the right hand side of the firm's value function.

By Proposition 2 then, in equilibrium the worker would go back to the labor market immediately after termination if and only if his severance compensation is sufficiently small; and the worker would withdraw from the labor force, either temporarily or permanently, if and only if his severance compensation is sufficiently large. This is easy to see. By Proposition 2, the worker who goes back to the market right after termination must have $s \in \widehat{\Psi}_U$. That is, he must have been terminated from the left hand side of the firm's value function U, or equivalently $v(s)$ must be sufficiently low. This, given that the worker's value function $v(\cdot)$ is strictly increasing, is the case if and only if s is sufficiently small. The worker who is terminated from the left hand side of the firm's value function, with $v(s) \leq V^*$ specifically, is given a severance compensation so small that he and the vacant firm that he could be matched with would be able to produce a joint surplus, and this, in turn, gives the worker incentives to participate in the labor market.

Proposition 2 builds a connection among all major components of the labor market equilibrium: the non-employed worker's decision on whether to participate in the labor market, the firm's decision on termination and severance compensation, the bargaining between non-employed workers and vacant firms, and the free entry/exit condition $\overline{U} = 0$. Observe especially that the non-employed worker's assets or severance compensation s plays a critical role in this connection. Specifically, it is the individual worker's asset holding or severance compensation that connects his employment history with his current decision on labor market participation, and this, in the aggregate, then divides the non-employed workers into those who are unemployed and those not in the labor force.

We now Let $\overline{s} = \max\{s : \; s \in S_U\}$. In words, $\overline{s}$ is the highest amount of assets an unemployed worker would hold in equilibrium. Our next proposition says that subject to a qualification on the worker's minimum working effort, this $\overline{s}$ is finite and so any asset holding or severance compensation above this level would lead the non-employed worker to quit the labor market at least temporarily. Moreover, given $S_N = \widehat{\Psi}_N$ by Proposition 2, that $\overline{s}$ is finite also implies that the equilibrium contract would terminate any worker with expected utility high enough - higher than $v(\overline{s})$.

Proposition 3 *Suppose* $\underline{a} > 0$. *Then* $\overline{s} < \infty$.

Given Proposition 3, we now proceed further to characterize the flow from the state of nonparticipation to the state of unemployment (participation). Among workers who do not participate in the labor force currently, there are those who have chosen to quit the labor force permanently and those who are just temporarily out of the labor force. We show that non-employed workers who are sufficiently rich (i.e., with a large enough asset holding) will choose to retire permanently from the labor market, otherwise he will return to the labor market in some future date, generating the flow from the state of nonparticipation to unemployment.

Proposition 4 *Suppose* $\beta = 1/(1+r)$*. Then there exists* $s^* \in [\overline{s}, \infty]$ *such that a non-employed worker with assets* $s \in (\overline{s}, s^*)$ *would return to the labor market after a finite number of periods, and a non-employed worker with assets* $s \in [s^*, \infty)$ *would stay permanently out of the labor market. In addition, if* $\lim_{c\to\infty} cu'(c) = 0$*, then* $s^* < \infty$*.*[17]

We sketch the proof of this intuitive result in the following, while leaving the formal proof for Appendix. Imagine a worker who is non-employed at the beginning of a period. He holds assets $s \notin S_U$ and is considering an optimal plan with which he would stay outside the labor market for the next $t(\geq 0)$ periods and then return to it with some asset holding $s' \in S_U$. We must show that there exists a cutoff for s, denoted s^*, below which the optimal t is finite and above which ∞. Obviously, the difficulty is that the optimal decision about t must be made jointly with that of s', and the worker's value at s' is not easy to calculate. To overcome this, we use the following strategy. We first show that for any given $s' \in S_U$, there is a cutoff asset level $\widetilde{s}(s') \in [s', \infty)$ such that for any $s \in [s', \widetilde{s}(s'))$, the optimal t (which depends on s and s' and may be denoted $t(s, s')$) is finite; and for any $s \geq \widetilde{s}(s')$, the optimal t is ∞. That is, the interval $[s', \widetilde{s}(s'))$ contains all s with which the non-employed worker returns to the labor market after a finite number of periods and with the assets s' is the optimal strategy, relative to staying permanently outside the labor market. It then follows that the union of these intervals,

$$\bigcup_{s' \in S_U} [s', \widetilde{s}(s')) \, ,$$

contains all s for which it is optimal for the non-employed worker to return to the labor market after *some* finite number of periods and with *some* assets $s' \in S_U$; and for all s not in this union, the non-employed worker is better off staying permanently outside the labor force. And of course this union, itself, is also an interval, and the desired cutoff is just the upper bound of this union.

[17] The condition $\lim_{c\to\infty} cu'(c) = 0$ admits a large family of utility functions, including for example the CARA utility function $u(c) = 1 - \exp(-\lambda c)$ with $\lambda > 0$.

5 Quantitative Analysis

5.1 Parameterization and Calibration

We set the time period to be one month. We set the monthly interest rate to be $r = 0.00417$ to obtain an annual interest rate of 5%. We then set the worker's discount factor to be $\beta = 1/(1+r)$. We set $\Delta = 0.99815$ so the worker's expected (working) lifetime is 45 years.

We set the worker's utility function to be

$$u(c) - \phi(a) = \log(\rho_0 + \rho_1 c) - a^2,$$

where ρ_0 is normalized to 1 and $\rho_1 > 0$.[18] We set $n = 2$ so output can be low (θ_1) or high (θ_2). We let $\underline{a} = 0$ and $\overline{a} = \infty$ and assume

$$\pi_1(a) = \exp(-\psi a),\ \pi_2(a) = 1 - \exp(-\psi a),\ \forall a \geq 0,$$

where ψ is a strictly positive constant.[19] [20] We follow the literature to assume a Cobb-Douglas matching function:

$$M(\eta_U, \gamma - \eta_E) = \alpha_0 {\eta_U}^{\alpha} (\gamma - \eta_E)^{1-\alpha}.$$

where α_0 and α are strictly positive parameters.

The above parameterization leaves us with nine free parameters for the calibration of the model:

$$\theta_1,\ \theta_2,\ \rho_1, \psi, \alpha_0, \alpha, c_0, \gamma, \omega.$$

We follow the literature to set $\alpha = 0.6$.[21] We follow Hosios (1990) and Shimer (2005) to set $\omega = 0.4$.[22] We then pick the values of the remaining parameters to target eight (essentially

[18] We assume $\rho_0 > 0$ so the utility function is bounded from below. Otherwise, the firm is able to punish the worker unlimitedly, by lowing his compensation towards zero, which implies that the firm never has incentives to terminate the worker even after observing a sequence of low outputs. Given $\rho_0 > 0$, the utility function exhibits decreasing absolute risk aversion and increasing relative risk aversion.

[19] If $\theta_2 > \theta_1 \geq 0$, then the expected output is non-negative regardless of the workers effort. As a result, the vacant firm would hire any non-employed worker regardless of his assets, implying that the labor force participation rate is one in equilibrium. The reason is as follows. The vacant firm could offer the non-employed worker with assets $s \in \mathbb{R}_+$ a one-period contract with a fixed wage $c(s)$ in the current period and a severance compensation $s'(s)$ in the next period, where $c(s)$ and $s'(s)$ are the optimal solutions in (15). Thus, the worker's expected utility is exactly $V_n(s)$ by (15), and the firm's expected profit is $\theta_1 - s + \frac{\overline{U}}{1+r}$, which implies $U(V_n(s)) + s \geq \theta_1 + \frac{\overline{U}}{1+r} \geq \frac{\overline{U}}{1+r}$. Hence, $s \in S_U$ by (13).

[20] Notice that if $\psi = 0$ or $\psi = \infty$, then output is constant in the worker's effort. This is the case of no moral hazard where the optimal contract degenerates to a fixed-wage contract with no endogenous termination.

[21] The literature reports a value of α between 0.5 and 0.7 (see Blanchard and Diamond, 1989; Petrongolo and Pissarides, 2001).

[22] We could alternatively set $\omega = 0.5$ without significantly changing the calibration outcome.

seven) measures of the U.S. labor market and to satisfy two equations of identity. The eight measures include (i) the stock of employment (E), (ii) the stock of unemployment (U), (iii) the stock of nonparticipation (N), (iv) the job opening rate, (v) the flow from E to U, (vi) the flow from U to E, (vii) the net flow from E to N, and (viii) the net flow from N to U. The two equations of identity are:

$$\text{flow from U to E} = \alpha_0 \left(\frac{\gamma - \eta_E}{\eta_U} \right)^{1-\alpha}, \tag{25}$$

$$\text{job opening rate} = \frac{\gamma - \mu_E}{\mu_E}. \tag{26}$$

The values of the eight targets are given in Table 2, in the Data column. In Table 2, the stock measures of employment, unemployment, and nonparticipation are derived from The Current Population Survey (CPS) for the period between January 1994 and December 2003.[23] These target measures imply an unemployment rate of 5.12% and a labor force participation rate of 66.78%. The job opening rate is from Davis, Faberman and Haltiwanger (2006) who reports a job opening rate of 3.4% for the period from December 2000 to January 2005.[24] The gross E to U and U to E flows are from Fallick and Fleischman (2004). The net E to N and N to U flows are calculated using the CPS measures of employment, unemployment and nonparticipation, and the flows from Fallick and Fleischman (2004).[25]

In order to generate a job finding probability (fraction of the unemployed to flow into employment) of 28.3%, from (25) the value of α_0 must satisfy

$$\alpha_0 \left(\frac{\gamma - \eta_E}{\eta_A} \right)^{1-0.6} = \alpha_0 \left(\frac{0.034 * 0.6336}{0.0342} \right)^{1-0.6} = 0.283,$$

which gives $\alpha_0 = 0.3405$. And from equation (26) it can be easily calculated that $\gamma = 1.034 \times 0.6338 = 0.6551$. The values of the remaining five parameters of the model are then chosen to match the eight targets above. The calibrated values of these parameters are:

Table 1

[23]This time period is chosen to be consistent with the time period over which Fallick and Fleischman (2004) calculate their flows using data from the same survey.

[24]Their measure is based on the Job Openings and Labor Turnover Survey (JOLTS).

[25]For example, the net flow from E to N is the difference between the measure of workers flowing from employment into nonparticipation (the gross flow from E to N times the measure of employment) and the measure of workers flowing from nonparticipation into employment (the gross flow from N to E times the measure of nonparticipation), divided by the measure of employment.

