Token-Based Platforms and Speculators*

Simon Mayer†

March 14, 2022

Abstract

In a dynamic model of cryptocurrencies and tokens, users hold tokens for transactions and speculators hold tokens for returns. Speculation is procyclical, i.e., high in a bull and low in a bear market. Speculators affect users via two opposing effects, adverse crowding-out and benign liquidity provision, and their token investments are static substitutes but dynamic complements. A dual token structure with a governance token and stablecoin attenuates the crowding-out effect, harnesses speculator sentiment, and stimulates adoption. The model has empirical implications regarding cryptocurrency usage and speculation, and normative implications for the optimal design of token-based or decentralized finance platforms.

---

*I especially thank Sebastian Gryglewicz for his continued guidance and helpful comments. Part of this paper was written during a research visit at the Kellogg School of Management. I am particularly grateful to Konstantin Milbradt for hosting and advising me. I thank Anat Admati, Tobias Berg, Bruno Biais, Dion Bongaerts, Andrea Canidio, Will Cong, Nicolas Crouzet, Ian Dew-Becker, Michael Fishman, Itay Goldstein, Barney Hartman-Glaser, Kose John, Sergei Kovbasyuk, Sukjoon Lee (discussant), Jiasun Li (discussant), Ye Li, Erwan Morellec, Charles Nathanson, Fahad Saleh, Michael Sockin, Alp Simsek, Bruno Strulovici, Spyros Terovitis, Quentin Vandeweyer, Vladimir Vladimirov, Neng Wang, and Kathy Yuan for valuable discussions and comments. I also thank the seminar participants at the University of Calgary, University of Chicago Booth, Erasmus University, HEC Paris, the CBER forum, the Digital Economy Workshop at HKBU/Monash, the Tokenomics conference 2021, and the MFA meeting 2022 for their comments.

†University of Chicago, Booth School of Business. E-mail: simon.mayer@chicagobooth.edu

Many cryptocurrencies and tokens — such as Ether, Ada, or Solana — are intended to serve as a means of payment on a token-based platform or within a decentralized finance ecosystem. At the same time, cryptocurrencies and tokens have become notorious as speculative assets. The fact that cryptocurrencies and tokens are held by both speculators and users raises several questions. How do speculators affect the adoption and pricing of cryptocurrencies and tokens? What are the potential conflicts of interest between speculators and users, and how to resolve them?

To address these and related questions, we analyze a dynamic model of a token-based platform that settles transactions among platform users with its native cryptocurrency (referred to as "tokens"). Users hold tokens to transact on the platform, in that they derive a convenience yield from holding tokens. Speculators representing financial investors trade tokens for returns (subject to a short selling constraint) but do not transact on the platform and do not derive a convenience yield. The benefits of transacting on the platform and token price increase with platform productivity capturing the general usefulness or technology of the platform. The growth rate of platform productivity follows a Markov chain, and is high in "good times," representing a bull market with rapidly rising token price, and low or even negative in "bad times," representing a bear market.

Crucially, users and speculators differ in their beliefs about the platform's future growth, as captured by the latent transition probabilities of the Markov chain. Speculators dynamically update their beliefs about transition probabilities and effectively extrapolate based on the past. In particular, when the growth rate increases or remains high, speculators become gradually more optimistic, i.e., speculator sentiment rises. When the platform's growth rate remains low, speculator optimism and sentiment dwindle. In contrast, users do not extrapolate or update their beliefs, which could capture overconfidence as in Scheinkman and Xiong (2003) or that users as the platform's insiders are informed about its fundamentals whereas the speculators as outside investors are not. Because speculator sentiment varies over time, our model generates rich equilibrium dynamics that shed light on the interactions between speculative trading, token pricing, and dynamic adoption.

The token price depends on both the transactional demand from users and the investments from speculators. Speculators' token investments increase with their sentiment, and so does token price. Importantly, speculators affect users via two opposing effects, adverse crowding-out and benign liquidity provision. When speculators buy tokens and drive up token price, expected token returns decrease and users' (opportunity) costs of holding tokens and transacting on the platform increase, which crowds out usage. Due to this crowding-out effect, speculative investment and token usage can be seen as static substitutes, consistent with the empirical findings in Silberholz and Wu (2021).

On the other hand, speculative investment also raises the resale option value of tokens in certain states. For instance, when speculator sentiment is sufficiently high, speculators provide liquidity in bad times and so dampen a token price crash following a regime shift that reduces the platform's growth rate. In particular, users can sell tokens after a token price drop at more favorable terms precisely because speculators buy. This benign liquidity provision in bad times limits token price risk, raises expected token returns, and stimulates adoption in good times. Symmetrically, the fact that speculators acquire tokens (i.e., provide liquidity) also in future periods of high platform growth (good times) boosts expected token returns and allows users to capitalize on platform growth, which fosters adoption. As, in addition, speculators only buy tokens due to the prospect of high future adoption, speculative and transactional token investments can be viewed as dynamic complements. Our model predicts that due to the static substitutes and dynamic complements features, the amount of speculative token investment correlates negatively with contemporaneous token usage but positively with future token usage.

As speculation has two opposing effects on token usage and adoption, token usage is — all else equal — lowest and token price volatility is highest when speculators' sentiment and investment take intermediate levels. To gain some intuition about this finding, consider a platform in its early stages without track record so that speculator optimism about the platform is low and speculators are unwilling to buy tokens. Then, the prospect that speculators buy tokens in the future boosts prevailing expected token returns, leading to a relatively high level of adoption. Intuitively, the presence of speculators helps early-stage platforms and tokens to gain adoption and to grow.

As platform adoption and token price increase over time, speculator sentiment (i.e., optimism) gradually improves. When speculator sentiment exceeds a certain cutoff, speculators start to buy tokens and, at first, the adverse crowding-out effect of rising speculative investment dominates. The reason is that for intermediate levels of sentiment, speculative demand for tokens remains fragile and, in particular, retreats following a negative shock, in that speculators do not provide liquidity following a token price crash. Therefore, an increase in speculative investment then reduces expected token returns and token usage, but increases token price and so the scope of a potential token price drop. As a result, for intermediate levels of speculator sentiment and investment, token price risk is high and usage is low, so tokens are mainly held as speculative asset.

When speculator sentiment and optimism are sufficiently high, speculative demand for tokens is stable. Then, speculators continue to remain optimistic about the tokens even after a negative shock and therefore provide liquidity following a regime shift causing the token price to drop. Under these circumstances, the rise of speculative investment in good times also implies increased liquidity provision in bad times, which reduces token price volatility and increases expected token returns and adoption in good times. That is, the benign liquidity provision effect of speculators dominates the crowding-out effect. As a result, for high levels of speculator sentiment and investment, tokens are relatively stable and to a relatively large extent held by users for transactions.

Consistent with empirical evidence in Liu and Tsyvinski (2021), speculative trading volume and investment is procyclical and follows a boom and bust cycle, with high speculative trading in bullish regimes and low speculative trading in bearish regimes. Speculative investment builds up in a bull market with rising token price. In contrast, speculators wind down their positions in bad times, limiting their liquidity provision in bad times. Upon a regime shift that ends a bull market, token price crashes and speculative trading volume drops sharply, possibly to the point that there is no more speculative trading. This bear market persists until the next regime shift in which case speculation starts to build up again and the above cycle repeats. As speculative trading is procyclical, it amplifies price volatility compared to a situation without speculators.

Our analysis also generates predictions on what types of tokens attract speculation and explains why cryptocurrencies are particularly prone to speculation. We show that speculators tend to invest in platforms and tokens with high growth potential and uncertain (i.e., volatile) fundamentals, such as the platform's technology or the demand for the platform's services or products. Because uncertainty regarding cryptocurrency fundamentals is generally large compared to uncertainty regarding fundamentals of traditional assets (e.g., stocks), this finding rationalizes the high levels of speculation in cryptocurrency markets. Notably, uncertainty regarding platform fundamentals implies token price fluctuations and thus invites speculative trading, boosting token demand and price. As, in addition, speculation tends to increase price volatility, our analysis suggests two-way causality regarding the positive correlation between speculation and token price volatility (Silberholz and Wu, 2021). Because speculators provide valuable liquidity in cryptocurreny markets, a high level of volatility in cryptocurrencies that invites speculative trading can actually increase both adoption and token price, and so can be seen as a feature rather than a bug.

The model also delivers a life-cycle theory for cryptocurrencies, consisting of an initial growth stage that is followed by a hype stage and eventually by a mature stage. In the growth stage, the interest for tokens from financial investors and speculators is low, and demand for tokens stems primarily from actual usage. This initial growth stage could represent the early years of cryptocurrencies and Bitcoin (e.g., before 2012) when cryptocurrencies and Bitcoin have been mostly known to a tight community. The hype stage begins following continued growth in cryptocurrency adoption. During this hype stage, speculators (e.g., retail investors) and, more broadly, financial investors (such as VCs) start to invest in cryptocurrencies, which crowds out usage and exacerbates price volatility. The hype stage characterized by high speculation may describe the current state of cryptocurrency markets. Our analysis predicts that, provided sufficient and continued future growth in adoption, cryptocurrencies eventually reach a mature stage which, compared with the hype stage, features lower price volatility and higher usage.

The adverse crowding-out effect arises because speculators compete with users for token ownership, thereby raising token price and the costs of using tokens for transactions. The benign liquidity provision effect, in turn, increases the resale value of tokens and allows users to benefit from speculator sentiment and optimism. To mitigate the adverse crowding-out effect whilst harnessing the benefits of speculator optimism, we then propose a dual token structure with two native tokens: i) a price-stable transaction token (i.e., stablecoin) held by users for transactions and ii) a governance (equity) token held by speculators (financial investors). Users are charged a transaction fee and the governance token pays out revenues from transaction fees and stablecoin issuance as dividends. The proceeds from governance token issuance are used to buy back (and burn) stablecoins to stabilize their price. This dual token structure resembles the one of leading decentralized finance (DeFi) platforms, such as Terra featuring the stablecoin Terra USD and the governance token Luna. Our model accordingly has implications for the optimal design of such DeFi platforms.

Unbundling the investment and utility function of tokens, the dual token structure removes the crowding-out effect speculators levy on users, harnesses speculator optimism, and therefore stimulates platform adoption. Compared to a traditional token-based structure with a single token, the dual token structure also increases platform dollar value, limits the speculators' stake in the platform, and stabilizes platform adoption and user base. These improvements brought about by a dual token structure are sizeable only for platforms that attract substantial speculator interest to begin with, such as platforms with high growth potential and uncertain fundamentals.

Crucially, a dual token structure is different to a fiat-based structure featuring dollars as the transaction medium. When tokens serve as the platform transaction medium, the platform earns payoff from issuing these tokens, that is, seigniorage. A fiat-based structure does not generate seigniorage revenues. Seigniorage allows the platform to be profitable *without* i) charging users a transaction fee or ii) adopting other monetization models that may harm users (such as exploiting user/transaction data or putting advertisements). Compared to a fiat-based platform, the dual token structure features lower transaction fees but achieves higher adoption and platform value.

Finally, to demonstrate how the presence of speculators generates conflicts of interest in platform development and token design, we consider an extension in which the platform has access to an investment technology that boosts the growth rate of productivity but hampers contemporaneous platform transactions by reducing tokens' convenience yield. To maximize its dollar value, the platform chooses its investment to cater to the needs of the *marginal* token investor so as to maximize token price. Hence, under the baseline token-based platform structure with one token, the platform's incentives are aligned with those of the marginal investor determining token price, which inevitably leads to con icts of interest in the triangular relationship between the platform (owners), its users, and speculators. When the marginal token investor is a user, the token price internalizes the negative impact of investment on platform transactions, so that investment is low. When the marginal token investor is a speculator, the token price does not internalize the impact of investment on transactions, so investment is \ine ciently" high and the platform caters to speculators at the expense of users.

We then consider two solution approaches to resolve these con icts of interest. First, we propose a decentralized governance structure in which token holders can vote for the choice of investment and voting rights are proportional to token holdings. However, when the majority of token holders are speculators, they may implement high investment at the expense of users. Thus, the decentralized governance structure can mitigate con icts of interest but only to a limited extent. Second, we show that the dual token structure is e ective at alleviating these con icts of interests and aligns the platform's incentives (and the incentives of governance token holders) with those of the users. The reason is that under a dual token structure, platform revenues and dividends consist of seigniorage and fee revenues and so depend on stablecoin demand re ecting user preferences.

Our work adds to the literature on blockchain economics and cryptocurrencies.[1] Chiu and Koeppl (2019) , Cong and He (2019), Huberman, Leshno, and Moallemi (2019), Ebrahimi, Routledge, and Zetlin-Jones (2020), John, Rivera, and Saleh (2020), and Halaburda, He, and Li (2021) study the economic implications of blockchains and decentralized ledger technology. Hu, Parlour, and Rajan (2019), Shams (2020), Liu and Tsyvinski (2021), and Liu, Tsyvinski, and Wu (2021) empirically analyze the cryptocurrency return structure. Biais, Bisiere, Bouvard, Casamatta, and Menkveld (2018), Schilling and Uhlig (2019), Pagnotta (2021), and Sockin and Xiong (2021) provide a theoretical analysis of the pricing of cryptocurrencies. Sockin and Xiong (2022) micro-found the network e ect inherent to cryptocurrency adoption and provide a conceptual framework for tokenization. Easley, O'Hara, and Basu (2019), Biais, Bisiere, Bouvard, and Casamatta (2019), Garratt and van Oordt (2020), Lehar and Parlour (2020), Hinzen, John, and Saleh (2021), Cong, He, and Li (2021), and Prat and Walter (2021) analyze the economic implications of the proof-of-work consensus algorithm and mining, and Rosu and Saleh (2020) and Saleh (2021) focus on proof-of-stake as alternative protocol. A large subset of the literature on cryptocurrencies focuses on initial coin o erings (ICOs) (see Li and Mann (2019) for a literature review). Empirical studies on ICOs include Lyandres, Palazzo, and Rabetti (2019). Howell, Niessner, and Yermack (2020), and Davydiuk, Gupta, and Rosen (2020). Recent theoretical contributions on ICOs include Mali-

[1] For an extensive literature review on blockchain economics, see Chen, Cong, and Xiao (2021), and for an empirical overview of the public blockchain ecosystem, see Irresberger, John, and Saleh (2020). Related, John, O'Hara, and Saleh (2021) provide a survey about research on consensus mechanisms, cryptocurrencies, and Bitcoin in particular.

nova and Park (2018), Catalini and Gans (2018), Bakos and Halaburda (2019a,b), Li and Mann (2020), Lee and Parlour (2020), Goldstein, Gupta, and Sverchkov (2021), Garratt and Van Oordt (2021), Gan, Tsoukalas, and Netessine (2021a,b), Gryglewicz, Mayer, and Morellec (2021), Lyandres (2021), and Chod and Lyandres (2021).

We model platform transactions following Cong, Li, and Wang (2021a,b) who analyze the token pricing implications of users' inter-temporal adoption decisions. Danos, Marcassa, Oliva, and Prat (2021) provide a valuation framework for utility tokens with endogenous token velocity and show that early on during the token's adoption phase, the marginal investor holds tokens for purely speculative purposes. Different to Cong et al. (2021a,b) and Danos et al. (2021) that feature a single type of token investors (users), our paper introduces speculators, who trade based on sentiment, as separate investor type so as to highlight the dynamic interactions between platform users and speculators. We find that speculators and users interact via the crowding-out and liquidity provision effect. These effects are not identified in Cong et al. (2021a,b) and Danos et al. (2021). Sockin and Xiong (2021) develop a model of cryptocurrencies in which speculator sentiment is i.i.d across periods and has unambiguously negative effect on adoption and users. The key difference to their analysis is that we explicitly model optimizing speculators, whereby speculator sentiment and demand is dynamic and endogenously linked across periods. Our analysis is therefore complementary to Sockin and Xiong (2021), who focus on the interactions between fragility and speculator sentiment induced by network effects, and delivers several novel results. First, speculators stimulate adoption via the liquidity provision effect, and so can benefit users. Second, speculation and usage are static substitutes but dynamic complements. Third, our model links speculation to platform characteristics and past performance, affecting sentiment, and, therefore, features procyclical speculative trading which is consistent with empirical studies.

1 The Model

Time $t \in [0, \infty)$ is continuous. We consider a platform that settles transactions among its users with its native cryptocurrency (“tokens”). More generally, the model applies to cryptocurrencies and tokens which grant access to services and products or facilitate economic activities among users, including general-purpose and non platform-specific cryptocurrencies. There is one generic consumption good (“dollars”) that serves as the numeraire. Tokens are in unit supply, and have equilibrium price P_t in dollars.

There are two types of risk-neutral representative agents who live in overlapping generations (OLG), as, e.g., in (John et al., 2020; John, Rivera, and Saleh, 2021): A representative platform user, also referred to as "users," and a representative speculator, also referred to as "speculators." For the sake of clarity in explaining the OLG environment in a continuous time model, we consider that each cohort t lives from time t until time $t+\Delta$ with $\Delta > 0$. We take the continuous time limit $\Delta \to dt$, once we have introduced the agents and their optimization.

Cohort t of the representative user (speculator) is born at time t with an endowment of $K_t^U > 0$ ($K_t^S > 0$) dollars, and lives until time $t+\Delta$ when a new cohort $t+\Delta$ is born. Cohort t derives utility from consumption only at time $t+\Delta$ and can invest her endowment at time t in tokens or in a risk-free asset paying interest at rate $r > 0$. Users and speculators differ in their motives to hold tokens and in their beliefs about the platform's and tokens' prospects. Users hold tokens for transactions. Speculators are financial investors who do not transact on the platform and hold tokens solely for investment/trading returns. In the following, we denote by E_t^U the time-t expectation under user beliefs and by E_t^S the time-t expectation under speculator beliefs.[2]

The representative user. At time t, cohort t of the representative user decides on her dollar token holdings $V_t \in [0, K_t^U]$, while she invests the remainder, $K_t^U - V_t$, in the risk-free asset paying interest at rate r. That is, cohort t holds V_t/P_t tokens. It is not possible to short sell tokens, in that $V_t \geq 0$, and the user cannot borrow and invest more than her endowment, in that $V_t \leq K_t^U$.[3] At time $t+\Delta$, cohort t sells all of her token holdings (i.e., V_t/P_t tokens) at price $P_{t+\Delta}$ to cohort $t+\Delta$ (of users and speculators) and consumes her entire wealth, leading to consumption at time $t+\Delta$ of

$$c_{t+\Delta}^U(V_t) \equiv V_t \frac{P_{t+\Delta}}{P_t} + (K_t^U - V_t)(1+r\Delta).$$

Cohort t's lifetime utility then reads

$$u_t^U(V_t) \equiv \underbrace{c_{t+\Delta}^U(V_t)}_{\text{Consumption}} + \underbrace{\left(\frac{V_t^{1-\alpha} A_t^{\alpha}}{1-\alpha}\right)\Delta}_{\text{Convenience yield}}, \tag{1}$$

where $A_t > 0$ is the platform's productivity which is discussed later in more detail, and $\alpha \in (0,1)$ is a constant. As in Cong et al. (2021b), users derive a convenience yield from holding tokens that depends on the dollar value V_t of user token holdings. We interpret the convenience yield broadly.

[2] That is, as is common in the literature on speculation in financial markets (see, e.g., Scheinkman and Xiong (2003), Simsek (2013), Caballero and Simsek (2020), Simsek (2021)), speculation arises in our model due to belief disagreement among different agents.

[3] Because the marginal convenience yield tends to $+\infty$ as $V_t \downarrow 0$, the constraint $V_t \geq 0$ never binds (i.e., $V_t > 0$) and is strictly speaking redundant. The constraint $V_t \leq K_t^U$ could also capture in reduced form a capacity constraint in the amount of transactions the platform can handle. Such a capacity constraint could be the consequence of blockchain congestion (Hinzen et al. (2021)).

The convenience yield represents in reduced form the bene ts of transacting with tokens, of using the services of the platform via its utility tokens, or the rewards derived from staking tokens, for instance, within decentralized nance protocols.[4] User token holdings V_t, in turn, capture platform transaction volume, token or platform usage and adoption.

Taking token price P_t as given, cohort t user maximizes the expected lifetime utility, that is,

$$\max_{V_t \in [0; K_t^U]} E_t^U[u_t^U(V_t)]. \tag{2}$$

The following Lemma characterizes the solution to the user's problem.

Lemma 1. The solution to the user's problem (2) reads

$$V_t = A_t \left(\frac{1}{r - E_t^U[dP_t]/(P_t dt)} \right)^{\frac{1}{1}} \wedge K_t^U; \tag{3}$$

where we denote the token price change over $[t, t + \;]$ by $dP_t := P_{t+} - P_t$ and write $= dt$. And, "$\wedge$" is shorthand notation for the "minimum" operator, i.e., $x \wedge y = \min\{x, y\}$.

According to (3), platform adoption V_t decreases with the opportunity cost of holding tokens, i.e., the interest rate r, but increases with productivity A_t and expected token returns from the user perspective, $E_t^U[dP_t]/(P_t dt)$. Expected token returns reduce the e ective costs of holding tokens and transacting on the platform. Intuitively, when tokens serve as the transaction medium, users can capitalize on platform growth by earning token returns when transacting on the platform.

The representative speculator At time t, cohort t of the representative speculator decides on her dollar token holdings $S_t \in [0; K_t^S]$, while she invests the remainder, $K_t^S - S_t$, in the risk-free asset. As in Scheinkman and Xiong (2003), the speculator cannot short sell tokens, in that $S_t \;\; 0$, and cannot borrow and invest more than her endowment, in that $S_t \;\; K_t^S$.[5] At time $t +$, cohort

[4] The reduced form modelling of transactions via a convenience yield is akin to the "money-in-the-utility-function approach" employed in the classical monetary economics literature (see, e.g., Feenstra (1986)). Note that, given the broad interpretation of the convenience yield, our model more generally applies to all types of tokens that grant users access to services/product or facilitate economic activities among users. Our theory also applies to general-purpose and non platform-speci c cryptocurrencies that serve both as a transaction medium and speculative asset, e.g., Bitcoin. Finally, note that locking up and staking tokens (such as Ether) in decentralized nance protocols, like Compound for decentralized lending or Uniswap for decentralized exchange (Lehar and Parlour (2021)), allows to earn staking rewards which could be represented by the convenience yield too.

[5] An alternative interpretation of the constraint $S_t \;\; K_t^S$ is that speculators have convex holding costs for tokens or a convex cost of capital, which could capture in reduced form risk-aversion as in Scheinkman and Xiong (2003). The convex holding cost implies a cost of holding a large undiversi ed stake in the platform: In our model, the marginal cost of holding tokens becomes $+1$ when $S_t > K_t^S$. Mathematically, such a convex holding cost would resemble the convex inventory cost of liquidity providers (market makers) as in Amihud and Mendelson (1980) and would similarly imply a bound on speculative investment S_t. Alternatively, the constraint $S_t \;\; K_t^S$ could arise due to a capacity constraint in the amount of speculative trading the platform can handle (e.g., due to blockchain congestion).

t sells all of her token holdings (i.e., S_t/P_t tokens) at price $P_{t+\Delta}$ to cohort $t+\Delta$ (of users and speculators) and consumes her entire wealth, leading to consumption at time $t+\Delta$ of

$$c^S_{t+\Delta}(S_t) \equiv S_t \frac{P_{t+\Delta}}{P_t} + (K^S_t - S_t)(1 + r\Delta).$$

Taking token price P_t as given, cohort t of the speculator maximizes her expected lifetime utility derived from consumption at $t+\Delta$, that is,

$$\max_{S_t \in [0, K^S_t]} E^S_t[c^S_{t+\Delta}(S_t)]. \tag{4}$$

The following Lemma characterizes the solution to the speculator's problem (4).

