Security Design in Non-Exclusive Markets with Asymmetric Information

Vladimir Asriyan and Victoria Vanasco*

April 23, 2022

Abstract

We study the problem of a seller (e.g., a bank) who is privately informed about the quality of her asset and wants to exploit gains from trade with uninformed buyers (e.g., investors) by issuing securities backed by her asset cash flows. In our setting, buyers post menus of contracts to screen the seller, but the seller cannot commit to trade with only one buyer, i.e., markets are *non-exclusive*. We show that non-exclusive markets behave very differently from exclusive ones: (i) separating contracts are never part of equilibrium; (ii) mispricing of claims faced by the seller is always greater than in exclusive markets; (iii) there is always a semi-pooling equilibrium where all sellers issue the same debt contract priced at *average*-valuation, and sellers of *low*-quality assets issue remaining cash flows at *low*-valuation; (iv) market liquidity can be higher or lower than in exclusive markets, but (v) the average quality of originated assets is always lower. Our model's predictions are consistent with empirical evidence on issuance and pricing of mortgage-backed securities, and we use the theory to evaluate recent reforms aimed at enhancing transparency and exclusivity in markets.

JEL Codes: G14, G18, D47, D82, D86.

Keywords: Adverse Selection; Security Design; Non-Exclusivity; Tranching; Liquidity; Securitization; Transparency; Complexity; Market Design; Regulation.

*CREi, UPF, BSE and CEPR. We thank Andrea Attar, Ulf Axelson, Bruno Biais, Peter DeMarzo, William Fuchs, Piero Gottardi, Denis Gromb, Qi Li, Alberto Martin, Thomas Noe, Giorgia Piacentino, Jason Roderick Donaldson, and seminar/conference participants at University of Zurich, Philadelphia Fed, Cambridge Corporate Finance Workshop 2020, Amsterdam Business School, CREI-UPF, SED Meetings in Mexico City, ESSFM in Gerzensee, Finance Theory Webinar and CEPR Advanced Forum for Financial Economics (CAFFE) for helpful feedback and suggestions. Asriyan and Vanasco acknowledge financial support from the Spanish Ministry of Economy and Competitiveness, through the Severo Ochoa Programme for Centres of Excellence in R&D (CEX2019-000915-S) and from the Generalitat de Catalunya, through CERCA and SGR Programme (2017-SGR-1393). Asriyan acknowledges financial support from the Spanish State Research Agency and the European Social Fund (RYC2019-027607-I/AEI/10.13039/501100011033). Vanasco acknowledges financial support from the European Research Council under the European Union's Horizon 2020 research and innovation program (INFOMAK - 948432). Joon Sup Park, Gerard Martín Escofet and Ilja Kantorovitch provided excellent research assistance.

1 Introduction

The question of how markets function in the presence of information asymmetries has been central in economics and finance since the seminal work of Akerlof (1970), who showed that information asymmetries between sellers and buyers can give rise to equilibrium multiplicity and market failures. In the study of asset markets, a large literature followed to explore the role of security design –the ability of sellers to optimally design and issue securities backed by asset cash flows– in ameliorating such information frictions. From it, we learned that by retaining exposure to an asset's cash flows a seller may be able to credibly convey to buyers information about her asset quality (e.g., Leland and Pyle (1977); DeMarzo and Duffie (1999)); and that standard debt emerges as the optimal security design since it minimizes the distortions due to adverse selection (e.g., Gorton and Pennacchi (1990); Nachman and Noe (1994); DeMarzo and Duffie (1999); Biais and Mariotti (2005); DeMarzo (2005); Daley et al. (2020b)). This literature, however, has mostly focused on exclusive contracting, i.e., environments where the buyer of claims can ensure that the seller does not engage in trade with other buyers. In many settings of interest, however, exclusivity may be difficult to enforce.

Exclusivity effectively requires that the seller be able to commit to trade with only one buyer, even if gains from trade arise with other buyers; or, alternatively, that the buyers are able to observe and contract upon the entire set of the seller's trades. In the context of modern financial markets, however, these requirements are unlikely to be fulfilled: there is little information about agents' trades, and the complexity of certain financial products makes it difficult for potential buyers to understand a seller's overall asset positions and resulting risk-exposures. This has been of particular concern to policymakers in the US and Europe, who are in the process of implementing policies aimed at enhancing exclusivity in financial markets.

Motivated by these observations, we revisit the classic problem of a seller who is privately informed about the quality of her asset and wants to exploit gains from trade with uninformed buyers by issuing securities backed by her asset cash flows. We consider a screening game, where buyers post menus of contracts (securities and prices) to be accepted by the seller, but where the seller cannot commit to trade with only one buyer, i.e., markets are *non-exclusive*. We use our framework to study the implications of non-exclusivity for equilibrium allocations, as characterized by issued securities and their prices, which we show are novel and consistent with recent empirical evidence from markets for mortgage-backed securities. We then investigate the theory's normative implications and relate our findings to proposed policies and reforms in the aftermath of the global financial crisis.

Our setup is as follows. There is a risk-neutral seller endowed with an asset that pays random cash flow, X, in the future; and, there is a large number of competitive, risk-neutral, deep-pocket buyers. Gains from trade arise because the seller is more impatient than the buyers. The seller, however, is privately informed about the quality of her asset, which could be *high*- or *low*-quality, and where higher cash flows are more likely to be obtained from a *high*-quality asset. Buyers compete by posting menus of bilateral contracts, where each contract is a security-price pair (F, p), where F maps cash flow X to a payment $F(X)$ to be received by the buyer in the future. The seller can accept contracts from multiple buyers but is subject to a capacity constraint, i.e., all issued securities must be fully backed by asset cash flows. To ensure equilibrium existence, we allow each buyer to withdraw his menu at a small cost after observing the initial menu offers available in the market.[1] Our equilibrium notion is perfect Bayesian.

Our setting features two frictions: the seller is privately informed about her asset quality and the securities market is non-exclusive. As a preliminary step, we consider two benchmark settings in which we shut down each of these frictions in turn. First, we study the full information benchmark in which the asset quality is public information but markets are non-exclusive. Here, we show that the first-best allocations are attained, as all asset cash flows are transferred from the seller to the buyers and they are priced at their full information expected value. By implication, the interaction of asymmetric information with non-exclusivity is crucial for the main findings of this paper.

Second, we study an *exclusive* markets benchmark in which the seller is restricted to trade with at most one buyer.[2] We show that an equilibrium exists, is unique, and can be separating or feature some cross-subsidization depending on the buyers' prior belief that the seller has a *high*-quality asset, i.e., that she is a *high*-type. When the belief is below a threshold, the equilibrium is separating: the *high*-type seller issues a debt security priced at *high*-valuation and retains the remaining cash flows, whereas the *low*-type seller issues a full claim to her asset priced at *low*-valuation. These separating allocations resemble closely those often obtained in the literature on security design with asymmetric information (e.g., DeMarzo (2005); Daley et al. (2020b)). In contrast, when the belief is above the threshold, the *high*-type seller continues to issue a debt security but with higher face value and priced below *high*-valuation, whereas the *low*-type seller continues to issue a full claim to her asset but priced above *low*-valuation.[3]

We then turn to our main setup and show that, in the presence of asymmetric information, non-exclusive markets behave very differently from exclusive ones: (i) separating contracts are never part of equilibrium; (ii) the mispricing of claims faced by the seller is larger than in

[1] This approach is in the spirit of the seminal papers by Wilson (1977) and Miyazaki (1977), and the more recent work by Netzer and Scheuer (2014) and Mimra and Wambach (2019).

[2] This restriction effectively captures the seller's ability to commit not to trade with other buyers.

[3] Buyers' ability to withdraw loss-making menus allows us to support this cross-subsidizing equilibrium, as "cream-skimming" deviations are rendered unprofitable.

exclusive markets; (iii) there is always a semi-pooling equilibrium in the sense that some, but not all, traded contracts are accepted by both seller types; (iv) market liquidity (i.e., realized gains from trade) can be higher or lower than in exclusive markets, but (v) the average quality of originated assets (which we endogenize in an extension) is always lower.

In non-exclusive markets, there is *always* some level of cross-subsidization from the *high*- to the *low*-type seller. Furthermore, such cross-subsidization, and the resulting mispricing of claims faced by the seller, is always larger than in exclusive markets. One might intuitively expect that, as cash flow retention is more costly for the *low*- than for the *high*-type, it should be possible for buyers to separate the *high*-type seller by offering her a contract that requires enough cash flow retention for it to be unattractive for the *low*-type. When markets are non-exclusive, however, retention of cash flows cannot be enforced: there is always a profitable deviation for a buyer to offer to buy the retention implied by the contract of the *high*-type seller, allowing the *low*-type to accept the contract meant for the *high*-type without having to retain the remaining cash flows.

Moreover, there is always an equilibrium where both seller types issue the same debt security (senior tranche). Standard debt emerges as the optimal security design for the *high*-type seller, as it minimizes the mispricing she faces due to adverse selection. Moreover, while the *high*-type optimally chooses to retain (i.e., not sell) her remaining cash flows (junior tranche), the *low*-type sells them to a distinct buyer. We refer to this as the star-equilibrium. We show that it is unique either when the buyers' prior belief is high enough or under an additional refinement that requires securities to be priced competitively off-equilibrium path. In the star-equilibrium, the senior tranche is *mispriced*, in the sense that its price reflects the average rather than the true asset quality; whereas the junior tranche is priced at its true, *low*-valuation.

The star-equilibrium is supported by the presence of *latent* contracts, i.e., contracts that are posted in some menus but not traded in equilibrium, which price all feasible securities at *low*-valuation. The role of such contracts is to allow the *low*-type seller to exploit all gains from trade with buyers, on and off equilibrium path. Intuitively, consider a "cream-skimming" deviation that attracts the *high*-type from her equilibrium contract. Then, the menu containing the senior tranche originally issued by both seller types must be withdrawn, as it would now be loss-making. But then, the *low*-type would find it optimal to accept the deviating contract together with a collection of latent contracts from other buyers, rendering the deviation unprofitable.[4]

Lastly, we provide conditions under which other equilibria arise, and we show that their allocations look a lot like the star-equilibrium, but with a twist. In particular, in such equilibria, the *high*-type seller issues more cash flows than in the star-equilibrium, but the entire surplus

[4]The role of latent contracts in supporting equilibria in non-exclusive markets is analyzed in Arnott and Stiglitz (1991) and Attar et al. (2011) in environments with moral hazard and adverse selection, respectively.

from this issuance accrues to the *low*-type seller, who thus gets a payoff above that in the star-equilibrium. Thus, besides ruling out perfectly separating allocations, non-exclusive competition can rationalize the existence of markets with different degrees of cross-subsidization.

Our theory's novel predictions are consistent with evidence from markets where exclusivity is difficult to enforce. First, we provide a new rationale for the practice of tranching underlying cash flows that are sold separately in markets. Indeed, within the context of markets for commercial mortgage-backed securities (CMBS), where tranching is common practice, Ashcraft, Gooriah, and Kermani (2019) argue that complex products like collateralized debt obligations (CDOs) enabled informed parties in the securitization pipeline to reduce their cash flow retention in a way not observable to other market participants, suggesting that exclusivity is hard to enforce. Second, and in sharp contrast to conventional models, our theory predicts that the amount of cash flows retained by a seller *should not* predict differential pricing in the market for her senior tranches, but that it *should* predict differential quality of these tranches. This is consistent with findings in Ashcraft et al. (2019) that, in the CMBS market, initially retained cash flows sold into CDOs in the twelve months following a transaction are not correlated with the prices of the more senior tranches, though they do predict a higher probability of default of these tranches.

After the 2008-09 financial crisis, a number of reforms were discussed in the US, which would either directly or indirectly enhance exclusivity in contracting. For instance, the Dodd-Frank Act explicitly prohibits the sellers of asset-backed securities to engage in trades that have any material conflicts of interest with the investors of trades completed within the previous year.[5] A natural obstacle to the enforcement of such rules is the complexity of balance sheets of financial institutions and the opacity of markets in which they trade. To address this, a number of complementary rules were implemented, primarily consisting of more stringent information disclosure requirements combined with the relocation of trading of certain securities from opaque over-the-counter markets to more transparent platforms. Our framework provides a natural laboratory within which one can evaluate the effects of such interventions.

First, we show that when the distribution of asset qualities in the market is exogenous (as in our baseline setting) non-exclusive markets increase ex-ante welfare vis-à-vis exclusive markets if and only if they generate higher market liquidity (i.e., more trade), which only occurs when the average asset quality is sufficiently high. Thus, our model suggests that to the extent that

[5]Statement at Open Meeting: Asset-Backed Securities Disclosure and Registration, by Commissioner Kara M. Stein on Aug. 27, 2014 states that "Section 621 prohibits an underwriter, placement agent, initial purchaser, sponsor, or any affiliate or subsidiary of any such entity, of an asset-backed security from engaging in any transaction that would involve or result in any material conflict of interest with respect to any investor in a transaction arising out of such activity for a period of one year after the date of the first closing of the sale of the asset-backed security."

market complexity/opacity inhibits exclusive contracting, its effects on liquidity and efficiency will depend on the average quality of assets available in the market.

Second, in some applications (e.g., loan origination), both the liquidity of markets and the manner by which claims are priced may impact efficiency by distorting investment incentives. To address this, we consider an extension where we allow the seller (who is now also the asset originator) to exert costly, unobservable effort to increase the likelihood of having a *high*-quality asset. Here, we uncover a robust result: the average quality of originated assets is *always* lower with non-exclusive markets than with exclusive ones, even when the former implement higher cash flow retention. This is due to the fact that in non-exclusive markets the seller always faces a greater mispricing of claims than in exclusive markets. Taking these results together, we conclude that non-exclusive markets increase ex-ante welfare vis-à-vis exclusive markets whenever the potential (though not guaranteed) gains from increased market liquidity more than compensate for the (guaranteed) fall in asset quality. Thus, our results suggest that complexity/opacity is desirable only when efficiency gains are mostly driven by reallocation of assets in markets *and* originators need not be too incentivized to produce *high*-quality assets.

We contribute to the literature on security design with asymmetric information (e.g., Nachman and Noe (1994); DeMarzo (2005); Axelson (2007); Daley et al. (2020b)) by studying the implications of non-exclusive competition. We conceptualize non-exclusivity following the literature on non-exclusive market competition by considering a screening game where buyers compete by posting menus of non-exclusive contracts. The relation between security design and non-exclusive competition has also been explored by Biais and Mariotti (2005) and Li (2019), albeit in different settings. In Biais and Mariotti (2005), securities are designed ex-ante (before private information is realized), and buyers compete ex-post by posting non-exclusive menus of quantities (rather than securities) and prices. Instead, in Li (2019), though securities are designed ex-interim (after private information is realized), they are traded in a segmented search market where exclusivity is enforced within each security-market.

We also contribute to the literature on non-exclusive competition in asset markets (e.g., Bisin and Gottardi (1999); Attar et al. (2011, 2014); Kurlat (2016)) by generalizing the contract space –from quantities of an asset to securities backed by asset cash flows– and studying the implications of security design. Attar et al. (2011) consider a setting similar to ours, but where buyers compete by posting menus of quantities and prices: they find that the equilibrium allocations of their game resemble those of Akerlof (1970), i.e., a given seller type either trades the entire asset at a pooling price or does not trade at all. Whereas our finding that non-exclusivity makes cross-subsidization a necessary feature of equilibrium is also present in Attar et al. (2011), in contrast to them we do not obtain Akerlof-like outcomes. The agents' ability to optimally design securities results in semi-pooling allocations, in which all seller types trade some of their cash flows.

In this sense, our star-equilibrium allocations resemble those studied in the literature on non-exclusive competition in insurance markets (Pauly, 1974; Jaynes, 1978; Hellwig, 1988; Glosten, 1994; Bisin and Gottardi, 2003; Dubey and Geanakoplos, 2019; Stiglitz et al., 2020).[6] A common finding in the literature is that, due to concavity of preferences, low- and high-risk types buy an interior amount of insurance at a pooling price, while the high-risk type also buys supplementary insurance at a higher price. Although the agents' preferences are linear in our setting (as in much of the literature on security design), the H-type's payoff inherits concavity in the face value of debt due to optimal security design. Our findings thus provide a bridge between the seemingly disparate results in the literature on non-exclusive competition in asset vs. insurance markets, and they yield novel predictions regarding issuance and pricing of asset-backed securities in line with empirical evidence.

Finally, on the normative front, we contribute to a growing literature that studies the costs and benefits of transparency in financial markets plagued with information asymmetries (e.g., Dang et al. (2010); Chemla and Hennessy (2014); Fuchs et al. (2016); Dang et al. (2017); Asriyan et al. (2017, 2021); Daley et al. (2020a,b)). This literature primarily focuses on how transparency affects the agents' ability to obtain additional information about the seller's asset quality (e.g., by observing signals). In contrast, we focus on the implications of transparency through its effects on exclusivity; that is, through the ability of an agent to observe and contract upon the set of trades that his counterparty enters into with others.

Our paper is organized as follows. In Section 2, we present the setup of our model, and we establish two useful benchmarks against which we compare our results. In Section 3, we characterize the equilibrium of our model. We consider the model's normative implications, which we relate to policy discussions, in Section 4; and its positive predictions, which we relate to empirical facts, in Section 5. All proof are relegated to the Appendix.

2 The Model

There are two dates, indexed by $t \in \{1, 2\}$. There is an asset seller (e.g. bank) and a large number N of "deep pocket" buyers (e.g., investors). The seller's preferences are:

$$U^S = c_1^S + \delta \cdot c_2^S, \tag{1}$$

[6] See, also, Riley (1979); Santos and Scheinkman (2001b); Ales and Maziero (2009, 2016).

where $\delta \in (0,1)$ and c_t^S is the cash flow she receives in period t. A buyer's preferences are:

$$U^B = c_1^B + c_2^B, \tag{2}$$

where c_t^B is the cash flow he receives in period t. Thus, gains from trade between the seller and the buyers arise due to heterogeneity in discount factors.[7]

The seller is endowed with an asset that delivers a random cash flow X in period $t = 2$. The asset can be of *high*- or *low*-quality, denoted by $\theta \in \{H, L\}$, and its cash flow is distributed according to cdf G_θ.[8] We assume that G_θ has an associated pdf g_θ with full support on the interval $\left[0, \bar{X}\right]$ for some $\bar{X} > 0$. The pdfs are in turn related by the monotone likelihood ratio property (MLRP); that is, $\frac{g_H(x)}{g_L(x)}$ is increasing in x. In the spirit of Akerlof (1970), asymmetric information arises because the seller knows the quality θ of her asset, whereas the buyers are uninformed and have a prior belief $\mu_0 = \mathbb{P}(\theta = H) \in (0,1)$.

To realize gains from trade, the seller raises funds at $t = 1$ by issuing securities fully backed by her asset cash flows to buyers. Formally, a security is a function $F : \left[0, \bar{X}\right] \to \mathbb{R}$ and its payoff is denoted by $F(x)$ when the realized cash flow is $X = x$. Let $\mathcal{F}$ be the collection of securities issued by the seller, then we say that this collection is *feasible* if:

- (Capacity Constraint - CC) $\sum_{F \in \mathcal{F}} F(x) \leq x$ and $F(x) \geq 0$ for all $F \in \mathcal{F}$.
- (Weak Monotonicity - WM) $F(x)$ and $x - \sum_{F \in \mathcal{F}} F(x)$ are weakly increasing in x.