Parameter	θ_1	θ_2	ψ	ρ_1	c_0
Value	-0.5000	2.5000	0.6386	1.2771	0.0096

and the outcomes of the calibration are summarized in

Table 2

Variable	Data	Calibrated Model
Measure of Employment	0.6336	0.6317
Measure of Unemployment	0.0342	0.0350
Measure of Nonparticipation	0.3320	0.3333
Job Opening Rate	3.4%	3.71%
Flow E to U	1.3%	1.26%
Flow U to E	28.3%	29.1%
Flow E to N (net)	0.18%	0.16%
Flow N to U (net)	$< 0.01\%$	0.12%

A remark on the calibration. In the model, since a non-employed worker must first participate in the labor market (i.e., become unemployed) before finding a job, workers don't flow directly from the state of nonparticipation to employment. That is, the gross N to E flow is zero in the model. Also, in the model, workers do not flow directly from unemployment to the state of nonparticipation either and hence the the gross U to N flow is also zero.[26] Given these, the counterpart of the model's E to N flow in the data should be the net rather than gross flow from E to N and the counterpart of the flow from N to U in the model should be the net, not gross, flow from N to U in the data. A formal justification for this is given in Appendix D.[27]

The model does a good job matching the targets. Note that since the model does not have a channel for the flow from U to N, to generate a net flow close to zero would require that model

[26] In the model, a non-employed worker would leave the labor force if and only if his reservation utility $V_n(s)$ is so large that in case he is matched with a vacant firm, a mutually beneficial contract could not be found. Now $V_n(s)$ is a strictly increasing function of s by Proposition 1. So in order for an unemployed worker to quit the labor force before finding a job, he must consume less than his annuity over some periods of time in order to accumulate assets. This is not optimal. Because by doing so the unemployed worker would not only enjoy less consumption smoothing, but also give up the potential surplus should he form a match with a vacant firm.

[27] In the U.S. data, there is a large flow from unemployment to not-in-the-labor-force, reflecting the movements of discouraged workers, and the movements from unemployment to education. These of course are not modeled in our environment. The U.S. data also shows a significant flow of workers from nonparticipation to employment. This reflects the fact that, in practice, firms search not only among workers that are unemployed, but also among workers that are not in the labor force. This too is missing in our model. In our model, workers must be actively looking for jobs before being matched with a firm.

generates a near zero gross flow from N to U. Note also that the low E to N and N to U flows in the calibrated model do not mean that the mechanisms generating these flows are not strong. They only reflect the fact that the net E to N and N to U flows are low in the data and the N to E and U to N flows are missing in our model. In fact, we can provide examples with reasonable parameter values in which the E to N and N to U flows in the calibrated model are comparable to the gross E to N and N to U flows in the data. The work of introducing into the model new channels through which workers can flow from N to E and U to N are left for future research.

5.2 Equilibrium

Figure 1 depicts the firm's net gains from retaining (rather than terminating) the worker as a function of the worker's expected utility. In order to deliver a given level of expected utility V to the worker, the firm's net profits are $U(V)$ if it retains the worker and $\overline{U} - v^{-1}(V)$ if it terminates the worker, and in equilibrium $\overline{U} = 0$. The value of the difference is shown in Figure 1. Obviously, termination is optimal if and only if V is sufficiently small or sufficiently large.

Figure 2 shows the (deterministic) law of motion for the non-employed worker's assets: $s_{t+1} - s_t$ as a function of s_t. As Proposition 3 predicted, there is a critical asset level (the $\overline{s}$ in Proposition 3) above which the non-employed worker chooses not to enter the labor market (i.e., $s \in S_N$). Consistent with Proposition 4, there is a critical level of assets or severance compensation (the s^* in Proposition 4) above which the non-employed worker would stay permanently out of the labor force while his assets remain constant (i.e., $s_{t+1} - s_t = 0$ for all t), and below which the non-employed worker would dissave and rejoin the labor force in a finite number of periods.

The equilibrium stationary distribution of employed workers is shown in Figure 3. The equilibrium distributions of non-employed workers (unemployed and those not in the labor force) are shown in Figures 4. Notice the large dispersion in expected utility among employed workers. Notice also that all unemployed workers have zero assets while there is a significant dispersion in asset holding among non-employed workers who are not in the labor force.

Figure 5 presents results that are based on simulation using the calibrated model. There, the first graph shows that the probability of transition from employment to unemployment decreases with the worker's tenure at the firm, the second graph shows the probability to transition from employment to nonparticipation increases with the worker's tenure at the firm. These predictions of our model are consistent with findings in Nagypal (2005).

Figure 6 shows the results of a simulation on the relationship between the worker's wage and his tenure at a firm. The simulation follows a large number of workers from the start of his career at a firm until termination (laid off or retired). The figure shows that on average wage increases with tenure.

Figure 7 plots the equilibrium wage dispersion that the calibrated model generates. Hornstein, Krusell and Violante (2011) (HKV) show that standard search matching models can generate only a very small, 3.6%, differential between the average wage and the lowest wage paid in the labor market, whereas the observed Mm ratio–the ratio between the average wage and lowest wage paid – is at least twenty times larger than what the model observes. As HKV argues, the logic of the search-matching model implies that a higher wage dispersion is associated with longer unemployment durations or a smaller probability of finding employment for the unemployed. Given that unemployment durations are typically short in the data, wage dispersion cannot be large in the model. This logic of the search-matching model does not apply in our model. In our model, wage dispersion is driven by the provision of intertemporal incentives and risk sharing. Wages of homogeneous workers who start with the same initial expected utility fan out over time as their outputs follow a stochastic process. In our model, workers who produce a high output not only receive a higher wage in the current period, but also will see their future utilities and wages increase. Likewise, workers who produce a low output will receive lower wages both in the current period and in the future. [28] In fact, our model generates a much larger wage dispersion, along with a job finding probability and hence an average unemployment duration that are consistent with U.S. data. In other words, large wage dispersions and short unemployment durations, which are not consistent to each other in standard search-matching model, are compatible with each other in our model. Specifically, the computed average wage is 0.4071, the lowest wage paid is 0.0166, and the Mm ratio is 24.5, similar to what HKV observes from the U.S. data. Suppose we use the average wage of the workers in the lowest wage percentile as the minimum wage in the calculation, then the computed Mm ratio is 13.89. Even if we use the average wage of the workers in the 5th wage percentile as the minimum wage in the calculation, the computed Mm ratio is 5.32, much larger than what the search/matching models permit.

6 Conclusion

In this paper, we have studied an equilibrium model of the labor market that unifies three existing classes of models for labor market analysis. Such a model has proven to have power in accounting simultaneously for several important empirical observations concerning modern labor markets. The model has a number of obvious extensions, among which perhaps the most impor-

[28] This effect of the dynamic contracting on compensation and utility dispersion was first discussed in Green (1987) and Atkeson and Lucas (1992). A similar observation was made in Wang (2011). This paper puts the same mechanism to work in an equilibrium search-matching model.

tant and challenging is to add aggregate uncertainty to our currently stationary environment. As discussed in the introduction of the paper, existing search-matching models have not been entirely successful in accounting for the observed patterns of wage dynamics over the business cycle. Adding aggregate uncertinty to our model would offer a promising new vehicle for understanding compensation dynamics and the business cycle, from the perspective that the dynamics of the worker's compensation is designed to achieve efficient intertemporal incentives and risk sharing.

Appendix

A Proof of Proposition 1

Let Φ' denote the interval on the right hand side of the equation we need to show to hold. To prove (i) of the proposition we need to show $\Phi = \Phi'$ and this takes two steps.

Step 1 We show $\Phi \subseteq \Phi'$.

This is the case because: (a) there does not exist $V \in \Phi$ such that $V < u(0) - \phi(\underline{a}) + \beta\Delta v(0)$, since the worker could always choose to exert the lowest effort $\underline{a}$ regardless of the contract offered, and he is guaranteed expected utility $v(0)$ next period no matter he is terminated or retained; and (b) there does not exist $V \in \Phi$ such that $V \geq u(\infty) - \phi(\underline{a}) + \beta\Delta V_{\max}$, which is obviously the case.

Step 2 We show that Φ' is self-generating with respective to the mapping B (defined in equation (12)) and hence $\Phi' \subseteq \Phi$.

Let $V \in \Phi'$. Let

$$a(V) = \underline{a},\ \Omega(V) = \emptyset,\ c_i(V) = x(V) \text{ and } s_i(V) = y(V),\ \forall i,$$

where $x(V), y(V) \in \mathbb{R}_+$ are chosen to satisfy

$$u(x(V)) - \phi(\underline{a}) + \beta\Delta v(y(V)) = V.$$

Obviously, such $x(V)$ and $y(V)$ exist given $u(\mathbb{R}_+) = [u(0), u(\infty))$ and $v(\mathbb{R}_+) = [v(0), V_{\max})$ as shown independently in (ii) of Proposition 1.Hence, $\{a(V), \Omega(V), \{c_i(V), s_i(V), V_i(V)\}_{i=1}^n\}$ satisfies (5)-(9), (11), and $V_i(V) \in \Phi'$, $\forall i \in \Omega(V) (= \emptyset)$, and thus it generates V. This completes the proof of (i) of the proposition. We now proceed to prove the remaining parts of the proposition.

Lemma 1 *For all $s \in S_U$, if $v(s)$ is well defined, then the solution to the Nash bargaining problem (18) exists and is unique.*

Proof. Fix $s \in S_U$. Let $O(V)$ denote the objective function in (18). We have

$$O'(V) = \left(\frac{U(V) + s - \frac{1}{1+r}\overline{U}}{V - V_n(s)} \right)^{\omega} \left[\omega U'(V) \frac{V - V_n(s)}{U(V) + s - \frac{1}{1+r}\overline{U}} + (1 - \omega) \right]. \tag{27}$$

The rest of the proof takes five steps.

Step 1 We show $V_m(s) \geq V^*$ where V^* is defined in (24) as the largest maximizer of U.

Suppose $V_m(s) < V^*$. Then $V^* > V_m(s) \geq V_n(s)$ and

$$U(V^*) + s - \frac{1}{1+r}\overline{U} \geq U(V_m(s)) + s - \frac{1}{1+r}\overline{U} \geq 0,$$

where the first inequality follows from the definition of V^* as the largest maximizer of U in (24). It then follows that $V^* \in \Phi$ is feasible with $O(V^*) > O(V_m(s))$, which contradicts with the definition of $V_m(s)$ in (18).

Step 2 We show that there exists $M(s) \geq V^*$ with $U(M(s)) + s = \frac{1}{1+r}\overline{U}$ such that the constraint set in (18) can be rewritten as $[\max\{V_n(s), V^*\}, M(s)]$.

Given $V_m(s) \geq V^* \in \Phi$ by Step 1 and $\Phi = [u(0) - \phi(\underline{a}) + \beta\Delta v(0), u(\infty) - \phi(\underline{a}) + \beta\Delta V_{\max})$ by (i) of Proposition 1, the constraint set in (18) can be rewritten as

$$\left\{ V \in [\max\{V_n(s), V^*\}, u(\infty) - \phi(\underline{a}) + \beta\Delta V_{\max}) : U(V) + s \geq \frac{1}{1+r}\overline{U} \right\}.$$

Given that the value function U is concave by Assumption 3 and V^* is the largest maximizer of U as defined in (24), U is strictly decreasing on $[V^*, u(\infty) - \phi(\underline{a}) + \beta\Delta V_{\max})$. The desired result then follows from the fact that $U(V) \to -\infty$ as $V \to u(\infty) - \phi(\underline{a}) + \beta\Delta V_{\max}$, where the latter being the case because $\overline{U} = 0$, U is bounded from above, and $c(V) \to \infty$ as $V \to u(\infty) - \phi(\underline{a}) + \beta\Delta V_{\max}$.

