Ideology as Opinion: Polarization in a Spatial Model of Common-value Elections

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Abstract

This paper analyzes a spatial voting model in which ideological differences stem not from competing private interests, but from a spectrum of private opinions regarding which policies will best achieve a common objective—in essence synthesizing the canonical approaches of Condorcet and Downs. In equilibrium, voters who lack confidence in their information remain ideologically moderate and abstain from voting, consistent with available empirical evidence. Office motivated candidates converge to the political center as in standard models, but instead of utilitarian compromise, this may reflect a spatial version of pandering to the uninformed. Alternatively, candidates who care about policy outcomes may be highly polarized, each anticipating that truth—and therefore voters—are on her side.

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1 Introduction

1.1 Overview

There are two fundamental paradigms by which elections and other democratic institutions can be understood. The first views democracy as a mechanism for resolving conflicts of interest. When policies such as wealth redistribution benefit one group at the expense of another, favoring the majority is desirable from a utilitarian perspective. Alternatively, democracy can be viewed as a mechanism for resolving differences of opinion regarding a common goal. Economic stability benefits essentially everyone, for example, but many believe that this is best achieved through active economic stimulus and tighter financial regulation, while others view such policies as counter-productive. In situations such as these, Condorcet’s (1785) classic jury theorem points out that a majority is more likely than a minority to correctly assess the truth.

Formal political theory has focused mainly on preference aggregation, with information playing a decidedly secondary role, if any. In the spatial literature begun by Hotelling (1929) and Downs (1957), for example, the importance of commonly-valued “valence” characteristics such as charisma or leadership are sometimes considered, but the policy tug-of-war between the liberal left and the conservative right is viewed almost universally as a contest of preferences (e.g. the poor against the rich). This literature has been extended and applied in a myriad of ways, while information models remain comparatively primitive, limited almost exclusively to binary environments, and to narrow applications such as committees and juries. One reason for this emphasis of preferences over information is that the latter seem inherently more fundamental: differences of opinion could in principle be resolved, but conflicts of interest would remain. A second reason is that conflicts of interest seem pervasive: even policies that benefit everyone will inevitably benefit some more than others, requiring a zero-sum choice between points on the Pareto frontier.

Notwithstanding these considerations, this paper explores the unconventional proposition that private opinions are actually more fundamental to political conflict than private incentives. In the model below, there are a spectrum of policy possibilities, as in Hotelling (1929) and Downs (1957), with candidates for political office choosing policy positions, and citizens voting for candidates. As in Condorcet (1785), however, one of these policies is ultimately optimal, in some objective sense. Ideology is not a preference parameter at all, but rather refers to a citizen’s opinion regarding the location of this optimum. Citizens

\footnote{Typically, ideological heterogeneity is either attributed to income, which determines the demand for public goods (Bergstrom and Goodman, 1973) and redistribution (Romer, 1975; Meltzer and Richard, 1981), or modeled without explanation as an exogenous preference parameter.}
are liberal if they believe that the optimal policy lies somewhere to the left of center and conservative if they think it is on the right. The analysis below then revisits several of the questions treated by canonical spatial models. In some cases, this creates incentives that differ from standard analyses, thus explaining empirical behavior that is puzzling from the standard perspective. In other cases, behavioral implications are the same, but have different normative implications, thus underscoring the importance of reconsidering model fundamentals.

1.2 Motivation for Information Paradigm

There are a number of reasons why, though it is non-standard, a common-value environment might actually be well suited for describing political conflict. For one thing, the most important ultimate political goals are universal: voters across the political spectrum share desires for world peace, economic growth and stability, and reducing crime, corruption, pollution, and poverty—even if they disagree widely over the means to these ends. Such disagreements are only natural, given the exceeding complexity of these issues: do financial instability and rising health care costs result from insufficient regulation, or excessive regulation? Does military aggression deter terrorist activity, or provoke it? Are the poor helped or harmed by minimum wage laws and workers’ unions? Preferences aside, different individuals—experts and non-experts alike—predict different outcomes for each of these policies. That they should also then favor different policies is unsurprising.

A second motivation for common values is that, in deciding how to vote, citizens seem already to take one another’s preferences into account. For example, policies such as minimum wages, food assistance, unemployment insurance, social security, and public education remain quite popular even among citizens who do not receive these services, presumably because they consider the benefits to those who do.\(^2\) Even wealth redistribution, which is inherently zero-sum, is typically discussed in the language of public goods: to liberals, including many who are wealthy, redistribution makes society better for all, by promoting fairness, providing a social “safety net” against uninsurable risks, and preserving democracy from the undue influence of wealthy elites; to conservatives, including many who are poor, redistribution makes society worse, by taking unfair advantage of a wealthy minority, squelching incentives for effort, investment, caution, and innovation, and subjecting democracy to abuse by political elites. According to data from the American National Election

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\(^2\)Summarizing earlier statistical studies, Caplan (2007, p. 149) writes that “The elderly are not more in favor of Social Security and Medicare than the rest of the population... The unemployed are at most a little more in favor of government-guaranteed jobs, and the uninsured at most a little more supportive of national health insurance... Males vulnerable to the draft support it at normal levels....”
Study (ANES), this is more than just rhetoric: in the 2012 U.S. presidential election, for example, 36% of citizens with below-median incomes reported voting Republican and 52% of wealthier citizens reported voting Democrat. Thus, as Figure 1 illustrates, vote choice is correlated with income, but only weakly: a regression on all 28 income categories produces an $R^2$ of only 0.03. More to the point, Fong (2001) reports that preferences for redistribution correlate with voters’ beliefs regarding the prevalence of poverty, and about the relative importance of luck and effort for economic success, more strongly than with demographic variables such as their own incomes. Even the correlation between income and voting could potentially have informational roots: if wealthy individuals are more keenly aware of the costs of redistribution and poor individuals are more keenly aware of its benefits, for example, then voting patterns should follow private interests even when everyone votes for what they perceive to be best for the group.

Sacrificing one’s own interests for the public good may seem like something that only heroically benevolent individuals would ever do, but this need not be the case: unlike other economic actions, policy decisions impact large numbers of individuals simultaneously, so

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3Piketty (1995) and Gelman et al. (2007) document similar trends in previous years and internationally, and Mueller (2003, ch. 14.4) cites several additional examples of groups who vote against their own interests.

4See also Alesina and Angeletos (2005), Piketty (1995), Buera, Monge-Naranjo, and Primiceri (2011), and Giuliano and Spilimbergo (2014).
a slight concern for others can be amplified substantially in political settings. Whatever the reason, voters seem to view political decisions as if through the eyes of social planners, looking beyond their own narrow self-interests to favor policies that they believe will be best for the group. Whatever is truly best for the group, then, constitutes a common objective. This is not to say that conflicts of interest do not exist or are unimportant, but rather that citizens may aggregate preferences internally, before ever casting their votes, so that the differences remaining on election day primarily reflect disagreements as to the true future impact of various policy proposals, or the true best interests of society.

A third consideration in favor of an information approach is that voters’ policy preferences continually change as they receive new information about the welfare consequences of various policies. Magleby (1984, ch. 9) documents the extreme volatility of policy views, and the field experiments of Gilens (2001), Luskin, Fishkin, and Jowell (2002), and Banerjee et al. (2010) reveal a causal impact of information. Broader ideological positions are more stable, but do drift, as Jennings and Markus (1984) document, especially for young voters—and sometimes shift quite abruptly, such as after market crashes, business or government scandals, or wars or acts of terrorism. According to Berelson, Lazarsfeld, and McPhee

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5 One way to formalize this logic is with a standard model of altruism, in which voter $i$’s utility function $U_i = u_i + \sum_{j \neq i} \alpha_i u_j$ places positive weight $\alpha_i$ on the well-being $u_j$ of each of his peers, and so can be rewritten as a weighted average $U_i = (1 - \alpha_i) u_i + \alpha_i \bar{u}$ of his own well-being $u_i$ and the average well-being $\bar{u} = \frac{1}{n} \sum_{j=1}^{n} u_j$ of the population. When the number $n$ of peers is large, the second term dominates, even if $\alpha_i$ is close to zero. In other words, even a citizen who is almost purely selfish should base political decisions almost entirely on his perception of the common good, rather than his private interest.

6 In principle, conflicts of interest could remain even between social planners, who prioritize common objectives differently, or aggregate preferences according to different social welfare functions. One might favor efficient policies, for example, while another gives higher priority to equity. If neither is merely pursuing his own private interests, however, it is not clear that such differences should be robust: proceeding to discuss reasons why one welfare function or the other more accurately reflects the true interests of society, for example, these citizen-planners might well reach a consensus.

7 In the words of Benjamin Franklin (1787) at the close of the U.S. Constitutional Convention, for example, “... there are several parts of this Constitution which I do not at present approve, but I am not sure I shall never approve them. For having lived long, I have experienced many instances of being obliged by better information, or fuller consideration, to change opinions even on important subjects, which I once thought right, but found to be otherwise.”

8 Page and Shapiro (1983) present survey evidence that major policy changes are preceded by shifts in public opinion. The field experiments of LaCour and Green (2014) appear to have influenced voters’ opinions even on the seemingly intractable issue of homosexual marriage.

9 In the wake of the 2008 financial crisis, for example, former U.S. Federal Reserve chairman Alan Greenspan famously testified before Congress of being suddenly convinced that the...
(1954, ch. 7), political conversions are most frequent among voters who interact and discuss politics with those of opposing views. Indeed, voters expend great resources trying to persuade one another through debates, endorsements, policy research, and so on. Such efforts reveal an optimism that political differences can be overcome; if political opinions derived from immutable preferences, persuasive efforts would be futile.  

A final motivation for an information model is simply the empirical inadequacy of the preference paradigm. Examples of this are discussed throughout this paper, but most notable is the ubiquitous prediction of spatial models that competition for office should drive candidates to the political center. This prediction is quite robust theoretically, as Section 1.4 discusses, but receives what Besley and Case (2003) dismiss as “little empirical support”. Using roll call voting, issue position statements, and surveys to estimate candidates’ ideological positions statistically, for example, Ansolabehere, Snyder, and Stewart (2001), McCarty and Poole (1995), Jessee (2009, 2010), Bafumi and Herron (2010), and Shor (2011) all find a substantial gap between Democratic and Republican candidates. Several of these papers estimate candidate and voter ideologies on the same scale, thus providing a picture of candidate polarization, relative to that of voters. By these estimates, the distribution of ideologies in Congress is clearly more extreme than the distribution of voter ideologies. State legislatures exhibit similar levels of polarization (Shor 2011), as do presidential candidates (McCarty and Poole 1995; Jessee 2009, 2010). The estimates of Bafumi and Herron (2010) are especially extreme: members of the House and Senate are as extreme as the most liberal and conservative voters in the electorate. Joint scalings such as these must be interpreted with some caution, as Lewis and Tausanovitch (2014) discuss, since politicians have reputation concerns that voters do not share, but voters’ own perceptions are similar: between 1972 and 2012, 90% of ANES survey respondents rated both major presidential candidates as weakly farther from the center of a standard 7-point ideological scale than they viewed themselves, while only 11% saw both candidates as weakly more moderate.  


10 In their review of empirical literature, DellaVigna and Gentzkow (2010) find patterns of political persuasion to be broadly consistent with Bayesian updating. Gerring (1997) traces the etymology of the word ideology, emphasizing its association with beliefs, ideas, and world views.  

11 This is also consistent with campaign rhetoric, where candidates emphasize their differences but rarely their similarities.
1.3 Preview of Results

The analysis below focuses first on voters and then on candidates. The central feature of voting behavior is simply that individuals who lack confidence in their knowledge of the optimal policy remain ideologically moderate, to avoid making errors on either side. Such behavior is quite intuitive, and is also consistent with the empirical correlation between information and ideology, which is difficult to reconcile with standard preference models, as explained below. The familiar left-right geometry of ideology emerges quite naturally because, even when the truth is ultimately binary, heterogeneous expertise produces a spectrum of voter opinions.

Following Condorcet (1785), the analysis below assumes that each voter’s private opinion is correlated with the truth. A consequence of this is that voters’ opinions are also correlated with each other. Especially on questions that have obvious answers, this can lead to widespread consensus and large margins of victory, which are rare in standard theoretical models but not uncommon empirically. Correlation also generates a “swing voter’s curse”, as in Feddersen and Pesendorfer (1996), leading citizens who lack expertise to abstain (even though voting is costless) in deference to those with superior information. Actually, ideological moderates face a swing voter’s curse as well, even if their expertise is high, and the resulting pattern of abstention is consistent with empirical evidence that existing literature has not explained.

Correlation between voter opinions is also important for candidate incentives, because it causes the candidate with a more extreme platform to win the election with nontrivial probability. This leads to the central result of the paper, which is that candidates may be highly polarized in equilibrium, consistent with the empirical evidence above. Put simply, neither candidate feels the need to moderate her policy stance because each expects the voters to be on her side.\(^\text{12}\) Polarization is especially pronounced if candidates have extreme beliefs about which policy is optimal. Even if candidates are ex-ante identical, however, extremeness can arise endogenously from an equilibrium calculus analogous to that of pivotal voting, as each candidate rationally presumes that her policy position will enjoy popular support. Because of the correlated voting discussed above, polarization is not highly sensitive to the exact specification of candidate incentives.

To candidates who care only about winning the election, it makes no difference whether ideology stems from information or preference: in either case, they converge to the center. While the behavior is the same, however, the normative implications differ starkly: in preference models, compromising between the competing interests at either extreme is attractive from a utilitarian perspective because it minimizes the total disutility inflicted on voters;

\(^{12}\)Throughout this paper, feminine pronouns refer to candidates and masculine pronouns refer to voters.
here, convergence is more akin to a spatial version of the pandering results of Canes-Wrone, Herron, and Shotts (2001) and Maskin and Tirole (2004), as candidates put popularity ahead of principles. At the same time, overconfident candidates are overly extreme in equilibrium, and this can have severe consequences for welfare, as well.

1.4 Related Literature

Condorcet’s (1785) original jury theorem has been extended to accommodate heterogeneous voter expertise (e.g. Nitzan and Paroush, 1982) and dependent private signals (e.g. Ladhja, 1992; Dietrich and Spiekermann, 2013), and to analyze alternative voting rules (e.g. List and Goodin, 2001; de Clippel and Eliaz, 2012). These models treat only binary issues, however, or decisions with small numbers of fixed alternatives. In particular, Condorcet’s model has not previously been extended to a spatial environment. In fact, even as they explore the information paradigm of democracy, most authors are careful to restrict application to juries and other committee settings, viewing the assumption of common values as implausible for general elections. Other models emphasize the interaction of information with private interests, where ideology is typically interpreted as the latter (e.g. Feddersen and Pesendorfer, 1997; 1999; Krishna and Morgan, 2011), or preferences are augmented by a common-value “shock” (e.g. Bernhardt, Duggan, and Squintani, 2009; Ortoleva and Snowberg, 2015).13

The empirical reality of polarization has long been recognized, but convergence has proven so robust, theoretically, that Roemer (2004) refers to the “tyranny of the median voter theorem”.14 If candidates want to win office, they should converge to the center to get votes (Hotelling, 1929; Downs, 1957).15 If they don’t care about winning but care about the policy outcome, they should still converge to the center (Calvert, 1985), because choosing policy requires winning first. Pundits often attribute polarization to the undue influence of extremists within either party (e.g. through primary elections or higher participation levels), but this is problematic because party extremists themselves should favor moderation, to avoid losing (Coleman, 1972). Wittman (1983) and Calvert (1985) show that policy motivated candidates do not converge when the location of the median voter is unknown, so subsequent literature has often attributed polarization to this combination of ingredients. Unless uncertainty is quite severe, however, this should produce only minimal polarization.