Lemma 2. The solution to the speculator's problem in (4) reads

$$S_t = \begin{cases} 0 & \text{if} \quad E^S_t[dP_t]/(P_t dt) < r \\ \hat{S}_t \in [0, K^S_t] & \text{if} \quad E^S_t[dP_t]/(P_t dt) = r \\ K^S_t & \text{if} \quad E^S_t[dP_t]/(P_t dt) > r; \end{cases} \tag{5}$$

where we denote the token price change over $[t, t+\Delta]$ by $dP_t = P_{t+\Delta} - P_t$ and write $\Delta = dt$.

Note that S_t captures speculative investment or speculative trading volume. Importantly, the sole purpose of the discrete time framework was to introduce overlapping generations of agents. As we have characterized agents' optimization and their optimal token holdings V_t and S_t, we can conduct the following analysis in the continuous time limit $\Delta \to dt$.

Platform productivity and beliefs. The platform and the convenience yield of its tokens are characterized by its productivity A_t which captures the general usefulness of the platform, the quality of the platform's technology, or user demand for platform services. Platform productivity grows at rate μ_t:

$$\frac{dA_t}{A_t} = \mu_t dt; \tag{6}$$

where $\mu_t = \mu \in \{\mu_L; \mu_H\}$ follows a Markov switching process with states L ("bad times") and H ("good times"). We assume $\mu_L < \mu_H$. That is, state H corresponds to a regime of rapidly growing demand for platform services and tokens, while this demand grows at lower pace or even shrinks in state L. Interpreted differently, state H may describe a bull market with rising token price (e.g., when $\mu_H > 0$) and state L may describe a bear market (e.g., when $\mu_L < 0$). As we show, a switch from state H to state L results in a token price drop (i.e., crash). The instantaneous transition probabilities are from state L to state H and $(1 \quad)$ from state H to state L, where $\quad 0$ is a constant and $\quad 2\ [0; 1]$ determines the platform's average (long-run) growth rate $\ _H + (1 \quad)\ _L$. Given $\ ;\ $, we have $P(\ _{t+dt} = \ _H \mid \ _t = \ _L) = \ dt$ and $P(\ _{t+dt} = \ _L \mid \ _t = \ _H) = (1 \quad)dt$.

Productivity A_t, its growth rate $\ _t$, and are public knowledge and observable, but is not. Notably, users and speculators form di erent beliefs about . Users are convinced that $= \ _B$, and do not update their beliefs. In contrast, speculators believe that either $= \ _B$ or $= \ _G$, with $\ _G \quad \ _B$. We denote by $q_t \quad P^S_t(\ = \ _G)$ speculators' belief at time t that $= \ _G$ which is conditional on all information up to time t. The fact that users do not update their beliefs at all could be interpreted as an extreme form of overcon dence (Scheinkman and Xiong, 2003), while speculators are less overcon dent and update their beliefs over time.[6] An alternative interpretation is that users as the platform's insiders are informed about the platform's fundamentals while speculators are not and therefore dynamically form beliefs about based on the past.

Finally, for technical reasons, we impose several parameter conditions. We assume

$$\ _H < r + \ (1 \quad \ _G) \tag{7}$$

and

$$\ _G\ _H + (1 \quad \ _G)\ _L < r: \tag{8}$$

Parameter condition (7) ensures that, even under the most optimistic beliefs (i.e., $q_t = 1$ and $(q_t) = \ _G$), the growth rate $\ _H$ lies below the \e ective discount rate" in state H, $r + \ (1 \quad \ _G)$. And, (8) ensures that the average (long-run) growth rate lies below the discount rate r, which combined with $\ _H \quad \ _L$ readily implies $\ _L < r$. We can combine conditions (7) and (8) as well as $\ _G < 1$ to obtain

$$\ _G < \quad 1 \quad \frac{\ _H \quad r}{} \ \wedge \ \frac{r \quad \ _L}{\ _H \quad \ _L} \ \wedge \ 1: \tag{9}$$

Dynamic learning, extrapolation, and sentiment. Cohort t of the speculator forms her beliefs q_t in a Bayesian manner based on all available information of the entire past (from time 0 to time t), starting with a prior at time $t = 0$, q_0. In other words, over $[t; t + dt)$, speculators have prior q_t and update their beliefs according to Bayesian learning, leading to the posterior q_{t+dt}.[7] Average

[6] Thus, our modelling of di erences in speculator and user beliefs is similar to the one in Scheinkman and Xiong (2003) where overcon dence causes belief di erences across di erent agent groups. However, we also note that the exact source of belief disagreement is not crucial to our ndings; our key results are likely carry through as long as belief di erences between user and speculators cause speculators to trade tokens even though they do not derive convenience yield.

[7] Notice that cohort t speculator forms a belief based on all past information. A di erent interpretation is that at birth at time t, cohort t inherits the belief from the previous cohort, and updates her belief according to Bayes' rule over her lifetime $[t; t + dt]$.

from speculators' point of view is then de ned as $_t = \; (q_t) \quad E^S_t[\;] = q_t \; _G + (1 \quad q_t) \; _B$, which increases with q_t, i.e., $^0(q_t) = \; _G \quad _B \quad 0$. Next, let $dJ^H_t = 1$ ($dJ^L_t = 1$) indicate a regime shift from state L (H) into state H (L); otherwise, $dJ^H_t = 0$ ($dJ^L_t = 0$). The following Lemma characterizes the dynamics of speculator beliefs.

Lemma 3. Speculator beliefs q_t evolve according

$$dq_t = \; ^q_t dt + \; q_H(q_t) \quad q_t \; dJ^H_t + \; q_L(q_t) \quad q_t \; dJ^L_t; \tag{10}$$

with

$$^q_t = \begin{cases} _H(q_t) \quad q_t(1 \quad q_t)(\; _G \quad _B) & \text{if} \quad _t = \; _H \\ _L(q_t) \quad q_t(1 \quad q_t)(\; _G \quad _B) & \text{if} \quad _t = \; _L: \end{cases} \tag{11}$$

and

$$q_H(q_t) = \frac{q_t \; _G}{q_t \; _G + (1 \quad q_t) \; _B} \quad \text{and} \quad q_L(q_t) = \frac{q_t(1 \quad _G)}{1 \quad q_t \; _G \quad (1 \quad q_t) \; _B}: \tag{12}$$

In state H (L), q_t drifts up (down) in that $^q_t = \; _H(q_t) > 0$ ($^q_t = \; _L(q_t) < 0$). And, a regime shift to state H (state L) implies a upward (downward) jump of the belief from q_t to $q_H(q_t)$ ($q_L(q_t)$).

Lemma 3 also demonstrates that unlike users, speculators extrapolate based on the past, in that they become more optimistic (pessimistic) following positive (negative) events. Formally, q_t increases over time when the growth $_t$ rate is high or jumps up, while q_t decreases over time when the growth rate is low or jumps down. Thus, assuming that speculators update their beliefs about platform prospects is similar to assuming extrapolative expectations. We refer to q_t as speculator sentiment or optimism. A high value of q_t implies that speculators are optimistic about future token returns and represents a bullish sentiment, which boosts perceived expected token returns $E^S_t[dP_t]=(P_t dt)$ and speculative investment.

Token price. Having characterized the sources of uncertainty in the model, we postulate that equilibrium token price in dollars P_t evolves according to

$$\frac{dP_t}{P_t} = \; ^P_t dt + \; ^H_t dJ^H_t + \; ^L_t dJ^L_t: \tag{13}$$

Price drift ^P_t and the jump loadings ^H_t and ^L_t are endogenous and determined in equilibrium, where ^H_t (^L_t) is the percentage change in token price upon a regime shift into state H (L).

Discussion: Different agent types and beliefs. Before proceeding, we discuss some of the differences between users and speculators in more detail. Notice that speculators are more optimistic than users, so they attach a higher (lower) likelihood to positive (negative) events than users. Our results would remain unchanged if we assumed that speculators believe $\in \{{}', {}_G\}$ with ${}' \neq {}_B$, which would allow for speculators to become (temporarily) more pessimistic than users when ${}' < {}_B$.[8] A possible interpretation of speculator optimism is that users are more risk-averse than speculators, and the transition probabilities from user perspective can be viewed as risk-adjusted probabilities: Compared with less risk-averse speculators, users attach a higher probability to negative outcomes (i.e., switch from H to L) and a lower probability to positive outcomes (i.e., a switch from L to H).[9]

Overall, the two types of agents differ i) in the utility they derive from tokens and ii) in their optimism/beliefs (due to dynamic learning), i.e., there are less optimistic users and more optimistic speculators. A more general model could feature two additional types of agents, namely, less optimistic speculators (e.g., with fixed belief $= {}_B$) and optimistic users who have the same beliefs q as speculators.[10] Note that speculators, who are no more optimistic than users, value tokens strictly less than users, who derive convenience yield, and generally do not participate. Thus, the presence of users, who have the same beliefs q and are as optimistic as speculators and have non-trivial endowment, would be akin to removing speculators from the model which is akin to a sufficient reduction in ${}_G$.[11] A tension therefore arises only between more optimistic speculators and less optimistic users; so we restrict attention to these two types of agents. Related, since we consider representative agents, our modelling is broadly consistent with differences in beliefs and utility derived from tokens among individual users and speculators.

1.1 Token pricing and equilibrium concept

Token market capitalization (in dollars) is P_t, and user and speculator dollar token holdings are V_t and S_t respectively. As a result, the token market clearing condition becomes

$$P_t = V_t + S_t. \tag{14}$$

[8] As will become clear later, speculators hold tokens only if they are sufficiently more optimistic than users and, in particular, never hold tokens when ${}_G$ is sufficiently close to ${}_B$. Thus, choosing ${}_G$ sufficiently small is akin to assuming ${}' < {}_G$ or to removing speculators from the model. Likewise, increasing ${}_G$ has similar effects as raising ${}'$.

[9] Users' reduced risk tolerance could reflect that price stability is an important feature of any transaction medium (Rocheteau, 2011; Doepke and Schneider, 2017).

[10] We would like to thank Jiasun Li for pointing this out.

[11] This finding will be formalized at a later stage, where we show that speculators hold tokens only if they are more optimistic than users.

Expected token returns reflect whether the marginal token investor is i) a speculator or ii) a user. When the speculator holds tokens and $S_t > 0$, we distinguish two different cases. First, when $S_t \in (0, K_t^S)$, then (5) implies $P_t = E_t^S[dP_t]/(r\,dt)$. In this case, the speculator is marginal in determining token price. Second, when $S_t = K_t^S$, then (14) implies $P_t = K_t^S + V_t$. In this case, the user is marginal in determining token price, as the speculator already invests her entire endowment. Altogether, when $S_t > 0$, token price satisfies

$$P_t = P_t^S \equiv \left(\frac{E_t^S[dP_t]}{r\,dt}\right) \wedge \left(K_t^S + V_t\right). \tag{15}$$

When there is no speculative investment (i.e., $S_t = 0$), then the marginal token investor is user. As a result, token market clearing implies $V_t = P_t$, and — using (3) — we obtain the following expression for the token price:

$$P_t = P_t^U \equiv \left(A_t \frac{1}{r - E^U[dP_t]/(P_t\,dt)}\right)^{\frac{1}{1-\alpha}} \wedge K_t^U. \tag{16}$$

We can combine expressions (15) and (16) to obtain

$$P_t = \max\left\{P_t^U, P_t^S\right\}. \tag{17}$$

Intuitively, P_t^S is the price that speculators are willing to pay and P_t^U is the price that users are willing to pay for tokens. The equilibrium token price P_t is the maximum price that users or speculators are willing to pay. The dollar value of speculative investment is $S_t = \max\{P_t^S - P_t^U, 0\}$.

If the constraints $V_t \leq K_t^U$ and $S_t \leq K_t^S$ do not bind, one can rearrange (17) and solve[12]

$$r = \max\left\{\left(\frac{A_t}{P_t}\right)^{1-\alpha} - \frac{E_t^S[dP_t] - E_t^U[dP_t]}{P_t\,dt}, 0\right\} + \frac{E_t^S[dP_t]}{P_t\,dt}. \tag{18}$$

Token pricing equation (18) resembles a traditional valuation equation for financial assets. The left-hand side is the required rate of return, r. The right-hand side depicts expected token returns (second term) from speculators' perspective and the adjusted (marginal) convenience yield to holding tokens that the marginal token investor derives (first term in curly brackets). This term is the marginal convenience yield users derive from holding tokens, $\left(\frac{A_t}{P_t}\right)^{1-\alpha}$, minus relative “speculator

[12] For a derivation, first solve (17) for r to obtain

$$r = \max\left\{\left(\frac{A_t}{P_t}\right)^{1-\alpha} + \frac{E_t^U[dP_t]}{P_t\,dt}, \frac{E_t^S[dP_t]}{P_t\,dt}\right\}.$$

Then, subtract $\frac{E_t^S[dP_t]}{P_t\,dt}$ on both sides and rearrange to obtain (18).

optimism" as captured by $E^S_t[dP_t] - E^U_t[dP_t] \geq 0$. The adjusted convenience yield, i.e., the term in curly brackets, is positive if and only if the marginal token investor is a user, as speculators do not derive convenience yield.

We now study a Markov equilibrium without rational bubbles and state variables A_t, $z \in \{L, H\}$ (or equivalently $\mu_t \in \{\mu_L, \mu_H\}$), and q_t which is characterized by the following conditions. First, all agents act optimally. The representative user solves at any time $t \geq 0$ the optimization in (2) so that user token holdings are characterized in (3). The representative peculator solves at any time $t \geq 0$ the optimization in (4) so that (5) holds. Second, the token market clears, in that (14) is satisfied. As shown above, the relations (3), (5), and (14) imply that the token price P_t satisfies the equilibrium pricing relationship (17), which simplifies to (18) when $V_t < K^U_t$ and $S_t < K^S_t$.

Following Sockin and Xiong (2021), we assume that the endowment of users K^U_t and speculators K^S_t grows with platform productivity A_t to capture that a platform with higher productivity attracts more users as well as more interest and capital from speculators.[13] For tractability, we assume $K^U_t = k^U A_t$ and $K^S_t = k^S A_t$, with constants $k^U > 0$ and $k^S > 0$.

1.2 Solving the Markov Equilibrium

We now solve for the Markov equilibrium described above. We demonstrate that token price P_t and platform adoption V_t can be expressed as functions of A_t and $z \in \{L, H\}$ (or equivalently $\mu \in \{\mu_L, \mu_H\}$) as follows. Token price and platform adoption V_t scale with A_t, so that $P_t = A_t p_z(q_t)$ and $V_t = A_t v_z(q_t)$ for $z \in \{L, H\}$, where $p_z(q)$ and $v_z(q)$ are functions of $(q; z)$ only.

The argument to solve for the equilibrium follows a conjecture-and-verify approach. That is, we conjecture that token price satisfies $P_t = A_t p_z(q_t)$ and token usage satisfies $V_t = A_t v_z(q)$. Given this conjecture, we can calculate the price drift in (13):

$$\mu^P_t = \frac{\partial P_t}{\partial A_t}\frac{A_t \mu_t}{P_t} + \frac{\partial P_t}{\partial q_t}\frac{\mu^q_t}{P_t} = \mu_z + \frac{p'_z(q_t)}{p_z(q_t)}\mu_z(q_t); \tag{19}$$

where $\mu^q_t = \mu_z(q_t)$ is described in Lemma 3. Thus, the price drift depends on $(q_t; z)$ only. We now characterize price changes upon a regime shift, that is, η^H_t and η^L_t. Itô's Lemma implies that percentage token price changes in response to jump shocks are characterized by

$$\eta^H_t = \frac{p_H(q_H) - p_L(q_t)}{p_L(q_t)} = \frac{p_H(q_H)}{p_L(q_t)} - 1 \quad \text{and} \quad \eta^L_t = \frac{p_L(q_L) - p_H(q_t)}{p_H(q_t)} = \frac{p_L(q_L)}{p_H(q_t)} - 1 \tag{20}$$

[13] The assumption that K^U_t and K^S_t increases with A_t may also reflect that the platform's capacity to facilitate transactions improves with its productivity/technology A_t, thereby relaxing capacity constraints on usage and speculative trading.

respectively, where $q_z \equiv q_z(q_t)$ is described in Lemma 3.

We observe that μ_t^P as well as η_t^H and η_t^L depend only on $q_t = q$ and $z \in \{L, H\}$ (and not on A_t). As a result, by the law of motion of the token price in (13), expected token returns of users and speculators read

$$\frac{E_t^x[dP_t]}{P_t dt} = \varepsilon_z^x(q_t) \begin{cases} \mu_t^P + \eta_t^H E_t^x[dJ_t^H]/dt; & \text{if} \quad z = L; \\ \mu_t^P + \eta_t^L E_t^x[dJ_t^L]/dt; & \text{if} \quad z = H; \end{cases} \tag{21}$$

with $x \in \{S, U\}$ where S stands for "speculator" and U stands for "user."

Because μ_t^P and η_t^H; η_t^L only depend on $(q; z)$, expected token returns $\frac{E_t^x[dP_t]}{P_t dt}$ for $x \in \{S, U\}$ only depend on $(q; z)$ too, in that we can write $\frac{E_t^x[dP_t]}{P_t dt} = \varepsilon_z^x(q_t)$. As such, (3) implies that $V_t = A_t v_z(q_t)$ where $v_z(q)$ only depends on $(q; z)$. Using (16), we obtain that $P_t^U = A_t v_z(q_t)$ scales with A_t, whereby $v_z(q) = \left(\frac{1}{r - \varepsilon_z^U(q)}\right)^{\frac{1}{1-\alpha}} \wedge k^U$. In addition, we can rewrite (15) as

$$P_t^S = \left(\frac{P_t}{r}\frac{E^S[dP_t]}{P_t dt} \wedge K_t^S\right) + V_t = A_t\left(\frac{p_z(q)\varepsilon_z^S(q_t)}{r} \wedge v_z(q_t) + k^S\right);$$

where we used that $V_t = A_t v_z(q_t)$ and $K_t^S = A_t k^S$. Hence, P_t^S scales with A_t. As token price satisfies (17), i.e., $P_t = \max\{P_t^U; P_t^S\}$, it follows that token price takes the form $P_t = A_t p_z(q_t)$.

We have now verified our conjecture that there is a Markov equilibrium in which $P_t = A_t p_z(q_t)$ and $V_t = A_t v_z(q_t)$. Speculator token holdings, capturing speculative investment and trading volume, read $S_t = P_t - V_t = A_t s_z(q_t)$ with $s_z(q_t) = p_z(q_t) - v_z(q_t)$. In what follows, we omit time subscripts unless necessary. We summarize our results in the following Proposition.

Proposition 1 (Markov Equilibrium). In a Markov equilibrium with state variables A_t and $\theta_t \in \{\theta_L; \theta_H\}$ or equivalently $z \in \{L; H\}$, platform adoption is characterized in (3), and satisfies $V_t = A_t v_z(q_t)$. Token price follows (13), and satisfies $P_t = A_t p_z(q_t)$ and (17). Speculative investment is characterized in (5), and satisfies $S_t = A_t s_z(q_t)$ with $s_z(q_t) = p_z(q_t) - v_z(q_t)$. Expected token returns $E^x[dP_t]/(P_t dt)$ for $x \in \{S; U\}$ satisfy (21), with price drift μ_t^P and jump loading η_t^z described in (19) and (20) respectively. Speculator beliefs q_t are characterized in Lemma 3.

In the following sections, we solve for the token prices and adoption more explicitly. To gain some intuition and highlight some key mechanisms, we start by discussing the solution without learning. After that, we turn to the complete model with dynamic learning.

2 Solution and Analysis

2.1 Benchmark: Solution without Learning

According to Lemma 3, there is no more learning and $dq = \ _z(q) = 0$, when speculator beliefs reach $q = 0$ or $q = 1$. Then, the only dynamics stem from the regime shifts, and the equilibrium is characterized by (scaled) token prices $p_L(q)$ and $p_H(q)$ as well as adoption levels $v_L(q)$ and $v_H(q)$. We now study the equilibrium without learning, and consider $q = 0$ or $q = 1$. For simplicity, when $q = 0$ or $q = 1$, our notation suppresses the dependency of equilibrium quantities on q, as $dq = 0$, and we write $\quad (q)$.

We now solve for equilibrium token prices $p_z \quad p_z(q) = p_z(q)$ and adoption levels $v_z \quad v_z(q) = v_z(q)$ for z 2 f $L; H$ g. First, note that by (19) with $\ _z(q) = 0$, we have $^P_t = \ _t = \ _z$. Using (3), (20), and (21), we obtain that in state G, scaled adoption reads

$$v_H(q) = v_H = \frac{1}{r \quad _H \quad (1 \quad _B)(p_L = p_H \quad 1)}^{\frac{1}{1}} \wedge k^U: \tag{22}$$

The pricing equation (17) then implies the scaled token price

$$p_H(q) = p_H = \max \quad v_H; \frac{(1 \quad)p_L}{r \quad _H + (1 \quad)} \wedge v_H + k^S \quad : \tag{23}$$

Likewise, in state L, scaled adoption reads

$$v_L(q) = v_L = \frac{1}{r \quad _L \quad _B(p_H = p_L \quad 1)}^{\frac{1}{1}} \wedge k^U; \tag{24}$$

and scaled token price is

$$p_L(q) = p_L = \max \quad v_L; \frac{p_H}{r \quad _L +} \wedge v_L + k^S \quad : \tag{25}$$

Speculative investment, in turn, reads $s_z(q) = s_z = p_z(q) \quad v_z(q)$. We characterize the equilibrium in the following Proposition.

Proposition 2. Consider $q = 0$ or $q = 1$, and set $\ = \ (q)$, so $\quad _G$. Then, scaled adoption and token prices are characterized in (22), (23), (24), and (25). Token price satis es $p_H \quad p_L$ where this inequality is strict if $v_z < k^U$ for $z = L; H$.

To solve for the equilibrium, one needs to solve (22), (23), (24), and (25) for $v_z = v_z(q)$ and $p_z = p_z(q)$ with $z = L; H$; in general, the solution is numerical. Notice that $(q) = \quad$ as a monotonic transformation of q with $^0(q) > 0$ captures speculator sentiment or optimism, thereby measuring speculators' willingness to invest. All else equal, an increase in should lead to more speculative investment. The following Proposition analytically proves some equilibrium properties in dependence of when k^U is su ciently large and the constraint v_z k^U does not bind.

Proposition 3. Consider $q = 0$ or $q = 1$. Suppose that(22), (23), (24), and (25) admit a unique solution v_z and p_z with $z = L; H$. Further, suppose that in equilibrium, $v_z < k^U$ for $z = L; H$. Then, the following holds:

1. There is speculative investment in statez if and only if $(\quad _B)(p_H \quad p_L) > (p_z)$.
2. Speculators either i) do not hold tokens at all (i.e.,$s_H = s_L = 0$), ii) hold tokens in state L only (i.e., $s_L > 0 = s_H = 0$), or iii) hold tokens in both states (i.e., $s_z > 0$ for $z = L; H$) in which case speculative investment is not interior (i.e.,$s_H = k^S$ or $s_L = k^S$). There exists unique $\_ \; 2 \; [\; _B ; \bar{} \;]$ such that $s_L = s_H = 0$ for $\_$ and $s_L > 0$ for $2 \; (\_ ; \bar{} \;]$.
3. Speculators hold tokens in both statesL and H (i.e., $s_L ; s_H > 0$) only if $_H > r$. Provided $_H > r$, $\_ < \bar{}$, and $\bar{} = 1 \quad \frac{_H \quad r}{}$ < 1, the following holds. First, $s_H = s_L = 0$ for $< \_$. Second,$s_L > 0 = s_H$ for $2 \; (\_ ; \tilde{}\,)$ with $\_ < \tilde{} < \bar{}$. Third, $s_H ; s_L > 0$ for su ciently large values of $< \bar{}$.

