Financial Intermediation and Business Cycles in Two-Period Lived OLG Models*

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November 25, 2022

Abstract

This article investigates a two-period lived OLG model with financial intermediation as a vehicle to share risk. Risk-averse agents subject to idiosyncratic income shocks prefer financial intermediation that implements the efficient allocation to capital markets. It is shown that the resulting dynamics is monotonic and qualitatively the same as the dynamics of the classical OLG model by Diamond (1965). These results contradict Banerji, Bhattacharya, and Van Long (2004) by demonstrating that in two-period lived OLG models with rational expectations, financial intermediation that provides complete risk sharing can neither trigger business cycles nor complex dynamics. Business cycles can only occur if banks offer inefficient contracts.

Keywords: Financial Intermediation, OLG Models, Risk Sharing, Business Cycles, Loan Contracts, Rational Expectations

JEL Classification: D53, E32, E44, G21, O41

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*Acknowledgements. We would like to thank participants of a research seminar at the University of Kaiserslautern for helpful comments and discussions. We are particularly grateful to Dominique Demougin, Philipp Weinschenk, Fabian Herweg, Tom Rauber, and Jan Münning for valuable comments.

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1 Introduction

Understanding the pace and patterns of economic growth is one of the central topics in economics. In light of the current global economic downturn, it is also a highly topical issue. Conventional wisdom in macroeconomics states that real economic development is supported by a well-functioning financial system (Levine, 1997).[1] Over the past decades, a small canvas of theoretical literature has emerged that successfully introduced financial intermediation to growth models. In these models, the promotion of economic growth is attributed to the fundamental functions that financial intermediaries fulfil in an economy (Pagano, 1993). The seminal contribution by Greenwood and Jovanovic (1990) highlights how risk sharing and the informational advantage of financial intermediaries encourage high-yield investments and economic growth. Bencivenga and Smith (1991), on the other hand, extend the arguments of Diamond and Dybvig (1983) by showing that liquidity provision and risk sharing induce savings behaviour of agents that enhances capital accumulation.

The risk-sharing function of financial intermediation plays a pivotal role in the overlapping generations (OLG) model of Banerji et al. (2004), in which loan and deposit contracts enable risk-averse agents to insure against idiosyncratic income shocks. While Banerji et al. (2004) acknowledge the growth-enhancing effect of financial intermediation, they argue that risk sharing may expose the economy to endogenous fluctuations in the form of real-sector business cycles and the full variety of complex dynamics. The authors therefore conclude that the promotion of economic growth by financial intermediaries comes at the cost of more volatility in growth patterns.

In this article, we address this issue and investigate the question of whether efficient risk sharing provided by financial intermediaries can induce endogenous business cycles in an economy that is otherwise known to experience monotonic growth only. Extending the setting in Banerji et al. (2004) to include a more general class of intertemporal preferences, we find that a collective bank can implement the efficient allocation by offering efficient loan and deposit contracts. Efficient contracts provide complete risk sharing but must enlarge the disposable income of an agent in order to be accepted. It turns out that the *,income effect‘* identified in Banerji et al. (2004) that is deemed responsible for generating endogenous fluctuations is, in fact, a consequence of a mere incentive problem without implications for the macroeconomic dynamics. We demonstrate that financial intermediation that implements efficient contracts *cannot* generate business cycles or even complex dynamics because the dynamics of the economy is always monotonic. Business cycles may only be triggered by inefficient contracts that would, however, not be accepted by rational agents.

[1]Excellent overviews of the empirical literature on the finance-growth nexus are provided by Levine (2005) and Aziakpono (2011).

Our approach is slightly more precise than in Banerji et al. (2004) since we have refined the decision problem of the bank by including economic feasibility as well as rationality constraints of agents. Moreover, we deduce our notion of an efficient contract from a social-planner benchmark. The analysis reveals that despite the bank maximising the welfare of the representative agent, incentive problems remain so that agents' acceptance of an efficient contract is not as straightforward as one would expect. Contrary to Banerji et al. (2004), who argue that financial intermediation may generate complex *backward* dynamics, we will focus on the *forward* dynamics of the economy. The reason is that the usefulness of the backward dynamics for a forward-time interpretation is limited and that the analysis is often restricted to a limited range of model parameters in the neighbourhood of a steady-state solution, see, e.g., Grandmont (1989) and Medio and Raines (2006).

The literature that examines the effects of financial intermediation on business cycles in OLG models is relatively scarce. Smith (1998) argues that monopolistic financial intermediaries can increase the severity of *existing* business cycles. In our article, however, we examine the question of whether a collective bank can induce these cycles or, in other words, whether financial intermediation is the reason for business cycles. The occurrence of business cycles in Williamson (1987), on the other hand, results from indivisibilities in the investment projects. These are also present in our model but not a source of fluctuations. Azariadis and Smith (1998) show that bank-loan financed capital investments cause business cycles if there is an adverse selection problem concerning the repayment probability of borrowers. Our contribution complements Azariadis and Smith (1998) by demonstrating that efficient bank loans cannot generate business cycles if informational asymmetries are absent. Finally, the OLG model with financial intermediation developed in Gersbach and Wenzelburger (2012, 2008, 2003) exhibits persistent business cycles. These are triggered by macroeconomic productivity shocks that cannot be diversified away so that the model has, as opposed to the model in Banerji et al. (2004) or ours, aggregate uncertainty.

The remainder of this article is organised as follows. The next section lays out the basic model including all essential assumptions. In Section 3, we formulate the decision problems of both agents and the bank. We then introduce our notion of an efficient contract and establish its existence and uniqueness. Section 4 is dedicated to the resulting dynamics of the model and states our main results. We revisit an example of Banerji et al. (2004) in Section 5 and thereby illustrate how inefficient contracts may trigger business cycles. Implications and limitations of the model are discussed in the conclusion.

2 Model Prerequisites

Following on from Banerji et al. (2004), we consider a two-period lived OLG model with financial intermediation. Time is discrete and indexed by $t = 0, 1, \ldots$. There exists a single perishable good that can be consumed and invested. Agents live for two periods, referred to as young and old. At the beginning of each period t, a new generation of homogeneous young agents is born. Every generation comprises a continuum with mass one, implying a stationary population profile. In period $t = 0$, there exists an initial old generation endowed with capital $K_0 > 0$.

Agents are risk-averse and value consumption in both periods of their life. Their intertemporal preferences are represented by an additive-separable lifetime utility function

$$U(c^1, c^2) \coloneqq u(c^1) + v(c^2),$$

where $c^1, c^2 \geq 0$ denote youthful and old-age consumption, respectively. The following assumptions on preferences are standard.

Assumption 1 (Preferences)**.**
The utility functions $u : \mathbb{R}_+ \to \mathbb{R}$ and $v : \mathbb{R}_+ \to \mathbb{R}$ are twice continuously differentiable, strictly increasing, strictly concave, and satisfy the Inada conditions.

A young agent may become an entrepreneur by undertaking a risky production project. Projects have a stochastic outcome and can either be successful or fail. The likelihood of a successful project depends on the amount of capital invested and is determined by an exogenously given *success function* $p : \mathbb{R}_+ \to (0, 1]$, so that $p(I)$ stipulates the success probability of the capital investment $I \geq 0$. The uncertainty about the outcome of a project resolves one period after capital is invested. If a project is successful, then it generates a verifiable gross rate of return $\varrho > 0$. If a project fails, then it is abandoned and the gross rate of return is zero.[2]

Invoking the law of large numbers, the productive capital stock of the economy is

$$\Omega(I) \coloneqq p(I)I.$$

The properties of the success function p determine the behaviour of the productive capital. These are of central importance for our analysis and are stipulated in terms of Ω in Assumption 2 below.

Assumption 2 (Productive Capital)**.**
The function $\Omega : \mathbb{R}_+ \to \mathbb{R}_+$ is strictly increasing and strictly concave.

[2]As an alternative interpretation, one may think of the entrepreneur as an investor who invests her capital into a firm. A failed project is then equivalent to a defaulting firm. Throughout this article, however, we adopt the entrepreneurial view.

Assumption 2 states that the productive capital stock is increasing in the amount of invested capital I and ensures that the capital income of the old generation is well behaved.

Example 1 (Success Probability)**.**
The productive capital stock $\Omega(I)$ corresponding to the success function

$$p(I) = \frac{\kappa}{1+I}, \quad \textit{where} \quad 0 < \kappa \leq 1,$$

satisfies Assumption 2.

The production sector of the economy is perfectly competitive. A neoclassical technology employs labour $N \geq 0$ and real capital $K \geq 0$ with constant returns to scale and full depreciation of capital. We denote by $k \coloneqq K/N$ the capital-labour ratio and by $f : \mathbb{R}_+ \to \mathbb{R}_+$ the production function of the representative firm in intensity form.

Assumption 3 (Technology)**.**
The production function $f : \mathbb{R}_+ \to \mathbb{R}_+$ is thrice continuously differentiable, strictly increasing, strictly concave, and satisfies the Inada conditions. Moreover, it holds that

$$\frac{f''(k)k}{f'(k)} > -1 \quad \textit{and} \quad \frac{f'''(k)k}{f''(k)} > -2 \quad \textit{for all} \quad k \geq 0.$$

The last two properties imposed on f in Assumption 3 are somewhat unusual. They imply that the capital income of the old generation, $f'(k)k$, is strictly increasing and strictly concave in the capital-labour ratio k. These properties facilitate the existence and uniqueness of efficient loan contracts and are satisfied by many standard production functions in the literature. Among these are the Cobb-Douglas production function and a wide range of parameterisations of the CES production function.

