Convertible Procurement Contracts*

Oleksii Birulin and Joshua Wong†

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Abstract

We study a convertible procurement contract that combines two prevalent fixed-price and cost-plus contracts. After the project begins, the contractor may invest in more accurate cost information that can be further used as evidence to convert his initial fixed-price contract into a cost-plus contract. We find that contract conversion and the associated cost overrun occur whenever the contractor's cost distribution is spread-out enough. We also find that despite the presence of both ex-ante and ex-post adverse selection the procurer does not benefit from sequential screening – it is optimal to offer the same static convertible contract to all contractors.

Keywords: procurement, cost overrun, regulation, contract, fixed-price, cost-plus, evidence, sequential screening, renegotiation

JEL codes: D82, D86, L14, L23

1 Introduction

Procurement of goods and services is a major part of both public and private spending. Public procurement alone amounted to \$11 trillion out of global GDP of nearly \$90 trillion in 2018.[1] We focus on procurement of a complex project such as a customised new technology or a building. Due to the complexity of the project, both the procurer and the contractor are uncertain of the project's cost at the time of contracting.[2] This uncertainty usually cannot be resolved until long after the project has started. Obtaining accurate cost information often

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†School of Economics, Social Sciences A02, Sydney University, NSW 2006, Australia.
E-mail: oleksii.birulin@sydney.edu.au and josh.wong@sydney.edu.au

[1] https://blogs.worldbank.org/developmenttalk/how-large-public-procurement.

[2] Many of the factors that can influence costs are largely out of the contractor's control. These changes include changes in regulation, unforeseen contingencies, shocks to input prices and concurrent project requirements. Such costs cannot be contracted upon ex-ante due to the complexities of forecasting and measuring risk, and the difficulty of writing complete contracts.

requires costly additional audit and consultation. If the contractor obtains new information on the cost, he may choose to hide it or he may bring it forward and renegotiate the contract.[3]

The vast majority of real-world contracts are variants of fixed-price and cost-plus contracts.[4] A cost-plus contract requires 'evidence' on the project's cost and specifies a mark-up over that cost. A fixed-price contract sets the price and any deviation from that price requires costly arbitration or renegotiation. While a cost-plus contract exposes the procurer to all the cost-related risk, the contractor's limited liability means that a fixed-price contract exposes the procurer to costly potential renegotiation.

Our model incorporates both the initial cost uncertainty and the option of resolving this uncertainty by obtaining costly evidence. The contractor initially has coarse information on his cost – he only knows the cost distribution. This distribution is the contractor's ex-ante type. The contractor's ex-post type is his true cost, but to discover it he has to make a non-verifiable investment. The contractor starts on the fixed-price contract and later has an option of acquiring evidence on his cost. Upon obtaining such evidence the contractor can choose to hide it and remain on the original fixed-price contract, or present the evidence to the procurer and *convert* his contract into the cost-plus contract.

The procurer may not know neither the ex-ante, nor the ex-post type of the contractor. Her only controls are the price in the fixed-price option and the cost mark-up in the cost-plus option of the convertible contract. The procurer cannot influence the cost of evidence. The presence of this cost, however, gives the procurer the leverage over the contractors' evidence acquisition decisions. She may, for example, discourage some of the ex-ante types from acquiring evidence, while encouraging other ex-ante types to do so.

The convertible contract that we study is analogous to the real-world provisional sum contract. This is a hybrid of the fixed-price and cost-plus contracts, where a fixed price is quoted, but there is an option for that price to be updated if the contractor reveals further cost information.[5] To an 'empiricist' the outcome of a convertible contract is equivalent to the outcome of either a fixed-price or a cost-plus contract. Our work explains why these types of contracts coexist in procurement practice and clarifies how a procurer chooses between these two prevailing types of contracts.[6]

[3]Construction industry is our main application, but similar issues also arise in IT procurement, transport and logistics, outsourced R&D, etc.

[4]Both types of contracts are extensively used in private procurement of construction projects, large high-tech procurement projects and software. Public sector uses cost-plus contracts less frequently, perhaps for political reasons, to avoid favouritism and corruption allegations. See the prominent work of Bajari and Tadelis (2001) and the references therein and Tadelis (2012). Decarolis and Palumbo (2015), Decarolis et al. (2020) and especially Bajari et al. (2009), Bajari et al. (2014) and Jung et al. (2019) provide empirical analysis.

[5]Such price update can be also called 'adjustment of compensation'. A variation of the provisional sum contract is a contract with a clause for extra scope, also called 'adjustment via change order'. In the latter, the total cost of the project can increase (or decrease) because the design of the project is amended. The provisional sum contract allows for the change in the cost without the change in the design, to accommodate the unforeseen contingencies, input prices fluctuations, etc.

[6]The trade-off between the fixed-price vs. the cost-plus contract is often analysed as the comparison between auctioning off the fixed-price contract and bilateral negotiation of the cost-plus contract, see e.g.,

While cost overruns are frequently reported and can reach scandalous proportions, cost underruns are virtually unheard of.[7] Our setup offers an explanation of this pattern. Cost overruns are noticed only when the contractor suspects that the actual project's cost greatly exceeds the initial estimate.[8] Next, the contractor will report the cost overrun to the procurer only if it is worth triggering costly renegotiation. A revealed true cost is thus typically much higher than was anticipated. Cost underruns can also occur (both in our model and, we believe, in practice), but since the contractors are strategic, these are never reported and thus remain unnoticed.

We derive many insights from the model with a single ex-ante type, where both the procurer and the contractor know the cost distribution but are equally (un)informed about the realisation of the true cost ex-ante. We find that the procurer prefers to encourage evidence acquisition and possible contract conversion if and only if the contractor's cost distribution is spread-out enough.[9] This is consistent with procurement practice, where the components of the project with less cost uncertainty are procured on the fixed-price contracts, while at the same time, the components with more cost uncertainty are quoted as provisional sums.[10]

Every contract conversion results in a cost overrun, but not allowing them may be worse for the procurer. To avoid cost overruns the procurer has to leak the rents to the contractor ex-ante. As we show, these *uncertainty* rents are larger when the contractor's cost distribution is more spread-out. Encouraging evidence acquisition opens the door for a contract conversion and the associated cost overrun. Bearing it may, in fact, result in a lower expected cost for the procurer because it lowers the rents conceded to the contractor. Contrary to the narrative in the popular press, a cost overrun is not a signal of the procurer's incompetence, nor is it a sign of corruption.

Our model explains why contractors may be eager to invest into cost-gathering technology such as new accounting software that reduces their cost of acquiring evidence. It also explains why contractors may be reluctant to decrease their cost uncertainty. As we show, both of these make contract conversions and cost overruns more likely. Hence our model suggests that the digitization of the construction industry, promoted in many countries, may result in more frequent cost overruns. We also explain how lowering the contractors' liability limits may benefit the procurer, although as a byproduct, it encourages contract conversion and

Bajari et al. (2009) and Herweg and Schwarz (2018). The selection of the contractor and the trade-off between one contractor (in negotiation) vs. many potential contractors (in the auction) is then important. Our focus is, instead, on how the choice of the contract is affected by the (one and only) contractor's own cost uncertainty.

[7]Flyvbjerg et al. (2002) report that in 90% of public infrastructure projects the cost exceeds the tender price, with the average overrun being 28%. A European Commission Report (2009) focusing on the rail, road, urban transport, water and energy sectors finds that project costs on average exceed the estimates by 21%. Kostka (2016) provides a cross-sectoral analysis of cost overruns in Germany.

[8]Equivalently, the contractor may be exerting costly effort on discovering the true cost of the project, and for tractability we reduce this to a fixed cost of effort.

[9]This is confirmed by the empirical studies, see Section 2 and especially Section 4.3 for further details.

[10]The variation in the total costs can be due either to the volatility of the per unit costs, or the difficulty with predicting the required quantities of the inputs. Currently, as supply chain crisis leads to large volatility in the costs of the materials, these are more likely to be quoted as provisional sums.

may lead to more cost overruns as well.

We further introduce asymmetric information at the ex-ante stage. There are two ex-ante types of the contractor, and the cost distribution of one of the types is a mean-preserving spread of the cost distribution of the other type. This setup is related to the sequential screening problem. In a sequential screening contract, agents are screened for their type at multiple stages, i.e., first for their ex-ante type then for their ex-post type. The menu of contracts offered ex-post may depend on the ex-ante types revealed earlier. We show that such complicated sequential screening does not benefit the procurer. Whatever outcome can be achieved by the sequential screening, can also be achieved by a much simpler *static* screening contract, where both ex-ante types are offered the same convertible contract.

Looking for the optimal contract the procurer now minimizes a combination of the usual informational rents with the uncertainty rents. Peculiarly, which of the ex-ante types receives an informational rent in its own turn depends on the contract the procurer intends to induce. Depending on the parameters, the procurer may offer a static pooling contract that discourages both ex-ante types from acquiring evidence, or conversely, encourages both types to acquire evidence. She may also prefer a separating equilibrium where only the higher variance type acquires evidence. There, only a fraction of the higher variance type actually converts their contract. Importantly, the mere presence of the lower variance type (who does not acquire evidence in the separating equilibrium) affects what fraction of the higher variance type converts their contracts. Specifically, ex-ante informational asymmetry lowers the fraction of contract conversions, which in turn lowers the frequency of the reported cost overruns.

The rest of the paper is structured as follows. Section 2 reviews the related literature. Section 3 sets up the model. Section 4 characterizes the optimal contract with a single ex-ante type. Section 5 studies the optimal (static) contract under ex-ante asymmetric information. Section 6 presents policy implications, and Section 7 some directions for future research. Proofs are relegated to the Appendix, which includes a characterization of the optimal sequential screening contract.

2 Related Literature

Bajari and Tadelis (2001) also stress the prominence of cost-plus and fixed-price contracts, but their emphasis is on the cost of specifying the project's design ex-ante. The procurer prefers to provide a more complete design for a simpler project and procures it on a fixed-price contract. A more complex project receives a less complete design specification and is procured on a cost-plus contract. The source of the cost overrun is the *adaptation cost*, i.e., the cost of adapting the project's design after the works have commenced. In a similar vein, in Herweg and Schwarz (2018) the procurer benefits from soliciting quotes for the basic design first, and then renegotiates with the winner towards a more expensive design. Similar post-allocation extension of project scope is studied in Huang et al. (2021). Herweg and Schmidt (2017) explores contractors' incentives to come up with design improvements and how these affect the choice of the procurement method (an auction or bilateral negotiation). Bajari et al. (2014) use the data on highway repair contracts and estimate the adaptation costs to be 7.5 - 14 percent of the initial project cost. At the same time, Flyvbjerg et al. (2002) and other literature in footnote 7 estimate the average cost overruns in infrastructure projects to be much larger, 21 - 28 percent of the initial cost estimate. We believe that informational advantage of the contractors and the associated rents can be partly responsible for the portion of the cost overruns 'not explained' by the adaptation costs, and hope that our approach is well suited for quantifying these rents. Furthermore, cost overruns are not connected with design upgrades in our setting. Instead, a conversion to the cost-plus contract and the associated cost overrun is a result of the procurer's optimal response to the uncertainty about the cost and her own informational disadvantage.

