Coordination Frictions and Public Communication*

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Abstract

Under global games' framework, we analyze the choice of information disclosure from a benevolent public agent (sender), who can send the message to a group of private players (receivers) before they take actions. It is shown that, in the presence of coordination frictions, conflict of interests endogenously. As a result, public information provision in equilibrium is coarse and presents partition structure. Interestingly, compared to benevolent public agent, introducing a small exogenous bias in the sender's preferences can mitigate the inefficient information provision problem and hence improve social welfare.

Keywords: endogenous conflict of interests, public information provision, optimal bias, global games, cheap-talk games.

JEL Classification Numbers: C72, D83.

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1 Introduction

Conventional wisdom suggests that transparent policy-making sometimes does more harm than good. Especially this concerns situations when a large group of people is expected to take action on the basis of public information, while the disclosing body is apt to making mistakes. For instance, it is often argued that at the onset of the 2008 sub-prime crisis, Credit Rating Agencies were responsible for fueling the housing bubble via over-rating the mortgage-backed securities. Had the private investors placed less weight on the reports published by CRAs, the stock-market collapse could have been less pronounced.

Information transmission between informed and uninformed parties is an important research question and has been long studied by economists. It is well documented that when the informed and uninformed parties' objectives diverge, conflict of interests prevents full information disclosure. Yet, a number of papers point out that transparency does not always improve social welfare.

In this paper, we look at a particular type of communication, in which an informed agent sends a public message to a large group of people. For example, credit rating agencies issue rating reports which contain information about financial securities to all potential investors; firms publish annual reports revealing their current performance to all stakeholders; managers review team performance in routine meetings with employees. Quite often, the informed party cannot commit to tell the truth. Therefore, rational audience needs to form expectations concerning the informativeness of the public messages.

Another feature of this type of communication is that the receivers may have strategic interaction among themselves. In a debt run context, depositors' withdrawal decisions generally depend on the choices of others. Peer pressure or synergy makes one employee more willing to exert effort when others are making effort. In the presence of such coordination frictions, public messages serve a dual role: on the one hand, they convey information, and on the other, they serve as a powerful *coordination device*. As a result, even when the informed party is benevolent, she is reluctant to reveal bad news. For example, in the recent crisis, credit rating agencies have been criticized for their "inflated ratings". However, one defending argument is that downgrading a security can have a feedback effect and trigger "multi-notch downgrades"[1].

The main message of our paper is that when the informed public agent cannot commit to the disclosure rule and when there exist coordination frictions among receivers, conflict of interests arises *endogenously*. As a result, public information provision in equilibrium is *coarse*,

[1] The feedback effect by credit ratings is studied by Manso (2011).

even if the sender is fully benevolent. Interestingly, introducing a small exogenous bias in the sender's preferences can mitigate inefficient information provision and improve social welfare. This sheds light on our understanding of the relationship between benevolence, transparency, and welfare.

To fix ideas, think about a group of investors, each of whom is considering whether or not to undertake a risky project with random payoff, which can be partially learned through noisy *private signals*. On top of that, there is a public agent who also has some information about the payoff and can share it by sending *public messages*. In addition, this random payoff from the risky project depends on the aggregate participation. In our model, we allow investors' action to be either *strategic complements* or *strategic substitutes*. Using global games' technique, we show that investors optimally use switching strategies, with the threshold being negatively related to the public prior.

Our core result is that, in comparison to the benchmark model, coordination friction places a wedge between the objectives of the benevolent public agent and of the marginal investor. When investors' actions are complements (substitutes), the public agent is willing to induce more aggressive (conservative) strategies for the individuals. It is also interesting to note that this endogenous conflict of interests in state-contingent: it is relatively negligible when the sender's signal is very high or very low, but may be quite substantial when the signal is medium. We show that in the absence of commitment (that is, when the public agent can only engage in cheap-talk), equilibrium information provision must be coarse in order to satisfy the sender's inventive compatibility condition.

Finally, we study the interaction between exogenous and endogenous conflict of interests. We provide numerical examples to show that it can be welfare-improving for the public agent to be subject to small exogenous bias in the appropriate direction. By mitigating endogenous conflict of interests, this bias may improve efficiency of information provision.

1.1 Related Literature

Our paper is related to the cheap-talk literature pioneered by Crawford and Sobel (1982) and Farrell and Gibbons (1989). Crawford and Sobel (1982) show that conflict of interests between an informed sender and an uninformed receiver prevents the former from fully disclosing her information. Their equilibrium communication strategy is represented by a partition structure.

Farrell and Gibbons (1989) generalize their model by considering two audiences and study the sender's optimal usage of public and private messages. In contrast, our model restricts the sender to use public messages. Another important distinction is that, instead of exogenously postulating the conflict of interest, we provide a micro-foundation for it.

Another building block we are borrowing from is the global games literature, which studies the role of public and private information in coordination games. Some papers have demonstrated that increasing the transparency of public information can reduce social welfare. In particular, Morris and Shin (2002) show that individuals tend to put too much weight (relative to the Pareto optimum) on the public signal, since it serves a coordination purpose. However, they assume the public agent can commit to the precision of her public message, whereas we do not allow commitment. Yet we obtain a similar result that social welfare is not necessarily increasing in the informativeness of equilibrium reporting strategy[2].

One more strand of global games literature studies endogenizing public information and equilibrium multiplicity. Angeletos et al. (2006) examine the informational role of policy in the global games' setting: they analyze a currency attack model along the lines of Morris and Shin (1998), which is preceded by the choice of the interest rate by the Central Bank. They show that such policy interventions generate multiple equilibria. The key difference here is that in our setup, payoff is determined by the fundamental rather than by coordination frictions. Hence, benevolent public agent in our model prefers interior instead of corner actions.

At a more applied level, our paper is related to a recent literature on Credit Rating Agencies. See Skreta and Veldkamp (2009); Bolton et al. (2012). Instead of addressing the question of what was done by the CRA's, we take a normative approach and pose the question of what they should do.

Finally, concerning the intuition for why an exogenous bias may be welfare-enhancing, our work is also related to Rogoff (1985), who shows that society can be better off by appointing a central banker placing "too large" a weight on inflation-rate stabilization relative to employment stabilization.

The rest of the paper is organized as follows. Section 2 outlines the general model setup. Section 3 first solves the investors' subgame then solve the full model where public information is endogenized. Section 4 discusses the welfare implication of the model and the relationship between the sender's exogenous bias, transparency and social welfare. Section 5 concludes.

[2]As in other cheap-talk models, equilibrium strategy in our framework is represented by a partition structure. Loosely speaking, a strategy with finer partition is considered to be more informative.

2 The Model

2.1 Environment

In this section, we outline the model and deliver our first result under a fairly general setup. There is a uniform continuum of risk-neutral players (the receivers), indexed by $i \in [0,1]$. Each of them has a choice between a safe and a risky action. We write $a_i \in \{0,1\}$ to denote receiver i's choice, with $a_i = 0$ when the safe action is taken, and $a_i = 1$ otherwise.

The payoff to undertaking the safe action is normalized to 1. The payoff to the risky action is denoted by R, and it depends on two variables: the state of economic fundamentals, which we denote by θ, and the overall measure of those who decide to take the risky action, $A \equiv \int_{\{i:a_i=1\}} a_i di$. Denoting the set of possible realizations of θ by Θ, we have

$$R : \Theta \times [0,1] \to \mathbb{R} \tag{2.1}$$

as the payoff function. In our setting, we focus on the case when the fundamental can take just two values: $\Theta = \{g, b\}$, where g and b stand for "good" and "bad", respectively. The commonly known prior probability of the good state is denoted by φ_0.

We suppose that the payoff from undertaking the risky action is always zero when $\theta = b$. On the other hand, when $\theta = g$, the payoff is given by some function of A. We have

$$R(\theta, A) = \begin{cases} R(A), & \text{if } \theta = g \\ 0, & \text{if } \theta = b \end{cases} \tag{2.2}$$

Two cases should be distinguished. We say that players' actions exhibit *strategic complementarities*, if $\partial R(\theta, A)/\partial A > 0$, whereas we have *strategic substitutabilities* for the case $\partial R(\theta, A)/\partial A < 0$. For most of the paper, we would assume that $R(\cdot)$ takes the following simple linear form:

$$R(A) = \rho(1 + rA) \tag{2.3}$$

Parameter ρ captures the private benefit from undertaking the risky action when the state is "good", whereas r might be referred to as the "complementarity term": it reflects the impact of a change in A on the payoff from choosing $a_i = 1$.