The young generation constitutes the workforce of the economy, whereas the old generation is retired and owns all capital. Each young agent supplies one unit of labour inelastically to a perfectly competitive labour market, implying that labour supply is fully allocated in every period t. Labour and capital are paid their respective marginal products. Given a capital investment of I, the productive capital stock of the subsequent period is $k = \Omega(I)$ and is paid its marginal product $\varrho = f'(\Omega(I))$. Capital income of the old generation thus becomes

$$g(I) \coloneqq f'(\Omega(I))\,\Omega(I). \tag{2.1}$$

Our assumptions lead to the following technical lemma.

Lemma 1 (Capital Income)**.**
Under the hypotheses of Assumptions 2 and 3, capital income g is strictly increasing, strictly concave, and satisfies $g(0) = 0$.

3 Financial Intermediation

Since agents work only when young, they need to transfer resources to the second period of their life. To do so, they may invest part of their wage income into a production project which bears the idiosyncratic risk of an old-age income shock. To mitigate this risk, young agents may form a coalition in the form of a risk-neutral collective bank in the sense of Freixas and Rochet (2008) that provides risk sharing. This bank offers young agents a loan contract (B_t, I_t, R_t), where $B_t \geq 0$ denotes the size of the loan, $I_t \geq 0$ is the capital investment into the project, and $R_t \geq 0$ is the gross interest rate on loans. By accepting a loan contract, agents are protected by limited liability as they do not have to repay the loan in case their project fails.[3] To finance its loans, the bank raises deposits from the public by offering a risk-free gross rate $r_t \geq 0$ on deposits.

3.1 Decision problems

Each young agent must decide whether to invest into a project by accepting a loan contract or to undertake the production project individually instead. In the latter case, the agent is fully exposed to the idiosyncratic risk of her project. Independently of her investment decision, however, a young agent is allowed to deposit part of her wage income at the bank.

The decision problem of the representative young agent is the following. Suppose the bank offers the loan contract (B_t, I_t, R_t) and the deposit rate r_t on savings in period t. Consider first the case in which the agent *accepts* the loan contract. Given her wage income w_t, the young agent must then decide on how much to consume and how much to save for retirement. By accepting the loan contract, her disposable income becomes $w_t^d := w_t + B_t - I_t$, so that youthful consumption is

$$c^1 = w_t^d - D, \tag{3.1}$$

where $D \geq 0$ is the amount saved and deposited at the bank. Denote by $c^{2g} \geq 0$ old-age consumption in case the project is successful and by $c^{2b} \geq 0$ in case it is a failure. Since agents have limited liability, the constraint for old-age consumption reads

$$\begin{cases} c^{2g} &= r_t D + \pi(I_t) - R_t B_t \\ c^{2b} &= r_t D \end{cases}, \tag{3.2}$$

where $\pi(I_t) := f'(\Omega(I_t))I_t$ is the revenue from a successful project and $R_t B_t$ the loan repayment obligation.

The objective of a young agent is to maximise her expected utility of lifetime consumption.

[3] We assume that the bank possess a monitoring technology that enables it to observe the investment behaviour and enforce the contract. A stipulated investment is a particular form of monitoring. For details we refer to the seminal contribution by Holmström and Tirole (1997) on the role of monitoring in settings with limited liability.

Inserting the budget constraints (3.1) and (3.2), the agent's objective function becomes[4]

$$\max_{0\le D\le w_t^d} \; u(w_t^d - D) + p(I_t)\, v(r_t D + \pi(I_t) - R_t B_t) + (1 - p(I_t))\, v(r_t D). \tag{3.3}$$

Given a loan contract (B_t, I_t, R_t), a deposit rate r_t, and a wage rate w_t, a solution to (3.3) is given by the agent's *savings function* S, which is well defined by

$$S(w_t, B_t, I_t, R_t, r_t) \coloneqq \operatorname*{argmax}_{0\le D\le w_t^d} \; u(w_t^d - D) + p(I_t)\, v(r_t D + \pi(I_t) - R_t B_t) + (1 - p(I_t))\, v(r_t D). \tag{3.4}$$

Observe that the optimal amount of savings depends on the loan contract offered by the bank. Inserting (3.4) into the objective function in (3.3) establishes the value function for Problem (3.3), which is denoted by

$$V(w_t, B_t, I_t, R_t, r_t). \tag{3.5}$$

The agent may, however, also *reject* the loan contract and decide to undertake the project without funding from the bank. To do so, she will invest the amount I^{A} into the project and deposit D^{A} at the bank in order to safeguard old-age consumption in case the project fails. The decision problem for this case is

$$\begin{aligned} \max_{I^{\mathrm{A}}, D^{\mathrm{A}}} \;& u(w_t - D^{\mathrm{A}} - I^{\mathrm{A}}) + p(I^{\mathrm{A}})\, v(r_t D^{\mathrm{A}} + \pi(I^{\mathrm{A}})) + (1 - p(I^{\mathrm{A}}))\, v(r_t D^{\mathrm{A}}) \\ \text{s.t.} \quad & I^{\mathrm{A}}, D^{\mathrm{A}} \ge 0 \;\text{ and }\; I^{\mathrm{A}} + D^{\mathrm{A}} \le w_t. \end{aligned} \tag{3.6}$$

The value function associated with Problem (3.6) is well defined and stipulates the reservation utility of the agent (i.e., her individual rationality level), which for any given w_t and r_t is

$$U_{\mathrm{res}}(w_t, r_t).$$

The decision problem of the bank is considered next. Since the bank is collectively owned, it offers a loan contract (B_t, I_t, R_t) and a deposit rate r_t so as to maximise the representative agent's expected utility V as defined by (3.5). Using the law of large numbers, the bank correctly anticipates that for any given I_t, the loan default rate is $1 - p(I_t)$. Therefore, the two feasibility constraints of the bank are the *profit constraint*

$$p(I_t) R_t B_t - r_t D_t \ge 0, \tag{PrC}$$

stating that bank profits must be non-negative, and the *resource constraint*

$$D_t \ge B_t, \tag{RC}$$

noting that the collective bank has no equity.

[4]Observe from the definition of $\pi(I)$ that both the bank and young agents are not price-takers with respect to the return on capital in the production sector. Here, we adopt the view put forward by Hellwig (1980) and Kyle (1989) who argue that the price-taking assumption is implausible in competitive markets with rational expectations because agents would exploit the informational advantage.

Both the loan and the deposit contract must be compatible with the savings behaviour of the agent. Because the amount saved is at the discretion of the agent, the bank has to fulfil the *incentive compatibility constraint*

$$D_t = S(w_t, B_t, I_t, R_t, r_t), \tag{IC}$$

in order to obtain the amount of deposits required in (RC). Moreover, since the agent may decide to invest without financing from the bank, the loan contract must be designed in such a way that the agent prefers the loan contract to undertaking the project individually. Formally, this *participation constraint* reads

$$V(w_t, B_t, I_t, R_t, r_t) \geq U_{\text{res}}(w_t, r_t), \tag{PC}$$

that is, the expected utility of accepting both the loan and the deposit contract is at least as high as the reservation utility.

Given the wage rate w_t, the maximisation problem of the bank now takes the form

$$\begin{aligned} \max_{B,I,R,r} \; & V(w_t, B, I, R, r) \\ \text{s.t.} \;\; & p(I)RB - rS(w_t, B, I, R, r) \geq 0, \quad S(w_t, B, I, R, r) \geq B, \\ \text{and} \;\; & V(w_t, B, I, R, r) \geq U_{\text{res}}(w_t, r). \end{aligned} \tag{3.7}$$

3.2 Efficient Contracts

We next establish the efficient allocation that a social planner would implement. To this end, we consider a myopic social planner. Given the wage rate w_t, the planner's objective in period t is to maximise the welfare of the generation born in t.[5] Applying the law of large numbers, the mass of successful agents of the subsequent period is $p(I)$, while the mass of failed agents is $1 - p(I)$. Capital income in the subsequent period is $g(I)$, independently of the state of nature. The social planner's maximisation problem thus becomes

$$\begin{aligned} \max_{I,c^1,c^{2g},c^{2b}} \; & u(c^1) + p(I)\, v(c^{2g}) + (1 - p(I))\, v(c^{2b}) \\ \text{s.t.} \;\; & I, c^1, c^{2g}, c^{2b} \geq 0, \quad c^1 + I \leq w_t, \\ \text{and} \;\; & p(I)\, c^{2g} + (1 - p(I))\, c^{2b} \leq g(I). \end{aligned} \tag{3.8}$$

The solution to Problem (3.8) is stated in following proposition.

Proposition 1 (Efficient Allocation)**.**
Let the hypotheses of Assumptions 1 – 3 be satisfied and $w_t > 0$ be given. Then Problem

[5] This modelling choice allows to link the loan contract introduced in Banerji et al. (2004) to the notion of an efficient allocation. Since our research question is not concerned with financial intermediation and intergenerational externalities, as for example, in Ennis and Keister (2003), a myopic social planner is justified.