Our approach emphasises both asymmetry of information and how this asymmetry accumulates gradually as the project progresses. While the contractor may have no superior information at the project's commencement, he can, perhaps with some investment, improve his knowledge of the cost once the project begins. Laying out this new information as the evidence of the true cost and triggering renegotiation is within the contractor's rights. By offering a contract with a conversion option the procurer attempts to harness the process of information acquisition to reduce the informational advantage of the contractor. Jung et al. (2019) empirically confirm that having a price adjustment mechanism in place results in savings to the procurer. They use procurement auctions data and their 'source' of the savings is different; contractors bid more aggressively, i.e. lower, when there is a price adjustment mechanism in the procurement contract.

In Laffont and Tirole (1987) and McAfee and McMillan (1986) the contractor can lower the project's cost by exerting costly effort. This effort is observable to the procurer and the contract aligns the contractor's compensation with the observed effort, hence it is always of the cost-plus variety. Cost-plus and fixed-price contracts provide different incentives for the contractor's cost reduction effort. This aspect is central to the adaptation cost literature mentioned at the beginning of this Section. In our model the contractor cannot affect the project's true cost. His choices are whether to acquire evidence on that cost and whether to reveal this evidence to the procurer. Despite this, the choice of the procurement contract affects the expected cost of the project from the standpoint of the procurer.

The literature on dynamic buyer-seller interaction with gradual information accumulation typically considers sales. In models of sales with refunds in Matthews and Persico (2005) and Matthews and Persico (2007) buyers can either acquire information and learn their values prior to purchase or buy the product and learn their values ex-post. Unlike in our model, buyers there always learn their true value via one of the above channels. The seller can design the process by which the buyer learns his value in Heumann (2020), but learning is costless hence the buyer always knows his true value before opting out of the sale contract. The feature that some ex-ante types may decide to learn their true cost, but others may choose to stay uninformed, is unique to our procurement model.

In our setup, if a contractor wants a better estimate of his cost, he has to invest in the new information rather than receive it for free simply as time passes. A complex project often involves tens of thousands of individuals working across different departments and on concurrent projects. It is reasonable to assume that there is a sizeable cost to estimating, collecting and combining the cost information for a given project.[11] The size of that cost is an exogenous parameter of the model that reflects the cost-gathering technology of contractors.

Control variables in our model also differ from the sales models. The procurer here faces an exogenous pressure to complete the project, whereas sellers do not have to sell to every potential customer.[12] Thus, in the sales models the seller explores the trade-off between the sale price and the probability of sale. Our procurer, purchases the project for sure, and instead, operates on the margin between the fixed price available to the contractor ex-ante and the mark-up in the cost-plus contract available to the contractor ex-post.

Courty and Li (2000) pioneered the work on sequential screening in a setting with ex-ante and ex-post types as here. Their optimal contract includes the menu of choices offered to the ex-ante types of the buyers, followed by further options at the ex-post stage. Which options are available ex-post may depend on the choices the buyers have made earlier. Krähmer and Strausz (2015) show that adding ex-post withdrawal rights to such model can restrict the ex-post options and as a result, as in our model, the static contract (a single contract offered to all ex-ante types) is optimal. Bergemann et al. (2020) provides an alternative analysis of such sales model with ex-post withdrawal rights.

The possibility to withdraw from sales contract ex-post is to an extent similar to the option of bringing the evidence to the procurer in our model. The difference is that withdrawal from the sales contract in Krähmer and Strausz (2015) or Bergemann et al. (2020) results in an *exogenously* given payoff, the same for every ex-ante and ex-post type of the buyer. In our model, the payoff from presenting evidence is controlled by the procurer. It is also the same for every ex-post type who presents his evidence, but serves as a lower bound on the expected payoff of the ex-ante type that chooses not to acquire evidence.[13]

Our concept of evidence is inspired by evidence games in Hart et al. (2017) and Ben-Porath et al. (2019) and research on voluntary disclosure, however, our motivation is different. As in these works, evidence is private information that the agent can choose to make public or hide. Importantly, in their models evidence is presented at the early stage with the aim of influencing the allocation. In Mylovanov and Zapechelnyuk (2017) evidence may be used after the allocation to assign a penalty for misreporting earlier. In our model evidence can only be acquired after the allocation and can only be used to renegotiate.

[11] Our private communications with construction industry experts and contracts administrators suggest the following. Even a small construction firm is a complex organization with employees and sub-contractors often involved in several projects at once. Gathering precise cost information for a given project requires time and effort. If contractors do not suspect that the project's cost departs too much from the initial estimate, they often prefer to stay uncertain and just finish the job risking running a loss. They can simply recoup potential losses on future projects or other components of the project.

[12] Elected officials can lose office if they fail to deliver or face major delays on the public procurement initiatives. Private households also feel the urge to complete construction projects once they begin.

[13] A more detailed comparison of our work with Krähmer and Strausz (2015) and Bergemann et al. (2020) is at the end of Section 5.

3 The Model

A procurer (she) acquires a single project from a contractor (he). The project needs to be completed and the cost of switching to another contractor is such that it is always in the procurer's interest to complete the project with the current contractor. Both parties are risk-neutral and have quasi-linear utility functions. Contractor i's *ex-post* type is c_i, his true cost of completing the project. Neither party knows the true cost before the project begins. Contractor i's *ex-ante* type is the cumulative distribution F_i of his c_i.[14] The contractor knows his ex-ante type. From the procurer's point of view there are two possible ex-ante types $i = H, L$ with the same *mean* cost μ but different (finite) variances of their cost distributions. More specifically, F_H is a mean-preserving spread of F_L.[15] The procurer attaches prior probabilities $\mathsf{Pr}(i = H) = \alpha$, and $\mathsf{Pr}(i = L) = 1 - \alpha$ to the two ex-ante types.

Converting the original fixed-price contract into the cost-plus type is a renegotiation and thus incurs a penalty. As the procurer designs the contract, she specifies the level of that penalty, p, and can use it to discourage renegotiation.[16] For simplicity, we only consider a constant mark-up over the cost, m, and a constant penalty, p. If the original contract is converted to the cost-plus, the contractor who reveals cost c_i receives $c_i + m - p$. The cost of the procurer correspondingly changes from q in the original fixed-price contract to $c_i + m - p$ after the contract conversion. A **convertible contract** is a pair (q, w) which sets a fixed price q and a (normalised) punishment $w := p - m$ for renegotiation. To reflect the contractor's limited liability, the punishment is limited, $p \leq \bar{p}$, or equivalently $w \leq \bar{w}$. The limit $\bar{w}$ is outside of the parties control and reflects, in particular, the legal protections and codes of the region.

The timing of events is presented on Figure 1. At the inception of the project the contractor only knows his ex-ante type and starts on a fixed-price contract that (in absence of further actions) would pay him q upon completion. After the project begins the contractor can choose to acquire evidence at cost e and learn his ex-post type c_i. The size of e is fixed exogenously, and the same e applies to every contractor. If the contractor acquires evidence, he has a further choice of whether to reveal his true cost or hide it. If the evidence is hidden, the contractor stays on the original fixed-price contract. If the contractor reveals his evidence on c_i to the procurer, the contract is converted into the cost-plus contract that pays $c_i - w$ to the contractor upon completion.

[14] The domain of F_i is $[0, \infty)$. To simplify the exposition we assume that F_i has no mass points and admits density f_i, strictly positive on the support $S \subseteq [0, \infty)$. Any of these assumptions can be relaxed, this will affect the illustrations and make the arguments heavier, but will not change our results qualitatively.

[15] See e.g., p. 262-263 in Krishna (2002) for the definition and useful properties. The common mean assumption highlights the role of the contractor's uncertainty about his own cost. It can be relaxed to second-order stochastic dominance at the expense of heavier notation, but this will not affect the results qualitatively.

[16] In practice both the contractor and the procurer may also suffer other monetary and non-monetary costs that result from arbitration and expert fees, psychological effects of arguments and delays, etc. As long as there are exogenously fixed, they can be incorporated in the model without changing our results qualitatively.

The contractor chooses whether to acquire or not acquire evidence based on which of these options promises him a higher expected payoff. Suppose contractor i is offered the contract (q, w). If he does not acquire evidence (NE), i's payoff is,

$$U^i_{NE}(q, w) = \mathsf{E}_c[q - c_i] = q - \mu, \tag{1}$$

where μ is the unconditional mean of the distribution with c.d.f. F_i.[17]

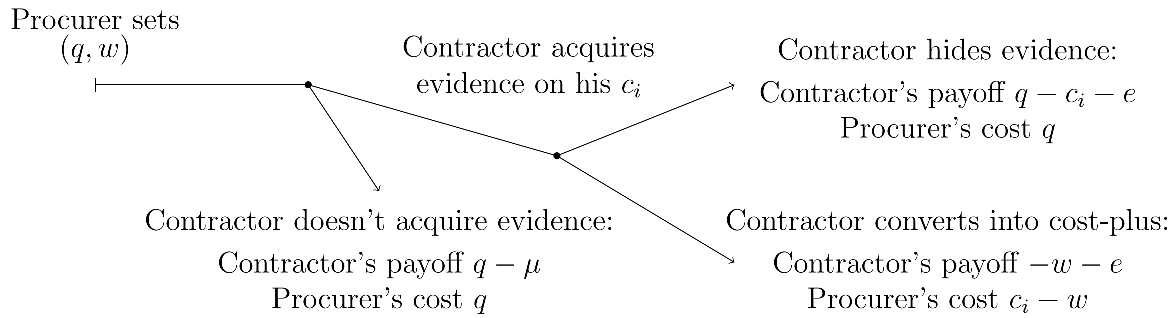


Figure 1: Timeline of the Convertible Contract

Now consider contractor i who is offered the same contract but acquires evidence and learns his ex-post cost c_i. By revealing the discovered c_i to the procurer, contractor i obtains the payoff of $-w$ upon completion. Hiding the evidence delivers i the payoff of $q - c_i$. Therefore, upon investing e into evidence, in expectation contractor i obtains payoff $U^i_E(q, w) = \mathsf{E}_c[\max\{q - c_i, -w\}] - e$. To facilitate the comparison with $U^i_{NE}(q, w)$ introduce

$$G_i(x) := \int_x^\infty \bar{F}_i(c)\, dc, \tag{2}$$

where $\bar{F}_i := 1 - F_i$. The mean-preserving spread assumption implies that $G_H(x) \geq G_L(x)$, for any $x > 0$. Integration by parts and some algebra provides

$$U^i_E(q, w) = \mathsf{E}_c[\max\{q - c_i, -w\}] - e = G_i(q + w) + q - \mu - e. \tag{3}$$

The ex-post incentive compatibility constraints govern which ex-post types will reveal their evidence. These are incorporated into the expected payoff $U^i_E(q, w)$. The ex-ante incentive compatibility constraints ensure that the contractors act as prescribed for their ex-ante types. In our context the procurer may provide certain ex-ante types with incentives to obtain evidence, but others to remain uninformed of their ex-post cost.