Step 3 We show

$$\omega U'(V_m(s))(V_m(s) - V_n(s)) + (1 - \omega) \left(U(V_m(s)) + s - \frac{1}{1+r}\overline{U} \right) = 0. \tag{28}$$

Suppose $\max\{V_n(s), V^*\} = M(s) = V_m(s)$. Then (28) holds given $U'(V^*) = 0$ by (24) and $U(M(s)) + s = \frac{1}{1+r}\overline{U}$ by Step 2.

Suppose $\max\{V_n(s), V^*\} < M(s)$. Then it suffices to show that $V_m(s)$ is an interior solution and the first order condition $O'(V_m(s)) = 0$ holds, which implies (28). Given that U is strictly decreasing on $[\max\{V_n(s), V^*\}, M(s)]$, there exists $V \in (\max\{V_n(s), V^*\}, M(s))$ such that $O(V) > 0 = O(V_n(s)) = O(M(s))$, which implies $V_m(s) \neq V_n(s)$ or $M(s)$. Furthermore, given $O'(V^*) > 0$ by (27) (in the case of $V^* > V_n(s)$), $V_m(s) \neq V^*$. Hence, $V_m(s)$ is an interior solution.

Step 4 We show the existence of the solution to the Nash bargaining problem.

The existence of the solution follows from the fact that the objective function is continuous given that U is continuous, and the constraint set is compact by Step 2.

Step 5 We show that the solution to the problem of Nash bargaining is unique.

Given that the constraint set is convex by Step 2, it suffices to show that the objective function is strictly quasi-concave, which, given that U is strictly decreasing and weakly concave on the constraint set, follows from (27). This concludes the proof of Lemma 1.

Lemma 2 *Under Assumption 2,* $0 \in S_U$*.*

Proof. Suppose $0 \notin S_U$. That is, suppose $0 \in S_N$. Then $v(0) = V_n(0) = \frac{u(0)-\phi(0)}{1-\beta\Delta}$ where the first equality follows from (17) and the second from (16). Now consider the following contract: it is σ_0 for the first period (remember σ_0 is the feasible and incentive compatible one period contract in Assumption 2), and then the worker is given $s = 0$ to leave the firm regardless of the output. This contract delivers an expected utility equal to $V_0 + \beta\Delta v(0)$ to the worker. Clearly, given $V_0 \geq u(0) - \phi(0)$ by Assumption 2, $V_0 + \beta\Delta v(0) \geq V_n(0)$ and $V_0 + \beta\Delta v(0) \in \Phi$. This contract delivers an expected profit equal to $\Pi(V_0) + \frac{1}{1+r}\overline{U}$ to the firm. Clearly, given $\Pi(V_0) \geq 0$ by Assumption 2, it holds that

$$U(V_0 + \beta\Delta v(0)) \geq \Pi(V_0) + \frac{1}{1+r}\overline{U} \geq \frac{1}{1+r}\overline{U}.$$

Hence, $s = 0 \in S_U$ by (14). A contradiction.

Lemma 3 *Suppose* v *is well defined and continuous. Then (i)* V_n *is well defined, continuous, and strictly increasing on* $\mathbb{R}_+$*; (ii)* V_m *is well defined, continuous, and strictly increasing on* S_U*; and (iii)* v *is strictly increasing on* $\mathbb{R}_+$*.*

Proof. Let v be well defined and continuous. Let $s_1 < s_2$.

(i) We show that V_n is well defined and continuous. This is the case because: (a) The objective function in (16) is continuous given that u and v are continuous. (b) The constraint correspondence in (16) is compact-valued and continuous. (c) The theorem of the maximum.

We then show that V_n is strictly increasing. Notice that with $s_2 > s_1$, the worker can always choose to have strictly more consumption in the current period while setting his future assets equal to that with s_1. The result then follows from the fact that u is a strictly increasing function.

(ii) We show that V_m is well defined and continuous. This is the case because: (a) The objective function in (18) is continuous given that U is continuous. (b) The constraint correspondence in (18) is compact-valued as shown in the proof of Lemma 1 and continuous given that U and V_n are continuous. (c) The theorem of the maximum.

We then show that V_m is strictly increasing. Suppose $V_m(s_1) \geq V_m(s_2)(\geq V^*$ as shown in the proof of Lemma 1). Then $U(V_m(s_1)) \leq U(V_m(s_2))$ given that U is strictly decreasing on $V \geq V^*$, and $V_n(s_1) < V_n(s_2)$ given that V_n is strictly increasing as shown in (i) of Lemma 3. Thus, (28) implies

$$\begin{aligned} U'(V_m(s_1)) &= -\frac{1-\omega}{\omega}\frac{U(V_m(s_1)) + s_1 - \frac{1}{1+r}\overline{U}}{V_m(s_1) - V_n(s_1)} \\ &> -\frac{1-\omega}{\omega}\frac{U(V_m(s_2)) + s_2 - \frac{1}{1+r}\overline{U}}{V_m(s_2) - V_n(s_2)} \\ &= U'(V_m(s_2)), \end{aligned}$$

which, given that U is concave, in turn implies $V_m(s_1) < V_m(s_2)$. A contradiction.

(iii) We show that v is a strictly increasing function. This is done by considering four cases. (a) Suppose $s_1, s_2 \in S_U$. Then

$$v(s_1) = \rho V_m(s_1) + (1-\rho)V_n(s_1) < \rho V_m(s_2) + (1-\rho)V_n(s_2) = v(s_2)$$

where the two equalities follow from (17) and the inequality follows from that V_m is strictly increasing by (ii) of Lemma 3 and V_n is strictly increasing by (i) of this lemma. (b) Suppose $s_1, s_2 \in S_N$. Then $v(s_1) = V_n(s_1) < V_n(s_2) = v(s_2)$. (c) Suppose $s_1 \in S_N$ and $s_2 \in S_U$. Then

$$v(s_1) = V_n(s_1) < \rho V_m(s_2) + (1-\rho)V_n(s_2) = v(s_2)$$

where the inequality follows from $V_n(s_1) < V_n(s_2) \leq V_m(s_2)$. (d) Suppose $s_1 \in S_U$ and $s_2 \in S_N$. Suppose $v(s_1) \geq v(s_2)$. Then

$$\rho V_m(s_1) + (1-\rho)V_n(s_1) = v(s_1) \geq v(s_2) = V_n(s_2)$$

which, given $V_n(s_1) < V_n(s_2)$ by (i) of this lemma, implies $V_m(s_1) > V_n(s_2)$, which, given

$$U(V_m(s_1)) + s_2 - \frac{1}{1+r}\overline{U} > U(V_m(s_1)) + s_1 - \frac{1}{1+r}\overline{U} \geq 0,$$

in turn implies $s_2 \in S_U$ by (14). A contradiction. Hence, we conclude $v(s_1) < v(s_2)$.

With Lemmas 1-3, we now proceed to prove (ii)-(iv) of the proposition.

1. Let (Y, d) denote the space of all bounded and continuous functions $f : \mathbb{R}_+ \to X$ under the *sup* norm, denoted d. Note that boundedness is needed since $\mathbb{R}_+$ is not compact. Y is a complete normed vector space.

2. Define a mapping Γ as follows:

$$\forall v \in Y \text{ and } \forall\, s \in \mathbb{R}_+,\ \Gamma(v)(s) = \begin{cases} \rho V_m(s) + (1-\rho)V_n(s), & \text{if } s \in S_U \\ V_n(s), & \text{if } s \in S_N \end{cases} \tag{29}$$

where S_U is defined by (14), S_N is defined by (15), V_n is defined by (16), and V_m is defined by (18). Notice that the function $\Gamma(v)$ is well defined for all $v \in Y$ by Lemma 3.

3. We show that Γ maps from Y to Y, that is, $\Gamma : Y \to Y$. It suffices to show that for all $v \in Y$, $\Gamma(v)$ is bounded and continuous, that is $\Gamma(v) \in Y$. The proof of the boundedness is straightforward. The proof of the continuity takes three steps.

Step 1 We show $[0, V_n^{-1}(V^*)] \subset S_U$. For all $s \in [0, V_n^{-1}(V^*)]$, given that V_n is strictly increasing by (i) of Lemma 3, $V_n(s) \leq V^*$ which, given $U(V^*) + s \geq \frac{1}{1+r}\overline{U}$, implies $s \in S_U$ by (14).

Step 2 We show that there exist b_0 and $\{a_i, b_i\}_{i=1}^m$ such that

$$S_U = [0, b_0] \bigcup \left(\bigcup_{i=1}^{m} [a_i, b_i] \right) \tag{30}$$

where $V_n^{-1}(V^*) \leq b_0 < a_1 \leq b_1 < \cdots < a_m \leq b_m$ and m may be infinity. Given that U is continuous and V_n is continuous by (i) of Lemma 3, (14) implies that S_U is a closed subset of $\mathbb{R}_+$, which can be written as a countable union of disjoint closed intervals.

Step 3 We show that $\Gamma(v)$ is continuous. Notice that given that V_n and V_m are continuous by (i) and (ii) of Lemma 3 respectively, $\Gamma(v)$ is continuous on $\mathbb{R}_+ \setminus \{b_0, a_1, b_1, ..., a_m, b_m\}$. Thus, it suffices to show that for all $s \in \{b_0, a_1, b_1, ..., a_m, b_m\}$, $\Gamma(v)$ is continuous, or equivalently, $V_n(s) = V_m(s)$. For this, we consider two cases.

(a) Suppose $V_n(b_i) < V_m(b_i)$ for some $i \in \{0, 1, ..., m\}$. Then, given that V_n is continuous by (i) of Lemma 3, there exists $\varepsilon > 0$ such that for all $s \in [b_i, b_i + \varepsilon]$, $V_n(s) < V_m(b_i)$ and

$$U(V_m(b_i)) + s - \frac{1}{1+r}\overline{U} \geq U(V_m(b_i)) + b_i - \frac{1}{1+r}\overline{U} \geq 0,$$

which imply $s \in S_U$ by (14), which contradicts with (30).

(b) Suppose $V_n(a_i) < V_m(a_i)$ for some $i \in \{1, 2, ..., m\}$. Notice $V_m(a_i) > V_n(a_i) \geq V^*$ which the second inequality follows from Step 2, which, given that U is strictly decreasing for $V \geq V^*$, implies

$$U(V_n(a_i)) + a_i - \frac{1}{1+r}\overline{U} > U(V_m(a_i)) + a_i - \frac{1}{1+r}\overline{U} \geq 0.$$

Then, given that V_n is continuous and strictly increasing by (i) of Lemma 3, there exists $\varepsilon > 0$ such that for all $s \in [a_i - \varepsilon, a_i]$, $V_n(s) \leq V_n(a_i)$ and $U(V_n(a_i)) + s > \frac{1}{1+r}\overline{U}$, which imply $s \in S_U$, which contradicts with (30).