(3.8) *admits a unique solution* $(I_t^\star, c_t^{1\star}, c_{t+1}^{2g\star}, c_{t+1}^{2b\star})$, *where the efficient consumption plan is*

$$c_t^{1\star} = w_t - I_t^\star, \qquad c_{t+1}^{2g\star} = c_{t+1}^{2b\star} = g(I_t^\star).$$

The efficient investment level satisfies $0 < I_t^\star < w_t$ *and solves*

$$\max_{0 \le I \le w_t} \; u(w_t - I) + v(g(I)).$$

The allocation $(c_t^{1\star}, c_{t+1}^{2g\star}, c_{t+1}^{2b\star})$ will henceforth be called efficient. Observe that an efficient allocation is characterised by the fact that the young generation consumes its wage income less the efficient investment level, while the old generation consumes aggregate capital income, which is completely smoothed out across the two possible states of nature.

Naturally, the question arises whether financial intermediation that offers loan and deposit contracts in line with Problem (3.7) can implement the efficient allocation determined in Proposition 1. As emphasised in Myerson (1979), in situations in which agents' private actions are difficult to control, the arising incentive problems and constraints make it questionable whether an efficient outcome can be achieved. To address this problem, we next define an *efficient contract* as a contract that implements the efficient allocation and is optimal for both agents and the bank.

Definition 1 (Efficient Contract)**.**
Given the wage rate w_t, *a loan contract* (B_t, I_t, R_t) *together with a deposit rate* r_t *is called an efficient contract (in period* t*) if the following holds true:*

(i) The quadruple (B_t, I_t, R_t, r_t) *solves Problem* (3.7).

(ii) The allocation induced by (B_t, I_t, R_t, r_t) *is efficient in the sense of Proposition 1.*

An efficient contract maximises the welfare of the representative agent, thereby respecting the participation constraint (PC), the incentive constraint (IC), the profit constraint (PrC), and the resource constraint (RC) of the bank. In particular, incentive compatibility of the efficient deposit rate r_t implies that agents decide at their own discretion to save the amount of funds required for complete risk sharing. With an efficient contract, agents are completely insured and old-age consumption is independent of the success of the project because the efficient loan portfolio of the bank diversifies away aggregate risk.

Our next proposition establishes the existence of a unique efficient contract.

Proposition 2 (Existence of an Efficient Contract)**.**
Let the hypotheses of Assumptions 1 – 3 be satisfied and $w_t > 0$ *be given. Then there exists a uniquely determined efficient contract* (B_t, I_t, R_t, r_t), *which is given by the following equations:*

(i) The investment level satisfies $0 < I_t < w_t$ *and solves*

$$\max_{0 \leq I \leq w_t} u(w_t - I) + v(g(I)). \tag{3.9}$$

(ii) The deposit rate is

$$r_t = g'(I_t). \tag{3.10}$$

(iii) The loan interest rate is

$$R_t = \frac{g'(I_t)}{p(I_t)}. \tag{3.11}$$

(iv) The loan size is

$$B_t = \frac{g(I_t)}{g'(I_t)}. \tag{3.12}$$

(v) The profit constraint (PrC) *and the resource constraint* (RC) *of the bank are binding with*

$$D_t = B_t = S(w_t, B_t, I_t, R_t, r_t). \tag{3.13}$$

An immediate implication of Proposition 2 can be deduced from (3.11) and (3.12): with an efficient contract, the bank extracts no rent as it seizes all the proceeds of successful projects,[6]

$$R_t B_t = \pi(I_t).$$

By a slight abuse of notation, the savings function (3.4) corresponding to an *efficient* loan contract thus takes the standard form

$$S(w_t^d, r_t) \coloneqq \operatorname*{argmax}_{0 \leq D \leq w_t^d} u(w_t^d - D) + v(r_t D). \tag{3.14}$$

Example 2 (Homothetic Preferences)**.**
For homothetic preferences, the savings function (3.14) *becomes* $S(w_t^d, r_t) = s(r_t)\, w_t^d$*, where* $0 \leq s(r_t) \leq 1$ *is the propensity to save. For the log-linear preferences* $u(c^1) = \ln(c^1)$ *and* $v(c^2) = \beta \ln(c^2)$ *with* $\beta > 0$*, the savings function is independent of the deposit rate so that* $S(w_t^d) = \frac{\beta}{1+\beta}\, w_t^d$*.*

A second implication of Proposition 2 resembles the *‚income effect‘* in Banerji et al. (2004). Namely, it follows from (3.12) that

$$B_t - I_t = \left(\frac{g(I_t)}{g'(I_t) I_t} - 1 \right) I_t > 0 \tag{3.15}$$

because the elasticity of g is less than unity.[7] As a consequence, the efficient contract

[6]Since consumers are awarded the full surplus, enjoy complete insurance, and the bank extracts no rent, our model can be reinterpreted as a model of perfect competition between banks.

[7]This is a well-known property of concave functions. The efficient contract can be interpreted as the agent selling her project to the bank for the amount $B_t - I_t > 0$ and then saving the amount $S(w_t^d, r_t)$.

enlarges the disposable income of the representative young agent, $w_t^d = w_t + B_t - I_t > w_t$, implying that she has no incentive to reject the loan contract and invest without funding from the bank. In the proof of Proposition 2, we establish that the expected utility of an efficient contract is strictly larger than the expected utility of an idiosyncratic investment combined with precautionary savings. Hence, each agent will accept the efficient contract and save the amount required to implement efficient risk sharing.

Remark 1. *For non-concave g, it is possible that $w_t^d < w_t$. In this case, the agent rejects the loan contract because she would be better off with saving out of wage income w_t, even without investing into the project. The bank could still implement the efficient allocation by tying the loan contract (B_t, I_t, R_t) to the deposit contract r_t and offering agents who want to save only a deposit rate $0 < \tilde{r}_t < r_t$ that makes them worse off. The existence of $\tilde{r}_t$ is seen as follows. The strict concavity of v implies*

$$U_{\text{res}}(w_t, \tilde{r}_t) < \max\Big\{ u(w_t - D^{\text{A}} - I^{\text{A}}) + v(\tilde{r}_t D^{\text{A}} + g(I^{\text{A}})) \;\Big|\; D^{\text{A}} \geq 0,\, I^{\text{A}} \geq 0,\, D^{\text{A}} + I^{\text{A}} \leq w_t \Big\}.$$

For $\tilde{r}_t = 0$, the objective function on the r.h.s. attains its maximum in $(D_t^{\text{A}} = 0, I_t^{\text{A}} = I_t)$, yielding the utility level $u(w_t - I_t) + v(g(I_t))$. This shows that for $\tilde{r}_t = 0$, the agent is strictly better off accepting the tying contract. However, since U_{res} is continuously increasing in r, there must exist a positive deposit rate $0 < \tilde{r}_t < r_t$ such that the agent still accepts,

$$U_{\text{res}}(w_t, \tilde{r}_t) = u(w_t - I_t) + v(g(I_t)) = V(w_t, B_t, I_t, R_t, r_t).$$

Remark 2. *Set $\eta(I) \coloneqq p'(I)I/p(I)$ and $\epsilon(I) \coloneqq f''(\Omega(I))\,\Omega(I)/f'(\Omega(I))$. Using the definition of g, Proposition 2 then implies*

$$\begin{aligned} R_t &= [1 + \epsilon(I_t)]\,[1 + \eta(I_t)]\, f'(\Omega(I_t)) \\ r_t &= [1 + \epsilon(I_t)]\,[1 + \eta(I_t)]\, p(I_t)\, f'(\Omega(I_t)) \\ B_t &= \big([1 + \epsilon(I_t)]\,[1 + \eta(I_t)]\big)^{-1} I_t. \end{aligned}$$

Apart from the multiplier $1 + \epsilon(I_t)$, these equations coincide with the functional form of the efficient contract in Banerji et al. (2004, p. 2225). The multiplier accounts for the financial intermediary exploiting the knowledge of how the invested amount I_t affects the return on investment. For a Cobb-Douglas production function, the multiplier is constant.

4 Capital Accumulation and Qualitative Dynamics

We analyse the qualitative dynamics induced by efficient contracts next. The aggregate capital stock of the economy is determined by the total capital endowment of successful projects. The law of large numbers implies that, on aggregate, the share of successful production projects in period $t + 1$ is $p(I_t)$, where I_t is the efficient investment level of period t. It follows from Proposition 2 that I_t is stipulated by the *investment function*

$\mathcal{I} : \mathbb{R}_+ \to \mathbb{R}_+$ that is well defined by setting

$$I_t = \mathcal{I}(w_t) \coloneqq \underset{0 \le I \le w_t}{\operatorname{argmax}} \; u(w_t - I) + v(g(I)). \tag{4.1}$$

Therefore, productive capital of the subsequent period $t+1$ is

$$k_{t+1} = \Omega(I_t) = \Omega(\mathcal{I}(w_t)).$$

In a perfectly competitive environment, labour is paid its marginal product. Letting $w_t = w(k_t) \coloneqq f(k_t) - f'(k_t)k_t$ denote the marginal product of labour, it follows that capital accumulation is driven by the *economic law* $G : \mathbb{R}_+ \to \mathbb{R}_+$, defined by

$$k_{t+1} = G(k_t) \coloneqq \Omega\big(\mathcal{I}(w(k_t))\big). \tag{4.2}$$

The economic law (4.2) gives an explicit time-forward representation of the dynamics. Indeed, the growth paths of the economy $\{k_t\}_{t=0}^{\infty}$ with initial capital $k_0 > 0$ are recursively generated by the time-one map G.