We make the following assumption concerning the ρ and r:

Assumption 1. *Parameters ρ and r satisfy*

$$\rho > 1 \quad \text{and} \quad r > \frac{1}{\rho} - 1 \tag{2.4}$$

Notice that in particular, Assumption 1 guarantees that $\min_{A\in[0,1]} R(A) > 1$: whatever others are doing, in the "good" state it pays to undertake the risky action.

Suppose that, before deciding on a_i, each player i observes noisy private signal $x_i \in X$, whose conditional distribution is denoted by $F(\cdot|\theta)$, with larger x_i corresponding to better news. We assume that these private signals are i.i.d.

Additionally, there is the public agent (the sender), who observes a noisy signal $y \in Y$, drawn from a conditional distribution $H(\cdot|\theta)$. Given θ, this public signal y is assumed to be independent from x_i's. In what follows, we set $X = Y = [0, 1]$.

In our basic framework, we assume that the public agent is benevolent: his payoff is given by the expected sum of the receivers' payoffs, conditional on her information y,

$$\Pi(y) = \mathbb{E}\left\{ \int \pi_i di \,\middle|\, y \right\} \tag{2.5}$$

The timing of the game is summarized below:

1. Nature draws θ from $\Theta = \{g, b\}$ with probabilities $(\varphi_0, 1 - \varphi_0)$.
2. The sender receives public signal $y \in Y$, while receivers get the private signals $x_i \in X$.
3. The sender sends a message, $m \in \mathcal{M}$.
4. Each receiver i chooses a_i, upon observing (x_i, m).
5. Payoffs are realized.

To ease exposition, we also assume that conditional distributions admit densities, which are conventionally denoted by lowercase letters: f for private signals and h for the public signal. Also, we denote the likelihood ratio by λ: $\lambda(x) \triangleq f(x|g)/f(x|b)$. We impose several regularity conditions on the distribution function F:

Assumption 2. *Conditional distribution of the private signal $F(\cdot|\theta)$ satisfies*

i. Monotone Likelihood Ratio Property (MLRP): $\lambda(x)$ is increasing in x.

ii. Precision at the extremes:

$$\lim_{x\to 0} \lambda(x) = 0 \quad \text{and} \quad \lim_{x\to 1} \lambda(x) = +\infty \tag{2.6}$$

iii. The elasticity of receiver's risky payoff (i.e. payoff from $a = 1$ conditional on $\theta = g$) with respect to the likelihood ratio $\lambda(x)$ is decreasing in x:

$$\frac{d}{dx}\left[\frac{d\log(\rho\left[1+r[1-F(x|g)]\right])}{d\log\lambda(x)}\right] < 0 \tag{2.7}$$

Condition (i) says that higher signals, quite naturally, correspond to "good news" in the sense of Milgrom (1981). Condition (ii) states that for the extreme signal realizations ($x = 1$ and $x = 0$, correspondingly), the signal precision is high enough, so that the agent is pretty sure that the state is "good" or "bad", regardless of the public information. Condition (iii) is a technical requirement, stating that as news become more pessimistic (lower x), the impact of the likelihood ratio on receiver's *expected* payoff conditional on x gradually decays.[3]

The sender can send public messages to individuals. We define the sender's strategy as a mapping from Y into the set of *probability distributions* over $\mathcal{M}$:

$$\mu : Y \to \Delta(\mathcal{M}) \tag{2.8}$$

which for any signal y received tells him to send the message m with probability $\mu(m, y)$. And since the public agent can not commit on her strategy, μ should satisfy incentive compatibility condition in equilibrium. $\mathcal{M}$ is the set that contains all messages, which are used with positive probability in equilibrium. In addition, we denote $\mathcal{M}(y)$ to be the set of messages, which are used with positive probability when the public agent's information is y.

Private agents cannot directly observe the sender's information $y \in Y$, but can only see the message $m \in \mathcal{M}$. So, receiver i's strategy is a mapping

$$a_i : X \times \mathcal{M} \to \{0, 1\} \tag{2.9}$$

which for any pair (x_i, m) tells him which action to take.

2.2 Equilibrium characterization

Let us specify the payoffs that accrue to the public and the private agents. Conditional on (x_i, m), receiver i chooses a_i, solving

$$\max_{a_i\in\{0,1\}} \mathbb{E}[\pi_i(a_i)|x_i, m)] = (1 - a_i) + a_i \cdot \varphi(g|x_i, m)\rho(1 + r\mathbb{E}[A(\theta)|x_i, m]) \tag{2.10}$$

[3]The elasticity in (2.7) is negative, hence "decreasing in x" means becoming more negative.

where $\varphi(g|x_i, m)$ denotes the posterior probability that $\theta = g$, given (x_i, m). Notice that the aggregate participation $A(\theta)$ may depend on the realized state and is itself considered to be a random variable from the viewpoint of the player who observes only x_i, but not θ. On the other hand, $A(\theta)$ is determined in equilibrium once individual agents' strategy profile is fixed and θ is known.

The sender evaluates expected social welfare, conditionally on the public signal y she gets. We denote the expected aggregate payoff conditional on the signal y by

$$\Pi(m; y) = \mathbb{E}\left\{ \int \pi_i(a_i; \theta, A) di \middle| y \right\} \tag{2.11}$$

where $\pi_i(\cdot; \theta, A)$ denotes the realized payoff to the receiver i, who chooses action a_i, conditional on the state being θ and the aggregate participation being A. This is given by

$$\pi_i(a_i; \theta, A) = \begin{cases} \mathbb{1}_{\{\theta=g\}} \cdot \rho(1 + rA), & \text{if } a_i = 1 \\ 1, & \text{if } a_i = 0 \end{cases} \tag{2.12}$$

where $\mathbb{1}_{\{\theta=g\}}$ is the indicator function of the event $\{\theta = g\}$.

At this point, we are ready to formally state the definition of the Bayesian Nash equilibrium for this game.

Definition 1. A Bayesian Nash equilibrium consists of (i) the decision rule for each individual agent $a_i(\cdot, \cdot)$, (ii) the information revelation strategy for the public agent $\mu(\cdot, \cdot)$, (iii) the conditional posteriors on states $\varphi(\cdot|\cdot)$, and (iv) the aggregate participation measurement $A(\cdot)$, such that

1. Given μ and $\forall \theta \in \{g, b\}$ and $\forall m \in \mathcal{M}$, $\varphi(\theta|m)$ is consistent with Bayesian rule.

2. Given A and φ, $\forall x_i \in X$ and $\forall m \in \mathcal{M}$, $a_i(x_i, m)$ solves (2.10).

3. Given a_i and $\forall \theta \in \{g, b\}$, $A(\theta) \equiv \int \mathbb{1}_{\{a_i=1\}} di$.

4. Given a_i, $\forall y \in Y$ and $\forall m \in \mathcal{M}(y)$, we should have

$$\Pi(m; y) \geq \Pi(m'; y), \quad \forall m' \in \mathcal{M} \tag{2.13}$$

The sender's value corresponding to the equilibrium strategy μ, or the ex-ante social welfare can be defined as

$$V(\mu) = \int_Y \mu(m, y) \Pi(m; y) dH(y) \tag{2.14}$$

where $H(y)$ stands for the unconditional distribution of the public signal y. Observe that there exists an upper bound for ex-ante social welfare, which is given by

$$\bar{V} = \varphi_0 R(1) + (1 - \varphi_0) \tag{2.15}$$

3 Cheap-talk Equilibrium

In this section, we characterize equilibria of the full game, in which the receivers take into account the fact that the sender may engage in potential manipulation of information to his own advantage. Those equilibria share the features of classical cheap-talk equilibria in Crawford and Sobel (1982).

Specifically, we look for a Bayesian Nash equilibrium, whereby simultaneously the sender chooses the reporting rule (as given by (2.8)) in order to maximize (2.11), while the receivers choose the action rule (as given by (2.9)) so as to maximize (2.10).

Prior to that, we characterize equilibria played by the receivers in the subgame following the message $m \in \mathcal{M}$ sent by the sender, and also analyze the conflict of interest that is endogenously created by the complementarities in actions.

3.1 Equilibrium in the receivers' subgame

We first solve the receivers' subgame by fixing the sender's strategy at $\bar{\mu}$. Under $\bar{\mu}$, when individuals receive a public message m, they will update their prior distribution about state according to

$$\varphi(g|m) = \frac{\varphi_0 \int \mu(m,y) dH(y|g)}{\varphi_0 \int \mu(m,y) dH(y|g) + (1-\varphi_0) \int \mu(m,y) dH(y|b)} \tag{3.1}$$

For example, if the sender were able to commit to truth-telling, then $m \equiv y$ and the receivers always obtain their updated prior as the sender's posterior. At the other extreme, if the sender is completely babbling, then for all messages, we have $\varphi(g|m) \equiv \varphi_0$.