Claim 1 of Proposition 2 states that there is speculative token investment if and only if

$$\frac{E^S[dP] \quad E^U[dP]}{P\,dt} = (\quad _B) \; \frac{p_H \quad p_L}{p_z} \; > (p_z) \quad ^1: \tag{26}$$

The interpretation of (26) is that speculators invest in tokens if and only if their relative optimism about future token returns on the left-hand-side outweighs the marginal convenience yield (i.e., $(p_z) \quad ^1$ in the scenario$v_z = p_z$) users derive from holding tokens. As (26) does not hold for $_B$, there is no speculative investment when is su ciently low. Put di erently, for speculative investment to arise in equilibrium, speculators must be more optimistic than users, i.e., $> \_ \; _B$. Because $(0) = \; _B$, there is no speculative investment when$q = 0$, so the case$q \,! \; 0$ is the benchmark without speculators as $\lim_{q! \; 0} dq = 0$ and $q = 0$ is an absorbing state.

Notably, speculators a ect users in two opposing ways, i) crowding-out and ii) liquidity provision. Claim 3 of Proposition 3 suggests that, when speculator sentiment takes intermediate levels, speculators acquire tokens in stateL but not in state H. That is, speculators provide liquidity in bad times and buy tokens after a price drop, which boosts token price in bad times. Increasing token price in bad times, speculative investment, however, reduces the potential for token price appreciation and so expected token returns. Lower expected token returns increase users' cost of holding tokens and transacting on the platform, which reduces platform adoption and transactions in bad times. In other words, speculators crowd out platform users in bad times. Due to this crowding-out effect, speculative investment and token usage can be viewed as static substitutes.

However, pushing up token price in bad times, speculators also decrease token price downside risk in good times, thereby raising expected token returns, platform adoption, and token price. Upon a token price crash when drops down to $_L$, user demand for tokens falls and users would like to sell tokens. Importantly, users can sell tokens in bad times at more favorable terms precisely because speculators buy. That is, speculative investment in bad times improves the resale option value of tokens and therefore increases users' incentives to hold tokens for transactions and to adopt the platform in good times. We interpret this effect as liquidity provision by speculators. As speculators buy tokens in bad times only due to the prospect of high future platform adoption, speculative and transactional token investments can be viewed as dynamic complements.

Finally, for sufficiently large values of and for $_H > r$, speculators buy tokens, boost token price, and crowd out usage in both states L and H. Then, the liquidity provision effect is present in both states too, which ceteris paribus stimulates adoption. That is, in state H, the liquidity provision effect insures users from downside risk, and, in state L, the fact that speculators buy tokens (i.e., provide liquidity) in state H boosts expected token returns. It then depends on the exact parameter values on whether crowding out or liquidity provision dominates.

An important technical detail is that, in an equilibrium without rational bubbles, token price must reflect user preferences in at least one of the states L or H, i.e., the user must be marginal in determining token price in at least one state L or H. The reason is that speculators do not fundamentally value tokens and, therefore, hold tokens only to resell them at a later point to users; if speculators only traded tokens among themselves ad infinitum, token price would be a rational bubble. As such, when $s_z(q) > 0$ for $z = L; H$, it must be that $s_L = k^S$, in which case the user is marginal in determining token price in state L, or $s_H = k^S$, in which case the user is marginal in determining token price in state H. As a consequence, token price p_z is potentially discontinuous in which reflects feedback effects inherent to speculation. Once becomes sufficiently large, speculators start buying tokens in both states and drive up prices to the point that their investment is constrained by their limited endowment, i.e., s_L or s_H reaches k^S.[14] The following Corollary concludes this section by analytically establishing the crowding-out and liquidity provision effects.

Corollary 1. [Crowding-out vs. liquidity provision] Consider $q = 0$ or $q = 1$, and suppose that $v_z < k^U$ and v_z is differentiable with respect to . Then, the following holds:

1. If there is no speculative investment (i.e., $s_H = s_L = 0$), then $\frac{\partial p_z}{\partial} = \frac{\partial v_z}{\partial} = 0$.

[14] This outcome can be interpreted as a static form of a speculative bubble with ever increasing prices which is only contained because speculators' endowment (wealth) is limited.

2. If there is speculative investment in state L, $s_L \in (0, k^S)$, but speculative investment in state H is either zero or at its maximum (i.e., $s_H \in \{0, k^S\}$), then: $\frac{\partial p_z}{\partial} > 0$, $\frac{\partial v_H}{\partial} > 0$, and $\frac{\partial v_L}{\partial} < 0$.

2.2 Dynamic Model

The benchmark with $q = 0$ or $q = 1$ is useful for understanding the boundary behavior of the system. We now turn to the more general model with learning. With dynamic learning, speculator sentiment evolves over time and, since users and speculators are forward-looking, affects dynamic adoption and token pricing. The analysis of the dynamic model leads to a set of additional results.

As shown in Appendix G, the token price $p_z(q)$ is determined by the system of coupled delayed ODEs characterized in (G.22) and (G.26). Note that q drifts up in state H and down in state L. In the limit $q \to 0$ or $q \to 1$, speculator sentiment remains constant ("stationary") at levels $q = 0$ or $q = 1$, that is, $q = 0$ or $q = 1$ imply $dq = 0$. As such, $p_z(q)$ solve (G.22) and (G.26) subject to the following two boundary conditions:

$$\lim_{q \to 1} p_H(q) = p_H(1) \quad \text{and} \quad \lim_{q \to 0} p_L(q) = p_L(0); \tag{27}$$

where $p_z(q)$ satisfies (23) and (25) for $z = H, L$, which completes the characterization of token pricing.[15] We characterize the equilibrium in the following Proposition.

Proposition 4. Equilibrium token prices $p_z(q)$ for $z = L, H$ solve on $(0, 1)$ the ODE system, characterized by (G.22) and (G.26) in Appendix G, subject to the boundary conditions in (27) and the dynamics of the state variable q in Lemma 3.

We now solve numerically for the equilibrium token price $p_z(q)$ under the following parameters. We set $r = 0.05$ and $= 0.1$, and normalize $= 1$. Next, we choose $_L = 0.2 < _H = 0.1$, and $_B = 0.2$ and $_G = 0.8$. Under these parametric assumptions, conditions (7) and (8) are satisfied. Scaled endowments are set to $k^U = k^S = 30$, which also implies that $v_z(q) < k^U$ for all all (q, z). The model's qualitative implications are robust to the choice of these parameters.

2.3 Equilibrium Dynamics

Figure 1 graphically illustrates the equilibrium dynamics under the numerical solution by plotting (scaled) equilibrium quantities against speculator sentiment/optimism q in state L (solid red line) and state H (solid black line). Panel A of Figure 1 shows that scaled token prices $p_z(q)$ increase with speculator optimism, notably, even if there is no speculative investment in state q and $s_z(q) = 0$.

[15] Recall that when $q = 0$ or $q = 1$, we suppress the dependency of equilibrium quantities on q. That is, we write $= (q)$, $v_z = v_z(q) = v_z(q)$, and $p_z = p_z(q) = p_z(q)$.

Figure 1: Equilibrium Dynamics. Solid black (red) lines correspond to quantities in state H (L). In Panel D, the horizontal dashed black (dotted red) line depicts the adoption level in the no-speculator benchmark for state H (state L), i.e., $v_H(0)$ ($v_L(0)$). The vertical dashed red line in Panel E depicts $\tilde{q} := \max\{q \quad 0 : s(q_L(q)) = 0\}$. The parameters are $r = 0.05$, $= 1$, $= 0.1$, $_B = 0.2$, $_G = 0.8$, and $k^U = k^S = 30$.

The reason is that prices are forward looking and account for prospective speculative demand that drives up token price in the future. Higher speculator sentiment q implies an increased prospective demand from speculators, raising resale option value of tokens and prevailing token price.

Panel B plots expected token returns from the user perspective, $"^U_z(q) := E^U[dP]=(P\,dt)$, in states $z = L$ (solid red line) and $z = H$ (solid back line). Panel C displays the absolute value of (percentage) token price changes upon a regime shift from state L to H and from state H and L respectively, i.e., $j\ ^H(q)j = jp_H(q)=p_L(q) \quad 1j$ (solid red line) and $j\ ^L(q)j = jp_L(q)=p_H(q) \quad 1j$ (solid black line). Note that the solid black line depicts the percentage token price drop when there is a negative shock in state H that causes to drop from $_H$ to $_L$ and so measures the token price downside risk in state H. Likewise, the solid red line depicts the percentage token price increase when there is a positive shock in state L that causes to increase from $_L$ to $_H$, capturing the potential for token price appreciation in state L. Taken together, Panel C plots measures of token price uctuation and/or volatility and, as both lines are hump-shaped in q, we conclude that token price volatility and risk are hump-shaped in q too.

Panels D and E display scaled platform adoption $v_z(q)$ and speculative investment $s_z(q)$. In Panel E, the horizontal black dashed (dotted red) line depicts the level of adoption in the model without speculators in state H (state L), that is, $v_H(0)$ ($v_L(0)$). As shown in Panel E, speculators do not hold tokens for su ciently low levels of q, and speculative investment increases with q. Eventually, when q is close to one, speculators invest their entire endowment in tokens in state $z = H$, i.e., $v_H(q) = k^S$. In Panel E, the vertical dashed red line depicts $\underline{q} := \max f\, q \quad 0 : s(q_L(q)) = 0\, g$, where $q_L(q)$ is the speculator sentiment after a regime shift from state H to state L. If $q < \underline{q}$ and $s_H(q) > 0$, then speculators sell all of their tokens upon a token price crash when drops from $_H$ to $_L$. In other words, speculators provide liquidity after a token price drop, in that $s_L(q_L(q)) > 0$, if and only if $q > \underline{q}$. Intuitively, speculative demand for tokens $s_H(q) > 0$ is fragile when $q < \underline{q}$. Last, Panel F shows speculative investment relative to usage, i.e., $s_z(q)=v_z(q)$. When $s_z(q)=v_z(q)$ is large (small), tokens are primarily held for speculation (usage).

To gain some intuition about the mechanisms at work, we start by considering state H. For the sake of interpreting the model dynamics, note that in recent years, the trend in the cryptocurrency market as well as for many individual tokens has been predominantly bullish, interrupted by several crashes. Through the lens of the model, such dynamics are best described by an extended period of time during which sentiment/optimism q has increased on average.[16]

When speculator sentiment q is su ciently low, for instance, because the platform is in its early stages and has no track record yet, there is no speculative investment. As the platform

[16] Notice that $_B$ and (q) determine the transition probabilities under user and speculator beliefs, but do not necessarily pin down physical transition probabilities.

and its adoption $V_t = A_t v_z(q_t)$ grow at high rate (i.e., $\ _t = \ _H$), speculators become gradually more optimistic, in that $q_t = q$ increases over time. The prospect that speculators buy tokens, i.e., provide liquidity, at future times raises expected token returns today, which reduces the cost of holding tokens for users and therefore increases scaled adoption $v_H(q)$. Put differently, due to speculators' future liquidity provision, $''_H(q)$ and $v_H(q)$ increase with q even when there is no speculative investment, that is, when $s_L(q) = s_H(q) = 0$ (see Panels B and D). In fact, scaled adoption in state H is highest once speculators start to buy tokens, i.e., at the largest q satisfying $s_H(q) = 0$. Intuitively, speculators help early-stage platforms and tokens, characterized by low q, to gain adoption and to grow, as they improve the liquidity of such tokens.

When platform growth has been high over an extended period of time and q takes intermediate levels, speculators start buying tokens in which case $s_H(q)$ increases with q. From this point on, expected token returns and platform adoption are "U-shaped" in q, i.e., they first decrease with q and then increase with q for larger values of q.[17] These patterns reflect both the crowding-out and liquidity provision effect. Buying tokens, speculators drive up token price and hence limit the potential for future token price appreciation, thereby reducing expected token returns and crowding out token usage. Once speculative trading starts to emerge, the crowding-out effect at first dominates: An increase in q then raises speculative investment $s_H(q)$ (Panel E) and token price risk (Panel C) but reduces expected token return $''_H(q)$ (Panel B) and adoption $v_H(q)$ (Panel D). Importantly, speculative demand remains initially fragile and retreats after a negative shock ("crash") triggering a drop of and q. Formally, if $s_H(q) > 0$ and $q < \bar{q}$, denoted by the vertical dashed red line in Panel E, speculators do not provide liquidity after a token price drop in which case q drops to $q_L(q)$ and $s_L(q_L(q)) = 0$. This fragility of speculative demand exacerbates token price (downside) risk in state H which increases in q and reaches its peak for intermediate levels of q (solid black line in Panel C).

For sufficiently large values of q, the liquidity provision effect dominates the crowding-out one, so that expected token returns and adoption increase with q while token price risk decreases with q. When speculator optimism is sufficiently high, speculators continue to remain optimistic even after a negative shock, i.e., a regime shift into state L. That is, speculative demand for tokens is stable and, specifically, does not fully retreat after a crash, in that $s_L(q_L(q)) > 0$ which holds for $q > \bar{q}$. As such, speculators hold tokens in state L and provide liquidity after a token price drop which reduces the extent of the token price drop and so the downside risk to holding tokens in state H. As this liquidity provision in bad times stabilizes token price, token price downside risk in state H decreases with q (Panel C), and expected token returns $''^U_H(q)$ and adoption $v_H(q)$ increase with

[17] While the "U-shaped" pattern is qualitatively robust across different parameter configuration, we emphasize that different parameterization (e.g., higher) lead to a more pronounced increase for larger values of q.

q (Panels B and D). Finally, when q is sufficiently close to one, the constraint $s_H(q) \le k^S$ limits speculative demand, which further boosts expected token returns and adoption.[18]

Interestingly, Panel F shows that the extent of speculative trading relative to token usage dwindles for sufficiently large values of q, in that $s_H(q)/v_H(q)$ decreases with q. That is, both token usage and speculative increase with q in absolute (dollar) terms, but token usage increases relatively more. The fraction $s_H(q)/v_H(q)$ indicates the extent to which tokens are held for speculation as opposed to for usage. The model therefore predicts that the speculative motive dominates for intermediate levels of q, while token usage is most pronounced for low and high levels of q.

Notably, the dynamics in state L are similar to the ones in state H. Speculative investment only prevails for larger values of q, that is, once speculators have become sufficiently optimistic. At this point, expected token returns and platform adoption are U-shaped in speculator sentiment, which again reflects the counteracting liquidity provision and crowding-out effects albeit in a slightly different way. As in state H, speculative investment in state L boosts token price and so reduces expected token returns and adoption, crowding out users. However, the prospect of high speculative investment following a regime shift into state H implies a potential for token price appreciation, increasing expected token returns and adoption. This potential for token price appreciation, depicted by the dotted red line in panel C, is inverted U-shaped in q. When there is speculative investment in state L, the crowding-out effect dominates for intermediate levels of q, while the latter liquidity provision effect dominates for sufficiently large q, so that $v_L(q)$ first decreases and then increases with q.

Also recall that by Lemma 3, speculator sentiment q in state L decreases over time. Thus, unlike in the model without learning, liquidity provision by speculators in bad times is transitory: If there is no recovery for an extended period of time, speculators loose faith and sell their token holdings. In contrast, in state H, q increases and speculative investment and trading build up over time. Due to these dynamics, speculative trading is on average most pronounced in good times (bull markets) and less pronounced in bad times (bear markets). In other words, speculative trading is procyclical. Figure 2 illustrates this pattern by plotting a simulated sample path of speculator sentiment q_t (Panel A), scaled token price p_t (Panel B), and speculative investment s_t (Panel C) against time t for $t \le 10$.

Panel A indicates that speculator sentiment exhibits several spikes which coincide with token price spikes. Notably, speculative investment builds up smoothly over time in a bull market (i.e.,

[18] An alternative interpretation is that when $s_H(q)$ is large, speculators hold a large undiversified stake in the platform, which makes it costly for them to further increase their investment. As such, a similar bound on speculative investment would be present if we had assumed a convex and increasing cost of holding tokens (or cost of capital) that could reflect risk-aversion.

Figure 2: Numerical Example of a Sample Path under Simulation. The parameters are $r = 0.05$, $= 1$, $= 0.1$, $_B = 0.2$, $k^U = k^S = 30$, and $_G = 0.8$. For the simulation, we assume a true value of of 0.5, so $_G = {}_B = 0.3$.

regime H) when token price rises. Upon a regime shift, token price crashes and speculative activity retreats. Then, a bear market begins in which token price continues to fall and speculators wind down their position, possibly, to the point that there is no more speculative trading at all. The bear market persists for a while, until the cycle repeats and speculators' sentiment and investment start to rise again. Moreover, as Panel C of Figure 2 illustrates, usage (dotted red line) spikes in periods of high growth but low speculative investment, reflecting the crowding-out effect. Finally, to further support the notion that speculative trading is procyclical, we simulate the stationary distribution of states $(q; z)$ under the parameterization from Figure 2 and obtain that on average, speculative investment is higher in state H than state L. In detail, we use the stationary distribution of states $(q; z)$ to calculate average speculative investment in state z, i.e., $avg(s_t|z = L)$ and $avg(s_t|z = H)$, and find $avg(s_t|z = H) = 18.78 > 7.95 = avg(s_t|z = L)$.[19]

Finally, we can assess the overall effects of speculators relative to the benchmark without speculators, which is achieved in the limit $q \to 0$. First, note that because $p_z(q)$ increases with q, the presence of speculators benefits the platform owners and platform dollar value by increasing token price. Panel D shows that, relative to the model without speculators, the presence of speculators increases adoption for low levels of q which may characterize early-stage tokens with low liquidity, adoption, and trading volume. In contrast, compared to the benchmark without speculators, the presence of speculators harms adoption for high levels of q which may characterize later-stage platforms and tokens with high trading volume and liquidity. That is, our analysis suggests that speculators tend to benefit early-stage platforms and tokens by stimulating their adoption, whilst crowding-out usage for more mature tokens. Moreover, as token price risk tends to be lowest in

[19] We also calculate $avg(s_t/p_t|z = H) = 0.53 > avg(s_t/p_t|z = H) = 0.34$. These patterns are robust with respect to the parameter choice.

the no-speculator limit $q \to 0$, we conclude that the presence of speculators tends to increase token price risk and volatility compared to the no-speculator benchmark. The reason is that speculative trading is procyclical and thus amplifies token price fluctuations and volatility.

While we do not model network effects that lead to strategic complementarities in users' adoption decisions, we note that in the presence of network effects, the risk of coordination failure would decrease and the convenience yield to holding tokens would increase with prevailing adoption $v_2(q)$ (Li and Mann, 2020; Sockin and Xiong, 2021; Cong et al., 2021b). Speculators stimulate the adoption and so reduce the risk of coordination failure for early-stage platforms that are particularly prone to coordination failures. On the other hand, speculators tend to crowd out user adoption for more mature platforms that are less prone to coordination failures. We conclude that the presence of speculators likely has a net negative effect on the risk of coordination failure, as the presence of speculators stimulates adoption and reduces the risk of coordination failure in the early stages which are key for long-term platform success and adoption.

2.4 Discussion and Empirical Implications

The previous analysis of our model has positive empirical implications for the dynamics of token price and platform adoption as well as usage and speculation in blockchain tokens. As indicated in Figure 2, speculative trading activity is procyclical and follows a boom and bust cycle, with high speculative trading in bullish regimes and low speculative trading in bearish regimes. This speculative trading pattern is consistent with the results in Liu and Tsyvinski (2021) who show that investor attention — which may proxy for speculative trading in cryptocurrencies — predicts future cryptocurrency returns. Figure 2 suggests that in a bull market, speculators' position in tokens builds up continuously, but may drop sharply at the inception of a bear market.

Our analysis reveals that speculative trading affects token usage via two opposing effects, crowding-out and liquidity provision. The crowding-out effect implies that speculative token investment and token usage are static substitutes. That is, speculative investment at a given point in time t correlates negatively with token usage at time t. Silberholz and Wu (2021) develop empirical measures of token usage and speculative token trading and provide supporting evidence for this crowding-out effect. In addition, speculators' role as liquidity providers implies that speculative investment and token usage are dynamic complements: Speculative investment at a given point in time t correlates positively with token usage at a future time $s > t$ and vice versa.

Our model predicts that speculators tend to invest in platforms and tokens with sufficiently good past performance and growth, in line with the findings in Silberholz and Wu (2021). Our analysis also suggests that for tokens with relatively low speculative investment, an increase in speculation triggers a decline in token usage, consistent with Silberholz and Wu (2021). Besides, token price risk is highest and token usage lowest when speculation takes intermediate levels. This indicates a negative correlation between token usage and token price risk/volatility, which is documented in Silberholz and Wu (2021).

Furthermore, our model predicts that, provided the cryptocurrency market continues to grow for sufficiently long time, token usage should increase again, possibly crowding out speculation. Applied to individual tokens: Given good fundamentals (i.e., a high growth rate of the A_t), token usage first decreases when speculation starts to rise, but eventually, following continued growth in token price, token usage increases again both in absolute terms and relative to speculation. It is plausible that the second part of this prediction regarding increasing token usage is not yet contained in the data, because most tokens are in their early stages.

2.5 A Life-Cycle Theory of Cryptocurrencies

Interpreted more broadly, our results suggest a life-cycle of cryptocurrencies and tokens with three stages: i) an early growth stage, ii) a hype stage, and iii) a mature stage. In this analogy, the variable q increases on average over time and the cryptocurrency life cycle. In fact, such an increase of q also describes the past of cryptocurrency markets which was characterized by persistent bull markets and growing investor interest which could be captured by time-increasing q.

First, in the early growth stage, investor optimism about cryptocurrencies q is low. The demand for tokens primarily stems from usage, and speculative or financially motivated investment is limited. Both sentiment and adoption increase over time at high pace. This early growth stage could represent the early years of cryptocurrencies and Bitcoin (e.g., before 2012) when cryptocurrencies and Bitcoin have been mostly known to a tight community.

Second, once usage and adoption have grown rapidly over an extended period of time, cryptocurrencies and tokens enter the hype stage. In this stage, they attract broader interest and, in particular, the interest from speculators and financial investors who start to buy tokens too. Then, demand for tokens stems to a large extent from speculators (e.g., retail investors) or, alternatively, financial investors (e.g., funds or VC investors). Moreover, the still fragile demand from speculators and financial investors exacerbates price volatility. This hype stage could describe the current state of cryptocurrency markets.

Third, going forward, our analysis predicts that eventually (i.e., in the future), cryptocurrencies and tokens enter the mature stage. During the mature stage, adoption and usage will increase again both in absolute terms and relative to speculation. Importantly, demand for tokens from speculators and financial investors is stable and limits token price risk, as speculators and financial investors act as liquidity providers after a crash and thus provide a backstop.

2.6 Platform characteristics, Usage, and Speculation

We now examine how platform characteristics determine usage and speculation. Crucially, speculation only arises when there is sufficient uncertainty regarding the platform's prospects. In particular, as the following Corollary demonstrates, $_H > {}_L$ is a necessary condition for speculation to arise.

Corollary 2. When $_H = {}_L$, then $v_L(q)$ and $v_H(q)$ are constant in q. And, there is no speculative investment, in that $s_z(q) = 0$ for all $(q; z)$.

The result of Corollary 2 is intuitive. Speculators hold tokens solely to earn returns and thus need price fluctuations to be able to realize returns from trading tokens. When $_H = {}_L$, there are no price fluctuations and there is no belief disagreement about the platform's future growth. Token price increases deterministically at rate $_L < r$, and expected token returns $_L$ then lie below speculators' required rate of return r. Conversely, uncertainty regarding the platform's growth, i.e., $_H > {}_L$, generates token price fluctuation and volatility, thereby inviting speculative trading. The model therefore predicts that speculation tends to be more pronounced for tokens with high volatility and uncertain fundamentals, in line with the results in Silberholz and Wu (2021) who document a positive relationship between price volatility and speculation. Our model suggests two-way causality in this context: Not only does fundamental uncertainty (volatility) invite speculation, but also does speculation boost token price volatility relative to the no-speculation benchmark.