13Principal-agent models (reviewed by Besley, 2006) feature a representative voter, which can be interpreted most naturally within a common-value context, but this is typically not made explicit.
14Krasa and Polborn (2014, p. 4) write that “the position choice of candidates and the determinants of policy convergence or divergence are arguably the central topics in political economy models of elections.”
15In the model of Davis, Hinich, and Ordeshook (1970), this logic applies even if alienated or indifferent citizens abstain from voting.
as Section 4, below, makes clear: straying too far from the expected position of the median voter merely surrenders policy control to one’s opponent.

Alesina (1988) attributes political extremism to candidates’ inability to make credible campaign promises: once elected, a candidate can be as extreme as she wishes. If candidates’ incentives are known by voters, however, then moderate candidates should still enjoy an electoral advantage, by the standard reasoning. This begs the question of which preference types have incentive to run for office. The entry models of Osborne and Slivinski (1996) and Besley and Coate (1997) exhibit equilibria with divergent candidates, but this is not a deliberate choice that candidates make; they simply lack the credibility to promise moderate policies and be believed. Moreover, equilibria with little or no divergence are equally plausible.

2 The Model

2.1 Uncertainty and Private Information

An electorate consists of $N$ citizens where, following Myerson (1998), $N$ is drawn from a Poisson distribution with mean $n$. Together, these citizens must choose and implement a policy from the interval $\mathcal{X} = [-1, 1]$ of alternatives, which will provide a common benefit to every citizen. Let $z \in \mathcal{Z} \subseteq \mathcal{X}$ denote an unknown state of the world, which designates the policy that in truth is best for society. For the most part, the analysis below assumes that $\mathcal{Z} = \mathcal{X}$, meaning that any policy that is feasible might also be optimal. For simplicity, the prior density of $z$ is uniform.

$$f(z) = \begin{cases} \frac{1}{2} & \text{if } z \in \mathcal{Z} \\ 0 & \text{otherwise} \end{cases}$$

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16 In repeated elections, even candidates with extreme preferences can converge.

17 Polarized equilibria also require near-ties (Eguia, 2007), which often are not observed empirically.

18 The above are the most widely cited explanations for political polarization, but recent literature has also proposed a growing list of alternatives: the threat of entry by a third party (Palfrey, 1984; Callander and Wilson, 2007; Brusco and Roy, 2011); heterogeneity of candidate valence or personality (Groseclose, 2001; Aragones and Palfrey, 2002; Gul and Pesendorfer, 2009); informational asymmetries (Glaeser, Ponzetto, and Shapiro, 2005; Bernhardt, Duggan, and Squintani, 2007); efforts to signal hidden types (Kartik and McAfee, 2007; Callander and Wilkie, 2007; Callander, 2008; Kartik, Squintani, and Tinn, 2012); and party ties across elections (Krasa and Polborn, 2014). Each of these theories, of course, has strengths and weaknesses. By comparison, the explanation below is notable for its simplicity: if truth is on her side, a candidate needs not move to the center because voters will instead come to her.

19 Alternatively, as Section 3.6 explains, $z$ can be interpreted as the policy that makes most efficient use of all available private information.
If policy \( x \in \mathcal{X} \) is implemented in state \( \zeta \) then each citizen receives utility \( u(x, \zeta) = -(x - \zeta)^2 \), which declines quadratically with the distance between \( x \) and \( \zeta \). This specification is standard both in political economics and in statistical theory, and is convenient because, conditional on information \( \Omega \), expected utility

\[
E[u(x, \zeta) | \Omega] = -[x - E(z | \Omega)]^2 - V(z | \Omega)
\]  

is similarly quadratic in the policy choice.\(^{20}\) That is, the optimal policy choice is the updated expectation of \( \zeta \), and preferences are single-peaked, as in standard spatial voting models. The concavity of \( u(x, \zeta) \) also implies that voters are risk averse, and thus favor moderate policies, which are guaranteed not to be too far from whatever is truly optimal.\(^{21}\) Ex ante, the optimal policy lies exactly at the center, \( E(\zeta) = 0 \).

For some applications, it is more reasonable to assume that \( \mathcal{Z} = \{-1, 1\} \), meaning that truth is binary. To end an economic recession, for example, Keynesian economic theory recommends a large policy of economic stimulus, while more classical economic theory views such stimulus efforts as wasteful. Thus, the optimal stimulus is known either to be very large or very small. To accommodate such issues, (1) can simply be reinterpreted as a mass function, meaning that \( z = -1 \) and \( z = 1 \) are equally likely. The densities below can be reinterpreted similarly, and proofs of formal results apply almost verbatim in that case, merely summing over \( \mathcal{Z} \) instead of integrating. Accordingly, the discussion below moves freely between examples of continuous and binary uncertainty. To be clear, binary uncertainty makes no restriction on the set \( \mathcal{X} \) of feasible policies: a moderate-sized stimulus is still feasible even if it is not optimal \textit{per se} in any state of the world. This is important because, in the face of uncertainty, risk-averse citizens might actually prefer such a policy as a way of avoiding the risk of a macroeconomic policy that is catastrophically misspecified.

Each citizen’s private opinion regarding the location of the optimal policy is represented by a private signal \( s_i \in \mathcal{S} \) which is positively correlated with \( \zeta \), and has the same domain \( \mathcal{S} = \mathcal{Z} \). Conditional on \( \zeta \), private signals are independent. Because citizens differ in expertise, their signals vary in quality. Specifically, a citizen’s expertise \( q_i \in \mathcal{Q} = [0, 1] \) is first drawn independently (from other citizens, and from \( \zeta \)) from a common distribution \( G \) which is differentiable and has a strictly positive density \( g \). Conditional on \( q_i = q \) and on \( \zeta \), the density of \( s_i = s \) is as follows.

\[
h(s | q, \zeta) = \frac{1}{2} (1 + qs) .
\]  

\(^{20}\)The final term in (2) does not depend on political outcomes, and so is suppressed in subsequent notation.\(^{21}\) The single-peakedness of \( u(x, \zeta) \) would make it nearly concave even if the utility loss function were linear or convex, so results would be similar to those below.
This specification is convenient because, for the case of binary $z$ and $s_i$, it can be reinterpreted as a mass function. Either way, the distributions $f$, $g$, and $h$ are common knowledge, but $q_i$ and $s_i$ are observed only privately. With binary uncertainty, $q_i$ gives the correlation coefficient between $s_i$ and $z$, meaning that a citizen with $q_i = 1$ observes $z$ perfectly; with continuous uncertainty, the correlation is only $\frac{1}{\pi} q_i$, so even the highest quality signals include substantial noise. With either specification, $s_i$ is uniform on $S$ and its precision increases with expertise. The lowest quality signal reveals nothing: if $q_i = 0$ then $s_i$ and $z$ are independent.

2.2 Voter and Candidate Strategies

With a continuum of policy possibilities, it is infeasible for citizens to vote for policies directly. Instead, there are two candidates, $A$ and $B$, who propose policy platforms $x_A, x_B \in X$ that they commit to implement if elected. Observing these platforms, citizens then vote (at no cost) for either candidate. A strategy $v : Q \times S \rightarrow \Delta (\{A, B\})$ in the voting subgame specifies a probability $v^j (q, s)$ of voting for each candidate $j \in \{A, B\}$, for every realization $(q, s) \in Q \times S$ of private information. Let $\mathcal{V}$ denote the set of such strategies. Abusing notation slightly, the pure strategy of voting for candidate $j$ is denoted simply by $v (q, s) = j$.

Votes are cast simultaneously, and a winning candidate $w \in \{A, B\}$ is determined by majority rule, breaking ties by coin toss. This winner takes office and implements her platform policy $x_w$. Expected utility can therefore be rewritten from (2) as follows, for any pair $(x_A, x_B) \in \mathcal{X}^2$ of candidate platforms.

$$E [u (x, z) | \Omega] = \int_z \left[ \sum_{j=A,B} u (x_j, z) \Pr (w = j | z) \right] f (z | \Omega) dz. \quad (4)$$

Implicitly, $\Pr (w = j | z)$ in (4) depends on the strategies used by every voter. If his peers all vote according to the strategy $v \in \mathcal{V}$, a citizen’s best response is the strategy $v^{br} \in \mathcal{V}$ that maximizes (4) for every realization $(q, s) \in Q \times S$ of private information. A (symmetric) Bayesian Nash equilibrium (BNE) in the voting subgame is a strategy $v^*$ that is its own best response.\(^{23}\)

\(^{22}\)Costly voting and costly information acquisition are important directions for future work, but beyond the scope of this paper. It is worth noting that ethical motivations such as altruism, which may provide a basis for common interest, are also routinely offered as explanations for costly voting (e.g. Edlin, Gelman, and Kaplan, 2007; Faravelli, Man, and Walsh, 2013).

\(^{23}\)In games of Poisson population uncertainty, equilibrium symmetry is inevitable because the distribution of opponent behavior is the same for any two individuals within the game (unlike a game between a finite set of players), implying that a best response for one citizen is a best response for all.
Like citizens, candidates prefer policies that are as close as possible to the optimum. However, candidates also hope to win office. Let \( 1_{w=j} \) denote an indicator of that equals 1 if candidate \( j \) wins the election, and zero otherwise, and let \( \beta \geq 0 \) denote the benefit of winning office, including legacy benefits as well as salary and other perquisites. With this notation, if candidate \( w \) wins the election and implements policy \( x \) in state \( z \) then candidate \( j \) receives utility
\[
     u_j (w, x, z) = u(x, z) + 1_{w=j} \beta,
\]
the expectation of which is
\[
     E [u_j (w, x, z) | \Omega] = E [u(x, z) | \Omega] + \beta \Pr (w = j | \Omega). \tag{5}
\]
If \( \beta = 0 \) then candidates are purely truth motivated, meaning that they share a common interest with voters.\(^{24}\) If \( \beta \) is sufficiently large then she is purely office motivated instead, just as in Hotelling (1929) and Downs (1957). For intermediate values of \( \beta \), neither incentive completely dominates.

It would be reasonable to assume that, like other citizens, candidates observe private signals of \( z \), which are informative but imperfect. This is a useful direction for future extension, but for simplicity and to maintain a focus on the aggregation of voter information, the analysis below instead treats the polar cases of biased candidates, who each have specific policies that they believe (with probability one) to be optimal, and unbiased candidates, who receive no information of their own, and must therefore base their opinions on whatever they can infer in equilibrium from voters.\(^{25}\)

A complete voting strategy \( \sigma : \mathcal{X}^2 \rightarrow \mathcal{V} \) specifies subgame behavior for every possible pair \((x_A, x_B) \in \mathcal{X}^2\) of candidate platforms. Let \( \Sigma \) denote the set of such strategies. A Perfect Bayesian Equilibrium (PBE) is a triple \((x^*_A, x^*_B, \sigma^*)\) such that \( \sigma^* (x_A, x_B) \) constitutes a BNE in the voting subgame associated with every platform pair \((x_A, x_B) \in \mathcal{X}^2\) and the platform choice \( x^*_j \) of each candidate maximizes (4), taking the platform position \( x^*_{-j} \) of her opponent as given.

\(^{24}\)The more standard label “policy motivation” would not be inappropriate here, since the candidates care about the policy outcome, but Section 4.2 uses that term to describe candidates who prefer specific policies independent of the true state of the world.

\(^{25}\)Actually, candidate signals would have little impact in the present model, because the informational content of one or two additional signals would be overwhelmed by what is inferred from the \( N \) citizens. For a recent model which emphasizes candidate over voter information, see Kartik, Squintani, and Tinn (2012).
3 Voter Behavior

This section analyzes equilibrium voting behavior, taking a pair \((x_A, x_B) \in \mathcal{X}^2\) of candidate platforms as given. Section 3.1 begins by defining voter ideology, and discussing how this formulation matches patterns documented in empirical studies. Sections 3.2 and Section 3.3 then discuss how this translates into voting behavior and can produce large margins of victory, and Section 3.4 considers the incentives for voter abstention. Section 3.5 analyzes voter welfare and Section 3.6 comments on the possibility of aggregate uncertainty. Section 4 then proceeds to endogenize the choice of campaign platforms by analyzing candidate incentives.

3.1 Information and Ideology

After observing his private information \((q, s) \in Q \times S\), an individual uses (1) and (3) to update his beliefs \(f(z|q, s)\) of the optimal policy by Bayes’ rule, as follows.

\[
f(z|q, s) = \frac{1}{2} (1 + qsz) \equiv \frac{1}{2} (1 + \theta z) = f(z|\theta).
\]

This posterior depends on \(q\) and \(s\) only through the product \(\theta(q, s) = qs\). Like the above densities, it can also be reinterpreted for binary \(z\) and \(s\) as a probability mass function.

As Section 2.1 discusses, a citizen’s preferred policy is his expectation of \(z\). Using (6) and summing or integrating over \(Z\), this is given simply by \(E(z|q, s) = \theta\) for the case of binary uncertainty and by \(E(z|q, s) = \frac{1}{3} \theta\) for the case of continuous uncertainty. Either way, the preferred policy is simply proportional to \(\theta\), which can therefore be interpreted as a citizen’s ideology. The set \(\Theta\) of ideologies ranges from \(-1\) to \(1\), which can be interpreted as ranging from strongly liberal to strongly conservative. That is, liberal citizens believe that the optimal policy is left of center, while conservatives believe it is right of center. When \(\theta\) is close to zero (on either side), a citizen is ideologically moderate.

The sign of a voter’s ideology \(\theta\) is the same as the sign of his signal realization \(s\). Recognizing that signals are noisy, however, an individual discounts his own information, in proportion to his uncertainty. Even if his hunch is that the optimal policy is quite extreme, therefore, a citizen who knows that he lacks information will tend to remain ideologically moderate. Remark 1 formalizes this insight, by stating that the magnitude of \(\theta\) increases not only with the magnitude of \(s\), but also with \(q\). The functional forms in (1) and (3) make this result especially easy to see, but a similar comparative static result would hold quite generally: posterior beliefs are a weighted average of the prior and the signal, and when the signal is noisier a citizen invariably places greater weight on the prior.

Remark 1 Ideological intensity \(|\theta| = q|s|\) increases both in \(|s|\) and in \(q\).
The density and conditional density of ideology can be derived from \( g(q) \) and \( h(s|q, z) \), as follows,

\[
\varphi(\theta) = \int_{Q} \int_{S} 1_{q_{s}=\theta} g(q) h(s) ds dq
\]

and

\[
\varphi(\theta|z) = \int_{Q} \int_{S} 1_{q_{s}=\theta} g(q) \frac{1}{2} (1 + qs z) ds dq = (1 + \theta z) \varphi(\theta),
\]

where \( 1_{q_{s}=\theta} \) is an indicator function. Analogous expressions can be derived for the case of binary uncertainty, by summing instead of integrating over \( S \). It is worth noting that ideology ranges continuously from \(-1\) to \(1\), even if uncertainty is binary, because of the continuity of expertise. The familiar geometry of ideology thus arises quite naturally in an informational setting: even if a proposition must be true or false, voters will form a spectrum of opinions, ranging from one extreme to the other.

The prediction of Remark 1 can be investigated empirically. Using four decades of ANES data, Figure 2 shows the distribution of self-reported ideology among citizens with different levels of information.\(^{26}\) Voters with the highest level of information hold a range of political opinions, but consistent with Remark 1, the least informed group are notably more moderate.\(^{27}\) Regressing ideological polarization on information yields a coefficient of 0.273 (standard error 0.029), meaning that each level of information makes a citizen more than one quarter-level more extreme, on average.

An alternative explanation for the pattern highlighted in Figure 2 is that the demand for news and political information is higher among extreme citizens than among moderates. On the surface, this may seem to be the natural prediction of a preference-based model, as ideological extremists have more at stake in an election than moderates, and so have stronger incentives to pay attention. This logic would only be valid, however, if the informational difficulty for voters were that of determining which candidate is on the left and which is on the right. In general elections with established parties, this is likely to be obvious. Moderate citizens must still take time to determine candidates’ (and their own) precise ideological positions, as well as non-policy “valence” characteristics such as leadership skill, honesty, or intelligence, because their vote choices are sensitive to this information, but extremists have a clear voting incentive regardless of these factors, and may just as well ignore them.