Without loss of generality, we can assume the sender reports her posterior as the message.[4] Hence, we will be solving the subgame equilibrium for any possible updated prior $\varphi \in [0,1]$. Furthermore, as in most of the global games' literature, we focus on equilibria, which are characterized by *threshold strategies*: that is, receiver i undertakes $a_i = 1$ iff his private signal x_i exceeds a given threshold, which we denote by $\hat{x}_i$.

[4] Sender's strategy μ must be consistent with Bayes' rule: for all $\varphi \in [0,1]$, (3.1) has to be satisfied.

For instance, if receiver i's threshold is x, while the others are playing around $\hat{x}$, then his expected payoff from the risky action is given by

$$\pi(x,\hat{x}) = \frac{\varphi f(x|g)}{\varphi f(x|g) + (1-\varphi) f(x|b)} \cdot \rho\left[1 + r(1 - F(\hat{x}|g))\right] \tag{3.2}$$

There are two conditions justifying the use of such strategies. First, receivers' payoffs have to be monotone in θ – this is satisfied, since R is assumed to be increasing in θ. Second, receivers' posterior beliefs on θ should be first-order stochastically increasing in their private signals, x_i – the latter property is guaranteed by MLRP.

We are looking for symmetric equilibrium, in which the receiver, whose private signal x is exactly equal to this threshold, is indifferent between choosing the risky and the safe action; he is the one whom we call the "marginal receiver". Given the induced prior $\varphi \in (0,1)$, the equilibrium threshold $\hat{x}(\varphi)$ has to satisfy the following indifference condition for the marginal receiver:

$$\pi(\hat{x},\hat{x}) = 1 \tag{3.3}$$

At this point, we are ready to state and prove the following proposition:

Proposition 1. *Given Assumptions 1 and 2, for any $\varphi \in (0,1)$ there exists a* unique *symmetric switching equilibrium, in which each receiver i undertakes a risky action if and only if his private signal x_i is larger than a threshold, denoted by $\hat{x}(\varphi)$.*

The threshold $\hat{x}(\varphi)$ is strictly decreasing in the posterior. Furthermore, when the posterior becomes extreme, every receiver ends up undertaking the same action:

$$\lim_{\varphi\to 0} \hat{x}(\varphi) = 1 \quad \textit{and} \quad \lim_{\varphi\to 1} \hat{x}(\varphi) = 0$$

Proof. In the Appendix. □

Existence of the threshold equilibrium is guaranteed by Assumption 1 imposed on ρ and r, coupled with part (ii) of Assumption 2. Together, they permit us to identify two dominance regions for receivers' strategies in the global game, given any induced prior $\varphi \in (0,1)$. Intuitively, as the payoff in the "bad" state is always zero, for the most pessimistic receivers (those whose x's are close to 0), conditional expected payoff from $a = 1$ is lower than 1 regardless of participation level A, – so $a = 0$ is a dominant strategy. Likewise, since due to Assumption 1, $R(A)$ always exceeds one, for the most optimistic receivers, undertaking the risky action ($a = 1$) is a dominant strategy.

However, unlike in the standard global games' literature, the function $\hat{\pi}(x) \triangleq \pi(x,x)$, even though satisfying $\hat{\pi}(0) < 1 < \hat{\pi}(1)$, is generically non-monotone, meaning that additional restrictions on the primitives have to be added in order to ensure uniqueness. This is where part (iii) of Assumption 2 is being used: technically, it guarantees that $\hat{\pi}(\cdot)$ is single-peaked, so that it crosses 1 only once.

Intuitively, $\hat{\pi}(\cdot)$ is a product of the receiver's posterior, $\varphi\lambda(x)/(\varphi\lambda(x) + 1 - \varphi)$, and the payoff when $\theta = g$, $\rho[1 + r(1 - F(x|g))]$, where the former is increasing, while the latter is decreasing in x. Condition (iii) says that the rate of change of the expected payoff for the marginal receiver is driven mainly by the increase in the posterior for small signal realizations, while for large x, it gets driven by the reduction in the payoff in the "good" state.

Concerning the inverse relationship between $\hat{x}$ and φ, notice that when the induced prior becomes more optimistic, receivers start to act more aggressively, that is to say, playing a lower threshold. When the induced prior moves to one of the extremes, e.g. close to 1, all receivers end up undertaking the same action (playing $a = 1$), except for those who are extremely pessimistic (i.e. whose x is close to 0).

3.2 Endogenous conflict of interest

Before we go on to solve for equilibria in the full game, it is worth to have a closer look on how coordination frictions impede communication. We will show that except for the knife-edge case where $r = 0$, truth-telling is never a part of equilibrium in the full game, in which the sender cannot commit. That way, a conflict of interest between the sender and the receivers arises *endogenously* and is *state-contingent*, even though the sender is fully benevolent.

We first show our benchmark result in the absence of coordination frictions.

Proposition 2. *Truth-telling is an equilibrium and is socially optimal, if and only if $r = 0$, i.e. there are no coordination frictions.*

Proof. In the Appendix. □

Intuitively, if the payoff to $a = 1$ does not depend on the aggregate measure A of those who choose the risky action ($r = 0$), the receiver, by his reluctance to undertake $a = 1$, does not impose any externality on others. Hence, on the margin, the sender would wish all the receivers whose signals are above $\hat{x}(y)$ to choose $a = 1$, whereas for those who have $x_i \leq \hat{x}(y)$ to choose $a = 0$ instead: the first order condition for the sender's problem equates the net gain of the marginal receiver to zero. For this to hold, the sender has no incentive to distort the information that she gets himself.

On the other hand, for $r > (<)0$, the receiver who chooses $a = 0$ imposes a negative (positive) externality on the others, as he increases (reduces) the payoff from the risky action. Hence, were receivers to believe that the sender reports truthfully ($m \equiv y$), the sender would have an incentive to locally manipulate the information of the marginal receiver so as to make him more optimistic (pessimistic).

If the sender were able to commit to report the signal truthfully for all $y \in Y$, then by Proposition 1, we know that the subgame equilibrium threshold played by individual agents would be given by

$$\hat{x}^T(y) = \hat{x}(\varphi(y)) \tag{3.4}$$

where

$$\varphi(y) = \frac{\varphi_0 h(y|g)}{\varphi_0 h(y|g) + (1 - \varphi_0)h(y|b)}$$

denotes the sender's posterior conditional on her signal y. We call $\hat{x}^T(y)$ the *truthful-telling threshold*: it is the threshold that would be played in a game, where the signal $y \in Y$ is drawn exogenously and publicly observed by everyone.

Now suppose that, instead of providing public messages and letting individuals choose their own strategy, the public agent is able to directly impose any participating threshold x for them, conditional on her information. Then the sender's expected payoff is given by

$$\begin{aligned}\Pi(x; y) &= \sum_{\theta \in \{g,b\}} \varphi(\theta|y) \int \pi_i(a_i, \theta, A(\theta))di \\ &= \varphi(y) \left[F(x|g) + (1 - F(x|g))r[1 - F(x|g)]\right] + (1 - \varphi(y))F(x|b)\end{aligned} \tag{3.5}$$

We call this hypothetical

$$\hat{x}^{FB}(y) \in \arg\max_x \Pi(x; y)$$

the *first-best threshold*: this is the one that would be picked by the planner, whose goal is to maximize aggregate payoff and who observes y, but does not know either θ or x_i's.

Proposition 2 states that $\hat{x}^{FB}(y) = \hat{x}^T(y)$ for all $y \in Y$ if and only if $r = 0$. When $r \neq 0$, we define

$$\Delta(y) \triangleq \hat{x}^{FB}(y) - \hat{x}^T(y) \tag{3.6}$$

as the *endogenous conflict of interests* between the sender and the receivers.

The characterization of $\Delta(y)$ is given by the following proposition.