4. We show that the mapping Γ is a contraction. Since Y is a normed vector space of bounded and continuous functions, it suffices to verify that the Blackwell sufficient conditions are satisfied.

Let $v_1, v_2 \in Y$ be given. Let S_U^i, S_N^i, V_n^i and V_m^i for $i = 1, 2$ denote the sets S_U and S_N and the functions V_n and V_m induced by v_i through (14), (15), (16) and (18) respectively. The proof takes two steps.

Step 1 (Monotonicity) We show that if $v_1 \leq v_2$, then $\Gamma(v_1) \leq \Gamma(v_2)$.

Let $v_1 \leq v_2$. Notice first that $V_n^1 \leq V_n^2$ by (16). Four cases must be considered.

(a) Suppose $s \in S_U^1 \cap S_U^2$. Then, using an argument that is similar to the one used for proving that V_m is a strictly increasing function in (ii) of Lemma 3, it is straightforward to show $V_n^1(s) \leq V_n^2(s)$ and $V_m^1(s) \leq V_m^2(s)$, which in turn imply $\Gamma(v^1)(s) \leq \Gamma(v^2)(s)$.

(b) Suppose $s \in S_N^1 \cap S_N^2$. Then $\Gamma(v^1)(s) = V_n^1(s) \leq V_n^2(s) = \Gamma(v^2)(s)$.

(c) Suppose $s \in S_N^1 \cap S_U^2$. Then $\Gamma(v^1)(s) = V_n^1(s) \leq \rho V_m^2(s) + (1-\rho)V_n^2(s) = \Gamma(v^2)(s)$ where the inequality follows from $V_n^1(s) \leq V_n^2(s) \leq V_m^2(s)$.

(d) Suppose $s \in S_U^1 \cap S_N^2$. Suppose $\Gamma(v^1)(s) > \Gamma(v^2)(s)$. Then

$$\rho V_m^1(s) + (1-\rho)V_n^1(s) = \Gamma(v^1)(s) > \Gamma(v^2)(s) = V_n^2(s)$$

which, given $V_n^1(s) \leq V_n^2(s)$, implies $V_m^1(s) > V_n^2(s)$, which, given $U(V_m^1(s)) + s \geq \frac{1}{1+r}\overline{U}$, in turn implies $s \in S_U^2$ by (14). A contradiction.

Hence, we conclude $\Gamma(v^1)(s) \leq \Gamma(v^2)(s)$ for all $s \in \mathbb{R}_+$.

Step 2 (Discounting) We show that if $v_2 = v_1 + a$ for some $a > 0$, then $\Gamma(v^2) \leq \Gamma(v^1) + \beta\Delta a$.

Notice, by (16), that for all $s \in \mathbb{R}_+$, $V_n^2(s) = V_n^1(s) + \beta\Delta a$. To derive the desired result, four cases are considered. Suppose first $s \in S_U^1 \cap S_U^2$. In this case, it suffices to show $V_m^2(s) \leq V_m^1(s) + \beta\Delta a$. Suppose $V_m^2(s) > V_m^1(s) + \beta\Delta a$. Then, (28) implies

$$\begin{aligned} U'(V_m^1(s)) &= -\frac{1-\omega}{\omega}\frac{U(V_m^1(s)) + s - \frac{1}{1+r}\overline{U}}{V_m^1(s) - V_n^1(s)} \\ &< -\frac{1-\omega}{\omega}\frac{U(V_m^2(s)) + s - \frac{1}{1+r}\overline{U}}{V_m^2(s) - V_n^2(s)} \\ &= U'(V_m^2(s)), \end{aligned}$$

which, given that U is concave, in turn implies $V_m^2(s) < V_m^1(s)$. A contradiction and thus the desired result holds.

The same result holds also in the remaining cases $s \in S_N^1 \cap S_N^2$, $s \in S_N^1 \cap S_U^2$ and $s \in S_U^1 \cap S_N^2$ which can be analyzed similarly, and we leave the details of the proof for the reader.

5. By the contraction mapping theorem then, Γ has a unique fixed point $v \in Y$ and v is continuous which implies that V_n, V_m and v are all well defined, continuous and strictly increasing, by Lemma 3. The proof is Proposition 1 is now complete.

B Proof of Proposition 2

Observe first that since $S_U \bigcup S_N = \widehat{\Psi}_U \bigcup \widehat{\Psi}_N \bigcup \widehat{\Psi}$, to prove the proposition we need only show $\widehat{\Psi}_U \subseteq S_U$, $\widehat{\Psi}_N \subseteq S_N$, and $\widehat{\Psi} \subseteq S_U$.

Suppose $s \in \widehat{\Psi}$. Then $U(v(s)) \geq \overline{U} - s$ which implies $U(v(s)) + s \geq \overline{U} = 0 = \frac{1}{1+r}\overline{U}$ which, given $v(s) \geq V_n(s)$, in turn implies $s \in S_U$ by (14). So $\widehat{\Psi} \subseteq S_U$.

Suppose $s \in \widehat{\Psi}_U$. Then, given $U(V^*) \geq 0$, it holds that $U(V^*) + s \geq 0 = \frac{1}{1+r}\overline{U}$ which, given $V^* \geq v(s) \geq V_n(s)$, implies $s \in S_U$ by (14). So $\widehat{\Psi}_U \subseteq S_U$.

Suppose $s \in \widehat{\Psi}_N$. Then, given that the firm's value function U is concave and peaks at V^*, for all $V \geq v(s) > V^*$, it holds that $U(V) \leq U(v(s))$, which implies $U(V) + s \leq U(v(s)) + s < \overline{U} = 0 = \frac{1}{1+r}\overline{U}$, which in turn implies $s \in S_N$ by (14) and (15). So $\widehat{\Psi}_N \subseteq S_N$ and the proposition is proven.

C Proof of Proposition 3

The promise-keeping constraint (5) implies

$$\begin{aligned} V &= \sum_{i \notin \Omega} \pi_i(a)(u(c_i) + \beta\Delta v(s_i)) + \sum_{i \in \Omega} \pi_i(a)(u(c_i) + \beta\Delta V_i) - \phi(a) \\ &\leq \sum_i \pi_i(a) u(c_i) + \beta\Delta V_{\max} - \phi(\underline{a}) \\ &\leq u\left(\sum_i \pi_i(a) c_i\right) + \beta\Delta V_{\max} - \phi(\underline{a}), \end{aligned}$$

where the first inequality follows from $v(s_i), V_i \leq V_{\max}$ and $a \geq \underline{a}$, and the second inequality holds because the utility function u is concave. We then have

$$\sum_i \pi_i(a) c_i \geq u^{-1}(V - \beta\Delta V_{\max} + \phi(\underline{a})),$$

which in turn implies

$$\lim_{V \to u(\infty) - \phi(\underline{a}) + \beta\Delta V_{\max}} \sum_i \pi_i(a) c_i = \infty.$$

In addition, remember

$$\lim_{V \to u(\infty) - \phi(\underline{a}) + \beta\Delta V_{\max}} U(V) = -\infty. \tag{31}$$

Now notice that a non-employed worker with assets $\frac{u^{-1}((1-\beta\Delta)V + \phi(0))}{1 - \frac{\Delta}{1+r}}$ is able to consume $u^{-1}((1-\beta\Delta)V + \phi(0))$ each period while staying permanently out of the labor force, and this gives him lifetime expected utility V. It should thus hold that

$$v\left(\frac{u^{-1}((1-\beta\Delta)V + \phi(0))}{1 - \frac{\Delta}{1+r}}\right) \geq V,$$

which, given that v is strictly increasing by (ii) of Proposition 1, implies

$$\begin{aligned} & \lim_{V \to u(\infty) - \phi(\underline{a}) + \beta\Delta V_{\max}} \overline{U} - v^{-1}(V) \\ \geq & \lim_{V \to u(\infty) - \phi(\underline{a}) + \beta\Delta V_{\max}} \overline{U} - \frac{u^{-1}((1-\beta\Delta)V + \phi(0))}{1 - \frac{\Delta}{1+r}} \\ = & \ \overline{U} - \frac{u^{-1}(u(\infty) - (\phi(\underline{a}) - \phi(0)))}{1 - \frac{\Delta}{1+r}} \\ > & -\infty \end{aligned} \tag{32}$$

where the second inequality follows from $\phi(\underline{a}) - \phi(0) > 0$ due to $\underline{a} > 0$. The desired result then follows from (31) and (32) and the proposition is proven.

D Proof of Proposition 4

For all $t \in \{1, 2, \cdots\} \cup \{\infty\}$, all $s \in [0, \infty) - S_U$, and all $s' \in S_U$, let $\widetilde{v}(s, t, s')$ denote the maximum expected utility for a non-employed worker who has assets $s > \overline{s}$ at the beginning of a period and has made the decision to stay out of the labor market for the next t periods and then rejoin the labor force with asset $s' \in S_U$ in period $t + 1$. For this worker, it is optimal for him to set consumption constant at $\frac{1-\gamma}{1-\gamma^t}(s - \gamma^t s')$, a non-negative amount given $s > \overline{s} \geq s'$, over the t periods in which he is not in the labor market. This gives the worker the expected utility equal to

$$\widetilde{v}(s, t, s') = \frac{1 - \gamma^t}{1 - \gamma} \left(u \left(\frac{1 - \gamma}{1 - \gamma^t}(s - \gamma^t s') \right) - \phi(0) \right) + \gamma^t v(s').$$

Observe that if $t = \infty$, then $\gamma^t = 0$, and the choice of s' is not relevant for the worker's value. Observe also that the case $t = \infty$ is the case where the worker consumes the annuity of his asset every period and never comes back to the labor market.

Notice that treating t as a variable that takes continuous values from the interval $[0, \infty)$ rather than discrete values from the set of $\{1, 2, ...\}$ would not change technically the nature of the problem. This is because for any decision the worker must make about t, the part which is relevant for our proof concerns only whether the optimal $t(s, s')$ is finite for the given s and s'. So, suppose t takes continuous values from the interval $[0, \infty)$. This allows us to take the partial derivative of $\widetilde{v}(s, t, s')$ with respect to t,

$$\frac{\partial \widetilde{v}(s, t, s')}{\partial t} = -\gamma^t \ln \gamma F(s, t, s') \tag{33}$$

where

$$F(s, t, s') \equiv \frac{1}{1 - \gamma} \left(u \left(\frac{1 - \gamma}{1 - \gamma^t}(s - \gamma^t s') \right) - \phi(0) \right) - \frac{s - s'}{1 - \gamma^t} u' \left(\frac{1 - \gamma}{1 - \gamma^t}(s - \gamma^t s') \right) - v(s') \tag{34}$$

with

$$\frac{F(s, t, s')}{\partial t} = -\frac{(1 - \gamma)\gamma^t \ln \gamma (s - s')^2}{(1 - \gamma^t)^3} u'' \left(\frac{1 - \gamma}{1 - \gamma^t}(s - \gamma^t s') \right) < 0, \tag{35}$$

$$F(s, \infty, s') = \frac{u((1 - \gamma)s) - \phi(0)}{1 - \gamma} - (s - s')u'((1 - \gamma)s) - v(s'). \tag{36}$$

With the above derivation, the remainder of the proof is organized in 4 steps, as follows.