\(^{26}\)This 9-point scale distinguishes survey respondents who originally placed themselves at the center of a 7-point scale, but later admitted leaning either liberal or conservative. The information variable reflects citizens’ “general level of information about politics and public affairs”, as assessed subjectively by ANES interviewers on a 5-point scale. Other measures of information are more objective but less comprehensive.

\(^{27}\)Palfrey and Poole (1987), Abramowitz and Saunders (2008), and Berelson, Lazarsfeld, and McPhee (1954) note similar patterns.
Thus, in a preference model, the most plausible sources of uncertainty should tend to lead to correlations opposite those observed in Figure 2.

In contrast with preference models, the model above gives a natural explanation for why intrinsic heterogeneity in the demand for news and political information should be correlated with ideological positions: namely, those citizens who enjoy consuming information develop greater expertise, and so are more confident in their opinions, and therefore less worried about hedging against errors. Mullainathan and Washington (2009) also present what could be interpreted as evidence of a causal influence of information on extremism: voters who were barely old enough to vote in the previous election hold more extreme views than those who were barely too young, perhaps because voting eligibility induced them to learn more about politics, and thus to develop stronger opinions. Consistent with the result that voting behavior is determined by the product of $q_i$ and $s_i$, Goren (1997) also reports that voters with extreme policy opinions but limited political knowledge vote similarly to moderates who are more knowledgeable.

3.2 Voting

If citizens vote according to the strategy $v \in \mathcal{V}$ then each votes for candidate $j \in \{A, B\}$ in state $z \in \mathcal{Z}$ with probability $\phi(j|z)$.

$$\phi(j|z) = \int_Q \int_S v^j(q, s) h(s|q, z) g(q) dsdq.$$  \hspace{1cm} (7)

By the decomposition property of Poisson random variables (Myerson, 1998), the numbers $N_A$ and $N_B$ of $A$ and $B$ votes are independent Poisson random variables with means $n\phi(A|z)$.
and \( n \phi (B|z) \), so the joint probability of vote totals \( N_A = a \) and \( N_B = b \) is given by

\[
\psi (a, b|z) = \frac{e^{-n \phi (A|z) - n \phi (B|z)} }{a!b!} [n \phi (A|z)]^a [n \phi (B|z)]^b .
\] (8)

In terms of (8), candidate A wins the election by a margin of exactly \( m \geq 0 \) votes (alternatively, B “wins” by \(-m \) votes) with probability

\[
\pi_A (m|z) = \pi_B (-m|z) = \sum_{k=0}^{\infty} \psi (k + m, k|z) \] (9)

and B wins by \( m \geq 0 \) votes (or A “wins” by \(-m \) votes) with probability

\[
\pi_B (m|z) = \pi_A (-m|z) = \sum_{k=0}^{\infty} \psi (k, k + m|z) .
\] (10)

Accordingly, candidate \( j \) takes office with probability

\[
\Pr (w = j|z) = \sum_{m=1}^{\infty} \pi_j (m|z) + \frac{1}{2} \pi_j (0|z)
\] (11)

where the second term reflects the event of winning the tie-breaking coin toss.

By the environmental equivalence property of Poisson games (Myerson, 1998), an individual from within the game reinterprets \( N_A \) and \( N_B \) as the numbers of \( A \) and \( B \) votes cast by his peers; by voting himself, he can add one to either total. If he votes for candidate \( j \), for example, that candidate will win with probability \( \Pr (w = j|z) = \sum_{m=0}^{\infty} \pi_j (m|z) + \frac{1}{2} \pi_j (-1|z) \) instead of (11). The difference between these two probabilities is simply the probability

\[
\Pr (piv_j|z) = \frac{1}{2} \pi_j (0|z) + \frac{1}{2} \pi_j (-1|z)
\] (12)

with which a vote for candidate \( j \) is pivotal (event \( piv_j \)), meaning that it reverses the election outcome. The two terms in (12) correspond to scenarios in which the candidates tie and \( j \) loses the tie-breaking coin toss and in which \( j \) wins the coin toss but loses the election by exactly one vote. In terms of (12), the difference \( \Delta_{AB}(q,s) \) in expected utility between voting \( B \) and voting \( A \) is given by

\[
\Delta_{AB}(q,s) = E_z \{ [u(x_B, z) - u(x_A, z)] \Pr (piv_B|z) |q, s \} \\
- E_z \{ [u(x_A, z) - u(x_B, z)] \Pr (piv_A|z) |q, s \} \\
= E_z \{ [u(x_B, z) - u(x_A, z)] \Pr (piv|z) |q, s \} \\
= E_z [2 (x_B - x_A) (z - \bar{x}) \Pr (piv|z) |q, s] \\
= 2 (x_B - x_A) [E (z|piv, q, s) - \bar{x}] \Pr (piv|q, s), \] (13)
where $\bar{x} = \frac{1}{2}(x_A + x_B)$ is the midpoint between the two candidates’ platforms and

$$\Pr(piv|z) = \Pr(piv|z) + \Pr(piv|z)$$ (14)

is the probability that either an $A$ vote or a $B$ vote is pivotal (event $piv$).28

Without loss of generality, suppose that $x_A \leq x_B$. In that case, (13) is positive if and only if $E(z|piv, q, s)$ is sufficiently high. As the proof of Lemma 1 shows, this expectation is increasing in a voter’s ideology. Accordingly, Lemma 1 characterizes best-response voting as an ideological strategy, defined in Definition 1. This behavior is simple: a citizen votes $A$ if his ideology is sufficiently to the left and votes $B$ if his ideology is sufficiently to the right. For an ideological strategy, vote shares can be notated as a function $v_j^i(\theta)$ of ideology, rather than of $q$ and $s$ separately.

**Definition 1** A strategy $v_\tau \in \mathcal{V}$ in the voting subgame is ideological, with ideology threshold $\tau \in [-1, 1]$, if $v_j^i(\theta) \equiv v_j^i(q, s) = 0$ unless

$$\begin{cases} j = A & \text{and} & \theta(q, s) \in [-1, \tau] \\ j = B & \text{and} & \theta(q, s) \in [\tau, 1] \end{cases}$$

**Lemma 1** If $x_A \neq x_B$ then, for any voting strategy $v \in \mathcal{V}$, the unique best response $v^{br} \in \mathcal{V}$ in the voting subgame is ideological.

When citizens follow an ideological strategy $v_\tau$, the expected vote shares $\phi(j|z)$ for each candidate reduce from (7) to the following.

$$\begin{align*}
\phi(A|z) &= \int_{-1}^{\tau} (1 + \theta z) \varphi(\theta) d\theta \\
\phi(B|z) &= \int_{\tau}^{1} (1 + \theta z) \varphi(\theta) d\theta.
\end{align*}$$ (15) (16)

The best response to an ideological strategy is another ideological strategy, so using a fixed point argument on the space of ideological thresholds, Proposition 1 builds on Lemma 1 to state the existence of an equilibrium strategy in every voting subgame. The location of the equilibrium ideology threshold depends only on the midpoint $\bar{x}$ of the candidates’ platform positions, and is monotonic. When candidates’ policy positions coincide, any voting strategy constitutes an equilibrium, but when platforms are distinct, equilibrium is unique. The symmetry of $\tau^*$ implies that if candidate platforms are symmetric, so that $\bar{x} = 0$, then $\tau^* = 0$, so that equilibrium voting is symmetric, as well. In that case, equilibrium voting is sincere: citizens simply vote $A$ if $\theta_i$ is negative and vote $B$ if $\theta_i$ is positive.

28Comparing electoral outcomes with and without his own vote is clearly the rational thing for a voter to do, but pivotal voting models remain somewhat controversial because, empirically, voters do not seem to be cognizant—or even capable—of such a calculus (Esponda and Vespa, 2014). For most of the results below, the issue is mute, since equilibrium voting turns out also to be sincere.
Proposition 1 There exists a function $\tau^*: \mathcal{X} \rightarrow \mathcal{X}$ satisfying $\frac{d\tau^*(\bar{x})}{dx} > 0$ and $\tau^*(-\bar{x}) = -\tau^*(\bar{x})$ such that, for any $x_A, x_B \in \mathcal{X}$ with midpoint $\bar{x} \in \mathcal{X}$, the ideological strategy $v_{\tau^*(\bar{x})}$ constitutes a BNE in the voting subgame. If $x_A \neq x_B$ then $v_{\tau^*(\bar{x})}$ is the only BNE.

3.3 Electoral Margins

Existing models of elections typically exhibit such symmetry that the expected election outcome is an exact tie. Contrary to such predictions, however, empirical margins of victory are often quite large. According to Mueller (2003, ch. 11), for example, the historic average margin of victory in U.S. gubernatorial elections is nearly 23%. Krehbiel (1997, ch. 1) points out that large, bipartisan majorities are also common in legislative votes.

Like existing models, the model above exhibits substantial symmetry. Indeed, according to Proposition 1, voters respond to symmetric candidate positions with symmetric voting. Since voters’ opinions are mutually correlated, however, symmetric voting behavior can produce highly asymmetric electoral outcomes. To see this, note that $x_A = -x_B$ implies that $\bar{x} = \tau^*(\bar{x}) = 0$, in which case (7) reduces to

$$\phi(B|z) = \phi(A|-z) = \int_0^1 \int_0^1 \frac{1}{2} (1 + qsz) g(q) dsdq$$

$$= \frac{1}{2} + \frac{1}{4} z E(q),$$

so the margin of victory is $\mu(z) = |\phi(B|z) - \phi(A|z)| = \frac{1}{2} |z| E(q)$ in state $z$. Across states of the world, the average margin

$$E[\mu(z)] = \int_{-1}^1 \mu(z) f(z) dz = \frac{1}{4} E(q)$$

is positive, as Remark 2 now states formally.

Remark 2 If $x_A = -x_B$ then, in equilibrium, $E[\mu(z)] > 0$.

Equation 17 makes clear that the margin of victory is especially likely to be large if $E(q)$ is high. This is consistent with the casual observation that large margins are most common for decisions that are in some sense obvious, such as retaining public officials with clear track records of quality governance, or revising archaic government procedures or constitutional language. Interpreting a large margin of victory as evidence that a decision was obvious is also natural in settings such as jury verdicts and constitutional amendments, where supermajority rules are explicitly utilized to prevent actions that are not obviously beneficial.\footnote{In an extension of the present model (McMurray, 2014a), candidates interpret large margins of victory as clear instructions from voters to adopt more extreme policies, consistent with empirical responses to electoral “mandates”.}
3.4 Voter Participation

The analysis above assumes that every citizen must vote, whereas in most elections, citizens are allowed to abstain, and many do so. Accordingly, this section redefines voting strategies \( v : Q \times S \rightarrow \{\{A, B, 0\}\} \) to allow abstention, denoted as action 0. Let \( V_0 \) denote the set of such subgame strategies, and let \( \Sigma_0 \) denote the set of complete voting strategies \( \sigma : X^2 \rightarrow V_0 \) that specify subgame behavior for every possible pair \((x_A, x_B) \in X^2\) of candidate platforms.

Definition 2 redefines an ideological strategy in this setting, using two thresholds instead of one. That is, a citizen whose ideology is sufficiently low votes for candidate \( A \), while one whose ideology is sufficiently high votes \( B \). If the two thresholds coincide, then everyone votes; if the thresholds are distinct, citizens between the thresholds abstain from voting for either candidate. With ideological strategies redefined, Lemma 2 reiterates Lemma 1 in stating that the best response to any voting strategy is ideological. Proposition 2 then reiterates Proposition 1 in stating that an equilibrium exists.\(^{30}\)

Definition 2 A strategy \( \nu_{\tau_1, \tau_2} \in V_0 \) in the voting subgame is ideological, with ideology thresholds \( \tau_1 \leq \tau_2 \), if \( \nu_{\tau_1, \tau_2}^j(q, s) = 0 \) unless \( j = A \) and \( \theta(q, s) \in [-1, \tau_1] \)
\( j = 0 \) and \( \theta(q, s) \in [\tau_1, \tau_2] \)
\( j = B \) and \( \theta(q, s) \in [\tau_2, 1] \).

Lemma 2 If \( x_A \neq x_B \) then, for any voting strategy \( v \in V_0 \), the unique best response \( v^{br} \in V_0 \) in the voting subgame is ideological.

With continuous distributions of \( q_i \) and \( s_i \), the probability of a citizen’s private information being completely uninformative is zero. If candidates take distinct platforms, therefore, then every citizen has at least a little information about which policy position is better, and thus has at least a slight preference for one candidate over the other. Since voting is assumed to be costless, this may seem to imply that everyone should vote. Contrary to this intuition, however, Proposition 2 states that \( \tau_1^* < \tau_2^* \), implying that a positive fraction of the electorate choose not to participate in equilibrium.

Proposition 2 (Swing voter’s curse) For any platform pairs \( x_A, x_B \in X \), a PBE \( v^* \in V_0 \) exists in the voting subgame. Moreover, if \( x_A \neq x_B \) then \( v^* \) is ideological, with distinct ideology thresholds \( \tau_1^* < \tau_2^* \). If \( x_A = -x_B \) then a PBE \( v_{\tau_1^*, \tau_2^*} \) exists with \( \tau_1^* = -\tau_2^* \).

\(^{30}\)The analysis is more complicated here, so no claim is made of uniqueness, but following the logic discussed in McMurray (2013), it seems reasonable to conjecture that multiple equilibria would require exotic distributions of private information.
The reason why citizens abstain in equilibrium is the *swing voter’s curse* identified by Feddersen and Pesendorfer’s (1996): since private opinions are correlated with the truth, the candidate with the truly superior policy platform is more likely to win the election by one vote than to lose by one vote, which means that a vote for the inferior candidate is more likely to be pivotal than a vote for the superior candidate. A citizen who is uncertain which platform is superior thus prefers to abstain. As noted above, the strategic complexity of the pivotal voting calculus makes it somewhat controversial. In a tie or near-tie, however, inferences drawn from a citizen’s peers largely cancel each other out, leaving essentially a simple comparison of a citizen’s own information quality with that of one other voter, chosen randomly, whose vote he will negate. Thus, relative to other pivotal inferences, the swing voter’s curse reduces to one that is rather intuitive and—perhaps for this reason—has stronger experimental support (e.g. Battaglini, Morton, and Palfrey, 2010).

Consistent with the prediction of Proposition 2, numerous studies document an empirical correlation between voter turnout and information variables such as voter education, political knowledge, age, access to news media, and contact from campaign workers. Lassen (2005), Banerjee et al. (2010), and Larcinese (2007) also present evidence that the relationship between information and voter participation is causal. A more traditional explanation for voter abstention is the time cost of traveling to the polls, waiting in line, and so on, but as Feddersen and Pesendorfer (1996) point out, this cannot explain why many citizens also cast incomplete ballots, even after voting costs have already been paid. The swing voter’s curse offers a natural explanation for this: a voter has clear, strong opinions on some issues, but not others, and so delegates the latter to those who know more than he does. In support of this explanation, Magleby (1984, ch. 6) and Wattenberg et al. (2000) find that educated and politically knowledgeable voters are the most likely to cast completed ballots.

Both in Feddersen and Pesendorfer (1996) and in the continuous extension of McMurray (2013), the sole source of the swing voter’s curse is a lack of expertise. Here, however, voter participation depends on ideology, which is the *product* of $q_i$ and $s_i$. Thus, citizens with moderate opinions regarding the optimal policy face a swing voter’s curse as well, even if their expertise is high, because they perceive only a slight difference between candidates. In contrast, a citizen with extreme policy preferences votes, even if his expertise is rather low, because even if the optimal policy is less extreme than he believes it to be, it probably at least lies in the same direction.