We denote the set of all feasible (collections of) securities by Φ. Conditions (CC) and (WM) are generalizations of the limited liability and the monotonicity constraints often used in the asset-backed security design literature (e.g., Nachman and Noe (1994); DeMarzo and Duffie (1999); Biais and Mariotti (2005); Daley et al. (2020b)) to a setting with multiple securities, and their role is to ensure tractability. We interpret these feasibility conditions as technological. For the interested reader, we provide a microfoundation that rationalizes these feasibility conditions in a non-exclusive market setting in Appendix B.1, where we approach security design as one of optimal bundling of basis assets.

The securities market is non-exclusive in the sense that trade between the seller and the buyers is bilateral, and a buyer cannot exclude the seller from trading with other buyers. Formally, we study the following three-stage screening game:

[7] The assumption that the seller is more impatient than the buyers is a common modeling device to rationalize gains from trade (e.g., DeMarzo and Duffie (1999); Biais and Mariotti (2005); DeMarzo (2005); Daley et al. (2020a,b)). For example, a bank that is financially constrained and has new profitable investment opportunities may benefit from selling a fraction of its loans to finance these new opportunities. Alternatively, asset securitization may allow loan originators to share-risks with market investors.

[8] We consider a setting with more types in Appendix B.3.

- Stage 1: buyers simultaneously post menus of contracts, where a menu $\mathcal{M}$ posted by a buyer is a (potentially infinite) set of contracts or security-price pairs (F, p).
- Stage 2: buyers observe all posted menus and simultaneously decide whether to remain in the market or to become inactive by withdrawing their menus at infinitesimal cost $\kappa > 0$.
- Stage 3: seller observes all active menus and accepts at most one contract from each menu, subject to the accepted collection of securities being feasible.[9]

Contracts are executed at the end of the game. Namely, if the seller has accepted contract (F, p) from a buyer at $t = 1$, then the buyer makes a transfer p to the seller at $t = 1$ and in exchange receives $F(x)$ at $t = 2$ when the realized cash flow is $X = x$.

Remarks on the modeling approach. We have made several assumptions to ensure model tractability, which are inspired by the literature on security design and on non-exclusive market competition. First, the assumptions of MLRP of cash flows combined with weak monotonicity of securities (WM) allows us to rank securities based on the seller's type: that is, for any non-trivial feasible security, $E_H[F] > E_L[F]$, which is useful when constructing equilibria and analyzing potential buyer deviations. Second, the (CC) condition, by restricting the seller to issue an asset-backed security, helps us isolate the mechanism arising due to asymmetric information from other mechanisms that may also be at play in non-exclusive settings, e.g., dilution.[10] Finally, our modeling approach of allowing buyers to withdraw menus is in the spirit of Wilson (1977) and Miyazaki (1977), and it allows us to ensure equilibrium existence in both exclusive and non-exclusive market settings (see Section 3 for a detailed discussion).[11] Note that the withdrawal stage only requires that the buyers, just like the seller, observe the set of contracts available in the market.

Payoffs. At Stage 3, the θ-type seller decides which contracts to accept from the active menus. Let $\mathcal{C}$ denote the set of contracts chosen by the seller, where we set $\mathcal{C} = \{(0, 0)\}$ if the seller chooses not to trade. Then, her payoff at Stage 3 is:

$$u_\theta \equiv \sum_{(F,p)\in\mathcal{C}} p + \delta \cdot \mathbb{E}_\theta \left[X - F(X)\right], \tag{3}$$

[9]That the seller accepts at most one contract from each menu is without loss of generality.

[10]A large literature has focused on the problem of dilution in financial markets, which arises when a firm is able to dilute existing claims by issuing new claims on the same set of cash flows (e.g., Parlour and Rajan (2001); Santos and Scheinkman (2001a,b); DeMarzo and He (2016); Admati, DeMarzo, Hellwig, and Pfleiderer (2018); Donaldson, Gromb, and Piacentino (2019)). Instead, we focus on asset sales by supposing that the transfer of asset cash flows in spot markets can be verified.

[11]That there is a small cost of menu withdrawal acts as a refinement of equilibrium, as in Netzer and Scheuer (2014). Such withdrawal cost can be interpreted directly as communication/administrative cost or indirectly as the loss of reputation associated with withdrawal of contracts posted in a marketplace.

where $\mathbb{E}_\theta[\cdot]$ is the expectations operator conditional on the seller's type θ.

At Stage 2, after observing all menus $\cup_j \mathcal{M}^j$ posted at Stage 1, buyer i decides whether to withdraw his menu $\mathcal{M}^i$. Let $\mu_2^i(F,p)$ denote buyer i's belief at Stage 2 that the seller is an H-type if she were to accept contract $(F,p) \in \mathcal{M}^i$. Buyer i's expected payoff at Stage 2 is:

$$(1-w^i) \cdot \sum_{(F,p)\in\mathcal{M}^i} \mathbb{P}\left(\text{seller accepts } (F,p) | \cup_j \mathcal{M}^j\right) \cdot \Big(-p + \mathbb{E}_{\mu_2^i}[F(X)]\Big) - w^i \cdot \kappa, \tag{4}$$

where $w^i \in \{0,1\}$, $w^i = 1$ if and only if buyer i withdraws, and $\mathbb{E}_\mu[F(X)] \equiv \mu \cdot \mathbb{E}_H[F(X)] + (1-\mu) \cdot \mathbb{E}_L[F(X)]$.

At Stage 1, buyer i decides which contracts to post in his menu. By inspection of the payoff in (4), it is immediate that buyer i will never post a menu that he expects to withdraw at Stage 2, since he can always ensure a payoff of zero by posting only the trivial contract $(0,0)$. Let $\mu_1^i(F,p)$ denote buyer i's belief at Stage 1 that the seller is an H-type if she were to accept contract $(F,p) \in \mathcal{M}^i$. Buyer i's payoff at Stage 1 is:

$$\sum_{(F,p)\in\mathcal{M}^i} \mathbb{P}\left(\text{seller accepts } (F,p)\right) \cdot \Big(-p + \mathbb{E}_{\mu_1^i}[F(X)]\Big). \tag{5}$$

We study pure strategy perfect Bayesian Nash equilibria (PBE) of the above screening game, which has the following implications. First, the seller's acceptance strategy must be optimal, given the menus that remain active after Stage 2 (*Seller Optimality*). Second, a buyer's menu chosen at Stage 1 and his decision to withdraw at Stage 2 must be optimal given his belief at each stage (*Buyer Optimality*). Finally, a buyer's belief at Stages 1 and 2 about the seller's type who is likely to accept a contract from his menu must be consistent with other buyers' posting and withdrawal strategies, the seller's acceptance strategy and Bayes' rule (*Belief Consistency*).

Our environment features two frictions: (i) the seller is privately informed about θ, and (ii) the securities market is non-exclusive. Before proceeding to the equilibrium analysis, we consider two benchmarks in which we shut down each of these frictions in turn.

2.1 Benchmark without Asymmetric Information

We first consider the allocations that would be attained in a setting without asymmetric information; that is, if the seller's asset quality, θ, were publicly observable.

Proposition 2.1 *Suppose that buyers observe asset quality θ before posting their menus. Then, the aggregate cash flows issued by the θ-type seller are $F_\theta(X) = X$, which are priced at their full information valuation $p_\theta = \mathbb{E}_\theta[X]$.*

In the absence of asymmetric information, first-best allocations are attained, as all gains from trade between the seller and the buyers are realized. Moreover, due to competition, the seller's cash flows are priced at their expected value, conditional on the true quality of the seller's asset. As we can see, in this setting, the fact that the securities market is non-exclusive has no bite. Therefore, all our novel findings will be due to the interaction of non-exclusivity with asymmetric information.

2.2 Benchmark with Exclusive Markets

We next consider the allocations that would be attained in a setting where the securities market is exclusive; that is, if the seller were restricted to accept contracts from *at most one* menu.

Consider the following optimization program, which will be useful in characterizing equilibria of this benchmark:

$$\max_{\{(F_\theta,p_\theta)\}_\theta;\ F_\theta\in\Phi} \quad p_H + \delta \cdot \mathbb{E}_H[X - F_H(X)] \tag{P1}$$

subject to the following constraints:

$$p_L + \delta \cdot \mathbb{E}_L[X - F_L(X)] \geq p_H + \delta \cdot \mathbb{E}_L[X - F_H(X)], \tag{6}$$

$$p_H + \delta \cdot \mathbb{E}_H[X - F_H(X)] \geq p_L + \delta \cdot \mathbb{E}_H[X - F_L(X)], \tag{7}$$

$$\mu_0 \cdot (\mathbb{E}_H[F_H(X)] - p_H) + (1-\mu_0) \cdot (\mathbb{E}_L[F_L(X)] - p_L) \geq 0, \tag{8}$$

$$p_L \geq \mathbb{E}_L[F_L(X)]. \tag{9}$$

Program P1 maximizes the H-type's payoff subject to: the seller's incentive compatibility constraints (6) and (7), which implicitly assume that when allocation (F, p) is accepted, the remaining cash flows $X - F_\theta(X)$ must be retained (i.e., not transferred to buyers); the buyers' participation constraint (8), which states that buyers do not make losses in expectation; and the participation constraint (9) for the L-type, which states that she must receive at least her full information payoff.

The following lemma characterizes the solution to program P1.

Lemma 2.1 *The unique solution to P1 is as follows. There exists $\widetilde{\mu} \in (0, 1)$ such that:*

1. *If $\mu_0 \leq \widetilde{\mu}$, the solution features perfect separation: there exists $d^S \in (0, \bar{X})$ such that:*

   (i) $F_H(X) = \min\{d^S, X\}$ and $p_H = \mathbb{E}_H[\min\{d^S, X\}]$;

   (ii) $F_L(X) = X$ and $p_L = \mathbb{E}_L[X]$.

2. *If $\mu_0 > \widetilde{\mu}$, the solution features cross-subsidization: there exists $d^C \in (d^S, \bar{X}]$ such that:*

(i) $F_H(X) = \min\{d^C, X\}$ *and* $p_H = E_{\mu_0}[F_H(X)] + (1-\mu_0)(1-\delta) E_L[X - F_H(X)] < \mathbb{E}_H[F_H(X)]$;

(ii) $F_L(X) = X$ *and* $p_L = E_{\mu_0}[F_H(X)] + [(1-\mu_0)(1-\delta) + \delta] E_L[X - F_H(X)] > \mathbb{E}_L[X]$.

The first part of Lemma 2.1 states that when the buyers' prior belief is low the solution to P1 is separating; i.e., constraint (9) binds. The seller's type is screened through two distinct contracts: (F_H, p_H), which offers to buy less cash flows, but at *high*-valuation; and (F_L, p_L), which offers to buy all cash flows, but at *low*-valuation. Because cash flow retention is more costly for the L-type seller, the security F_H is designed so that the L-type is indifferent between issuing all of her cash flows at *low*-valuation or mimicking the H-type by accepting contract (F_H, p_H). Consistent with this, debt is the optimal security design, as it relaxes the incentive compatibility constraint of the L-type relative to other feasible securities. In particular, d^S is pinned down by requiring that the incentive compatibility constraint of the L-type binds.[12]

The second part of Lemma 2.1 states that when the buyers' prior belief is high the solution features cross-subsidization from the H- to the L-type seller; i.e., constraint (9) is slack. In this scenario, the H-type is better off by selling more cash flows, as $d^C > d^S$, even though this comes at the cost of receiving less than *high*-valuation for them. Debt continues to be the optimal security design, as it now minimizes the subsidy that the H-type has to give to the L-type by being the feasible security for which the difference in valuations between the types is the smallest. Furthermore, d^C is exactly chosen to optimally trade off the marginal benefit for the H-type of selling more cash flows with the marginal cost of subsidizing the L-type.

Proposition 2.2 (Equilibrium in Exclusive Markets) *Suppose that the seller can accept contracts from at most one menu. Then, an equilibrium always exists and it is unique.*[13] *In it, the H-type seller accepts contract (F_H, p_H), while the L-type seller accepts contract (F_L, p_L), which are given by the solution to program P1 in Lemma 2.1.*

The proof of Proposition 2.2 consists of, first, showing that any equilibrium allocation must solve program P1. That is, if either of the constraints is violated or the H-type's payoff is not maximized, there is a profitable deviation for a buyer. Note that, as markets are exclusive, any cash flow that is not transferred to the buyer must be retained by the seller, as stated by the incentive compatibility constraints (6)-(7).[14] Second, we show that the solution to program P1

[12] We provide the expressions defining d^S and d^C in the Appendix; see equations (20) and (21) respectively.

[13] Since there are many buyers who compete, a given buyer's equilibrium menu is not pinned down. We say that the equilibrium is unique when the allocation of the transfer at $t = 1$ and of the asset cash flows at $t = 2$ between each seller type and the buyers is uniquely pinned down.

[14] We highlight this, as understanding what is the appropriate incentive compatibility constraint will prove to be essential in the study of non-exclusive markets.

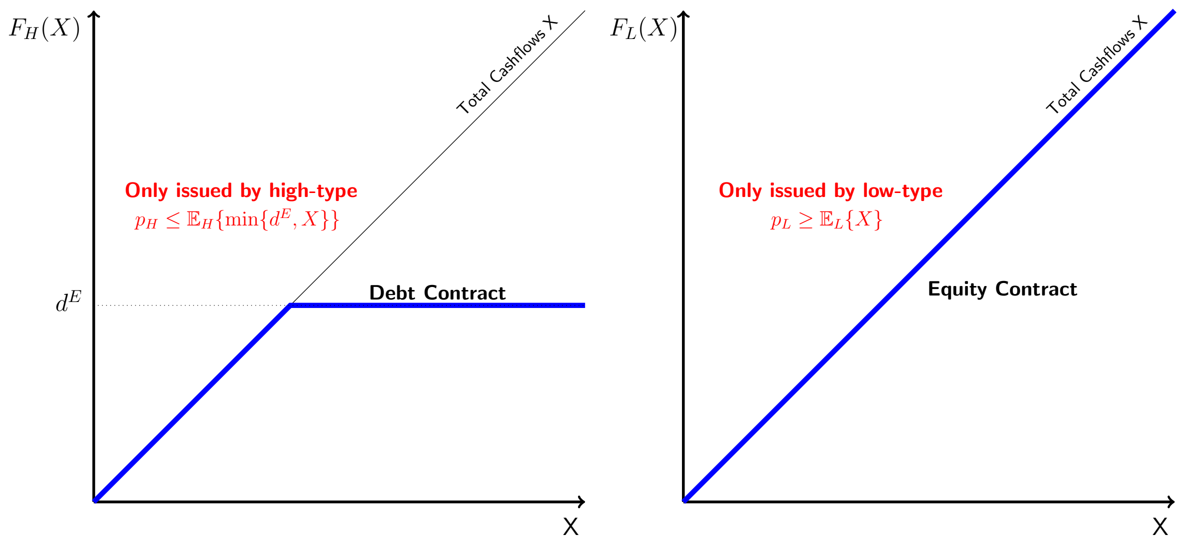

Figure 1: **Security Design in Exclusive Markets.** The left-panel depicts the security issued by the H-type seller (where $d^E \equiv \max\{d^S, d^C\}$), which is priced at or below *high*-valuation; whereas the right-panel depicts the security issued by the L-type seller, which is priced at or above *low*-valuation.

can be supported as a PBE, i.e., there are no profitable deviations for the buyers nor for the seller. We depict the contracts traded in equilibrium in Figure 1.

It is worth noting that when the buyers' prior belief is low, i.e., $\mu_0 \leq \widetilde{\mu}$, the unique equilibrium is separating and its allocations coincide with those of the least-costly separating equilibrium (LCSE) typically studied in signaling games (DeMarzo, 2005; DeMarzo et al., 2015; Daley et al., 2020b). On the other hand, when the buyers' prior belief is high, i.e., $\mu_0 > \widetilde{\mu}$, the unique equilibrium features some cross-subsidization from the H-to the L-type. In contrast to Rothschild and Stiglitz (1978), existence of a cross-subsidizing equilibrium in our setting is ensured by buyers' ability to withdraw loss-making menus, as was first pointed out by Wilson (1977) and Miyazaki (1977) in the context of insurance markets.

3 Equilibrium

We are now ready to characterize equilibria of our model, where the seller is able to accept contracts from multiple menus.

Consider the following optimization program, which will be useful in characterizing equilibria in non-exclusive markets:

$$\max_{\{(F_\theta, p_\theta)\}_\theta \text{ s.t. } F_\theta \in \Phi} \quad p_H + \delta \cdot \mathbb{E}_H[X - F_H] \tag{P2}$$

subject to the following constraints:

$$p_L + \mathbb{E}_L[X - F_L] \geq p_H + \mathbb{E}_L[X - F_H], \tag{10}$$

$$p_H + \delta \cdot \mathbb{E}_H[X - F_H] \geq p_L + \delta \cdot \mathbb{E}_H[X - F_L], \tag{11}$$

$$\mu_0 \cdot (\mathbb{E}_H[F_H] - p_H) + (1 - \mu_0) \cdot (\mathbb{E}_L[F_L] - p_L) \geq 0, \tag{12}$$

$$p_L \geq \mathbb{E}_L[F_L]. \tag{13}$$

Program P2 maximizes the H-type's payoff subject to the same constraints as in program P1 except for the L-type's incentive compatibility constraint (10). Now, if the L-type were to mimic the H-type seller, she would also be able to issue her remaining cash flows at *low*-valuation rather than have to retain them.

The following lemma characterizes the solution to program P2.

Lemma 3.1 *The unique solution to P2 is as follows. There exists $d^* \in (0, \bar{X}]$ such that*

(i) $F_H(X) = \min\{d^*, X\}$ *and* $p_H = \mathbb{E}_{\mu_0}[F_H(X)]$*;*

(ii) $F_L(X) = X$ *and* $p_L = \mathbb{E}_{\mu_0}[F_H(X)] + \mathbb{E}_L[X - F_H(X)]$.

Moreover, there exists $\widehat{\mu} \in (0, 1)$ such that $d^* = \bar{X}$ if and only if $\mu_0 \geq \widehat{\mu}$.

By comparing Lemma 3.1 with Lemma 2.1, it follows that when the L-type is able to sell her remaining cash flows at *low*-valuation, the solution is never separating, i.e., there is always cross-subsidization from the H- to the L-type seller. Furthermore, the cross-subsidy is such that the H-type seller receives the average valuation for all the cash flows that she issues. The optimal security issued by the H-type is debt, as it is the design that maximizes the H-type's payoff by minimizing the subsidy given to the L-type among all feasible securities.

The allocations that solve P2 can be conveniently implemented through a set of non-cross-subsidizing contracts, i.e., contracts that earn zero expected profits for the buyer:

(i) A *senior tranche*, $F_S(X) = \min\{d^*, X\}$, priced at average valuation, $p_S = \mathbb{E}_{\mu_0}[F_S(X)]$, and accepted by all seller types.

(ii) A *junior tranche*, $F_J(X) = \max\{X - d^*, 0\}$, priced at *low*-valuation, $p_J = \mathbb{E}_L[F_J(X)]$, and accepted only by the L-type seller.

In the remainder of the paper, we use $\mathcal{C}^* \equiv \{(F_S, p_S), (F_J, p_J)\}$ to represent the set of contracts whose resulting allocations solve P2.[15]

[15] Focusing on $\mathcal{C}^*$ is without loss of generality, since the solution to program P2 is unique in terms of allocations between each seller type and the buyers.