Step 1 We show that for any given s and s', the optimal t is finite (that is, he prefers going back to the labor market after being out of it for a finite number of periods to staying permanently out of it) if and only if $F(s, \infty, s') < 0$.

Suppose $F(s, \infty, s') < 0$. Then there exists $\tau < \infty$ such that for all $t \in [\tau, \infty)$, $F(s, t, s') < 0$, which, given $-\gamma^t \ln \gamma > 0$, implies $\partial \widetilde{v}(s, t, s')/\partial t < 0$ by (33), which in turn implies $\widetilde{v}(s, t, s') > \widetilde{v}(s, \infty, s')$.

Suppose $F(s, \infty, s') \geq 0$. Then for all $t < \infty$, given $F_t(s, t, s') < 0$ by (35), $F(s, t, s') > 0$ which, given $-\gamma^t \ln \gamma > 0$, implies $\widetilde{v}_t(s, t, s') > 0$ by (33), which in turn implies $\widetilde{v}(s, t, s') < \widetilde{v}(s, \infty, s')$.

Step 2 We show that for all $s' \in S_U$, there exists $\widetilde{s}(s') \in [s', \infty]$ such that $F(s, \infty, s') < 0$ for all $s \in (s', \widetilde{s}(s'))$, and $F(s, \infty, s') \geq 0$ for all $s \in [\widetilde{s}(s'), \infty)$. Furthermore, if $\lim_{c \to \infty} [u(c) - cu'(c)] = u(\infty)$, then $\widetilde{s}(s') < \infty$;

Fix $s' \in S_U$. Given

$$F(s', \infty, s') = \frac{u((1-\gamma)s') - \phi(0)}{1-\gamma} - v(s') \leq 0$$

which follows from (34) and the definition of $v(s')$, and

$$F_s(s, \infty, s') = -(1-\gamma)(s - s')u''((1-\gamma)s) > 0$$

which follows from (34) and $s > \overline{s} \geq s'$, the result then follows. Furthermore, if $\lim_{c \to \infty} u(c) - cu'(c) = u(\infty)$, then (34) implies $F(\infty, \infty, s') > 0$, which in turn implies $\widetilde{s}(s') < \infty$.

Step 3 We show that $\widetilde{s}(s')$ is continuous in s'.

By Step 2, $\widetilde{s}(s')$ is defined by $F(\widetilde{s}(s'), \infty, s') = 0$. Given that $u(\cdot)$ is twice differentiable and $v(\cdot)$ is continuous, the needed result follows.

Step 4 We show that there exists $s^* \in [\overline{s}, \infty)$ such that the worker with asset $s > \overline{s}$ would return to the labor market in a finite number of periods if and only if $s < s^*$.

Fix $s > \overline{s}$. Define $s^* = \max_{s' \in S_U} \widetilde{s}(s')$. Given that $\widetilde{s}(s')$ is a continuous function of s' by Step 3 and S_U is a compact set by Proposition 3, $s^* \in [\overline{s}, \infty)$ is well-defined.

Suppose $s < s^*$. Then by the definition of s^*, there exists $s' \in S_U$ such that $s \in (s', \widetilde{s}(s'))$, which implies $F(s, \infty, s') < 0$ by Step 2, which in turn implies that the worker prefers going back to the labor market in a finite number of periods with asset $s' \in S_U$ than staying out of the labor market forever by Step 1.

Suppose $s \geq s^*$. Then by the definition of s^*, for all $s' \in S_U$, $s \in [\widetilde{s}(s'), \infty)$, which implies $F(s, \infty, s') \geq 0$ by Step 2, which in turn implies that the worker prefers staying out of the labor market forever than going back to the labor market in a finite number of periods with asset $s' \in S_U$ by Step 1. The proof of the proposition is complete.

E Calibration

Let $\widetilde{R}_{ij} \in [0,1]$ with $i, j \in \{E, U, N\}$ denote the i to j flow in the data (the probability with which a worker is in state j next month conditional on being in state i this month). Let $\widetilde{\eta}_E$ be the percentage of employed workers, $\widetilde{\eta}_U$ that of unemployed workers, and $\widetilde{\eta}_N$ that of workers not participating in the labor market. The stocks and flows and their relations in the US labor market are summarized in the following equations.

$$\widetilde{R}_{UE}\widetilde{\eta}_U + \widetilde{R}_{NE}\widetilde{\eta}_N = \widetilde{R}_{EU}\widetilde{\eta}_E + \widetilde{R}_{EN}\widetilde{\eta}_E + \varepsilon_E\widetilde{\eta}_E, \tag{37}$$

$$\widetilde{R}_{EU}\widetilde{\eta}_E + \widetilde{R}_{NU}\widetilde{\eta}_N = \widetilde{R}_{UE}\widetilde{\eta}_U + \widetilde{R}_{UN}\widetilde{\eta}_U + \varepsilon_U\widetilde{\eta}_U, \tag{38}$$

$$\widetilde{R}_{EN}\widetilde{\eta}_E + \widetilde{R}_{UN}\widetilde{\eta}_U = \widetilde{R}_{NE}\widetilde{\eta}_N + \widetilde{R}_{NU}\widetilde{\eta}_N + \varepsilon_N\widetilde{\eta}_N, \tag{39}$$

where the error terms, $\varepsilon_E\widetilde{\eta}_E$ in (37), $\varepsilon_U\widetilde{\eta}_U$ in (38), and $\varepsilon_N\widetilde{\eta}_N$ in (39), reflect measurement errors and the fact that the data is not entirely stationary.[29] [30]

As discussed in the main body of the paper, in the model the N to E and the U to N flows are zero. The stocks and flows in the model thus must satisfy the following equations:

$$R_{UE}\eta_U = R_{EU}\eta_E + R_{EN}\eta_E, \tag{40}$$

$$R_{EU}\eta_E + R_{NU}\eta_N = R_{UE}\eta_U, \tag{41}$$

$$R_{EN}\eta_E = R_{NU}\eta_N. \tag{42}$$

Now the question is: should the model be calibrated to generate R_{EN} and R_{NU} to match the gross flows $\widetilde{R}_{EN}$ and $\widetilde{R}_{NU}$ respectively, or to match the net flows from E to N and from N to U respectively? The answer is: the net, not the gross. This is for two reasons.

First, viewing R_{EN} and R_{NU} as corresponding to the net flows provides a consistent view of the flows in the data from the model's perspective. To see this, rearrange terms in equations (37)-(39) to get:

$$\widetilde{R}_{UE}\widetilde{\eta}_U = \widetilde{R}_{EU}\widetilde{\eta}_E + \left(\widetilde{R}_{EN}\widetilde{\eta}_E - \widetilde{R}_{NE}\widetilde{\eta}_N\right) + \varepsilon_E\widetilde{\eta}_E, \tag{43}$$

[29] We use the transition probabilities calculated in Fallick and Fleischman (2004) with Current Population Survey 1994 and 1996-2003 (excluding incomplete data from 1995). For the sake of consistency, we calculate the percentages with Current Population Survey for the same period (1994-2003).

[30] According to our calculations, $\varepsilon_E = 0.13\%$, $\varepsilon_U = -4.22\%$ and $\varepsilon_N = 0.35\%$. Here, $|\varepsilon_E| = 0.13\%$ is much smaller than $\widetilde{R}_{EU} = 1.3\%$ and $\widetilde{R}_{EN} = 2.7\%$, which implies that the error term $\varepsilon_E\widetilde{\eta}_E$ in (37) is not quantitatively significant; $|\varepsilon_U| = 4.22\%$ is much smaller than $\widetilde{R}_{UE} = 28.3\%$ and $\widetilde{R}_{UN} = 23.3\%$, which implies that the error term $\varepsilon_U\widetilde{\eta}_U$ in (38) is not quantitatively significant; $|\varepsilon_N| = 0.35\%$ is much smaller than $\widetilde{R}_{NE} = 4.8\%$ and $\widetilde{R}_{NU} = 2.4\%$, which implies that the error term $\varepsilon_N\widetilde{\eta}_N$ in (39) is not quantitatively significant.

$$\widetilde{R}_{EU}\widetilde{\eta}_E + \left(\widetilde{R}_{NU}\widetilde{\eta}_N - \widetilde{R}_{UN}\widetilde{\eta}_U\right) = \widetilde{R}_{UE}\widetilde{\eta}_U + \varepsilon_U\widetilde{\eta}_U, \tag{44}$$

$$\left(\widetilde{R}_{EN}\widetilde{\eta}_E - \widetilde{R}_{NE}\widetilde{\eta}_N\right) = \left(\widetilde{R}_{NU}\widetilde{\eta}_N - \widetilde{R}_{UN}\widetilde{\eta}_U\right) + \varepsilon_N\widetilde{\eta}_N, \tag{45}$$

Viewing R_{EN} in the model as the counterpart of $\left(\widetilde{R}_{EN}\widetilde{\eta}_E - \widetilde{R}_{NE}\widetilde{\eta}_N\right)/\widetilde{\eta}_E$ in the data and R_{NU} as the counterpart of $\left(\widetilde{R}_{NU}\widetilde{\eta}_N - \widetilde{R}_{UN}\widetilde{\eta}_U\right)/\widetilde{\eta}_N$ establishes a clear correspondence between the model and the data.

Second, suppose we use the model generated R_{EN} and R_{NU} to match the the gross flows $\widetilde{R}_{EN}$ and $\widetilde{R}_{NU}$ in the data. Then (39) and (42) imply

$$0 = -\frac{\widetilde{R}_{UN}\widetilde{\eta}_U}{\widetilde{\eta}_N} + \widetilde{R}_{NE} + \varepsilon_N$$

and (39) and (42) imply

$$0 = -\widetilde{R}_{NE} + \widetilde{R}_{UN}\frac{\widetilde{\eta}_U}{\widetilde{\eta}_N} - \varepsilon_N.$$

There are no reasons why the data should meet (or be close to meeting) these conditions.

References

[1] Abreu, D., D. Pearce and E. Stacchetti (1990), Toward a Theory of Discounted Repeated Games with Imperfect Monitoring, *Econometrica*, 58(5), 1041-1063.

[2] Alvarez, F. and M. Veracierto (1999), Labor-Market Policies in an Equilibrium Search Model, *NBER Macroeconomics Annual*, 14, 265-304.

[3] Athey, S. and K. Bagwell (2001), Optimal Collusion with Private Information, *Rand Journal of Economics*, 32(3), 428-465.