Proposition 3. *For any $y \in Y$, the first-best threshold $\hat{x}^{FB}(y)$ is unique and is implicitly defined by the sender's first-order condition with respect to x: $\partial\Pi(\hat{x}^{FB}(y); y)/\partial x = 0$. Moreover,* *the endogenous conflict of interest* $\Delta(y)$ *possesses the following properties:*

i. When receivers' actions exhibit strategic complementarity (substitutability), $\Delta(y)$ *is negative (positive), for all* $y \in Y$*.*

ii. $\Delta(y)$ *is non-monotonic in* y*. As* $\varphi(y) \to 0$ *or* 1*,* $\Delta(y) \to 0$*.*

Proof. In the Appendix. □

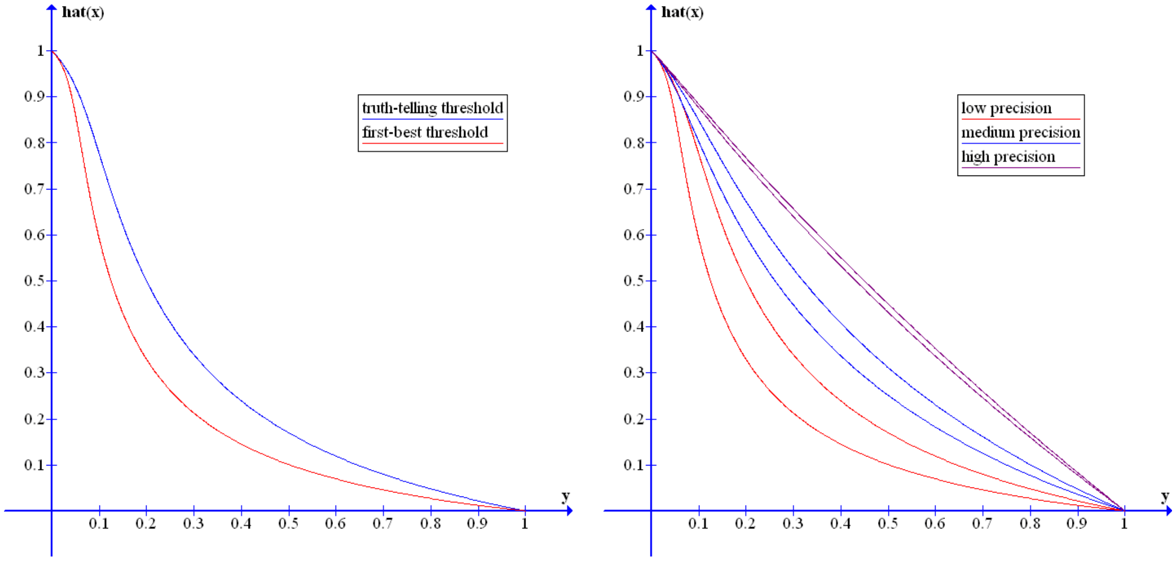
The intuition behind Proposition 3 is straightforward. When receivers face coordination concerns about their actions (which can either be strategic complements or strategic substitutes), each individual's action imposes externalities – positive or negative – on others' payoffs. From the viewpoint of the benevolent public agent, when $r > (<)0$ individual agents are under-participating (over-participating) under the truth-telling thresholds in any subgame. Hence, public agent's objective is no longer in line with the objective of the marginal receiver, and the endogenous conflict of interests emerges.

Furthermore, as shown in Figure 1a, this endogenous conflict of interests in *state-contingent*: it is stronger for intermediate signals y than for extreme signals. The reason is that, when the public agent's signal is at extremes (y being close to 0 or 1), her posterior is more revealing the true state, as well as receivers' updated prior, if she tells the truth. Then, even though individual participation level is still away from socially optimal, they become closer, since more individuals will (not) participate when they are more certain that the state is good (bad). This creates higher incentives to deviate from truth-telling when y is intermediate. In the limit when y becomes fully revealing, all individuals coordinate on the "right" strategy and the conflict of interests disappears.

3.3 Equilibrium in the full game

In general, the public sender may not be able to commit on truth-telling strategy or any other strategies. Hence, in an full game equilibrium, there should be no deviation for any message used in her reporting strategy. In this section, we are going to solve the equilibrium strategies and relate equilibria characterization with our result of endogenous conflict of interests.

Consider any strategy used by the sender and a message $m \in \mathcal{M}$, the essential thing learned by receivers, through Bayesian updating, is an updated prior $\varphi(m)$. For any $\varphi \in (0, 1)$, we know that the equilibrium in the receivers' subgame is a function $\{\hat{x}(\cdot)\}_{\varphi \in [0,1]}$ that solves (3.3). By revelation principle, we can focus on *direct communication*. Under direct communication, the reporting rule is given by a mapping from sender's signal set into the



(a) Endogenous Conflict of Interests

(b) Conflict of Interests Under Different Signal Precision

Figure 1: Truth-Telling Threshold and First-best Threshold

receivers' switching strategy set. In particular, when the sender's signal realization is y, she recommends the receivers to play the threshold $\hat{x}(y)$. The recommendation rule should satisfy incentive compatibility for the receivers.

Since the sender can not commit herself, our game is a cheap-talk game. And as is always the case in cheap-talk models, there exists the babbling equilibrium, in which receivers completely disregard the signals and play their threshold corresponding to the induced prior, whereas the sender uniformly randomizes over $[0, 1]$. In that case, for any $y \in Y$, we have $\varphi(y) \equiv \varphi_0$. As long as the distribution of θ is non-degenerate, i.e. if $0 < \varphi_0 < 1$, this threshold is always interior: $\hat{x}(\varphi_0) \in (0, 1)$.

The questions we raise are:

1. Under what conditions there exist informative equilibria, in which the sender's signal is partially transmitted to the receivers?

2. How are these equilibria altered when the precision of the private or the public signal, or the profitability parameters (ρ, r), change?

3. Finally, whether the public agent is always better off (from an ex-ante point of view) when equilibria under the finer partition are played?

We call a full game equilibrium a *K-partition equilibrium*, which consists of a K-interval partition of the message space $\{y_0, \ldots, y_k, \ldots, y_K\}$, with $y_0 = 0$ and $y_K = 1$, and a sequence of thresholds $1 \geq \hat{x}_1 \geq \cdots \geq \hat{x}_K \geq 0$. The sender, when receiving signal $y \in [y_{k-1}, y_k]$, will only report her posterior via a coarse information, $\varphi_k \equiv \varphi(g|y \in [y_{k-1}, y_k])$.[5]

In response to the sender's message, the receivers are optimally playing a strategy with the corresponding threshold, which is given by

$$1 = \frac{\varphi_k f(\hat{x}_k|g)}{\varphi_k f(\hat{x}_k|g) + (1-\varphi_k) f(\hat{x}_k|b)} \cdot \rho \left[1 + r(1 - F(\hat{x}_k|g))\right] \tag{3.7}$$

for all $k = 1, \cdots, K$, where

$$\varphi_k = \frac{\varphi_0 \left[H(y_k|g) - H(y_{k-1}|g)\right]}{\varphi_0 \left[H(y_k|g) - H(y_{k-1}|g)\right] + (1-\varphi_0)\left[H(y_k|b) - H(y_{k-1}|b)\right]} \tag{3.8}$$

Let us define the likelihood ratio induced by reporting the interval $[y_{k-1}, y_k]$ by

$$\Lambda(y_{k-1}, y_k) \triangleq \frac{H(y_k|g) - H(y_{k-1}|g)}{H(y_k|b) - H(y_{k-1}|b)}$$

Taken together, (3.7) and (3.8) implicitly define $\hat{x}(y_{k-1}, y_k)$ as

$$1 = \frac{\varphi_0 \lambda(\hat{x}(y_{k-1}, y_k))}{\varphi_0 \lambda(\hat{x}(y_{k-1}, y_k)) + \frac{1-\varphi_0}{\Lambda(y_{k-1}, y_k)}} \cdot \rho \left[1 + r(1 - F(\hat{x}(y_{k-1}, y_k)|g))\right] \tag{3.9}$$

We also need to check the incentive compatibility condition for the sender. Since there is no commitment, the sender's strategy should be interim optimal. This means that at any realized signal y, the sender should not benefit by reporting y' from another partition, or equivalently for all k and all $y \in [y_{k-1}, y_k]$,

$$\hat{x}_k \in \underset{x \in \{\hat{x}_1, \cdots, \hat{x}_K\}}{\arg\max}\ \Pi(x; y) \tag{3.10}$$

where $\Pi(x; y)$ is given by (3.5). The next lemma simplifies verification of the sender's ICs by showing that it is sufficient to consider only *local* constraints.

Lemma 1. *For any $x \in X$ and $y \in Y$, we have* $\dfrac{\partial^2 \Pi(x; y)}{\partial x \partial y} < 0$.

Proof. In the Appendix. □

According to Lemma 1, the sender's global incentive conditions are guaranteed by local indifference conditions for each signal at the partition boundaries. In particular, for all $k = 1, \cdots, K-1$, we should have

$$\Pi(\hat{x}_k; y_k) = \Pi(\hat{x}_{k+1}; y_k) \tag{3.11}$$

[5] Equivalently, she will report a noisy signal $m \sim \mathcal{U}[y_{k-1}, y_k]$.

This implies that the sender who receives the signal at the boundary of the partition is indifferent between recommending the closest actions from either left or right. And for all interior realizations $y \in (y_{k-1}, y_k)$, the sender strictly prefers to recommend the corresponding threshold $\hat{x}_k$ rather than any other threshold that can be induced in equilibrium.