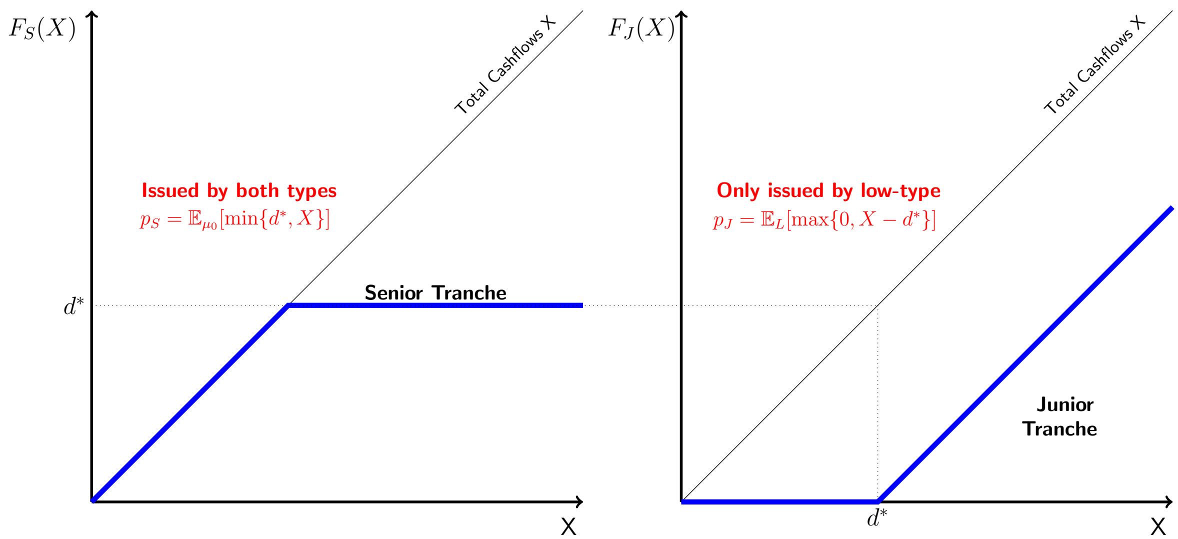

Figure 2: **Security Design in Non-Exclusive Markets.** The left-panel depicts the security issued by both seller types, which is priced at average valuation; whereas the right-panel depicts the security issued by the L-type seller only, which is priced at *low*-valuation.

Proposition 3.1 (Equilibrium in Non-Exclusive Markets) *Suppose that the seller can accept contracts from multiple menus. Then, there exists an equilibrium in which all seller types accept contract (F_S, p_S), and in addition the L-type seller accepts contract (F_J, p_J).*

Proposition 3.1 states that there always exists an equilibrium, with semi-pooling allocations, in which both seller types issue the same, non-trivial debt security. In addition to accepting the same contract as the H-type, the L-type seller issues her remaining cash flows $F_J(X) = X - F_S(X)$ at *low*-valuation to a distinct buyer, in order to further exploit gains from trade. We refer to this equilibrium as the star-equilibrium, we denote the sellers payoffs in it by u^*_θ, and we depict the contracts traded in it in Figure 2.

The star-equilibrium is supported by the presence of *latent contracts*, which are contracts that are offered, though not accepted, on equilibrium path and whose only role is to deter deviations by buyers at Stage 1. In what follows, we sketch the proof of Proposition 3.1 and show how the presence of latent contracts, combined with the ability of buyers to withdraw menus at Stage 2, ensures equilibrium existence.

Consider the following candidate equilibrium strategies. At Stage 1, buyers 1 and 2 offer to purchase the senior tranche at average valuation, $\{(F_S, p_S)\}$, buyers 3 and 4 offer to purchase any claim at *low*-valuation $\{F, E_L[F]\}_{F\in\Phi}$, and the remaining buyers offer the trivial contract, $\{(0, 0)\}$. At Stage 2, there are no menu withdrawals. At Stage 3, the H-type accepts contract (F_S, p_S) from buyer 1 whereas the L-type accepts (F_S, p_S) from buyer 1 and (F_J, p_J) from buyer 3. Note that buyers 3 and 4, in addition to contract (F_J, p_J), are posting latent contracts, allowing the L-type to issue any feasible claim at *low*-valuation following a deviation.

By construction, there are no profitable deviations for any seller at Stage 3. Moreover, since in equilibrium buyers break-even, there are no profitable deviations at Stage 2 either. Thus, to establish that the above strategies constitute a PBE, it suffices to rule out deviations by buyers at Stage 1. First, any deviation to attract the L-type alone would need to price her cash flows above *low*-valuation and must therefore be loss-making. Second, any deviation to attract only the H-type ends up also attracting the L-type, who would prefer to mimic the H-type and issue her remaining cash flows using a corresponding latent contract. Finally, any deviation that attracts both types is loss-making since $\mathcal{C}^*$ already maximizes the H-type's payoff subject to the seller's incentive compatibility and participation constraints, which incorporate the presence of latent contracts, and buyers not making losses (see program P2). The last two arguments rely on the fact that the menus containing the contract (F_S, p_S) would be withdrawn at Stage 2 if only the L-type were to pick it up following a deviation.

A natural next question is whether the star-equilibrium is unique. To this end, we show that in any equilibrium in non-exclusive markets, buyers must break-even and all gains from trade between the L-type and the buyers must be realized (see Lemma A.7 in the Appendix). As a result, an equilibrium is fully characterized by the H-type's allocation: the cash flows that she transfers to the buyers and how the buyers price them. As we show next, there are indeed conditions under which this allocation is pinned down uniquely.

Proposition 3.2 *If $\mu_0 \geq \widehat{\mu}$, the star-equilibrium is unique. Otherwise, the star-equilibrium is unique under the refinement that any feasible security be priced weakly above low-valuation.*

The first part of the proposition shows that the star-equilibrium is unique when in it the H-type seller transfers all of her cash flows to the buyers, i.e., $d^* = \bar{X}$, which occurs when $\mu_0 \geq \widehat{\mu}$ (see Lemma 3.1). Note that any other equilibrium would require that the H-type retains some cash flows. But then, since the H-type seller is willing to trade all of her cash flows at average valuation, such equilibria can be destroyed with a deviation that exploits gains from trade on the retained cash flows. Moreover, the second part of the proposition shows that the star-equilibrium is also unique if we refine the set of equilibria by imposing a lower bound, $E_L[F]$, on the price of any security F that could be issued off-equilibrium. Such a refinement effectively requires that buyers price securities competitively also off-equilibrium path. Intuitively, when the seller can issue any security priced weakly above *low*-valuation, equilibrium allocations must solve program P2, which has a unique solution.

For the interested reader, in Appendix B.2, we show that when the conditions stated in Proposition 3.2 are not satisfied other equilibria also arise. Moreover, these equilibria look a lot like the star-equilibrium, but with a twist: the H-type issues cash flows F_{Mz} in addition to

F_S (i.e., a mezzanine tranche), but the entire surplus from their issuance, $(1-\delta)\cdot\mu_0\cdot E_H[F_{Mz}]$, accrues to the L-type seller, who thus gets a payoff above u_L^*, which is her payoff in the star-equilibrium. A corollary to this result is that, in contrast to exclusive markets, non-exclusive markets must always feature cross-subsidization as $u_L^* > E_L[X]$, i.e., separation is not possible.

Remarks on the star-equilibrium allocations. It is useful to compare the star-equilibrium allocations with those obtained in the literature on non-exclusive competition with asymmetric information.[16] Attar et al. (2011) consider a setting similar to ours, but restrict buyers to post menus of quantities and prices (rather than of securities and prices): they find that the equilibrium allocations of their game resemble those of Akerlof (1970), i.e., a given seller type either trades the entire asset at a pooling price or does not trade at all. Whereas our finding that non-exclusivity makes cross-subsidization a necessary feature of equilibrium is also present in Attar et al. (2011), in contrast to them we do not obtain Akerlof-like outcomes. The agents' ability to optimally design securities results in semi-pooling allocations, in which all seller types trade some of their cash flows, so that there is pooling in the market for the senior tranche but separation in the market for the junior tranche. In fact, our star-equilibrium allocations are closer to those obtained in the literature on non-exclusive competition in insurance markets (e.g., Jaynes (1978); Hellwig (1988); Glosten (1994); Dubey and Geanakoplos (2019)). In that literature, due to concavity of preferences, low- and high-risk types buy an interior amount of insurance at a pooling price, while the high-risk type also buys supplementary insurance at a higher price. Although the agents' preferences are linear in our setting (as in much of the literature on security design), the H-type's payoff becomes endogenously concave due to optimal security design. Our findings thus provide a bridge between the seemingly disparate findings in the literature on non-exclusive competition in asset vs. insurance markets.

Finally, as noted in Attar et al. (2014), cross-subsidizing allocations may be difficult to support as PBE in the presence of non-linearities, which in our setting arise naturally due to security design. We are able to ensure equilibrium existence by introducing the withdrawal stage. In this sense, our equilibrium notion is close in spirit to the Miyazaki-Wilson-Spence equilibrium (Wilson, 1977; Miyazaki, 1977; Spence, 1978), where buyers are able to withdraw offers upon observing the set of offers available in the market.

4 Costs and Benefits of Non-Exclusivity

As we already discussed in the introduction, after the 2008-09 financial crisis, a number of exclusivity- and transparency-enhancing financial market reforms were discussed in the US and

[16] For a comparison with the traditional literature on security design, see Section 2.2 and Section 4.

Europe, which would either directly or indirectly enhance exclusivity in contracting. Despite of these efforts of policymakers and regulators, there is surprisingly little theoretical work on the policy implications of non-exclusivity in markets with asymmetric information. Motivated by this, in this section we consider the normative implications of our theory by studying the potential costs and benefits of non-exclusivity. First, we study the welfare properties of our baseline model, where the distribution of asset qualities is exogenous. Second, we consider an extension of the model that endogenizes asset quality.

4.1 Non-Exclusivity and Market Liquidity

We begin by introducing the notion of efficiency/welfare in our setting. Since buyers break-even in any equilibrium, efficiency is determined by the ex-ante expected payoff to the seller:

$$W(\mu_0) \equiv \mu_0 \cdot u_H(\mu_0) + (1-\mu_0) \cdot u_L(\mu_0). \tag{14}$$

where $u_\theta(\mu_0)$ denotes the equilibrium payoff of a θ-type seller when the buyers' prior belief that the seller is H-type is μ_0. In what follows, we will sometimes use superscripts to indicate the outcomes in the first-best (FB), exclusive (E) or non-exclusive (NE) market settings. Unless specified otherwise, by an equilibrium in non-exclusive markets we refer to the star-equilibrium.

In the presence of asymmetric information, equilibrium allocations may be distorted away from first-best for two reasons. First, due to retention of cash flows by the H-type seller, some gains from trade remain unrealized. We say that the market is more *liquid* when more gains from trade between the seller and the buyers are realized, i.e., when cash flow retention is lower. Second, because the prices of claims need not reflect true underlying asset quality θ, the H-type seller may effectively subsidize the L-type seller. When this occurs, we say that there is *mispricing*. To illustrate the effects of these distortions, the equilibrium payoff of a θ-type seller can be expressed as follows:

$$u_L(\mu_0) = \underbrace{\mathbb{E}_L[X]}_{=u_L^{FB}} + \underbrace{\Delta(\mu_0)}_{\text{Mispricing Subsidy}}, \tag{15}$$

$$u_H(\mu_0) = \underbrace{\mathbb{E}_H[X]}_{=u_H^{FB}} - \underbrace{(1-\delta)\cdot\mathbb{E}_H[X - F_H(X)]}_{\text{Cost of Retention}} - \underbrace{\frac{1-\mu_0}{\mu_0}\cdot\Delta(\mu_0)}_{\text{Mispricing Tax}}. \tag{16}$$

where F_H denotes the cash flows issued by the H-type seller in equilibrium.

Since the mispricing of claims for the seller generates a transfer from the H-type to the L-type seller, equilibrium welfare is distorted away from first-best only due to inefficient cash

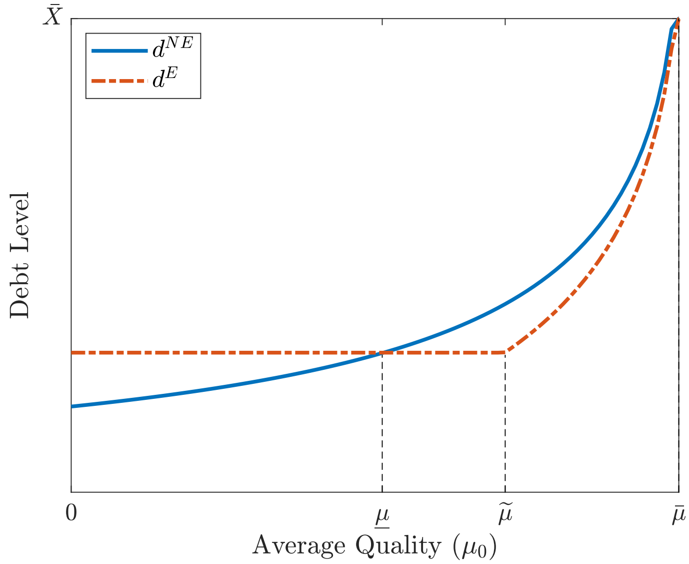

Figure 3: **Which market structure is more efficient for an exogenously given average quality μ_0?** The figure depicts the debt levels $d^E \equiv \max\{d^S, d^C\}$ and $d^{NE} \equiv d^*$ that arise with exclusive and non-exclusive markets, respectively, as a function of μ_0.

flow retention:

$$W(\mu_0) = \underbrace{\mu_0 \cdot \mathbb{E}_H[X] + (1-\mu_0) \cdot \mathbb{E}_L[X]}_{=W^{FB}(\mu_0)} - \underbrace{\mu_0 \cdot (1-\delta) \cdot \mathbb{E}_H[X - F_H(X)]}_{\text{Expected Cost of Retention}}. \tag{17}$$

Thus, when asset quality is exogenous, a more liquid market is also more efficient. Hence, non-exclusive markets are more efficient than exclusive markets whenever they implement higher liquidity, i.e., when $d^{NE} \equiv d^* > d^E \equiv \max\{d^S, d^C\}$. The next proposition states when this is the case.

Proposition 4.1 *There exist $0 < \underline{\mu} < \bar{\mu} < 1$, such that welfare in exclusive markets is greater than in non-exclusive markets for $\mu_0 < \underline{\mu}$, lower for $\mu_0 \in \left(\underline{\mu}, \bar{\mu}\right)$, and equal for $\mu_0 \geq \bar{\mu}$.*

Figure 3 depicts the debt levels d^E and d^{NE} in exclusive and non-exclusive markets respectively as a function of μ_0. And, recall that there is a one-to-one mapping between these debt-levels and welfare. Consider the region $\mu_0 \leq \widetilde{\mu}$, where $\widetilde{\mu}$ is specified in Lemma 2.1. Here, d^E is pinned down by the L-type's incentive compatibility constraint (6) and, thus, it is independent of μ_0. Instead, d^{NE} starts below d^E, is continuously increasing in μ_0, and crosses d^E at some $\underline{\mu} < \widetilde{\mu}$. Consider next the region $\mu_0 > \widetilde{\mu}$. Here, d^E is also continuously increasing in μ_0, since the H-type is willing to cross-subsidize the L-type even in exclusive markets; however, we show that d^{NE} is always greater than d^E. Lastly, after μ_0 increases above some $\bar{\mu}$, the seller issues a full claim to her asset in both markets structures and, thus, welfares coincide.[17]

These findings contrast with the by-now conventional 'ignorance is bliss' view of Dang et al. (2010) and Dang et al. (2017), according to which market liquidity and efficiency are maximized through complexity of assets and opacity of issuers' balance sheets. Our results instead suggest that to the extent that complexity/opacity also inhibits exclusive contracting, it may actually reduce market liquidity and efficiency whenever the underlying asset quality is low.

Lastly, here, we focused on a notion of ex-ante welfare. One may wonder, however, about the welfare implications of non-exclusivity at an ex-interim stage, i.e., conditional on the seller's type. Indeed, it is straightforward to show that whereas the L-type weakly prefers non-exclusive markets, the H-type has a weak preference for exclusive markets. Thus, even though the two market structures can be ranked from an ex-ante perspective, neither dominates the other in an ex-interim Pareto sense.

4.2 Non-Exclusivity and Origination Incentives

In the previous section, we showed that non-exclusive markets may either increase or decrease market liquidity, and therefore efficiency, depending on the average quality of assets. However, due to larger cross-subsidization, non-exclusive markets always induce a larger mispricing of claims for the seller than exclusive markets. In our baseline setting, such mispricing was irrelevant for efficiency, as the distribution of asset quality was exogenous. In many applications, however, such mispricing may impact efficiency by distorting agent's decisions, e.g. distorting investment decisions. To address this, we now explore how non-exclusivity, through its effects on market liquidity and the pricing of claims, affects incentives to originate high quality assets.

We extend our baseline setting to allow the seller (who is now also an asset originator) to exert costly, unobservable effort $C(q)$ to ensure that her asset is of high quality with probability $q \in [0,1]$. For interior solution, we suppose that $C(0) = C'(0) = 0$, $C'(q) > 0$ and $C''(q) > 0$ for $q \in (0,1)$, and $\lim_{q\to 1} C'(q) = \infty$.[18] Given the buyers' prior belief μ_0, efficiency is given by the ex-ante payoff of the seller:

$$W(\mu_0) = \max_{q} \; q \cdot u_H(\mu_0) + (1-q) \cdot u_L(\mu_0) - C(q), \tag{18}$$

where $u_\theta(\mu_0)$ denotes the payoff of a θ-type seller in the equilibrium of the *trading stage* as

[17] Note that, since the star-equilibrium allocations provide a lower bound on welfare in non-exclusive markets (see Proposition B.1 in Appendix B.2), it follows that welfare in any equilibrium with non-exclusive markets must be greater than with exclusive markets when $\mu_0 \in (\underline{\mu}, \bar{\mu})$.

[18] This formulation is standard and has been employed in several papers that study the effect of secondary market liquidity on origination incentives (e.g., Chemla and Hennessy (2014); Vanasco (2017); Neuhann (2017); Fukui (2018); Asriyan et al. (2019); Caramp (2020); Daley et al. (2020a)).

defined in Section 2, for given belief μ_0. The solution q^* to problem (18) exists, is unique, and satisfies:

$$C'(q^*) = \underbrace{\mathbb{E}_H[X] - \mathbb{E}_L[X] - (1-\delta) \cdot \mathbb{E}_H[X - F_H(X)] - \frac{\Delta(\mu_0)}{\mu_0}}_{=u_H(\mu_0)-u_L(\mu_0)}. \tag{19}$$

An equilibrium of the entire game now in addition requires that (i) given the seller's payoffs $\{u_\theta(\mu_0)\}_\theta$ at the trading stage, her effort choice is optimal, i.e., solves (19); and (ii) the buyers' prior belief is consistent with the seller's optimal effort choice, i.e., $\mu_0 = q^*$. It is straightforward to show that an equilibrium exists both in exclusive and non-exclusive markets.

From equation (19), we see that both market liquidity (as captured by the cash flows sold by the H-type at the *trading stage*, F_H, which varies with μ_0) and the extent to which the claims are mispriced (as captured by $\Delta(\mu_0)$) are relevant for determining the originator's effort incentives. Moreover, even though market liquidity may be higher or lower in non-exclusive markets (see Proposition 4.1), the mispricing of claims faced by the seller is always larger in non-exclusive markets, which, as we show next, is crucial for understanding origination incentives.