Remark 3. *The evolution of investment levels is driven by*

$$I_t = H(I_{t-1}) \coloneqq \mathcal{I}\big(w(\Omega(I_{t-1}))\big), \tag{4.3}$$

noting that $w_t = w(\Omega(I_{t-1}))$. *The dynamics induced by* G *and* H *are qualitatively the same if Assumption 2 is satisfied so that* Ω *is invertible. Inserting* (3.10) – (3.12) *into* (3.13) *yields an implicit difference equation describing the same dynamics as* (4.3). *This difference equation corresponds to Equation (21) in Banerji et al. (2004), which the authors use to analyse the dynamics of their model.*

We are now in the position to state the main result of this article.

Theorem 1 (Monotonic Growth)**.**
Let Assumptions 1 – 3 be satisfied and assume the bank offers efficient contracts. Then $G' > 0$ *so that the dynamics of the economy is monotonic.*

Theorem 1 demonstrates that if financial intermediation allocates efficient contracts, then the resulting dynamics is always monotonic. All growth paths $\{k_t\}_{t=0}^{\infty}$ generated by (4.2) are either monotonically increasing or monotonically decreasing, as depicted in Figure 1 below.

The result that efficient contracts rule out business cycles and complex dynamics contradicts the findings in Banerji et al. (2004) who argue that efficient risk sharing through financial intermediation may generate endogenous fluctuations. Our theorem implies that the enlargement of disposable income is, in fact, a mere byproduct of an incentive problem without implications for the qualitative nature of the macroeconomic dynamics. More specifically, an immediate consequence of Theorem 1 is the following corollary.

Corollary 1 (Endogenous Fluctuations).
If $G'(k_t) < 0$ for some k_t, then the contract (B_t, I_t, R_t, r_t) minimises the expected utility of the representative agent.

Corollary 1 states that endogenous fluctuations can only occur if contracts are *inefficient*.[8]

Remark 4. *The discrepancy between Theorem 1 and the findings in Banerji et al. (2004) is that the authors seem to have overlooked that the first-order condition of their decision problem may admit more than one solution. We will illustrate this observation along with Corollary 1 in Section 5 below by showing that ‚inefficient' contracts are responsible for complex dynamics rather than financial intermediation.*

The qualitative dynamics induced by efficient contracts may be classified by the stability properties of the steady states $k_\star = G(k_\star)$ of G.

Proposition 3 (Properties of Steady States).
Let Assumptions 1 – 3 be satisfied and assume the bank offers efficient contracts. Then the following holds.

(i) The origin $k_\star = 0$ is a steady state of G if and only if $w(0) = 0$.
(ii) If either $w(0) > 0$ or $\lim_{k\to 0} G'(k) > 1$, then there exists at least one positive steady state $k_\star > 0$ of G. The largest one of these steady states is asymptotically stable.

Proposition 3 implies that the forward dynamics of our model is qualitatively equivalent to the dynamics of the standard two-period lived OLG model, cf. De La Croix and Michel (2002). At this point, it is worthwhile considering a short example that illustrates our main results using a standard parameterisation.

Example 3 (Monotonic Dynamics).
Consider the success function $p(I) = \frac{1}{1+I}$ combined with the log-linear utility function $u(c^1) = \ln(c^1)$ and $v(c^2) = \beta \ln(c^2)$, where $\beta > 0$; cf. Examples 1 and 2. Let the technology be given by a Cobb-Douglas production function of the form $f(k) = Ak^\alpha$, where $A > 0$ and $0 < \alpha < 1$. In this case, the investment function (4.1) takes the form

$$\mathcal{I}(w_t) = \sqrt{\left(\tfrac{1+\alpha\beta}{2}\right)^2 + \alpha\beta w_t} - \tfrac{1+\alpha\beta}{2}$$

and the evolution of capital-labour ratios is driven by

$$k_{t+1} = G(k_t) = 1 - \left(\frac{1-\alpha\beta}{2} + \sqrt{\left(\frac{1+\alpha\beta}{2}\right)^2 + \alpha\beta(1-\alpha)Ak_t^\alpha}\right)^{-1}.$$

[8] Inefficient contracts are, however, ruled out if the strict concavity of Ω stated in Assumption 2 is satisfied because in this case, the objective function of agents is strictly concave.

Since $G' > 0$, the dynamics is monotonic and all growth paths converge to a unique positive steady state $k_\star$ that is asymptotically stable, cf. Figure 1.

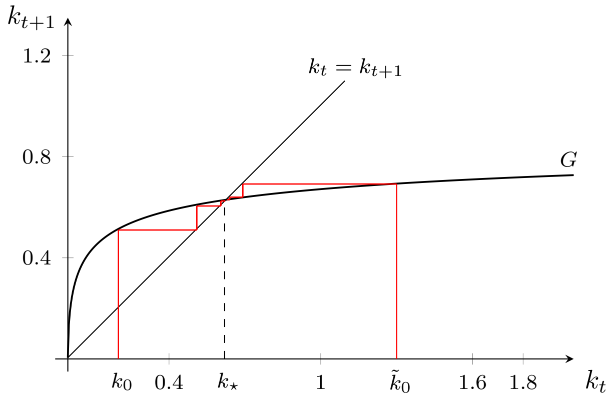


Figure 1: Monotonic dynamics with steady state $k_\star > 0$ ($A = 30$, $\alpha = 0.6$, $\beta = 1$).

We will show next that the steady states of the economy will generically be *dynamically inefficient* in the sense used in macroeconomics. Given a wage rate w_t, youthful consumption in period t is

$$c_t^1 = w_t - I_t \tag{4.4}$$

and old-age consumption in period t

$$c_t^{2b} = c_t^{2g} = g(I_{t-1}) = c_t^2. \tag{4.5}$$

Adding (4.4) and (4.5), total consumption per capita in period t becomes

$$c_t := c_t^1 + c_t^2 = w_t - I_t + g(I_{t-1}) = f(k_t) - I_t. \tag{4.6}$$

Since $k_{t+1} = \Omega(I_t)$, stationary allocations $(\bar{k}, \bar{c})$, where $\bar{c}$ denotes stationary total consumption with $\bar{c} = \bar{c}^1 + \bar{c}^2$, are given by

$$\bar{c} = \phi(\bar{I}) := f(\Omega(\bar{I})) - \bar{I}.$$

Observe that $\bar{c}$ is maximal if the function ϕ attains its maximum.

Lemma 2 (Maximal Consumption)**.**
Under the hypotheses of Assumptions 1 – 3, the map ϕ attains its global maximum at the golden-rule investment level $I_G > 0$, which is uniquely determined by

$$f'(\Omega(I_G))\,\Omega'(I_G) = 1. \tag{4.7}$$

Lemma 2 resembles the golden rule of capital accumulation of the standard Solow (1956) growth model. Denote the capital-labour ratio corresponding to I_G by $k_G = \Omega(I_G)$. If capital depreciates fully and the population profile is stationary, as is the case in our model, then Solow's golden-rule capital-labour ratio k_G^{S} is determined by $f'(k_G^{\mathrm{S}}) = 1$.

Since f is strictly concave and Assumption 2 implies $0 < \Omega' < 1$, it follows from (4.7) that $k_G < k_G^{\mathrm{S}}$. The lower golden-rule value in our model is due to a positive failure rate of the production projects.

Observe that k_G is solely determined by the production technology and the probability distribution. Since any steady state will, by construction, depend on agents' preferences, k_G will generally not be a steady state of the dynamical system (4.2). Indeed, k_G is a steady state of G if and only if $I_G = \mathcal{I}(w(k_G))$. Put differently,

$$c_G^1 = w(k_G) - I_G, \quad c_G^2 = g(I_G)$$

is a steady-state consumption plan of the dynamical system (4.2) if and only if I_G solves

$$-u'(w(\Omega(I)) - I) + v'(g(I))\, g'(I) \stackrel{!}{=} 0.$$

Finally, observe from (3.15) that

$$c_G^1 + \frac{c_G^2}{r_G} = w(k_G) + \left(\tfrac{g(I_G)}{g'(I_G) I_G} - 1\right) I_G =: w_G^d,$$

so that in a golden-rule steady state, disposable income is fully consumed. In general, however, a golden-rule steady state will not obtain so that, generically, the economy will be dynamically inefficient.

5 Inefficient Contracts

Banerji et al. (2004) discuss an example with a success function p for which Ω violates Assumption 2. As a consequence, the objective function of agents is not strictly concave. In this section we discuss their example from our perspective. To this end, we analyse the forward dynamics instead of the backward dynamics as done in Banerji et al. (2004) because the former is economically more meaningful.