Even though contractors are risk-neutral, the payoff of those who learn their true cost is not linear in c_i. The option of converting the contract into the cost-plus effectively truncates the potential losses from below and makes the contractor's payoff convex in c_i. Hence the contractors with the same mean but different spreads of their cost distributions can make

[17] Contractor i's actual cost may differ from μ, but he will never know this without acquiring evidence.

different evidence acquisition decisions when facing the same contract (q, w). We further call these ex-ante IC constraints the *evidence acquisition* constraints. The EA constraint ensures that the payoff from the option where evidence is acquired is higher. The NEA constraint ensures the opposite. Given (1) and (3) these can be stated succinctly as,

$$e \geq G_i(q + w), \tag{NEA}$$
$$G_i(q + w) \geq e. \tag{EA}$$

When presented with the contract (q, w), contractor i anticipates whether he will later acquire evidence and participates if and only if the relevant expected payoff exceeds his outside option, normalised to zero. The same (1) and (3) then provide the corresponding *individual rationality* (IR) constraints - with the evidence (EIR) and without it (NEIR),

$$q \geq \mu, \tag{NEIR}$$
$$G_i(q + w) + q \geq e + \mu. \tag{EIR}$$

Limited liability (LL) ensures that the punishment $w \leq \bar{w}$.

The objective of the procurer is to minimize her expected cost subject to the above (N)EA, (N)EIR and LL constraints. The procurer may, in principle, intend to screen ex-ante and offer different contracts (q_H, w_H), and (q_L, w_L) to different ex-ante types. She commits to the contract terms. No such commitment is required from the contractors, they behave optimally at every stage: deciding on participation, acquiring evidence and deciding on whether to reveal or hide it.

The Unconstrained Outcome and the Variance

To set a benchmark, we relax the LL constraint, effectively allowing for $\bar{w} = \infty$. The procurer can then discourage the contractor from acquiring evidence while setting $q = \mu$. Intuitively, this is ideal for the procurer as she concedes no rent to the contractor and bears no risk of renegotiation. We further call the outcome where the contractor does not acquire evidence and receives $q = \mu$ the *unconstrained outcome*. When the cost of evidence is sufficiently high, $e \geq G_i(\mu)$, the procurer can set $q = \mu$ and discourage evidence acquisition without any punishment. To rule out this trivial case we further assume $e \leq G_i(\mu)$, for $i = H, L$.

Definition 1 (w_0^i)**.** *Let w_0^i be the minimal punishment needed to induce the unconstrained outcome. The contract (μ, w_0^i) satisfies both the NEA and the NEIR constraints of contractor i with equality,*

$$e = G_i(\mu + w_0^i). \tag{4}$$

Since $G_i(\mu + w)$ is non-increasing, and $G_H(\mu + w) \geq G_L(\mu + w)$, the minimal punishment needed to induce the unconstrained outcome must be higher for the type whose cost distribution is more spread-out, $w_0^L \leq w_0^H$. With this in mind, we further call w_0^i the *variance* of a contractor with ex-ante type i.

4 A Single Ex-ante Type

In this Section the contractor's ex-ante type, (but not his ex-post type) is commonly known. We first derive the optimal contracts in two scenarios: i) where the contractor is induced to acquire evidence on his ex-post cost and ii) where such evidence acquisition is discouraged. We then compare the resulting expected costs to formulate the conditions under which the procurer prefers to induce each scenario.

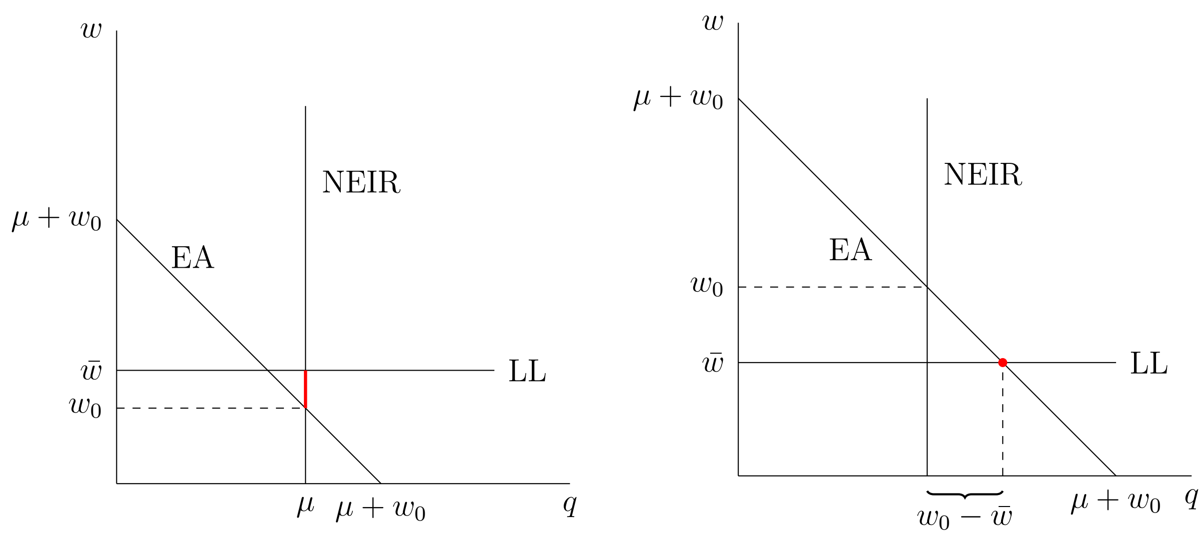
4.1 Evidence is Not Acquired

Suppose the contractor does not acquire evidence and introduce the corresponding expected cost of the procurer, $P_{NE} := \mathsf{E}_c[P \,|\, \text{evidence not acquired}] = q$. The procurer solves:

$$\begin{aligned} \min_{q,w}\ & P_{NE} = q & \\ \text{s.t. } & q \geq \mu & \text{NEIR} \\ & e \geq G(q+w) & \text{NEA} \\ & \bar{w} \geq w & \text{LL} \end{aligned}$$

Figure 2 presents the constraints with the controls q and w on the axes. The contracts (q, w) such that $q + w = G^{-1}(e)$ are on the the EA line, hence its slope is -1 and the intercept is $\mu + w_0$. The NEIR line is vertical at $q = \mu$ and the LL line is horizontal at $w = \bar{w}$. To satisfy the constraints the procurer needs to set (q, w) such that the contract is below the LL line, above the EA line, and to the right of the NEIR line. When $w_0 \leq \bar{w}$, the intersection of the EA and NEIR lines is below the LL line, see the left panel of Figure 2, and the unconstrained outcome is attainable, recall Definition 4. The procurer then sets some $q = \mu$ and w such that the contract is on the NEIR line, shown by the bold red line on the left panel. The contractor's rent is zero.

The spread of the distribution affects w_0 and thus the relative position of the EA line. When $w_0 > \bar{w}$, the intersection of the NEIR and EA lines is above the LL line, see the right panel of Figure 2. The LL constraint is now binding and the procurer then sets $q = q^*$ as low as possible, such that both EA and LL constraints bind, but the NEIR constraint is slack. At the optimum, $G(\mu + w_0) = e = G(q^* + \bar{w})$ which implies $q^* = \mu + (w_0 - \bar{w})$. The procurer leaves rent $q^* - \mu = w_0 - \bar{w}$ to the contractor to discourage him from acquiring evidence. This *uncertainty* rent scales up with the contractor's variance, as more own cost uncertainty makes the contractor more inclined to acquire evidence.



(a) $w_0 \leq \bar{w}$ (b) $w_0 > \bar{w}$

Figure 2: Incentive and participation constraints when evidence is not acquired

Lemma 1. *Suppose the contractor is discouraged from acquiring evidence.*

i. *If* $w_0 \leq \bar{w}$*, the procurer sets* $q^* = \mu$ *and some* $w^* \in [w_0, \bar{w}]$.

ii. *If* $w_0 \geq \bar{w}$*, the procurer sets* $q^* = \mu + (w_0 - \bar{w})$ *and* $w^* = \bar{w}$.

The contractor stays on the fixed-price contract. He receives uncertainty rent

$$U^*_{NE} = (w_0 - \bar{w})_+ := \max\{(w_0 - \bar{w}), 0\}.$$

The procurer's expected cost is $P^*_{NE} = \mu + U_{NE}(q, w)$.

4.2 Evidence is Acquired

Now suppose the contractor is induced to acquire evidence. The ex-post types with $c_i \geq q+w$ reveal their evidence to the procurer and receive a negative ex-post payoff $-w$. The ex-post types with $c_i < q + w$ strategically hide their evidence and receive the ex-post payoff $q - c_i > -w$. The procurer's controls q and w and can determine which ex-post types will reveal their evidence, but this threshold structure applies for any (q, w). The procurer's ex-post cost is $P = c_i - w$ when the evidence is revealed, and $P = q$ when it is hidden. Given the contractor reveals his evidence strategically, $P = \max\{c_i - w, q\}$. Using (3) the expected cost of the procurer can be written as $P_E := \mathsf{E}_c[P \mid \text{evidence acquired}] = U_E + \mu + e = G(q+w) + q$.

The procurer therefore solves:

$$\begin{aligned}
&\min_{q,w} P_E = G(q+w) + q \\
&\text{s.t. } G(q+w) + q \geq e + \mu && \text{EIR} \\
&\quad\;\; G(q+w) \geq e && \text{EA} \\
&\quad\;\; \bar{w} \geq w && \text{LL}
\end{aligned}$$

Figure 3 presents these constraints. The EA and LL lines are as before. The EIR curve is now convex.[18] By definition of w_0 the EIR curve intersects the EA line at (μ, w_0). This intersection is above the LL line implying that the EIR curve intersects the LL line to the left of $q = \mu$.[19] The procurer minimizes $G(q+w) + q$ in the region below the EA, EIR and LL curves. She optimally reduces the contractor's expected payoff until the EIR constraint binds. Any contract on the EIR curve that satisfies the other constraints minimises the procurer's expected cost. Such contracts are shown on Figure 3 in bold.

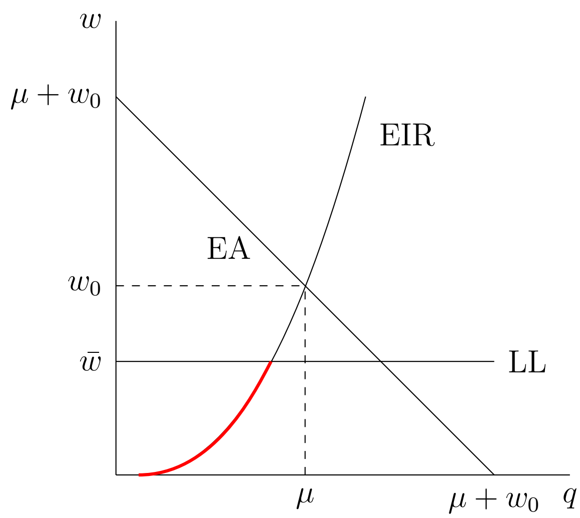


Figure 3: Incentives and participation constraints when evidence is acquired

Lemma 2. *Suppose the contractor is encouraged to acquire evidence. The procurer then offers a contract $q^* < \mu$, $w^* \leq \bar{w}$ such that the EIR constraint binds, $G(q^* + w^*) + q^* = e + \mu$. The procurer's expected cost is $P_c^* = \mu + e$ and the contractor receives an expected payoff of zero. Contractors with ex-post cost $c < q^* + w^*$ hide their evidence and stay on the fixed-price contract, those with $c \geq q^* + w^*$ present their evidence and convert their contract to the cost-plus.*

[18]It has a slope of $F(q+w)/\bar{F}(q+w) > 0$ and increasing in q and a positive intercept with the q-axis. At $q = w = 0$, $G(0) = \mu$, which implies that for all $w \geq 0$ we must have $q > 0$ to satisfy the EIR.