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31 For references, see McMurray (2011). Voter turnout is also higher in national and general elections than in local races and primaries, perhaps because partisan labels and media exposure give voters greater confidence in their opinions. Similarly, participation tends to be low for policy initiatives, which are typically non-partisan, and which many find “too long and hard to understand” (Magleby 1984, ch. 6).

32 Abstention also often occurs in committee settings, where voting requires only the raise of a hand.
The prediction that voter participation depends jointly on information and ideology has strong empirical support. Using ANES data, for example, Figure 3 displays voter participation levels by information level and ideology. According to probit estimates in Table 1, each information level makes the average citizen 9% more likely to vote in the primary election, 9% more likely to register in the general election, 6% more likely to vote, conditional on registering, and 4% more likely to cast a complete ballot, conditional on voting. Similarly, moving one ideological category increases these participation levels by 2%, 1%, 1%, and 1%, respectively. These estimates are consistent with the findings of Palfrey and Poole (1987), Keith et al. (1992, ch. 3), and Abramowitz and Saunders (2008), that voter participation and information are jointly correlated with ideology. Bade and Rice (2009) suggest an alternative theoretical link between information and ideology based on persuasion and the demand for information, but find the additional link with participation inexplicable. Sobbrio and Navarra (2010) find evidence that supports the strategic theory even more specifically, pointing out that participation is correlated with information, even conditional on ideology.33

33Those authors actually interpret their finding as evidence against the swing voter’s curse, but this is because, following Feddersen and Pesendorfer (1996), they conceptualize ideology as a preference parameter, independent of voter beliefs.
3.5 Welfare

Before analyzing incentives for candidate behavior, this section analyzes the implications of equilibrium voting for social welfare. Let $x^*_n$ denote the equilibrium policy outcome of the game when the expected number of citizens is $n$. With common preferences, it is uncontroversial to measure social welfare simply by the expected utility $E[u(x^*_n,z)]$ of an individual citizen, which averages over the various realizations both of the state variable and of each citizen’s private information, but depends on the strategies adopted by voters and candidates in equilibrium. Like the equilibrium characterizations above, these results below apply whether uncertainty is continuous or binary.

Condorcet’s (1785) classic jury theorem—which Krishna and Morgan (2011) laud as the “first welfare theorem of political economy”—states that public opinion almost surely favors the better of two alternatives. Proposition 3 merely affirms that this holds for two arbitrary policies within this spatial environment, and also notes that the voting strategy adopted by citizens in equilibrium uniquely maximizes social welfare.

**Proposition 3 (Jury Theorem)** If $x_A < x_B$ then, for any $n$, there exists a unique voting strategy $v^*_n \in V$ that maximizes $E[u(x,z)]$. Moreover, (1) $v^*_n$ is ideological and is the unique BNE of the voting subgame, and (2) $x^*_n \rightarrow a.s. \max_{x \in \{x_A, x_B\}} u(x,z)$. 

<table>
<thead>
<tr>
<th>Information, Ideology, and Voting</th>
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</thead>
<tbody>
<tr>
<td><strong>Independent Variable</strong></td>
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<tr>
<td>Voted in primary</td>
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<tr>
<td><strong>Means</strong></td>
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<tr>
<td>Information</td>
</tr>
<tr>
<td>(0.006)</td>
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<tr>
<td>Polarization</td>
</tr>
<tr>
<td>(0.006)</td>
</tr>
<tr>
<td>Observations</td>
</tr>
<tr>
<td>Pseudo R square</td>
</tr>
</tbody>
</table>

1972-2012 ANES. Probit marginal effects are evaluated at average values of independent variables. Standard errors are in parentheses.

Table 1
The existence of an optimal voting strategy may not be surprising, but is also not trivial, because the set of voting strategies is not compact under the standard topology. The proof of Proposition 3 first points out that there is an optimal ideological strategy, and then shows by construction that every non-ideological voting strategy is inferior to another that is ideological. That a socially optimal strategy is also individually optimal, and therefore constitutes an equilibrium, follows from the common value assumption, as in McLennan (1998). Uniqueness then follows from Lemma 1.

Proposition 3 is a normative result, but also sheds light on empirical facets of voter behavior. For one thing, it gives a rationale for the popularity of democratic institutions: even citizens whose private opinions differ from the majority opinion are broadly supportive of following majority rule. Similarly, it explains why people often appeal to public opinion as evidence in favor of their rhetorical positions—especially when the margin of victory is large, which as noted above, is evidence that a decision was obvious. The result that private opinions and public opinion are both correlated with $z$ also means that they are correlated with each other, which could explain the so-called “consensus effect” documented by Ross, Green, and House (1977), whereby individuals on both sides of an issue expect to belong to the majority. In essence, a citizen who perceives candidate $A$ to be superior to candidate $B$ predicts that others will come to the same conclusion. In the 2012 U.S. presidential election, for example, 96% of ANES survey respondents who planned to vote Democrat also predicted a Democratic victory, while 83% of those who planned to vote Republican predicted a Republican victory. As Figure 4 illustrates, predictions are strongly monotonic in ideology. A least-squares regression of forecasts on ideologies yields a coefficient of 0.107 (standard error 0.003), implying that moving one category to the right on a 9-point ideological scale makes a citizen 11% more likely to predict a Republican presidential victory.
Proposition 3 restricts attention to strategies in $\mathcal{V}$, which do not allow voter abstention. As Theorem 2 states, however, some citizens will abstain if allowed to do so. Since every citizen receives an informative signal, it may seem that this merely throws away valuable information, thus vindicating efforts to penalize non-voters with stigma or fines. Contrary to this intuition, however, Proposition 4 states that the optimal voting strategy in $\mathcal{V}_0$ also constitutes an equilibrium, implying that social welfare is higher in voluntary elections than under compulsory voting.\textsuperscript{34}

**Proposition 4** If $x_A < x_B$ then, for any $n$, there exists a voting strategy $\nu^*_n \in \mathcal{V}_0$ that maximizes $E[u(x, z)]$. Moreover, (1) $\nu^*_n$ is ideological, with distinct ideology thresholds $\tau_1 < \tau_2$, and is a BNE of the voting subgame, and (2) $x^* \to_{a.s.} \arg\max_{x \in \{x_A, x_B\}} u(x, z)$.

As McMurray (2013) explains, one way to understand how aggregating additional signals could reduce the quality of the electoral decision is to note that the decision of whether to vote or abstain conveys private information beyond the content of the vote itself; when voting is made mandatory, this additional information is lost. Ideally, each citizen’s opinion would be utilized, but would be weighted according to its reliability, whereas majority rule instead weights each vote equally.\textsuperscript{35} When voting is mandatory, the sign of $\theta_i$ is recorded for each citizen, but its magnitude is not utilized. Abstention provides a crude mechanism by which citizens with weak opinions (i.e. those with moderate $\nu^*_n$) can shift weight from their own votes to those of their peers who, in equilibrium, are more confident, whether because of greater expertise or more extreme signals.

### 3.6 Aggregate Uncertainty

The result that large electorates are essentially infallible follows from the structure of the model of Section 2, where the state of the world can be almost perfectly deduced from a large number of private signals. An alternative possibility is aggregate uncertainty, meaning that there is relevant information about the state of the world that no citizen possesses, so that even if all private information were made public, the location of the optimal policy would remain uncertain. For example, suppose that the truth is binary, so that the optimal policy $z^* \in \{-1, 1\}$ lies at one of the extremes of the policy interval, but that even after observing all private information, the most that could be learned is that $z^*$ is positive with probability

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\textsuperscript{34}The optimality of equilibrium voting is also relevant to the controversy over pivotal voting models noted above. In deciding whether to vote or abstain, for example, a citizen might adopt a planner’s perspective, and ask himself which information types should optimally be participating. If he then followed this recommendation, he would be behaving as if he had conditioned on the event of a pivotal vote.

$p$ and negative with probability $1 - p$. This could be modeled simply by assuming that voter signals are correlated with $p$, rather than with $z^*$. That is, citizens would form beliefs not about which extreme is optimal, per se, but rather about how strongly the available evidence favors $z^* = -1$ or $z^* = 1$.

This setup may seem quite different from the model above, but actually it is not. To see this, note that the policy in $[-1,1]$ that is optimal, conditional on aggregate information, is simply the expectation of $z^*$, which is given by $z \equiv p(1) + (1 - p)(-1)$. If the prior distribution of $p$ is uniform on $[0,1]$, for example, meaning that any distribution of $z^*$ is equally likely ex ante, then $z$ is uniform on $[-1,1]$. This is precisely the model of Section 2, except that $z$ can no longer be interpreted as the policy that is truly optimal, but rather as the policy that is conditionally optimal, given all available private information. This observation is easiest to see for binary $z^*$, but of course applies more generally: whatever its prior distribution, if the policy $z^*$ that is truly optimal cannot be determined even from aggregate information then the best an electorate can do is determine its expectation $z \equiv E(z^*)$. Reinterpreting $z$ in this way, all of the formal results above (and below) remain valid.

While it does not change the analysis, aggregate uncertainty does change the interpretation of some of the results above. For example, Propositions 3 and 4 no longer imply that electorates are infallible, but only that electorates do the best that can be done, given available information. At the individual level, citizens have a new reason to remain ideologically moderate, which can be characterized as skepticism. A citizen whose signal of $p$ is close to 0 or 1 views the election as black and white, and so champions one extreme or the other. A citizen whose signal of $p$ is close to $\frac{1}{2}$ views the world as truly gray. Such a citizen might favor a moderate-sized stimulus, for example, not because he lacks expertise or believes that moderate stimulus is ultimately optimal, but because neither Keynesian nor classical economic theory strikes him as sufficiently convincing.

## 4 Candidate Behavior

Having characterized voters’ equilibrium response to any pair of candidate platforms, this section proceeds to analyze the incentives of candidates as they choose which policy positions to adopt. Section 4.1 begins with candidates who are truth motivated, just like citizens, first for biased and then for unbiased beliefs. Section 4.2 then illustrates these results by computing equilibrium platform positions for small electorates, and comparing them with the equilibrium positions predicted by standard private-value models with probabilistic voting. Section 4.3 next analyzes the case of office motivation (i.e. $\beta$ large), as well as the general case of mixed motivations, and Section 4.4 analyzes social welfare for the various model
specifications.

4.1 Truth Motivation

If \( \beta = 0 \) then (5) reduces to (4) and a candidate is purely truth motivated, just like voters, preferring policies that are as close as possible to the true optimum. What this implies for behavior depends on what a candidate believes about \( z \). Section 4.1 treats the simplest case, which is that of \textit{biased} candidates, who each believe they have identified the true optimum. Section 4.1 treats the opposite case of candidates who have no information of their own, and must base their beliefs on whatever they can infer from equilibrium voting.

**Biased Beliefs**

Candidates have \textit{biased} beliefs if there are policies \( \theta_A < \theta_B \) such that candidate \( j \in \{A, B\} \) believes that \( z = \theta_j \) with probability one. For candidate \( A \), for example, (4) reduces to

\[
E [u(x, z) | z = \theta_A] = \sum_{j=A,B} u(x_j, \theta_A) \Pr (w = j | z = \theta_A).
\]

(18)

If candidates are biased then, as Theorem 1 now states, they adopt distinct policy platforms in equilibrium. This follows from the logic of Wittman (1983) and Calvert (1985): if policy positions were the same, a candidate could deviate toward the policy that she perceives to be optimal, making herself better off if she wins and no worse off if she loses. If biases are symmetric then equilibrium platforms may be symmetric, as well. In fact, there is exactly one such equilibrium.\(^{36}\)

**Theorem 1** If \( \beta = 0 \) and candidates have biased beliefs with \( \theta_A < \theta_B \) then \( x^*_A, x^*_B, \sigma^* \in X^2 \times \Sigma \) is a PBE only if \( \theta_A < x^*_A < x^*_B < \theta_B \). If \( \theta_A = -\theta_B \) then, for any \( n \), there is exactly one PBE with \( x^*_A = -x^*_B \). Moreover, \( \lim_{n \to \infty} x^*_j = \theta_j \) for \( j = A, B \).

Since she believes she knows which policy is socially optimal, a biased candidate would like to commit to that policy as her campaign platform. If she does so, many voters will recognize her position as superior (as she believes that it is), and will vote for her. As she becomes more extreme, however, a wider range of voter opinions will mistakenly favor her opponent’s platform, and her risk of losing the election will increase. She does not care about winning per se, but must win in order to influence the policy outcome. Thus, her

\(^{36}\)It is not clear whether equilibria exist with asymmetric platforms, but if biases are symmetric, the unique symmetric equilibrium seems a likely one for candidates to coordinate on. A similar remark applies to Theorem 2, below.
equilibrium policy position must balance the desire to do what she believes is right, with the need to attract enough voters to secure victory. As the electorate grows large, however, mistakes in public opinion become less likely (by Proposition 3). Thus, as the last part of Theorem 1 states, equilibrium policy positions converge to $\theta_A$ and $\theta_B$.

**Unbiased Beliefs**

Unbiased candidates receive no private information of their own. Like citizens, therefore, they choose their platforms to maximize (4). Differentiating this expression with respect to $x_A$, for example, yields

$$E_z \left[ -2 \left( x_A - z \right) \Pr (w = A | z) \right] + E_z \left[ \sum_{j = A, B} u (x_j, z) \frac{\partial \Pr (w = j | z) \partial \bar{x} \partial x_A}{\partial \bar{x} \partial x_A} \right] = 2 \Pr (w = A) \left[ E (z | w = A) - x_A \right] + \frac{\partial E [u (x, z)] \partial \bar{x} \partial x_A}{\partial \bar{x} \partial x_A}$$

$$= 2 \Pr (w = A) \left[ E (z | w = A) - x_A \right]. \quad (19)$$

This derivation proceeds as it does because the probability with which a candidate wins depends on the strategy used by voters, and according to Proposition 1, voters’ equilibrium response to candidate platforms is ideological. The final equality follows because, by Proposition 3, the equilibrium ideology threshold maximizes $E [u (x, z)]$. (19) is zero if and only if $x_A = E (z | w = A)$, so after ruling out corner solutions, Lemma 3 establishes this as a necessary condition for equilibrium, along with the analogous condition for candidate $B$.

**Lemma 3** If $\beta = 0$ and candidates are unbiased then $(x_A^*, x_B^*, \sigma^*) \in X^2 \times \Sigma$ is a PBE only if $x_j^* = E (z | w = j)$ for $j = A, B$.

Since unbiased candidates are ex ante identical, and receive no private information of their own, their basic inclination would be to both adopt the same centrist platform, as a hedge against error. The implication of Lemma 3, however, is that candidates base their equilibrium positions on additional information, namely the event of winning the election. This may seem counterintuitive, since the timing of the game is such that the election winner can only be determined once policy positions are established and votes are cast. The key insight is that even though a candidate doesn’t yet know whether she will win or lose, she optimally chooses a policy position that will be desirable if she wins, because otherwise her policy choice doesn’t matter.\(^{37}\)

\(^{37}\)This is the essential logic of the lemma, but the proof is complicated by the fact that moving a candidate’s platform shifts the subgame that voters will be playing. In responding to candidate $B$’s platform position,
Restricting attention to events in which a candidate wins the election is analogous to the result in Section 3.2 that citizens should rationally restrict attention to situations in which a vote is pivotal. Beginning with Austen-Smith and Banks (1996), several studies have demonstrated how, in informational settings, this pivotal voting calculus can have important consequences for voting behavior; similarly, Theorem 2 now shows how the asymmetry between winning and losing can have important consequences for candidate polarization. When the policy that is truly optimal turns out to be on the left, citizens tend to receive signals on the left and to vote \( \mathbf{1} \), so that candidate \( \mathbf{1} \) tends to win the election; similarly, candidate \( \mathbf{2} \) tends to win when the optimal policy is on the right. Conditional on winning the election, therefore, candidate \( \mathbf{1} \) expects the optimal policy to be negative, and in anticipation of this, she adopts a liberal policy platform. Analogously, candidate \( \mathbf{2} \) adopts a conservative policy platform, anticipating that if she wins, it will be because \( \zeta \) is positive.