As, in addition, speculators only invest in platforms with sufficient growth potential, our analysis predicts that speculators tend to invest in tokens that both offer high growth potential and are sufficiently risky. Notably, as the presence of speculators can stimulate adoption, high token price volatility that invites speculative trading can be seen as a feature rather than a bug. To formalize this result, we simulate the stationary distribution of states $(q; z)$ to calculate average equilibrium quantities in steady state, assuming a true value of $= 0.5$. The choice of $= 0.5$ seems natural since it implies that speculators have noisy beliefs around the true value of , whereby upward (downward) deviation $_G \quad = 0.3$ ($\quad _B = 0.3$) are symmetric. We then calculate (scaled) average adoption, $avg(v_t)$, average (scaled) speculative investment, $avg(s_t)$, average (scaled) token price $avg(p_t)$, and average scaled token price volatility $vol(p_t)$. Next, we analyze the effects of a mean-preserving spread in $(_L; {}_H)$: That is, we write $_H = {}_H + (\;)$ and $_L = {}_L \quad (\;)$, and conduct comparative statics in the "volatility" $(\;)$, whilst holding the average growth rate $0.5 {}_H + 0.5 {}_L = 0.5({}_H + {}_L)$ fixed starting with $_H = 0.05$ and $_L = \quad 0.075$.

Figure 3 plots average adoption $avg(v_t)$ (Panel A), average speculative investment $avg(s_t)$ (Panel B), average token price, i.e., platform value, $avg(p_t)$ (Panel C), and average token price volatility

Figure 3: Comparative Statics. The parameters are $r = 0{:}05$, $= 1$, $= 0{:}1$, $_B = 0{:}2$, $_G = 0{:}8$, and $k^U = k^S = 30$. For the simulation, we assume a true value of of 0:5, so $_G$ = $_B = 0{:}3$.

$vol(p_t)$ (Panel D) against ( ). To the left of the vertical solid red line for low values of ( ), there is no speculative investment in equilibrium (i.e., $s_z(q) = 0$ for all $(q; z)$), mirroring the statement from Corollary 2. Once \fundamental" volatility ( ) increases beyond the vertical solid red line, speculative trading volume $avg(s_t)$ jumps up and increases thereafter in ( ). The discontinuity re ects feedback e ects inherent to speculation. When ( ) crosses a critical threshold and sentiment q is large, speculators buy tokens in both states and so drive up prices to the point that their investment is constrained by their limited endowment.

To the right of the vertical solid red line, the increased demand from speculators boosts average platform value and token price, so $avg(p_t)$ increases with ( ) too. Interestingly, the average adoption level $avg(v_t)$ is lowest for intermediate levels of ( ). In particular, the rise of speculative trading harms adoption via the crowding-out e ect in that $avg(v_t)$ jumps down at the solid red line. However, for su ciently large values of ( ), the benign liquidity provision e ect dominates (on average) the adverse crowding-out e ect of speculation, and an increase in( ) increases average speculation (Panel B) and adoption (Panel A).

Another interesting e ect is that average token price and adoption increase (decreases) with ( ) to the right (left) of the solid red line. That is, fundamental volatility generally harms adoption and token price (platform value) in the absence of speculators, but the opposite is true in the presence speculators. The reason is that to the right of the solid red line, an increase in fundamental volatility ( ) boosts speculative trading and demand for tokens, which increases both price and resale value of tokens. The improved resale value, in turn, raises expected token returns and so stimulates adoption.

3 Platform Structure and Token Design

3.1 Dual token structure

As the previous analysis has shown, speculators harm users and platform adoption via the crowding-out effect, but benefit them via the liquidity provision effect. In essence, the adverse crowding-out effect arises from the dual function of tokens in that they serve both as financial investment asset and medium of exchange. A natural approach to mitigate crowding-out and thereby to stimulate adoption is to separate ("unbundle") these two functions, which is achieved via a so-called dual token structure with two tokens. In this section, we show that such a dual token structure improves upon the baseline token-based platform by leveraging the beneficial liquidity provision effect and eliminating the adverse crowding out effect.

The proposed dual token structure features two native tokens: i) a price-stable transaction token (i.e., stablecoin) held by users and ii) a governance token which is held by speculators and pays dividends. The price of the stablecoin is stable, and normalized to one dollar. That is, the stablecoin serves as a transaction medium and holding it entails no risk or returns, while governance tokens are risky and held for financial returns. The dual token structure resembles the one of major decentralized finance (DeFi) platforms such as Terra, with the stablecoin Terra USD and the governance token Luna. Accordingly, our model has normative implications for the optimal design and implementation of such DeFi platforms.

We assume that users are charged a dollar (flow) fee f_t which is proportional to their stablecoin holdings in dollars denoted V_t. As such, the representative user solves

$$\max_{V_t \in [0; K_t^U]} E_t^U \left(\underbrace{\frac{V_t^{\alpha} A_t^{1-\alpha}}{\alpha}}_{\text{Convenience yield}} dt + V_t \Big(\underbrace{- f_t dt}_{\text{Fee}} \underbrace{- r dt}_{\text{Opportunity cost}} \Big) \right); \tag{28}$$

leading to

$$V_t = A_t \left(\frac{1}{r + f_t} \right)^{\frac{1}{1-\alpha}} \wedge K_t^U. \tag{29}$$

Thus, user stablecoin/token holdings decrease with the interest rate, capturing the opportunity cost of holding tokens, and with the fees f_t. Because stablecoins are price-stable, users cannot capitalize on platform growth by earning token returns. As in the baseline, users cannot invest more than their endowment $K_t^U = A_t k^U$, i.e., $V_t \leq K_t^U$.[20] For the stablecoin market to clear, stablecoin supply must equal users' aggregate stablecoin holdings in dollars V_t, so V_t is stablecoin supply both in nominal and dollar terms.

Governance tokens are in unit supply and pay revenues from transaction fees and stablecoin issuance as dividends $dDiv_t$ and thus resemble an equity stake in the platform. The platform can always issue new governance tokens to raise funds (without any frictions), in which case $dDiv_t < 0$. These funds are used to buy back stablecoins to adjust supply to maintain price stability. In particular, governance token issuance ($dDiv_t < 0$) and buybacks, i.e., dividends $dDiv_t > 0$, is such that stablecoin price remains stable at one dollar. Such stability mechanisms underlie major algorithmic stablecoins, like Terra USD or DAI. Thus, governance token dividends read

$$dDiv_t = \underbrace{dV_t}_{\text{Stablecoin supply adjustment}} + \underbrace{f_t V_t dt}_{\text{Fee revenues}}; \tag{30}$$

where dV_t is the (dollar) payoff from issuing new stablecoins, which is negative when the platform buys back tokens, and $f_t V_t dt$ is the (dollar) fees collected from users over $[t, t+dt)$. Also, note that fee revenues can be negative, in case the platform stimulates usage by paying users subsidies or interest on their stablecoin holdings (i.e., $f_t < 0$).

Governance tokens have dollar value G_t, and are only held by speculators. The market clearing condition for governance tokens reads $S_t = G_t$. As in the baseline, speculators' investment S_t must be positive. Thus, market clearing requires $S_t = G_t \geq 0$ which can also be seen as limited liability constraint.[21] In addition, speculators cannot invest more than their endowment, i.e, $S_t \leq K_t^S$, and governance token price is bounded from above by K_t^S, i.e., $G_t \leq K_t^S$. For the market for governance tokens to clear, speculators must have incentives to buy governance tokens, in that

$$E_t^S[dDiv_t + dG_t] \geq rG_t dt: \tag{31}$$

Condition (31) states that in equilibrium, the expected returns to holding governance tokens must exceed speculators' required rate of return; the inequality (31) holds in equality if $S_t = G_t < K_t^S$.[22]

At inception at time $t = 0$, there is an initial dividend of $dDiv_0 = V_0$ from minting V_0 stablecoins

[20] Expression (29) shows that $V_t > 0$, so the constraint $V_t \geq 0$ does not bind.

[21] One alternative way to motivate that constraint $S_t = G_t \geq 0$ is that speculators are protected by limited liability and cannot commit to hold a claim in the platform that has negative value $G_t < 0$. For instance, when $G_t < 0$, speculators are better-off walking away from their claim instead of holding or selling it.

[22] If (31) were violated, the speculator would not find it optimal hold governance tokens.

which is the platform's seigniorage. With a slight abuse of notation, G_0 denotes the governance token value at time $t = 0$ after the dividend $dDiv_0$ is paid. Thus, under a dual token structure, total platform (dollar) value at time 0 reads $G_0 + V_0$ and stems from two sources: i) seigniorage (i.e., the proceeds from issuing stablecoins) yielding V_0 dollars at $t = 0$ and ii) equity or governance token issuance yielding G_0 dollars at $t = 0$. The platform or, equivalently, its initial owners (governance token holders) choose fees $f(\cdot)_{t \geq 0}$ at time $t = 0$ to maximize total platform value, that is,

$$\max_{(f_t)_{t \geq 0}} G_0 + V_0; \tag{32}$$

whilst $G_t \in [0; K_t^S]$ and $V_t \in [0; K_t^U]$. By the dynamic programming principle, the objective (32) implies that optimal fees $(f_s)_{s \geq t}$ at any time $t \geq 0$ are chosen to maximize continuation platform value $G_t + V_t$.[23] As in the baseline, we look for a Markov equilibrium with state variables A_t, q_t, and z in which quantities scale with A_t, so that $G_t = A_t e_z(q_t)$ and $V_t = A_t v_z(q_t)$. The detailed solution to the model with dual token structure is deferred to Appendix I and described in the following Proposition.

Proposition 5 (Dual Token Structure)**.** *Under the dual token structure, stablecoin holdings are characterized in (29) and governance token value G_t satisfies (31). In a Markov equilibrium with state variables A_t, q_t, and z, governance token value satisfies $G_t = A_t g_z(q_t)$ and stablecoin value satisfies $V_t = A_t v_z(q)$ for $z = L; H$. The functions $g_z(q)$ and $v_z(q)$ are characterized via a system of delayed first order ODEs on the domain $(0; 1)$ — characterized in (I.43), (I.45), (I.42), and (I.44) — subject to the boundary conditions (I.47) and (I.46). In optimum, scaled adoption (i.e., the user's scaled stablecoin holdings) equals*

$$v_H(q) = \left(\frac{1}{r - E^S[dV]/(V\,dt) - (E^S[dJ^L]/dt)\frac{g_L(q_L(q))}{v_H(q)}} \right)^{\frac{1}{1-\alpha}} \wedge k^U \tag{33}$$

$$v_L(q) = \left(\frac{1}{r - E^S[dV]/(V\,dt) - (E^S[dJ^H]/dt)\frac{g_H(q_H(q))}{v_L(q)}} \right)^{\frac{1}{1-\alpha}} \wedge k^U: \tag{34}$$

When $v_z(q) \in (0; k^U)$, then $g_z(q) = 0$, so the dual token structure minimizes the speculator's stake.

[23] Moreover, note that at any time $t \geq 0$, governance token holders are the platform owners and so residual claimants to any change in total platform value. Thus, governance token holders would like to maximize total platform value $G_t + V_t$ too, i.e., the platform's objective at time $t = 0$ in (32) is consistent with governance token holders' objective.

Figure 4: Numerical Illustration of the Equilibrium Dynamics. The parameters are $r = 0{:}05$, $= 1$, $= 0{:}1$, $_B = 0{:}2$, $_G = 0{:}8$, and $k^S = k^U = 30$.

3.2 Equilibrium Dynamics under Dual Token Structure

Next, we solve for the equilibrium under the dual token structure and contrast the outcomes to the ones under the baseline token-based structure. The equilibrium under the dual token structure | characterized in Proposition 5 | has to be solved numerically, with the details of the solution provided in Appendix I. We use our baseline parameters from Figure 1.

Figure 4 plots scaled total platform value, scaled token usage/adoption, and the scaled speculator investment against q both under the baseline token-based platform structure (solid black line) and under a dual token structure (dotted red line). The upper three panels A, B, and C depict quantities in state $z = H$ and the lower three panels D, E, and F depict the corresponding quantities in state $z = L$. Panel A and Panel D plot total platform value in states H and L respectively. Scaled total platform value in state $(q; z)$ is scaled token market capitalization $p_z(q)$ under the baseline token-based structure and the scaled sum of transaction and governance token value, $v_z(q) + g_z(q)$, under a dual token structure. Panel B and Panel E plot scaled token usage and adoption $v_z(q)$ in states H and L respectively. Finally, Panel C and Panel F display scaled speculator investment in states H and L respectively. Speculators' investment reads $s_z(q) = p_z(q) \quad v_z(q)$ in the baseline and $g_z(q)$ under the dual token structure.

Figure 4 highlights three main results regarding the optimal dual token structure. First, a dual token structure improves total platform value relative to the baseline token-based structure. Second, the dual token structure unambiguously boosts token usage and adoption, thereby benefiting users. Third, on average, the dual token structure reduces investments by financial investors and speculators: Compared to the baseline token-based structure, the dual token structure reduces speculative investment in state L but leaves it more or less unchanged in state H.

To understand these results, we now discuss the changes brought about by a dual token structure. To start with, recall that under the baseline token-based structure, more optimistic speculators compete with less optimistic users for token ownership, thereby raising token price and crowding out users. In particular, the fact that optimistic speculators are willing to pay a high price for tokens, which represent a stake in the platform, hampers adoption and usage. The dual token structure harnesses speculators' optimism, i.e., their willingness to pay a high price for a stake in the platform, whilst avoiding crowding-out.

Under the dual token structure, speculators buy the platform's governance tokens which are akin to an equity stake in the platform. The proceeds from governance token issuance are effectively redistributed to users via low transaction fees or even subsidies (in case $f_t < 0$), which stimulates adoption and, notably, leads to endogenous reinforcements effects. Lower individual fees f_t raise token usage V_t and allow the platform to mint additional transaction tokens, which boosts seigniorage revenue and allows the platform to further reduce its fees. The optimal dual token structure maximizes token usage and so minimizes transaction fees subject to the constraint $g_z(q) \geq 0$, and the constraint that user adoption is bounded, i.e., $v_z(q) \leq k^U$.[24] The optimal dual token structure therefore minimizes the value of the stake that speculators hold in the platform, $g_z(q)$.

As a result, the optimal dual token structure harnesses speculator sentiment and allows users to benefit from speculator optimism. Formally, the expressions for scaled adoption in (33) illustrate that users effectively apply the more optimistic speculator beliefs to evaluate payoffs. For instance, when $g_L(q_L(q)) = 0$, then $V_t = v_H(q)$ resembles the expression for adoption under the token-based structure in (3) with $V_t = P_t$ (i.e., no speculative investment) with one important difference: Under the dual token structure, the expectation over platform growth dV_t is formed under the optimistic speculator beliefs.

The improvements brought about by a dual token structure (in terms of adoption and platform value) only pertain when speculators are more optimistic than users and there is sufficient belief disagreement. Therefore, a dual token structure creates most value for platforms that attract sufficient interest from speculators, such as platforms facing high uncertainty going forward or platforms with uncertain/risky fundamentals and demand. In particular, when ${}_H = {}_L$, there is

[24] If it were $g_z(q) < 0$, speculators would be unwilling to hold governance tokens as a stake in the platform, as they would be better off parting from the platform. That is, $g_z(q) \geq 0$ can be interpreted as a limited liability constraint.

no speculation and belief disagreement under the baseline token-based structure, and a dual token structure induces the exact same level of adoption and platform value.

Corollary 3. When $_{H} = {}_{L}$, adoption and platform value are the same under the baseline token-based structure with one token and the dual token structure with two tokens.

While our previous results have shown that the dual token structure dominates the baseline token-based structure, implementing a dual token structure in practice may be challenging, costly, or even infeasible due to various factors omitted in our analysis. Price stability of the stablecoin may be difficult to achieve, for instance, because the platform cannot credibly commit to token price stability or the issuance of governance tokens is subject to frictions (Routledge and Zetlin-Jones, 2021; Li and Mayer, 2021).

Discussion: Fiat-based platform. The dual token structure is fundamentally different from a fiat-based structure with dollars as the platform transaction medium. When tokens serve as the platform transaction medium, the platform earns payoff from issuing these tokens which is akin to seigniorage. Seigniorage allows the platform to be profitable without i) charging users a transaction fee or ii) adopting other monetization models that may harm users (such as exploiting user/transaction data or putting advertisements). Since fiat money is issued by a central bank which collects the seigniorage, a fiat-based platform does not earn seigniorage and must charge transaction fees (or adopt aforementioned monetization models that are outside of our model) to be profitable. Transaction fees increase the cost of transacting on the platform and hence reduce platform transactions and adoption.

Importantly, a dual token structure combines the advantages of both fiat-based and token-based structure. The dual token structure allows for seigniorage revenues and low transaction fees, whilst eliminating the adverse crowding-out effect of speculation. As such, the dual token structure dominates a fiat-based and standard token-based structure. Likewise, the baseline token-based platform structure with one token and no transaction fees leads to higher adoption and platform value than a fiat-based structure under our baseline parameters. Appendix J provides a formal model of a fiat-based platform within our framework, and its comparison to other platform structures.

Discussion: Governance token value and reserves. Finally, note that as shown in Figure 4, governance tokens can be a claim with zero value in certain states in which case $g_z(q_t) = 0$. While this outcome seems at first glance extreme and counterfactual, we note that this anomaly could be fixed in several ways. First, we could impose the constraint $G_t \quad \underline{g}A_t$ for some constant

$\underline{g} > 0$. Our findings would remain qualitatively unchanged under this alternative specification, and the baseline can be understood as the limit case with $\underline{g} \to 0$. Second, the platform could maintain some (risk-free) reserves M_t, for instance, to back their stablecoin. Such a reserve is common for both centralized stablecoins (e.g., USDC) and decentralized algorithmic stablecoins with a dual token structure, such as DAI.[25] Appendix I.8 provides the solution under a dual token structure with platform reserves and shows that, under an appropriate implementation, adoption is unchanged by the introduction of reserves, but governance tokens have strictly positive value.

4 Conflicts of Interest in Platform Development and Token Design

In this Section, we highlight conflicts of interests that can arise on a token-based platform because the platform caters to the needs of speculators at the expense of its users. To allow for this tension in the most simple and tractable way, we consider that the platform can choose an action $b_t \in \{0; 1\}$ that boosts the platform's growth rate by $b_t \varepsilon$ for $\varepsilon > 0$ and > 0 in that the drift of dA_t/A_t becomes $_t + b_t \varepsilon$, but hampers current platform operations and reduces the convenience yield to holding tokens, in that the convenience yield becomes $\frac{V_t\ A_t^{1}}{}$ $V_t b_t \varepsilon$ dt. Under these circumstances, user token holdings become[26]

$$V_t = A_t \left(\frac{1}{r \quad _z + \varepsilon b_t(1 \quad) \quad E_t^U[dp_z(q_t)]/(p_z(q_t)dt)} \right)^{\frac{1}{1}} \wedge K_t^U; \tag{35}$$

where $p_z(q_t)$ is scaled token price (i.e., $P_t = A_t p_z(q_t)$) and $dp_z(q_t)$ is the change in scaled token price in this model variant.

As such, if interior (i.e., $V_t \in (0; K_t^U)$), V_t increases with b_t if and only if 1. We assume that choosing $b_t = 1$ is inefficient from the user perspective, in particular, we consider $\to 0$. Thus, when users are marginal, i.e., $S_t = 0$ or $S_t = K_t^S$, then token price reads $P_t = V_t + S_t$ and therefore decreases with b_t (as $\to 0$). Next, observe that when $S_t \in (0; K_t^S)$, then speculators are marginal

[25] The stablecoin DAI is issued by MakerDAO, a decentralized autonomous organization (DAO) with the governance token MKR. MakerDAO maintains a so-called "System Surplus Buffer" which is akin to the reserves we consider here.

[26] To derive (35), first note that $dP_t = d(A_t p_z(q_t)) = dA_t p_z(q_t) + A_t dp_z(q_t)$. As $dA_t/A_t = (\ _z + b_t \varepsilon \ )dt$, it follows that $\frac{E_t^U[dP_t]}{P_t\, dt} = \ _z + b_t \varepsilon \ + E[dp_z(q_t)]/(p_z(q_t)dt)$. Next, note that the representative user maximizes

$$\max_{V_t \in [0; K_t^U]} V_t \quad \frac{E_t^U[dP_t]}{P_t} \quad rdt \quad + \quad \frac{V_t\ A_t^{1}}{} \quad dt \quad V_t b_t \varepsilon dt \ :$$

Going through the optimization, we obtain (35).

in determining token price P_t which becomes[27]

$$P_t = \tilde{P}^S_t = \frac{A_t E^S_t[dp_z(q_t)]}{r - \mu_z - \epsilon b_t} \tag{36}$$

and unambiguously increases with b_t. For tractability, we consider that ϵ is small, i.e., the limit $\epsilon \to 0$. In the limit $\epsilon \to 0$, the equilibrium quantities under the model variant from this section coincide with the ones of the baseline regardless of the choice of b_t. Nevertheless, the study of the limit case $\epsilon \to 0$ allows us to identify the conflicts of interests on a token-based platform.[28]

Under the baseline token-based structure, the platform dynamically maximizes platform dollar value (i.e., token market capitalization), and chooses at each time $t \geq 0$ the level $b_t \in \{0, 1\}$ to maximize token price $P_t = \max\{P^S_t, P^U_t\}$. Here, $P^U_t = V_t$ is from (35) and $P^S_t = \tilde{P}^S_t \wedge V_t + K^S_t$, with $\tilde{P}^S_t$ from (36). When $V_t = K^U_t$, choosing $b_t = 1$ has no adverse effects on adoption, as users are constrained by their limited endowment, so $b_t = 1$ is optimal. We focus now — unless otherwise mentioned — on the more interesting case, $V_t < K^U_t$.

When $P_t = P^U_t > P^S_t$, there is no speculative investment and the marginal token investor is user. Then, the platform chooses $b_t = 0$ according to users' preferences. In contrast, when $P_t = P^S_t \in (P^U_t, V_t + K^S_t)$ and $S_t \in (0, K^S_t)$, the speculator is marginal in determining token price and the platform caters to the speculator by choosing $b_t = 1$ at the expense of user. When $P_t = P^S_t = V_t + K^S_t$ and $S_t = K^S_t$, then users are again the marginal token investors, leading to $b_t = 0$. Taken together, we obtain $b_t = b^{Token}_z(q_t)$, with

$$b^{Token}_z(q) = \mathbb{I}\{s_z(q) \in (0, k^S) \quad \text{or} \quad v_z(q) = k^U\}$$

where $\mathbb{I}\{\cdot\}$ is the indicator function which equals one if $\{\cdot\}$ and zero otherwise. Under our baseline parameters, we can infer from Figure 1: $b_H(q) = 1$ for intermediate levels of q when $s_H(q) \in (0, k^S)$, while $b_H(q) = 0$ for low and high q when $s_H(q) \in \{0, k^S\}$. And, $b_L(q) = 0$ ($b_L(q) = 1$) for low (high) q when $s_L(q) = 0$ ($s_L(q) > 0$).[29]

Above analysis has several implications. First, under a token-based structure, the platform owner chooses platform development and/or token design to maximize the platform's dollar value,

[27] For a derivation, note that as in (5), when $S_t \in (0, K^S_t)$, expected token returns $E^S_t[dP_t]/(P_t dt)$ must compensate speculators for the required rate of return r, i.e., $E^S_t[dP_t] = rP_t dt$. As $dA_t/A_t = \mu_z dt + \epsilon b_t dt$ and $P_t = A_t p_z(q_t)$, it follows that $E^S_t[dP_t] = A_t\,p_z(q_t)(\mu_z + \epsilon b_t)dt + E^S_t[dp_z(q_t)]$. Inserting this relation and $P_t = A_t p_z(q_t)$ into $E^S_t[dP_t] = rP_t dt$ and rearranging, one obtains (36). Also notice that b_t does not directly affect $dp_z(q_t)$.