Following on from Banerji et al. (2004, Example 2, p. 2228), consider the case of a log-linear utility function with $\beta = 1$ combined with a Cobb-Douglas production function. Let the success function be

$$p(I) = \frac{0.32 e^I}{1.2 + 1.901885 e^I - I^3}.$$

The objective function in Problem (3.9) now takes the form

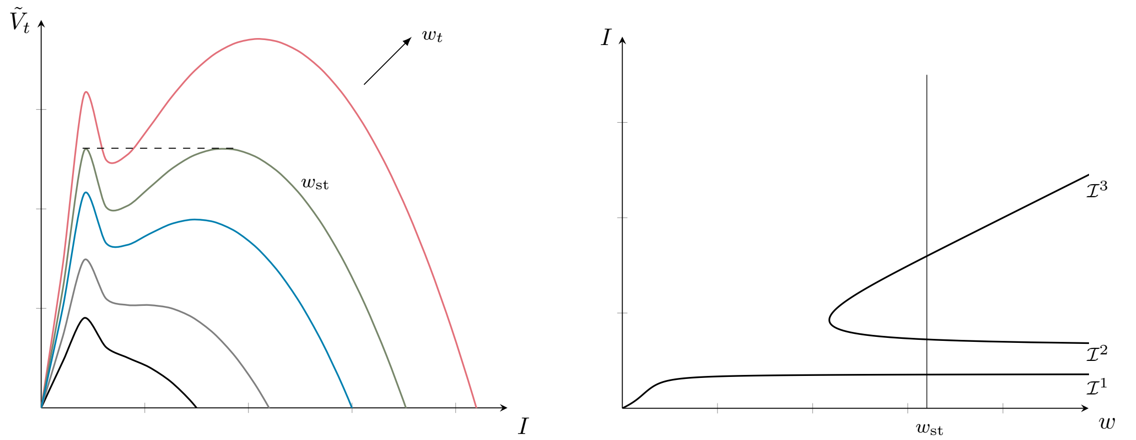
$$V_t(I) = \ln\big((w_t - I)\big) + \ln\big(g(I)\big)$$

and is monotonically transformed into

$$\tilde{V}_t(I) = (w_t - I)\, g(I). \tag{5.1}$$

Observe that $\tilde{V}_t(0) = \tilde{V}_t(w_t) = 0$ because $g(0) = 0$ by Lemma 1. Since, for any given w_t, $\tilde{V}_t$ is continuous on the compact interval $[0, w_t]$, there exists at least one investment level

$0 < I_t < w_t$ that maximises $\tilde{V}_t$. In this case, however, it turns out that neither g nor $\tilde{V}_t$ are concave.[9]



(a) Expected utility $\tilde{V}_t(I)$ over $0 \leq I \leq w_t$ **(b)** Solutions to the first-order condition (5.2)

Figure 2: Expected utility levels and critical points.

The objective function $\tilde{V}_t$ is portrayed in Figure 2a for different levels of w_t. The first-order condition for a maximum of (5.1) takes the form

$$w_t \stackrel{!}{=} \left(1 + \frac{g(I_t)}{g'(I_t)I_t}\right) I_t. \tag{5.2}$$

A numerical investigation reveals that, depending on the level of w_t, (5.2) admits up to three distinct solutions, cf. Figure 2b. For all sufficiently low wage rates w_t, there exists a unique solution $\mathcal{I}^1(w_t)$ to (5.2) in which the expected utility (5.1) attains its global maximum, cf. the black line in Figure 2a. For all sufficiently large wage rates w_t, there exist three distinct solutions $\mathcal{I}^1(w_t) < \mathcal{I}^2(w_t) < \mathcal{I}^3(w_t)$. From Figure 2a, we can infer that for all wage rates $w_t < w_{\text{st}}$, with w_{st} defined by

$$\tilde{V}_t(\mathcal{I}^1(w_{\text{st}})) = \tilde{V}_t(\mathcal{I}^3(w_{\text{st}})),$$

the lowest investment level $\mathcal{I}^1(w_t)$ is the global maximum. The global maximum obtains for the largest investment level $\mathcal{I}^3(w_t)$ whenever $w_t > w_{\text{st}}$, cf. the red line in Figure 2a. Observe that $\mathcal{I}^2(w_t)$ is always a local minimum.

Since an *efficient* contract must *maximise* the expected utility (5.1), the investment function $\mathcal{I}$ may be formally defined by

$$I_t = \mathcal{I}(w_t) := \operatorname{argmax}\Big\{\tilde{V}_t(I^i) \;\Big|\; I^i = \mathcal{I}^i(w_t),\ i = 1, \ldots, n(w_t)\Big\}, \tag{5.3}$$

where $\mathcal{I}^i(w_t)$ is a solution to (5.2) and $n = n(w_t)$ denotes the total number of solutions

[9]We slightly extend the example in Banerji et al. (2004) by allowing the investment levels to become arbitrarily small. However, this change does not alter the main point of this section.

to (5.2), given w_t.[10] Using (5.3), the forward dynamics is governed by the time-one map

$$k_{t+1} = G(k_t) = \Omega\big(\mathcal{I}(w(k_t))\big),$$

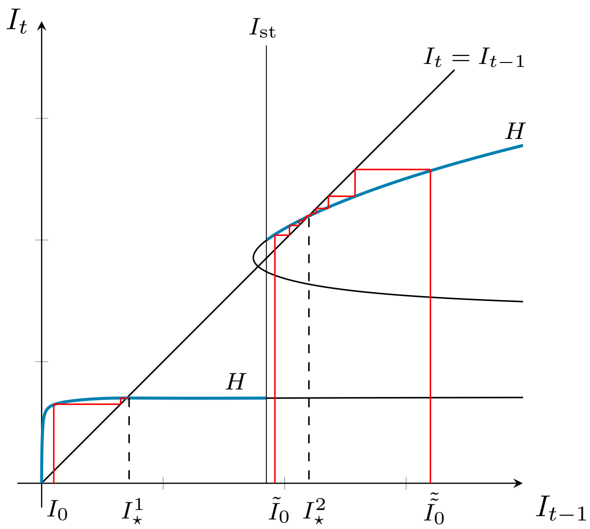
or, in terms of investment levels, by

$$I_t = H(I_{t-1}) = \mathcal{I}\big(w(\Omega(I_{t-1}))\big). \tag{5.4}$$

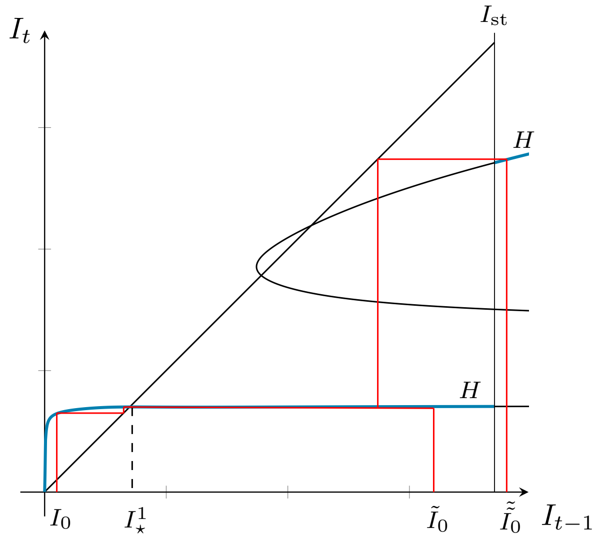
The map $I \mapsto H(I)$ is continuous, except for I_{st}, defined by

$$w(\Omega(I_{\mathrm{st}})) = w_{\mathrm{st}},$$

and strictly increasing. Depending on the magnitude of I_{st}, the dynamical system (5.4) has at least one and at most two positive steady states, denoted by $I_\star^1$ and $I_\star^2$, respectively. Contingent on the initial investment level $I_0 = \mathcal{I}(w(k_0))$, the growth paths of the economy either converge to $I_\star^1$ or to $I_\star^2$. Since H is monotonically increasing, convergence is monotone so that business cycles and complex dynamics cannot occur, see Figure 3.



(a) Two stable steady states.



(b) Unique stable steady state.

Figure 3: Dependence of the qualitative dynamics on initial conditions and technology.

This analysis demonstrates that efficient loan and deposit contracts cannot trigger business cycles, let alone complex dynamics. Corollary 1 implies that these may only occur if the bank mistakenly offers the inefficient contract corresponding to $\mathcal{I}^2(w_t)$. In the case of agents accepting inefficient contracts, we find numerical evidence for three possible scenarios regarding endogenous fluctuations. First, convergence to an inefficient steady state $I_\star^{\mathrm{IE}}$, as illustrated in Figure 4.

[10]Since in this case g is not strictly concave, the bank might be required to tie the loan and the deposit contract, cf. Remark 1.

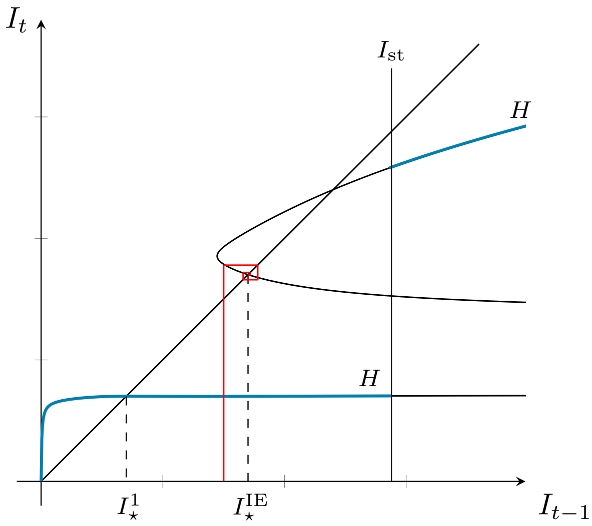
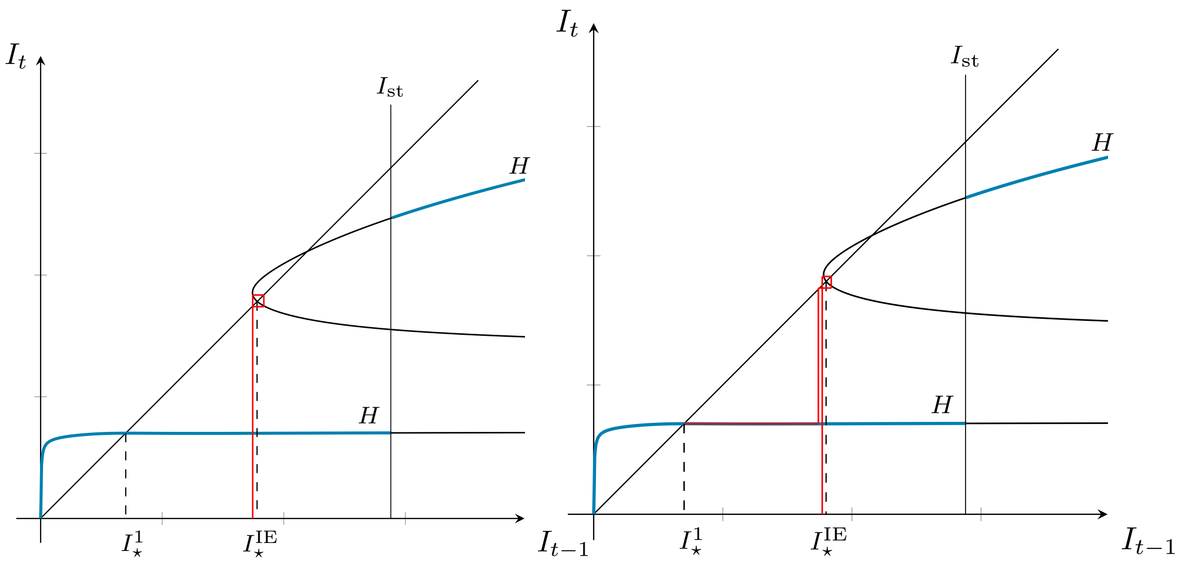