Proposition 4.2 *The average quality of originated assets in non-exclusive markets is lower than in exclusive markets:* $0 < \mu_0^{NE} < \mu_0^E < \mu_0^{FB} < 1$.

Proposition 4.2 establishes a very strong result: non-exclusive markets always implement lower equilibrium asset quality than exclusive markets. The reason behind it is intuitive, and it is driven by fundamental differences between exclusive and non-exclusive markets. Recall that the originator's incentive to exert effort increases with the payoff gap between seller types, $u_H(\mu_0) - u_L(\mu_0)$ (see equation (19)). The proof then consists of showing that this gap is always smaller in non-exclusive markets.[19]

By combining the results of Propositions 4.1 and 4.2, we conclude that non-exclusive markets can only be more efficient if the potential (though not guaranteed) gains from increased market liquidity more than compensate for the (guaranteed) fall in asset quality. Figure 4 illustrates that this happens when the gains from trade (as captured by $1-\delta$) are large relative to the cost of exerting effort (as captured by χ), where we use a simple parameterization, $C(q) = \chi \cdot \frac{q^2}{1-q}$. It is important to highlight that when asset quality is endogenous, there is never an equilibrium in which all seller types sell a full claim to their cash flows: there is always retention in equilibrium.

These findings thus suggest that complexity/opacity is desirable only in environments where efficiency gains are mostly driven by reallocation of assets in markets *and* at the same time the originators need not be too incentivized to produce high-quality assets.

[19] We note that this result holds for any equilibrium in non-exclusive markets (see proof of Proposition 4.2).

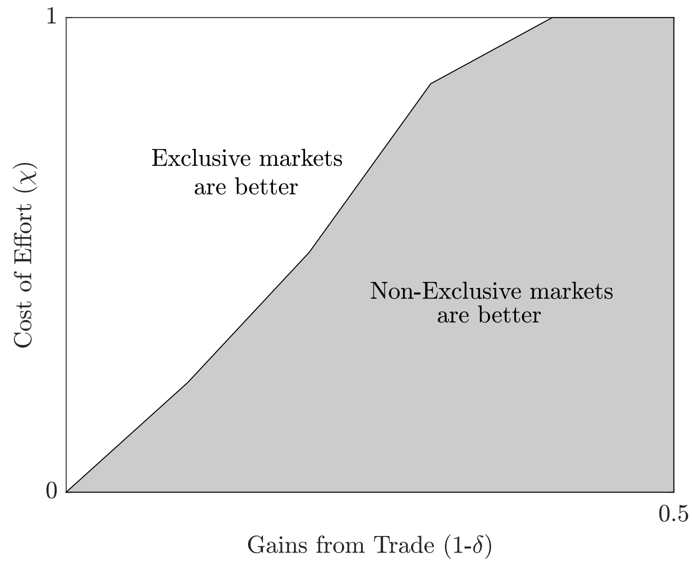

Figure 4: **Which market structure is more efficient when average quality is endogenous?** The unshaded region depicts parameter values for which welfare in the equilibrium with exclusive markets is greater than welfare in the star-equilibrium. The shaded region depicts parameter values for which welfare in the star-equilibrium is greater than welfare in the equilibrium with exclusive markets.

5 Empirical Implications

Our model has important implications for markets in which exclusivity is difficult to enforce. This is likely to be the case in markets where sellers' (e.g., firms', banks') risk exposures or trades are either not observable or hard to understand by other market participants (e.g., investors, regulators, courts). Understanding the implications of non-exclusivity is particularly relevant for the study of modern financial markets, where the increasing complexity of assets and balance sheets of financial intermediaries combined with the opacity of markets where these assets are traded makes it virtually impossible for outsiders to ensure that a seller retains a particular risk-exposure (Ashcraft et al., 2019). In what follows, we present the novel empirical implications of our model and relate them to empirical evidence in the market for mortgage-backed securities. When doing so, we focus on the *star* equilibrium in non-exclusive markets.

Prediction 1. As exclusivity becomes harder to enforce, the practice of splitting asset cash flows into different tranches that are sold separately in markets is more likely to occur.

Indeed, in recent decades, the expansion of securitization and of the practice of tranching loan cash flows coincided with an increase in the complexity of financial intermediaries' balance sheets, whose risk-exposures became harder to understand and contract upon. As argued in a recent paper by Ashcraft et al. (2019), the complexity of collateralized debt obligations (CDOs) "enabled informed parties in the commercial mortgage-backed securitization pipeline to reduce their skin-in-the-game [retention] in a way not observable to other market participants."

Prediction 2. In non-exclusive markets, the amount of cash flows retained **should not** *predict differential pricing of securities in the market for senior tranches.*

This prediction follows from Proposition 3.1, which states that the senior tranche, issued by all seller types in equilibrium is priced at average valuation. As a result, whether the seller retains (H-type) or sells (L-type) her junior tranche does not affect its pricing. This result is consistent with findings in Ashcraft et al. (2019), who study cash flow retention and its relation to security performance in the conduit segment of the commercial mortgage-backed securities market.[20] They find that the fraction of initially retained cash flows sold into CDOs in the twelve months following a transaction, i.e., not observed at the time of the transaction, is not correlated with the prices of the more senior tranche.

Prediction 3. In non-exclusive markets, the amount of cash flows retained **should** *predict differential quality of the senior tranches.*

This prediction also follows from Proposition 3.1, which states that while the H-type seller only issues a senior tranche, and thus retains a junior tranche, the L-type seller issues both tranches to distinct buyers, and thus does not retain cash flows in equilibrium. This result is consistent with evidence in Ashcraft et al. (2019), who find that a higher fraction of initial cash flow retention sold into CDOs predicts a higher probability of default of the more senior tranches, even after controlling for all information available at issuance. To the best of our knowledge, the predictions 2 and 3 combined are not consistent with other models of security design where cash flow retention operates as a signal/screening device.

Prediction 4. As exclusivity becomes harder to enforce, the quality of originated assets declines.

This prediction is effectively a re-statement of Proposition 4.2. Though we are not aware of a formal test of this prediction, it is broadly consistent with the well-known stylized fact that the US credit boom of the early 2000s, fueled by securitization and financial engineering of complex assets traded in opaque markets, has been associated with falling lending standards and a decline in the quality of originated assets (e.g., Mian and Sufi (2009); Keys et al. (2010); Dell'Ariccia

[20] They study the retention of B-piece investors, who are buyers that perform due-diligence and re-underwrite all of the loans in a given pool, indicating there is no asymmetric information between the actual seller and the B-piece investors. Importantly, even though the size of the B-piece is disclosed to other (uninformed) buyers, how much the B-piece buyer actually retains over time is not transparent to these buyers. See Ashcraft et al. (2019) for a more detailed description of the environment and empirical strategy.

et al. (2012)). This is commonly attributed to the observed decline in the originators' cash flow retention (i.e., less skin-in-the-game) that the securitization process had apparently enabled (Parlour and Plantin, 2008; Chemla and Hennessy, 2014; Vanasco, 2017). As we showed in Section 4, however, the manner by which secondary markets price claims is also an essential determinant of the originators' incentives, above and beyond overall cash flow retention.

6 Concluding Remarks

We revisit the classic problem of a seller who is privately informed about the quality of her asset and wants to exploit gains from trade with uninformed buyers by issuing securities backed by her asset cash flows. We depart from the traditional literature by positing that the securities market is non-exclusive; that is, the seller cannot commit to trade with only one buyer. We show that non-exclusive markets behave very differently from exclusive ones in the presence of information asymmetries: (i) separating contracts are never part of equilibrium; (ii) mispricing of claims faced by the seller is always larger than in exclusive markets; (iii) there is always a semi-pooling equilibrium where a senior debt security is issued by both seller types; (iv) market liquidity can be higher or lower than in exclusive markets, but (v) the average quality of originated assets is always lower. Our model's predictions are consistent with empirical evidence on issuance and pricing of mortgage-backed securities, and we use the theory to evaluate some recent reforms aimed at enhancing transparency and exclusivity in financial markets.

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A Proofs

Proof of Proposition 2.1. The proof is straightforward. ■

Proof of Lemma 2.1. We prove this result by establishing a set of intermediate lemmas. We begin by guessing that the incentive compatibility constraint (7) in program P1 is slack at the optimum and, thus, dropping it from the program. We then verify that this is indeed the case.

The first lemma shows that the L-type transfers all of her cash flows to the buyers.

Lemma A.1 $F_L(x) = x \ \forall x$.

Proof. Suppose to the contrary that the solution to P1 consists of allocations $\{(F_\theta, p_\theta)\}$ with $F_L(x) < x$ for some $x \in [0, \bar{X}]$. Consider next the allocations $\{(F'_\theta, p'_\theta)\}$ where: (i) $F'_H(x) = F_H(x) \ \forall x$ and $p'_H = p_H + \frac{1-\mu_0}{\mu_0} \cdot \varepsilon$; and (ii) $F'_L(x) = x \ \forall x$ and $p'_L = p_L + E_L[X - F_L(X)] - \varepsilon$. For $\varepsilon > 0$ sufficiently small, with the new allocations $\{(F'_\theta, p'_\theta)\}$, the constraints (6), (8), and (9) are satisfied, but the objective of the program has increased, a contradiction. ■

The second lemma shows that at the optimum the incentive compatibility constraint of the L-type, given by (6), and the buyers' participation constraint, given by (8), bind.

Lemma A.2 *The constraints* (6) *and* (8) *are satisfied with equality.*

Proof. Suppose not. If the constraint (8) were slack, then the objective of the program could be increased by raising p_H and p_L by a small amount $\varepsilon > 0$, which leaves the constraints (6) and (9) satisfied, a contradiction.

If instead the constraint (6) were slack, then there are two possibilities. If also the constraint (9) is slack, then decreasing p_L by ε and increasing p_H by $\frac{1-\mu_0}{\mu_0} \cdot \varepsilon$ increases the objective while, for $\varepsilon > 0$ sufficiently small, still satisfying the constraints (6), (8) and (9), a contradiction. On the other hand, if the constraint (9) binds and thus $p_L = E_L[X]$, it must be that $F_H(x) < x$ for some x; otherwise, since the constraint (8) binds, the constraint (6) would have to be violated. But then, replacing allocation (F_H, p_H) with (F'_H, p'_H), where $F'_H(x) = \min\{F_H(x) + \epsilon, x\} \ \forall x$ and $p'_H = E_H[F'_H(x)]$ and $\epsilon > 0$ is small, increases the objective of the program while still satisfying the constraints (6), (8) and (9), a contradiction. ■

The third lemma shows that the H-type issues a non-trivial debt security.

Lemma A.3 $F_H(x) = \min\{d, x\} \ \forall x$ *and some* $d \in (0, \bar{X}]$.

Proof. Suppose to the contrary that F_H is not a debt security, and let d' be such that $\mathbb{E}_H[F_H(X)] = \mathbb{E}_H[\min\{d', X\}]$. Since $F_H(\cdot)$ is monotonic, there exists $\widehat{x} \in [0, \bar{X}]$ such that $F_H(x) > d'$ if and only if $x > \widehat{x}$. By MLRP, it must then be that $\mathbb{E}_L[X - F_H(X)] > \mathbb{E}_L[X - \min\{d', X\}]$, since both securities have the same *high*-valuation:

$$\begin{aligned}\mathbb{E}_L[F_H(X) - \min\{d', X\}] &= \mathbb{E}_H\left[(F_H(X) - \min\{d', X\}) \cdot \frac{g_L(X)}{g_H(X)}\right] \\ &< \mathbb{E}_H[F_H(X) - \min\{d', X\}] \cdot \frac{g_L(\widehat{x})}{g_H(\widehat{x})} = 0.\end{aligned}$$

Thus, whereas the H-type is indifferent between (F_H, p_H) and (F', p_H) with $F'(X) = \min\{d', X\}$, the L-type strictly prefers her allocation (X, p_L) to (F', p_H). Next, consider changing the allocation of the H-type to (F'', p'') with $F''(X) = \min\{d'', X\}$, $p'' = p_H + \mathbb{E}_H[\min\{d'', X\} - \min\{d', X\}]$, and $d'' > d'$. If d'' is close to d', then the L-type still prefers to accept (X, p_L) to (F'', p''), and the H-type strictly prefers (F'', p'') to (F_H, p_H). Hence, such a change increases the objective of the program while leaving the constraints (6), (8) and (9) satisfied, a contradiction. ■

Next, we determine the optimal debt level. Define $d^C : [0,1] \to \left[0, \bar{X}\right]$ as follows:

$$d^C(\mu_0) = \arg\max_{d \in \left[0,\bar{X}\right]} E_{\mu_0}\left[\min\{d, X\}\right] - (1-\mu_0)(1-\delta)\, E_L\left[\min\{d, X\}\right] - \delta \cdot E_H[\min\{d, X\}]. \tag{20}$$

From Lemmas A.1-A.3, this is the debt level that solves program P1 when constraint (9) is slack. Define $d^S \in (0, \bar{X})$ as follows:

$$(1-\delta) \cdot E_L[X] = E_H\left[\min\left\{d^S, X\right\}\right] - \delta \cdot E_L\left[\min\left\{d^S, X\right\}\right]. \tag{21}$$

From Lemmas A.1-A.3, this is the debt level that solves program P1 when constraint (9) binds.

The derivative of the expression on the right-hand side of equation (20) w.r.t. d is:

$$H(d, \mu_0) = (\mu_0 - \delta) \cdot (1 - G_H(d)) + (1 - \mu_0) \cdot \delta \cdot (1 - G_L(d)), \tag{22}$$

which satisfies the following properties:

(i) $H(\cdot,\cdot)$ is continuous in both arguments and $H(0,0) = H(\bar{X}, \cdot) = 0$.

(ii) For $\mu_0 > 0$, when d is small, $H(d, \mu_0) > 0$ and $H_1(d, \mu_0) < 0$. Moreover, $H(\cdot, \mu_0)$ crosses zero at an interior d at most once, and iff $\mu_0 < \delta \cdot \lim_{x \to \bar{X}} \frac{\frac{g_H(x)}{g_L(x)} - 1}{\frac{g_H(x)}{g_L(x)} - \delta} \equiv \bar{\mu}$.

(iii) since $H_2(\cdot,\cdot) > 0$, this interior value of d is continuously increasing in μ_0 for $\mu_0 < \bar{\mu}$, and it goes to zero as $\mu_0 \downarrow 0$ and to $\bar{X}$ as $\mu_0 \uparrow \bar{\mu}$.

It follows that the solution $d^C(\mu_0)$ is unique for any μ_0, and that $d^C(0) = 0$, $d^C(\cdot)$ is continuous and increasing on $[0, \bar{\mu}]$ and $d^C(\cdot) = \bar{X}$ for $\mu_0 \geq \bar{\mu}$. On the other hand, d^S is also unique and satisfies $d^S \in (0, \bar{X})$ for any μ_0. It is then straightforward to show that the constraint (9) is slack if and only if $d^C(\mu_0) > d^S$, which is equivalent to μ_0 being greater than some $\widetilde{\mu} \in (0, \bar{\mu})$. Once the optimal debt level is found, the expressions for the prices $\{p_\theta\}$ follow directly from Lemmas A.1-A.3 and the fact that the constraint (9) is slack if and only if $\mu_0 > \widetilde{\mu}$.

Finally, the result stated in the proposition follows from Lemmas A.1-A.3 together with the following verification that the constraint (7) is always slack. The latter holds if:

$$p_H + \delta \cdot E_H\left[X - \min\left\{d^E, X\right\}\right] \geq p_L = p_H + \delta \cdot E_L\left[X - \min\left\{d^E, X\right\}\right],$$

which holds, since MLRP implies that:

$$E_H\left[X - \min\left\{d^E, X\right\}\right] \geq E_L\left[X - \min\left\{d^E, X\right\}\right].$$

■

Proof of Proposition 2.2. We first show that any equilibrium must consist of allocations that solve program P1. To this end, consider an equilibrium with allocations $\{(F_\theta, p_\theta)\}$; that is, the θ-type transfers cash flows $F_\theta(x)$ to the buyers at $t = 2$ when $X = x$, and the buyers transfer p_θ to the seller at $t = 1$.

In any equilibrium, each buyer must make zero expected profits. Suppose to the contrary that the buyers' aggregate profits are positive. Suppose now that buyer i who were earning less than half of the aggregate profits were to deviate and add the contracts $\{(F_\theta, p_\theta + \varepsilon)\}$ to his menu. Clearly, the seller would pick these contracts instead of the equilibrium allocation $\{(F_\theta, p_\theta)\}$, independently of whether the other buyers withdraw their menus or not. Moreover, for ε small enough, such a deviation is profitable as buyer i effectively captures all of the aggregate expected profits, a contradiction.

It is clear that the assumption that equilibrium allocations satisfy the incentive compatibility constraints (6) and (7) is without loss of generality.

It must also be that in any equilibrium $p_L \geq E_L[X]$. Suppose to the contrary that $p_L < E_L[X]$, and consider a buyer who were to deviate and add the following contract to his menu: (F', p') with $F'(X) = X$ and $p' = E_L[X] - \varepsilon$. For ε small, the low-type prefers this contract to his equilibrium allocation and, thus, would pick it up. Moreover, this contract makes strictly positive profits even if it is also picked up by the H-type. Thus, independently of whether other buyers withdraw their menus or not, such a deviation is profitable; a contradiction.

Lastly, the equilibrium allocations must also maximize H-type's payoff. Suppose to the contrary, and let $\{(F_\theta^{P1}, p_\theta^{P1})\}$ denote the unique solution to program P1, which note cannot be offered in the candidate equilibrium. Suppose that the candidate allocations are picked up from buyer i (and possibly j), and consider a deviation by buyer $k \neq i, j$ to post a menu that consists of the contracts $\{(F_\theta^{P1}, p_\theta^{P1} - \varepsilon)\}$. For $\varepsilon > 0$ small enough, such a deviation attracts the H-type seller to contract $(F_H^{P1}, p_H^{P1} - \varepsilon)$, and the deviation is profitable whether or not the L-type is attracted to the contract $(F_L^{P1}, p_L^{P1} - \varepsilon)$, a contradiction.

We next show that an equilibrium exists when the cost of withdrawal κ is small enough. That equilibrium allocations are unique (in terms of allocations between each seller type and the buyers) will follow from the uniqueness of the solution to P1.

Consider the following candidate equilibrium strategies. At Stage 1, buyers 1 and 2 offer the contracts $\{(F_\theta^{P1}, p_\theta^{P1})\}$, whereas the remaining buyers post the trivial contract. At Stage 2, there are no menu withdrawals. At Stage 3, the H-type accepts contract (F_H, p_H) and the L-type accepts contract (F_L, p_L) from buyer 1 (or both from buyer 2).

By construction, there are no deviations for the seller at Stage 3. Also, since in the candidate equilibrium buyers break even, there are also no deviations at Stage 2. We are therefore left to rule out deviations at Stage 1. There are three types of deviations to consider.

(i) Consider a deviation that attracts only the L-type. Such a deviation clearly cannot be profitable as it would need to offer the L-type a price $p_L > E_L[X]$.