[28] The assumption $\epsilon \to 0$ is for simplicity, as we can readily use the baseline model solution to describe the model solution in this extension, whilst being able to identify the conflicts of interests between the platform, its users, and speculators.

[29] Recall that in the limit $\epsilon \to 0$, the equilibrium quantities coincide with the corresponding equilibrium quantities from the baseline regardless of the choice of b_t.

i.e., token market capitalization. As a result, the platform owner's incentives are aligned with those of the marginal token investor, which inevitably leads to conflicts of interest in the triangular relationship between the platform (owners), its users, and speculators. Second, over its life-cycle, the platform's incentives tend to be aligned with those of the users in the early and later stages (i.e., for low and large q), but less so in the intermediate stages.

A natural question is whether this problem could be solved through decentralized governance mechanisms in which speculators and users can vote for the choice of b_t proportional to their token holdings and the majority choice is implemented. Under these circumstances, we consider that speculators would always vote $b_t = 1$ as long as $s_z(q) < k^S$, as they do not have transaction needs and therefore prefer growth, while users would always vote $b_t = 0$.[30] As a result, the decentralized governance structure would mandate

$$b_z^{Decentral}(q) = I\{s_z(q) \in (v_z(q); k^S)\}.$$

All else equal, $b_z^{Decentral}(q) \leq b_z^{Token}(q)$, meaning that decentralized can alleviate conflicts of interest. However, since $b_z^{Decentral}(q)$ can still be positive when $v_z(q) < k^U$ and so can hamper adoption, the decentralized governance structure can only alleviate but only to limited extent.

In contrast, the dual token structure from Section 3.1 can more effectively solve the problem and align the incentives of the platform and its governance token holders with those of the users. Under the dual token structure, the platform dynamically maximizes platform dollar value $V_t + G_t$, consisting of stablecoin value (user token holdings) V_t and governance token value (speculator token holdings) G_t. The platform therefore internalizes the negative impact of investment on adoption. Notably, the platform never caters to the speculators at the expense of users by boosting growth at the expense of adoption.

In particular, when $V_t < K_t^U$, then $G_t = 0$ and the platform chooses $b_t = 0$ to maximize $V_t + G_t = V_t$. When $V_t = K_t^U$ and adoption has reached its limit, the choice of $b_t = 1$ has no more adverse effects on adoption and $b_t = 1$. Either way, under the dual token structure, the choice of b_t does not compromise adoption unlike under the baseline token-based structure.[31] In particular, the fact that governance token holders' dividend payouts stem from fees and new token issuance incentivizes the governance token holders to act in the users' interest: If they were to cater to speculators at the expense of users, fee revenues and token usage would drop, curbing dividends to governance token holders. As a result, under a dual token structure, the platform optimally

[30] When $s_z(q) \geq k^S$, speculators are not marginal and would vote $b_t = 0$.

[31] A more involved trade-off would reappear, if we had not assumed that the action $b_t = 1$ is highly inefficient as $\to 0$. We leave this possibility for future research.

chooses the e cient action from user perspective.

5 Conclusion

We develop dynamic a model of cryptocurrencies and tokens in which tokens are held by both users and speculators. Users and speculators di er both in their motives to hold tokens and in their beliefs about the token's prospects. Users hold tokens for transactions and speculators hold tokens for returns. In addition, speculators update their beliefs and thus, unlike users, extrapolate based on the past. Speculation is procyclical, i.e., high in a bull and low in a bear market, and ampli es token price uctuations. Speculators a ect users via two opposing e ects, adverse crowding-out and benign liquidity provision. The crowding-out e ect arises, as speculators compete with users for token ownership, thereby raising token price and the cost of transacting with tokens. On the other hand, speculators buy tokens and provide liquidity especially, improving the resale value of tokens and stimulating adoption. In light of the crowding-out and liquidity provision e ects, token usage and speculation are static substitutes but dynamic complements.

Overall, our model suggests a two-way positive relationship between speculation and token price volatility: Price volatility invites speculative trading, which, in turn, makes token price more volatile. As speculative trading can stimulate adoption, high token price volatility that invites speculative trading can actually be bene cial and be seen as a feature rather than a bug. We also show that a dual token structure with a governance token and stablecoin, resembling the one of major decentralized nance platforms (e.g., Terra), attenuates the crowding-out e ect and stimulates adoption. Finally, the model generates several empirical implication regarding speculation and usage of cryptocurrencies. It also delivers a life-cycle theory for cryptocurrencies, consisting of a growth, hype, and mature stage.

References

Amihud, Y. and H. Mendelson (1980). Dealership market: Market-making with inventory. Journal of nancial economics 8(1), 31{53.

Bakos, Y. and H. Halaburda (2019a). Funding new ventures with digital tokens: Due diligence and token tradability. Working Paper.

Bakos, Y. and H. Halaburda (2019b). The role of cryptographic tokens and icos in fostering platform adoption. Working Paper.

Biais, B., C. Bisiere, M. Bouvard, and C. Casamatta (2019). The blockchain folk theorem. The Review of Financial Studies 32(5), 1662{1715.

Biais, B., C. Bisiere, M. Bouvard, C. Casamatta, and A. J. Menkveld (2018). Equilibrium bitcoin pricing. Working Paper.

Caballero, R. J. and A. Simsek (2020). A risk-centric model of demand recessions and speculation. The Quarterly Journal of Economics 135(3), 1493{1566.

Catalini, C. and J. S. Gans (2018). Initial coin o erings and the value of crypto tokens. Working Paper.

Chen, L., L. W. Cong, and Y. Xiao (2021). A brief introduction to blockchain economics. In Information for E cient Decision Making: Big Data, Blockchain and Relevance , pp. 1{40. World Scienti c.

Chiu, J. and T. V. Koeppl (2019). Blockchain-based settlement for asset trading. The Review of Financial Studies 32(5), 1716{1753.

Chod, J. and E. Lyandres (2021). A theory of icos: Diversi cation, agency, and information asymmetry. Management Science

Cong, L. W. and Z. He (2019). Blockchain disruption and smart contracts. The Review of Financial Studies 32(5), 1754{1797.

Cong, L. W., Z. He, and J. Li (2021). Decentralized mining in centralized pools. The Review of Financial Studies 34(3), 1191{1235.

Cong, L. W., Y. Li, and N. Wang (2021a). Token-based platform nance. Journal of Financial Economics (forthcoming).

Cong, L. W., Y. Li, and N. Wang (2021b). Tokenomics: Dynamic adoption and valuation. The Review of Financial Studies 34(3), 1105{1155.

Danos, V., S. Marcassa, M. Oliva, and J. Prat (2021). Fundamental pricing of utility tokens. Working Paper.

Davydiuk, T., D. Gupta, and S. Rosen (2020). De-crypto-ing signals in initial coin o erings: Evidence of rational token retention. Working Paper.

Doepke, M. and M. Schneider (2017). Money as a unit of account. Econometrica 85(5), 1537{1574.

Easley, D., M. O'Hara, and S. Basu (2019). From mining to markets: The evolution of bitcoin transaction fees. Journal of Financial Economics.

Ebrahimi, Z., B. Routledge, and A. Zetlin-Jones (2020). Getting blockchain incentives right. Working paper.

Feenstra, R. C. (1986). Functional equivalence between liquidity costs and the utility of money. Journal of Monetary Economics 17(2), 271 { 291.

Gan, J., G. Tsoukalas, and S. Netessine (2021a). Initial coin o erings, speculation, and asset tokenization. Management Science 67(2), 914{931.

Gan, R., G. Tsoukalas, and S. Netessine (2021b). To in nity and beyond: Financing platforms with uncapped crypto tokens. Working Paper.

Garratt, R. and M. R. van Oordt (2020). Why xed costs matter for proof-of-work based cryptocurrencies. Working Paper.

Garratt, R. J. and M. R. Van Oordt (2021). Entrepreneurial incentives and the role of initial coin o erings. Journal of Economic Dynamics and Control, 104171.

Goldstein, I., D. Gupta, and R. Sverchkov (2021). Initial coin o erings as a commitment to competition. Working Paper.

Gryglewicz, S., S. Mayer, and E. Morellec (2021). Optimal nancing with tokens. Journal of Financial Economics (forthcoming).

Halaburda, H., Z. He, and J. Li (2021). An economic model of consensus on distributed ledgers. Technical report, National Bureau of Economic Research.

Hinzen, F. J., K. John, and F. Saleh (2021). Bitcoin's fatal aw: The limited adoption problem. Working Paper.

Howell, S. T., M. Niessner, and D. Yermack (2020). Initial coin o erings: Financing growth with cryptocurrency token sales. The Review of Financial Studies 33(9), 3925{3974.

Hu, A. S., C. A. Parlour, and U. Rajan (2019). Cryptocurrencies: Stylized facts on a new investible instrument. Financial Management 48(4), 1049{1068.

Huberman, G., J. Leshno, and C. C. Moallemi (2019). An economic analysis of the bitcoin payment system. Working Paper.

Irresberger, F., K. John, and F. Saleh (2020). The public blockchain ecosystem: An empirical analysis. Working Paper.

John, K., M. O'Hara, and F. Saleh (2021). Bitcoin and beyond. Annual Review of Financial Economics 14

John, K., T. J. Rivera, and F. Saleh (2020). Economic implications of scaling blockchains: Why the consensus protocol matters. Working paper.

John, K., T. J. Rivera, and F. Saleh (2021). Equilibrium staking levels in a proof-of-stake blockchain. Available at SSRN 3965599

Lee, J. and C. A. Parlour (2020). Consumers as financiers: crowdfunding, initial coin offerings and consumer surplus. The Review of Financial Studies (forthcoming).

Lehar, A. and C. A. Parlour (2020). Miner collusion and the bitcoin protocol. Working Paper.

Lehar, A. and C. A. Parlour (2021). Decentralized exchanges. Technical report, working paper, University of Calgary and University of California, Berkeley.

Li, J. and W. Mann (2019). Initial coin offerings: Current research and future directions. Palgrave-MacMillan Handbook of Alternative Finance.

Li, J. and W. Mann (2020). Digital tokens and platform building. Working Paper.

Li, Y. and S. Mayer (2021). Money creation in decentralized finance: A dynamic model of stablecoins and crypto shadow banking. Working Paper.

Liu, Y. and A. Tsyvinski (2021). Risks and returns of cryptocurrency. The Review of Financial Studies 34(6), 2689–2727.

Liu, Y., A. Tsyvinski, and X. Wu (2021). Common risk factors in cryptocurrency. The Journal of Finance (forthcoming).

Lyandres, E. (2021). Product market competition with crypto tokens and smart contracts. Working Paper.

Lyandres, E., B. Palazzo, and D. Rabetti (2019). Do tokens behave like securities? an anatomy of initial coin offerings. Working Paper.

Malinova, K. and A. Park (2018). Tokenomics: when tokens beat equity. Working Paper.

Pagnotta, E. S. (2021, 01). Decentralizing Money: Bitcoin Prices and Blockchain Security. The Review of Financial Studies. hhaa149.

Prat, J. and B. Walter (2021). An equilibrium model of the market for bitcoin mining. Journal of Political Economy 129(8).

Rocheteau, G. (2011). Payments and liquidity under adverse selection. Journal of Monetary Economics 58(3), 191–205.

Rosu, I. and F. Saleh (2020). Evolution of shares in a proof-of-stake cryptocurrency. Management Science (forthcoming).

Routledge, B. and A. Zetlin-Jones (2021). Currency stability using blockchain technology. Journal of Economic Dynamics and Control, 104155.

Saleh, F. (2021). Blockchain without waste: Proof-of-stake. The Review of Financial Studies 34(3), 1156–1190.

Scheinkman, J. A. and W. Xiong (2003). Overconfidence and speculative bubbles. Journal of Political Economy 111(6), 1183–1220.

Schilling, L. and H. Uhlig (2019). Some simple bitcoin economics. Journal of Monetary Economics 106, 16–26.

Shams, A. (2020). The structure of cryptocurrency returns. Fisher College of Business Working Paper (2020-03), 011.

Silberholz, J. and D. A. Wu (2021). Measuring utility and speculation in blockchain tokens. Available at SSRN 3915269

Simsek, A. (2013). Belief disagreements and collateral constraints. Econometrica 81(1), 1{53.

Simsek, A. (2021). The macroeconomics of nancial speculation. Annual Review of Economics 13

Sockin, M. and W. Xiong (2021). A model of cryptocurrencies. Working Paper.

Sockin, M. and W. Xiong (2022). Decentralization through tokenization. Journal of Finance (forthcoming).

Appendix

A Agent optimization | Proofs of Lemma 1 and 2

We provide details on the optimization problems that the representative user and speculator solve.

A.1 User optimization | Proof of Lemma 1

Consider cohort t of the representative user that is born at time t and lives until time $t+$. Cohort t only consumes at time $t+$, and is endowed with K_t^U dollars at birth at time t.

Cohort t invest her entire endowment either in the risk-free asset bearing interest at rate r or in tokens which have price P_t at time t and price P_{t+} at time $t+$. We denote by V_t the amount of dollars invested in tokens at time t, so the user holds $V_t{=}P_t$ units of tokens. Thus, cohort t invests $K_t^U \;\; V_t$ dollars in the risk-free asset, which yields interest at time $t+$ of $(K_t^U \;\; V_t)r$ dollars. At time $t+$, cohort t sells $V_t{=}P_t$ tokens at price P_{t+} and consumes the proceeds of $V_t P_{t+} {=} P_t$ dollars. She also consumes her initial deposit in the risk-free asset plus interest, yielding $(K_t^U \;\; V_t)(1+r \;)$ dollars.

Thus, consumption in dollars at time $t+$ reads

$$c_{t+}^U (V_t) \quad V_t \;\; \frac{P_{t+}}{P_t} \;\; + (K_t^U \;\; V_t)(1+r \;): \tag{A.1}$$

We de ne $dP_t = P_{t+} \;\; P_t$ (i.e., $P_{t+} = dP_t + P_t$), and rewrite (A.1) as

$$c_{t+}^U (V_t) = V_t \;\; \frac{dP_t}{P_t} \;\; + V_t + (K_t^U \;\; V_t)(1+r \;) = V_t \;\; \frac{dP_t}{P_t} \;\; r \;\; + K_t^U(1+r \;):$$

Next, cohort t's lifetime utility is de ned in (1), that is,

$$u_t^U(V_t) = c_{t+} (V_t) + \;\; \frac{V_t \;\; A_t^1}{} \;\; ! \;\; :$$

Thus, the optimization in (2) becomes

$$\max_{V_t \, 2\, [0;K_t^U]} E_t^U[u_t^U(V_t)] = K_t^U(1+ \;\;) + \max_{V_t\, 2\, [0;K_t^U]} \;\; " \;\; V_t \;\; \frac{E_t^U[dP_t]}{P_t} \;\; r \;\; + \;\; \frac{V_t \;\; A_t^1}{} \;\; ! \;\; \# \;\; :$$

If the solution V_t is interior (i.e., $V_t \; 2 \; (0;K_t^U)$), the following rst-order condition (FOC) must hold:

$$\frac{@}{@V_t} E_t^U[u_t^U(V_t)] = 0 \quad () \quad \frac{E_t^U[dP_t]}{P_t} \;\; r \;\; + V_t \;\; {}^1 A_t^1 \;\; = 0;$$

which we can solve for

$$V_t = V_t \;\; A_t \;\; \frac{1}{r \;\; E_t^U[dP_t]{=}(P_t \;)} \;\; ^{\frac{1}{1}} :$$

The second order condition is $\frac{@^2}{@V_t^2} E_t^U[u_t^U(V_t)] = (\;\; 1)V_t \;\; {}^2 A_t^1 \;\; < 0$, as <1, so the solution to the FOC is a maximum.

If $V_t \;\; > K_t^U$, then the solution V_t to (2) is not interior and $V_t = K_t^U$. As the marginal convenience yield to holding tokens, that is, $V_t \;\; {}^1 A_t^1$, tends to $+1$ as $V_t \; ! \; 0$, it is clear that $V_t > 0$. Thus, overall

$$V_t = A_t \left(\frac{1}{r - E_t^U[dP_t]/(P_t \Delta)} \right)^{\frac{1}{1-\alpha}} \wedge K_t^U;$$

which becomes (3) after replacing "Δ" by "dt." This concludes the argument.

A.2 Speculator optimization — Proof of Lemma 2

Consider cohort t of the representative speculator that is born at time t and lives until time $t+\Delta$. Cohort t only consumes at time $t+\Delta$, and is endowed with K_t^S dollars at birth at time t.

Cohort t invest her entire endowment either in the risk-free asset paying interest at rate r or in tokens which have price P_t at time t and price $P_{t+\Delta}$ at time $t+\Delta$. We denote by S_t the amount of dollars invested in tokens at time t, so the speculator holds S_t/P_t units of tokens. Thus, cohort t invests $K_t^S - S_t$ dollars in the risk-free asset, which yields interest at time $t+\Delta$ of $(K_t^S - S_t)r\Delta$ dollars. At time $t+\Delta$, cohort t sells S_t/P_t tokens at price $P_{t+\Delta}$ and consumes the proceeds of $S_t P_{t+\Delta}/P_t$ dollars. She also consumes her initial deposit in the risk-free asset plus interest, yielding $(K_t^S - S_t)(1+r\Delta)$ dollars.

As such, cohort t's consumption at $t+\Delta$ reads

$$c_{t+\Delta}^S(S_t) \equiv S_t \frac{P_{t+\Delta}}{P_t} + (K_t^S - S_t)(1+r\Delta). \tag{A.2}$$

We define $dP_t = P_{t+\Delta} - P_t$ (i.e., $P_{t+\Delta} = dP_t + P_t$), and rewrite (A.2) as

$$c_{t+\Delta}^S(S_t) = S_t \frac{dP_t}{P_t} + S_t + (K_t^S - S_t)(1+r\Delta) = S_t\left(\frac{dP_t}{P_t} - r\Delta\right) + K_t^S(1+r\Delta).$$

Next, cohort t's lifetime utility is her consumption $c_{t+\Delta}(S_t)$. As such, the cohort t speculator solves (4) and therefore solves

$$\max_{S_t \in [0, K_t^S]} E_t^S[c_{t+\Delta}^S(S_t)] = K_t^S(1+r\Delta) + \max_{S_t \in [0, K_t^S]} S_t\left(\frac{E_t^S[dP_t]}{P_t} - r\Delta\right)$$

Thus, the speculator's choice of S_t is a linear optimization problem, whereby

$$\frac{\partial}{\partial S_t} E_t^S[c_{t+\Delta}^S(S_t)] = \frac{E_t^S[dP_t]}{P_t} - r\Delta.$$

The optimal choice of S_t therefore satisfies

$$S_t = \begin{cases} 0 & \text{if } E_t^S[dP_t]/(P_t\Delta) < r \\ \hat{S}_t \in [0, K_t^S] & \text{if } E_t^S[dP_t]/(P_t\Delta) = r \\ K_t^S & \text{if } E_t^S[dP_t]/(P_t\Delta) > r, \end{cases}$$

which becomes (5) after replacing "Δ" by "dt."

B Proof of Lemma 3

We derive the law of motion of q_t when speculators update their beliefs according to Baye's rule. We consider state L and H separately.

B.1 State L

First, we consider time t with speculator belief q_t and ${}_t = {}_L$. Then, if there is a regime switch over $[t; t + dt)$ from L to H, we have by Bayes' rule:

$$q_{t+dt} = q_H(q_t) = \frac{q_t \ {}_G}{q_t \ {}_G + (1 \quad q_t) \ {}_B}:$$

Absent a regime switch, Bayes' rule implies

$$q_{t+dt} = \frac{(1 \quad {}_G dt) q_t}{q_t(1 \quad {}_G dt) + (1 \quad q_t)(1 \quad {}_B dt)}:$$

Thus,

$$\begin{aligned} dq_t = q_{t+dt} \quad q_t &= \frac{(1 \quad {}_G dt) q_t}{q_t(1 \quad {}_G dt) + (1 \quad q_t)(1 \quad {}_B dt)} \quad q_t \\ &= \frac{(1 \quad {}_G dt) q_t (1 \quad q_t) \quad q_t (1 \quad q_t)(1 \quad {}_B dt)}{q_t(1 \quad {}_G dt) + (1 \quad q_t)(1 \quad {}_B dt)} \\ &= \frac{q_t (1 \quad q_t)(\ {}_G \quad {}_B) dt}{1 \quad q_t \ {}_G dt \quad (1 \quad q_t) \ {}_B dt} \end{aligned}$$

With $dq_t = q_{t+dt} \quad q_t = {}_L(q_t) dt$, we obtain

$${}_L(q_t) \ 1 \ q_t \ {}_G dt \quad (1 \ q_t) \ {}_B dt \ dt = \quad q_t(1 \ q_t)(\ {}_G \quad {}_B) dt \quad () \quad {}_L(q_t) = \quad q_t(1 \ q_t)(\ {}_G \quad {}_B);$$

where we discard higher order terms, i.e. $o(dt^2) = 0$, and divide through dt. Thus,

$$dq_t = {}_L(q_t) dt + (q_H(q_t) \quad q_t) dJ_t^H;$$

with ${}_L(q_t) = \quad (\ {}_G \quad {}_B) q_t (1 \quad q_t)$.

B.2 State H

Consider state H. That is, consider t with speculator belief q_t and ${}_t = {}_H$. Then, if there is a regime switch from H to L, Bayes' rule implies

$$q_{t+dt} = q_L(q_t) = \frac{q_t(1 \quad {}_G)}{q_t(1 \quad {}_G) + (1 \quad q_t)(1 \quad {}_B)}:$$

Absent a regime switch, Bayes' rule implies

$$q_{t+dt} = \frac{1 \quad (1 \quad {}_G) dt \ q_t}{1 \quad (1 \quad {}_G) dt \ q_t + \ 1 \quad (1 \quad {}_B) dt \ (1 \quad q_t)}:$$

Thus,

$$dq_t = q_{t+dt} \quad q_t = \frac{1 \quad (1 \quad {}_G)dt \; q_t}{1 \quad (1 \quad {}_G)dt \; q_t + 1 \quad (1 \quad {}_B)dt \; (1 \quad q_t)} \quad q_t$$
$$= \frac{1 \quad (1 \quad {}_G)dt \; q_t(1 \quad q_t) \quad 1 \quad (1 \quad {}_B)dt \; q_t(1 \quad q_t)}{1 \quad (1 \quad {}_G)dt \; q_t + 1 \quad (1 \quad {}_B)dt \; (1 \quad q_t)}$$
$$= \frac{q_t(1 \quad q_t)({}_G \quad {}_B)dt}{1 \quad q_t \; (1 \quad {}_G)dt \quad (1 \quad {}_B)(1 \quad q_t)dt}$$

With $dq_t = {}_H(q_t)dt$, we obtain

$${}_H(q_t) = \; q_t(1 \quad q_t)({}_G \quad {}_B);$$

after discarding higher order terms, i.e., $o(dt^2) = 0$. Altogether, we have

$$dq_t = {}_H(q_t)dt + (q_L(q_t) \quad q_t)dJ_t^L;$$

with ${}_H(q_t) = \; q_t(1 \quad q_t)({}_G \quad {}_B)$.

C Proof of Proposition 1

Follows from the arguments of the main text.