Proposition 4. *For each $1 \leq K \leq \bar{K}$, there is a unique K-partition equilibrium. Also, if $h(g|y)/h(b|y) \to 0$ as $y \to 0$, then we have $\bar{K} \to +\infty$.*

Proof. In the Appendix. □

This result shows that communication remains imperfect, even though the public agent is benevolent: only coarse information revelation can be sustained in all equilibria, due to the endogenous conflict of interests and the fact that the sender is unable to commit to the disclosure rule prior to learning y.

On the other hand, equilibrium information provision becomes more efficient as the precision of the public or the private signal increases: as shown in Figure 1b, endogenous conflict of interests vanishes as signals become more precise.[6] The reason is that with the increased precision, individual agents' actions become more responsive to their posterior. As a result, this reduces the incentive for the public agent to induce additional participation. In the limit, the first-best threshold coincides with the truth-telling threshold, and full information revelation becomes possible.

4 Welfare Implications

So far, from equilibrium analysis, we know that due to the endogenous conflict of interests, information provision from the public agent must be coarse in all equilibria. We still yet to answer the question that which equilibrium corresponds to the highest social welfare. In other words, if the benevolent public agent may pick the desirable equilibrium, which one should she choose?

It is useful to first introduce an upper bound for social welfare. $\bar{V}$ is defined as the highest possible aggregate welfare for individuals. Given our assumption on payoffs, it is achieved when all choose the risky action when state is good and all choose safe action when state is bad. Hence we have,

$$\bar{V} = \varphi_0 \rho (1 + r) + 1 - \varphi_0 \tag{4.1}$$

[6] In this example, we increase the precision for both the public and the private signals, while keeping the relative precision constant.

Following (2.14), the ex-ante social welfare, or sender's value, from a K-partition equilibrium is given by

$$V(K) = \sum_{1 \le k \le K} \{\varphi_0[H(y_k|g) - H(y_{k-1}|g)][F(\hat{x}_k|g) + (1 - F(\hat{x}_k|g))R(1 - F(\hat{x}_k|g))] + (1 - \varphi_0)[H(y_k|b) - H(y_{k-1}|b)]F(\hat{x}_k|b)\} \tag{4.2}$$

Proposition 5. *$V(K)$ is increasing in K. As $K \to +\infty$, $V(K) \to \widehat{V} < \bar{V}$.*

Proof. In the Appendix. □

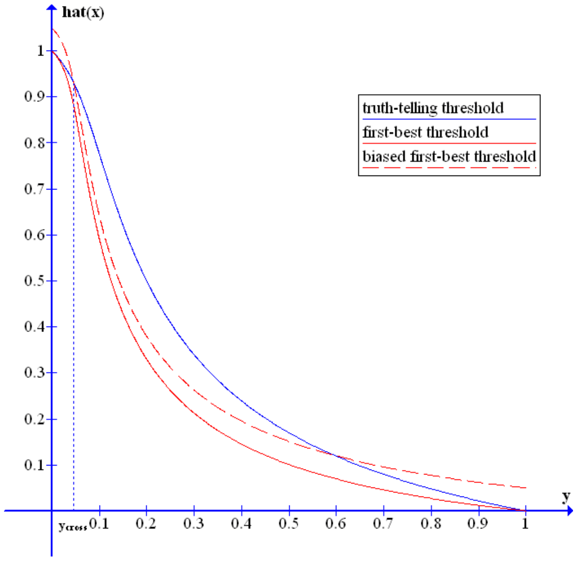
Our first result about welfare suggests that when the public agent is benevolent, equilibrium welfare increases in the number of partitions, but is still bounded strictly below $\bar{V}$ even in the limit. The intuition behind is straightforward. More precise public information is valuable for individuals to make better decision. Due to the fact that the public agent can not commit, only cheap-talk equilibria can be supported, where the equilibrium public information provision is coarse. Hence, social welfare is higher in equilibrium with higher number of partitions. However, even in the most informative equilibrium, public information provision is still inefficient. So the highest equilibrium welfare is strictly below the upper bound.

4.1 Biased Public Agent

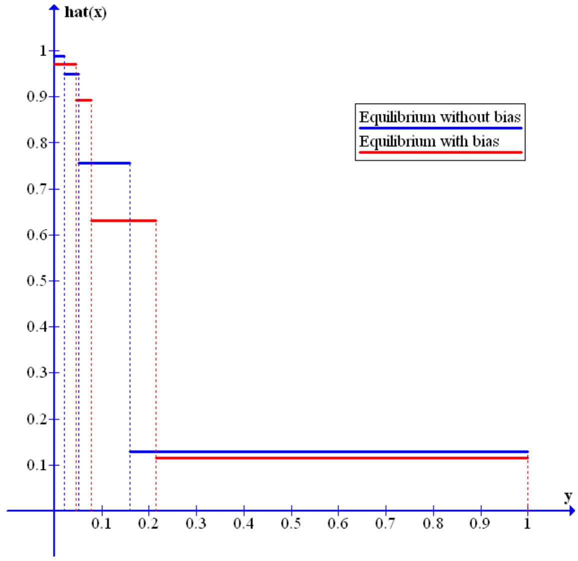
Traditional cheap-talk literature shows that when the sender can not commit and has exogenous conflict of interests with the receiver, the equilibrium information provision will be inefficient and receiver's welfare will be substantially lower than the full information case. Usually, an increase in the bias term will exaggerate the problem and further reduce receiver's welfare. In our work, instead of exogenously given, the conflict of interests arises endogenously. We are going to show that by introducing a small exogenous conflict of interests can indeed reduce the inefficiency and increase the social welfare.

Now, we consider the case that the public agent can be subject to exogenous bias regarding aggregate participation level. For example, she may personally prefer either the individuals behave more conservative or more aggressive, according to the first-best participating threshold. Compared to our original model where the public agent is benevolent, we get an *exogenous* conflict of interests on top of the *endogenous* one.

We assume that the type of the public agent is known by the individuals. For simplicity, we also assume a biased public agent will maximize her conditional payoff $\Pi(x - b; y)$. We call b the *subjective bias* for the public agent, which represents how much more participation she wants to have exogenously, compared to a pure benevolent public agent.



(a) Endogenous Conflict of Interests with Sender's Bias



(b) Equilibrium Partition for Unbiased and Biased Sender

Figure 2: The Impact of Sender's Bias on Conflict of Interests and Cheap-talk Equilibrium

Indeed, we have $\hat{x}^{FB}(y;b) = \hat{x}^{FB}(y;0) + b$, meaning a parallel upward (or downward, if $b < 0$) shift of the first-best threshold by a constant equal to b.

One of the key insights from the cheap-talk literature is that the conflict of interests between the informed and the uninformed player impedes communication. However, in our model, adding exogenous bias, which interferes with the one that arises endogenously due to coordination frictions, may actually *improve* information provision in equilibrium.

To see this, suppose that the sender wants the receivers to be more conservative in their actions, i.e. her bias term is positive: $b > 0$. Figure 2a shows that, by introducing a small positive bias to the sender's objective, endogenous conflict of interests becomes smaller in absolute scale generally, compared to the original model where the sender is purely benevolent. The comparison of cheap-talk equilibria is illustrated on Figure 2b.

Compared to equilibrium partition without sender's subjective bias, the first partition is wider since the biased first-best threshold and truth-telling threshold no long converge as the sender gets extreme bad signals. Indeed, since the conflict of interest changes sign as the biased first-best threshold and truth-telling threshold cross, the first partition boundary is given by the signal y_{cross} of the intersection point. In addition, the partitions on the right become narrower due to the fact that endogenous conflict of interests is smaller. Interestingly, even though the public agent's objective is biased towards conservative individual actions, we can see that in the equilibrium participation thresholds are uniformly decreased for each partition, which means that aggregate participation is indeed increases.

Proposition 6. *Compared to when the sender is benevolent ($b = 0$), social welfare is higher when the sender has a small and downward bias ($b > 0$), if actions are strategic complement ($r > 0$); social welfare is higher when the sender has a small and downward bias ($b < 0$), if actions are strategic substitute ($r < 0$).*

Proof. In the Appendix. □

5 Extensions

In this section, we discuss informally several extensions to the general model outlined in Section 2. First, we turn to analyzing the situation where the sender can commit to the reporting strategy prior to learning the signal. Then, we introduce the possibility for the sender to use monetary transfers in order to promote or discourage investment. Finally, we consider the case where the sender can impose binding constraints for individuals' minimum or maximum participation level.

5.1 Persuasion game with commitment

In our setting, the sender communicates the information after the signal $y \in Y$ has already been observed. One potentially interesting extension would be to consider the game, where the sender *commits* to the reporting strategy prior to getting private information. Quite unsurprisingly, such commitment can substantially improve upon the outcome.