[19]If not, we have $w_0 \leq \bar{w}$, the unconstrained outcome would be attainable and the contract that induces evidence acquisition would not be optimal.

A contract conversion increases the procurer's cost from the original q^* to $c_i - w^*$. Since only the contractors with $c_i \geq q^* + w^*$ request a contract conversion, every conversion from a fixed-price to the cost-plus contract here and further in the text leads to a cost overrun. The procurer may be willing to allow these cost overruns since precluding them requires conceding the uncertainty rent to the contractor.

4.3 The Optimal Contract

One one hand, to discourage the contractor from acquiring evidence the procurer has to concede him the uncertainty rent $(w_0 - \bar{w})_+$. This rent is larger when the cost distribution is more spread-out. On the other hand, by encouraging the contractor to acquire evidence, the procurer reduces the contractor's payoff to zero, but also decreases the total expected surplus by the evidence cost e. Following Lemma 1, by discouraging evidence acquisition, the procurer reduces her expected cost to $P_n^* = \mu + (w_0 - \bar{w})_+$. The alternative, where the evidence is acquired, delivers her the expected cost of $P_c^* = \mu + e$, Lemma 2. For a given e and $\bar{w}$, the contractor's variance w_0 ultimately determines the optimal contract. The contractor is offered a contract that induces him to acquire evidence if and only if $e \leq (w_0 - \bar{w})$.

Proposition 1. *Suppose the procurer knows the contractor's ex-ante type.*

i. *If $(w_0^i \leq \bar{w})$, the procurer discourages the contractor from acquiring evidence. The procurer's expected cost is $P_{NE}^* = \mu$ and the contractor's expected payoff is zero.*

ii. *If $(w_0^i \in [\bar{w}, e + \bar{w}])$, the procurer discourages the contractor from acquiring evidence. Her expected cost is $P_{NE}^* = \mu + (w_0^i - \bar{w})$ and the contractor's expected payoff is the uncertainty rent $(w_0^i - \bar{w})$.*

All the above types stay on the fixed-price contract.

iii. *If $(w_0^i \geq e + \bar{w})$, the procurer encourages the contractor to acquire evidence. The procurer's expected cost is $P_E^* = \mu + e$ and the contractor's expected payoff is zero. Only the ex-post types with $c \leq q^* + w^*$, as determined in Lemma 2, stay on the fixed-price contract. The higher ex-post types convert their contracts to the cost-plus.*

Bajari and Tadelis (2001) associate cost-plus contracts with complex projects that have designs that are more costly to specify ex-ante. We use an alternative interpretation of what a complex project is. We emphasize how the procurer's choice of the contract is affected by the cost uncertainty, and suggest that projects for which the contractor's cost uncertainty is high are more likely to be procured on cost-plus contracts. Bajari et al. (2009), to our knowledge, provide the most direct empirical test of the use of fixed-price and cost-plus contracts. The project complexity in their data is proxied by (the log of) the project dollar value, (the log of) the project floor area, and the contractor's number of divisions. In summary, a complex project in Bajari et al. (2009) is a large project, their regressions include the above proxies in linear terms, hence they essentially test the hypothesis that cost-plus contracts are more likely to be used for larger projects.

We stress that a complex project, or equivalently a large project, is also a project with a large cost uncertainty. With this interpretation in mind, Bajari et al. (2009) also confirms our hypothesis that cost-plus contracts are more likely to be used on the projects with large cost variation. Jung et al. (2019) provide a list of items in the procurement contracts with the highest frequency of the change orders or price adjustments. Their data includes highway repair contracts auctioned off at fixed prices. That list confirms our insight; items with higher cost uncertainty are more likely to be the subject of contract renegotiation.

More pointedly, Krasnokutskaya and Seim (2011) analyse the effects of bid preference programs in procurement. One of their aims is to empirically recover cost distributions of large and small bidders in tenders for highway repair contracts. Krasnokutskaya and Seim (2011) estimation favors the hypothesis that small bidders have lower variance of the cost distribution than that of the large bidders. In addition, Bajari et al. (2009) estimation finds that larger contractors, measured for example, by the number of the divisions in their firms, are more likely to be employed on cost-plus contracts. These two results taken together are clearly aligned with our conclusions in Proposition 1.

That Proposition is also consistent with procurement practice, where the inputs that have large variations in their total cost are often quoted as provisional sums, or are subjected to a change order or an adjustment of compensation. At the same time, the inputs that have less variable total costs are quoted at fixed prices. The cost variation can be due either to the volatility of the per unit cost, or the difficulty with predicting the required quantity of the input. Recently, as supply chain crisis leads to large volatility in the costs of the materials, they are more likely to be quoted as provisional sums. The stylized fact that larger projects experience larger and more frequent cost overruns is also consistent with Proposition 1.[20]

Prior research suggests that cost overruns are associated with project design changes.[21] As an alternative explanation, our work suggests that the procurer may prefer to bear cost overruns to reduce the informational advantage the contractor can choose to accumulate. Cost overruns may occur in equilibrium when there is enough uncertainty about the cost of the design, or the cost of implementing that design on a particular site, even if the design per se is unaltered.[22]

[20]See the report by the Grattan Institute https://grattan.edu.au/news/34b-and-counting-beware-cost overruns-in-an-era-of-megaprojects/.

[21]See e.g., Herweg and Schmidt (2017), Herweg and Schwarz (2018) and Huang et al. (2021).

[22]In Herweg and Schmidt (2020) the contractor(s) are informed about the project's design flaws before the contract is signed, but may prefer to conceal such information. In practice, such behavior may be playing a role as well. Unlike ours, such model does not emphasize the spread of the cost distribution, hence it is harder to relate to the empirical evidence in this Section.

5 Private Information at the Ex-ante Stage

In this Section the contractor is privately informed about his ex-ante type. From the procurer's perspective there are two possible contractor's types $i = H, L$ with prior probabilities $\mathsf{Pr}(i = H) = \alpha$, and $\mathsf{Pr}(i = L) = 1 - \alpha$. The H type's cost distribution is more spread-out than the L type's cost distribution, hence $G_H(q + w) \geq G_L(q + w)$, for any (q, w). As a result, $w_0^H > w_0^L$, and the H type has a greater incentive to acquire evidence. This Section focuses on the *static* contract, where both types are offered the same contract (q, w). This greatly simplifies the exposition but does not reduce the generality. Proposition 2 shows that the static contract allows to achieve the same outcome as the optimal sequential screening contract. For a characterisation of that sequential screening contract see Appendix C.

The procurer can potentially induce three scenarios: neither type acquires evidence, both types acquire evidence, and the separating equilibrium where only one of the types acquires evidence. We first examine the optimal static contract for the separating equilibrium. As the H type is more inclined to acquire evidence, in such equilibrium the H type makes the corresponding investment and the L type does not. Introduce the corresponding expected cost, $P_{SE} := \mathsf{E}_c[P \,|\, \text{only the } H \text{ type acquires evidence}]$. The procurer solves:

$$\begin{aligned}
&\min_{q,w} P_{SE} = q + \alpha G_H(q + w) && \\
&\text{s.t. } q \geq \mu && \text{NEIR}^L \\
&\qquad G_H(q + w) + q \geq e + \mu && \text{EIR}^H \\
&\qquad G_H(q + w) \geq e \geq G_L(q + w) && \text{EA}^H,\ \text{NEA}^L \\
&\qquad \bar{w} \geq w && \text{LL}
\end{aligned}$$

Figure 4 presents the constraints under the assumption that $w_0^H > \bar{w}$, as otherwise the unconstrained outcome is feasible and the separating contract cannot be optimal. The L type does not acquire evidence, hence his NEIR constraint is vertical, as in Section 4.1. The H type acquires evidence, hence his EIR constraint is convex, as in Section 4.2. Since $\mu_0^H > \mu_0^L$ the EA constraint for the H type is parallel and above the the EA constraint for the L type. Also, since $G_i(\mu + w_0^i) = e$, EIR^i and EA^i always intersect at the $q = \mu$ vertical line with $w = w_0^i$ at the intersection.

The procurer optimally offers the L type's first-best contract $(q^*, w^*) = (\mu + (w_0^L - \bar{w})_+, \bar{w})$, described in Section 4.1, to both ex-ante types. Here and further in the text when we refer to a first-best contract we mean one of the optimal contracts from Sections 4.1 and 4.2 where the procurer knows the ex-ante type of the contractor. The optimal contract (q^*, w^*) discourages the L type from acquiring evidence. Whether the L type receives a rent depends on his own variance. If $w_0^L < \bar{w}$, then the NEIR and the LL constraints bind for the L type and his rent is zero. This case is on the left panel of Figure 4. The case with $w_0^L > \bar{w}$ is on the right panel of Figure 4. There the NEA and the LL constraints bind for the L type and his uncertainty rent U_{NE}^L is given by the horizontal distance between the optimal q^* and μ, as shown on the right panel on Figure 4.

The *same* contract (q^*, w^*) induces the H type to acquire evidence, and regardless of the variance of the L type, the H type receives an informational rent. The magnitude of this rent U_E^H is proportional to the horizontal distance between the optimal q^* and the intersection of the EIR^H and LL constraints. This rent is shown on the left panel on Figure 4, but of course the H type's always collects such rent in the separating equilibrium. Moreover, the H type's (informational) rent increases with the variance of the L type, and the H type's rent always exceeds the L type's (uncertainty) rent.

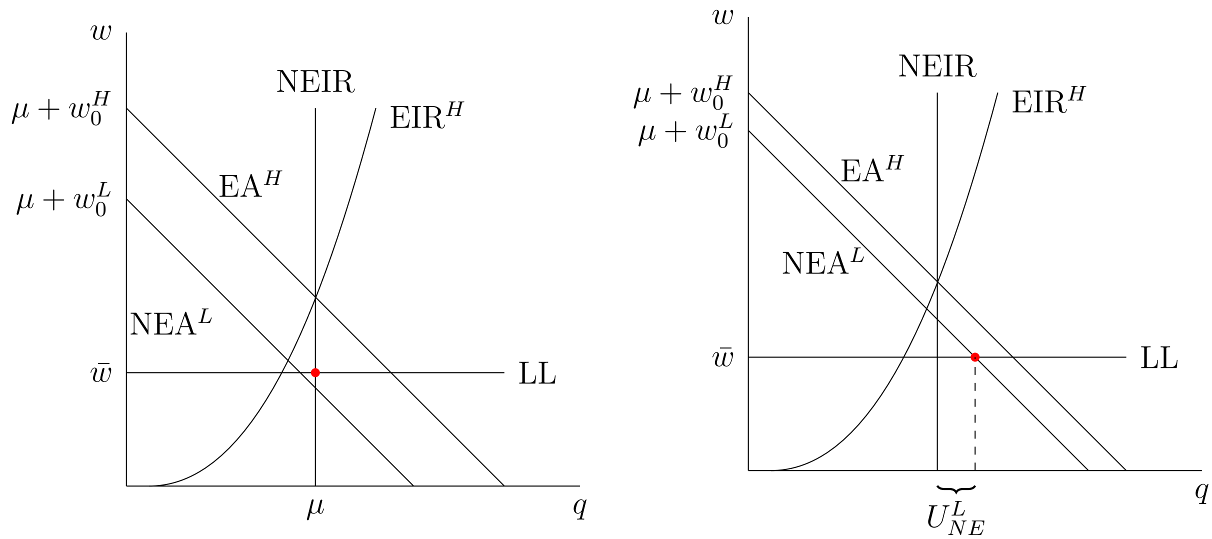


Figure 4: Incentive and participation constraints in the Separating Equilibrium

Lemma 3. *In a* ***separating equilibrium*** *only the H type is encouraged to acquire evidence. The procurer optimally offers the L type's first-best contract,*

$$(q^*, w^*) = (\mu + (w_0^L - \bar{w})_+, \bar{w}),$$

to both ex-ante types. The L type receives an uncertainty rent of $(w_0^L - \bar{w})_+$, while the H type receives an informational rent of $G_H(q^* + w^*) + (w_0^L - \bar{w})_+ - e$. Only the ex-post types $c \geq q^* + w^*$ of the H type convert their fixed-price contract to the cost-plus contracts.