In this way, candidates polarize in equilibrium even though they are ex ante identical.38

**Theorem 2** If \( \beta = 0 \) and candidates are unbiased then \((x^*_A, x^*_B, \sigma^*) \in X^2 \times \Sigma \) is a PBE only if \( x^*_A < 0 < x^*_B \). There is exactly one PBE with symmetric platforms \( x^*_A = -x^*_B \).39 For this sequence \((x^*_A, x^*_B, \sigma^*)_n \) of PBE, \( \lim_{n \to \infty} x^*_A = E (z|z < 0) \) and \( \lim_{n \to \infty} x^*_B = E (z|z > 0) \).

The final claim of Theorem 2 states that platforms in large elections tend toward \( E (z|z < 0) \) and \( E (z|z > 0) \). With continuous uncertainty, recall from Section 3.1 that citizens with the most extreme signal realizations favored policies \(-\frac{1}{3}\) and \(\frac{1}{3}\), so with platforms approaching \( E (z|z < 0) = -\frac{1}{2} \) and \( E (z|z > 0) = \frac{1}{2} \), candidates are more extreme than even the most extremely ideological voters. With binary uncertainty, \( E (z|z < 0) = -1 \) and \( E (z|z > 0) = 1 \), implying that candidates diverge to opposite extremes of the policy space.

It is intuitive that candidates should be emboldened by popular support. Sobel (2006) uses essentially this logic, for example, to explain why group decisions in experimental settings are often more extreme than individual opinions.40 It is less clear whether candidates

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38 If candidates adopt identical platforms, the equilibrium response from voters need not be ideological, and could therefore leave candidates with identical beliefs. However, an equilibrium of this form is not sub-game perfect, because deviating to a distinct platform would prompt an ideological response from voters, thus vindicating the deviation.

39 Uniqueness here is up to the relabeling of candidates and the specification of behavior for indifferent voters, both right at the equilibrium ideology threshold and in subgames associated with convergent platform pairs \( x_A = x_B \), off the equilibrium path.

40 Glaeser and Sunstein (2009) make similar arguments, while also emphasizing the possible importance of non-Bayesian cognitive mistakes that are not modeled here.
are truly influenced by the anticipation of public support (just as it is unclear to what extent voters are influenced by the anticipation of a pivotal event): politicians rarely admit, for example, that their platform policies might ultimately turn out to be sub-optimal. One possibility is that candidates are biased, as in Section 4.1, and so restrict attention to certain states of the world out of ex-ante confidence, rather than strategic inference. Alternatively, it may be that candidates are indeed emboldened by the anticipation of public support, as in Theorem 2, but that this effect is sub-conscious. To the extent that candidates are not certain of the optimal policy, however, it would be irrational to ignore the informational content of winning the election, just as it is irrational to ignore the informational content of a pivotal vote. The analysis of this section makes clear how remarkably strong such theoretical considerations can be.

4.2 Examples

In demonstrating that candidates adopt distinct platforms in equilibrium, Theorem 1 draws directly on the logic of Wittman (1983) and Calvert (1985), who derive the same result in private-value settings with probabilistic voting. The purpose of this section is to compare the degree of polarization in the two types of models. To that end, examples are computed for elections of various size, first for two canonical specifications of probabilistic voting with private values and then for the information model above, for both binary and continuous uncertainty and for truth-motivated candidates who are both biased and unbiased, as well as for candidates whose preferences differ from those of voters. To facilitate computation, these examples drop the assumption of population uncertainty, assuming instead that the number $2n+1$ of voters is known, and is odd, and for simplicity, abstention is not allowed.\footnote{Thus, (11) and (14) must be replaced by $\Pr(w = B|z) = \sum_{k=n+1}^{2n+1} \binom{2n+1}{k} \phi(A|z)^k \phi(B|z)^{2n+1-k}$ and $\Pr(piv|z) = \binom{2n}{n} \phi(A|z)^n \phi(B|z)^n$, respectively. For precise details about the computations below, a Mathematica code file is available from the author upon request.}

Private Values

As Duggan (2013) notes, private-value literature offers two main specifications of probabilistic voting, both of which are computed below. In both specifications, policy preferences are represented by the following utility function,

$$u(x, \theta_i) = -(x - \theta_i)^2,$$

where ideology $\theta_i$ is an exogenous preference parameter, and candidates have bliss points $\theta_A = -1$ and $\theta_B = 1$ at the two extremes of the policy interval. The stochastic preference

$$29$$
model of probabilistic voting, which is essentially the framework of Wittman (1983) and Calvert (1985), assumes that candidates cannot observe voters’ precise policy positions, but only know that $\theta_i$ are drawn from a known distribution—for the examples below, a uniform distribution on $[-1, 1]$.

For elections of various sizes, candidates’ equilibrium policy positions are displayed in Column 1 of Table 2. In small electorates, candidates indulge in some extremism. Moderating slightly would be beneficial if doing so attracted the median voter, but with substantial probability, the median voter’s ideology is either far left or far right, so that small adjustments in the position of one candidate have no impact. As the number of citizens grows, the probability of an extreme median voter declines, so the incentive to moderate increases. The consequence of this is that, while equilibrium platforms never coincide, polarization all but vanishes in large elections. With at least one thousand voters, for example, candidates are more moderate in equilibrium than 96% of the electorate.

<table>
<thead>
<tr>
<th>Voters</th>
<th>Private value</th>
<th>Continuous uncertainty</th>
<th>Binary Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Stochastic Preference</td>
<td>Stochastic Partisanship</td>
<td>Biased Candidates</td>
</tr>
<tr>
<td>3</td>
<td>0.400</td>
<td>0.549</td>
<td>0.142</td>
</tr>
<tr>
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<td>0.511</td>
<td>0.227</td>
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<tr>
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<tr>
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<tr>
<td>25</td>
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</tr>
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<td>51</td>
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<td>-</td>
</tr>
<tr>
<td>100,001</td>
<td>0.004</td>
<td>0.061</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 2

Hinich (1978) and Lindbeck and Weibull (1987) introduce what Duggan (2013) refers to as the stochastic partisanship model: in addition to policy utility, each voter is biased in favor of one candidate or the other, receiving a benefit $\alpha_i$ if candidate $A$ is elected (where a negative $\alpha_i$ reflects a bias for $B$). Such a citizen votes for candidate $B$ if and only if $u(x_A, \theta_i) + \alpha_i > u(x_B, \theta_i)$. For simplicity, the computations below assume that $\alpha_i$ is an i.i.d. draw from a uniform distribution. For the sake of emphasis, the domain of $\alpha_i$ is the interval $[-4, 4]$, so that biases can be large enough to make even the most extreme citizens vote for a candidate at the opposite extreme. For this case, equilibrium platforms are displayed in column 2 of Table 2. For any $n$, polarization is substantially higher than in the simpler stochastic preference model, but polarization again declines as the electorate
grows large: with at least 100,000 citizens, candidates should be more moderate than 94% of voters, in spite of the huge partisan biases.

For an electorate of a given size, of course, any degree of polarization can be sustained by adding enough uncertainty to the model. In a preference framework, however, it is unclear why uncertainty about voters should be so severe. Voter polls with moderate sample sizes should estimate the median preference rather precisely, for example. As uncertainty about the location of the median voter declines, policy positions should quite generally converge relatively quickly to the predictions of a deterministic model. Ultimately, then, a standard probabilistic voting model is hard pressed to explain the empirical degree of polarization noted in Section 1.

**Truth Motivation**

The remaining columns of Table 2 display equilibrium policy positions for candidates in a common-value environment. For simplicity, these examples assume that the distribution $G$ of voter expertise is uniform on $[0, 1]$. With continuous or binary uncertainty, respectively, this implies that $s_i$ or $\theta_i$ is distributed uniformly on $[-1, 1]$. Examples with binary uncertainty are simpler, and so can be computed for larger electorates. For truth-motivated candidates with extremely biased beliefs (i.e. $\theta_A = -1$ and $\theta_B = 1$), platform positions are listed in columns 3 and 6 of Table 2 for elections of various size. Evidently, platforms are much more polarized than in the probabilistic voting model, even with the extreme formulation of stochastic partisanship. Theorem 1 implies that, as the electorate grows large, candidate positions will diverge to $-1$ and $1$. In the case of binary uncertainty, column 6 makes clear that this divergence is quite rapid.

Columns 4 and 7 display equilibrium platform positions for unbiased candidates. By Lemma 3, these positions reflect candidates’ expectations $E(z|w = A)$ and $E(z|w = B)$ conditional on winning the election. This event becomes more strongly informative as the electorate grows large, so that as in the case of biased beliefs, polarization increases with $n$. With continuous uncertainty, polarization increases more slowly with unbiased candidates: by Theorem 2, it only ever approaches $E(z|z < 0) = -\frac{1}{2}$ and $E(z|z > 0) = \frac{1}{2}$, whereas biased candidates approach $-1$ and $1$. With binary uncertainty, however, the impact of candidates’ equilibrium calculus is almost just as strong as the impact of extremely biased beliefs, and candidates of either type quickly diverge to the polar extremes. Thus, consistent even with Bafumi and Herron’s (2010) more extreme empirical estimates, candidates in large elections are as extreme as the most extremely ideological citizens.
Policy motivation

The examples of Sections 4.1 and 4.1 differ from the examples of Section 4.2 both in the specification of voter behavior and in the specification of candidate behavior, making it difficult to know which source is more important for increasing the equilibrium level of polarization. Accordingly, this section computes equilibrium platforms for a model in which citizens follow the common-value model of Section 2 but candidates are policy motivated, meaning that each prefers a specific policy, as in Section 4.2, regardless of the true state of the world. Specifically, candidate preferences are given by (20), with $\theta_A = -1$ and $\theta_B = 1$.

For this variation on the basic model, the resulting equilibrium platform positions are listed in columns 5 and 8 of Table 2. Since candidates do not care about the state of the world, it may seem intuitive that equilibrium platforms should be just as they are in Section 4.2. To the contrary, however, candidate positions in column 8 are almost as polarized as those in columns 6 and 7, where candidates were truth-motivated. Candidates in column 5 are more highly polarized than the unbiased candidates of column 5, and almost as polarized as the biased candidates of column 4. Evidently, polarization stems more from the assumption of informative voting than from particular assumptions about candidate motivations.$^{42}$

The reason that polarization is so much higher here than in the private-value specification of Section 4.2 is that candidates are influenced by the state of the world, even though they do not care about it directly. If the state of the world is on her side, a candidate will likely win the election by a comfortable margin. In that case, moving her policy position toward her bliss point will cost her some votes, but not cost her the election. Similarly, if the state of the world favors her opponent then moving to a less extreme position will pick up a few votes, but will not be enough to secure victory. Another way of viewing the same phenomenon is to note that citizens’ votes are correlated with one another, since each is individually correlated with $z$. This correlation makes the location of the median voter more highly variable, even in large elections. Thus, the structure of commonly-valued information provides a rationale for why elections should be so unpredictable. As noted in Section 4.2, this was the missing link for standard probabilistic voting models to be able to explain high levels of polarization.$^{43}$

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$^{42}$Truth-motivated candidates would converge, for example, if citizen ideologies were a preference parameter, as in Section 4.2: since they could learn nothing from voters in that case, candidates with unbiased beliefs would converge to the political center; candidates with biased beliefs would be behaviorally equivalent to the privately-motivated candidates of Section 4.2, and would polarize in small electorates but not when $n$ is large.

$^{43}$This same mechanism underlies results by Londregan and Romer (1991) and Boleslavsky and Cotton (2015), that the platform positions of policy-motivated parties become more polarized as commonly-valued
Available empirical evidence seems to support the hypothesis that candidate polarization stems from voter behavior, rather than the inability to reign in rogue candidates. In the 2012 U.S. presidential primaries, for example, many Republicans reported viewing Mitt Romney as the most likely candidate to beat President Barack Obama in the general election, but “not conservative enough”, and so voted for Rick Santorum, Newt Gingrich, or Ron Paul instead. Apparently, voters seek not a moderate who will attract swing voters, but a bold and confident champion, who can recognize the truth and then convince the electorate accordingly. The latter is the popular perception of many prominent U.S. presidential candidates, such as Franklin Roosevelt, Barry Goldwater, George McGovern, Ronald Reagan, and Barack Obama. Ansolabehere, Snyder, and Stewart (2001) and Canes-Wrone, Brady, and Cogan (2002) present statistical evidence from U.S. House elections that, indeed, political extremism imposes only a minor handicap on candidates’ electoral fortunes.

4.3 Office Motivation

The analysis above has treated the case of $\beta = 0$, meaning that candidates care only about the policy outcome, regardless of whether they themselves win or lose the election, whereas much of the spatial voting literature has started from the opposite extreme, assuming that candidates are willing to make any policy concessions necessary to win. Most likely, candidate have mixed motivations, desiring good policies but at the same time hoping to be the one to implement them. This section accommodates mixed motivation by allowing $\beta > 0$. It also accommodates the case of pure office motivation, wherein $\beta$ tends to infinity.

In private-value models, each citizen votes for the candidate whose platform is closest to his own bliss point. That is, citizens vote $A$ and $B$, respectively, whose bliss points are to the left of $\bar{x} = \frac{x_A + x_B}{2}$. By moving toward her opponent, therefore, a candidate shifts $\bar{x}$ and thus (in expectation) attracts some of her opponent’s supporters. The voting calculus is more complicated here because citizens take into account the informational content of a pivotal vote, but Lemma 1 states that the equilibrium ideology threshold is monotonic in $\bar{x}$, so the standard logic applies. As Theorem 3 now states, the equilibrium consequence of this is that office-motivated candidates are less extreme than purely truth-motivated candidates and, for strong enough office motivation, candidates converge to the ideological center, just as in the canonical median voter theorems of Hotelling (1929) and Downs (1957). Equilibrium valence characteristics becomes more important relative to privately-valued policy considerations. The former are typically considered to receive much less weight in voter utility, however, limiting the ability of valence to generate substantial polarization in a model with conflicts of interest over policy.

44 For example, see http://elections.nytimes.com/2012/primaries/states/ohio/exit-polls.

45 More precisely, since voters’ ideologies are private information, platforms converge to the median of the
platforms may be symmetric, and in that case Theorem 3 states that polarization strictly decreases with $\beta$. These results hold whether candidates’ beliefs are biased or unbiased.\footnote{An analogous result could also be stated for candidates who are policy motivated, as supposed in Section 4.2.}

**Theorem 3 (Median Opinion Theorem)** If $\beta > 0$ then, whether candidate beliefs are biased or unbiased, $(x_A^*, x_B^*, \sigma^*) \in X^2 \times \Sigma$ is a PBE only if $|x_j^*| < |E(z|w = j)|$ for $j = A, B$. Also, for every $\varepsilon > 0$ there exists a $\beta$ sufficiently large that if $(x_A^*, x_B^*, \sigma^*) \in X^2 \times \Sigma$ is a PBE then $|x_j^*| < \varepsilon$ for $j = A, B$. Moreover, for any $\beta \geq 0$ there exists a unique PBE with symmetric platforms $x_A^* = -x_B^*$, where $|x_A^*| = |x_B^*|$ strictly decreases in $\beta$ until $x_A^* = x_B^* = 0$.

The logic that office-motivated candidates should move their platforms toward each other, and toward the center of the policy space, is quite familiar from the private-value spatial voting literature. Theorem 3 is labeled as the median opinion theorem to emphasize that voter ideologies, which are all-important in determining political behavior, are in this model actually only approximations of a more fundamental preference. Thus, the behavioral prediction of Theorem 3 is familiar, but, as Section 4.4 emphasizes, has dramatically different implications for social welfare.