As was shown by Kamenica and Gentzkow (2011), perfect communication can be achieved in a wide class of environments. However, their analysis is confined to the setting with discrete prior. Su and Yamashita (2012) extend their model to the case with continuous prior and show that truth-telling remains optimal as long as the sender's bias is non-decreasing in his type.

Unfortunately, the results that we obtain are more complicated, since the sender's incentives to inflate the public signal are non-monotone in y, being the highest for intermediate y's. Hence, it remains an open question whether perfect communication would be an equilibrium in the persuasion-game version of our model.

5.2 Monetary transfers

Another way for the sender to improve upon the allocation would be to use monetary instruments, which may be made contingent on the action taken by receivers. That is, investment can be subsidized and non-investment can be taxed.

In particular, consider the setting, in which the sender along with the message $m \in \mathcal{M}$ chooses the mapping

$$t : [0, 1] \to \mathbb{R} \tag{5.1}$$

where $t(a_i)$ is the transfer given to receiver i, in case he undertakes the action a_i. The transfers have to satisfy the following two conditions. First, for any $i \in [0, 1]$,

$$(1 - a_i) + a_i \cdot \mathbb{1}_{\theta > \hat{\theta}(m)} R(\theta) + t(a_i) \geq 0 \tag{5.2}$$

which is investor i's limited liability condition. Second,

$$\int_0^1 t(a_i) di \leq 0 \tag{5.3}$$

which is the sender's budget balance condition.

5.3 Cap and Floor for Individual Participation

5.4 Continuum State Space

6 Concluding Remarks

We developed a model that describes the interaction between the benevolent public agent and a group of private agents, who face coordination frictions, in a form of a cheap-talk game. It was shown that payoff externalities in individuals' actions endogenously generate conflict of interest between the sender and the receivers.

Viewed as an application to the CRA industry, our model provides rationale for rating inflation, suggesting that it might be in the public interest to actually overrate securities in order to prevent panics and inefficient liquidation. Our theory also predicts that the investors are more likely to keep track of ratings during busts than during booms, since in the former case ratings are more informative.

This stands in contrast to Veldkamp (2005), where the news are more abundant during economic upturns, because more investment projects get undertaken, generating more information.

Still, our model remains highly stylized, as it abstracts from a number of important features of the CRA market, such as reputation concerns (which is the focus of Bolton et al. (2012)), or competition, or the degree of asset complexity (as in Skreta and Veldkamp (2009)). Despite this fact, we believe that our story sheds light on the role played by the information generated from the credit ratings over the business cycle.

References

Angeletos, G., C. Hellwig, and A. Pavan (2006): "Signaling in a Global Game: Coordination and Policy Traps," *Journal of Political Economy*, 114, 452–484.

Bolton, P., X. Freixas, and J. Shapiro (2012): "The Credit Ratings Game," *Journal of Finance*, 67, 85–112.

Crawford, V. P. and J. Sobel (1982): "Strategic Information Transmission," *Econometrica*, 50, 1431–1451.

Farrell, J. and R. Gibbons (1989): "Cheap Talk with Two Audiences," *American Economic Review*, 79, 1214–1223.

Kamenica, E. and M. Gentzkow (2011): "Bayesian Persuasion," *American Economic Review*, 101, 2590–2615.

Manso, G. (2011): "Feedback Effect of Credit Ratings," Mimeo.

Milgrom, P. R. (1981): "Good News and Bad News: Representation Theorems and Applications," *Bell Journal of Economics*, 12, 380–391.

Morris, S. and H. S. Shin (1998): "Unique Equilibrium in a Model of Self-Fulfilling Currency Attacks," *American Economic Review*, 88, 587–597.

——— (2002): "Social Value of Public Information," *American Economic Review*, 92, 1521–1534.

Rogoff, K. (1985): "The Optimal Degree of Commitment to an Intermediate Monetary Target," *Quarterly Journal of Economics*, 100, 1169–1189.

Skreta, V. and L. Veldkamp (2009): "Ratings Shopping and Asset Complexity: A Theory of Ratings Inflation," *Journal of Monetary Economics*, 56, 678–695.

Su, T. and T. Yamashita (2012): "Optimal Persuasion and Conflict of Interests," Mimeo.

Veldkamp, L. (2005): "Slow Boom, Sudden Crash," *Journal of Economic Theory*, 124, 230–257.

Appendices

A Proofs

A.1 Proof of Proposition 1

Let us rewrite the $\pi(x,\hat{x})$ function as

$$\pi(x,\hat{x}) = \frac{\varphi\lambda(x)}{\varphi\lambda(x)+(1-\varphi)} \cdot \rho\left[1 + r(1 - F(\hat{x}|g))\right] \tag{A.1}$$

where we have divided the numerator and the denominator by $f(x|b)$ and used the definition of $\lambda(x)$.

Differentiating $\pi(x,\hat{x})$ with respect to its first argument, we get

$$\begin{aligned}\frac{\partial\pi(x,\hat{x})}{\partial x} &= \frac{\varphi\lambda'(x)\left[\varphi\lambda(x)+(1-\varphi)\right] - \varphi^2\lambda'(x)\lambda(x)}{\left[\varphi\lambda(x)+(1-\varphi)\right]^2} \cdot \rho\left[1 + r(1 - F(\hat{x}|g))\right] \\ &= \frac{\varphi(1-\varphi)\lambda'(x)}{\left[\varphi\lambda(x)+(1-\varphi)\right]^2} \cdot \rho\left[1 + r(1 - F(\hat{x}|g))\right]\end{aligned} \tag{A.2}$$

which is strictly positive, since due to Assumption 2, we have $\lambda'(x) > 0$. Given that $\pi(\cdot,\hat{x})$ is strictly increasing in x, for any $\hat{x}$, there exists at most one $x^\star$, which solves

$$\pi(x^\star,\hat{x}) = 1 \tag{A.3}$$

That is, there exists at most one signal realization $x = x^\star$, which makes the receiver indifferent between undertaking the risky and the safe action: for any $x > x^\star$, he would strictly prefer $a = 1$, while for any $x < x^\star$, he would strictly prefer $a = 0$.[7] Hence, adopting the threshold strategy is optimal when everyone else is playing it.

Now consider the function

$$\hat{\pi}(x) = \frac{\varphi\lambda(x)}{\varphi\lambda(x)+(1-\varphi)} \cdot \rho\left[1 + r[1 - F(x|g)]\right] \tag{A.4}$$

which denotes the expected payoff from playing the threshold strategy around x, *given that all the receivers also play around* x. We will show that there is a unique *interior* solution to $\hat{\pi}(x) = 1$. Our proof proceeds in 2 steps:

[7] In case $\pi(0,\hat{x}) > 1$ we set $x^\star = 0$, while for $\pi(1,\hat{x}) < 1$ we set $x^\star = 1$.

1. Prove that $\hat{\pi}(0) < 1$ and $\hat{\pi}(1) > 1$.

2. Prove that there exists at most one $\tilde{x}$ solving $\hat{\pi}'(\tilde{x}) = 0$.

Together, this would imply that there exists a unique $\hat{x}$, such that $\hat{\pi}(\hat{x}) = 1$.

For step 1, notice that

$$\hat{\pi}(0) - 1 = \frac{\varphi\lambda(0)}{\varphi\lambda(0) + (1-\varphi)} \cdot \rho(1+r) - 1 = -1 < 0 \tag{A.5}$$

where the inequality follows from part (iv) of Assumption 2: the fact that $\lim_{x\to 0}\lambda(x) = 0$. So, $\hat{\pi}(0) < 1$. Likewise, we have

$$\hat{\pi}(1) - 1 = \frac{\varphi\lambda(1)}{\varphi\lambda(1) + (1-\varphi)} \cdot \rho - 1 = \frac{\varphi}{\varphi + (1-\varphi)/\lambda(1)} \cdot \rho - 1 = \rho - 1 > 0 \tag{A.6}$$

because $\lim_{x\to 1}\frac{1}{\lambda(x)} = 0$ and $\rho > 1$ due to Assumption 1.

For step 2, let us calculate the derivative of $\hat{\pi}(x)$:

$$\frac{\partial\hat{\pi}(x)}{\partial x} = \frac{\varphi(1-\varphi)\lambda'(x)}{[\varphi\lambda(x) + (1-\varphi)]^2} \cdot \rho\,[1 + r[1 - F(x|g)]] - \frac{\varphi\lambda(x)f(x|g)}{\varphi\lambda(x) + (1-\varphi)} \cdot \rho r \tag{A.7}$$

First, observe that if $r \leq 0$, that is when agents' actions are strategic substitutes, $\hat{\pi}(\cdot)$ is always strictly increasing: $\hat{\pi}'(x) > 0$. So, let us focus on the case when $r > 0$.