Importantly, only a fraction of the H type contractors convert their contract into the cost-plus, but the presence of the L type affects that fraction. It is always the case that the contractors who learn that their ex-post types $c \geq q^* + w^*$ convert the contract. When the L type is not present, the optimal contract that prompts the H type to acquire evidence is on that type's EIR constraint below the LL constraint and to the left of μ, recall Figure 3. Suppose now the L type is present, but does not acquire evidence, as in the separating equilibrium in Lemma 3. Then the optimal contract for the H type is on the L type's EA constraint, to the right of μ, as on Figure 4. Regardless of the underlying variance of the

L type, the value of $q^* + w^*$ at the optimal contract is higher when the L type is present, which corresponds to a lower fraction of the ex-post types with $c \geq q^* + w^*$. In summary, the procurer accommodates the ex-ante informational asymmetry by lowering the fraction of contract conversions, which lowers the frequency of reported cost overruns. Therefore, the absence of cost overruns may be a result of the (ex-ante) informational disadvantage of the procurer.

In practice, procurers are often committed to the completion of a project, e.g., a city mayor may pledge to complete an infrastructure project by the next election cycle. This commitment is incorporated in our model. In the absence of such commitment, the procurer could potentially offer a contract that would preclude participation of certain types and trade-off the expected cost vs. the probability of completion. Such setup would be close to a sales contract where a seller ignores the 'bad' types when there is enough 'good' types in the population. Returning to the above separating equilibrium, pledging to complete for sure can be costly in itself. Suppose that both the variance of the H type and their fraction in the population are high. If the procurer can ignore the L types, i.e., offer a contract that they will reject, she could extract all of the H type's informational rents by encouraging him to acquire evidence as in Section 4.2.

In addition to the separating equilibrium above, depending on the parameters of the model, the procurer may benefit from encouraging both ex-ante types to acquire evidence, or discouraging both types from doing so. The analysis of such cases is similar to Sections 4.1 and 4.2 above. We describe the corresponding optimal static contracts in the following two Lemmas (with the proofs in Appendix B) and then comment.

Lemma 4. *Suppose* ***both types are discouraged from acquiring evidence****. The procurer offers both ex-ante types the H type's first-best contract as in Section 4.1*

i. If $(w_0^H \leq \bar{w})$, then $q^* = \mu$ and $w^* \in [w_0^H, \bar{w}]$. Both ex-ante types receive an expected payoff of zero.

ii. If $(w_0^H \geq \bar{w})$, then $q^* = \mu + (w_0^H - \bar{w})$ and $w^* = \bar{w}$. Both ex-ante types receive an uncertainty rent of $(w_0^H - \bar{w})$.

All of the types remain on the fixed-price contract.

Lemma 5. *Suppose* ***both types are encouraged to acquire evidence****. The procurer offers both ex-ante types a static contract such that the L type's EIR binds and the H type's rents are minimised. Formally, she sets the contract (q^*, w^*) that solves*

$$\min_{q \leq \mu, w \leq \bar{w}} U^H = G_H(q+w) - G_L(q+w) \tag{5}$$
$$s.t. \quad G_L(q+w) = e + \mu - q$$

The H type receives an informational rent of $U^H(q^*, w^*) = G_H(q^*+w^*) - G_L(q^*+w^*)$, while the L type receives an expected payoff of zero. With either ex-ante type, the ex-post types

$c \geq q^* + w^*$ convert their fixed-price contract to the cost-plus contract. The lower ex-post types remain on the fixed-price contract.

Which equilibrium the procurer prefers to induce depends on the parameters. Consider first the separating equilibrium in Lemma 3 and the pooling equilibrium in Lemma 4. Start with $w_0^H > w_0^L > \bar{w}$. Then the expected cost in the separating equilibrium is $P_{SE} = \mu + \alpha G_H\left(\mu + w_0^L\right) + w_0^L - \bar{w}$. At the same time the expected cost in pooling equilibrium where no evidence is acquired is $P_{pool}^{NE} = \mu + w_0^H - \bar{w}$. Since P_{SE} is increasing in w_0^L and independent of w_0^H, while P_{pool}^{NE} is instead increasing in w_0^H and independent of w_0^L, the space $\left(w_0^L, w_0^H\right)$ can be partitioned into the regions where the separating equilibrium and correspondingly the pooling equilibrium with no evidence deliver the lower expected cost to the procurer.

These regions are labeled SE and PNE on Figure 5. Since $P_{SE} > P_{pool}^{NE}$ at $w_0^L = w_0^H$, (and both costs are continuous) the SE region is separated from the $w_0^L = w_0^H$ line by the PNE region. The boundary between the SE and PNE regions is given by the curve $w_0^L(w_0^H)$ that satisfies $\alpha G_H\left(\mu + w_0^L\right) + w_0^L = w_0^H$. Since the LHS is increasing in w_0^L and its derivative wrt to w_0^L is $\alpha F_H\left(\mu + w_0^L\right) + 1 - \alpha < 1$, the curve $w_0^L(w_0^H)$ is upward sloping and steeper than the 45 degree line. The region where the separating equilibrium is preferred to the pooling equilibrium (with no evidence) lies to the right of that curve. To complete this comparison consider $w_0^H > \bar{w} > w_0^L$. Then the expected cost in the separating equilibrium is $P_{SE} = \mu + \alpha G_H\left(\mu + \bar{w}\right)$. The expected cost in the pooling equilibrium is still $P_{pool}^{NE} = \mu + w_0^H - \bar{w}$. Hence with $w_0^L < \bar{w}$, the boundary between the regions of separating and pooling equilibrium with no evidence is vertical.

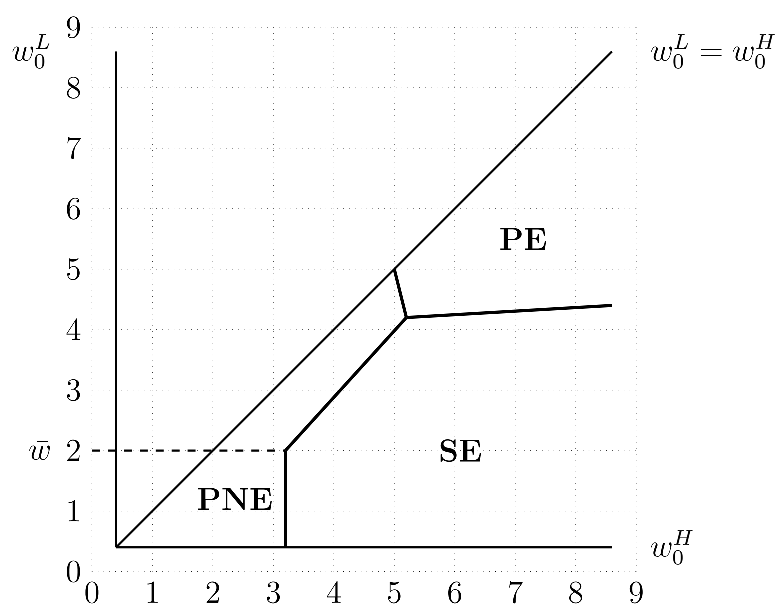


Figure 5: Regions where separating (SE) and pooling equilibria with (PE) and without evidence (PNE) are optimal

Figure 5 presents the results of the numerical simulations. The two ex-ante types are two log-normal distributions on $[0, \infty)$ with the mean $\mu = 1$ and variances σ_H and $\sigma_L < \sigma_H$. Since the c.d.f.s of such log-normal distributions cross only once, the type with the variance σ_H is a mean-preserving spread of the type with the variance σ_L. Moreover, $G_H(x) - G_L(x)$ for such distributions is single-peaked. The values of σ_L, σ_H were varied on a dense grid on $[1, 3]$ interval. We have chosen other parameters, i.e., $e = 3$, $\bar{w} = 2$ and $\alpha = 1/3$ such that the optimal contract in Lemma 5 is to the right of that peak, hence at the optimum $w^* = \bar{w}$, and q^* satisfies $G_L(q^* + \bar{w}) + q^* = e + \mu$. The expected cost in the equilibrium where both types are encouraged to acquire evidence is then

$$P^E_{pool} = e + \mu + \alpha G_H(q^* + \bar{w}) - \alpha G_L(q^* + \bar{w}).$$

The comparison of this expected cost with the P^{NE}_{pool} and P_{SE} given above results in the regions where separation (SE), pooling with no evidence (PNE) and pooling with evidence (PE) are optimal. (With $w^H_0 \leq \bar{w}$ the PNE results in the unconstrained outcome where none of the types acquires evidence and the expected cost is μ.) The numerical values on the axis on Figure 5 correspond to the actual values of w^H_0 and $w^L_0 \leq w^H_0$, which in turn correspond to the values of σ_H and σ_L used in the simulations. The boundaries between the SE and the PNE regions are as predicted above. These and other boundaries on Figure 5 appear to be piece-wise linear, although we were unable to argue this point analytically for the log-normal distributions and we do not expect this to be true in general.

The log-normality assumed in our numerical simulations allows a precise characterization of the optimal contract in Lemma 5. Without such distributional assumptions we cannot pin that contract down.[23] Each $G_i(q + w)$ is a non-increasing function of its argument. The fact that F_H is a mean-preserving spread of F_L guarantees that the difference in (5) is positive, however that difference need not be monotonic and can have multiple minima. Despite this, is any pooling equilibrium in Lemma 5 the rent of the L type is zero, and the rent of the H type depends on the difference between the variances of the two ex-ante types. We therefore expect the procurer to induce the pooling equilibrium where both types acquire evidence whenever the variances of both types are large, but not too dissimilar.

Which Ex-Ante Type Benefits from Adverse Selection

In each of the Lemmas 3 - 5 the procurer optimally offers the first-best contract to the type that has more stringent constraints. This delivers the other type an informational rent. In standard static models of adverse selection with two privately known types, such as a monopolist implementing second degree price discrimination, a 'high' type always benefits from being able to hide his type and receives an informational rent while the other 'low' type expects zero payoff. *In our setting, which type has the incentive to mimic the other type and thus benefit from his private information, depends on the outcome the procurer intends to induce.*

[23] The existence of the optimal contract is guaranteed by the continuity of G_i.