[4] Atkeson, A. and R.E. Lucas (1992), On Efficient Distribution with Private Information, *Review of Economic Studies*, 59(3), 427-453.

[5] Blanchard, O.J. and P. Diamond (1989), The Beveridge Curve, Brookings Papers on Economic Activity, 1, 1-76.

[6] Brown, J.N. (1989), Why Do Wages Increase with Tenure? On-the-Job Training and Life-Cycle Wage Growth Observed within Firms, *American Economic Review*, 79(5), 971-991.

[7] Burdett, K. and M. Coles (2003), Equilibrium Wage-Tenure Contracts, *Econometrica*, 71(5), 1377-404.

[8] Davis, S.J., R.J. Faberman and J. Haltiwanger (2006), The Flow Approach to Labor Markets: New Data Sources and Micro-Macro Links, NBER Working Paper No. 12167.

[9] Den Haan, W.J., G. Ramey and J. Watson (1999), Contract-Theoretic Approaches to Wages and Displacement, NBER Working Paper No. 6972, Federal Reserve Bank of St. Louis Review, May/June, 55-68.

[10] Farber, H.S. (1999), Mobility and Stability: The Dynamics of Job Change in Labor Markets, Handbook of Labor Economics Volume 3B, 2439-2483.

[11] Fallick, B. and C.A. Fleischman (2004), Employer-to-Employer Flows in the U.S. Labor Market: The Complete Picture of Gross Worker Flows, Finance and Economics Discussion Series, Board of Governors of the Federal Reserve System.

[12] Green, E.J. (1987), Lending and the Smoothing of Uninsurable Income, Contractual Agreements for Intertemporal Trade (Minnesota Papers in Macroeconomics, No.1), E. Prescott and N. Wallace, eds., University of Minnesota Press.

[13] Guerrieri, V. (2008), Inefficient Unemployment Dynamics under Asymmetric Information, *Journal of Political Economy*, 116(4), 667-708.

[14] Guerrieri, V., R. Shimer and R. Wright (2010), Adverse Selection in Competitive Search Equilibrium, *Econometrica*, 78(6), 1823-1862.

[15] Hall, R. (2005), Employment Fluctuations with Equilibrium Wage Stickiness, *American Economic Review*, 95(1), 50-65.

[16] Hopenhayn, H.A. and J.P. Nicolini (1997), Optimal Unemployment Insurance, *Journal of Political Economy*, 105(2), 412-438.

[17] Hornstein, A., P. Krusell, G.L. Violante (2011), Frictional Wage Dispersion in Search Models: A Quantitative Assessment, *American Economic Review*, 101(7), 2873-98.

[18] Hosios, A.J. (1990), On the Efficiency of Matching and Related Models of Search and Unemployment, *Review of Economic Studies*, 57(2), 279-298.

[19] Jovanovic, B. (1979), Job Matching and the Theory of Turnover, *Journal of Political Economy*, 87(5), 972-990.

[20] Lazear, E.P. (1979), Why is There Mandatory Retirement?, *Journal of Political Economy*, 87(6), 1261-1284.

[21] Kim, S. (2001), Labor Force Participation, Worker Flows and Labor Market Policies, Working Paper, University of Pennsylvania.

[22] Krusell, P., T. Mukoyama, R. Rogerson and A. Sahin (2011), A Three State Model of Worker Flows in General Equilibrium, *Journal of Economic Theory*, 146(3), 1107-1133.

[23] Moen, E.R. and A. Rosen (2011), Incentives in Competitive Search Equilibrium, *Review of Economic Studies*, 78(2), 733-761.

[24] Mortensen, D.T. and Pissarides, C.A. (1994), Job Creation and Job Destruction in the Theory of Unemployment, *Review of Economic Studies*, 61(3), 397-415.

[25] Mortensen, D. T. and C. A. Pissarides (1998), New Developments in Models of Search in the Labor Market, in O. Ashenfelter and D. Card, eds., Handbook of Labor Economics, North Holland, Amsterdam.

[26] Moscarini, G. (2003), Skill and Luck in the Theory of Turnover, Working Paper, Yale University.

[27] Moscarini, G. (2005), Job Matching and the Wage Distribution, *Econometrica*, 73(2), 481-516.

[28] Nagypal, Eva (2005), On the Extent of Job-to-Job Transitions, Working Paper, Northwestern University.

[29] Petrongolo, B. and C.A. Pissarides (2001), Looking into the Black Box: A Survey of the Matching Function, *Journal of Economic Literature*, 39(2), 390-431.

[30] Rocheteau, G. (2001), Equilibrium Unemployment and Wage Formation with Matching Frictions and Worker Moral Hazard, *Labour Economics*, 8(1), 75-102.

[31] Rudanko, L. (2009), Labor Market Dynamics under Long-Term Wage Contracting, *Journal of Monetary Economics*, 56(2), 170-183.

[32] Sannikov, Y. (2008), A Continuous-Time Version of the Principal-Agent Problem, *Review of Economic Studies*, 75(3), 957-984.

[33] Shapiro, C. and J.E. Stiglitz (1984), Equilibrium Unemployment as a Worker Discipline Device, *American Economic Review*, 74(3), 433-444.

[34] Shavell, S. and L. Weiss (1979), The Optimal Payment of Unemployment Insurance Benefits over Time, *Journal of Political Economy*, 87(6), 1347-1362.

[35] Shimer, R. (2005a), The Cyclical Behavior of Equilibrium Unemployment, Vacancies, and Wages: Evidence and Theory, *American Economic Review*, 95(1), 25-49.

[36] Shimer, R.(2005b), Reassessing the Ins and Outs of Unemployment, Working Paper, University of Chicago.

[37] Stiglitz, J.E. and A. Weiss (1983), Incentive Effects of Terminations: Applications to the Credit and Labor Markets, *American Economic Review*, 73(5), 912-927.

[38] Spear, S.E. and S. Srivastava (1987), On Repeated Moral Hazard with Discounting, *Review of Economics Studies*, 54, 599-617.

[39] Spear, S. and C. Wang (2005), When to Fire a CEO: Optimal Termination in Dynamic Contracts, *Journal of Economic Theory*, 120(2), 239-256.

[40] Veracierto, M. (2008), On the Cyclical Behavior of Employment, Unemployment and Labor Force Participation, *Journal of Monetary Economics*, 55(6), 1143-1157.

[41] Wang, C. (1995): Dynamic Insurance with Private Information and Balanced Budgets, *Review of Economic Studies*, 62(4), 577-95.

[42] Wang, C. (2011), Termination of Dynamic Contracts in an Equilibrium Labor Market Model, *Journal of Economic Theory*, 146(1), 74-110.

[43] Wang, C. (2013), Incentives and the Cost of Firing in an Equilibrium Labor Market Model, *International Economic Review*, 54(2), 443-472.

[44] Wang, C. and S. Williamson (1996), Unemployment Insurance with Moral Hazard in a Dynamic Economy, Carnegie-Rochester Conference Series on Public Policy, Spring, 1-41.