(ii) Consider a deviation that attracts both types. Let $\{(F_\theta, p_\theta)\}$ denote the allocations of the seller when she accepts a contract from the deviating menu. For it to be profitable, $\{(F_\theta, p_\theta)\}$ must satisfy the constraints of P1. Additionally, in order to attract the H-type, the deviating buyer would need to offer her a higher payoff than the solution to P1, which is not possible.

(iii) Consider a deviation that attracts only the H-type. There are two cases to consider. Case (1). Suppose that $\mu_0 \leq \widetilde{\mu}$, so that the solution to P1 is separating, i.e., $p_L^{P1} = E_L[X]$. A profitable deviation that attracts only the H-type to some contract $\{(F_H, p_H)\}$ would have to satisfy the constraints:

$$p_H + \delta \cdot E_H[X - F_H] \geq E_L[X],$$
$$E_L[X] \geq p_H + \delta \cdot E_L[X - F_H],$$
$$p_H \leq E_H[F_H],$$

as, after the deviation, the equilibrium menus do not make losses and are therefore not withdrawn at Stage 2. But note that these are exactly the constraints in program P1 (where the last constraint is effectively the buyers' participation constraint as $p_L^{P1} = E_L[X]$). Therefore, it is impossible that such a deviation yields the H-type a higher payoff than the equilibrium allocation, a contradiction.

Case (2). Suppose that $\mu_0 > \widetilde{\mu}$, so that at the solution to P1 involves cross-subsidization, i.e., $p_L^{P1} > E_L[X]$. Consider a deviation by a buyer to attract the H-type to an allocation (F_H, p_H). If κ is small enough (i.e., $\kappa < (1-\mu_0) \cdot \left(p_L^{P1} - E_L[X]\right)$), non-deviating buyers offering (X, p_L^{P1}) withdraw their menus at Stage 2 as these are now loss-making. But then, the L-type must also be picking a non-trivial contract from the deviating menu, a contradiction. ■

Proof of Lemma 3.1. We prove this result by establishing a set of intermediate lemmas. We begin by guessing that the incentive compatibility constraint (11) in program P2 is slack at the optimum and, thus, dropping it from the program. We then verify that this is indeed the case.

The first lemma shows that it is without loss to assume that the L-type transfers all of her cash flows to the buyers.

Lemma A.4 *It is without loss of generality to set $F_L(x) = x \ \forall x$.*

Proof. Suppose that at a solution to program (P2), (F_L, p_L) is such that $F_L(x) < x$ for some x. Consider changing the L-type's allocation to (F', p') with $F'(x) = x$ for all x and $p' = p_L + E_L[X - F_L(X)]$. Observe that the H-type's payoff is unaffected, and the remaining constraints are still satisfied. ■

The second lemma shows that at the optimum the incentive compatibility constraint of the L-type, given by (10), and the buyers' participation constraint, given by (12), bind. However, constraint (13) is slack.

Lemma A.5 *The constraints* (10) *and* (12) *are satisfied with equality, whereas the constraint* (13) *is slack.*

Proof. Suppose not. If the constraint (12) were slack, then the objective of the program could be increased by raising p_H and p_L by a small amount $\varepsilon > 0$, which leaves the constraints (10) and (13) satisfied, a contradiction.

If instead the constraint (10) were slack, then there are two possibilities. If also the constraint (13) is slack, then decreasing p_L by ε and increasing p_H by $\frac{1-\mu_0}{\mu_0} \cdot \varepsilon$ increases the objective while, for $\varepsilon > 0$ sufficiently small, still satisfying the constraints (10), (12) and (13), a contradiction. On the other hand, if the constraint (13) binds and thus $p_L = E_L[X]$, it must be that $F_H(x) = 0$ for all x; otherwise, since the constraint (12) binds, the constraint (10) would have to be violated. But then, the constraint (10) is satisfied with equality as both sides are equal to $E_L[X]$, a contradiction.

Finally, suppose that the constraint (13) is satisfied with equality. From the constraints (10) and (12), it must be that $p_H = E_H[F_H]$ and therefore $F_H(x) = 0$ for all x, which yields an expected payoff $\delta \cdot E_H[X]$ to the H-type. But then, it is straightforward to show that the allocations proposed in the statement of Lemma 3.1 both raise the objective of the program and satisfy the constraints (10)-(13), a contradiction. ■

The third lemma shows that the H-type issues a non-trivial debt security.

Lemma A.6 $F_H(x) = \min\{d, x\}$ $\forall x$ *and some* $d \in (0, \bar{X}]$.

Proof. Suppose to the contrary that F_H is not a debt security, and let d' be such that $\mathbb{E}_H[F_H(X)] = \mathbb{E}_H[\min\{d', X\}]$. Since $F_H(\cdot)$ is monotonic, there exists $\widehat{x} \in [0, \bar{X}]$ such that $F_H(x) > d'$ if and only if $x > \widehat{x}$. By MLRP, it must then be that $\mathbb{E}_L[X - F_H(X)] > \mathbb{E}_L[X - \min\{d', X\}]$, since both securities have the same *high*-valuation:

$$\begin{aligned}\mathbb{E}_L[F_H(X) - \min\{d', X\}] &= \mathbb{E}_H\left[(F_H(X) - \min\{d', X\}) \cdot \frac{g_L(X)}{g_H(X)}\right] \\ &< \mathbb{E}_H[F_H(X) - \min\{d', X\}] \cdot \frac{g_L(\widehat{x})}{g_H(\widehat{x})} = 0.\end{aligned}$$

Thus, whereas the H-type is indifferent between (F_H, p_H) and (F', p_H) with $F'(X) = \min\{d', X\}$, the L-type strictly prefers her allocation (X, p_L) to (F', p_H). Next, consider changing the allocation of the H-type to (F'', p'') with $F''(X) = \min\{d'', X\}$, $p'' = p_H + \mathbb{E}_H[\min\{d'', X\} - \min\{d', X\}]$, and $d'' > d'$. If d'' is close to d', then the L-type still prefers to accept (X, p_L) to (F'', p''), and the H-type strictly prefers (F'', p'') to (F_H, p_H). Hence, such a change increases the objective of the program while leaving the constraints (6), (8) and (9) satisfied, a contradiction. ■

Next, we determine the optimal debt level. Define $d^* : [0, 1] \to \left[0, \bar{X}\right]$ as follows:

$$d^*(\mu_0) = \arg\max_{d \in \left[0, \bar{X}\right]} E_{\mu_0}[\min\{d, X\}] - \delta \cdot E_H[\min\{d, X\}]. \tag{23}$$

From Lemmas A.4-A.6, this is the debt level that solves program P2. The derivative of the expression on the right-hand side of equation (23) w.r.t. d is:

$$\widehat{H}(d, \mu_0) = (\mu_0 - \delta) \cdot (1 - G_H(d)) + (1 - \mu_0) \cdot (1 - G_L(d)). \tag{24}$$

Note the similarity between equations (24) and (22), where the only difference is that the last term does not have a δ multiplying it. Therefore, by arguments similar to those in the proof of Lemma 2.1, it follows that $d^*(\mu_0)$ is continuously increasing in μ_0 until it reaches $\bar{X}$ at $\widehat{\mu} = \lim_{x \to \bar{X}} \frac{\delta \cdot \frac{g_H(x)}{g_L(x)} - 1}{\frac{g_H(x)}{g_L(x)} - 1}$, which note is strictly smaller than $\bar{\mu}$, which was defined in the proof of Lemma 2.1. Moreover, by inspection of equation (24), $d^*(\mu_0) > d^C(\mu_0)$ for any $\mu_0 < \bar{\mu}$.

Finally, the result stated in the proposition follows from Lemmas A.4-A.6 together with the following verification that the constraint (11) is always slack. The latter holds if:

$$p_H + \delta \cdot E_H\left[X - \min\left\{d^*, X\right\}\right] \geq p_L = p_H + E_L\left[X - \min\left\{d^*, X\right\}\right],$$

which holds if and only if:

$$\delta \cdot E_H\left[X - \min\left\{d^*, X\right\}\right] \geq E_L\left[X - \min\left\{d^*, X\right\}\right].$$

But the last inequality follows by the observation that, if it were violated, then it would be possible to increase the payoff to the H-type by simply increasing the debt level above d^*, contradicting the optimality of the security $F_H(X) = \min\{d^*, X\}$. ■

Proof of Proposition 3.1. Consider the following candidate equilibrium strategies. At Stage 1, buyers 1 and 2 post contract (F_S, p_S), buyers 3 and 4 offer to purchase any feasible security at *low*-valuation $\{F, E_L[F]\}_{F\in\Phi}$, while the remaining buyers offer the trivial contract, $\{(0,0)\}$. At Stage 2, there are no menu withdrawals. At Stage 3, the H-type accepts contract (F_S, p_S) from buyer 1 whereas the L-type accepts (F_S, p_S) from buyer 1 and (F_J, p_J) from buyer 3.

By construction, there are no profitable deviations for the seller at Stage 3. Also, since in the candidate equilibrium buyers break even, there are also no deviations at Stage 3 as $\kappa > 0$. We are therefore left to rule out deviations at Stage 1. For this, note that the menus of buyers 3 and 4 are never withdrawn at Stage 2, since they are never loss-making.

(i) Consider a deviation that attracts only the L-type. Such a deviation clearly cannot be profitable as it would need to price L-type's cash flows above *low*-valuation.

(ii) Consider a deviation that attracts only the H-type. There are two possibilities.

First, suppose that H-type no longer accepts contract (F_S, p_S). As the menu of any buyer who posted contract (F_S, p_S) at Stage 1 now only attracts the L-type, it makes losses equal to $(1-\mu_0)\cdot(E_{\mu_0}[F_S] - E_L[F_S]) > 0$; thus, if κ is smaller than these losses (which we assume), such a menu would be withdrawn at Stage 2. But then, the deviating buyer would also attract the L-type, who would be strictly better off by picking up a contract from the deviating menu and selling the remaining cash flows by accepting a latent contract from other buyers, a contradiction.

Second, suppose that the H-type also accepts contract (F_S, p_S). But then, the deviation cannot only attract the H-type, since the L-type could obtain a higher payoff by simply mimicking the H-type and issuing any residual cash flows using latent contracts, a contradiction.

(iii) Consider a deviation that attracts both types. There are again two possibilities.

First, suppose that the H-type no longer accepts contract (F_S, p_S). By the same argument as in (ii), it must be that the L-type also does not accept contract (F_S, p_S) following the deviation. Note that the allocations of such a deviation must satisfy the seller incentive and participation constraints (10), (11) and (13) of program P2, as the L-type has access to the menus of buyers 3 or 4, which would not be withdrawn at Stage 2. But then, such a deviation cannot at the same time attract the H-type and be profitable for the deviating buyer.

Second, suppose that the H-type continues to accept contract (F_S, p_S). But then, once again, the allocations following the deviation must satisfy constraints (10), (11) and (13) of program P2, such a deviation cannot at the same time attract the H-type and be profitable for the deviating buyer. ■

The following two lemmas will be useful to establish Proposition 3.2.

Lemma A.7 *In any equilibrium, each buyer earns zero expected profits, and the L-type seller transfers all of her cash flows to the buyers.*

Proof. Let $F_\theta(x)$ denote the aggregate cash flows that the θ-type seller transfers to buyers in state $X = x$; and let p_θ denote the aggregate transfer that she receives from the buyers.

Suppose to the contrary that the buyers' aggregate expected profits are positive (not that a buyer cannot earn negative profits, as she can always earn zero by posting only the trivial contract). Suppose now that buyer i who were earning less than half of the aggregate profits were to deviate and add the contracts $\{(F_\theta, p_\theta + \varepsilon)\}$ to his menu. Clearly, the seller would pick these contracts instead of her equilibrium allocation $\{(F_\theta, p_\theta)\}$, independently of whether the other buyers withdraw their menus or not. Moreover, for ε small enough, such a deviation is profitable as buyer i effectively captures all of the aggregate expected profits, a contradiction.

Suppose to the contrary that there exists an equilibrium in which $F_L(x) < x$ for some x. Consider a deviation by a buyer to add to his menu the contracts $(F_H, p_H + \varepsilon)$ and (F, p) with $F(x) = x$ for all x and $p = p_L + \delta \cdot E_L[X - F_L(X)] + 2 \cdot \varepsilon$. Clearly, this deviation attracts both types for any $\varepsilon > 0$. We next show that these allocations are also incentive compatible and profitable for ε small enough. That it attracts the L-type to contract (F, p) follows from the fact that L-type preferred (F_L, p_L) to (F_H, p_H) prior to the deviation:

$$p_L + \delta \cdot E_L[X - F_L] \geq p_H + \widehat{p}(X - F_H),$$

where $\widehat{p}(X - F_H)$ is the L-type's maximal payoff from selling or retaining the residual cash flows $X - F_H$, if she were to mimic the H-type. But then, we have that:

$$p_L + \delta \cdot E_L[X - F_L] + 2 \cdot \varepsilon > p_H + \varepsilon + \widehat{p}(X - F_H),$$

for any $\varepsilon > 0$. Analogously, for the H-type:

$$p_H + \delta \cdot E_H[X - F_H] \geq p_L + \delta \cdot E_H[X - F_L].$$

Since $E_H[X - F_L] > E_L[X - F_L]$, for ε small enough, we have:

$$p_H + \varepsilon + \delta \cdot E_H[X - F_H] > p_L + \delta \cdot E_L[X - F_L] + 2 \cdot \varepsilon.$$

Lastly, the expected profits of the deviating buyer are given by

$$\Pi = (1 - \mu_0) \cdot [(1 - \delta) \cdot E_L[X - F_L] - 2 \cdot \varepsilon] - \mu_0 \cdot \varepsilon,$$

which are positive for ε small enough. ■

Lemma A.8 *In any equilibrium, $u_H \geq u_H^*$.*

Proof. Suppose to the contrary that there is an equilibrium in which u_H is strictly below $u_H^* = p_S + \delta \cdot E_H[X - F_S]$. By Lemma A.7, buyers must break even in this equilibrium. But then, consider a deviation for a buyer to replace his menu with a menu containing the contract

$(F_S, p_S - \varepsilon)$. For ε small enough, this contract would attract the H-type (independently of whether other buyers withdraw at Stage 2) and possibly the L-type. As a result, the deviation would be profitable, a contradiction. ■

Proof of Proposition 3.2. Let $F_\theta(x)$ denote the aggregate cash flows that the θ-type seller transfers to buyers in state $X = x$; and let p_θ denote the aggregate transfer that she receives from the buyers. By Lemma A.7, we have that $F_L(x) = x$ for all x and:

$$\mu_0 \cdot (E_H F_H - p_H) + (1 - \mu_0) \cdot (E_L X - p_L) = 0. \tag{25}$$

Thus, to demonstrate that the allocations between each seller type and the buyers are unique, it suffices to show that $F_H = F_S$ and $p_H = p_S$.

Suppose that $\mu_0 \geq \widehat{\mu}$. This implies that for all non-trivial $F \in \Phi$, we have $E_{\mu_0}[F] > \delta \cdot E_H[F]$. Then, $F_S = X$ and $u^*_\theta = E_{\mu_0} X$. Thus, the star-equilibrium is unique if and only if $F_H = X$, since then we also have $p_H = p_L$. Suppose to the contrary that $F_H \neq X$.

By Lemma A.8, $u_H \geq u^*_H$. Since $F_S = X = \arg\max_{F \in \Phi} E_{\mu_0}[F] - \delta \cdot E_H[F]$ and $u^*_H = E_{\mu_0}[X]$, it must be that $p_H > E_{\mu_0}[F_H]$. Because buyers break even in equilibrium, it must also hold that $p_L < E_{\mu_0}[X]$. Consider a deviation by a buyer to replace his menu with a menu containing the contracts $(F, E_{\mu_0}[F] - \varepsilon_F)$ for all $F \in \Phi$ with $\varepsilon_F = \alpha \cdot (E_{\mu_0}[F] - \delta \cdot E_{\mu_0}[F])$ for $\alpha \in (0, 1)$ satisfying $E_{\mu_0}[X] - \varepsilon_X > p_L$. Let $(\widetilde{F}, \widetilde{p})$ denote the allocation that the H-type picks up from the (remaining) equilibrium menus after the deviation. Note that $\widetilde{F} \neq X$; otherwise, since it must also hold that $\widetilde{p} \geq E_{\mu_0}[X] - \varepsilon_X > p_L$, the L-type would have mimicked the H-type prior to the deviation. By construction, the H-type will sell the remaining cash flows $X - \widetilde{F}$ to the deviating buyer. Further, the L-type will mimic the H-type, since both types are maximizing prices at which they sell their entire cash flow X. But then, the deviation is profitable since $E_{\mu_0}[X - \widetilde{F}] > \delta \cdot E_{\mu_0}[X - \widetilde{F}]$

Suppose instead that $\mu_0 < \widehat{\mu}$, and that the L-type is able to issue any feasible security at weakly above *low*-valuation. With this, equilibrium allocations must satisfy the following incentive compatibility constraint:

$$\begin{aligned} & p_L \geq p_H + E_L[X - F_H] \\ & \iff \\ & p_H \leq E_{\mu_0}[F_H], \end{aligned}$$

where we again made use of equation (25). But then, it must be that:

$$\begin{aligned} u_H &= p_H + \delta \cdot E_H[X - F_H] \\ &\leq E_{\mu_0} F_H + \delta \cdot E_H[X - F_H] \\ &\leq u^*_H, \end{aligned}$$

where the last inequality holds since $F_S = \arg\max_{F \in \Phi} E_{\mu_0} F + \delta \cdot E_H[X - F]$. Thus, if $F_H \neq F_S$ or $p_H \neq p_S$, then $u_H < u^*_H$, which contradicts Lemma A.8. ■

Proof of Proposition 4.1. Under both exclusive and non-exclusive market structures, the buyers break even. So the entire trading surplus accrues to the seller.

Given buyers prior belief μ_0, when markets are non-exclusive, the expected trading surplus (i.e., the expected welfare of the seller) is given by:

$$W^{NE}(\mu_0) \equiv \mathbb{E}_{\mu_0}[X] - \mu_0 \cdot (1-\delta) \cdot \mathbb{E}_H[X - \min\{d^*(\mu_0), X\}], \tag{26}$$

whereas the expected trading surplus with exclusive markets is given by:

$$W^{E}(\mu_0) \equiv \mathbb{E}_{\mu_0}[X] - \mu_0 \cdot (1-\delta) \cdot \mathbb{E}_H[X - \min\{\max\{d^S, d^C(\mu_0)\}, X\}]. \tag{27}$$

Hence, we have that:

$$W^{NE}(\mu_0) - W^{E}(\mu_0) = \mu_0 \cdot (1-\delta) \cdot \mathbb{E}_H[\min\{d^*(\mu_0), X\} - \min\{\max\{d^S, d^C(\mu_0)\}, X\}]. \tag{28}$$

The result then follows from the following three observations:

(i) $d^*(\mu_0) < d^S$ for μ_0 sufficiently small. This is because by revealed preference in non-exclusive markets the H-type is willing to issue the security $\min\{d^*(0), X\}$ at low valuation. However, in exclusive markets, the H-type strictly prefers not to issue any additional cash flows (i.e., more than $d^E = d^S$) at low valuation (otherwise, in equilibrium she would had done so), i.e., $d^S > d^*(0)$. By continuity of $d^*(\cdot)$ (see proof of Proposition 3.1), this must also be the case for μ_0 small enough.

(ii) $d^*(\mu_0)$ is continuously increasing and crosses d^S at some $\underline{\mu} < \widetilde{\mu}$, i.e., when $d^E = d^S$. See the proof of of Lemma 3.1.