The H ex-ante type has more spread-out distribution and always has higher incentive to acquire evidence. When both types are discouraged from acquiring evidence, the H type is given his first-best contract. In such equilibrium, the H type's constraints are more stringent. The L type is less reluctant to acquire evidence, but can pretend to be the H type, hence the L type receives an informational rent.

Conversely, when both types are encouraged to acquire evidence, the L type's constraints are more stringent. The H type is more inclined to acquire evidence, but can pretend to be the L type, hence the H type receives the informational rent. In the separating equilibrium, a single contract offered to both types induces the H type to acquire evidence while discourages such acquisition by the L type. Here the L type's constraints are binding in equilibrium and the H type receives the informational rent.

The Optimality of the Static Contract

In general, when performing sequential screening, the procurer can offer different ex-ante types different contracts. As the conversion to the cost-plus contract may depend on what contract has been accepted ex-ante, two contractors with the same ex-post type but different ex-ante type can potentially be also treated differently. Proposition 2, however, shows that a simple static contract where the same pair (q, w) is offered to both ex-ante types performs as well as the general sequential screening contract.

Proposition 2. *The expected cost that can be achieved by the optimal sequential screening contract can also be achieved by the static convertible contract.*

The proof and the characterisation of the optimal sequential screening contract are in Appendix C. Here we provide the intuition, again focusing on the separating equilibrium. The L type is always less inclined to acquire evidence. Therefore, the contract that just satisfies the L type's participation constraint and discourages L from acquiring evidence, already delivers the separating outcome. Type H will acquire evidence if offered the same contract, recall Figure 4. The procurer then optimally offers the L type a contract, say (q^*, w^*), that leaves the L type with zero rent. Any contract attractive to the H type has to provide him with the payoff at least as large as the payoff from the contract (q^*, w^*) because the H type can obtain that payoff by pretending to be the L type. The procurer therefore cannot gain from offering another contract (q', w') to the H type.[24]

Krähmer and Strausz (2015) also find the static contract to be optimal when sequential screening is possible.[25] In their model the contract is a pair of the ex-ante payable entry fee F for the option of buying the good at the price R ex-post. The buyer learns his ex-post value (for free), and can withdraw from the contract at that stage. The ex-post withdrawal

[24]Clearly randomization between the (q^*, w^*) and (q', w') contracts does not help the procurer. The scenario where both types are encouraged to acquire evidence is treated along the same lines. The type that obtains the informational rents varies from one scenario to another, but the procurer can always minimize the rents conceded to the contractors by offering both types the same contract.

[25]See also Bergemann et al. (2020).

right implies $F = 0$ for every ex-ante type of the buyer. Further, the static option contract with the smallest R (above the cost of production) is optimal because it delivers the highest surplus and concedes the lowest rent to the buyer.

The role of the controls (F, R) in the above sales models is not the same as the role of the controls (q, w) in our model. Whether the buyer withdraws from the sale contract depends only on the ex-post price, R. Whether the contractor converts his contract into the cost-plus depends on $q + w$. The essential difference between the inner workings of our models boils down to the feature that here the payments q and w are assigned at different decision nodes; the contractor cannot possibly receive both. In the sales models, in contrast, the buyer's ex-post payoff includes $F + R$. That said, the ex-post participation constraints in Krähmer and Strausz (2015) and Bergemann et al. (2020) and the option of providing evidence in our model play somewhat similar roles. They both provide a lower bound on the ex-post payoff of the contractor's type who has acquired evidence. In our model, however, this lower bound holds only in expectation for the contractor's type who does not acquire evidence on his ex-post type.

6 Uncertainty Rents and Implications for Regulation

Contractors in our setting derive rents from two sources. First, as always in adverse selection models, they may receive informational rents. As we have stressed above, which one of the ex-ante types receives this rent in our setting depends on the contract offered by the procurer in equilibrium. Second, the contractors may receive *uncertainty* rents. These rents stem from their ability to acquire further information and at the same time being able to *credibly abstain* from acquiring it. This latter point is somewhat subtle, and related to the way evidence cost is modelled in our paper.

Crucially, the evidence cost here is not the price of 'certifying' the information on the project's cost to further convey this information to the procurer. It is the price of *obtaining* the precise information on that cost that the contractor may or may not choose to pay. It is easy to repeat the analysis in Section 4 under the alternative assumption that the contractor learns his ex-post cost c for free, and e is the cost of certifying the evidence to present it to the procurer. In such model some contractor's ex-post types will reveal their evidence and convert the contract to the cost-plus, but none of the contractors will receive any expected rent ex-ante. This finding would be hard to reconcile with reality, where construction contractors usually earn large profits.

For the contractor to earn the uncertainty rent, he has to possess both the option of staying uninformed about the true cost and the option of learning that cost. The procurer then may concede the uncertainty rent to the contractor, essentially bribing him to stay uninformed. This Section concentrates on the uncertainty rents. It considers several extensions; we informally allow some parameters, fixed in the main analysis, to be influenced by contractors or outside regulators. The arguments are largely driven by the insights derived from the single ex-ante type model in Section 4, but of course, contractors enjoy similar uncertainty rents in the main model in Section 5 as well.

The left panel of Figure 6 presents the contractor's expected payoffs as a function of his own variance w_0^i for a fixed e and $\bar{w}$ as described in Proposition 1. The right panel presents this payoff as a function of the evidence cost e when $w_0^i(e)$ varies with e according to (4). As $w_0^i(e)$ is decreasing, high e corresponds to low w_0^i and vice versa. Introduce $\bar{e} = G_i(\mu, \bar{w})$ such that $w_0^i(\bar{e}) = \bar{w}$ and let $\hat{e} \equiv w_0^i(\hat{e}) - \bar{w}$. The right panel of Figure 6 plots these $\hat{e}$ and $\bar{e}$ on the X-axis. When e is small, $e \leq \hat{e}$, the contractor acquires evidence and does not receive the uncertainty rent. When e is moderate, $e \in [\hat{e}, \bar{e}]$, the contractor receives an uncertainty rent that discourages him from acquiring evidence. This rent decreases with e. When e is large, $e \geq \bar{e}$, the contractor does not acquire evidence and receives no uncertainty rent.

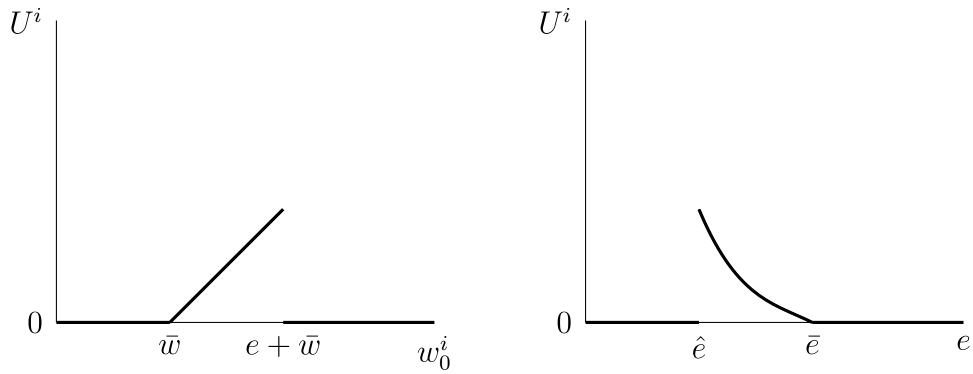


Figure 6: Expected payoff of the contractor as a function of his variance and evidence cost

The contractor's 'sweet spot' is a moderate variance w_0^i, ideally just below $e + \hat{w}$ on the left panel of Figure 6, or the moderate level of evidence cost, ideally just above $\hat{e}$ on the right panel of Figure 6, which maximizes the uncertainty rent. A contractor benefits from his private information, hence has no incentive to reduce his variance too much. Nor he has the incentive to lower his evidence cost below $\hat{e}$. This explains why contractors who have the capacity to be more certain of their costs often make those costs difficult to decipher even for the insiders. Perhaps counter intuitively, our analysis also suggests that a contractor with high cost uncertainty has an intrinsic incentive to moderate it, and reduce his variance to just below $e + \bar{w}$, see Figure 6.

Implications for Regulation

Our analysis explains how 'digitization' of construction industry can lead to an increased frequency of cost overruns. In many countries construction firms are encouraged to adopt more efficient accounting software which will reduce their evidence cost, see World Economic Forum Report (2016). Contractors may embrace such incentives and lower their evidence cost to just above $\hat{e}$ as described above. Once their evidence costs are reduced, contractors are more inclined to acquire evidence. As a result, cost overruns can become more frequent because contractors are now more likely to convert their contracts into the cost-plus. The push towards the digitization, indeed, provides the contractors with more precise information on their costs, but since this information is released to the procurers strategically, it may also increase the frequency of cost overruns. At the same time, assuming e is fixed, contractors may have little incentive to decrease their cost uncertainty as explained above. Facing a contractor with high cost uncertainty, the procurer optimally provides greater incentives to acquire evidence, which increases the frequency of cost overruns as well.

Further, consider a regulator whose role is to set the maximum punishment $\bar{w}$. By setting the $\bar{w}$ the regulator effectively determines the range of variances that are considered moderate, refer to the left panel of Figure 6. Assume for the moment that the regulator aims to benefit the contractors. To increase their expected payoffs, the regulator does not lower their liability as much as possible, and instead sets the $\bar{w}$ at the level that ensures the contractor receives the uncertainty rent. To illustrate, suppose that contractor i's ex-ante type is such that his $w_0^i \in (\bar{w}, e + \bar{w}]$ on the left panel of Figure 6, so that i receives the uncertainty rent. Lowering the liability limit of the contractor, i.e., lowering the $\bar{w}$, increases the (relative) variance. This can result in $w_0^i > e + \bar{w}$, and the corresponding loss of the uncertainty rent.

This suggests, perhaps counter-intuitively, that by lowering the liability limit of the contractor the regulator may benefit the procurer. Importantly, the liability level here does not determine the 'fine' the contractor pays to the procurer in case of a default or delay in the project, as it is, e.g., in Birulin (2020) or Chillemi and Mezzetti (2014). Instead, the $\bar{w}$ here affects the circumstances under which the fixed-price contract is converted into the cost-plus contract. Such conversion does not lead to a default in our model, and we believe, helps averting defaults in procurement practice, although as any renegotiation it may lead to a delay in the project. Lowering the $\bar{w}$, the regulator encourages contract conversion. As we have explained, this may reduce the uncertainty rents of the contractors, but as a byproduct, more frequent contract conversions lead to more frequent cost overruns.

In the EU and in Australia it is common to split very large infrastructure projects into multiple smaller projects and contract out each of these separately.[26] This measure is designed to support small contractors and allow them to compete with larger firms for the projects they can handle, essentially leveling the playing field. We argue that such split should be considered with caution and the relative cost uncertainty on the larger and smaller projects should be accounted for. It is natural to assume that by splitting a large project a regulator obtains smaller sub-projects with lower cost uncertainty. The original large project may deliver uncertainty rents to the contractor. The split may result is multiple projects with small cost uncertainty that leave contractors with no uncertainty rents. The regulator ends up harming the contractors she was trying to empower. In this scenario, however, due to their low variance the contractors are more likely to stay on the fixed-price contract, hence cost overruns may become less frequent.

[26] https://www.enotrans.org/article/the-case-against-mega-contracts-in-mega-transit-projects/ and https://www.dlapiper.com/en/us/insights/publications/2020/04/projects-global-insight-issue-3/breaking-mega-projects-into-smaller-contract-packages/ provide examples of such projects.