We have $\hat{\pi}'(x) = 0$ whenever

$$\frac{(1-\varphi)\lambda'(x)}{[\varphi\lambda(x) + (1-\varphi)]^2} \cdot (1 + r[1 - F(x|g)]) = \frac{\lambda(x)f(x|g)}{\varphi\lambda(x) + (1-\varphi)} \cdot r \tag{A.8}$$

or

$$\frac{\lambda'(x)}{\lambda(x)} \cdot \frac{1 + r[1 - F(x|g)]}{rf(x|g)} = \frac{\varphi}{1-\varphi} \cdot \lambda(x) + 1 \tag{A.9}$$

The right-hand side of (A.9) is strictly increasing in x due to MLRP condition. On the other hand, observe that

$$\frac{d\log(\rho\,[1 + r[1 - F(x|g)]])}{d\log\lambda(x)} = \frac{d\log(\rho\,[1 + r[1 - F(x|g)]])/dx}{d\log\lambda(x)/dx}$$

$$= -\frac{rf(x|g)}{1 + r[1 - F(x|g)]} \cdot \frac{\lambda(x)}{\lambda'(x)} = -\frac{1}{\frac{\lambda'(x)}{\lambda(x)} \cdot \frac{1+r[1-F(x|g)]}{rf(x|g)}} \tag{A.10}$$

which is decreasing in x as long as the denominator, $\frac{\lambda'(x)}{\lambda(x)} \cdot \frac{1+r[1-F(x|g)]}{rf(x|g)}$, is decreasing. But the fact that the left-hand side of (A.9) is decreasing in x implies that there exists at most one

$\tilde{x}$, which solves $\hat{\pi}'(\tilde{x}) = 0$.

Taken together, this tells us that the function $\hat{\pi}(\cdot)$ is either everywhere increasing (for non-positive r), or else has a unique local maximum. The intuition carried by the elasticity term (condition (iii) in Assumption 2) is clear: for small x, the sensitivity of $\lambda(x)$ (and hence, of $\varphi\lambda(x)/[\varphi\lambda(x) + (1-\varphi)]$) to changes in x is higher than the sensitivity of the $\rho[1+r(1-F(x|g))]$ term – and so overall, as x increases, the product of the two, which equals $\hat{\pi}(x)$, rises as well. For large x, however, $\hat{\pi}(x)$ is dominated by the change in the second component, $\rho[1+r(1-F(x|g))]$.

Furthermore, since $\hat{\pi}(\cdot)$ starts below 1 and ends above 1, it must intersect 1 exactly once, and from below. That is, there exists a unique $\hat{x}$, such that $\hat{\pi}(\hat{x}) = 1$ and

$$\left.\frac{\partial \hat{\pi}(x)}{\partial x}\right|_{x=\hat{x}} > 0 \tag{A.11}$$

For the second part of Proposition 1, notice that $\hat{x}(\varphi)$ is implicitly defined by $\hat{\pi}(\hat{x}(\varphi)) = 1$. Observe that

$$\begin{aligned}\frac{\partial \hat{\pi}(x)}{\partial \varphi} &= \frac{\lambda(x)\cdot[\varphi\lambda(x) + (1-\varphi)] - \varphi\lambda(x)\cdot[\lambda(x)-1]}{[\varphi\lambda(x)+(1-\varphi)]^2}\cdot \rho\,[1+r[1-F(x|g)]] \\ &= \frac{\lambda(x)\cdot\rho\,[1+r[1-F(x|g)]]}{[\varphi\lambda(x)+(1-\varphi)]^2} > 0\end{aligned} \tag{A.12}$$

Making use of the Implicit Function Theorem, we have

$$\frac{d\hat{x}(\varphi)}{d\varphi} = -\left.\frac{\partial\hat{\pi}/\partial\varphi}{\partial\hat{\pi}/\partial x}\right|_{x=\hat{x}} < 0 \tag{A.13}$$

because of (A.11), so that $\hat{x}(\varphi)$ is strictly decreasing in φ, as was claimed.

FInally, observe that as $\varphi \to 0$, we have $\hat{\pi}(x) \to 0$ for any $x \in (0,1)$: whatever the threshold played by others, payoff to undertaking $a_i = 1$ becomes uniformly equal to zero. Hence, the only threshold consistent with equilibrium is given by $\hat{x} = 1$. On the other hand, in the limit when $\varphi \to 1$, we have $\hat{\pi}(x) = \rho\,[1+r[1-F(x|g)]]$, which due to Assumption 1 exceeds the safe payoff of one regardless of x. Hence, it pays to play $\hat{x} = 0$ as $\varphi \to 1$.

This completes the proof.

A.2 Proof of Proposition 2

Consider the communication strategy $\mu : Y \to \Delta\mathcal{M}$ defined by

$$\mu^\star(m, y) = \begin{cases} 1, & \text{if } m = y \\ 0, & \text{otherwise} \end{cases} \tag{A.14}$$

for all $y \in Y$. That is, $\mu^\star(m, y)$ corresponds to truth-telling. Observe that the induced prior φ corresponding to $\mu^\star$ would satisfy

$$\varphi(g|y) = \frac{\varphi_0 h(y|g)}{\varphi_0 h(y|g) + (1 - \varphi_0)h(y|b)} \tag{A.15}$$

Given the induced prior φ, receivers' threshold $\hat{x}(\varphi)$ would satisfy

$$\frac{\varphi\lambda(\hat{x}(\varphi))}{\varphi\lambda(\hat{x}(\varphi)) + (1 - \varphi)} \cdot \rho = 1 \tag{A.16}$$

where we used the fact that $r = 0$.

Define the sender's payoff given she has observed y and the receivers are playing the threshold strategy around x by

$$\Pi(x; y) = \varphi(g|y) \cdot \{F(x|g) + (1 - F(x|g)) \cdot \rho\} + \varphi(b|y) \cdot F(x|b) \tag{A.17}$$

Suppose the sender were able to choose x himself. Then, differentiating the above expression with respect to x, we get

$$\frac{\partial\Pi(x; y)}{\partial x} = \varphi(g|y) \cdot (1 - \rho)f(x|g) + \varphi(b|y) \cdot f(x|b) \tag{A.18}$$

Equating $\partial\Pi(x; y)/\partial x$ to zero and denoting $\varphi \triangleq \varphi(g|y) \equiv 1 - \varphi(b|y)$, after rearranging some terms we get

$$\varphi f(x|g) \cdot \rho = \varphi f(x|g) + (1 - \varphi)f(x|b) \tag{A.19}$$

Dividing both sides of the equation by the expression on the right-hand side and using the definition of $\lambda(x)$, we arrive exactly at (A.16). So, the equilibrium threshold $\hat{x}(\varphi)$ induced by the prior $\{\varphi(g|y), \varphi(b|y)\}$ is the same as what the sender himself would have chosen.

Next, consider the case when $r \neq 0$. In that situation, the marginal receiver's indifference condition is written as

$$\frac{\varphi\lambda(\hat{x}(\varphi))}{\varphi\lambda(\hat{x}(\varphi))+(1-\varphi)}\cdot\rho\,[1+r(1-F(\hat{x}(\varphi)|g))]=1 \tag{A.20}$$

while the sender's expected payoff would be given by

$$\Pi(x;y)=\varphi(g|y)\cdot\{F(x|g)+(1-F(x|g))\cdot\rho\,[1+r[1-F(x|g)]]\}+\varphi(b|y)\cdot F(x|b) \tag{A.21}$$

Maximizing (A.21) with respect to x and substituting $\varphi\equiv\varphi(g|y)\equiv 1-\varphi(b|y)$, we get

$$\varphi\,[1-\rho-2\rho r[1-F(x|g)]]\cdot f(x|g)+(1-\varphi)f(x|b)=0 \tag{A.22}$$

which after rearrangement and substitution of $\lambda(x)$ yields

$$\frac{\varphi\lambda(x^\star)}{\varphi\lambda(x^\star)+(1-\varphi)}\cdot\rho\,[1+2r(1-F(x^\star|g))]=1 \tag{A.23}$$

where $x^\star$ denotes the maximizer. As can be seen, this is clearly different from (A.20). So, unless $r=0$, the sender would have an incentive to choose different threshold: $x^\star\neq\hat{x}(\varphi)$.

This completes the proof.