(iii) $d^*(\mu_0) > d^C(\mu_0)$ for all $\mu_0 < \bar{\mu}$, where $\bar{\mu}$ is such that $d^*(\mu_0) = \bar{X}$. See the proof of Lemma 3.1. ■

Proof of Proposition 4.2. Recall that the L-type's payoff u_L^{NE} in non-exclusive markets is bounded below by her payoff u_L^* in the *star equilibrium* (Proposition B.1), which is strictly greater than her payoff u_L^E in exclusive markets as long as $\mu_0 < \bar{\mu}$ (the only possibility when the asset quality is endogenous). It is therefore sufficient to show that the H-type's payoff u_H^{NE} in non-exclusive markets is weakly lower than her payoff u_H^E in exclusive markets, and thus $u_H^E - u_L^E > u_H^{NE} - u_L^{NE}$. But, the latter follows from the observation that, whereas u_H^E is the maximum of program P1, any equilibrium allocation in non-exclusive markets must satisfy the constraints (6) and (8) with $F_L(x) = x$ for all x (Lemma A.7) and $p_L \geq u_L^*$; that is, the L-type's participation constraint (9) is tightened. ■

B Supplementary Appendix

B.1 Microfounding feasibility conditions: Bundling assets in anonymous markets

A central friction that we consider throughout our analysis is that of market non-exclusivity. A way to view non-exclusivity is that it captures a form of market anonymity or opacity, whereby agents cannot observe (and therefore contract upon) all the trades that their counterparties enter into. The reader may wonder, however, whether the feasibility conditions (CC) and (WM), which we have exogenously imposed on the seller's strategy, are consistent with that view of the world. In what follows, we describe such an environment, which has two key features.

Limited commitment. Suppose that neither the seller nor the buyers have the commitment to deliver goods to their contracting counterparties at $t = 2$, i.e., upon learning X an agent can always default and walk away with all the goods he or she has. This immediately implies that any credible contract between the seller and a buyer must take the form of a spot trade at $t = 1$, consisting of a transfer of assets in exchange for a payment (price).

Financial innovation. We have assumed that the seller is endowed with an asset at $t = 1$ that delivers risky cash flows X at $t = 2$. Suppose that, due to financial innovation, the seller is able to split the asset's cash flows into separate assets that can be individually traded in spot markets. In particular, the seller has one unit of each of the following so-called basis assets, indexed by $a \in [0, \bar{X}]$, with payoffs:

$$a\left(x\right) = \begin{cases} 0 & \text{if } x < a \\ 1 & \text{if } x \geq a \end{cases}. \tag{29}$$

Thus, $X = \int_0^{\bar{X}} a(X) \cdot da$.

In this context, security design simply means that the seller can choose which bundle of basis assets to sell to the buyers in the spot market: $F(X) = \int_0^{\bar{X}} g_F(a) \cdot a(X) \cdot da$ where $g_F(\cdot) \in [0, 1]$ determines the amount of each basis asset sold in the bundle.[21] Observe that the feasibility conditions hold naturally in this setting:

- (CC) If the seller transfers a bundle of basis assets to a buyer, she cannot transfer the cash flows of these assets to another buyer since she no longer has them.
- (WM) If the seller sells a collection of securities $\mathcal{F} = \{F\}$ to the buyers, then by construction $x - \sum_{F \in \mathcal{F}} F(x)$ and, for each F, $F(x)$ are weakly increasing in x.

Finally, our notion of non-exclusivity is equivalent to assuming that each buyer observes the bundle of basis assets that the seller transfers to him, but he cannot observe the assets or prices at which the seller trades with other buyers.

B.2 Other equilibria

In this Appendix, we provide a further characterization of the equilibrium set; the formal proofs can be found further below. The following result will be useful in the characterization. Let $\{u^*_\theta\}$ denote the seller's payoffs in the star-equilibrium. Then:

Proposition B.1 *There exists an $\omega > 0$ such that in any equilibrium the seller's payoffs, $\{u_\theta\}_\theta$, satisfy the following bounds: $u^*_H \leq u_H \leq u^*_H + \omega \cdot \kappa$ and $u^*_L \leq u_L$.*

Thus, we see that as the menu withdrawal cost, κ, becomes small, the H-type's equilibrium payoff must coincide with her payoff in the star-equilibrium. In what follows, therefore, we will focus on the construction of equilibria in which the H-type's payoff is equal to u^*_H whereas the L-type's payoff is strictly greater than u^*_L. By Proposition 3.2, this requires that $\mu_0 < \widehat{\mu}$ and $F_S \neq X$.

[21] See also Axelson (2007) for such a representation of monotone securities.

Recall that, in any equilibrium, buyers earn zero expected profits and all gains from trade with the L-type are exhausted (Lemma A.7). Let the allocations of this candidate equilibrium be denoted by (F_H, p_H) and (X, p_L), and note that these allocations must satisfy:

$$p_H = p_S + \delta \cdot E_H [F_H - F_S], \tag{30}$$

$$p_L = p_S + E_L [X - F_S] + (1 - \delta) \cdot \frac{\mu_0}{1 - \mu_0} \cdot E_H [F_H - F_S], \tag{31}$$

where we imposed both buyer-zero profits and $u_H = u_H^*$. Note that, since $F_S = \arg\max_{F \in \Phi} E_{\mu_0} [F] - \delta \cdot E_H [F]$ and $u_L > u_L^*$, we can already conclude that the following properties must hold: (i) $p_H > E_{\mu_0} [F_H]$, (ii) $E_H [F_H] > E_H [F_S]$, and (iii) $p_H > p_S$.

Moreover, the allocations (F_H, p_H) and (X, p_L) must be incentive compatible:

$$p_L \geq p_H + \delta \cdot E_L [X - F_H], \tag{32}$$

$$p_H + \delta \cdot E_H [X - F_H] \geq p_L. \tag{33}$$

We next show that if also $F_H(x) \geq F_S(x)$ for all x and κ is sufficiently small, then the above allocations can be supported as a PBE. It is also straightforward to show that the set of allocations (F_H, p_H) and (X, p_L) satisfying the properties (30)-(33) and $F_H \geq F_S$ with $F_H \neq F_S$ is non-empty.

Consider the following candidate equilibrium. Suppose that the contracts (F_H, p_H) and (X, p_L) are posted by buyers 1 and 2; in addition, buyers 3 and 4 also post the contract (F_S, p_S), buyers 5 and 6 post the contracts $\{(F, p(F))\}_{F \in \Phi}$ satisfying:

$$p(F) = E_L [X - F_S] - \frac{\delta - \mu_0}{1 - \mu_0} \cdot E_H [X - F - F_S],$$

and all other buyers post the trivial contract. Note that $p(F) \leq E_L [F]$ with equality if $F = X - F_S$, as $F_S = \arg\max_{F \in \Phi} E_{\mu_0} [F] - \delta \cdot E_H [F]$.[22]

We first rule out deviations by the seller at Stage 3. First, the L-type seller prefers her equilibrium allocation to any other allocation available in the candidate equilibrium menus. To see this, note first that the L-type does not want to pick up (F_H, p_H) together with any feasible collection of latent contracts, i.e.,

$$p_L \geq p_H + p(F) + \delta \cdot E_L [X - F_H - F]$$

for all $F \leq X - F_H$. This follows from the observations (i)-(iii) below:

[22]An equivalent implementation of the allocations (F_H, p_H) and (X, p_L) is for the buyers 1 and 2 to offer instead the contracts (F_{Mz}, p_{Mz}) and (F_J, p_J) with (i) $F_{Mz} \equiv F_H - F_S$ and $p_{Mz} \equiv p_H - p_S$, and (ii) $F_J \equiv X - F_S$ and $p_J \equiv p_L - p_S$. Both seller types accept (F_S, p_S) from buyer 3 (or 4), and in addition the H-type and the L-type accept respectively the contracts (F_{Mz}, p_{Mz}) and (F_J, p_J) from buyer 1 (or 2).

(i) $p(F_1) + p(F_2) \leq p(F_1 + F_2)$ for any F_1 and F_2 that can be issued jointly:

$$\begin{aligned} p(F_1) + p(F_2) &= 2 \cdot E_L[X - F_S] - \frac{\delta - \mu_0}{1 - \mu_0} \cdot E_H[2 \cdot X - F_1 - F_2 - 2 \cdot F_S] \\ &= p(F_1 + F_2) + \frac{1}{1 - \mu_0} \cdot [E_{\mu_0}[X - F_S] - \delta \cdot E_H[X - F_S]] \\ &< p(F_1 + F_2), \end{aligned}$$

where the inequality follows since $F_S = \arg\max E_{\mu_0}[F] - \delta \cdot E_H[F] \neq X$.

(ii) $p(F) + \delta \cdot E_L[X - F_H - F] \leq p(X - F_H)$ for any $F \leq X - F_H$, since the optimal security that maximizes $p(F) + \delta \cdot E_L[X - F_H - F]$ is given by $F = X - F_H$:

$$\max_{F \in \Phi} \frac{\delta - \mu_0}{1 - \mu_0} \cdot E_H[F] - \delta \cdot E_L[F]$$

$$\text{s.t. } 0 \leq F(x) \leq x - F_H(x), \forall x.$$

The solution to this problem, $F(x) = \int_0^x s(z) \cdot dz$, satisfies:

$$s(x) = \begin{cases} 1 & \text{if } \frac{\delta - \mu_0}{1 - \mu_0} \cdot (1 - G_H(x)) > \delta \cdot (1 - G_L(x)) \\ 0 & \text{otherwise.} \end{cases}$$

We know that $\frac{\delta - \mu_0}{1 - \mu_0} \cdot (1 - G_H(x)) \geq 1 - G_L(x) > \delta \cdot (1 - G_L(x))$ for all $x \in [d^*, \bar{X})$ (see proof of Lemma A.6). Since $F_H(x) \geq F_S(x) = \min\{d^*, x\}$ for all x, it follows that $F = X - F_H$.

(iii) By construction, $p_L = p_H + p(X - F_H)$.

Similarly, the L-type does not want to pick up (F_S, p_S), i.e.,

$$p_L \geq p_S + p(F) + \delta \cdot E_L[X - F_S - F]$$

for all $F \leq X - F_S$. This follows from observations (i)-(iii) below:

(i) $p(F_1) + p(F_2) \leq p(F_1 + F_2)$ for any securities F_1 and F_2 that is feasible to issue together (see the argument above).

(ii) $p(F) + \delta \cdot E_L[X - F_S - F] \leq p(X - F_S)$ for any $F \leq X - F_S$, since the optimal security that maximizes the payoff $p(F) + \delta \cdot E_L[X - F_S - F]$ is given by $F = X - F_S$ (see the argument above).

(iii) $p_L > p_S + p(X - F_S)$, since $F_H(x) \geq F_S(x), \forall x$ with $F_H \neq F_S$, and thus:

$$\begin{aligned} p_L &= p_H + p(X - F_H) \\ &= p_S + \delta \cdot E_H[F_H - F_S] + E_L[X - F_S] - \frac{\delta - \mu_0}{1 - \mu_0} \cdot E_H[F_H - F_S] \\ &> p_S + p(X - F_S). \end{aligned}$$

Second, the H-type prefers her equilibrium allocation to any other allocation available in the candidate equilibrium menus. This follows from the fact that the H-type does not want to sell any cash flows beyond F_S at or below *low*-valuation (and she is already selling the cash flows $F_H \geq F_S$), and the H-type strictly prefers her allocation to the L-type's:

$$\begin{aligned} u_H^* &= p_H + \delta \cdot E_H[X - F_H] \\ &> p_H + E_L[X - F_H] \\ &\geq p_H + p(X - F_H) \\ &= p_L. \end{aligned}$$

We have thus ruled out deviations by the seller at Stage 3. Since the buyers break even in the candidate equilibrium, there are no profitable deviations at Stage 2. We are therefore left to rule out deviations by the buyers at Stage 1. We do so next.

Observe that following a deviation by a buyer at Stage 1, (i) the menus containing the contracts $(F, p(F))$ are never loss-making and therefore are not withdrawn, (ii) the menus containing (F_S, p_S) are withdrawn if contract (F_S, p_S) is only picked up by the L-type, and (iii) the menus containing the contracts $(F_H, p_H), (X, p_L)$ are withdrawn if either these attract the L-type to either contract but not the H-type or if both types are attracted to (F_H, p_H). The observations (ii) and (iii) follow from:

$$\begin{aligned} p_H - E_L[F_H] &\geq p_L - E_L[X] \\ &= p_H + p(X - F_H) - E_L[X] \\ &= E_{\mu_0}[F_S] - E_L[F_S] + \left(\delta - \frac{\delta - \mu_0}{1 - \mu_0}\right) \cdot E_H[F_H - F_S] \\ &> E_{\mu_0}[F_S] - E_L[F_S] \end{aligned}$$

since $F_H \geq F_S$ and, in addition, if we assume that κ is smaller than (a) the expected losses $(1-\mu_0)\cdot(E_{\mu_0}[F_S] - E_L[F_S]) > 0$ if contract (F_S, p_S) were only accepted by the L-type at Stage 3, and (b) the expected losses $p_H - E_{\mu_0}[F_H] > 0$ if both types were attracted to the contract (F_H, p_H) at Stage 3. Given this, any contract posted in the candidate equilibrium menus that remains posted after Stage 2 must earn non-negative expected profits. It is therefore without loss of generality to focus on deviations at Stage 1 where the deviating buyer attracts both seller types without them also picking up contracts from the equilibrium menus.

Suppose that there is a profitable deviation that attracts the H-type to some contract $(\widetilde{F}_H, \widetilde{p}_H)$ and the L-type to some contract $(\widetilde{F}_L, \widetilde{p}_L)$. It is also without loss of generality to assume that $\widetilde{F}_L = X$, as the buyer's profits can only increase by exploiting more gains from trade with the L-type. Thus, to attract the H-type, it must be that:

$$\begin{aligned} \widetilde{p}_H + E_H[X - \widetilde{F_H}] &> u_H^* \\ \iff \widetilde{p}_H &> p_S + \delta \cdot E_H[\widetilde{F}_H - F_S]. \end{aligned}$$

since the H-type always has the option to accept (F_S, p_S) and obtain u_H^*.[23] In turn, the L-type's

[23] Note that the menus containing contract (F_S, p_S) are not be withdrawn if, following the deviation, (F_S, p_S)

incentive compatibility requires that:

$$\widetilde{p}_L \geq \widetilde{p}_H + p(X - \widetilde{F}_H) = \widetilde{p}_H + E_L[X - F_S] - \frac{\delta - \mu_0}{1 - \mu_0} \cdot E_H[\widetilde{F}_H - F_S].$$

But then, the deviating buyer's expected profits are given by:

$$\begin{aligned}
\widetilde{\Pi} &= \mu_0 \cdot (E_H[\widetilde{F}_H] - \widetilde{p}_H) + (1 - \mu_0) \cdot (E_L[X] - \widetilde{p}_L) \\
&< \mu_0 \cdot (E_H[\widetilde{F}_H] - p_S - \delta \cdot E_H[\widetilde{F}_H - F_S]) + \\
&+ (1 - \mu_0) \cdot \left(E_L[\widetilde{F}_H] - p_S - \delta \cdot E_H[\widetilde{F}_H - F_S] - E_L[\widetilde{F}_H - F_S] + \frac{\delta - \mu_0}{1 - \mu_0} \cdot E_H[\widetilde{F}_H - F_S] \right) \\
&= E_{\mu_0}[\widetilde{F}_H - F_S] - \delta \cdot E_H[\widetilde{F}_H - F_S] + (1 - \mu_0) \cdot \left(-E_L[\widetilde{F}_H - F_S] + \frac{\delta - \mu_0}{1 - \mu_0} \cdot E_H[\widetilde{F}_H - F_S] \right) \\
&= 0.
\end{aligned}$$

Hence, the deviation could not have been profitable, a contradiction.

B.2.1 Additional Proofs for Appendix B.2

The following three lemmas will be useful in the proof of Proposition B.1.

Lemma B.1 *In any equilibrium, $u_L \geq u_L^*$.*

Proof. By Lemma A.8, in any equilibrium $u_H \geq u_H^*$. Suppose to the contrary that $u_L < u_L^*$. Let (F_θ, p_θ) denote the equilibrium allocations of the θ-type seller. By Lemma A.7, $F_L = X$, $p_L = u_L$ and buyers make zero expected profits. As buyers make zero expected profits and $u_H \geq u_H^*$, it must be that the IC constraint of Program P2 is violated, i.e., $u_L < p_H + E_L[X - F_H]$. Given this, we consider the following deviation by a buyer. We will construct a profitable deviation recursively, so we let $(\widetilde{F}_{\theta 0}, \widetilde{p}_{\theta 0}) \equiv (F_\theta, p_\theta)$ and $\widetilde{u}_{\theta 0} \equiv u_\theta$.

Step 0. Consider a deviation by a buyer to replace his menu with a menu containing contract $\mathcal{C}_0^d = (X - \widetilde{F}_{H0}, E_L[X - \widetilde{F}_{H0}] - \epsilon_0)$ for $\epsilon_0 > 0$ such that $\widetilde{u}_{L0} < \widetilde{p}_{H0} + E_L[X - \widetilde{F}_{H0}] - \epsilon_0$. If allocation $(\widetilde{F}_{H0}, \widetilde{p}_{H0})$ is still available at Stage 3, then such a deviation attracts the L-type who would combine the deviating contract with $(\widetilde{F}_{H0}, \widetilde{p}_{H0})$. If contract C_0^d is picked up at Stage 3, then the deviation is profitable. Thus, the deviating menu is $\{\mathcal{C}_0^d\}$.

If instead contract $\mathcal{C}_0^d$ is not picked up at Stage 3, it must be that allocation $(\widetilde{F}_{H0}, \widetilde{p}_{H0})$ is not available at Stage 3, i.e., due to withdrawal of some menus. Thus, the seller must be obtaining allocation $(\widetilde{F}_{\theta 1}, \widetilde{p}_{\theta 1})$ from equilibrium menus (but different from equilibrium allocations) and obtaining payoff $\widetilde{u}_{\theta 1} \leq \widetilde{u}_{\theta 0}$. Finally, let $\Delta_0 = p_H + E_L[X - F_H] - \epsilon_0 - \widetilde{u}_{L0}$. With $\{\Delta_0, (\widetilde{F}_{\theta 1}, \widetilde{p}_{\theta 1}), \widetilde{u}_{\theta 1}\}$, we proceed to Step 1.

Step $n \geq 1$. Let $(\widetilde{F}_{\theta n}, \widetilde{p}_{\theta n})$ denote the θ-type's allocation at Stage n, and $\widetilde{u}_{\theta n}$ her payoff. We can assume without loss that $\widetilde{F}_{L,n} = X$ and, therefore, that $\widetilde{p}_{Ln} = \widetilde{u}_{Ln}$. Also, note that $\widetilde{u}_{\theta n} \leq \widetilde{u}_{\theta n-1}$. With this, we adjust the candidate deviation in Step $n - 1$ as follows.

is to be picked up by the H-type.