### 4.4 Welfare

The analysis above has derived equilibrium behavior for candidates with various motivations. This section analyzes the implications of such behavior for social welfare. Standard private-value models highlight the utilitarian benefits of convergence: centrist policies compromise between the competing interests of the left and right, thus minimizing the total disutility that voters suffer from a policy that is far from their bliss points.\footnote{This is formalized by Davis and Hinich (1968). If utility functions are tent-shaped or quadratic, for example, then total utility is maximized at the median voter’s or mean voter’s ideal point, respectively; generically, the utilitarian optimum lies in the interior of the policy space. Utilitarian considerations are also implicit in May’s (1952) axioms, which endorse the median voter’s ideal policy as the one that is majority-preferred to any other (i.e. the Condorcet winner).} In that light, the theoretical prediction that competition for office should drive candidates—who might otherwise prefer extreme policies—toward each other and toward the political center could be viewed as the “invisible hand” of politics. On the other hand, as Ansolabehere, Snyder, and Stewart (2001) note, empirical evidence of polarization must then be interpreted as evidence of political failure.

To the extent that candidates are office motivated, election outcomes are zero-sum. Since citizens and candidates otherwise share a common objective, it is therefore uncontroversial distribution of possible median voters, as in Calvert (1985).
to equate social welfare with expected utility. In contrast with private-value models, Proposition 5 states that the policy positions adopted here in equilibrium by truth motivated candidates are socially optimal, despite the result of Theorem 2 that such policy positions are highly polarized.\textsuperscript{48} In the case of binary uncertainty, in fact, platforms diverge to the polar ends of the policy space, but $x^*_n$ converges in probability to precisely the optimal policy.

**Proposition 5** If candidates are truth motivated then, for any $n$, there exists a strategy vector $(x^*_{A,n}, x^*_{B,n}, \sigma^*_n) \in \mathcal{X}^2 \times \pm$ that maximizes $E_{x,z} [u(x, z); x_A, x_B, \sigma, n]$. Moreover, (i) $\sigma^*_n$ is ideological and $(x^*_{A,n}, x^*_{B,n}, \sigma^*_n)$ constitutes a PBE of the game, and (ii) if $Z = \{-1, 1\}$ then $|x^*_n - z| \to_p 0$.

The benefit of candidate polarization is that it tailors the policy choice to the situation: if the optimal policy is on the left then citizens can elect the candidate on the left; if the optimal policy is on the right, they can elect the candidate on the right. The same logic drives a similar result in the model of Bernhardt, Duggan, and Squintani (2009), where standard spatial preferences are shifted by a common “shock”, so that the median voter prefers a menu of two similar alternatives, rather than one. While the intuition is similar, the implications here are much more stronger, because the optimal policy may turn out to be far from the center.

A particularly stark example of this is the case of binary uncertainty, such as the economic stimulus example of Section 2.1: political compromise may produce a moderate level of economic stimulus, but this is known ex ante to be far away from what is ultimately optimal. In that sense, the convergence result of Theorem 3 is reminiscent of the models of Canes-Wrone, Herron, and Shotts (2001) and Maskin and Tirole (2004), where candidates adopt inferior policies in an effort to “pander” to the premature preferences of uninformed voters. Those models are all binary, however, and so lack the geometric interpretation that can be seen in the present model, namely that pandering involves timidly avoiding the extreme policy changes that would actually provide the greatest social benefit, at the sacrifice of some popularity.

While overlooked by academic literature, the logic of Proposition 5 seems implicit in many informal discussions of politics. For example, ideological moderates are often given pejorative labels such as DINO and RINO (i.e. Democrats and Republicans-in-name-only) for compromising their principles in pursuit of popularity. In 2000, third-party U.S. presidential candidate Ralph Nader famously criticized Republicans and Democrats for being “look alike parties”, “tweedledum and tweedledee”.\textsuperscript{49} These concerns are not new: over a

\textsuperscript{48}Proposition 5 restricts attention to the set $\Sigma$ of voting strategies with no abstenion, but an analogous result can be stated for strategies in $\Sigma_0$.

century ago, Tocqueville (1835, p. 175) wrote in praise of political parties that “cling to principles rather than to their consequences”. The American Political Science Association (1950, p. 15) issued a manifesto calling for “responsible parties” who believe that “putting a particular candidate into office is not an end in itself”, and advocating to “keep parties apart” in order to “provide the electorate with a proper range of choice”. Such recommendations are odd if convergence to the center is optimal for voters, as in standard preference models.

In the context of the information model above, however, they can be easily understood as a call for truth-over-office-motivation.

It is remarkable that the prediction of political convergence could arise in both an information model and a standard private-value model, but carry such opposite implications for social welfare. This is important for policy: worried about polarization, for example, political pundits might blame the inattention and non-participation of ideological moderates, who fail to protect their own interests from the extreme activists on either side. As a remedy, they might propose making voting mandatory. In contrast, Proposition 4 implies that welfare would be lower, not higher, if voting were mandatory, and Proposition 5 shows that convergence would be lower, not higher, if voting were mandatory, and Proposition 5 shows that convergence amounts to a spatial version of pandering.

Of course, being too moderate is not the only sin a candidate can commit: in recent years, candidates have often been criticized for being too extreme and uncompromising. This also has a clear rationale within the context of the model above: as Section 4.2 demonstrates, candidates who are overconfident in their political opinions are more extreme than those with unbiased beliefs, and the implication of Proposition 5 is that, from the perspective of social welfare, this is undesirable. In fact, since the electorate is roughly evenly divided between liberals and conservatives, the optimal policy positions \( E(z|w = A) \) and \( E(z|w = B) \) are likely not very extreme, suggesting that the high levels of polarization described in Section 1 are indeed over-polarized, given the information of voters.

With binary uncertainty, overextremism may seem to be of minor concern, because as columns 6 and 7 of Table 2 demonstrate, the platform positions of biased and unbiased candidates are similar in that case. However, such a view is incomplete: if there is aggregate uncertainty—for example, because even the totality of available macroeconomic evidence is inconclusive—then as Section 3.6 notes, uncertainty is continuous even though truth is ultimately binary. Thus, the relevant columns of Table 2 are 3 and 4, which exhibit a large disparity between the optimal platforms of unbiased candidates, which by Theorem 2 converge to \(-1\) and \(1\), and the overly extreme positions of biased candidates, which in that case would diverge to \(-1\) and \(1\). Put simply, the problem is that candidates view the world as black and white, when in fact it is gray.

The result that office motivation and overconfidence push candidates in opposite direc-
tions suggests that one can at least partly compensate for the other: the ideal candidate is truth motivated and unbiased, but a candidate who is both overconfident and office motivated is better than a candidate who is one or the other. Which types of candidates are likely to run for office, and how this is shaped by candidate salaries or other policy instruments, are important questions for future work.

5 Conclusion

From the perspective of preference aggregation, democracy has excellent theoretical appeal, as competition for office drives candidates toward the utilitarian optimum at the political center, but empirical polarization is then both puzzling and alarming, reflecting some inexplicable political failure. This paper has explored an alternative paradigm, in which the familiar left-right geometry of ideology reflects the myriad of private opinions regarding what is truly optimal, consistent with evidence of socially-minded voting and evolving political opinions. Polarization then occurs quite naturally: candidates need not go to voters, instead expecting voters to come to them. Over-polarization is still a potential problem, but for informational rather than utilitarian reasons, and convergence can have negative consequences as well, as it merely panders to voters’ premature preferences. The information paradigm also explains empirical patterns that are difficult to reconcile with preference models.

In addition to the results above, McMurray (2014a) shows how voting in an information environment may take on a signaling role, conveying electoral mandates to elected officials and providing a rationale for supporting minor parties, who are unlikely to win elections. McMurray (2014b) extends the model above to multiple dimensions, which is a well-known limitation of preference models, and shows how logical correlations across issues shape the endogenous bundling of policy positions, so that a single ideological dimension emerges. Another important implication of an information paradigm is the inherent difficulty in forecasting individual political behavior: to know whether a citizen will support wealth redistribution, it is not sufficient to know how wealthy he is, or even whether he supported wealth redistribution in the past, because his opinion may differ from his private interest, and is

50 List and Goodin (2001) point out that common values can also explain the empirical rarety of voting cycles, which preference models such as Arrow’s (1950) suggest should be ubiquitous.

51 Other directions that would be useful to explore include voting and information costs and deviations from common value, as well as endogenous candidate entry: moderate candidates should be more attractive to voters, for example, just as in the median opinion theorem above, but extremists should be more anxious to influence policy, and also more confident of winning. If confidence signals competence, extremism might also be rewarded by voters, as in Kartik, Squintani, and Tinn (2013).
liable to drift over time.

Both for simplicity and for consistency with existing literature, the analysis above preserves most of the information structure of Condorcet (1785): Bayesian rationality, accurate prior beliefs, and informative and conditionally independent signals. Clearly, these constitute a best-case scenario for information aggregation, making majority opinion essentially infallible in large elections. If this were literally true, a citizen who learns that he holds a minority opinion should immediately change sides and join the majority; empirically, of course, this does not happen. One possibility is that, as citizens communicate with one another and are prone to similar mistakes, their private opinions are correlated; this can reduce the accuracy of majority opinion, as Dietrich and Spiekermann (2013) discuss, which could give independent thinkers a legitimate reason to maintain minority opinions. Another possibility is that voters make cognitive mistakes that are inconsistent with Bayesian rationality, as argued by Caplan (2007) and Ortoleva and Snowberg (2015). Indeed, the latter seems likely: communication should resolve any disagreements based on private signals alone, by the logic of Aumann (1976), but heterogeneous beliefs about the signal-generating process can sustain persistent disagreement, as Acemoglu, Chernozhukov, and Yildiz (2009) show. Such beliefs could result from non-Bayesian cognitive mistakes such as forgetfulness, as in Brandenburger, Dekel, and Geanakoplos (1989) or Fryer, Harms, and Jackson (2013).

A complete analysis of errors in information processing is beyond the scope of this paper, but just as overconfident candidates are overly extreme, incorrect beliefs seem generally to be bad for social welfare. On the other hand, Dietrich and Spiekermann (2013) point out in a binary setting that, even when the accuracy of public opinion is not perfect, it may still be high. How this translates to a continuum of policy alternatives is an important question, but there seems to be scope for optimism, as long as individual opinions are not completely divorced from the truth. In any case, a fully rational model is useful as a benchmark by which information impediments or other political failures can be measured, just as the idyllic assumption of truth-motivated and unbiased candidates provides a foil against which overconfidence and office motivation can be assessed. Exogenously subjective beliefs may better predict certain behaviors, but determining the welfare implications of such behavior requires an explicit connection to an underlying truth variable.

In contrast with welfare, behavioral predictions seem likely to be very robust: quite gen-

52 Conflicts of interest constitute another obvious impediment to unanimity. Once policies have been decided, however, preferences are no longer relevant, but individuals nevertheless continue to predict opposite policy effects (e.g. some predicting that a recession will end, others that it will worsen), suggesting that informational impediments remain, as well.

53 Sunstein (2002, p. 188) finds this eminently reasonable: “it is sensible to say that as a statistical matter, though not an invariable truth, people who are confident are more likely to be right”.
erally, individuals who lack confidence in their information are likely to remain ideologically moderate and reluctant to participate. Conversely, confident voters and candidates should tend to be more extreme, and to expect others to be on their side. At the same time, moving toward the center should quite generally give a candidate an electoral advantage with ideological moderates, whether this is socially desirable or not. That behavioral predictions depend on the source of ideological differences is another general implication of the above, and the fact that similar behavior can arise in either model, but carries such opposite welfare consequences, underscores the importance of not taking the preference paradigm for granted.

A Appendix

Proof of Lemma 1. The expectation

$$E(z|\pi v, q, s) = \frac{\int_{-1}^{1} z \Pr(\pi v|z) \frac{1}{2} (1 + qs z) dz}{\int_{-1}^{1} \Pr(\pi v|z) \frac{1}{2} (1 + qs z) dz} = \frac{E(z|\pi v) + \theta E(z^2|\pi v)}{1 + \theta E(z|\pi v)}$$

is increasing in $\theta$, and exceeds $\bar{x}$ if and only if $\theta$ exceeds

$$\tau^{br}_{AB} = \frac{\bar{x} - E(z|\pi v)}{E(z^2|\pi v) - \bar{x} E(z|\pi v)}.$$ (21)

When $\theta < \tau^{br}_{AB}$, therefore, (13) is negative, and a citizen prefers to vote $A$; when $\theta > \tau^{br}_{AB}$, (13) is positive, and a citizen prefers to vote $B$. Therefore, the best response to any voting strategy is an ideological strategy, with threshold $\tau^{br}_{AB}$ defined by (21).

Proof of Proposition 1. Implicitly, (15) and (16) are continuous functions of the ideology threshold $\tau$, and therefore so is (21), since (8) through (14) are continuous. Thus, the best response threshold $\tau^{br}_{AB}(\tau)$ can be viewed as a continuous function from the compact interval $[-1, 1]$ of thresholds into itself, and by Brower’s theorem, a fixed point $\tau^* = \tau^{br}_{AB}(\tau^*)$ exists, which characterizes an ideological strategy $\nu_{\tau^*}$ that is its own best response, thus constituting a Bayesian Nash equilibrium.

From (16) it is clear that $\phi(B|z)$ is increasing in $z$, which implies the existence of a state of the world $\hat{z}$ that solves $\phi(A|\hat{z}) = \phi(B|\hat{z}) = \frac{1}{2}$, thereby dividing the electorate equally (in expectation). This state is important because, using (8) through (12), the probability (14)

54This is true of the overconfident voters of Ortoleva and Snowberg (2015), for example.
of a pivotal vote can be rewritten as

\[
\Pr (pv|z) = \sum_{k=0}^{\infty} \left[ \psi (k, k|z) + \frac{1}{2} \psi (k, k+1|z) + \frac{1}{2} \psi (k+1, k|z) \right]
\]

\[
= \sum_{k=0}^{\infty} \psi (k, k|z) \left[ 1 + \frac{1}{2} n \phi (A|z) + \frac{1}{2} n \phi (B|z) \right]
\]

\[
= \sum_{k=0}^{\infty} \psi (k, k|z) \left( 1 + \frac{n}{2k+1} \right)
\]

\[
= \sum_{k=0}^{\infty} \frac{e^{-n}}{k!} n^{2k} k! \left[ \phi (A|z) \phi (B|z) \right]^k \left( 1 + \frac{n}{2k+1} \right),
\]

which is an increasing function of the product \( \phi (A|z) \phi (B|z) = [1 - \phi (B|z)] \phi (B|z) \), which increases in \( \phi (B|z) \) if and only if \( \phi (B|z) < \frac{1}{2} \). In other words, \( \Pr (pv|z) \) is increasing in \( z \) for all \( z < \hat{z} \), and then decreasing in \( z \) for all \( z > \hat{z} \).

It is also clear from (16) that \( \phi (B|z) \) is decreasing in \( \tau \), for any \( z \). Therefore, \( \Pr (pv|z) \) is decreasing in \( \tau \) for \( z < \hat{z} \) and increasing in \( \tau \) for \( z > \hat{z} \). In other words, an increase in \( \tau \) causes an increase in \( \Pr (pv|z) \), in the sense of first-order stochastic dominance, implying that \( E (z|pv) \) and \( E (z|pv, \theta) \) both increase in \( \tau \) (for any \( \theta \in \times \)). The latter of these implies, in turn, that \( \tau_{AB}^{br} (\tau) \) is decreasing in \( \tau \), because \( \theta = \tau_{AB}^{br} (\tau) \) if and only if \( E (z|pv, \theta; \tau) = \bar{x} \), in which case raising the threshold to \( \tau' > \tau \) would imply that \( E (z|pv, \theta; \tau') > \bar{x} \), and therefore that \( \tau_{AB}^{br} (\tau') < \theta = \tau_{AB}^{br} (\tau) \).