D Proof of Proposition 2

D.1 Equilibrium quantities

We start by providing a detailed derivation for expressions (24), (25), (22), and (23). To begin with, recall (3), which can be rewritten as $V_t = A_t v_z(q_t)$ with

$$v_z(q_t) = \left(\frac{1}{r \quad E^U[dP_t]=(P_t dt)} \right)^{\frac{1}{1}} \; \wedge \; k^U; \tag{D.3}$$

Due to $q = 0$ or $q = 1$, Lemma 3 implies ${}_z(q) = 0$ and $q_z(q) = 0$. We now use (19) to obtain ${}^P_t = {}_t = {}_z$. Next, (20) implies

$${}^H_t = \frac{p_H}{p_L} \quad 1 \quad \text{and} \quad {}^L_t = \frac{p_L}{p_H} \quad 1; \tag{D.4}$$

where we adopt the notation $p_z = p_z(q) = p_z(q)$ and $v_z = v_z(q) = v_z(q)$.

Inserting these relations into (21), we obtain

$$\frac{E_t^x[dP_t]}{P_t dt} = "^x_z(q) = \begin{cases} {}_L + \frac{p_H}{p_L} \quad 1 \;\; E_t^x[dJ_t^H]=dt; & \text{if} \quad z = L; \\ {}_H + \frac{p_L}{p_H} \quad 1 \;\; E_t^x[dJ_t^L]=dt; & \text{if} \quad z = H; \end{cases}$$

for $x = S; U$. Also note that when $z = L$ ($z = H$), $E_t^U[dJ_t^H] = \; {}_B dt$ ($E_t^U[dJ_t^L] = (1 \quad {}_B)dt$). Likewise, when $z = L$ ($z = H$), $E_t^S[dJ_t^H] = \; dt$ ($E_t^S[dJ_t^L] = (1 \quad)dt$).

For states $z = L; H$, we now insert above expression for $E^U[dP]=(P\,dt)$ into (D.3) to obtain

$$v_L(q) = v_L = \left(\frac{1}{r \quad {}_L \quad {}_B(p_H = p_L \quad 1)} \right)^{\frac{1}{1}} \wedge \quad k^U;$$

and

$$v_H(q) = v_H = \left(\frac{1}{r \quad {}_H \quad (1 \quad {}_B)(p_L = p_H \quad 1)} \right)^{\frac{1}{1}} \wedge \quad k^U;$$

which are (24) and (22) respectively, as desired.

Next, we derive scaled token prices p_z and start by considering $z = L$. Under these circumstances, we can calculate $\frac{E_t^S[dP_t]}{dt} = P_t \quad {}_L + \quad {}_t^H$. As $P_t = A_t p_L$, we have $P_t \quad {}_t^H = A_t(p_H \quad p_L)$. Inserting these relations into (15), we calculate

$$\frac{P_t^S}{A_t} = \frac{p_H}{r \quad {}_L +} \quad \wedge \quad k^S + v_L:$$

In addition, using (16), we obtain $P_t^U = A_t = v_L$. Thus, by (17), we have

$$p_L = \max \left(\frac{P_t^S}{A_t}; \frac{P_t^U}{A_t} \right) = \max \left(v_L(q); \frac{p_H}{r \quad {}_L +} \quad \wedge \quad v_L + k^S \right);$$

which is (25), as desired.

Finally, we derive scaled token price p_H in state $z = H$. Under these circumstances, we have $\frac{E_t^S[dP_t]}{dt} = P_t \quad {}_H + \quad (1 \quad) \quad {}_t^L$. Inserting this relation and $P_t = A_t p_H$ and $P_t \quad {}_t^L = A_t(p_L \quad p_H)$ into (15), we calculate

$$\frac{P_t^S}{A_t} = \frac{(1 \quad)p_L}{r \quad {}_H + \quad (1 \quad)} \quad \wedge \quad k^S + v_L:$$

In addition, using (16), we obtain $P_t^U = A_t = v_H$. Thus, by (17), we have

$$p_H = \max \left(\frac{P_t^S}{A_t}; \frac{P_t^U}{A_t} \right) = \max \left(v_H(q); \frac{(1 \quad)p_L}{r \quad {}_H + \quad (1 \quad)} \quad \wedge \quad k^S + v_L \right);$$

which is (23), as desired.

D.2 Token Price Properties

Here, we prove $p_H \quad p_L$.

Define $\quad {}_U \quad {}_B$ and $\quad {}_S \quad = \quad (q)$. Expected token returns from the perspective of agent $x \in \{S; U\}$ in state L can be written as

$$"_L^x = \quad {}_L + \quad {}_x \left(\frac{p_H}{p_L} \quad 1 \right) \tag{D.5}$$

and in state H can be written as

$$"_H^x = \quad {}_H + \quad (1 \quad {}_x) \left(\frac{p_L}{p_H} \quad 1 \right); \tag{D.6}$$

for $_x \in \{_U, _S\}$, where $x = S$ stands for speculator and $x = U$ stands for user. Notice that

$$(\varepsilon^S_z - \varepsilon^U_z) p_z = (\quad _B)(p_H - p_L) \Longleftrightarrow \varepsilon^S_z = \varepsilon^U_z + \frac{(\quad _B)(p_H - p_L)}{p_z}. \tag{D.7}$$

To show $p_H \quad p_L$, suppose to the contrary $p_L > p_H$. Then, from both user and speculator perspective, expected returns are strictly higher in state H than in state L. Formally, for $x \in \{S, U\}$, we have $\varepsilon^x_H > \varepsilon^x_L$. Note that by (3), we can rewrite scaled adoption as

$$v_z = \left(\frac{1}{r - \varepsilon^U_z}\right)^{\frac{1}{1}} \wedge k^U. \tag{D.8}$$

As a result, because of $\varepsilon^U_H > \varepsilon^U_L$, it follows that $v_H \quad v_L$. Due to $p_L > p_H \quad v_H$, it must be that $p_L > v_L$ and thus $s_L > 0$ by means of market clearing (14). However, as $_L < r$ and $p_L > p_H$, we have $\varepsilon^S_L < r$ and — by (5) — $S_t = s_t = s_L = 0$, a contradiction. Thus, $p_H \quad p_L$.

Notably, when $v_z < k^U$, then we even have $p_H > p_L$. To show this, suppose to the contrary $p_L \quad p_H$. Then, $\varepsilon^U_H > \varepsilon^U_L$ and therefore it follows by (D.8) and $v_z < k^U$ that $v_H > v_L$. Due to $p_L \quad p_H \quad v_H$, it must be that $p_L > v_L$ and $s_L > 0$. However, as $_L < r$ and $p_L \quad p_H$, we have $\varepsilon^S_L < r$ and — by (5) — $S_t = s_t = s_L = 0$, a contradiction. Thus, $p_H > p_L$.

E Proof of Proposition 3

For a better overview, we prove all three claims of the Proposition separately.

E.1 Claim 1

We now characterize under what circumstances speculative investment emerges and, in particular, show that speculative investment in state H, that is, $s_H > 0$, implies speculative investment in state L, that is, $s_L > 0$.

Consider $v_z < k^U$ which is assumed to hold throughout. We can rewrite (24) and (22) compactly as

$$v_z = \left(\frac{1}{r - \varepsilon^U_z}\right)^{\frac{1}{1}};$$

whereby ε^x_L and ε^x_H are defined in (D.5) and (D.6) respectively.

We now solve above equation for expected token returns ε^U_z, i.e.,

$$\varepsilon^U_z = r \quad (v_z)^{\quad 1}.$$

Using (D.7), we can solve for expected token returns from speculator perspective, i.e.,

$$\varepsilon^S_z = r \quad (v_z)^{\quad 1} + \frac{(\quad _B)(p_H - p_L)}{p_z}. \tag{E.9}$$

Suppose now $s_z = 0$, so by means of market clearing $p_z = v_z$. Then, $s_z = 0$ and $p_z = v_z$ can be an equilibrium if and only if speculators have no strict incentives to invest in tokens. By (5), this requires $E[dP^S]/(P\,dt) = \varepsilon^S_z \quad r$, that is,

$$(\quad _B)\left(p_H(q) \quad p_L(q) \right) \quad (p_z) \; :$$

By contrast, if $(\quad {}_B)\ p_H(q) \quad p_L(q) \ > (p_z)$, $s_z = 0$ is no equilibrium and it must be $s_z > 0$.

On the other hand, if speculators invest in tokens and $s_z > 0$ and $v_z < p_z$, then $"^S_z \quad r$ by (5). As such, (E.9) and $"^S_z \quad r$ imply

$$(\quad {}_B)\ p_H \quad p_L \quad (v_z)\ {}^1 p_z > (p_z)\ ;$$

which is (26). As such, $s_z > 0$ if and only if (26) holds, i.e., if and only if $(\quad {}_B)(p_H \quad p_L) > (p_z)$.

E.2 Claim 2

The proof of Claim 3 contains three parts. Part I shows that there are three cases: i) no speculative investment at all (i.e., $s_z = 0$), ii) speculative investment only in state L (i.e., $s_L > 0 = s_H$), or iii) speculative investment in both states (i.e., $s_z > 0$). Part II shows that if $s_z > 0$ for $z = L; H$, then $s_H = k^S$ or $s_L = k^S$. Part III shows that there exists unique $\_ \ 2\ ({}_B; \bar{\ }]$, so that $s_L > 0$ for $> \_$.

E.2.1 Part I

Recall that speculators hold tokens in state z if and only if (26) holds. Note that, due to $p_H \quad p_L$, we have $(p_H) \quad (p_L)$. Thus, $s_H > 0$ implies $s_L > 0$. In other words, there are three di erent possibilities regarding speculative investment. First, speculators invest in both states L and H, i.e., $s_z > 0$ for $z = L; H$. Second, speculators hold tokens in state L but not in state H, i.e., $s_L > 0 = s_H$. Third, speculators do not hold tokens at all, i.e., $s_z = 0$ for $z = L; H$. We treat these three cases separately.

E.2.2 Part II

We consider that there is speculative investment, $s_z > 0$, in both states $z = L$ and $z = H$. Thus, by means of (5), it must be that $"^S_z \quad r$ for $z = L; H$. As $p_H \quad p_L$, a necessary condition for $s_H > 0$ is $\ {}_H > r$. Thus, consider $\ {}_H > r$.

Suppose that speculative investment satis es $s_z(q)\ 2\ (0; k^S)$ in both states $z = L; H$. Thus, by (25) and (23), we obtain

$$p_L = \frac{p_H}{r \quad {}_L +} \quad \text{and} \quad p_H = \frac{(1 \quad)p_L}{r \quad {}_H + \ (1 \quad)}:$$

Next, inserting p_H into the expression for p_L and dividing by p_L, we have

$$\frac{}{r \quad {}_L +} = \frac{r \quad {}_H + \ (1 \quad)}{(1 \quad)};$$

which cannot hold as the right-hand-side exceeds one due ${}_H > r$ and the left-hand-side is strictly less than one due ${}_L < r$, a contradiction. Thus, $s_H = k^S$ or $s_L = k^S$.

E.2.3 Part III

Suppose that speculators do not invest in tokens, in that $s_L = s_H = 0$. Then, token prices p_z and adoption levels v_z are independent of for $z = L; H$, so we can write $p_z = \hat{p}_z$ and $v_z = \hat{v}_z$ with $\hat{p}_z$ and $\hat{v}_z$ being independent of .

Recall that speculators hold tokens in state z and $s_z > 0$ if and only if (26) holds, that is, if and only if

$$(\quad {}_B) \, p_H \quad p_L \; > (p_z) \; ;$$

whereby $(p_H) \; > (p_L)$. That is, $s_H = 0$ implies $s_L = 0$, i.e., above condition is more demanding to meet in state H than in state L. Thus, when $s_L = s_H = 0$, the left-hand-side of (26) increases with , while the right-hand-side remains constant. As such, the function

$$7! \quad (\quad {}_B) \, \hat{p}_H \quad \hat{p}_L \quad (\hat{p}_L)$$

has exactly one root $\_$ on $[0;1\,)$. As the left-hand-side is negative for $\quad {}_B$, we have $\_ > \; {}_B$. Thus, there exists unique $\_$, so that there is speculative investment $s_L > 0$ if and only if $\bar{\ } > \_$.[32] If $\_ > \bar{\ }$, then the set of admissible facilitating speculative investment is empty.

E.3 Claim 3

As argued above in the proof of Claim 2 (Part II), a necessary condition for $s_H > 0$ is $\;{}_H > r$. Consider that $\;{}_H > r$ and $\bar{\ } = 1 \quad \frac{{}_H \;\; r}{} < 1$. Suppose that $s_L > 0 = s_H$.

Then, speculators value tokens at (scaled) price

$$p_H^S = \frac{(1 \qquad)}{r \qquad {}_H + \;\; (1 \qquad)} \quad p_L :$$

It follows that as $\;\; " \bar{\ }$, then $(1 \qquad) \# \;\; {}_H \quad r$. As a result, $\lim_{\,"\bar{\ }} p_H^S = +1$.

As such, when $\;{}_H > r$, $\bar{\ } = 1 \quad \frac{{}_H \;\; r}{}$, and is su ciently close to $\bar{\ }$, then there is no equilibrium with $s_H = 0 < s_L$. Due to $\bar{\ } > \_$, the equilibrium for values of that are su ciently close to $\bar{\ }$ must feature speculative investment in both states L and H, i.e., $s_H; s_L > 0$.

Finally, suppose $\_ < \bar{\ }$. As $v_z < k^U$, Proposition 2 implies $p_H > p_L$. Recall that $s_z > 0$ if and only if $(\quad {}_B) \, p_H \quad p_L \; > (p_z)$. Then, due to $p_H > p_L$, continuity implies the existence of an interval $(\_; \_ + ")$, so that $s_L > 0 = s_H$ for $\; 2 \; (\_; \_ + ")$.

F Proof of Corollary 1

When there is no speculative investment in either state L and H, then the expressions in (24), (22), (25), and (23) reveal that that v_z and p_z are independent of , so $\frac{@v_z}{@} = \frac{@p_z}{@} = 0$.

Next, consider that $s_L \; 2 \; (0; k^S)$ and $s_H \; 2 \; f \, 0; k^S g$, so $\frac{@s_H}{@} = 0$. Then, (25) implies

$$p_L = \frac{p_H}{r \qquad {}_L +} :$$

Thus,

$$\frac{p_L}{p_H} \quad 1 = \frac{{}_L \quad r}{r \qquad {}_L +} \quad \text{and} \quad \frac{p_H}{p_L} \quad 1 = \frac{r \qquad {}_L}{} : \tag{F.10}$$

[32] Note that when $s_L = s_H = 0$, token price p_z reads $p_z = \hat{p}_z$ where $\hat{p}_z$ is independent of . If there existed $\; > \_$ for which $s_L = s_H = 0$, then $p_z = \hat{p}_z$ and $(\quad {}_B) \, \hat{p}_H \quad \hat{p}_L \quad (\hat{p}_L) \; > 0$, so $s_L > 0$ yielding a contradiction.

Inserting above expression for $\frac{p_L}{p_H}$ 1 from ([F.10](#)) into ([22](#)) yields

$$v_H = \left(\frac{1}{r \quad _H + \ (1 \quad _B) \ \frac{r \quad _L}{r \quad _L +}} \right)^{\frac{1}{1}} \wedge \ k^U :$$

As a result, v_H increases with and does so strictly if $v_H < k^U$ (i.e., $\frac{\partial v_H}{\partial}$ 0 with the inequality being strict for $v_H < k^U$).

Market clearing implies $p_H = v_H + s_H$, so that p_H increases with as $\frac{\partial s_H}{\partial} = 0$ (i.e., $\frac{\partial p_H}{\partial}$ 0 with the inequality being strict for $v_H < k^U$). Due to $_L < r$, it then also follows that $p_L = \frac{p_H}{r \quad _L +}$ increases strictly with , i.e., $\frac{\partial p_L}{\partial} > 0$.

Next, we use ([F.10](#)) to plug in for $\frac{p_H}{p_L}$ 1 in ([24](#)), which yields

$$v_L(q) = v_L = \left(\frac{1}{r \quad _L \quad _B \ \frac{r \quad _L}{}} \right)^{\frac{1}{1}} \wedge \ k^U;$$

As such, v_L decreases with and does so strictly if $v_L < k^U$ (i.e., $\frac{\partial v_L}{\partial}$ 0 with the inequality being strict for $v_L < k^U$).

G Proof of Proposition 4

We derive the token pricing equation in states L and H separately.

As a preparation, we rewrite (3) as

$$V_t = A_t v_z(q_t) = A_t \left(\frac{1}{r + \ _t^U \quad E^U[dP_t]=(P_t dt)} \right)^{\frac{1}{1}} \tag{G.11}$$

where we de ne

$$_t^U = \max \left((k^U) \quad ^1 + \frac{E_t^U[dP_t]}{P_t dt} \quad r; 0 \right) : \tag{G.12}$$

In equilibrium, $_t^U$ is a function of $(q; z)$, in that $_t^U = \ _z^U(q_t)$. Notice that $_z^U(q_t) > 0$ if and only if $v_z(q_t) = k^U$. That is, $_z^U(q_t)$ can be seen as the user's \Lagrange Multiplier" in the optimization (2) in state $(q; z)$.

Likewise, we de ne

$$_t^S = \max \left(\frac{E_t^S[dP_t]}{(V_t + K_t^S)dt} \quad r; 0 \right) = \max \left(\frac{P_t}{V_t + K_t^S} \quad \frac{E_t^S[dP_t]}{P_t dt} \quad r; 0 \right) : \tag{G.13}$$

Notice from (15) that $P_t^S = \frac{E_t^S[dP_t]}{r + \ _t^S} = V_t + K_t^S$ if and only if $_t^S > 0$. It follows from (5) that

$$S_t = \begin{cases} 0 & \text{if} \quad E_t^S[dP_t]=(P_t dt) < r + \ _t^S \\ \hat{S}_t \ 2 \ [0; K_t^S] & \text{if} \quad E_t^S[dP_t]=(P_t dt) = r + \ _t^S : \end{cases} \tag{G.14}$$

In equilibrium, ${}^S_t$ is a function of $(q; z)$, in that ${}^S_t = {}^S_z(q_t)$. Notice that ${}^S_t = {}^S_z(q_t)$ can be seen as the speculator's \Lagrange multiplier" in the optimization (4). Thus, ${}^S_t > 0$ if and only if $S_t = K^S_t$; otherwise, ${}^S_t = 0$.

We can rewrite (17) as

$$P_t = \max \left\{ A_t \left(\frac{1}{r + {}^U_t \quad E^U_t[dP_t]=(P_t dt)} \right)^{\frac{1}{1}} ; \frac{E^S_t[dP_t]}{r + {}^S_t} \right\} : \tag{G.15}$$

We can solve (G.15) forr to obtain

$$r = \max \left\{ \left(\frac{A_t}{P_t} \right)^{1} \quad {}^U_t + \frac{E^U_t[dP_t]}{P_t dt} ; \frac{E^S_t[dP_t]}{P_t dt} \quad {}^S_t \right\} : \tag{G.16}$$

Rearranging (G.16), we obtain the token pricing equation

$$r + {}^S_t = \max \left\{ \left(\frac{A_t}{P_t} \right)^{1} \quad ({}^U_t \quad {}^S_t) \quad \frac{E^S_t[dP_t] \quad E^U_t[dP_t]}{P_t dt} ; 0 \right\} + \frac{E^S_t[dP_t]}{P_t dt} ; \tag{G.17}$$

which reduces to (18) for ${}^S_t = {}^U_t = 0$.

We now consider both states $z = L$ and $z = H$ separately, and we omit time subscripts unless needed. Using (G.11) and substituting $E^U[dP_t]=(P_t dt)$ and ${}^U_t$, we can also write

$$v_z(q) = \left(\frac{1}{r + {}^U_z(q) \quad "^U_z(q)} \right)^{\frac{1}{1}} ; \tag{G.18}$$

so that $V_t = A_t v_z(q_t)$. The market clearing condition (14), i.e., $P_t = V_t + S_t$, then becomes

$$p_z(q) = v_z(q) + s_z(q);$$

and does not depend on A_t but only on $(q; z)$.

G.1 State L

We start by analyzing state L, and we use (19), (20), and (21) to calculate

$$"^U_L(q) = \frac{E^U[dP]}{P dt} = {}_L + \frac{p^0_L(q) \quad {}_L(q)}{p_L(q)} + {}_B \left(\frac{p_H(q_H(q))}{p_L(q)} \quad 1 \right) : \tag{G.19}$$

and

$$"^S_L(q) = \frac{E^S[dP]}{P dt} = {}_L + \frac{p^0_L(q) \quad {}_L(q)}{p_L(q)} + (q) \left(\frac{p_H(q_H(q))}{p_L(q)} \quad 1 \right) ; \tag{G.20}$$

where $q_H = q_H(q)$ is from Lemma 3. Next, we combine (G.19) and (G.20) to obtain

$$\frac{E^S[dP] \quad E^U[dP]}{P dt} = (\; (q) \quad {}_B) \left(\frac{p_H(q_H(q))}{p_L(q)} \quad 1 \right) :$$

Then, we use (G.12) to calculate

$$^{U} = {}^{U}_{L}(q) = \max \quad 0; (k^{U}) \quad ^{1} + \quad _{L} + \frac{p^{0}_{L}(q) \quad _{L}(q)}{p_{L}(q)} + \quad _{B} \quad \frac{p_{H}(q_{H}(q))}{p_{L}(q)} \quad 1 \quad r$$

and (G.13) to calculate

$$^{S} = {}^{S}_{L}(q) = \max \quad \frac{p_{L}(q)}{v_{L}(q) + k^{S}} \quad _{L} + \frac{p^{0}_{L}(q) \quad _{L}(q)}{p_{L}(q)} + \quad (q) \quad \frac{p_{H}(q_{H}(q))}{p_{L}(q)} \quad 1 \quad r; 0 \quad :$$

We insert $E^{U}[dP]=(P\,dt) = \;"^{U}_{L}(q)$ into (G.18) to obtain

$$v_{L}(q) = \frac{1}{r + \quad ^{U}_{L}(q) \quad _{L} \quad _{B}(p_{H}(q_{H}(q))=p_{L}(q) \quad 1) + \quad _{L}(q)p^{0}_{L}(q)=p_{L}(q)}^{\frac{1}{1}} ; \tag{G.21}$$

Next, inserting the expressions for $^{x}_{L}(q)$ and $"^{x}_{L}(q)$ for $x\ 2\ f\ U; Sg$ into (G.17) yields

$$r \quad _{L} + {}^{S}_{L}(q) = \max \quad (p_{L}(q)) \quad ^{1} \quad ^{U}_{L}(q) \quad ^{S}_{L}(q) \quad (\quad (q) \quad _{B}) \frac{p_{H}(q_{H}(q))}{p_{L}(q)} \quad 1 \quad ; 0$$
$$+ \quad (q) \quad \frac{p_{H}(q_{H}(q))}{p_{L}(q)} \quad 1 \quad + \frac{p^{0}_{L}(q) \quad _{L}(q)}{p_{L}(q)} : \tag{G.22}$$

Above ODE (G.22) is a rst order ODE for token price.