Figure 4: Business cycles with inefficient contracts ($A = 60, \alpha = 0.33$).

Second, a persistent period-2 cycle around an inefficient steady state $I_\star^{\text{IE}}$, as portrayed in Figure 5a. Third, initial fluctuations around an inefficient steady state $I_\star^{\text{IE}}$, which eventually converge to the stable steady state with the low efficient investment level $I_\star^1$. This scenario is portrayed in Figure 5b.



(a) Stable period-2 cycle ($\alpha = 0.32$). **(b)** Stable efficient steady state ($\alpha = 0.315$).

Figure 5: Qualitative dynamics with inefficient contracts ($A = 60$).

6 Conclusion

Following on from Banerji et al. (2004), this article examined an OLG model with idiosyncratic risk in which financial intermediation arises endogenously as a vehicle to share risk. Welfare of agents is maximised by efficient contracts that provide complete insurance and perfect consumption smoothing while the bank extracts zero rent. To implement efficient contracts, the bank must enlarge the disposable income of agents because savings decisions are at the agents' discretion. The central result of this article is that in any two-period lived OLG model, efficient financial intermediation with complete insurance can neither generate business cycles nor complex dynamics. The resulting forward dynamics is monotonic so that qualitatively, it is indistinguishable from the dynamics of the standard two-period lived OLG model. This finding contradicts Banerji et al. (2004), who claim the exact opposite, namely, that financial intermediation can be responsible for business cycles. Our analysis revealed that business cycles and thus complex dynamics in their example can only be triggered by inefficient loan contracts. The assumption of homogeneous two-period lived agents with rational expectations is, of course, a benchmark scenario and therefore limited. Real-world phenomena involve a wide variety of heterogeneous agents who, for example, differ in preferences, income, and subjective beliefs. This opens up an avenue for further research in which we will investigate the question of which features of financial intermediation might trigger business cycles.

Appendix: Proofs of Main Results

Proof of Lemma 1. Differentiating $g(I) = f'(\Omega(I))\,\Omega(I)$ yields

$$g'(I) = \Omega'(I)\, f'(\Omega(I)) \left[1 + \frac{f''(\Omega(I))\,\Omega(I)}{f'(\Omega(I))}\right].$$

By Assumption 2, $\Omega' > 0$. By Assumption 3, $f' > 0$, $f'' < 0$, and $f''(k)k/f'(k) > -1$ for each $k \in \mathbb{R}_+$. Hence, g is strictly increasing. Since $\lim_{k\to 0} f'(k)k = 0$ and $\Omega(0) = 0$, it follows that $g(0) = 0$.[11] Strict concavity of g holds because

$$g''(I) = \Omega''(I)\, f'(\Omega(I)) \left[1 + \frac{f''(\Omega(I))\,\Omega(I)}{f'(\Omega(I))}\right] + \Omega'(I)^2\, f''(\Omega(I)) \left[2 + \frac{f'''(\Omega(I))\,\Omega(I)}{f''(\Omega(I))}\right] < 0,$$

noting that $\Omega'' < 0$ by Assumption 2 and $f'''(k)k/f''(k) > -2$ by Assumption 3. □

Proof of Proposition 1. Let w_t be arbitrary but fixed. By Lemma 1, g is strictly increasing and strictly concave with $g(0) = 0$. Assumption 1 implies that in an optimum, $c^1 = w_t - I$. The first-order conditions are therefore

$$-u'(w_t - I) + p'(I)[v(c^{2g}) - v(c^{2b})] + \lambda[g'(I) - p'(I)(c^{2g} - c^{2b})] = 0 \qquad \text{(A.1)}$$

$$p(I)\, v'(c^{2g}) - \lambda p(I) = 0 \qquad \text{(A.2)}$$

$$(1 - p(I))\, v'(c^{2b}) - \lambda(1 - p(I)) = 0 \qquad \text{(A.3)}$$

[11] For a formal proof that Assumption 3 implies $\lim_{k\to 0} f'(k)k = 0$, we refer to De La Croix and Michel (2002, p. 308).

and the complementary slackness condition is

$$\lambda[g(I) - p(I)\, c^{2g} - (1 - p(I))\, c^{2b}] = 0. \tag{A.4}$$

Assumption 1 implies that in an optimum, the constraint on the consumption plan must hold with equality. Since $g(0) = 0$, the Inada conditions on v imply that $I > 0$ because otherwise $c^{2g} = c^{2b} = 0$ by (A.4). Therefore, $0 < p(I) < 1$. Conditions (A.2) and (A.3) then imply $v'(c^{2g}) = v'(c^{2b}) = \lambda > 0$. Since $v'' < 0$, it follows from (A.4) that a social optimum requires

$$c^{2g} = c^{2b} = g(I). \tag{A.5}$$

Moreover, (A.1) reduces to

$$-u'(w_t - I) + v'(g(I))\, g'(I) = 0. \tag{A.6}$$

Observe that (A.6) is the first-order condition of the maximisation problem

$$\max_{0 \le I \le w_t} \; u(w_t - I) + v(g(I)). \tag{A.7}$$

The objective function in (A.7) is either already a continuous function or can be transformed into a continuous function on the compact interval $[0, w_t]$ using the exponential function. Hence, a solution $I_t^\star$ to (A.7) exists. It follows that any solution $(I_t^\star, c_{t+1}^{2g\star}, c_{t+1}^{2b\star})$ to the first-order conditions (A.1) – (A.3) must satisfy (A.5) with $I_t^\star$ being a maximiser of Problem (A.7). In other words, the social planner's problem (3.8) reduces to Problem (A.7), so that any maximiser $I_t^\star$ of (A.7) together with (A.5) and $c_t^{1\star} = w_t - I_t^\star$ is a maximiser of (3.8). The concavity of g implies that the objective function in (A.7) is strictly concave so that the social optimum is uniquely determined. □

Proof of Proposition 2. The proof works in **four steps**.

Step 1 (Relaxed problem). We establish the existence and uniqueness of a solution to Problem (3.7) without the participation constraint. The Lagrangian of Problem (3.7) without the participation constraint is

$$\begin{aligned} \mathcal{L}(B, I, R, r, \lambda_1, \lambda_2) \coloneqq{}& u(w_t + B - I - D) + p(I)\, v(rD + \pi(I) - RB) \\ &+ ((1 - p(I))\, v(rD) + \lambda_1(p(I)RB - rD) + \lambda_2(D - B), \end{aligned} \tag{A.8}$$

where $\lambda_1, \lambda_2 \ge 0$ are the Lagrange multipliers and $D = S(w_t, B, I, R, r)$ to simplify notation. The four first-order conditions for a solution (B_t, I_t, R_t, r_t) are:

$$0 = p(I_t)R_t\Big[v'(r_tD_t + \pi(I_t) - R_tB_t) - \lambda_1\Big] + \lambda_2 - u'(w_t + B_t - I_t - D_t) + (\lambda_1 r_t - \lambda_2)\,\frac{\partial S}{\partial B}(w_t, B_t, I_t, R_t, r_t) \tag{A.9}$$

$$0 = h'(I_t)\,p(I_t)\,v'(r_tD_t + \pi(I_t) - R_tB_t) + p'(I_t)\Big[v(r_tD_t + \pi(I_t) - R_tB_t) - v(r_tD_t)\Big] + \lambda_1 p'(I_t)R_tB_t - u'(w_t + B_t - I_t - D_t) - (\lambda_1 r_t - \lambda_2)\,\frac{\partial S}{\partial I}(w_t, B_t, I_t, R_t, r_t) \tag{A.10}$$

$$0 = \Big[\lambda_1 - v'\big(r_tD_t + \pi(I_t) - R_tB_t\big)\Big]p(I_t)B_t - (\lambda_1 r_t - \lambda_2)\,\frac{\partial S}{\partial R}(w_t, B_t, I_t, R_t, r_t) \tag{A.11}$$

$$0 = \Big[p(I_t)\,v'\big(r_tD_t + \pi(I_t) - R_tB_t\big) + (1 - p(I_t))\,v'(r_tD_t) - \lambda_1\Big]D_t - (\lambda_1 r_t - \lambda_2)\,\frac{\partial S}{\partial r}(w_t, B_t, I_t, R_t, r_t), \tag{A.12}$$

where $D_t = S(w_t, B_t, I_t, R_t, r_t)$. The two complementary slackness conditions are:

$$\lambda_1(p(I_t)R_tB_t - r_tD_t) = 0 \tag{A.13}$$

$$\lambda_2(D_t - B_t) = 0. \tag{A.14}$$

Assume that $\lambda_1 r_t - \lambda_2 = 0$. We will show below with (A.34) that in an optimum, this identity must hold. As a consequence, all terms involving derivatives of S in the first-order conditions (A.9) – (A.12) are zero. Since $p > 0$, (A.11) is equivalent to

$$\Big[\lambda_1 - v'(r_tD_t + \pi(I_t) - R_tB_t)\Big]B_t = 0. \tag{A.15}$$

By Assumption 1, $v' > 0$ so that two cases can occur in (A.15). First,

$$B_t > 0 \quad \text{and} \quad \lambda_1 = v'(r_tD_t + \pi(I_t) - R_tB_t) > 0. \tag{A.16}$$

Second, $B_t = 0$.