7 Conclusion

Our model relates the optimal choice of the procurement contract to an intuitively appealing and simple measure — the contractor's own cost uncertainty. The data on prices and bids in procurement tenders is available and we hope that our model can be tested empirically. An instance of conversion to the cost-plus contract can present itself in the data as a contract price adjustment. The measure of the contractor's cost distribution spread or variance that our model emphasises can also be recovered empirically.

Several simplifications were important to derive our conclusions. Relaxing either of them is beyond the scope of this paper, and is deferred for future research. Our model allows only the contractor to acquire evidence. In practice the procurer also can hire an expert to investigate the true cost of the project. Such model would have different incentive constraints and would require a separate analysis. The choice between acquiring evidence herself of delegating this to the contractor would create an additional trade-off for the procurer. On one hand, by acquiring evidence herself, the procurer reduces the contractor's informational advantage. The latter can no longer acquire and hide unfavourable evidence. On the other hand, acquiring evidence is costly, and by delegating this task to the contractor, the procurer shares that cost.

A contractor can hide his evidence but he cannot fake it. Hence the procurer can offer different mark-ups to different ex-post types revealed to her. This would change the ex-post incentive constraints. The interval of the low ex-post types who hide their evidence can be followed by an interval of the types who reveal their evidence, while yet higher types can be also hiding the evidence. The procurer may control this partition by varying how the mark-up depends on the revealed cost. The expected payoff from acquiring evidence would reflect this partition and the distribution of the ex-post types. It appears that even a simple choice between the fixed-price and cost-plus contracts cannot then boil down to a simple measure of variance as in Proposition 1, and will depend on a finer detail of the cost distribution.

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A Appendix

Contractor's Expected Payoff

We first derive a few useful identities.

$$\int_0^A F(c)dc = (c-A)F(c)|_0^A - \int_0^A (c-A)f(c)\,dc = \int_0^A (A-c)\,dF(c) \tag{6}$$

$$\mu \equiv \mathsf{E}(c) = \int_0^\infty c\,dF(c) = c(F(c)-1)|_0^\infty + \int_0^\infty (1-F(c))\,dc = \int_0^\infty \bar{F}(c)\,dc$$

$$\int_{q+w}^\infty \bar{F}(c)\,dc - \mu = \int_0^{q+w} (F(c)-1)\,dc = \int_0^{q+w} F(c)\,dc - w - q \tag{7}$$

We can now apply these to derive the contractor's expected payoff when he acquires evidence on his cost

$$
\begin{aligned}
U_E(q,w) &= -e + \mathsf{E}[\max\{q-c,-w\}] = -e + \mathsf{E}[\max\{q+w-c,0\}] - w \\
&= -e + \int_0^{q+w} (q+w-c)\, dF - w \\
&= -e + \int_0^{q+w} F(c)\, dc - w \qquad\qquad \text{using (6)} \\
&= -e + \int_{q+w}^{\infty} \bar{F}(c)\, dc - \mu + q \qquad\qquad \text{using (7)}
\end{aligned}
$$

B Appendix: Proofs of Lemmas in Section 5

Proof of Lemma 4. With a given contract (q,w) the H type has a greater incentive to acquire evidence, and that type's IA constraint is more stringent when the procurer discourages evidence acquisition. The procurer solves

$$
\begin{aligned}
&\min_{q,w} P_n = q \\
&\text{s.t. } q \geq \mu && \text{NEIR} \\
&\quad\; e \geq G_H(q+w) \geq G_L(q+w) && \text{NEA} \\
&\quad\; \bar{w} \geq w, && \text{LL}
\end{aligned}
$$

As the procurer discourages both types from acquiring evidence, whereas the H is always more inclined to do so, the analysis follows Section 4.1 with w_0^H taking the role of w_0. Figure 2 in Section 4.1 is a useful reference. When $w_0^H \leq \bar{w}$ the procurer can discourage both types from acquiring evidence with sufficiently large w and attain the unconstrained outcome with $q = \mu$, as on the left panel of Figure 2. Neither type receives any rent in such case and the procurer's expected cost is μ.

When $w_0^H > \bar{w}$, as on the right panel of Figure 2, both ex-ante type receive a positive uncertainty rent, and the optimal contract is determined by the variance of the H type. Both types are offered $q^* = (w_0^H - \bar{w})_+$ and $w^* = \bar{w}$ such that the LL and the NEA constraints bind for the H type. Both types receive the same uncertainty rent $q^* - \mu$. This rent is determined by the variance of the H type since his LL constraint is more stringent. When both types are discouraged from acquiring evidence, the optimal contract (q^*, w^*) is the first-best contract for the H type.

All constraints are slack for the L type, and the value of the L type's variance does not affect the optimal contract. The L type is the one who benefits from the asymmetric information. The rent that the L type receives due to the presence of the H type, $(w_0^H - \bar{w})_+ - \mu$ exceeds $(w_0^L - \bar{w})_+ - \mu$ that the L type would get upon revealing his ex-ante type. Hence in the scenario where neither type acquires evidence the L type receives informational rent on top of the uncertainty rent. None of the constraints bind for the L type, but reducing either of these rents would violate either the LL or the NEA constraint of the H type. $\square$

Proof of Lemma 5. Assume the procurer induces both ex-ante types to acquire evidence. Since with a given contract (q, w) the L type is less inclined to acquire evidence, that type's EA constraint is more stringent. The procurer then solves

$$\begin{aligned}
&\min_{q,w} P_c = q + \alpha G_H(q+w) + (1-\alpha)G_L(q+w) && \\
&\text{s.t. } G_H(q+w) + q \geq G_L(q+w) + q \geq e + \mu && \text{EIR} \\
&\qquad G_H(q+w) \geq G_L(q+w) \geq e && \text{EA} \\
&\qquad \bar{w} \geq w && \text{LL}
\end{aligned}$$

Since the L type's constraints are more stringent, the procurer optimally offers to both ex-ante types the L type's first-best contract, as in Section 4.2. With such contract the EIR constraint of the L type binds, which leaves the L type with no rents. The H type receives informational rents when both types are induced to acquire evidence. If the H type was induced to acquire evidence, but the L type was not present, the procurer would have optimally reduced the H type rent to zero, recall Section 4.2.

Since any contract on the EIR constraint of the L type that satisfies this type's EA and LL constraints leaves no rent to the L type, the procurer can choose one of such contracts to minimize the informational rents conceded to the H type. This rent amounts to

$$\begin{aligned}
U_E^H(q,w) &= G_H(q+w) + q - e - \mu \\
&= G_H(q+w) - G_L(q+w),
\end{aligned}$$

since $G_L(q+w) = e + \mu - q$, as the L type EIR constraint binds. $\square$

C Appendix: Sequential Screening Contract Characterisation

We characterise the optimal sequential screening contract where the procurer is able to set different contracts for different ex-ante types. She faces two ex-ante types $i = H, L$ with prior probabilities $\mathsf{Pr}(i = H) = \alpha$, $\mathsf{Pr}(i = L) = 1 - \alpha$. By the revelation principle, we need only consider incentive compatible contracts, such that a contractor of ex-ante type i has no incentive to accept the contract intended for type $j \neq i$. The ex-ante incentive compatibility (IC) constraints are then given by

$$U^i(q_i, w_i) \geq U^i(q_j, w_j), \text{ for } i = H, L \text{ and } j \neq i \tag{IC}$$

where $U^i(q,w) := \max\{U_E^i(q,w), U_{NE}^i(q,w)\}$ is the contractor's utility as he is strategic about evidence acquisition. There are three scenarios the procurer can induce: discourage both types from acquiring evidence, encourage both types to do so, or induce the separating equilibrium where only the H type acquires evidence.

Separating Equilibrium

Suppose the procurer induces the H type to acquire evidence and the L type to not acquire evidence.[27] The expected cost is then $P_{SE} := \mathsf{E}[P \,|\, \text{only the high type acquires evidence}] = \alpha\left(\int_{q+w}^{\infty} \bar{F}_H(c)\,dc + q_H\right) + (1-\alpha)q_L$. The procurer solves the following optimisation problem

$$
\begin{aligned}
\min_{q_L,q_H,w_L,w_H} \quad & P_{SE} = \alpha\left(\int_{q+w}^{\infty} \bar{F}_H(c)\,dc + q_H\right) + (1-\alpha)q_L \\
\text{s.t.} \quad & q_L \geq \mu && (\text{NEIR}^L) \\
& e \geq \int_{q_L+w_L}^{\infty} \bar{F}_L(c)\,dc && (\text{NEA}^L) \\
& \int_{q_H+w_H}^{\infty} \bar{F}_H(c)\,dc + q_H \geq e + \mu && (\text{EIR}^H) \\
& \int_{q_H+w_H}^{\infty} \bar{F}_H(c)\,dc \geq e && (\text{EA}^H) \\
& \bar{w} \geq w_i, \; i = H, L \\
& U^i(q_i, w_i) \geq U^i(q_j, w_j), \; \text{for } i = H, L, j \neq i && (\text{IC})
\end{aligned}
$$

Suppose the L type is offered a first-best contract to acquire evidence as if the procurer was certain there were no H types, $(q_L, w_L) = (\mu + (w_0^L - \bar{w})_+, \bar{w})$, where $(w_0^L - \bar{w})_+ := \max\{(w_0^L - \bar{w}), 0\}$ is the minimum uncertainty rent needed to prevent him from acquiring evidence.

The H type requires a higher uncertainty rent to stay uninformed and thus will always acquire evidence if he accepts the contract intended for the L type. In such scenario the H type's payoff would be

$$
\begin{aligned}
U_E^H(q_L, w_L) &= \int_{\mu+(w_0^L-\bar{w})_++\bar{w}}^{\infty} \bar{F}_H\,dc + (\mu + (w_0^L - \bar{w})_+) - e - \mu \\
&= \left(\int_{\mu+(w_0^L-\bar{w})_++\bar{w}}^{\infty} \bar{F}_H\,dc - e\right) + (w_0^L - \bar{w})_+
\end{aligned}
$$

The procurer offers the H type a contract that makes the H type indifferent, $U_E^H(q_H, w_H) = U_E^H(q_L, w_L)$, that is:

$$
\int_{q_H^*+w_H^*}^{\infty} \bar{F}_H\,dc + q_H^* = e + \mu + \left(\int_{\mu+(w_0^L-\bar{w})_++\bar{w}}^{\infty} \bar{F}_H\,dc - e\right) + (w_0^L - \bar{w})_+
$$

[27] As the more dispersed H type always has greater incentives to acquire evidence for a given contract, a menu of contracts that conversely induces the L to acquires evidence but the H to stay uncertain cannot be incentive compatible.