G.2 State H

Next, we consider state H. To begin with, we use (19), (20), and (21) to calculate

$$"^{U}_{H}(q) = \frac{E^{U}[dP]}{P\,dt} = \quad _{H} + \frac{p^{0}_{H}(q)}{p_{H}(q)} \quad _{H}(q) + \quad (1 \quad _{B}) \quad \frac{p_{L}(q_{L}(q))}{p_{H}(q)} \quad 1 \quad : \tag{G.23}$$

and

$$"^{S}_{H}(q) = \frac{E^{S}[dP]}{P\,dt} = \quad _{H} + \frac{p^{0}_{H}(q)}{p_{H}(q)} \quad _{H}(q) + \quad (1 \quad (q)) \quad \frac{p_{L}(q_{L}(q))}{p_{H}(q)} \quad 1 \quad ; \tag{G.24}$$

where $q_{L} = q_{L}(q)$ is from Lemma 3. Next, we combine (G.23) and (G.24), and calculate

$$\frac{E^{S}[dP] \quad E^{U}[dP]}{P\,dt} = \quad (\quad (q) \quad _{B}) \quad 1 \quad \frac{p_{L}(q_{L}(q))}{p_{H}(q)} \quad :$$

We use (G.12) to calculate

$$^{U}_{H}(q) = \max \quad (k^{U}) \quad ^{1} + \quad _{H} + \frac{p^{0}_{H}(q)}{p_{H}(q)} \quad _{H}(q) + \quad (1 \quad _{B}) \quad \frac{p_{L}(q_{L}(q))}{p_{H}(q)} \quad 1 \quad r; 0$$

and (G.13) to calculate

$$^{S}_{H}(q) = \max \quad 0; \quad \frac{p_{H}(q)}{v_{H}(q) + k^{S}} \quad _{H} + \frac{p^{0}_{H}(q)}{p_{H}(q)} \quad _{H}(q) + \quad (1 \quad (q)) \quad \frac{p_{L}(q_{L}(q))}{p_{H}(q)} \quad 1 \quad r \quad :$$

We can then insert above expression for $E^U[dP]/(P\,dt)$ into (3) to solve for

$$v_H(q) = \left[\frac{1}{r + {}^{U}_{H}(q) \quad _H \quad (1 \quad _B)(p_L(q_L(q))/p_H(q) \quad 1) + \quad _H(q)p^0_H(q)/p_H(q)}\right]^{\frac{1}{1}} : \quad \text{(G.25)}$$

Inserting above expressions for ${}^{x}_{H}(q)$ and $"^{x}_{H}(q)$ into (G.17) yields

$$r \quad _H + {}^{S}_{H}(q) = \max \quad (p_H(q)) \quad ^1 \quad {}^{U}_{H}(q) \quad {}^{S}_{H}(q) \quad (\ (q) \quad _B) \quad 1 \quad \frac{p_L(q_L(q))}{p_H(q)} \ ; 0$$

$$+ \ (1 \quad (q)) \quad \frac{p_L(q_L(q))}{p_H(q)} \quad 1 \ + \frac{p^0_H(q) \quad _H(q)}{p_H(q)}; \quad \text{(G.26)}$$

which is a rst order ODE for token price.

G.3 Solving the equilibrium

The two rst order ODEs (G.22) and (G.26) are solved subject to the boundary conditions

$$\lim_{q! \ 1} p_H(q) = p_H(1) \quad \text{and} \quad \lim_{q! \ 0} p_L(q) = p_L(0);$$

which is (27). Here, the boundary conditions $p_L(0)$ and $p_H(1)$ are characterized in Proposition 1.

The Picard Lindeloef theorem ensures that under mild regularity conditions, there exists a unique, continuously di erentiable solution $(p_L(q); p_H(q))$ to (G.22) and (G.26) subject to (27) on $(0; 1)$. A formal existence and uniqueness proof is beyond the scope of the paper and we simply assume that such a unique solution and equilibrium exists. We numerically verify that this is indeed the case under our baseline parameters as well as all other parameter con gurations considered throughout the analysis.

H Proof of Corollary 2

Consider $_H = \quad _L$. Then, clearly, $v_L(q) = v_H(q)$ and $p_L(q) = p_H(q)$ with $v^0_L(q) = v^0_H(q) = 0$, which implies

$$\frac{E^S_t[dP_t]}{P_t dt} = \quad _L < r:$$

Thus, $s_L(q) = s_H(q) = 0$ and $v_z(q) = p_z(q)$.

I Solution under Dual Token Structure

We present the detailed solution under a dual token structure, whereby | in equilibrium (to be characterized) | $V_t = A_t v_z(q)$ and $G_t = A_t g_z(q)$ with functions $v_z(q)$ and $g_z(q)$. As we express all quantities in terms of $(q; z)$, we omit time subscripts unless confusion is likely to arise.

We proceed as follows. We rst describe the problems that the user and the speculator solve under the dual token structure model variant. Then, we characterize a Markov equilibrium with state variables $A_t; z\ 2\ f\ L; H\ g$, and q_t in which all agents act optimally and the markets for tokens clear.

I.1 User problem

As in the baseline, cohort t of the representative user lives from t to $t +$ and derives utility from consumption only at time $t +$. The user can invest her endowment of K_t^U dollars either in transaction tokens (i.e., stablecoins) or the risk-free asset with interest rate r. The user's transaction token holdings equal V_t and she sells these transaction token holdings at time $t +$ for V_t dollars. The user also incurs dollar fees from holding transaction tokens, $V_t f_t$.

The user's consumption at time $t +$ reads

$$c^U_{t+} (V_t) = V_t \quad V_t f_t \quad + (K_t^U \quad V_t)(1 + r \ ) = V_t (1 \quad r \quad f_t \ ) + K_t^U (1 + r \ ):$$

Next, cohort t's lifetime utility is de ned as

$$u_t^U (V_t) = c_{t+} \ (V_t) + \quad \frac{V_t \ A_t^1}{} \quad ! \quad :$$

Thus, the representative user solves

$$\max_{V_t \, 2 \, [0; K_t^U]} E_t^U [u_t^U (V_t)] = K_t^U (1 + \ ) + \max_{V_t \, 2 \, [0; K_t^U]} \ " \ V_t (1 \quad r \quad f_t \ ) + \frac{V_t \ A_t^1}{} \ ! \ \# \ ; \qquad \text{(I.27)}$$

yielding the solution

$$V_t = A_t \quad \frac{1}{r + f_t} \ ^{\frac{1}{1}} \quad ^ \quad K_t^U;$$

which is (29). Notice that by (29), we have

$$v_z(q_t) = \quad \frac{1}{r + f_t} \ ^{\frac{1}{1}} \quad ^ \quad k^U; \qquad \text{(I.28)}$$

where we will show that $f_t = f_z(q_t)$ is a function of $(q; z)$.

I.2 Speculator problem

As in the baseline, cohort t of the representative speculator lives from t to $t +$ and derives utility from consumption only at time $t +$. The speculator can invest her endowment of K_t^S dollars either in governance tokens or the risk-free asset with interest rate r. The speculator's token holdings in dollars equal S_t and the nominal token holdings read $\hat{S}_t$ (i.e., the speculator holds $\hat{S}_t$ units of governance tokens), so $S_t = G_t \hat{S}_t$. The speculator buys governance tokens at time t at price G_t and sells them at time $t +$ at price G_{t+} . We write $dG_t \quad G_{t+} \quad G_t$. In addition, the speculator receives a dividend $dDiv_t$ per unit of governance tokens, i.e., the dividend reads $\hat{S}_t dDiv_t$.

The consumption at time $t +$ is therefore

$$c^S_{t+} \ (\hat{S}_t) = \hat{S}_t (G_{t+} \ + dDiv_t) + (K_t^S \quad \hat{S}_t G_t)(1 + r \ ) = \hat{S}_t (dG_t + dDiv_t \quad r \ ) + K_t^S (1 + r \ ): \quad \text{(I.29)}$$

The speculator chooses $\hat{S}_t$ to maximize

$$\max_{\hat{S}_t} E_t^S [c^S_{t+} \ (\hat{S}_t)] \quad \text{s.t.} \quad \hat{S}_t G_t \ 2 \ [0; K_t^S]: \qquad \text{(I.30)}$$

The constraint $\hat{S}_t G_t \leq K_t^S$ means that the speculator cannot invest more than her endowment, and the constraint $\hat{S}_t G_t \geq 0$ reflects that the speculator cannot hold claims on the platform that have (strictly) negative value. And, the speculator cannot short sell tokens ($\hat{S}_t \geq 0$).

The solution to (I.30) is[33]

$$\hat{S}_t = \begin{cases} 0 & \text{if} \quad E_t^S[dDiv_t + dG_t] < rG_t \\ \tilde{S} \in [0,1] & \text{if} \quad E_t^S[dDiv_t + dG_t] = rG_t \\ K_t^S/G_t & \text{if} \quad E_t^S[dDiv_t + dG_t] > rG_t \, . \end{cases} \tag{I.31}$$

For the governance token market to clear, it must be that $\hat{S}_t = 1$, so by (I.31):

$$E_t^S[dDiv_t + dG_t] \geq rG_t \, ,$$

with the inequality holding in equality if $\hat{S}_t < K_t^S$. After replacing “ ” by “dt”, above relation becomes (31), which was to show.

I.3 Equilibrium valuation and pricing equations

We consider a Markov equilibrium with state variables A_t, $z \in \{L, H\}$, and q_t in which the representative user solves the optimization in (I.27) and the representative speculator solves the optimization in (I.30). In this Markov equilibrium, the token markets for transaction and governance tokens must clear. We conjecture and verify that in this Markov equilibrium, V_t and G_t scale with A_t, in that $V_t = v_z(q_t)$ and $G_t = g_z(q_t)$. And, fees are a function of (q, z) only, i.e., $f_t = f_z(q_t)$. We omit time subscripts unless needed.

We assume that such a Markov equilibrium exists and is well-behaved: In particular, we assume that, within the Markov equilibrium, the set of points on $(0, \infty)$ on which the functions $g_z(q)$ and $v_z(q)$ are not differentiable with respect is countable. In what follows, we follow — with a slight abuse of notation — the convention that if $g_z(q)$ (resp. $v_z(q)$) is not differentiable with respect to q, then $g_z'(q)$ (resp. $v_z'(q)$) denotes the left-limit $\lim_{x \uparrow q} g_z'(x)$ (resp. $\lim_{x \uparrow q} v_z'(x)$), if $\mu_z(q) < 0$, and the right-limit $\lim_{x \downarrow q} g_z'(x)$ (resp. $\lim_{x \downarrow q} v_z'(x)$), if $\mu_z(q) \geq 0$. These left and right limits exist as the set of points of non-differentiability is non-dense in $(0, \infty)$.

To begin with, we can use (29) to calculate

$$v_z(q) = \left(\frac{1}{r + f_z(q)} \right)^{\frac{1}{1-\alpha}} \wedge k^U.$$

Dividend payouts to governance token holders read

$$dDiv_t = dV_t + f_t V_t dt = v_z(q_t) dA_t + A_t dv_z(q_t) + A_t f_t v_z(q_t) dt = A_t \left((\mu_t + f_t) v_z(q_t) dt + dv_z(q_t) \right)$$

Next, note that

$$dv_L(q_t) = v_L'(q_t) \mu_L(q_t) dt + (v_H(q_H) - v_L(q_t)) dJ_t^H$$
$$dv_H(q_t) = v_H'(q_t) \mu_H(q_t) dt + (v_L(q_L) - v_H(q_t)) dJ_t^L \, ;$$

[33] If $G_t = 0$, we define K_t^S/G_t as $+\infty$.

where $q_z = q_z(q_t)$ is defined in Lemma 3. As a result, we obtain (omitting time subscripts)

$$\frac{dDiv}{A} = \begin{cases} (f_H(q) + \delta_H)v_H(q)dt + v_H'(q)\mu_H(q)dt + \left(v_L(q_L) - v_H(q)\right)dJ^L & \text{if} \quad z = H \\ (f_L(q) + \delta_L)v_L(q)dt + v_L'(q)\mu_L(q)dt + \left(v_H(q_H) - v_L(q)\right)dJ^H & \text{if} \quad z = L. \end{cases}$$

Thus,

$$\frac{E^S[dDiv]}{Adt} = \begin{cases} (f_H(q) + \delta_H)v_H(q) + v_H'(q)\mu_H(q) + \lambda(1 - \pi(q))\left(v_L(q_L) - v_H(q)\right) & \text{if} \quad z = H \\ (f_L(q) + \delta_L)v_L(q) + v_L'(q)\mu_L(q) + \lambda\pi(q)\left(v_H(q_H) - v_L(q)\right) & \text{if} \quad z = L. \end{cases} \tag{I.32}$$

Using $G = Ag_z(q)$ and $dA = A\mu_z dt$, we have $dG = Adg_z(q) + g_z(q)dA$ and we can rewrite (31) as

$$rAg_z(q)dt \geq A\left(E^S\left[\frac{dDiv}{A} + \mu_z g_z(q)dt + dg_z(q)\right]\right);$$

where the inequality holds in equality for $G = S < K^S = Ak^S$. Thus,

$$(r - \mu_z)g_z(q)dt \geq E^S\left[\frac{dDiv}{A} + dg_z(q)\right].$$

Next, note that

$$dg_L(q) = g_L'(q)\mu_L(q)dt + \left(g_H(q_H) - g_L(q)\right)dJ^H;$$
$$dg_H(q) = g_H'(q)\mu_H(q)dt + \left(g_L(q_L) - g_H(q)\right)dJ^L;$$

where $q_z = q_z(q)$ is defined in Lemma 3. As a result, we obtain

$$(r - \mu_z)g_z(q) \geq \frac{E^S[dDiv]}{Adt} + \begin{cases} g_H'(q)\mu_H(q) + \lambda(1 - \pi(q))\left(g_L(q_L) - g_H(q)\right); & \text{if } z = H \\ g_L'(q)\mu_L(q) + \lambda\pi(q)\left(g_H(q_H) - g_L(q)\right); & \text{if } z = L; \end{cases} \tag{I.33}$$

where $\frac{E^S[dDiv]}{Adt}$ is characterized above in (I.32). In state $z = L, H$, the inequality holds as equality if $g_z(q) < k^S$.

Finally, we recall the time-0 optimization (32), which is

$$F_0 := \max_{(f_t)_{t \geq 0}} G_0 + V_0 \quad \text{s.t.} \quad G_t \in [0, K_t^S] \quad \text{and} \quad V_t \in [0, K_t^U].$$

That is, the platform dynamically maximizes joint value of governance and transaction tokens. By the dynamic programming principle, at any point in time $t \geq 0$, the choice of the fee f_t maximizes $G_t + V_t$ or, equivalently, $g_z(q_t) + v_z(q_t)$. Thus, optimal fees are a function of (q, z) only as $g_z(q)$ and $v_z(q)$ depend on (q, z) only.

I.4 Optimal Fees in State H

Optimal Fees. We start by analyzing state $z = H$ to determine the optimal fees $f_H(q)$ that maximize $g_H(q) + v_H(q)$, whilst $g_H(q) \in [0, k^S]$ and $v_H(q) \in [0, k^U]$ for $z = H$.

We first consider $g_H(q) = k^S$. Then, the fee $f_H(q)$, maximizing $g_H(q) + v_H(q) = k^S + v_H(q)$, maximizes $v_H(q) = \left(\frac{1}{r + f_H(q)}\right)^{\frac{1}{1-\alpha}} \wedge k^U$. Note that $\lim_{f_H(q) \downarrow -r} v_H(q) = +\infty \wedge k^U$. Thus, the optimal fee $f_H(q)$ induces $v_H(q) = k^U$. And, $g_H(q) = k^S$ implies $v_H(q) = k^U$.

Second, we consider $g_H(q) < k^S$, so (I.33) holds as equality. We evaluate (I.33) for $z = H$ and under the optimal fee schedule $f_H(q)$ and add on both sides $rv_H(q)$ to obtain

$$\begin{aligned}(r \quad {}_H + \ (1 \quad (q))(g_H(q) + v_H(q)) \qquad \text{(I.34)}\\ = (r + f_H(q))v_H(q) + (g_H^0(q) + v_H^0(q)) \quad {}_H(q) + \quad (1 \quad (q)) \ v_L(q_L) + g_L(q_L) \ :\end{aligned}$$

The optimal fee $f_H(q)$ is set to maximize $g_H(q) + v_H(q)$ and thus according to (I.34)

$$f_H(q) = \arg\max_{f_H} \Big[(r + f_H)v_H(q) + (g_H^0(q) + v_H^0(q)) \quad {}_H(q) + \quad (1 \quad (q)) \ v_L(q_L) + g_L(q_L) \Big] ;$$

subject to $g_H(q) \in [0; k^S]$ and $v_H(q) \in [0; k^U]$. Thus, the optimal fee $f_H(q)$ maximizes

$$(r + f_H(q))v_H(q) = \left(\frac{1}{r + f_H(q)}\right)^{\frac{1}{\ }} \wedge \ k^U (r + f_H(q));$$

whilst $g_H(q) \in [0; k^S]$ and $v_H(q) \in [0; k^U]$.

Noting that $v_H(q) = \left(\frac{1}{r+f_H(q)}\right)^{\frac{1}{1}} \wedge k^U$, we see that controlling $f_H(q)$ is akin to controlling $v_H(q)$. The objective $f_H(q) \mapsto \left(\frac{1}{r+f_H(q)}\right)^{\frac{1}{\ }}$ decreases with $f_H(q)$, and tends to $+\infty$, as $f_H(q) \downarrow \ r$. Thus, ignoring bounds on $g_H(q)$ and $v_H(q)$, it would be optimal to set $f_H(q) \downarrow \ r$. As $\lim_{f_H(q) \downarrow \ r} v_H(q) = +\infty \wedge \ k^U$, there cannot be a solution in which $v_H(q) \in (0; k^U)$ and $g_H(q) \in (0; k^S)$, and it must hold that $f_H(q) > \ r$.

In essence, the platform optimally sets fees $f_H(q)$ to maximize $v_H(q)$, subject to $v_H(q) \quad k^U$ and $g_H(q) \in [0; k^S]$. The solution is therefore constrained by these bounds, $v_H(q) \quad k^U$ and $g_H(q) \in [0; k^S]$. As such, it must be $g_H(q) \in \{0; k^S\}$ or $v_H(q) = k^U$. By assumption, $g_H(q) < k^S$, so we are left with $g_H(q) = 0$ or $v_H(q) = k^U$. Thus, $v_H(q) < k^U$ implies $g_H(q) = 0$ or, equivalently, $g_H(q) = 0$ is a necessary condition for $v_H(q) < k^U$.

If $v_H(q) = k^U$, then we can solve (I.28) for

$$f_H(q) = F_H^1(q) := (k^U)^{\quad 1} \quad r;$$

which is the maximum fee implementing $v_H(q) = k^U$. If $v_H(q) < k^U$, and $g_H(q) = 0$ and $g_H^0(q) = 0$, then $f_H(q) = F_H^2(q)$, with

$$F_H^2(q) := \quad (1 \quad (q)) \quad {}_H \quad \frac{v_H^0(q) \ {}_H(q)}{v_H(q)} \quad (1 \quad (q)) \ \frac{v_L(q_L(q)) + g_L(q_L(q))}{v_H(q)} \ :$$

Altogether, since $v_H(q)$ clearly decreases with $f_H(q)$, we have

$$f_H(q) = \max\{F_H^1; F_H^2(q)\}:$$

Finally, recall that when $g_H(q) = k^S$, then $v_H(q) = k^U$ and the fees are $f_H(q) = F_H^1$.

Adoption under Optimal Fees. As a next step, we reinsert the fees $f_H(q)$ into (I.28) to obtain equilibrium platform adoption. We consider that $g_H(q)$ and $v_H(q)$ are differentiable with respect to q: In this case, $g_H(q) = k^S$ ($v_H(q) = k^U$) implies $g_H^0(q) = 0$ ($v_H^0(q) = 0$). First, consider

$g_H(q) = g_H^0(q) = 0$, and thus $f_H(q) = F_H^2(q)$. As such, we obtain

$$v_H(q) = \left(\frac{1}{r \quad {}_H \quad \frac{v_H^0(q) \quad {}_H(q)}{v_H(q)} \quad (1 \quad (q)) \quad \frac{v_L(q_L(q)) + g_L(q_L(q))}{v_H(q)} \quad 1} \right)^{\frac{1}{1}} \wedge k^U; \tag{I.35}$$

which we can rewrite as

$$v_H(q) = \left(\frac{1}{r \quad E^S[dV]/(V\,dt) \quad (E^S[dJ^L]/dt) \quad \frac{g_L(q_L(q))}{v_H(q)}} \right)^{\frac{1}{1}} \wedge k^U; \tag{I.36}$$

where $E^S[dV]/(V\,dt) = \; {}_H + \frac{v_H^0(q) \quad {}_H(q)}{v_H(q)} + \; (1 \quad (q))(v_L(q_L)/v_H(q) \quad 1)$ and $E^S[dJ^L]/dt) = \; (1 \quad (q))$.

When $v_H(q) < k^U$, then $g_H(q) = 0$. Next, consider that $v_H(q) = k^U$ in which case $f_H(q) = F_H^1$ and $v_H^0(q) = 0$ (provided $v_H(q)$ is differentiable). Then, we can use (I.33) to derive an ODE for $g_H(q)$. Note that when $g_H(q) = k^S$ and $g_H(q)$ is differentiable with respect to q, then simply $g_H^0(q) = 0$. Otherwise, (I.33) holds as equality and, equivalently, (I.34) applies. Then, we obtain

$$(r \quad {}_H)g_H(q) = (F_H^1 + \; {}_H)k^U + \; (1 \quad (q))(v_L(q_L(q)) \quad k^U) + g_H^0(q) \; {}_H(q) + \; (1 \quad (q)) \; g_L(q_L(q)) \quad g_H(q) \; ; \tag{I.37}$$

provided $v_H(q) = k^U$ and $v_H(q)$ is differentiable with respect to q in which case $v_H^0(q) = 0$.

I.5 Optimal Fees in State L

Optimal Fees. Next, we consider state L. The determination of the fees $f_L(q)$ is analogous to the determination of fees in state H. The fee $f_L(q)$ in state $(q; L)$ dynamically maximizes $g_L(q) + v_L(q)$, whilst respecting $g_L(q) \in [0; k^S]$ and $v_L(q) \in [0; k^U]$.

We first consider $g_L(q) = k^S$. Then, the fee $f_L(q)$, maximizing $g_L(q) + v_L(q) = k^S + v_L(q)$, maximizes $v_L(q) = \left(\frac{1}{r + f_L(q)}\right)^{\frac{1}{1}} \wedge k^U$. Note that $\lim_{f_L(q) \downarrow \; r} v_L(q) = +\infty \wedge k^U$. Thus, the optimal fee $f_L(q)$ induces $v_L(q) = k^U$. And, $g_L(q) = k^S$ implies $v_L(q) = k^U$.

Second, consider $g_L(q) < k^S$, so (I.33) holds as equality for $z = L$. The fee $f_L(q)$ in state $(q; L)$ dynamically maximizes $g_L(q) + v_L(q)$, whilst respecting $g_L(q) \in [0; k^S]$ and $v_L(q) \in [0; k^U]$. We use (I.33) for $z = L$ to calculate under the optimal transaction fees $f_L(q)$:

$$\begin{aligned} &(r \quad {}_L + \quad (q))(g_L(q) + v_L(q)) \\ &= \Big[(r + f_L(q))v_L(q) + (g_L^0(q) + v_L^0(q)) \; {}_L(q) + \quad (q) \; v_H(q_H) + g_H(q_H) \Big] \end{aligned} \tag{I.38}$$

Equation (I.38) reveals that the optimal fee $f_L(q)$ maximizes

$$(r + f_L(q))v_L(q) = \left(\frac{1}{r + f_L(q)}\right)^{\frac{\;}{1}} \wedge k^U(r + f_L(q));$$

whilst $g_L(q) \in [0; k^S]$ and $v_L(q) \in [0; k^U]$.