- If $\widetilde{u}_{Ln} < \widetilde{p}_{Hn} + E_L[X - \widetilde{F}_{Hn}]$, then adjust the menu by adding the contract $\mathcal{C}_n^d = (X - \widetilde{F}_{Hn}, E_L[X - \widetilde{F}_{Hn}] - \epsilon_n)$ such that $\widetilde{u}_{Ln} < \widetilde{p}_{Hn} + E_L[X - \widetilde{F}_{Hn}] - \epsilon_n \equiv \widetilde{u}_{Ln} + \Delta_n < \widetilde{u}_{Ln-1} + \Delta_{n-1}$. If contract $\mathcal{C}_n^d$ is picked up at Stage 3, then the deviation is profitable. Thus, the deviating menu is $\left\{\mathcal{C}_0^d, ..., \mathcal{C}_n^d\right\}$.

  If instead contract $\mathcal{C}_n^d$ is not picked up at Stage 3, it must be that allocation $(\widetilde{F}_{\theta n}, \widetilde{p}_{\theta n})$ is not available at Stage 3. Thus, the seller must be obtaining allocations $(\widetilde{F}_{\theta n+1}, \widetilde{p}_{\theta n+1})$ from equilibrium menus with corresponding payoffs $\widetilde{u}_{\theta n+1}$. With $\{\Delta_n, (\widetilde{F}_{\theta n+1}, \widetilde{p}_{\theta n+1}), \widetilde{u}_{\theta n+1}\}$, we proceed to Step $n+1$.

- If $\widetilde{u}_{Ln} \geq \widetilde{p}_{Hn} + E_L[X - \widetilde{F}_{Hn}]$, then there are two cases to consider:

  1. Suppose $\widetilde{F}_{Hn}(x) \geq F_S(x)$ for all x. If $\widetilde{u}_{Hn} \geq u_H^*$, then $\widetilde{p}_{Hn} \geq E_{\mu_0}[\widetilde{F}_{Hn}]$ since $F_S = \arg\max_{F \in \Phi} E_{\mu_0}[F] - \delta \cdot E_H[F]$ and therefore:

     $$\widetilde{u}_{Ln} \geq E_{\mu_0}[\widetilde{F}_{Hn}] + E_L[X - \widetilde{F}_{Hn}] \geq E_{\mu_0}[F_S] + E_L[X - F_S] = u_L^*,$$

     which contradicts the fact that $\widetilde{u}_{Ln} \leq ... \leq \widetilde{u}_{L0} < u_L^*$. Thus, since $\widetilde{u}_{\theta n} < u_\theta^*$, the buyers aggregate profits must now be positive:

     $$\begin{aligned}
     &\mu_0 \cdot [E_H[\widetilde{F}_{Hn}] - \widetilde{p}_{Hn}] + (1-\mu_0) \cdot [E_L[X] - \widetilde{u}_{Ln}] > \\
     &\mu_0 \cdot [E_H[\widetilde{F}_{Hn}] - u_H^* + \delta \cdot E_H[X - \widetilde{F}_{Hn}]] + (1-\mu_0) \cdot [E_L[X] - u_L^*] = \\
     &\mu_0 \cdot [E_H[F_S] - u_H^* + \delta \cdot E_H[X - F_S] + (1-\delta) \cdot E_H[\widetilde{F}_{Hn} - F_S]] + (1-\mu_0) \cdot [E_L[X] - u_L^*] \geq \\
     &\mu_0 \cdot [E_H[F_S] - u_H^* + \delta \cdot E_H[X - F_S]] + (1-\mu_0) \cdot [E_L[X] - u_L^*] = 0,
     \end{aligned}$$

     where we used the fact that buyers make zero expected profits with the allocations of the star-equilibrium and that $p_S = u_H^* - \delta \cdot E_H[X - F_S]$. But then, adjust the deviating menu by adding contracts $(\widetilde{F}_{Hn}, \widetilde{p}_{Hn} + \upsilon)$ and $(X, \widetilde{u}_{Ln} + \upsilon)$ with $\upsilon > 0$ such that:

     $$\widetilde{u}_{Ln} + \upsilon < \widetilde{u}_{Ln-1} + \Delta_{n-1}.$$

     This inequality ensures that the L-type prefers to mimic the H-type's allocation in all previous steps instead of picking up contract $(X, \widetilde{u}_{Ln} + \upsilon)$. This deviation would attract the H-type to contract $(\widetilde{F}_{Hn}, \widetilde{p}_{Hn} + \upsilon)$ and is thus profitable for υ small enough. Thus, the deviating menu is $\{\mathcal{C}_0^d, ..., \mathcal{C}_{n-1}^d, (\widetilde{F}_{Hn}, \widetilde{p}_{Hn} + \upsilon), (\widetilde{F}_{Ln}, \widetilde{p}_{Ln} + \upsilon)\}$.

  2. Suppose instead that $\widetilde{F}_{Hn}(x) < F_S(x)$ for some x. Then, add to the deviating menu contract $(\widehat{F}, \widehat{p})$ where $\widehat{F}(X) = \min\{d, X - \widetilde{F}_{Hn}(X)\}$, $0 < d < d^* - \widetilde{F}_{Hn}(d^*)$ and $\widehat{p} = E_{\mu_0}[\widehat{F}] - \nu > \delta \cdot E_H[\widehat{F}]$ where (d, ν) are such that the L-type prefers to mimic the H-type's allocation in all previous steps instead of picking up $(\widetilde{F}_{Hn}, \widetilde{p}_{Hn})$ and $(\widehat{F}, \widehat{p})$, that is:

     $$\widetilde{p}_{Hn} + \widehat{p} + p(X - \widetilde{F}_{Hn} - \widehat{F}) < \widetilde{u}_{Ln-1} + \Delta_{n-1},$$

     where $p(X - \widetilde{F}_{Hn} - \widehat{F})$ is the maximal payoff that the L-type can obtain by retaining and/or selling the cash flows $X - \widetilde{F}_{Hn} - \widehat{F}$. Such a pair exists as:

$$\lim_{\nu,d\to 0} \widetilde{p}_{Hn} + E_{\mu_0}\{\min\{d, X - \widetilde{F}_{Hn}(X)\}\} - \nu + p(X - \widetilde{F}_{Hn} - \min\{d, X - \widetilde{F}_{Hn}(X)\}) \leq \widetilde{u}_{Ln-1}.$$

Note that the allocation $(\widetilde{F}_{Hn}, \widetilde{p}_{Hn})$ is still available at Stage 3 to the seller, since the profits of the non-deviating buyers weakly increase after the addition of contract $(\widehat{F}, \widehat{p})$. Hence, after the deviation, the H-type (and possibly the L-type) are attracted to pick up contracts $(\widetilde{F}_{Hn}, \widetilde{p}_{Hn})$ and $(\widehat{F}, \widehat{p})$, which renders the deviation profitable. Thus, the deviating menu is $\{\mathcal{C}_0^d, ..., \mathcal{C}_{n-1}^d, (\widehat{F}, \widehat{p})\}$.

After at most $N-2$ steps, this procedure yields a deviating menu that is profitable. ■

Lemma B.2 *Consider an allocation (F_H, p_H) and (X, u_L) such that:*

$$\mu_0 \cdot (E_H F_H - p_H) + (1-\mu_0) \cdot (E_L X - u_L) \geq -\xi,$$

for some $\xi \geq 0$. If $u_H \equiv p_H + \delta \cdot E_H[X - F_H] > u_H^* + \xi$, then:

$$u_L < p_H + E_L[X - F_H].$$

Proof. This follows from the observation that the solution to Program P2, with the adjusted participation constraint of the buyers allowing them to earn ξ expected losses, implies a payoff to the H-type of $u_H^* + \xi$. Hence, if the H-type's payoff is above $u_H^* + \xi$ and the buyers' losses do not exceed ξ, the L-type's incentive constraint in Program P2 must be violated. ■

Lemma B.3 *In any equilibrium, $u_H \leq u_H^* + \omega \cdot \kappa$ for $\omega = \frac{N\cdot(N-3)}{2} + 2$.*

Proof. Suppose to the contrary that $u_H > u_H^* + \omega \cdot \kappa$. Let (F_H, p_H) and (X, u_L) denote the equilibrium allocations. Since these allocations must earn zero expected profits for the buyers, and since $u_H > u_H^*$, by Lemma B.2 it follows that:

$$u_L < p_H + E_L[X - F_H].$$

We will construct a a profitable deviation for a buyer recursively, so we let $(\widetilde{F}_{\theta 0}, \widetilde{p}_{\theta 0}) \equiv (F_\theta, p_\theta)$ and $\widetilde{u}_{\theta 0} \equiv u_\theta$.

Step 0. Consider a deviation by a buyer to replace his menu with a menu that contains contract $\mathcal{C}_0^d = (X - \widetilde{F}_{H0}, E_L[X - \widetilde{F}_{H0}] - \epsilon_0)$ for $\epsilon_0 > 0$ such that $\widetilde{u}_{L0} < \widetilde{p}_{H0} + E_L[X - \widetilde{F}_{H0}] - \epsilon_0$. Note that if allocation $(\widetilde{F}_{H0}, \widetilde{p}_{H0})$ is still available at Stage 3, then such a deviation attracts the L-type who would combine the deviating contract with $(\widetilde{F}_{H0}, \widetilde{p}_{H0})$. If contract $\mathcal{C}_0^d$ is picked up at Stage 3, then the deviation is profitable. Thus, the deviating menu is $\{\mathcal{C}_0^d\}$.

If instead contract $\mathcal{C}_0^d$ is not picked up at Stage 3, it must be that the allocation $(\widetilde{F}_{H0}, \widetilde{p}_{H0})$ is not available at Stage 3, i.e., due to withdrawal of some menus. Thus, the seller must be obtaining allocation $(\widetilde{F}_{\theta 1}, \widetilde{p}_{\theta 1})$ from some equilibrium menus (but different from the equilibrium allocations) and obtaining a payoff $\widetilde{u}_{\theta 1} \leq \widetilde{u}_{\theta 0}$. Note that there are now at most $N-2$ equilibrium menus available to the seller. Let $\Delta_0 = \widetilde{p}_{H0} + E_L[X - \widetilde{F}_{H0}] - \epsilon_0 - \widetilde{u}_{H0}$. With $\{\Delta_0, (\widetilde{F}_{\theta 1}, \widetilde{p}_{\theta 1}), \widetilde{u}_{\theta 1}\}$, we proceed to Step n.

Step $n \in \{1, ..., N-2\}$. Let $(\widetilde{F}_{\theta n}, \widetilde{p}_{\theta n})$ denote the θ-type's allocation at Stage n, and $\widetilde{u}_{\theta n}$ her payoff. We can assume without loss that $\widetilde{F}_{L,n} = X$ and, therefore, that $\widetilde{p}_{Ln} = \widetilde{u}_{Ln}$. Also, note that $\widetilde{u}_{\theta n} \leq \widetilde{u}_{\theta n-1}$. With this, we adjust the candidate deviation in Step $n-1$ as follows.

- If $\widetilde{u}_{Hn-1} - \widetilde{u}_{Hn} > (N-n-1) \cdot \kappa$, then adjust the deviating menu by adding the contracts $(\widehat{F}, \widehat{p})$ and $(X, \widehat{u})$ with $\widehat{p} \leq \widetilde{p}_{Hn-1}$, $\widehat{F}(X) = \min\left\{\widetilde{F}_{Hn-1}(X) + \upsilon, X\right\}$ and $\widehat{u} = \widetilde{u}_{Ln-1} + \widehat{p} - \widetilde{p}_{Hn-1}$ satisfying:

  $$\begin{aligned} \widetilde{u}_{Hn} &< \widehat{p} + \delta \cdot E_H[X - \min\{\widetilde{F}_{Hn-1}(X) + \upsilon, X\}] \\ &= \widetilde{u}_{Hn-1} + \widehat{p} - \widetilde{p}_{Hn-1} - \delta \cdot E_H[\min\{\upsilon, X - \widetilde{F}_{Hn-1}(X)\}]. \end{aligned}$$

  The H-type prefers $(\widehat{F}, \widehat{p})$ to the allocation $(\widetilde{F}_{Hn}, \widetilde{p}_{Hn})$. The L-type also prefers $(X, \widehat{u})$ to picking up the contract $(\widehat{F}, \widehat{p})$ since otherwise she would have also preferred $(\widetilde{F}_{Hn-1}, \widetilde{p}_{Hn-1})$ to $(X, \widetilde{u}_{Ln-1})$. Lastly, note that the contract $(\widehat{F}, \widehat{p})$ is less attractive to the seller than $(\widetilde{F}_{Hn-1}, \widetilde{p}_{Hn-1})$ (as it has more cash flows sold at a lower price) and is therefore only picked up if the menus containing the latter are withdrawn. Moreover, as $\upsilon \downarrow 0$ and $\widehat{p} \downarrow \widetilde{p}_{Hn-1} + \widetilde{u}_{Hn} - \widetilde{u}_{Hn-1}$, the deviating buyer's profits converge to:

  $$\Pi_n^{-i} + \widetilde{u}_{Hn-1} - \widetilde{u}_{Hn} \geq -(N-n-1) \cdot \kappa + \widetilde{u}_{Hn-1} - \widetilde{u}_{Hn} > 0,$$

  where Π_n^{-i} are the aggregate profits of the $N-n-1$ remaining non-deviating buyers if allocations $(\widetilde{F}_{\theta n}, \widetilde{p}_{\theta n})$ were accepted by the seller at Stage 3. Thus, the deviating menu is $\{\mathcal{C}_0^d, ..., \mathcal{C}_n^d, (\widehat{F}, \widehat{p}), (X, \widehat{u})\}$.

- If $\widetilde{u}_{Hn-1} - \widetilde{u}_{Hn} \leq (N-n-1) \cdot \kappa$, then:

  $$\begin{aligned} \widetilde{u}_{H,n} &\geq \widetilde{u}_{H,0} - \sum_{j=1}^{n} (N-j-1) \cdot \kappa \\ &> u_H^* + \omega \cdot \kappa - \sum_{j=1}^{n} (N-j-1) \cdot \kappa \\ &= u_H^* + \left(\omega - n \cdot \left[N - 1 - \frac{n+1}{2}\right]\right) \cdot \kappa \\ &\geq u_H^* + (N-n-1) \cdot \kappa \end{aligned}$$

  where the last inequality follows from the fact that $\omega \geq n \cdot \left(N - 1 - \frac{n+1}{2}\right) + N - n - 1$ for all $n \leq N-2$. By Lemma B.2, we therefore conclude that:

  $$\widetilde{u}_{Ln} < \widetilde{p}_{Hn} + E_L[X - \widetilde{F}_{Hn}].$$

  Given this, we adjust the deviating menu by adding the contract $\mathcal{C}_n^d = (X - \widetilde{F}_{Hn}, E_L[X - \widetilde{F}_{Hn}] - \epsilon_n)$ such that $\widetilde{u}_{Ln} < \widetilde{p}_{Hn} + E_L[X - \widetilde{F}_{Hn}] - \epsilon_n \equiv \widetilde{u}_{Ln} + \Delta_n < \widetilde{u}_{Ln-1} + \Delta_{n-1}$. If contract $\mathcal{C}_n^d$ is picked up at Stage 3, then the deviation is profitable. Thus, the deviating menu is $\{\mathcal{C}_0^d, ..., \mathcal{C}_n^d\}$.

If instead contract $\mathcal{C}_n^d$ is not picked up at Stage 3, it must be that allocation $(\widetilde{F}_{\theta n}, \widetilde{p}_{\theta n})$ is not available at Stage 3. Note that there are now at most $N-n-2$ equilibrium menus available to the seller. Thus, the seller must be obtaining allocations $(\widetilde{F}_{\theta n+1}, \widetilde{p}_{\theta n+1})$ from equilibrium menus with corresponding payoffs $\widetilde{u}_{\theta n+1}$. With $\{\Delta_n, (\widetilde{F}_{\theta n+1}, \widetilde{p}_{\theta n+1}), \widetilde{u}_{\theta n+1}\}$, we proceed to Step $n+1$.

After at most $N-2$ steps, this procedure yields a deviating menu that is profitable. ■

Proof of Proposition B.1. The proof follows from Lemmas A.8, B.1 and B.3. ■

B.3 Going beyond two types

In this Appendix, we extend the analysis of our baseline model in Section 2 beyond two types. In particular, we show how to construct a candidate equilibrium – in the spirit of the star-equilibrium in Section 3, – for a three-type setting, and we provide conditions under which it can be supported as a PBE. We then discuss how this analysis can be extended to any finite number of types.

We now suppose that the seller's asset can be of *high-*, *medium-*, or *low-*quality, denoted by $\theta \in \{H, M, L\}$, with $\mu_\theta \in (0,1)$ being the probability that the asset is of θ-quality. The pdfs continue to be related by MLRP: $\frac{g_H(x)}{g_M(x)}$ and $\frac{g_M(x)}{g_L(x)}$ are both increasing in x. We use subscripts $\{H, M, L\}$ to indicate the seller's type and $E_\theta[\cdot]$ to indicate the corresponding conditional expectation operator.

We begin the analysis by characterizing the solution to two programs, which will be useful in the construction of an equilibrium.

ML-Program. For any $F \in \Phi$, let $(\widehat{F}_M, \widehat{p}_M)$ and $(\widehat{F}_L, \widehat{F}_L)$ be given by the solution to the following program:

$$\max_{\{(\widehat{F}_M, \widehat{p}_M), (\widehat{F}_L, \widehat{p}_L)\}; \widehat{F}_M, \widehat{F}_L \in \Phi} \widehat{p}_M - \delta \cdot E_M[\widehat{F}_M] \tag{P2$_{ML}$}$$

subject to the following constraints:

$$\begin{aligned} \widehat{p}_M - E_L[\widehat{F}_M] &\leq \widehat{p}_L - E_L[\widehat{F}_L] \\ \frac{\mu_M}{\mu_L + \mu_M} \cdot (\widehat{p}_M - E_M[\widehat{F}_M]) + \frac{\mu_L}{\mu_L + \mu_M} \cdot (\widehat{p}_L - E_L[\widehat{F}_L]) &\leq 0 \\ E_L[\widehat{F}_L] &\leq \widehat{p}_L \\ \widehat{F}_j &\leq X - F, \ j \in \{M, L\}. \end{aligned}$$

Note that the solution to the program depends on the security F. We will omit this dependence in what follows, however, in order to economize on notation.

For a given $F \in \Phi$, Program P2$_{ML}$ is isomorphic to the Program P2 in Section 3, with the exception that the H-type has been replaced with the M-type and the securities are defined over cash flows $X-F$. As $X-F$ is monotonic, MLRP continues to hold over these cash flows. Thus, using the results from from Lemma 3.1, we have that:

1. $\widehat{F}_M = \min\{d_{Mz} - F(d_{Mz}), X-F\}$ and $\widehat{p}_M = E_{ML}[\widehat{F}_M]$.
2. $\widehat{F}_L = X$ and $\widehat{p}_L = E_{ML}[\widehat{F}_M] + E_L[X-\widehat{F}_M]$,

for some $d_{Mz} \in (0,\bar{X}]$ and where $E_{ML}[F] \equiv \frac{\mu_M}{\mu_L+\mu_M} \cdot E_M[F] + \frac{\mu_L}{\mu_L+\mu_M} \cdot E_L[F]$. Moreover,

Lemma B.4 *The debt level d_{Mz} does not depend on F.*

Lemma B.4 states that the M-type is willing to issue up to cash flows $\min\{d_{Mz}, X\}$ at *medium-low*-valuation. That is, for any cash flows F that the M-type issues, she also benefits from selling cash flows $L_F \equiv \min\{d_{Mz} - F(d_{Mz}), X-F\}$ priced at or above *medium-low*-valuation. Moreover, d_{Mz} is given by the solution to Program P2$_{ML}$ for $F=0$.