Note that $\int_{\mu+(w_0^L-\bar{w})_+ +\bar{w}}^{\infty} \bar{F}_H \, dc \geq \int_{\mu+w_0^H}^{\infty} \bar{F}_H \, dc = e$. Then, given the above we require that $q_H^* \leq \mu + (w_0^L - \bar{w})_+$ to ensure that $\int_{q_H^*+w_H^*}^{\infty} \bar{F}_H \, dc \geq e$ and thus that H type's EA constraint is satisfied. The same assumption $q_H^* \leq \mu + (w_0^L - \bar{w})_+$ is also necessary to satisfy the L type's ex-ante IC constraint. Without it, the L type prefers to accept the H type contract with the higher fixed price, but will not later acquire evidence. In addition, to satisfy the L type's IC, we require that the L type prefers to not mimic the H type (i.e. accept the contract intended for the H type and also acquire evidence), $U_c^L(q_H^*, w_H^*) \leq U_n^L(q_L^*, w*_L)$:

$$\int_{q_H^*+w_H^*}^{\infty} \bar{F}_L \, dc + q_H^* \leq e + \mu + (w_0^L - \bar{w})_+$$

The procurer has no incentive to raise $(w_0^L - \bar{w})_+$ or lower w_L as that only raises the price he pays to either type. Clearly, the outcome where the L type does not acquire evidence, while the H type facing the same contract does acquire evidence can be implemented by a static contract.

Proposition 3 (Separating Contract)**.** *Suppose it is optimal for the procurer to induce the separating equilibrium. Let $(w_0^L - \bar{w})_+ := \max\{(w_0^L - \bar{w}), 0\}$ be the uncertainty rent the L type gets for not acquiring evidence when there is no H types. The procurer optimally sets $(q_L^*, w_L^*) = (\mu + (w_0^L - \bar{w})_+, \bar{w})$ such that the L type does not acquire evidence. She also sets any $q_H^* \leq \mu + (w_0^L - \bar{w})_+$, $w_H^* \leq \bar{w}$ such that*

i. $U_E^H(q_H^*, w_H^*) = U_E^H(q_L^*, w_L^*)$: $\int_{q_H^*+w_H^*}^{\infty} \bar{F}_H \, dc + q_H^* = \mu + (w_0^L - \bar{w})_+ + \int_{\mu+(w_0^L-\bar{w})_+ +\bar{w}}^{\infty} \bar{F}_H \, dc$

ii. $U_E^L(q_H^*, w_H^*) \leq U_{NE}^L(q_L^*, w_L^*)$: $\int_{q_H^*+w_H^*}^{\infty} \bar{F}_L \, dc + q_H^* \leq e + \mu + (w_0^L - \bar{w})_+$

The L type does not acquire evidence and expects a payoff of $(w_0^L - \bar{w})_+$, while the H type acquires evidence and expects a payoff of $\int_{q_L^*+w_L^*}^{\infty} \bar{F}_H \, dc - e + (w_0^L - \bar{w})_+$.

Neither Type Acquires Evidence

Assume the procurer discourages both types from acquiring evidence. The procurer solves

$$\begin{aligned}
\min_{q_L, q_H, w_L, w_H} \quad & P_{NE} = (1-\alpha)q_L + \alpha q_H \\
\text{s.t.} \quad & q_i \geq \mu, \ i = 1, 2 && \text{(NEIR)} \\
& e \geq \int_{q_i+w_i}^{\infty} \bar{F}_i(c) \, dc, \ i = H, L && \text{(NEA)} \\
& \bar{w} \geq w_i, \ i = H, L \\
& U_{NE}^i(q_i, w_i) \geq U_{NE}^i(q_j, w_j), \text{ for } i = H, L, j \neq i && \text{(IC)}
\end{aligned}$$

We will argue that a static contract is a solution to the above problem. Assume that there exists an incentive compatible menu of contracts with, say, $q_L < q_H$ that induces neither type to acquire evidence. As both types do no acquire evidence, they both prefer the contract with q_H, contradicting IC. Thus any IC menu of contracts that induces neither type to acquire evidence must set $q_L = q_H$.

Recall that the H type requires a higher uncertainty rent to not acquire evidence. Consider first the case where the H type has relatively lower variance, $w_0^H \leq \bar{w}$. The procurer is able to punish sufficiently to attain the unconstrained outcome for the H type and thus also for the L type. She sets $q_L = q_H = \mu$ such that NEIR is binding for both types and $w_L = w_H \in [w_0^H, \bar{w}]$ such that NEA is satisfied for both types. Both types expect a payoff of zero.

Consider now the case where the H type has higher variance such that $w_0^H \geq \bar{w}$. The procurer sets $q_L = q_H = \mu + w_0^H - \bar{w}$ and $w_H = \bar{w}$ such that the H type's NEA is satisfied with equality, giving both types the uncertainty rent the H types would get if the procurer was certain there were no L types. She also sets $w_L = \bar{w}$ to ensure that the H type does not have an incentive to accept the L type's contract. The procurer has no incentive to raise the price or lower the punishment as that would only increase the rents being paid to the contractors. She cannot also lower the price or raise the punishment without violating some of the constraints. Both types expect a payoff of $w_0^H - \bar{w}$.

Proposition 4. *Suppose it is optimal to discourage both types from acquiring evidence.*

i. *If* $(w_0^H \leq \bar{w})$, *the procurer can attain the unconstrained outcome. She sets* $q_L^* = q_H^* = \mu$ *and* $w_L^*, w_H^* \in [w_0^H, \bar{w}]$. *Both types expect a payoff of zero.*

ii. *If* $(w_0^H \geq \bar{w})$, *the procurer sets* $q_L^* = q_H^* = \mu + (w_0^H - \bar{w})$ *and* $w_H^* = w_L^* = \bar{w}$. *Both types expect a payoff of* $(w_0^H - \bar{w})$.

Notably, when neither type acquires evidence, even when the procurer can sequentially screen, the optimal static contract for the H type is an optimal contract.

Both Types Acquire Evidence

Assume the procurer induces both types to acquire the evidence. She solves the following problem

$$\min_{q_L, q_H, w_L, w_H} P_E = (1-\alpha)\left(\int_{q_L+w_L}^{\infty} \bar{F}_L(c)\,dc + q_L\right) + \alpha\left(\int_{q_H+w_H}^{\infty} \bar{F}_H(c)\,dc + q_H\right)$$

$$\text{s.t.} \int_{q_i+w_i}^{\infty} \bar{F}_i(c)\,dc + q_i \geq e + \mu,\ i = H, L \tag{EIR}$$

$$\int_{q_i+w_i}^{\infty} \bar{F}_i(c)\,dc \geq e,\ i = H, L \tag{EA}$$

$$\bar{w} \geq w_i,\ i = H, L$$

$$U_E^i(q_i, w_i) \geq U_E^i(q_j, w_j),\ \text{for } i = H, L, j \neq i \tag{IC}$$

The H type must have a high enough variance $w_0^H > \bar{w}$ as otherwise the unconstrained outcome is attainable and inducing both types to acquire evidence is never optimal. The IR constraint for the L type must bind. To see this, suppose instead the procurer sets a contract for the low type that leaves the L type with a payoff of $M > 0$ in expectation,

$$U_E^L(q_L, w_L) = \int_{q_L+w_L}^{\infty} \bar{F}_L \, dc + q_L - e - \mu = M$$

If the H type accepts the L type's contract, he expects a non-negative payoff

$$\begin{aligned} U_E^H(q_L, w_L) &= \int_{q_L+w_L}^{\infty} \bar{F}_H \, dc + q_L - e - \mu \\ &= \int_{q_L+w_L}^{\infty} (\bar{F}_H - \bar{F}_L) \, dc + \left(\int_{q_L+w_L}^{\infty} \bar{F}_L \, dc + q_L - e - \mu \right) \\ &= \int_{q_L+w_L}^{\infty} (\bar{F}_H - \bar{F}_L) \, dc + M \geq M \end{aligned}$$

To avoid conceding unnecessary rents to the H type, the H type's ex-ante IC constraint binds, $U_E^H(q_H, w_H) = U_E^H(q_L, w_L)$, so that on his contract the H type gets just as much expected payoff as he would get by pretending to be the L type. Therefore, by reducing M to 0 the procurer reduces the rents she concedes to both types. From the fact that IC binds for the H type, it follows that

$$U_E^H(q_H, w_H) = \int_{q_H+w_H}^{\infty} \bar{F}_H(c) \, dc + q_H - e - \mu = \int_{q_L+w_L}^{\infty} (\bar{F}_H - \bar{F}_L) \, dc$$

Given the above, to satisfy the H type's EA constraint we require $q_H^* \leq \mu + \int_{q_L+w_L}^{\infty} (\bar{F}_H - \bar{F}_L) \, dc$. With $M = 0$ the procurer optimally offers the L type his first-best contract as described in Section 4.2. Such contract is not unique, and from the set of contracts that induce the L type to acquire evidence and leave him no rent, the procurer chooses the contract (q_L^*, w_L^*) to minimise the rent she concedes to the H type, $\int_{q_L^*+w_L^*}^{\infty} (\bar{F}_H - \bar{F}_L) \, dc$.

Finally, to ensure the L type's ex-ante IC constraint, the optimal H contract must satisfy $U_E^L(q_H^*, w_H^*) \leq 0$. Clearly such contract exists, take the optimal static contract. The outcome of the optimal sequential screening contract is such that both types acquire evidence. The L type receives no rent. The H type receives just enough rent to prevent him from mimicking the L type.

Proposition 5. *Suppose it is optimal to induce both types to acquire evidence. The procurer then optimally sets* $q_L^* \leq \mu$, $q_H^* \leq \mu + \int_{q_L^*+w_L^*}^{\infty} (\bar{F}_H - \bar{F}_L) \, dc$ *and* $w_L^*, w_H^* \leq \bar{w}$ *such that the following are met*

i. $U_E^L(q_L^*, w_L^*) = 0 \iff \int_{q_L^*+w_L^*}^{\infty} \bar{F}_L(c) \, dc + q_L^* = e + \mu$

ii. $(q_L^*, w_L^*) = \mathsf{argmin} U_E^H(q_L, w_L) = \int_{q_L+w_L}^{\infty} (\bar{F}_H(c) - \bar{F}_L(c))\, dc$

iii. $U_E^H(q_H^*, w_H^*) = U_E^H(q_L^*, w_L^*) \iff \int_{q_H^*+w_H^*}^{\infty} \bar{F}_H(c)\, dc + q_H^* = e + \mu + \int_{q_L^*+w_L^*}^{\infty} (\bar{F}_H(c) - \bar{F}_L(c))\, dc$

iv. $U_E^L(q_H^*, w_H^*) \leq U_E^L(q_L^*, w_L^*) \iff \int_{q*_L+w_L^*}^{q_H^*+w_H^*} (\bar{F}_H(c) - \bar{F}_L(c))\, dc \leq 0$

The L type contractor expects a zero payoff while the H type contractor expects a payoff of $\int_{q_L^*+w_L^*}^{\infty} (\bar{F}_H(c) - \bar{F}_L(c))\, dc$.

(i) ensures that the L type is given the contract that extracts all his surplus, (ii) ensures that the same contract minimises the H type's information rent, (iii) and (iv) are the ex-ante incentive compatibility constraints.

Optimality of The Static Contract

The optimal static contracts achieve the same outcomes as the optimal sequential screening contracts and are thus equally optimal. The optimal static contracts are either equivalent to or they are the special cases of the optimal sequential contract. Specifically,

i. In the separating equilibrium the optimal sequential contract is the optimal static contract in which both types are offered the L type's first-best contract.

ii. When neither type acquires evidence, the optimal sequential contract is the optimal static contract in which both types are offered the H type's first-best contract.

iii. When both types acquire evidence, the optimal sequential contract is the optimal static contract in which both types are offered the L type's first-best contract that minimises the H type's rent.