Case 1. Since $B_t > 0$ and $\lambda_1 > 0$, (A.13) requires

$$p(I_t)R_tB_t = r_tD_t, \tag{A.17}$$

stating that the profit constraint is binding. Inserting (A.16), it follows that (A.9) holds with

$$\lambda_2 = u'(w_t + B_t - I_t - D_t) > 0. \tag{A.18}$$

Since $\lambda_2 > 0$, (A.14) implies that the resource constraint is binding,

$$B_t = D_t > 0. \tag{A.19}$$

Using (A.19), it follows from (A.17) that

$$r_t = p(I_t)R_t. \tag{A.20}$$

Inserting (A.16) into (A.12) yields

$$(1 - p(I_t))D_t\Big[v'(r_tD_t) - v'(r_tD_t + \pi(I_t) - R_tB_t)\Big] = 0. \tag{A.21}$$

(A.21) has two possible solutions. First, since $0 < p(I) < 1$ for $I > 0$, $I_t = 0$ is a solution whenever $p(0) = 1$. In this case, (A.20) implies $R_t = r_t$ and thus $R_tB_t = r_tD_t$ so that the attained utility level is $u(w_t) + v(0)$. By Assumption 1, this level cannot be optimal. Since $v'' < 0$, the second solution to (A.21) is

$$R_tB_t = \pi(I_t). \tag{A.22}$$

It follows from (A.16) and (A.18) that

$$\lambda_1 = v'(r_tD_t) > 0 \quad \text{and} \quad \lambda_2 = u'(w_t - I_t) > 0. \tag{A.23}$$

Combining (A.20) with (A.22) yields

$$B_t = \frac{g(I_t)}{r_t}. \tag{A.24}$$

Since $D_t = B_t$, (A.24) implies

$$r_tD_t = g(I_t) \tag{A.25}$$

and therefore,

$$\lambda_1 = v'(g(I_t)). \tag{A.26}$$

Inserting (A.19), (A.22), (A.25), and (A.26), Condition (A.10) reduces to

$$-u'(w_t - I_t) + v'(g(I_t))\, g'(I_t) = 0. \tag{A.27}$$

Condition (A.27) determines the optimal investment level I_t. Observe that (A.27) is the first-order condition for the maximisation problem

$$\max_{0 \le I \le w_t} \; u(w_t - I) + v(g(I)). \tag{A.28}$$

Equations (A.22) and (A.25) imply that any utility-maximizing consumption plan of the relaxed problem (A.8) has to satisfy

$$c_t^1 = w_t - I_t \quad \text{and} \quad c_{t+1}^{2g} = c_{t+1}^{2b} = r_tD_t = g(I_t). \tag{A.29}$$

Hence, any solution to (A.8) is already determined by a solution I_t to Problem (A.28). Since Problem (A.28) coincides with Problem (3.9), existence and uniqueness of $0 < I_t < w_t$ obtain from the same arguments as presented in the proof of Proposition 1.

Case 2. If $B_t = 0$, then the profit constraint (PrC) implies that $r_tD_t = 0$. The strict concavity of v yields

$$u(w_t - I - D) + p(I)\, v(\pi(I)) + (1 - p(I))\, v(0) < u(w_t - I) + v(g(I)) \tag{A.30}$$

for all $I, D > 0$ with $I + D \le w_t$. Note that the r.h.s. of (A.30) contains the objective function of Problem (A.28), which assumes its maximum in $0 < I_t < w_t$ with I_t being determined by (A.27). Hence, $B_t = 0$ cannot be optimal.

It follows that $B_t > 0$ is optimal and that the optimal solution to the relaxed problem (A.8) is uniquely determined by (A.27) together with (A.29).

Step 2 (Incentive compatibility). In Step 1, the optimal deposit rate r_t has not yet been determined. Given the loan contract (B_t, I_t, R_t) determined in Step 1, the incentive constraint (IC) implies that deposits $D_t = S(w_t, B_t, I_t, R_t, r_t)$ must satisfy the first-order condition

$$u'(w_t + B_t - I_t - D_t) = \Big[p(I_t)\, v'(r_t D_t + \pi(I_t) - R_t B_t) + (1 - p(I_t))\, v'(r_t D_t)\Big] r_t. \quad \text{(A.31)}$$

Inserting (A.19), (A.22), and (A.25), Condition (A.31) simplifies to

$$-u'(w_t - I_t) + v'(g(I_t))\, r_t = 0. \quad \text{(A.32)}$$

A comparison of (A.27) with (A.32) shows that the optimal deposit rate is

$$r_t = g'(I_t). \quad \text{(A.33)}$$

Using (A.23), (A.26), and (A.32), it follows that r_t satisfies

$$\lambda_1 r_t - \lambda_2 = v'(g(I_t))\, r_t - u'(w_t - I_t) = 0, \quad \text{(A.34)}$$

thus justifying the assumption made at the outset of the proof. Finally, inserting (A.33) into (A.24) and (A.20) yields

$$B_t = \frac{g(I_t)}{g'(I_t)} \quad \text{and} \quad R_t = \frac{g'(I_t)}{p(I_t)}.$$

Step 3 (Efficiency). To see that the (B_t, I_t, R_t, r_t) computed above implements the efficient allocation, recall that the first-order conditions (A.6) and (A.27) coincide, so that $I_t = I_t^\star$ is the efficient investment level. It follows from (A.22) and (A.25) that

$$c_{t+1}^{2g} = c_{t+1}^{2b} = r_t S(w_t, I_t, B_t, R_t, r_t) = g(I_t) = g(I_t^\star) = c_{t+1}^{2g\star} = c_{t+1}^{2b\star}.$$

Thus, (B_t, I_t, R_t, r_t) implements the efficient allocation.

Step 4 (Participation constraint). We prove that the relaxed problem without the participation constraint (A.8) leads to the same solution as Problem (3.7) by showing that agents will accept the efficient contract (B_t, I_t, R_t, r_t) computed above.

An agent who rejects the efficient loan contract may save and invest with idiosyncratic risk, solving Problem (3.6). By the strict concavity of v, the objective function in (3.6) satisfies

$$\begin{aligned} &u(w_t - I^{\mathrm{A}} - D^{\mathrm{A}}) + p(I^{\mathrm{A}})\, v(r_t D^{\mathrm{A}} + \pi(I^{\mathrm{A}})) + (1 - p(I^{\mathrm{A}}))\, v(r_t D^{\mathrm{A}}) \\ \le\, &u(w_t - I^{\mathrm{A}} - D^{\mathrm{A}}) + v(r_t D^{\mathrm{A}} + g(I^{\mathrm{A}})) \end{aligned} \quad \text{(A.35)}$$

for all $I^{\mathrm{A}}, D^{\mathrm{A}} \geq 0$ with $I^{\mathrm{A}} + D^{\mathrm{A}} \leq w_t$. Replacing the objective function in Problem (3.6) with the r.h.s. of Inequality (A.35), an auxiliary problem obtains. We next establish that the uniquely determined maximiser of this auxiliary problem is $(I_t^{\mathrm{A}} = I_t, D_t^{\mathrm{A}} = 0)$, where I_t is the efficient investment level. This then proves that agents will never be better off by rejecting the efficient contract.

Observe first that the auxiliary objective function is strictly concave if g is strictly concave. The Inada conditions on u and v imply that any solution $(I_t^{\mathrm{A}}, D_t^{\mathrm{A}})$ to Problem (3.6) must satisfy $0 < I_t^{\mathrm{A}} + D_t^{\mathrm{A}} < w_t$. Thus, there are three cases left.

Case 1: $I_t^{\mathrm{A}} = 0$, $D_t^{\mathrm{A}} > 0$. The resulting first-order conditions in this case read

$$-u'(w_t - D^{\mathrm{A}}) + v'(r_t D^{\mathrm{A}})\, g'(0) + \lambda_1 = 0 \tag{A.36}$$

$$-u'(w_t - D^{\mathrm{A}}) + v'(r_t D^{\mathrm{A}})\, r_t = 0. \tag{A.37}$$

The Inada conditions imply that a solution $0 < D_t^{\mathrm{A}} < w_t$ to (A.37) exists. Inserting (A.37) into (A.36), we see that $(I_t^{\mathrm{A}} = 0, D_t^{\mathrm{A}})$ is a possible maximum if

$$\lambda_1 = v'(r_t D_t^{\mathrm{A}})[r_t - g'(0)] \geq 0,$$

Since $r_t = g'(I_t)$ and $g'' < 0$, it follows that λ_1 must be negative, implying that $(I_t^{\mathrm{A}} = 0,\ D_t^{\mathrm{A}})$ does not satisfy the first-order conditions.