HML-Program. Let $\{(F_\theta, p_\theta)\}$ be given by the solution to the following program:

$$\max_{\{(F_\theta,p_\theta):\ F_\theta\in\Phi\}} \quad p_H - \delta \cdot E_H[F_H] \tag{P2$_{HML}$}$$

subject to the following constraints:

$$p_H + E_{ML}[L_{F_H}] - \delta\cdot E_M[F_H + L_{F_H}] \le p_M - \delta\cdot E_M[F_M] \tag{34}$$
$$p_H + E_{ML}[L_{F_H}] - E_L[F_H + L_{F_H}] \le p_L - E_L[F_L] \tag{35}$$
$$p_M - E_L[F_M] \le p_L - E_L[F_L] \tag{36}$$
$$\mu_L\cdot(p_L - E_L[F_L]) + \mu_M\cdot(p_M - E_M[F_M]) + \mu_H\cdot(p_H - E_H[F_H]) \le 0 \tag{37}$$
$$E_{ML}[\min\{d_{Mz}, X\}] - \delta\cdot E_M[\min\{d_{Mz}, X\}] \le p_M - \delta\cdot E_M[F_M] \tag{38}$$
$$E_L[X] \le p_L \tag{39}$$

where $L_{F_H} \equiv \min\{d_{Mz} - F_H(d_{Mz}), X - F_H\}$.

In the same vein as Program P2, Program P2$_{HML}$ solves for the allocations that maximize the H-type's payoff subject to incentive compatibility constraints, (34)-(35)-(36), buyers' participation constraint (37), and the $\theta\in\{M,L\}$-types' participation constraints, (38)-(39). The seller's incentive and participation constraints suppose that (i) the L-type seller can always issue any feasible security at *low*-valuation, and (ii) the M-type seller can always issue any remaining cash flows "below" F_{Mz}, i.e., L_F for any $F\in\Phi$, at *medium-low*-valuation. Whereas (i) is just as in Program P2, (ii) is now added due to the presence of the M-type. For any F, define $E_{HML}[F] \equiv \sum_{j=H,M,L} \mu_j \cdot E_j[F]$. Then:

Lemma B.5 *The unique solution to P2$_{HML}$ is as follows:*

1. *$F_H(X) = \min\{d_S, X\}$, $p_H = \mathbb{E}_{HML}[F_H]$, and d_S solves*

$$\max_{d\in[0,\bar{X}]} E_{HML}[\min\{d,X\}] - \delta\cdot E_H[\min\{d,X\}];$$

2. *$F_M(X) = \min\{d_{Mz}, X\}$, $p_M = \mathbb{E}_{HML}[F_H] + \mathbb{E}_{ML}[F_M - F_H]$, and d_{Mz} solves*

$$\max_{d\in[0,\bar{X}]} E_{ML}[\min\{d,X\}] - \delta\cdot E_M[\min\{d,X\}];$$

3. $F_L(X) = X$, $p_L = \mathbb{E}_{HML}[F_H] + \mathbb{E}_{ML}[F_M - F_H] + \mathbb{E}_L[X - F_M]$.[24]

Consider next the following set of contracts:

1. *Senior Tranche:* (F_S, p_S), *where*

$$F_S = \min\{d_S, X\},\ p_S = E_{HML}[F_S].$$

2. *Mezzanine Tranche:* (F_{Mz}, p_{Mz}), *where*

$$F_{Mz} = \min\{d_{Mz}, X\} - \min\{d_S, X\}, \quad p_{Mz} = E_{ML}[F_{Mz}].$$

3. *Junior Tranche:* (F_J, p_J), *where*

$$F_J = X - F_{Mz} - F_S,\ p_J = E_L[F_J].$$

4. *Set of low-valuation securities:*

$$\mathcal{L}_L = \{(F, p) : F \in \Phi,\ p = E_L[F]\}.$$

5. *Set of medium-low-valuation securities:*

$$\mathcal{L}_{ML} = \{(F, p) : \exists \widetilde{F} \in \Phi\ s.t.\ F = \min\{d_{Mz} - \widetilde{F}(d_{Mz}), X - \widetilde{F}\},\ p = E_{ML}[F]\}.$$

Given these contracts, we propose the following as an equilibrium candidate. At Stage 1, buyer 1 and 2 offer the senior tranche, (F_S, p_S), buyers 3 and 4 offer the mezzanine tranche, (F_{Mz}, p_{Mz}), buyers 5 and 6 offer the set of latent contracts $\mathcal{L}_L$, buyers 7 and 8 offer the set of latent contracts $\mathcal{L}_{ML}$, all other buyers offer the trivial menu, $\{(0, 0)\}$.[25] At Stage 2, there are no withdrawals. At Stage 3, the H-type accepts contract (F_S, p_S) from buyer 1, the M-type accepts contract (F_S, p_S) from buyer 1 and (F_{Mz}, p_{Mz}) from buyer 3, and the L-type accepts contract (F_S, p_S) from buyer 1, (F_{Mz}, p_{Mz}) from buyer 3, and (F_J, p_J) from buyer 5.

Equilibrium existence. In what follows, we show that the above candidate can be supported as a PBE when contract withdrawal is costless for the buyers at Stage 2. Note that the star-equilibrium of our baseline setting in Section 3 is also a PBE when buyers can withdraw contracts (rather than menus). While in the two-type setting introducing costly menu withdrawal allowed us to narrow down the set of equilibria and provide a sharp characterization of the equilibrium set, with more than two types menu withdrawal may be too restrictive and, therefore, we allow buyers to withdraw individual contracts. As our goal in this Appendix is not to characterize the entire equilibrium set beyond two types, we also suppose that contract withdrawal is costless. This greatly simplifies our arguments due to the following result; the formal proof can be found further below.

[24] Note that we implicitly assume that $d_{Mz} > d_S$. A sufficient condition is that $1 - \frac{\mu_M}{1-\mu_H} \cdot G_M(d_S) - \frac{\mu_L}{1-\mu_H} \cdot G_L(d_S) > \delta \cdot (1 - G_M(d_S))$, which holds if μ_H or μ_L are not too large. We impose this condition throughout to streamline the analysis.

[25] Note that the junior tranche is posted at *low*-valuation by buyers 5 and 6.

Lemma B.6 *When contract withdrawal is costless for buyers at Stage 2, it is without loss of generality to consider deviations at Stage 1 that fully attract all types to the deviating menu.*

We now prove that the above candidate is indeed an equilibrium. We first rule out deviations by the seller at Stage 3.

By construction, there are no profitable deviations for the H- and the M-type at Stage 3, as the senior and mezzanine contracts, $\{(F_S, p_S), (F_{Mz}, p_{Mz})\}$ maximize the H- and M-type's payoffs subject to securities being priced at HML- and ML- valuations, respectively, and all other available contracts price securities at weakly lower valuations. Analogously, the L-type has no incentives to deviate to accept contracts different from (F_J, p_J) and priced at low-valuation, i.e., from the menu $\mathcal{L}_L$. Next, we show that the L-type has no incentives to deviate to accept contracts priced at *medium-low*-valuation, i.e., from $\mathcal{L}_{ML}$, either.

First, it is not feasible to issue any non-trivial contract $(F, p) \in \mathcal{L}_{ML}$ together with equilibrium contracts $\{(F_S, p_S), (F_{Mz}, p_{Mz})\}$, since any such contract must satisfy $F(d_{Mz}) > 0$ and therefore $F_S(d_{Mz}) + F_{Mz}(d_{Mz}) + F(d_{Mz}) > d_{Mz}$, which violates feasibility.

Second, the L-type would also not want to accept any collection $\{(F_k, p_k)\}_k \in \mathcal{L}_{ML}$ instead of equilibrium contract (F_S, p_S) (whether (F_{Mz}, p_{Mz}) is dropped or not is irrelevant since $(F_{Mz}, p_{Mz}) \in \mathcal{L}_{ML}$). To see this, suppose she were to do so. For this deviation to be attractive to the L-type, it must be that such a collection $\{(F_k, p_k)\}_k$ allows her to sell more cash flows than the equilibrium contracts priced at *medium-low*-valuation, i.e., there exists $\bar{x}$ such that:

$$\sum_k F_k(x) > d_{Mz}, \forall x \geq \bar{x}.$$

Next, observe that:

$$\sum_k F_k(x) = \sum_k \min\left\{d_{Mz} - \widetilde{F}_k(d_{Mz}), x - \widetilde{F}_k(x)\right\},$$

for some $\widetilde{F}_k$'s $\in \Phi$. Hence, for $x \leq d_{Mz}$, we have:

$$\sum_k \min\left\{d_M - \widetilde{F}_k(d_M), x - \widetilde{F}_k(x)\right\} \leq x \leq d_{Mz},$$

which also holds for $x = d_{Mz}$. It follows that for $x > d_{Mz}$:

$$\sum_k \min\left\{d_{Mz} - \widetilde{F}_k(d_{Mz}), x - \widetilde{F}_k(x)\right\} \leq \sum_k \left(d_{Mz} - \widetilde{F}_k(d_{Mz})\right) \leq d_{Mz}.$$

Thus, for any collection $\{(F_k, p_k)\}_k \in \mathcal{L}_{ML}$, $\sum_k F_k(x) \leq d_{Mz}$, and such a deviation is not attractive to the L-type.

Since buyers break even, there are no profitable deviations at Stage 2. We are thus left to rule out profitable deviations by buyers at Stage 1. We do so next.

By Lemma B.6, it suffices to consider deviations that fully attract all three types from equilibrium menus. As (F_S, p_S) maximizes the H-type seller's payoff subject to incentive compatibility constraints (in the presence of latent contracts in $\mathcal{L}_{ML}$ and $\mathcal{L}_L$), and buyer break-even conditions, it follows that any deviation to offer a set of contracts to attract all three types that manages to attract the H-type seller must be loss-making to the deviating buyer. That is, when contract withdrawal is costless, any set of allocations that attract the three types must give the H-type seller a higher payoff than the allocations that solve Program P2$_{HML}$, which satisfy incentive compatibility constraints in the presence of contracts in $\mathcal{L}_{ML}$ – which are always available to the M-type – and of contracts in $\mathcal{L}_L$ – which are always available to the L-type, – and be profitable to the buyer, which is a contradiction.

$T \geq 3$ *types.* Our goal in this Appendix was to show how the star-equilibrium, which we constructed for a two-type setting, could be extended beyond two types. We have chosen to do so by characterizing the analogue of the star-equilibrium for a three-type setting, as its construction sheds light on how to extend the analysis to any finite number of types.

To see this, consider a setting with $\theta \in \{1, ..., T\}$ types, ordered by quality with $\theta = 1$ being the highest and $\theta = T$ being the lowest. From the previous analysis, we learn that the construction of a (star-)equilibrium candidate requires solving $T-1$ programs, starting from a program that maximizes the payoff to the $(T-1)$-type, subject to incentive and participation constraints that only consider the presence of the $T-1$- and T-types, and where the latter has access to a set of contracts priced at the lowest, i.e., $T-$, valuation (as in our Program P2$_{ML}$). With the solution to this program, a corresponding set of latent contracts is constructed (analogous to the construction of the set $\mathcal{L}_{ML}$) and the next program is solved, which maximizes the payoff of the $(T-2)$-type subject to incentive and participation constraints of $\theta \in \{T-1, T\}$, as in Program P2$_{HML}$. This procedure continues until the last program, which maximizes the payoff to the 1-type, subject to incentive and participation constraints for all other types, and considering the presence of the previously constructed sets of latent contracts. The solution to the last program provides the candidate equilibrium allocations for all types, while the solutions to the previously solved $T-2$ programs are used to construct the set of latent contracts needed to support the candidate as an equilibrium. When contract withdrawal is costless, due to Lemma B.6, the existence proof is isomorphic to the case with three types.

Thus, our analysis suggests that in a setting with T-types, when contract withdrawal is costless, there always exists an equilibrium in which T-tranches are traded. While the most senior tranche would be issued by all seller types and priced at average valuation, there are also $T-2$ mezzanine tranches that are issued by ordered subsets of seller types, with better types dropping out as tranches become more junior, and that are priced at average valuation conditional on the seller-types issuing the tranche. Finally, there is a junior tranche priced at the lowest valuation and issued only by the lowest type.

B.3.1 Additional Proofs for Appendix B.3

Proof of Lemma B.4. From the solution to Program P1, we know that the incentive compatibility for the L-type and buyers' participation constraint must bind, and that the participation constraint of the L-type is slack. With this, the solution to Program P2$_{ML}$

reduces to solving:

$$\max_{d\in[0,\bar{X}-F(\bar{X})]} E_{ML}\left[\min\left\{d, X-F(X)\right\}\right]-\delta\cdot E_M\left[\min\left\{d, X-F(X)\right\}\right] \tag{40}$$

$$=\max_{d'\in[0,\bar{X}]} E_{ML}\left[\min\left\{d'-F\left(d'\right), X-F\left(X\right)\right\}\right]-\delta\cdot E_M\left[\min\left\{d'-F\left(d'\right), X-F\left(X\right)\right\}\right] \tag{41}$$

The solution for $d'=d_{Mz}$ must satisfy:

$$\left(1-F'\left(d_{Mz}\right)\right)\cdot\left(\left(\frac{\mu_M}{\mu_L+\mu_M}-\delta\right)\cdot\left(1-G_M\left(d_{Mz}\right)\right)+\frac{\mu_L}{\mu_L+\mu_M}\cdot\left(1-G_L\left(d_{Mz}\right)\right)\right)\geq 0,$$

with equality if $d_{Mz}<\bar{X}$, and where $F'(d_{Mz})\leq 1$. But then, setting $d'=d_{Mz}$ such that:

$$\left(\frac{\mu_M}{\mu_L+\mu_M}-\delta\right)\cdot\left(1-G_M\left(d_{Mz}\right)\right)+\frac{\mu_L}{\mu_L+\mu_M}\cdot\left(1-G_L\left(d_{Mz}\right)\right)\geq 0$$

with equality if $d_{Mz}<\bar{X}$, is optimal and implies that d_{Mz} is independent of F. ■

Proof of Lemma B.5. With analogous arguments to those in the proofs of Lemmas A.4 and A.5, we obtain that $F_L=X$ and that the buyers' participation constraint must bind. Next, we guess (and later verify) that the the M- and L-type's the participation constraints, (38)-(39), and the incentive compatibility constraint that ensures the L-type does not mimic the H-type, (35), are slack. With this, it is immediate that the incentive compatibility constraint, (36), binds, as the objective increases in p_H. Thus, we have that:

$$F_L=X, \tag{42}$$

$$p_L=E_L\left[X\right]+\frac{\mu_M}{\mu_L}\cdot\left(E_M\left[F_M\right]-p_M\right)+\frac{\mu_H}{\mu_L}\cdot\left(E_H\left[F_H\right]-p_H\right), \tag{43}$$

$$p_M=E_{ML}\left[F_M\right]+\frac{\mu_H}{1-\mu_H}\cdot\left(E_H\left[F_H\right]-p_H\right), \tag{44}$$

$$p_H=E_{HML}\left[F_H\right]+\left(1-\mu_H\right)\cdot\left(E_{ML}\left[F_M-F_H-L_{F_H}\right]-\delta\cdot E_M\left[F_M-L_{F_H}-F_H\right]\right), \tag{45}$$

with F_H and F_M being given by the solution to:

$$\max_{\{F_M,F_H\}\in\Phi} E_{HML}\left[F_H\right]-\delta\cdot E_H\left[F_H\right]+\left(\mu_L+\mu_M\right)\cdot\left(E_{ML}\left[F_M-F_H-L_{F_H}\right]-\delta\cdot E_M\left[F_M-F_H-L_{F_H}\right]\right)$$

But then, it is immediate that $F_M=\min\{d_{Mz},X\}$, and that F_H solves:

$$\max_{F_H\in\Phi} E_{HML}\left[F_H\right]-\delta\cdot E_H\left[F_H\right] \tag{46}$$
$$-\left(\mu_M+\mu_L\right)\cdot\left(E_{ML}\left[\max\left\{0, F_H\left(X\right)-F_H\left(d_{Mz}\right)\right\}\right]-\delta\cdot E_M\left[\max\left\{0, F_H\left(X\right)-F_H\left(d_{Mz}\right)\right\}\right]\right),$$

which is equivalent to solving:

$$\max_{\{s(z)\}_{z\in[0,\bar{X}]}:s(z)\in[0,1]} \quad \int_0^{\bar{X}} F_H(x)\cdot(g_{HML}(x)-\delta\cdot g_H(x))\cdot dx \tag{47}$$
$$-(\mu_M+\mu_L)\cdot\left(\int_{d_{Mz}}^{\bar{X}}(F_H(x)-F_H(d_{Mz}))\cdot(g_{ML}(x)-\delta\cdot g_M(x))\cdot dx\right)$$

subject to:

$$F_H(x)=\int_0^x s(z)\cdot dz.$$

Let d_S denote the maximizer of $E_{HML}[\min\{d_S,X\}]-\delta\cdot E_H[\min\{d_S,X\}]$. Then, for $z\le d_{Mz}$, the derivative of the objective in (47) with respect to $s(z)$ is given by:

$$(1-G_{HML}(z))-\delta\cdot(1-G_H(z))\begin{cases}>0 & z<d_S\\ =0 & z=d_S\\ <0 & z>d_S\end{cases}.$$

Instead, for $z>d_{Mz}$, it is given by:

$$(1-G_{HML}(z))-\delta\cdot(1-G_H(z))-(\mu_M+\mu_L)\cdot((1-G_{ML}(z))-\delta\cdot(1-G_M(z)))<0,$$

and is it negative since it is negative at $z=d_{Mz}$, it is equal to zero at $z=\bar{X}$, and its derivative,

$$\left[-(\mu_H-\delta)\cdot\frac{g_H(z)}{g_M(z)}-\delta\cdot(1-\mu_H)\right]\cdot g_M(z),$$

is increasing in z by MLRP.

Thus, we have that $s(z)=1$ for all $z\le d_S$ and zero otherwise, i.e., $F_H(X)=\min\{d_S,X\}$. At the obtained solution, it is straightforward to verify that constraints (35)-(38)-(39) are satisfied, and thus the claim is verified. ■

Proof of Lemma B.6. Suppose that there exists a profitable deviation that attracts each seller type to contract $(\widetilde{F}_\theta,\widetilde{p}_\theta)$ in the deviating menu, but that it does not "fully" attract all seller types, i.e., additional allocations (F_θ,p_θ) are obtained through equilibrium menus, where $F_\theta\neq 0$ for at least one type. Then, we show that there exists a profitable deviation that fully attracts all seller types to the deviating menu. To see this, suppose that instead the deviating buyer offers $(\widetilde{F}_\theta+F_\theta,\widetilde{p}_\theta+p_\theta)$. As these are the allocations obtained by the seller following the original deviation, we know that participation and incentive compatibility constraints are unaffected. As the original deviation was profitable, we have:

$$\sum_\theta \mu_\theta\cdot(E_\theta[\widetilde{F}_\theta]-\widetilde{p}_\theta)>0. \tag{48}$$

and allocations (F_θ,p_θ) are not loss-making, as they were available at Stage 3 (i.e., they were not withdrawn):

$$\sum_{\theta} \mu_{\theta} \cdot \left(E_{\theta}\left[F_{\theta}\right] - p_{\theta}\right) \geq 0, \tag{49}$$

then the deviation to "fully" attract all seller types is also profitable to the seller. ■