The objective $f_L(q) \mapsto \left(\frac{1}{r + f_L(q)}\right)^{\frac{\;}{1}}$ decreases with $f_L(q)$, and tends to $+\infty$, as $f_L(q) \downarrow \; r$. Ignoring the constraints on $g_L(q)$ and $v_L(q)$, the optimal fee $f_L(q)$ would satisfy $f_L(q) \downarrow \; r$. As such, the optimal fee $f_L(q)$ maximizes $v_L(q) = \left(\frac{1}{r+f_L(q)}\right)^{\frac{1}{1}} \wedge k^U$ whilst $g_L(q) \in [0;k^S]$. As $\lim_{f_L(q)\downarrow r} v_L(q) = +\infty \wedge k^U$, the solution is therefore constrained by the bounds. As such, it must be $g_L(q) \in \{0;k^S\}$ or $v_L(q) = k^U$. By assumption $g_L(q) < k^S$, which leaves us with $g_L(q) = 0$ or $v_L(q) = k^U$. Thus, $v_L(q) < k^U$ necessarily implies $g_L(q) = 0$.

If $v_L(q) = k^U$, then we can solve (I.28) for

$$f_L(q) = F_L^1 := (k^U)^{\quad 1} \quad r;$$

which is the highest fee implementing $v_L(q) = k^U$. If, on the other hand, $g_L(q) = 0$ and $g_L^0(q) = 0$, then $f_L(q) = F_L^2(q)$, with

$$F_L^2(q) := \quad (q) \qquad {}_L \quad \frac{v_L^0(q) \quad {}_L(q)}{v_L(q)} \qquad (q) \quad \frac{v_H(q_H) + g_H(q_H)}{v_L(q)}$$

Altogether, we have

$$f_H(q) = \max\{F_L^1; F_L^2(q)\}:$$

Finally, recall that when $g_L(q) = k^S$, then $v_L(q) = k^U$ and the fee reads $f_H(q) = F_L^1$.

Adoption under Optimal Fees. As a last step, we reinsert the fees $f_L(q)$ into (I.28) to obtain equilibrium platform adoption. We consider that $g_L(q)$ and $v_L(q)$ are differentiable with respect to q: In this case, $g_L(q) = k^S$ ($v_L(q) = k^U$) implies $g_L^0(q) = 0$ ($v_L^0(q) = 0$).

First, consider $g_L(q) = g_L^0(q) = 0$, and thus $f_L(q) = F_L^2(q)$. As such, we obtain

$$v_L(q) = \left(\frac{1}{r \quad {}_L \quad \frac{v_L^0(q) \quad {}_L(q)}{v_L(q)} \qquad (q) \quad \frac{v_H(q_H)+g_H(q_H)}{v_L(q)} \quad 1}\right)^{\frac{1}{1}} \wedge \quad k^U; \tag{I.39}$$

which we can rewrite as

$$v_L(q) = \left(\frac{1}{r \quad E^S[dV]=(V\,dt) \quad (E^S[dJ^H]=dt) \quad \frac{g_H(q_H)}{v_L(q)}}\right)^{\frac{1}{1}} \wedge \quad k^U; \tag{I.40}$$

where $E^S[dV]=(V\,dt) = \quad {}_L + \frac{v_L^0(q) \quad {}_L(q)}{v_L(q)} + \quad (q)(v_H(q_H)=v_L(q) \quad 1)$ and $E^S[dJ^H]=dt = \quad (q)$.

When $v_L(q) < k^U$, $g_L(q) = 0$. Next, consider that $v_L(q) = k^U$ in which case $f_L(q) = F_L^1$ and $v_L^0(q) = 0$ (provided differentiability). Then, we can use (I.33) to derive an ODE for $g_L(q)$. Note that when $g_L(q) = k^S$ and $g_L(q)$ is differentiable with respect to q, then simply $g_L^0(q) = 0$. Otherwise, (I.33) holds as equality and, equivalently, (I.38) applies. Then, we obtain

$$(r \quad {}_L)g_L(q) = (F_L^1 + \quad {}_L)k^U + \quad (q)(v_H(q_H) \quad k^U) + g_L^0(q) \quad {}_L(q) + \quad (q) \quad g_H(q_H) \quad g_L(q) \; ; \tag{I.41}$$

when $v_L(q) = k^U$ and $v_L(q)$ is differentiable in which case $v_L^0(q) = 0$.

I.6 Solving the equilibrium

We show that the numerical solution for the equilibrium under a dual token structure is characterized by a system of four coupled and delayed first order ODEs. To do so, we consider that $g_z(q)$ and

$v_z(q)$ are differentiable with respect to q. In this case, $g_z(q) = k^S$ ($v_z(q) = k^U$) implies $g_z'(q) = 0$ ($v_z'(q) = 0$).

First, provided $v_H(q) < k^U$, we can invert (I.35) to solve for

$$v_H'(q) \quad _H(q) = v_H(q) \quad r \quad _H \quad v_H(q)^{1} \quad (1 \quad (q)) \quad \frac{v_L(q_L) + g_L(q_L)}{v_H(q)} \quad 1 \quad .$$

In case, $v_H(q) = k^U$, we have $v_H'(q) = 0$. Thus,

$$v_H'(q) \quad _H(q) = \begin{cases} v_H(q) \quad r \quad _H \quad v_H(q)^{1} \quad (1 \quad (q)) \quad \frac{v_L(q_L) + g_L(q_L)}{v_H(q)} \quad 1 & \text{if } v_H(q) < k^U \\ 0 & \text{if } v_H(q) = k^U. \end{cases} \tag{I.42}$$

Likewise, when $v_L(q) < k^U$, we can solve (I.39) for

$$v_L'(q) \quad _L(q) = v_L(q) \quad r \quad _L \quad v_L(q)^{1} \quad (q) \quad \frac{v_H(q_H) + g_H(q_H)}{v_L(q)} \quad 1 \quad .$$

When $v_L(q) = k^U$, then $v_L'(q) = 0$. Thus, altogether,

$$v_L'(q) \quad _L(q) = \begin{cases} v_L(q) \quad r \quad _L \quad v_L(q)^{1} \quad (q) \quad \frac{v_H(q_H) + g_H(q_H)}{v_L(q)} \quad 1 & \text{if } v_L(q) < k^U \\ 0 & \text{if } v_L(q) = k^U. \end{cases} \tag{I.43}$$

Next, we derive an ODE for the governance token value $g_z(q)$ for $z = L, H$.

When $g_z(q) \in \{0, k^S\}$, then $g_z'(q) = 0$. Otherwise, we can invert (I.37) to solve for

$$g_H'(q) \quad _H(q) = G_H(q) \quad (r \quad _H) g_H(q) \quad (F_H^{1} + \quad _H) k^U \quad (1 \quad (q))(v_L(q_L) \quad k^U + g_L(q_L) \quad g_H(q)) \quad .$$

Thus,

$$g_H'(q) \quad _H(q) = \begin{cases} G_H(q) & \text{if } g_H(q) \in (0, k^S) \\ 0 & \text{if } g_H(q) \in \{0, k^S\}. \end{cases} \tag{I.44}$$

Likewise, we can use (I.41) to obtain

$$g_L'(q) \quad _L(q) = G_L(q) \quad (r \quad _L) g_L(q) \quad (F_L^{1} + \quad _L) k^U \quad (q)(v_H(q_H) + g_H(q_H) \quad k^U \quad g_L(q)) \quad .$$

$$g_L'(q) \quad _L(q) = \begin{cases} G_L(q) & \text{if } g_L(q) \in (0, k^S) \\ 0 & \text{if } g_L(q) \in \{0, k^S\}. \end{cases} \tag{I.45}$$

Thus, the equilibrium and the functions $v_z(q)$ and $g_z(q)$ are characterized by the coupled system of four first order ODEs in (I.42), (I.43), (I.44), and (I.45) on $(0, 1)$. It remains to determine the boundary behavior as $q \to 0$ and $q \to 1$. When $q \in \{0, 1\}$, then $_z(q) = 0$ and $q_z(q) = q$.

Boundary behavior. We start with the lower boundary, $q = 0$. Note that when $q \in \{0, 1\}$, then $_z(q) = 0$ and $q_z(q) = q$ and the relations (I.42), (I.43), (I.44), and (I.45) determine the equilibrium, with the left-hand-side being zero. For any state $z = L, H$ and $q \in \{0, 1\}$, we can solve

(I.42), (I.43), (I.44), and (I.45) to obtain $v_z(q) = v_z(q)$ and $g_z(q) = g_z(q)$. We assume that such a solution exists and is unique.

Having obtained $v_z(q) = v_z(q)$ and $g_z(q) = g_z(q)$ $v_z(q) = v_z(q)$ and $g_z(q) = g_z(q)$, the following boundary conditions apply:

$$\lim_{q \to 1} g_H(q) = g_H(1) \quad \text{and} \quad \lim_{q \to 1} v_H(q) = v_H(1) \tag{I.46}$$

as well as

$$\lim_{q \to 0} g_L(q) = g_L(0) \quad \text{and} \quad \lim_{q \to 0} v_L(q) = v_L(0): \tag{I.47}$$

Taking stock, to solve for the equilibrium, one has to solve the system a system of four rst order ODEs | characterized by (I.42), (I.43), (I.44), and (I.45) | on (0 ; 1) subject to four boundary conditions (I.46) and (I.47) as well as the dynamics of q in Lemma 3.

Given the four boundary conditions in (I.46) and (I.47), the Picard-Lindeloef theorem ensures under mild conditions the existence of a unique solution $(g_z(q); v_z(q))$ for $z = L; H$ to the ODE system | characterized by (I.42), (I.43), (I.44), and (I.45). A formal existence and uniqueness proof is beyond the scope of the paper and we simply assume that such a unique solution and equilibrium exists. We numerically verify that this is indeed the case under our baseline parameters as well as all other parameter con gurations considered throughout the analysis.

I.7 Proof of Corollary 3

Take $_H = \ _L$, so $p_H(q) = p_L(q)$ and $v_L(q) = v_H(q)$ under the baseline token-based structure, with $v_L^0(q) = p_L^0(q) = 0$. It is clear that, under the token-based structure, $s_z(q) = 0$ and $p_z(q) = v_z(q)$, with

$$v_z(q) = \left(\frac{1}{r \quad _L}\right)^{\frac{1}{1}} \wedge k^U: \tag{I.48}$$

Next, consider the dual token structure. It is clear that $v_L(q) = v_H(q)$ and $g_L(q) = g_H(q)$, with $g_L^0(q) = v_L^0(q) = 0$. First, suppose that $v_z(q) < k^U$. Then, Proposition 5 implies $g_z(q) = 0$, and (I.40) implies $v_z(q) = \left(\frac{1}{r \quad _L}\right)^{\frac{1}{1}} \wedge k^U$ due to $E^S[dV] = (V\,dt) = \ _L$. Altogether, adoption under the dual token structure is therefore characterized in (I.48), which was to show.

I.8 Dual Token Structure and Reserves

In this Section, we show how to value governnace tokens when the platform maintains (part of) the seigniorage revenue as reserves. We denote the dollar value of reserves by M_t which evolves according to

$$dM_t = rM_t dt \quad d\widetilde{Div}_t; \tag{I.49}$$

where $d\widetilde{Div}_t$ is the payout from the reserves to the governance token holders; this payout can be negative. The reserves accrue interest at the risk-free rate and must remain positive, i.e., $M_t \quad 0$. Imposing the transversality condition $\lim_{s \to 1} \ e^{r(s \ t)} E_t^S[M_s] = 0$, we can integrate over time to obtain

$$M_t = E_t^S \int_t^1 e^{r(s \ t)} d\widetilde{Div}_s \ :$$

The baseline dual token structure without reserves is achieved by choosing $d\widetilde{Div}_t = 0$ at all times $t \quad 0$, so $M_t = 0$.

Governance token holders receive both the dividend $dDiv_t$ from (30) and the payout from the reserve $d\widehat{Div}_t$. Governance token value under this specification, denoted $\hat{G}_t$, satisfies analogously to (31)

$$r\hat{G}_t dt \geq E_t^S[dDiv_t + d\widehat{Div}_t + d\hat{G}_t]; \tag{I.50}$$

where the inequality holds as equality if $\hat{G}_t < K_t^S$. Integrating over time, we obtain

$$\hat{G}_t = \underbrace{E_t^S \int_t^\infty e^{-r(s-t)} dDiv_s}_{G_t} + \underbrace{E_t^S \int_t^\infty e^{-r(s-t)} d\widehat{Div}_s}_{=M_t} \wedge K_t^S;$$

where we call $G_t = \hat{G}_t - M_t$ the "net governance token value." Limited liability requires now $G_t \geq 0$, as otherwise governance token holders would be better shutting down the platform and paying a dividend of M_t dollars to themselves. The constraint is analogously to the baseline.

Consider $\hat{G}_t < K_t^S$. Then, we have $\hat{G}_t = G_t + M_t$ and (I.50) holds as equality, so $d\hat{G}_t = dG_t + dM_t$. Thus, (I.50) reduces to

$$rG_t dt = E_t^S[dDiv_t + dG_t];$$

which is (31) holding as equality. Thus, the asset pricing equation (I.50) becomes independent of $d\widehat{Div}_t$ and M_t.

Under these circumstances, the platform could always increase (decrease) the level of reserves by a marginal unit, $dM_t = \delta$ ($dM_t = -\delta$), by stipulating $d\widehat{Div}_t = -\delta$ ($d\widehat{Div}_t = \delta$), which leaves the total change in payoff of governance token holders, i.e., $dDiv_t + d\widehat{Div}_t + d\hat{G}_t$ unchanged but raises governance token token value by δ. In other words, provided $\hat{G}_t < K_t^S$, M_t is a control variable that does not affect the optimal levels of fees f_t, adoption V_t, or "net governance token value" G_t.

One possible choice of M_t is $M_t = \max\{K_t^S - G_t; 0\} = A_t \max\{k^S - g_z(q_t); 0\}$, where $g_z(q_t)$ is the governance token value under the baseline dual token structure without reserves (i.e., $M_t = 0$). Under this choice of M_t, the governance token value is always at scaled level k^S, whilst the optimization and results remain unchanged relative to the baseline dual token structure and, notably, lead to the same equilibrium levels of $g_z(q)$ (net governance token value), $f_z(q)$ (fees), and $v_z(q)$ (adoption).

J Fiat-Based Platform

To model the fiat-based platform in our framework, we assume that users derive a utility from transacting on the platform and holding V_t dollars for platform transactions and that users are charged a dollar fee f_t that is proportional to their transaction level V_t. As a consequence, given f_t, the representative user solves the same optimization as under the dual token structure, that is, (I.27). The solution to this problem reads

$$V_t = A_t \left(\frac{1}{r + f_t}\right)^{\frac{1}{1-\alpha}} \wedge K_t^U; \tag{J.51}$$

The platform's equity value E_t held by speculators is the expected discounted stream of fee revenues $f_t V_t dt$ which are paid out as dividends. For speculators to be willing to hold the platform's equity, it must be that the returns of doing so exceed the required rate of return, in that

$$\mathrm{E}_t^S[V_t f_t dt + dE_t] \quad rE_t dt; \tag{J.52}$$

where the inequality is strict for $E_t < K_t^S$.

The optimal transaction fee maximizes equity value and thus transaction fee revenues:

$$f_{Fiat} = \arg\max_f \left[\quad \frac{1}{r+f} \quad ^{\frac{1}{1}} \right] :$$

Solving this maximization problem yields closed-form expressions for the optimal transaction fee and transaction volume:

$$f_{Fiat} = \frac{(1 \quad)r}{} \quad \text{and} \quad v_{Fiat} = \quad \frac{}{r} \quad ^{\frac{1}{1}} : \tag{J.53}$$

Note that the optimal transaction fee does not depend on the state $(q; z)$, so — by (J.57) — transaction volume or platform usage v_E does not depend on $(q; z)$ either. This also implies that under a fiat-based structure, users are not exposed to any platform-specific risk or upside which is entirely absorbed by speculators as the platform's equity holders. Unlike in a token-based structure, users do not benefit from speculator optimism and platform growth, which curbs adoption.

We now provide the detailed solution under a fiat-based platform structure.

J.1 User problem

As in the baseline, cohort t of the representative user lives from t to $t+$ and derives utility from consumption only at time $t+$. The user can hold her endowment of K_t^U dollar as fiat money on the platform, in which case she derives convenience yield but does not earn interest, or in the risk-free asset with interest rate r. The user's dollar holdings on the platform over $[t; t+ \ ]$ equal V_t; at time $t+$, she can consume V_t. The user also incurs dollar fees from holding transaction tokens, $V_t f_t$.

The user's consumption at time $t+$ reads

$$c_{t+}^U(V_t) = V_t \quad V_t f_t \quad + (K_t^U \quad V_t)(1 + r \quad) = V_t(1 \quad r \quad f_t \quad) + K_t^U(1 + r \quad):$$

Next, cohort t's lifetime utility is defined in (1), that is,

$$u_t^U(V_t) = c_{t+}(V_t) + \left(\frac{V_t \ A_t^1}{} \right) :$$

Thus, the optimization in (2) becomes

$$\max_{V_t \in [0; K_t^U]} \mathrm{E}_t^U[u_t^U(V_t)] = K_t^U(1 + \quad) + \max_{V_t \in [0; K_t^U]} \left[V_t(1 \quad r \quad f_t \quad) + \left(\frac{V_t \ A_t^1}{} \right) \right] ;$$

yielding the solution

$$V_t = A_t \quad \frac{1}{r + f_t} \quad ^{\frac{1}{1}} \quad \wedge \quad k^U;$$

which is (29). Notice that by (29), we have

$$v_t = v_z(q_t) = \left(\frac{1}{r+f_t}\right)^{\frac{1}{1-\alpha}} \wedge k^U;$$

where we will show that $f_t = f_z(q_t)$ is a function of $(q; z)$.

J.2 Speculator problem

As in the baseline, cohort t of the representative speculator lives from t to $t+\Delta$ and derives utility from consumption only at time $t+\Delta$. The speculator can invest her endowment of K_t^S dollars either in equity or the risk-free asset with interest rate r. The speculator's equity holdings in dollars equal S_t and the nominal token holdings read $\hat{S}_t$ (i.e., the speculator holds $\hat{S}_t$ units of equity), so $S_t = E_t\hat{S}_t$. The speculator buys equity at time t for price E_t and sells it at time $t+\Delta$ for price $E_{t+\Delta}$. We write $dE_t \equiv E_{t+\Delta} - E_t$. In addition, the speculator receives a dividend equal to transaction fee revenues $f_tV_t\Delta$ per unit of governance tokens, i.e., the dividend reads $\hat{S}_tf_tV_t\Delta$.

The consumption at time $t+\Delta$ is therefore

$$c_{t+\Delta}^S(\hat{S}_t) = \hat{S}_t(E_{t+\Delta} + f_tV_t\Delta) + (K_t^S - \hat{S}_tE_t)(1+r\Delta) = \hat{S}_t(dE_t + f_tV_t\Delta - r\Delta) + K_t^S(1+r\Delta). \quad \text{(J.54)}$$

The speculator chooses $\hat{S}_t$ to maximize

$$\max_{\hat{S}_t \geq 0} \mathrm{E}_t^S[c_{t+\Delta}^S(\hat{S}_t)] \quad \text{s.t.} \quad \hat{S}_tE_t \in [0; K_t^S]. \quad \text{(J.55)}$$

The constraint $\hat{S}_tE_t \leq K_t^S$ means that the speculator cannot invest more than her endowment, and the constraint $\hat{S}_tE_t \geq 0$ reflects that the speculator cannot hold claims on the platform that have (strictly) negative value. And, the speculator cannot short sell tokens ($\hat{S}_t \geq 0$).

The solution to (J.55) is[34]

$$\hat{S}_t = \begin{cases} 0 & \text{if} \quad \mathrm{E}_t^S[f_tV_t\Delta + dE_t] < rE_t\Delta \\ \tilde{S} \in [0; 1] & \text{if} \quad \mathrm{E}_t^S[f_tV_t\Delta + dE_t] = rE_t\Delta \\ K_t^S/E_t & \text{if} \quad \mathrm{E}_t^S[f_tV_t\Delta + dE_t] > rE_t\Delta. \end{cases} \quad \text{(J.56)}$$

For the equity market to clear, it must be that $\hat{S}_t = 1$, so by (J.56):

$$\mathrm{E}_t^S[f_tV_t\Delta + dE_t] \geq rE_t\Delta$$

with the inequality holding in equality if $S_t = K_t^S$. After replacing "Δ" by "dt", above relation becomes (J.52), which was to show.

J.3 Optimal Fees

Platform usage under a fiat-based structure equals $V_t = A_tv_z(q_t)$ with

$$v_z(q_t) = \left(\frac{1}{r+f_t}\right)^{\frac{1}{1-\alpha}} \wedge k^U; \quad \text{(J.57)}$$

[34] If $E_t = 0$, we define K_t^S/E_t as $+\infty$.

The optimal transaction fee maximizes equity value and fee revenues, $f_t v_z(q_t)dt$, so

$$f_{Fiat} = \arg\max_f \left[f \left(\frac{1}{r+f} \right)^{\frac{1}{1}} \wedge f k^U \right] :$$

We now solve this optimization and distinguish two cases.

First, when $v_z(q) = k^U$, then it is optimal to set the fee as large as possible and consistent with $v_z(q) = k^U$, so $f = 1(k^U)^{\ 1}\ r$. Second, when $v_z(q) < k^U$, the $f = f_{Fiat}$ must solve the first order condition

$$\frac{1}{r+f}^{\frac{1}{1}} \quad f \quad \frac{1}{r+f}^{\frac{1}{1}\ 1} \quad \frac{1}{1} \quad \frac{1}{(r+f)^2} = 0;$$

which we can solve for $f_{Fiat} = \frac{(1\ \ )r}{}$. Altogether,

$$f_{Fiat} = \max \ \frac{(1\ \ )r}{} ; (k^U)^{\ 1}\ r \tag{J.58}$$

The optimal fee does not depend on the state $(q; z)$.

Next, can insert the optimal fee $f_t = f_{Fiat}$ from (J.58) into (J.57) to calculate scaled adoption/transaction volume

$$v_{Fiat} = \ \frac{}{r}^{\frac{1}{1}} \wedge \ k^U;$$

which also does not depend on the state $(q; z)$.

J.4 Solving for equity value

The equity value E_t take the form $E_t = A_t e_z(q_t)$, where scaled equity value $e_z(q_t)$ depends on $(q; z)$ only. As such,

$$dE_t = dA_t e_z(q_t) + A_t de_z(q_t)$$

and

$$\begin{aligned} de_L(q_t) &= e_L'(q_t)\ \ _L(q_t)dt + (e_H(q_H(q_t)) \quad e_L(q_t))dJ_t^H \\ de_H(q_t) &= e_H'(q_t)\ \ _H(q_t)dt + (e_L(q_L(q_t)) \quad e_H(q_t))dJ_t^L : \end{aligned}$$

Inserting these relations into (J.52), we obtain

$$(r \quad\ _z)e_z(q) \quad v_{Fiat} f_{Fiat} + \begin{cases} e_H'(q)\ \ _H(q) + \ (1 \quad (q))(e_L(q_L(q)) \quad e_H(q)); & \text{if } z = H \\ e_L'(q)\ \ _L(q) + \quad (q)(e_H(q_H(q)) \quad e_L(q)); & \text{if } z = L; \end{cases} \tag{J.59}$$

where the inequality holds as equality in state $z = L; H$ if $e_z(q) < k^S$. When $e_z(q) = k^S$, then $e_z'(q) = 0$.

At the boundaries of the state space, i.e., when $q = 0$ or $q = 1$, we have $\ _z(q) = 0$ and $q_z(q) = q$. Then, we can insert $\ _z(q)$ and $q_z(q)$ into (J.59), and we can solve (J.59) for $e_z(q) = e_z(q)$ for $z = L; H$ and $q\ 2\ f0; 1g$. Next, we can solve the system of ODEs in (J.59) subject to the boundary conditions

$$\lim_{q\,!\,0} e_L(q) = e_L(0) \quad \text{and} \quad \lim_{q\,!\,1} e_H(q) = e_H(1): \tag{J.60}$$