A.3 Proof of Proposition 3

The first-best threshold $\hat{x}^{FB}(y)$ can be derived by solving, for each $y\in Y$,

$$\max_{x\in X}\{\varphi(y)\,[F(x|g)+(1-F(x|g))r[1-F(x|g)]]+(1-\varphi(y))F(x|b)\} \tag{A.24}$$

As we have established in the proof to Proposition 2 above, this threshold has to satisfy

$$\frac{\varphi(y)\lambda(x^{FB}(y))}{\varphi(y)\lambda(x^{FB}(y))+(1-\varphi(y))}\cdot\rho\left[1+2r(1-F(x^{FB}(y)|g))\right]=1 \tag{A.25}$$

On the other hand, $\hat{x}^T(y)$ has to satisfy

$$\frac{\varphi(y)\lambda(x^{T}(y))}{\varphi(y)\lambda(x^{T}(y))+(1-\varphi(y))}\cdot\rho\left[1+r(1-F(x^{T}(y)|g))\right]=1 \tag{A.26}$$

Let us define

$$\hat{\pi}(x,r)\triangleq\frac{\varphi\lambda(x)}{\varphi\lambda(x)+(1-\varphi)}\cdot\rho\,[1+r[1-F(x|g)]] \tag{A.27}$$

Then $x^T(y)$ is given by $\hat{\pi}(x,r)=1$ and $x^{FB}(y)$ is given by $\hat{\pi}(x,2r)=1$. So the first-best threshold can be seen as the truth-telling threshold, but with coordination friction parameter r being doubled. Therefore, the relationship between the thresholds $\hat{x}^T(\cdot)$ and $\hat{x}^{FB}(\cdot)$ can be captured by the comparative statics study with respect to r.

By Proposition 1, we know $\hat{x}^{FB}(0) = 1$ and $\hat{x}^{FB}(1) = 0$. Hence, when either $\varphi(y) = 0$ or $\varphi(y) = 1$, $\Delta(y) = 0$. Now, we have

$$\frac{\partial \hat{x}}{\partial r} = -\frac{\varphi\lambda(\hat{x})}{\varphi\lambda(\hat{x}) + (1-\varphi)} \cdot \frac{\rho(1 - F(\hat{x}|g))}{\hat{\pi}_x(\hat{x})} < 0 \tag{A.28}$$

for all $x \in (0,1)$, since $\hat{\pi}_x > 0$ when evaluated at $x = \hat{x}$. So, when $r > (<)0$, we have $\forall y \in Y$, $\hat{x}^{FB}(y) \leq (\geq)\hat{x}^T(y)$, and thus $\Delta(y) \leq (\geq)0$.

This completes the proof.

A.4 Proof of Lemma 1

This Lemma shows the local indifference conditions for the sender with boundary signals are sufficient for no global deviation in the full game equilibrium. By no deviation, it means that

$$\begin{aligned}
&\text{Given a partition and } \forall k \neq k', \forall y \in (y_{k-1}, y_k), \text{ we have } \Pi(x_k; y) \geq \Pi(x_{k'}; y) \\
\Longleftarrow &\forall k, \Pi(x_k; y) > \Pi(x_{k+1}; y), \forall y < y_k \text{ and } \Pi(x_k; y) < \Pi(x_{k+1}; y), \forall y > y_k \\
\Longleftarrow &\forall y \text{ and } \forall x_1 > x_2, \Pi(x_1; y) - \Pi(x_2; y) = \int_{x_2}^{x_1} \Pi_x(\tilde{x}; y) d\tilde{x} \text{ is decreasing in } y \\
\Longleftarrow &\forall y, \forall x, \Pi_{xy} < 0
\end{aligned}$$

In fact, $\Pi_{xy} = \varphi'(y)\left(f(x|g)[1 - R(A) - 2AR'(A)] - f(x|b)\right) < 0$.

This completes the proof.

A.5 Proof of Proposition 4

In order to prove this proposition, we shall make use of the following lemma.

Lemma 2. *For any* $0 \leq \underline{y} < \hat{y} < 1$, *if there exists* $\overline{y} \in (\hat{y}, 1]$, *such that* $\Pi(\underline{x}; \hat{y}) = \Pi(\overline{x}; \hat{y})$, *where* $\overline{x} = \hat{x}(\underline{y}, \hat{y})$ *and* $\underline{x} = \hat{x}(\hat{y}, \overline{y})$, *as given by* (3.9), *then* $d\overline{y}/d\hat{y} > 0$.

Proof. Define the function

$$\eta(\underline{y}, \hat{y}, \overline{y}) \triangleq \Pi(\hat{x}(\underline{y}, \hat{y}); \hat{y}) - \Pi(\hat{x}(\hat{y}, \overline{y}); \hat{y}) = 0 \tag{A.29}$$

By the Implicit Function Theorem, we have

$$\frac{d\overline{y}}{d\hat{y}} = -\frac{\partial\eta/\partial\hat{y}}{\partial\eta/\partial\overline{y}} = \frac{\overbrace{\frac{\partial\Pi(\overline{x};\hat{y})}{\partial\hat{y}} - \frac{\partial\Pi(\underline{x};\hat{y})}{\partial\hat{y}}}^{<0} + \overbrace{\frac{\partial\Pi(\overline{x};\hat{y})}{\partial x}}^{<0}\cdot\overbrace{\frac{\partial\hat{x}(\underline{y},\hat{y})}{\partial\hat{y}}}^{<0} - \overbrace{\frac{\partial\Pi(\underline{x};\hat{y})}{\partial x}}^{>0}\cdot\overbrace{\frac{\partial\hat{x}(\hat{y},\overline{y})}{\partial\hat{y}}}^{<0}}{\underbrace{\frac{\partial\Pi(\underline{x};\hat{y})}{\partial x}}_{>0}\cdot\underbrace{\frac{\partial\hat{x}(\hat{y},\overline{y})}{\partial\overline{y}}}_{<0}} > 0$$

Observe that the first two terms in the numerator can be rewritten as

$$\frac{\partial\Pi(\overline{x};\hat{y})}{\partial\hat{y}} - \frac{\partial\Pi(\underline{x};\hat{y})}{\partial\hat{y}} = \int_{\underline{x}}^{\overline{x}} \frac{\partial^2\Pi(x;\hat{y})}{\partial x\partial\hat{y}}dx < 0$$

since by Lemma 1, we have $\frac{\partial^2\Pi(x;y)}{\partial x\partial y} < 0$ for any $x \in X$ and $y \in Y$.

Next, for any $\hat{y}$, marginally increasing $\underline{x}$ or reducing $\overline{x}$ increases the sender's payoff:

$$\frac{\partial\Pi(\underline{x};\hat{y})}{\partial x} > 0 \quad \text{and} \quad \frac{\partial\Pi(\overline{x};\hat{y})}{\partial x} < 0$$

Finally, for any $y_1, y_2 \in (0,1)$ with $y_1 < y_2$ and $\hat{x}(y_1, y_2)$ as defined by (3.9), increasing either y_1 or y_2 reduces $\hat{x}(y_1, y_2)$. So, we have

$$\frac{\partial\hat{x}(\underline{y},\hat{y})}{\partial\hat{y}} < 0 \quad \text{and} \quad \frac{\partial\hat{x}(\hat{y},\overline{y})}{\partial\overline{y}} < 0$$

Taken together, this implies that $d\overline{y}/d\hat{y} > 0$, what was to be shown. □

Now, let $\underline{y} = y_0 = 0$ and $\hat{y} = y_1$. If there exists $\overline{y} = y_2 \in (y_1, 1]$, then certainly there exists a two-partition ($K = 2$) equilibrium, and moreover, this equilibrium is unique – since $\partial y_2/\partial y_1 > 0$ and $y_2(y_1) > 1$ as $y_1 \to 1$, there exists a unique y_1 such that $y_2(y_1) = 1$.

Next, for any $K \geq 3$, applying Lemma 2 iteratively on k, by taking $\underline{y} = y_{k-2}$ and $\hat{y} = y_{k-1}$, we show that if there exists $y_k \in (y_{k-1}, 1]$, then this y_k will be increasing in y_{k-1}, – and hence, in y_{k-2}, y_{k-3}, ..., and y_1 (by induction). So, for any K such that $y_K = 1$, there exists a unique sequence $\{y_1, \ldots, y_{K-1}\}$. This proves uniqueness.

For existence, we use backwards induction. We first prove that there exists $\bar{K} \geq 1$ such that we have a $\bar{K}$-partition equilibrium but for all $K > \bar{K}$, K-partition equilibrium does not exists. <TO BE COMPLETED>

Assume that there exists a K-partition equilibrium. Then, we claim that there also exist $(K-1)$-partition equilibrium. To see, this, write y_{K-1} as a function of y_1; by appropriately increasing y_1, we eventually reach the point where $y_{K-1}(y_1) = 1$, which delivers a $(K-1)$-

partition equilibrium. By induction, for each $1 \leq k \leq K$, there exists a unique k-partition equilibrium

A.6 Proof of Proposition 5

A.7 Proof of Proposition 6