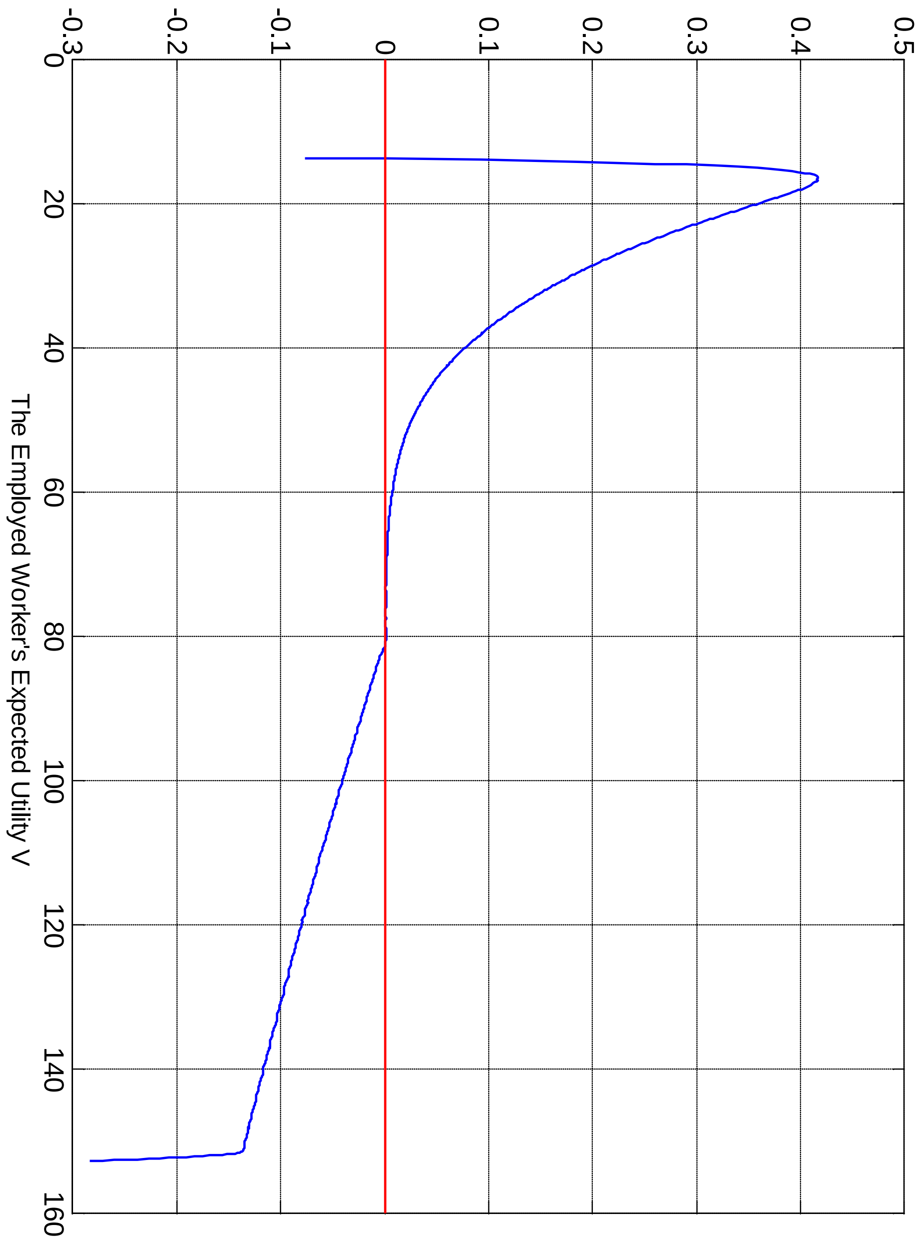

Figure 1: The firm's net gains from retaining the worker

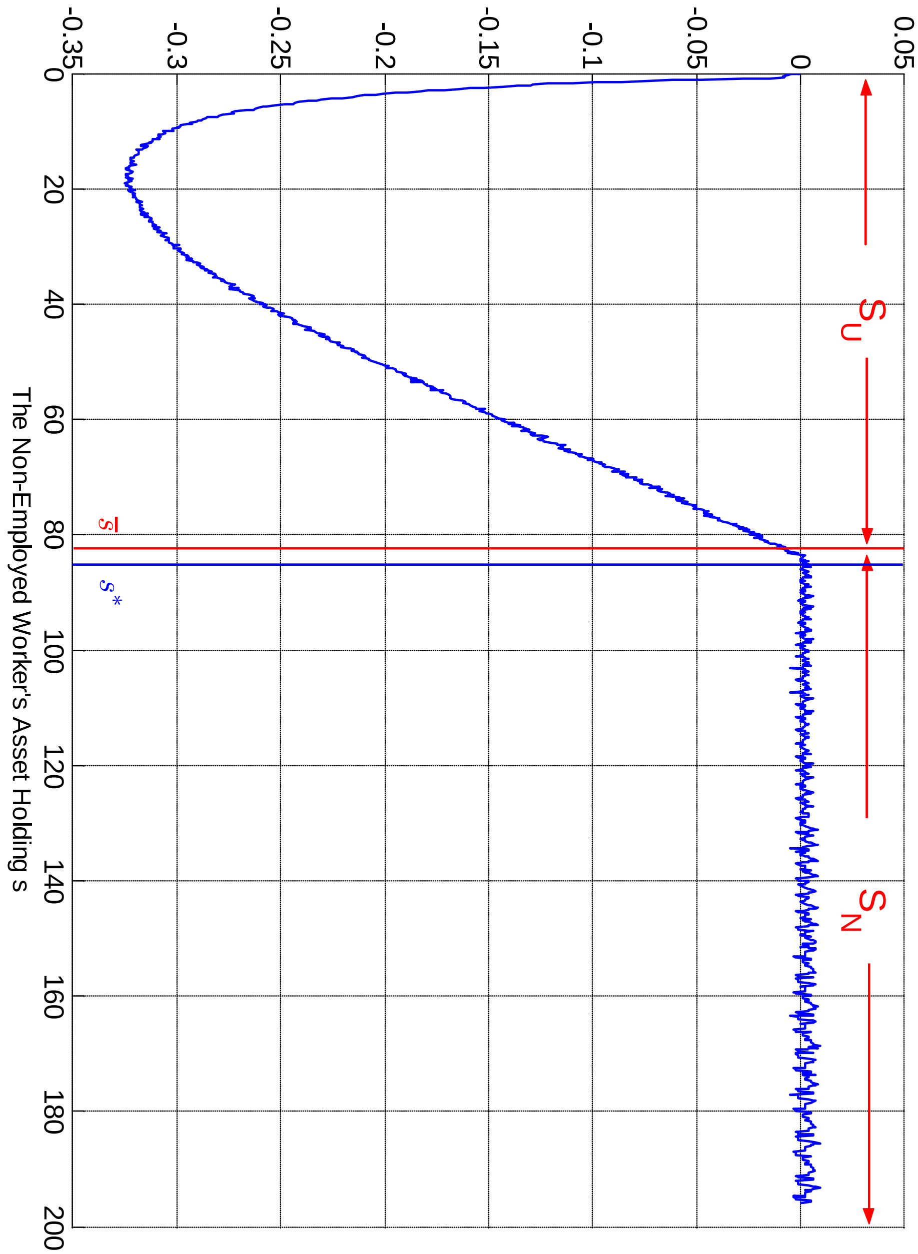

Figure 2: The law of motion for the non-employed worker's assets

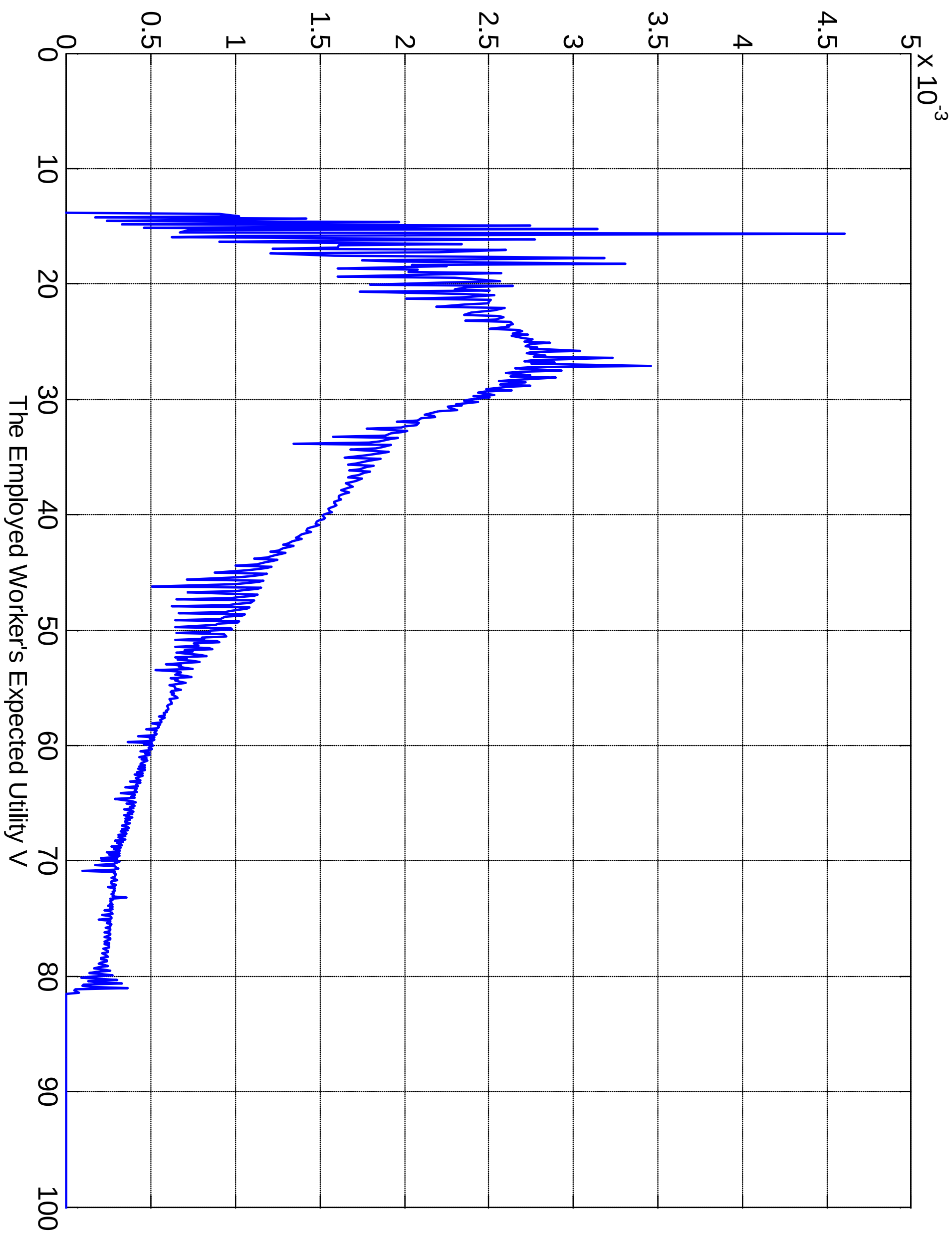

Figure 3: The distribution of employed workers

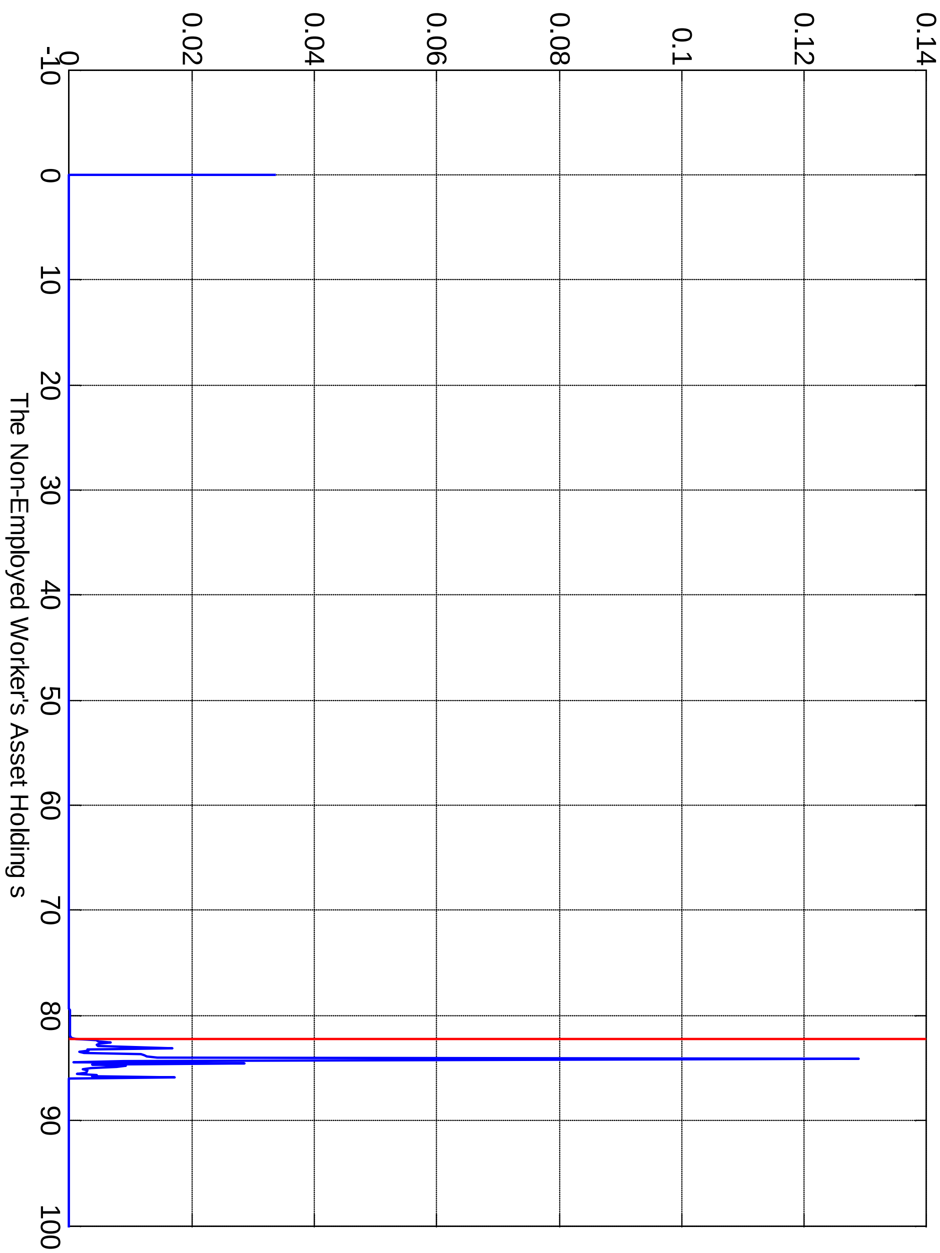