Since \( \tau_{AB}^{br} (\tau) \) is decreasing in \( \tau \), it has a unique fixed point \( \tau^* = \tau_{AB}^{br} (\tau^*) \), therefore constituting (as long as \( x_A \neq x_B \)) a unique BNE \( v^* = v_{\star} \), as claimed. From (21) it can be seen that \( \tau_{AB}^{br} (\tau) \) is the same for any \((x_A, x_B)\) pairs with the same midpoint \( \bar{x} \), and is increasing in \( \bar{x} \). This implies that the same ideological strategy \( v_{\star} \) constitutes a BNE in the subgames associated with any such platform pair, and that \( \tau^* (\bar{x}) \) is increasing in \( \bar{x} \), as claimed.

Given the symmetry of ideology, symmetric ideology thresholds imply symmetric vote probabilities (15) and (16): \( \phi (A|z; -\tau) = \phi (B|z; \tau) \). By (8) through (14), this translates into symmetric pivot probabilities (i.e. \( \Pr (pv|z; -\tau) = \Pr (pv|z; \tau) \)) and symmetric expectations (i.e. \( E (z|pv; -\tau) = -E (z|pv; \tau) \) and \( E (z^2|pv; -\tau) = E (z^2|pv; \tau) \)). If \( \tau^{br} (\tau^*; \bar{x}) = \tau^* \), therefore, then from (21) it is clear that

\[
\tau^{br} (-\tau^*; -\bar{x}) = \frac{-\bar{x} + E (z|pv)}{E (z^2|pv; -\bar{x}E (z|pv))} = -\tau^{br} (\tau^*; \bar{x}) = -\tau^*.
\]

In other words, \( \tau^* (-\bar{x}) = -\tau^* (\bar{x}) \).

**Proof of Proposition 2.** If candidates platforms \( x_A \neq x_B \) are distinct then, without loss of generality, assume \( x_A < x_B \). If a citizen’s peers vote according to the subgame strategy
$v \in \mathcal{V}_0$ then, given his own private information $(q, s) \in Q \times S$, the difference $\Delta_{AB}(q, s)$ in expected utility between voting $B$ and voting $A$ is given by (13), as before, and is positive if and only if $\theta(q, s)$ exceeds $\tau_{AB}$, as defined in (21). Similarly, the benefit of voting $B$ instead of abstaining is given by

$$
\Delta_{0B}(q, s) = \int \mathbb{E} [u(x_B, z) - u(x_A, z)] \Pr(piv_B|z) \frac{1}{2} (1 + qs) \, dz
$$

which is positive if and only if $\theta(q, s)$ exceeds

$$
\tau^{br}_{0B} = \frac{\bar{x} - \mathbb{E}(z|piv_B)}{\mathbb{E}(z^2|piv_B) - \bar{x} \mathbb{E}(z|piv_B)}.
$$

and the benefit

$$
\Delta_{A0}(q, s) = \int \mathbb{E} [u(x_B, z) - u(x_A, z)] \Pr(piv_A|z) \frac{1}{2} (1 + qs) \, dz
$$

of abstaining instead of voting $A$ is positive if and only if $\theta(q, s)$ exceeds

$$
\tau^{br}_{A0} = \frac{\bar{x} - \mathbb{E}(z|piv_A)}{\mathbb{E}(z^2|piv_A) - \bar{x} \mathbb{E}(z|piv_A)}.
$$

Thus, the best response to $v$ is an ideological voting strategy $v^{br} = v^{br}_{1}, \tau^{br}_{2}$, with thresholds $\tau^{br}_{1} = \min \{\tau^{br}_{A0}, \tau^{br}_{AB}\}$ and $\tau^{br}_{2} = \max \{\tau^{br}_{0B}, \tau^{br}_{AB}\}$. ■

**Proof of Proposition 2.** According to Lemma 2, the best response to any voting strategy is an ideological strategy, with ideology thresholds $\tau^{br}_{1} \leq \tau^{br}_{2}$. In particular, the best response to an ideological strategy $v_{\tau_1, \tau_2}$ is another ideological strategy $v_{\tau^{br}_1, \tau^{br}_2}$. Together, then, the pair $(\tau^{br}_1, \tau^{br}_2)$ of best-response thresholds can be viewed as a single continuous function from the compact set $\{(\tau_1, \tau_2) : -1 \leq \tau_1 \leq \tau_2 \leq 1\}$ of ideological threshold pairs into itself. A fixed point $(\tau^{*}_1, \tau^{*}_2)$ exists by Brouwer’s theorem, which characterizes an ideological strategy $v_{\tau^{*}_1, \tau^{*}_2}$ that is its own best response.

To see that $\tau^{*}_1 < \tau^{*}_2$ in equilibrium, it suffices to show that an ideological strategy $v_{\tau, \tau}$ with full participation (i.e. $\tau_1 = \tau_2 = \tau$) is not its own best response. Since $v_{\tau, \tau}$ is ideological it is straightforward to show that $\phi(A|z)$ and $\phi(B|z)$ are decreasing and increasing in $z$, respectively, which implies that

$$
E(z|N_A = k, N_B = k + 1, q, s) = \frac{\int_{-1}^{1} z \psi(k, k|z) f(z|q, s) \, dz}{\int_{-1}^{1} \psi(k, k|z) f(z|q, s) \, dz} > \frac{\int_{-1}^{1} z \psi(k, k|z) f(z|q, s) \, dz}{\int_{-1}^{1} \psi(k, k|z) f(z|q, s) \, dz} = E(z|N_A = a, N_B = b, q, s)
$$
Proof. Write the difference in win probabilities for the two candidates as

\[
\Pr(w = B) - \Pr(w = A) = \sum_{a=0}^{\infty} \Pr(N_A = a) [\Pr(N_B > a) - \Pr(N_B < a)]
\]

\[
= \sum_{b=0}^{\infty} \Pr(N_B = b) [\Pr(N_A < b) - \Pr(N_A > b)].
\]

Since the distributions of \( N_A \) and \( N_B \) are increasing in \( \phi(A) \) and \( \phi(B) \), respectively, in the sense of first-order stochastic dominance, the first of these expressions is increasing in \( \phi(B) \) and the second is decreasing in \( \phi(A) \). Since \( \Pr(w = A) + \Pr(w = B) = 1 \), this establishes the first claim.

---

Lemma A1 A candidate’s win probability \( \Pr(w = j) \) increases with her expected vote share \( \phi(j) \) (and decreases with her opponent’s expected vote share \( \phi(-j) \)). If voting is ideological then (1) for any \( z \), \( \phi(A|z) \) and \( \phi(B|z) \) increase and decrease, respectively, in the ideology threshold \( \tau \), (2) \( E(z|w = j) \) increases in \( \tau \), and (3) for any \( \tau \), \( E(z|w = A) < 0 < E(z|w = B) \).

Proof. Write the difference in win probabilities for the two candidates as

\[
\Pr(w = B) - \Pr(w = A) = \sum_{a=0}^{\infty} \Pr(N_A = a) [\Pr(N_B > a) - \Pr(N_B < a)]
\]

\[
= \sum_{b=0}^{\infty} \Pr(N_B = b) [\Pr(N_A < b) - \Pr(N_A > b)].
\]

Since the distributions of \( N_A \) and \( N_B \) are increasing in \( \phi(A) \) and \( \phi(B) \), respectively, in the sense of first-order stochastic dominance, the first of these expressions is increasing in \( \phi(B) \) and the second is decreasing in \( \phi(A) \). Since \( \Pr(w = A) + \Pr(w = B) = 1 \), this establishes the first claim.
If voting is ideological and \( \tau \geq 0 \) then \( \phi (B|z) \) reduces from (16) to

\[
\int_{\tau}^{1} (1 + \theta z) \varphi (\theta) = \Pr (\theta > \tau) [1 + z E(\theta|\theta > \tau)],
\]

which is clearly decreasing in \( \tau \). This implies that \( \phi (A|z) = 1 - \phi (B|z) \) increases in \( \tau \), thus establishing claim (1). Moreover, for positive \( \tau \) and \( z \), the ratio

\[
\frac{\phi (B|z)}{\phi (B|z)} = \frac{1 + E(\theta|\theta > \tau)}{1 - E(\theta|\theta > \tau)}
\]

exceeds 1 and increases in \( \tau \). Therefore, by the first part of the lemma, \( \frac{\Pr (w = B|z)}{\Pr (w = B|z)} \) exceeds 1 and increases in \( \tau \) as well, implying that \( E(z|w = B, |z|) = \frac{\Pr (w = B|z) - \Pr (w = B|z)}{\Pr (w = B|z) + \Pr (w = B|z)} \) is positive and increases in \( \tau \). Integrating over \( |z| \), \( E (z|w = B) \) is positive and increases in \( \tau \), as well. Analogous reasoning shows that \( E (z|w = B) \) is positive and increases in \( \tau \) for negative \( \tau \), as well, and symmetric arguments establish that \( E (z|w = A) \) is negative for any \( \tau \), and increasing in \( \tau \), thus establishing claims (2) and (3). ■

**Lemma A2** If \( x_A < x_B \) then, for every non-ideological strategy \( v \in V \), there exists an ideology threshold \( \tau \in [-1, 1] \) and corresponding ideological strategy \( v_{\tau} \) such that \( E[u(x, z); v_{\tau}] > E[u(x, z); v] \). For every non-ideological strategy \( v \in V_0 \), there exist ideology thresholds \( -1 \leq \tau_1 \leq \tau_2 \leq 1 \) and a corresponding ideological strategy \( v_{\tau_1, \tau_2} \) such that \( E[u(x, z); v_{\tau_1, \tau_2}] > E[u(x, z); v] \).

**Proof.** From (7), the vote share of candidate \( j \in \{A, B\} \) in state \( z \) is given by

\[
\phi (j|z) = \int_{-1}^{1} v^j (\theta) (1 + \theta z) \varphi (\theta) d\theta.
\]

Integrating over states gives

\[
\phi (j) = \int_{-1}^{1} \int_{-1}^{1} v^j (\theta) (1 + \theta z) \varphi (\theta) dz d\theta
= \int_{-1}^{1} v^j (\theta) \varphi (\theta) d\theta.
\]

(26)

Given these preliminary observations, this proof proceeds by construction, in a series of steps. To begin, let \( v \in V_0 \) be given, and define strategies \( v', v'', v''' \in V_0 \) as described below, maintaining the same unconditional vote shares \( \phi (j; v) = \phi (j; v') = \phi (j; v'') = \phi (j; v''') \) for each, which will imply that \( \Pr (w = j; v) = \Pr (w = j; v') = \Pr (w = j; v'') = \Pr (w = j; v''') \) for \( j \in \{A, B\} \), as well. Define the first of these strategies, \( v' \), so that voter participation
patterns are the same as under strategy \( v \) (i.e. \( v'_j(q, s) = v_0(q, s) \) for all \( (q, s) \in Q \times S \)) but, conditional on voting at all, \( v' \) is ideological.\(^{55}\) That is, for some \( \tau_{AB} \in [-1, 1] \),

\[
v'_A(q, s) = \begin{cases} 1 - v_0(q, s) & \text{if } \theta(q, s) \leq \tau_{AB} \\ 0 & \text{otherwise} \end{cases}
\]

\[
v'_B(q, s) = \begin{cases} 1 - v_0(q, s) & \text{if } \theta(q, s) > \tau_{AB} \\ 0 & \text{otherwise} \end{cases}
\]

Without loss of generality, assume that \( \tau_{AB} > 0 \). The next strategy, \( v'' \), is identical to \( v' \) (i.e. \( v''_j(q, s) = v'_j(q, s) \) for all \( j \in \{A, B, 0\} \)) for all ideologies above \( \tau_{AB} \), but for some \( \tau_{A0} \leq \tau_{AB} \),

\[
v''(q, s) = \begin{cases} A & \text{if } \theta(q, s) \leq \tau_{A0} \\ 0 & \text{if } \tau_{A0} \leq \theta(q, s) \leq \tau_{AB} \end{cases}
\]

Finally, define \( v''' \) to be identical to \( v'' \) (i.e. \( v'''_j(q, s) = v''_j(q, s) \) for all \( j \in \{A, B, 0\} \)) for all ideologies below \( \tau_{AB} \), but for some \( \tau_{0B} \geq \tau_{AB} \),

\[
v'''(q, s) = \begin{cases} B & \text{if } \theta(q, s) \geq \tau_{0B} \\ 0 & \text{if } \tau_{AB} \leq \theta(q, s) \leq \tau_{0B} \end{cases}
\]

Constructed this way, \( v''' \) is ideological, with thresholds \( \tau_{A0} \leq \tau_{0B} \).

Defined this way, voting according to \( v' \) instead of \( v \) changes candidate \( B \)'s vote share by the following amount.

\[
\phi(B|z; v') - \phi(B|z; v) = \int_{-1}^{1} [v'_B(\theta) - v_B(\theta)] (1 + \theta z) \varphi(\theta) \, d\theta.
\]

Differentiating this with respect to \( z \) then yields

\[
\frac{d}{dz} [\phi(B|z; v') - \phi(B|z; v)]
= \int_{-1}^{1} [v'_B(\theta) - v_B(\theta)] \theta \varphi(\theta) \, d\theta
= \int_{-1}^{0} [v'_B(\theta) - v_B(\theta)] \theta \varphi(\theta) \, d\theta
+ \int_{0}^{\tau_{AB}} [v'_B(\theta) - v_B(\theta)] \theta \varphi(\theta) \, d\theta
+ \int_{\tau_{AB}}^{1} [v'_B(\theta) - v_B(\theta)] \theta \varphi(\theta) \, d\theta
\]

\[
\geq \int_{0}^{\tau_{AB}} [v'_B(\theta) - v_B(\theta)] \tau_{AB} \varphi(\theta) \, d\theta
+ \int_{\tau_{AB}}^{1} [v'_B(\theta) - v_B(\theta)] \tau_{AB} \varphi(\theta) \, d\theta
= -\tau_{AB} \int_{-1}^{0} [v'_B(\theta) - v_B(\theta)] \varphi(\theta) \, d\theta
> 0,
\]

\(^{55}\)This construction is always possible because vote shares are monotonic in the threshold \( \tau_{AB} \). Similar monotonicity facilitates the constructions below, as well.
where the inequalities both follow because \( v'_B (\theta) \leq v_B (\theta) \) if and only if \( \theta < \tau_{AB} \), and the final equality follows because \( \phi (B; v') = \phi (B; v) \). Since candidate \( B \)'s vote share increases with \( z \), so does her probability \( \Pr (w = j | z) \) of winning. In other words, moving from \( v \) to \( v' \) causes the distribution of \( z \), conditional on candidate \( B \) winning the election, to increase in the sense of first-order stochastic dominance, implying that \( E (z | w = B; v') > E (z | w = B; v) \).

By similar derivations, \( E (z | w = B; v''') > E (z | w = B; v''') > E (z | w = B; v') \) as well.