Figure 4: The distribution of non-employed workers

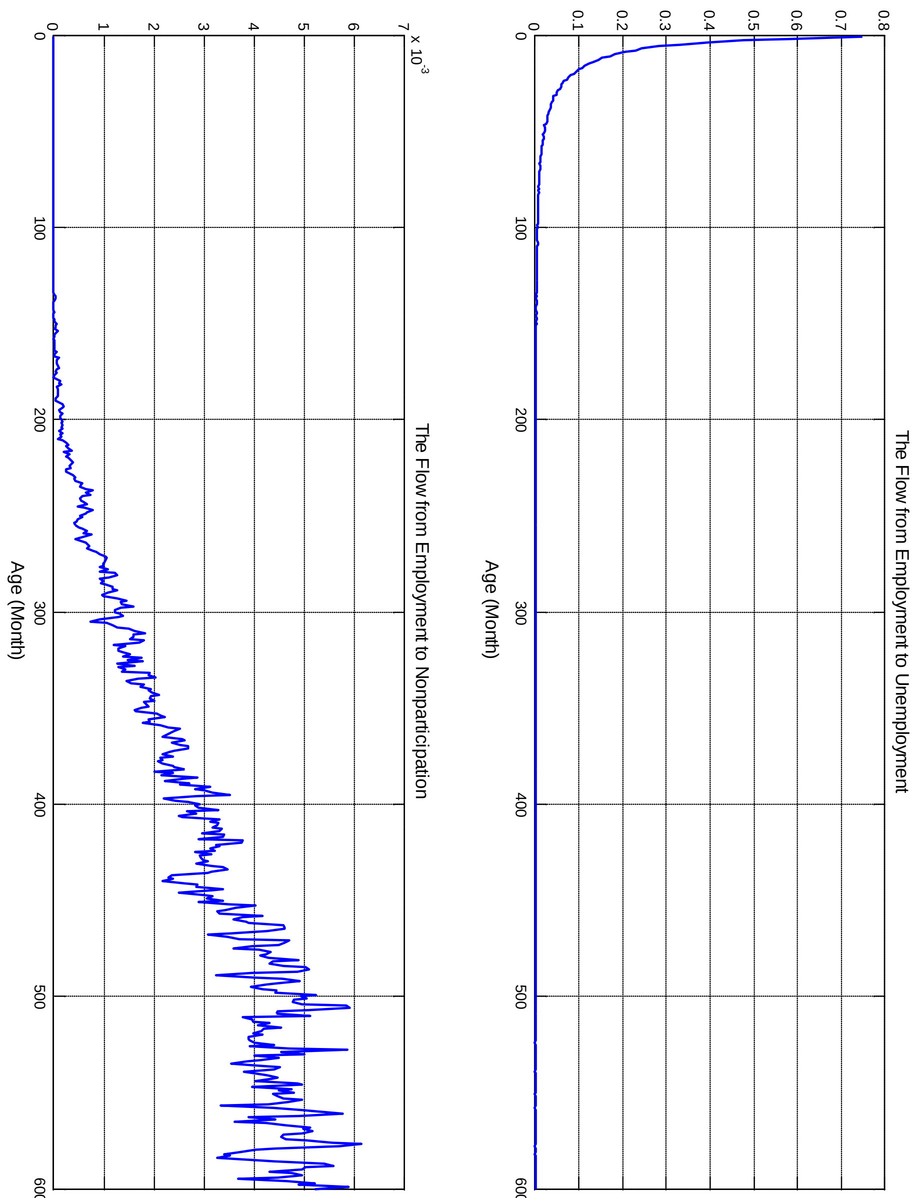

Figure 5: The probabilities of termination

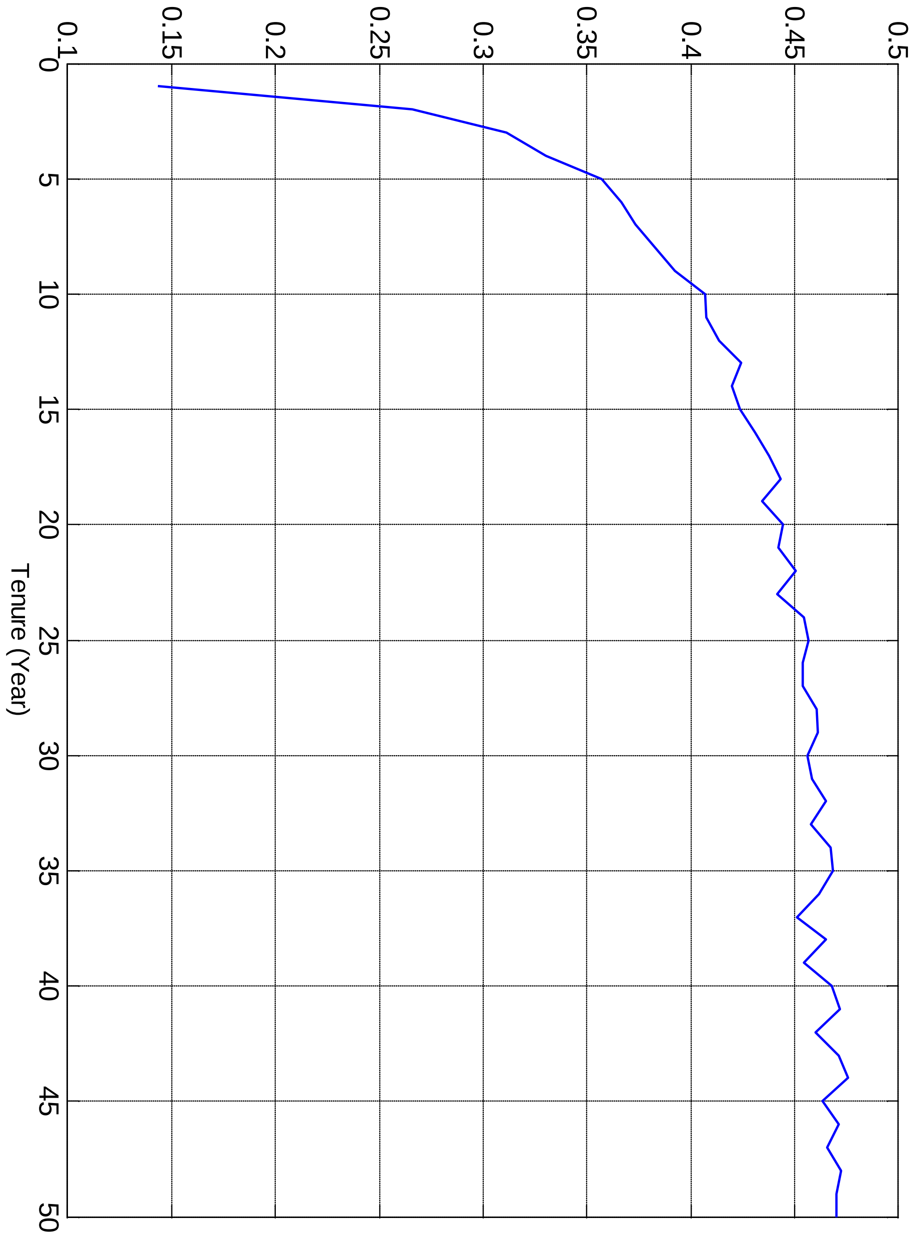

Figure 6: Average wage as a function of tenure

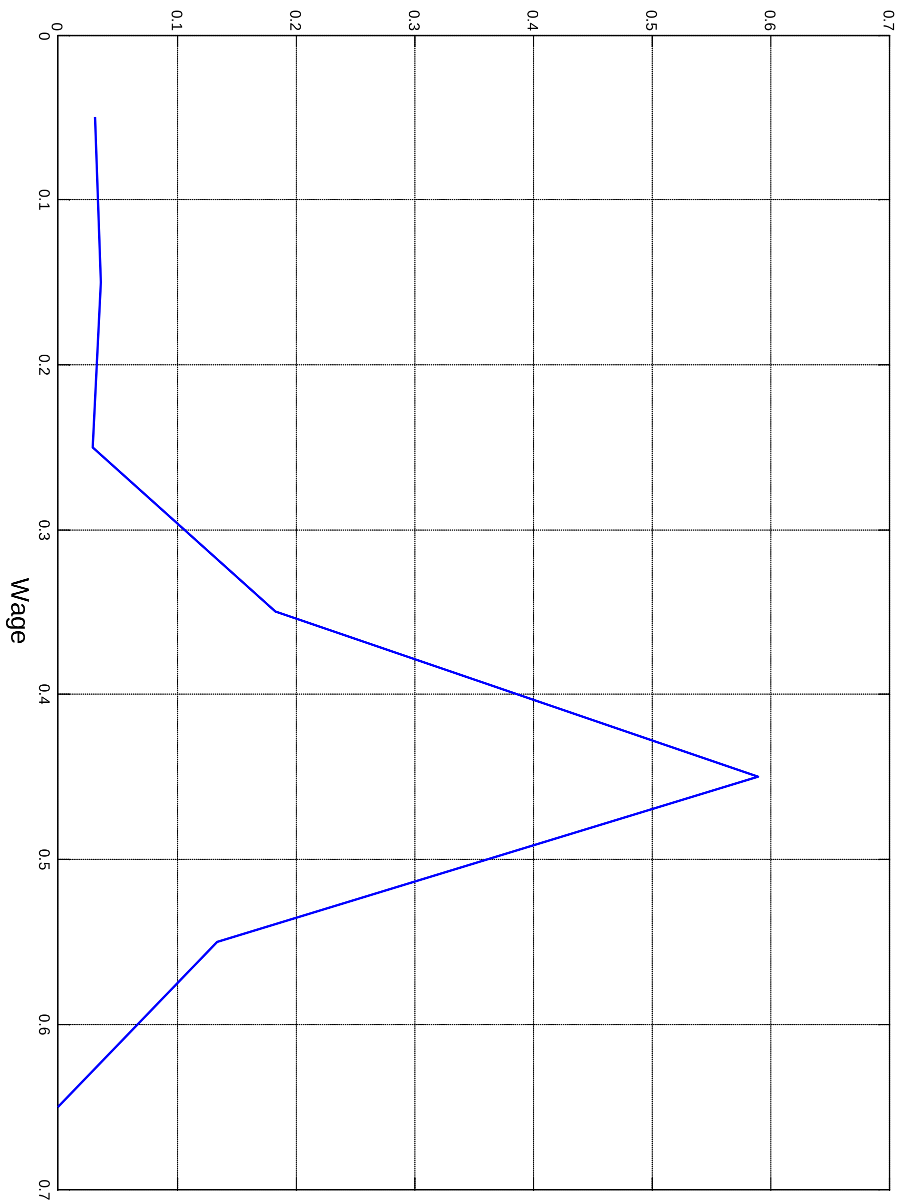

Figure 7: The wage distribution