The expectation formed by candidate \( A \) upon winning the election is related to the expectation formed by candidate \( B \), in the following way.

\[
\Pr (w = A) E (z | w = A) = \int_{-1}^{1} z [1 - \Pr (w = B | z)] f (z) dz
\]

\[
= - \int_{-1}^{1} z \Pr (w = B | z) f (z) dz
\]

\[
= - \Pr (w = B) E (z | w = B).
\]

Accordingly, expected utility can be written as

\[
E [u (x, z)] = \int_{-1}^{1} \sum_{j=A,B} [-x_j^2 + 2x_j z - z^2] \Pr (w = j | z) dz
\]

\[
= \sum_{j=A,B} \Pr (w = j) [-x_j^2 + 2x_j E (z | w = j)] - E (z^2)
\]

\[
= - \sum_{j=A,B} \Pr (w = j) x_j^2 + 2 (x_B - x_A) \Pr (w = B) E (z | w = B) - E (z^2).
\]

Since \( \Pr (w = j) \) is the same for \( v \) and \( v' \), the difference in expected utility for the two strategies is

\[
E [u (x, z); v'] - E [u (x, z); v] = 2 (x_B - x_A) \Pr (w = B) [E (z | w = B; v') - E (z | w = B; v)] > 0.
\]

Similarly, \( E [u (x, z); v'''] > E [u (x, z); v'''] > E [u (x, z); v'] \). This establishes the lemma for a strategy in \( V_0 \). The same result holds for strategy in \( v \in V \), simply by reinterpreting \( v \) as an element of \( V_0 \), which doesn’t happen to include any voter abstention. ■

**Proof of Proposition 3.** Within the class of ideological strategies, there is at least one, say \( v_{r^{* *}} \), that maximizes expected utility. This follows from the extreme value theorem, since expected utility is a continuous function of the threshold \( \tau \), and the set \([-1, 1]\) of thresholds is compact. Since \( v_{r^{* *}} \) is optimal within the class of ideological strategies, it is also globally optimal, because by Lemma A2, every non-ideological strategy in \( V \) is dominated by another strategy that is ideological. \( v_{r^{* *}} \) therefore also constitutes a BNE, by the logic of McLennan (1998): in common-interest games such as this, a symmetric strategy that is socially optimal
is also individually optimal. According to Lemma 1, therefore, it is also unique. This establishes Part (1).

To see Part (2), let \( \hat{\tau} \) denote the median value of \( \theta \), conditional on \( z = \bar{x} \), and consider the (non-equilibrium) ideological strategy \( v_{\hat{\tau}} \). When \( z = \bar{x} \), this strategy is such that \( \phi (A|z; v_{\hat{\tau}}) = \frac{1}{2} = \phi (B|z; v_{\hat{\tau}}) \), implying that \( \Pr (w = A|z; v_{\hat{\tau}}) = \frac{1}{2} = \Pr (w = B|z; v_{\hat{\tau}}) \). For \( z > \bar{x} \), then, \( \phi (A|z; v_{\hat{\tau}}) < \frac{1}{2} < \phi (B|z; v_{\hat{\tau}}) \). As \( n \) grows large, \( \frac{N_B}{N_A + N_B} \) therefore converges in probability to \( \phi (B|z; v_{\hat{\tau}}) > \frac{1}{2} \), so \( \Pr (x_n^* = B) = \Pr \left( \frac{N_B}{N_A + N_B} > \frac{1}{2} \right) \to 1 \). This is good, because \( z > \bar{x} \) implies that \( \arg \max_{x \in \{ x_A, x_B \}} u (x, z) = x_B \). If \( z < \bar{x} \) then \( \arg \max_{x \in \{ x_A, x_B \}} u (x, z) = x_A \), but the inequalities above are reversed, so \( \Pr (x_n^* = A) = \Pr \left( \frac{N_A}{N_A + N_B} > \frac{1}{2} \right) \to 1 \), which is optimal in that case. Together, these arguments have shown that \( x_{\hat{\tau}, n} \to a.s. \arg \max_{x \in \{ x_A, x_B \}} u (x, z) \), where \( x_{\hat{\tau}, n} \) is the policy chosen when citizens vote according to \( v_{\hat{\tau}} \). The desired claim then follows from the observation that \( x_n^* \) is optimal, so \( E[u (x_n^*, z); n] \geq E[u (x_{\hat{\tau}, n}, z); n] \).

**Proof of Proposition 4.** The proof of this proposition is nearly identical to that of Proposition 3, except that no claim is made of uniqueness. The result that equilibrium ideology thresholds \( \tau_1 < \tau_2 \) are distinct follows from Theorem 2.

**Proof of Theorem 1.** It cannot be the case in equilibrium that \( x_A \) is closer to \( \theta_B \) than \( x_B \) is, because in that case, candidate \( B \) could improve her welfare by mimicking \( A \)'s platform. It also cannot be the case in equilibrium that \( x_B \) is more extreme than \( \theta_B \) because then by moderating her position to \( \theta_B \), candidate \( B \) could improve her odds of winning, and also her utility conditional on winning. Symmetrically, \( x_A \) cannot be more extreme than \( \theta_A \). Differentiating (18) yields \( \frac{\partial E[u(x, z)|z = \theta_B]}{\partial x_B} \) equal to

\[
\begin{align*}
= & -2 (x_B - \theta_B) \Pr (w = B|z = \theta_B) + \sum_{j=B,B} u (x_j, \theta_B) \frac{\partial}{\partial x_B} \Pr (w = j|z = \theta_B) \\
= & (\theta_B - x_B) \Pr (w = B|z = \theta_B) + [u (x_B, \theta_B) - u (x_A, \theta_B)] \frac{\partial}{\partial x_B} \Pr (w = B|z = \theta_B)
\end{align*}
\]

The result that \( \theta_A \leq x_A^* \leq x_B^* \leq \theta_B \) implies that the first term in this sum is weakly positive while the second term is weakly negative. If \( \theta_B = x_B^* \), however, then the first term is zero, so the sum is strictly negative; if \( x_A^* \leq x_B^* \) then the second term is zero, so the sum is strictly positive. Thus, \( \theta_A < x_A^* < x_B^* < \theta_B \) in equilibrium.

If \( \theta_A = -\theta_B \) and \( x_A = -x_B \) then the two candidates’ incentives are symmetric, implying that their best response strategies satisfy \( x_{BR}^A = -x_{BR}^B \). Thus, a best response to the platform pair \((-x_B, x_B)\) is another platform pair \((-x_{BR}^B, x_{BR}^B)\). Since utility is continuous over the compact set \([0, 1]\) of \( B \) platforms (each representing a platform pair) and since, as noted above, \( B \)'s utility is increasing if \( x_A = x_B \) but decreasing if \( x_A = \theta_A \) and \( x_B = \theta_B \), an equilibrium with \( 0 < x_B^* < \theta_B \) and \( x_A^* = -x_B^* \) exists by the intermediate value theorem.
Uniqueness follows because \( x_A = -x \) and \( x_B = x \) implies that \( \bar{x} = 0 \) for any \( x \), so that neither \( \Pr(w = A|z) \) nor \( \frac{\partial}{\partial x_A} \Pr(w = A|z = \theta_A) \) changes with \( x \). Substituting into (27) and differentiating with respect to \( x \) therefore yields
\[
-x \Pr(w = B|z = \theta_B) + [\frac{2(x - \theta_B) + 2(x + \theta_B)}{\partial x_B}] \Pr(w = B|z = \theta_B)
\]
\[
= -x \Pr(w = B|z = \theta_B) + 4\theta_B \frac{\partial}{\partial x_B} \Pr(w = B|z = \theta_B),
\]
which is strictly negative. Thus, there exists only one pair \((-x, x)\) such that \( \frac{dE[u(x,z)]_{z=\theta_B}}{dx_B} = 0 \).

For any \( n \), candidate \( B \) could deviate to \( x_B = \theta_B \) and receive utility
\[
u(x_A^{*}, \theta_B) \Pr(w = A|z = \theta_B; x_A = x_A^{*}, x_B = \theta_B) + u(\theta_B, \theta_B) \Pr(w = B|z = \theta_B; x_A = x_A^{*}, x_B = \theta_B).
\]
Since \( \lim_{n \to \infty} \Pr(w = B|z = \theta_B; x_A = x_A^{*}, x_B = \theta_B) = 1 \), a sequence of such deviations would yield utility \( u(\theta_B, \theta_B) \) in the limit. But equilibrium utility is weakly higher for every \( n \), implying that equilibrium utility approaches \( u(\theta_B, \theta_B) \) as well. This is possible only if \( \lim_{n \to \infty} x_A^{*} = \theta_B \) (and, symmetrically, \( \lim_{n \to \infty} x_A^{*} = \theta_A \)).

**Proof of Lemma 3.** Proposition 1 states that, for every pair \( x_A \neq x_B \) of distinct candidate platforms, there is a unique Bayesian Nash equilibrium in the voting subgame. As Proposition 3 states below, the ideology threshold strategy adopted in equilibrium is also the voting strategy that maximizes expected utility. Thus, \( \frac{dE[u(x,z)]_{z=\theta_B}}{dx} = 0 \) in that case. The same equality holds for platform pairs \( x_A = x_B \), since utility doesn’t depend on the voting strategy in that case. Either way, then, differentiating \( EU_A = \mathbb{E} \sum_{j=A,B} u(x_j, z) \Pr(w = j|z) f(z) dz \) with respect to \( x_A \) yields (19) for any \(-1 \leq x_A \leq x_B \).

The derivative (19) is positive if \( x_A = -1 \), because \( E(z|w = A) > -1 \) for any voting strategy, implying that the corner solution \( x_A = -1 \) is never optimal for candidate \( A \). Similarly, \( x_B = 1 \) is never optimal for candidate \( B \). The opposite corner solution would require \( x_A = x_B \). To see that this cannot occur in equilibrium, suppose that \( x_A = x_B \geq 0 \). In that case, by Lemma A1, ideological voting would generate expectations \( E(z|w = A) < 0 \leq x_A \), implying from (19) that candidate \( A \) should prefer to move to the left. Voting need not be ideological when the platforms coincide, but any other voting strategy produces the same utility (since the policy outcome is then invariant to voting behavior), so if citizens follow some other voting strategy candidate \( A \) still has an incentive to move left. Thus, equilibrium requires an interior solution, in which (19) equals zero, or \( x_A = E(z|w = A) \).

Symmetrically, equilibrium also requires that \( x_B = E(z|w = B) \).

**Proof of Theorem 2.** Proposition 1 characterizes equilibrium voting as being ideological. Given this voting strategy, Lemma 3 states that \( x_A^{*} = E(z|w = A) \) and \( x_B^{*} = E(z|w = B) \).
and Lemma A1 states that $E (z|w = A) < 0 < E (z|w = B)$, thus establishing the first result. For any pair of symmetric platforms $x_A = -x_B$ the midpoint $\bar{x} = 0$ lies exactly at the center of the policy interval, so by Lemma 1, voters’ equilibrium response is characterized by $\tau^* (x_A, x_B) = 0$. By the symmetry of the model, this implies that candidates form symmetric expectations $E (z|w = A) = -E (z|w = B)$, and therefore symmetric platforms $x_A^* = -x_B^*$. Together with the equilibrium voting strategy $\sigma^*$, these constitute a PBE, and by Lemma 3, this is the only pair of symmetric platforms that can be sustained when $\tau = 0$.

The limit result follows because with symmetric platforms for all $n$, $\tau^* = 0$ for all $n$. The expression (7) therefore reduces to $\phi (A|z) = \Pr (s < 0|z)$ and $\phi (B|z) = \Pr (s > 0|z)$. If $z < 0$, therefore, then $\phi (A|z) > \frac{1}{2} > \phi (B|z)$, implying that $\lim_{n \to \infty} \Pr (w = A|z) = 1$. Similarly, if $z > 0$ then $\lim_{n \to \infty} \Pr (w = A|z) = 0$. Thus, $f (z|w = A)$ converges to $f (z|z < 0)$, and $E (z|w = A)$ converges to $E (z|z < 0)$. Similarly, $E (z|w = B)$ converges to $E (z|z > 0)$.

**Proof of Theorem 3.** If $(x_A^*, x_B^*, \sigma^*) \in \mathcal{X}^2 \times \Sigma$ is a PBE then $x_A^* = x_B^*$ only if $x_A^* = x_B^* = 0$, because when platforms coincide, candidates do not care on policy grounds who wins the election, but each wishes to be marginally closer to the center, so that her probability of winning exceeds $\frac{1}{2}$. For a candidate with unbiased beliefs, differentiating (5) with respect to $x_A$ yields

$$
\frac{\partial E U_A}{\partial x_A} = 2 \Pr (w = A) [E (z|w = A) - x_A] + \beta \frac{\partial}{\partial x_A} \Pr (w = A),
$$

where the first term on the right-hand side is given by (19). For a candidate with biased beliefs, this expression is unchanged, except that candidate $A$ is certain that $z = \theta_A$, so $E (z|w = A) = \theta_A$ in that case.

By Lemma A1, the second term in (28) is positive, so no matter how negative the first term is, the sum is positive for large enough $\beta$. This establishes the second claim. The first claim follows because an equilibrium with distinct platforms requires that $\frac{\partial E U_A}{\partial x_A} = 0$, and therefore that $x_A > E (z|w = A)$, so that the first term in (28) is negative. Symmetrically, $\frac{\partial E U_B}{\partial x_B} = 0$ only if $x_B < E (z|w = B)$.

The existence of a PBE with symmetric platforms follows just as above, because symmetric platforms prompt sincere voting (by Lemma 1), and the symmetry of the model is then such that best-response platforms $x_A^{br} = -x_B^{br}$ are symmetric, so a fixed point pair $(-x_B^{br}, x_B^{br})$ exists by Brouwer’s theorem. Uniqueness follows because any symmetric platform pair $x_A = -x_B$ produces the same voting behavior, characterized by $\tau^* = 0$, and therefore the same values for $\Pr (w = A)$, $E (z|w = A)$, and $\frac{\partial}{\partial x_A} \Pr (w = A) = \frac{\partial \Pr (w = A)}{\partial \tau} \frac{\partial \tau^*(\theta)}{\partial \theta} \frac{\partial \theta}{\partial x_A}$. Thus, if $\frac{\partial E U_A}{\partial x_A} = 0$ for one symmetric platform pair then $\frac{\partial E U_A}{\partial x_A} > 0$ for any more extreme pair and $\frac{\partial E U_A}{\partial x_A} < 0$ for any more moderate pair.

(28) is increasing in $\beta$, implying that if $\frac{\partial E U_A}{\partial x_A} = 0$ for one value of $\beta$ then $\frac{\partial E U_A}{\partial x_A} > 0$ for
any $\beta' > \beta$, and and equilibrium with the higher office motivation $\beta'$ requires a higher value of $x_A$. Symmetrically, it requires a lower value of $x_B$. In other words, $|x_A|$ and $|x_B|$ both decline, as claimed. If $\beta$ is sufficiently large, $\frac{\partial \text{EU}_A}{\partial x_A} > 0$ and $\frac{\partial \text{EU}_B}{\partial x_B} < 0$ for any symmetric platform pair, implying that the unique equilibrium is characterized by $x_A^* = x_B^* = 0$.

**Proof of Proposition 5.** By Proposition 3, the ideological strategy defined by $\sigma^*_n (x_A, x_B) = v^*_n(x)$ constitutes a uniquely optimal response to any platform pair $(x_A, x_B) \in X^2$. A strategy vector $(x_{A,n}^*, x_{B,n}^*, v^*_n) \in X^2 \times \pm$ is therefore socially optimal if the platform pair $(x_{A,n}^*, x_{B,n}^*)$ is an optimal response to $\sigma^*_n$. Such a platform pair exists by the extreme value theorem, since (given $\sigma^*_n$) expected utility is a continuous function over the compact set $X^2$ of platform pairs. Part (1) of the proposition then follows from the logic of McLennan (1998): a candidate platform that is socially optimal is also individually optimal for both candidates, and together with $\sigma^*$, thus constitutes a PBE.

Part (2) follows because $x_A^*$ and $x_B^*$ converge to $-1$ and $1$, respectively, by Theorem 2, and $\Pr (w = A|z = -1)$ and $\Pr (w = B|z = 1)$ both converge to 1, by Proposition 3. These results were for one particular equilibrium strategy vector, but the result must then hold for the socially optimal strategy vector, as well, since it provides weakly higher welfare.

**References**