Case 2: $I_t^{\mathrm{A}} > 0$, $D_t^{\mathrm{A}} = 0$. The corresponding first-order conditions are

$$-u'(w_t - I^{\mathrm{A}}) + v'(g(I^{\mathrm{A}}))\, g'(I^{\mathrm{A}}) = 0 \tag{A.38}$$

$$-u'(w_t - I^{\mathrm{A}}) + v'(g(I^{\mathrm{A}}))\, r_t + \lambda_2 = 0. \tag{A.39}$$

As shown in the proof of Proposition 1, the unique solution to (A.38) is the efficient investment level $I_t^{\mathrm{A}} = I_t$. Since $r_t = g'(I_t)$, it follows that $(I_t^{\mathrm{A}} = I_t, D_t^{\mathrm{A}} = 0)$ together with $\lambda_2 = 0$ solves the first-order conditions.

Case 3: $I_t^{\mathrm{A}} > 0$, $D_t^{\mathrm{A}} > 0$. The resulting first-order conditions are

$$-u'(w_t - I^{\mathrm{A}} - D^{\mathrm{A}}) + v'(r_t D^{\mathrm{A}} + g(I^{\mathrm{A}}))\, g'(I^{\mathrm{A}}) = 0 \tag{A.40}$$

$$-u'(w_t - I^{\mathrm{A}} - D^{\mathrm{A}}) + v'(r_t D^{\mathrm{A}} + g(I^{\mathrm{A}}))\, r_t = 0. \tag{A.41}$$

A comparison of (A.40) with (A.41) shows that any solution I_t^{A} requires $r_t = g'(I_t^{\mathrm{A}})$. Since $r_t = g'(I_t)$ and $g'' < 0$, it follows that $I_t^{\mathrm{A}} = I_t$. A comparison with Case 2 shows that $(I_t^{\mathrm{A}} = I_t, D_t^{\mathrm{A}} = 0)$ solves (A.40) and (A.41). Since both equations are strictly decreasing in D, no solution $(I_t^{\mathrm{A}} = I_t, D_t^{\mathrm{A}} > 0)$ exists.

These considerations show that Case 2 is decisive and that the maximum of the auxiliary problem is $u(w_t - I_t) + v(g(I_t))$. Hence,

$$U_{\text{res}}(w_t, r_t) \leq\ u(w_t - I_t) + v(g(I_t)) = V(w_t, B_t, I_t, R_t, r_t),$$

showing that agents are indeed willing to accept the efficient contract (B_t, I_t, R_t, r_t). $\square$

Proof of Theorem 1. Endogenous fluctuations are ruled out if $G' > 0$. Differentiating (4.2) yields

$$G'(k) = \Omega'\big(\mathcal{I}(w(k))\big)\,\mathcal{I}'(w(k))\,w'(k). \tag{A.42}$$

By Assumption 2, $\Omega' > 0$. Moreover, $w' > 0$ by Assumption 3. We show next that $\mathcal{I}' > 0$ such that $G' > 0$ holds. The investment function $\mathcal{I}$ is defined by the first-order condition

$$-u'(w - \mathcal{I}(w)) + v'\big(g(\mathcal{I}(w))\big)\,g'(\mathcal{I}(w)) = 0. \tag{A.43}$$

Differentiating (A.43) yields

$$\mathcal{I}'(w) = \frac{u''(w - \mathcal{I}(w))}{u''(w - \mathcal{I}(w)) + v''\big(g(\mathcal{I}(w))\big)\,g'(\mathcal{I}(w))^2 + v'\big(g(\mathcal{I}(w))\big)\,g''(\mathcal{I}(w))}. \tag{A.44}$$

By Assumption 1, $u'' < 0$ such that the numerator in (A.44) is strictly negative. Since $0 < \mathcal{I}(w) < w$ is a maximum, the second-order condition for the objective function in (A.28) is satisfied, implying that the denominator in (A.44) is strictly negative. Thus, we conclude that (A.44) is strictly positive so that $G' > 0$. $\square$

Proof of Corollary 1. Since $\Omega' > 0$ by Assumption 2 and $w' > 0$ by Assumption 3, we can read off (A.42) that $G'(k_t) < 0$ if and only if $\mathcal{I}'(w(k_t)) < 0$. However, (A.44) implies that $\mathcal{I}'(w_t) < 0$ if and only if $\mathcal{I}(w_t)$ satisfies the second-order condition for a local *minimum* of the objective function in (A.28). Thus, the contract (B_t, I_t, R_t, r_t) *minimises* the agent's expected utility and hence cannot be an efficient contract. $\square$

Proof of Proposition 3. (i) Steady states of G are determined by solutions $k_\star \geq 0$ to

$$k \overset{!}{=} \Omega\big(\mathcal{I}(w(k))\big). \tag{A.45}$$

Note that $\Omega(0) = 0$. If $w(0) = 0$, then $\mathcal{I}(w(0)) = 0$ and, consequently, $k_\star = 0$ solves (A.45). On the contrary, if $w(0) > 0$, then the Inada conditions stated in Assumption 1 imply $0 < \mathcal{I}(w(0)) < w(0)$. Since $\Omega' > 0$, it follows that $\Omega\big(\mathcal{I}(w(0))\big) > 0$, showing that $k_\star = 0$ cannot solve (A.45). Hence, $k_\star = 0$ solves (A.45) if and only if $w(0) = 0$.

(ii) It follows from the definition of Ω and Proposition 2 that

$$0 \leq G(k) = \Omega\big(\mathcal{I}(w(k))\big) \leq f(k) \quad \text{for all} \quad k \geq 0. \tag{A.46}$$

If $w(0) > 0$, then $G(0) > 0$, so that in the local neighborhood of zero, we have $G(k) > k$. On the other hand, if $w(0) = 0$, then $G(0) = 0$. In this case, it follows from the property

$\lim_{k \to 0} G'(k) > 1$ that in the local neighborhood of zero, $G(k) > k$ holds. Inequality (A.46), the strict concavity of f, and the Inada condition $\lim_{k \to \infty} f'(k) = 0$ then imply that there exists at least one $k_\star > 0$ that solves (A.45). The largest of these solutions must satisfy $0 < G'(k_\star) < 1$ and thus be asymptotically stable. □

Proof of Lemma 2. Assumptions 2 and 3 imply that $I \mapsto f(\Omega(I))$ is strictly increasing and strictly concave. Hence, a solution I_G to

$$\max_{I \geq 0} f(\Omega(I)) - I \tag{A.47}$$

is unique, if it exists. Observe that $f(\Omega(I)) - I < f(I) - I$ for all $I > 0$. If follows from Assumption 3 that the function $I \mapsto f(I) - I$ has a unique maximum. Hence, Problem (A.47) admits a unique solution $I_G > 0$, determined by the first-order condition (4.7). □

References

Azariadis, C., & Smith, B. (1998). Financial intermediation and regime switching in business cycles. *American Economic Review*, *88*(3), 516–536.

Aziakpono, M. (2011). Financial development and economic growth: Theory and a survey of evidence. *Studies in Economics and Econometrics*, *35*(1), 15–43.

Banerji, S., Bhattacharya, J., & Van Long, N. (2004). Can financial intermediation induce endogenous fluctuations. *Journal of Economic Dynamics and Control*, *28*(11), 2215–2238.

Bencivenga, V. R., & Smith, B. D. (1991). Financial intermediation and endogenous growth. *Review of Economic Studies*, *58*(2), 195–209.

De La Croix, D., & Michel, P. (2002). *A theory of economic growth: Dynamics and policy in overlapping generations*. Cambridge, UK: Cambridge University Press.

Diamond, D. W., & Dybvig, P. H. (1983). Bank runs, deposit insurance, and liquidity. *Journal of Political Economy*, *91*(3), 401–419.

Diamond, P. A. (1965). National debt in a neoclassical growth model. *American Economic Review*, *55*(5), 1126–1150.

Ennis, H. M., & Keister, T. (2003). Economic growth, liquidity, and bank runs. *Journal of Economic Theory*, *109*(2), 220–245.

Freixas, X., & Rochet, J.-C. (2008). *Microeconomics of banking*. MIT press.

Gersbach, H., & Wenzelburger, J. (2003). The workout of banking crises: A macroeconomic perspective. *CESifo Economic Studies*, *49*(2), 233–258.

Gersbach, H., & Wenzelburger, J. (2008). Do risk premia protect against banking crises? *Macroeconomic Dynamics*, *12*(S1), 100-111.

Gersbach, H., & Wenzelburger, J. (2012). Interest-rate policies and stability of banking systems. In J. Chadha & S. Holly (Eds.), *Lessons for monetary policy from the financial crisis* (pp. 71–107). Cambridge University Press.

Grandmont, J.-M. (1989). Local bifurcations and stationary sunspots. In W. Barnett (Ed.), *Chaos, sunspots, bubbles, and nonlinearities: Proceedings of the fourth international symposium in economic theory and econometrics.* Cambridge, MA: Cambridge University Press.

Greenwood, J., & Jovanovic, B. (1990). Financial development, growth, and the distribution of income. *Journal of Political Economy*, *98*(5, Part 1), 1076–1107.

Hellwig, M. F. (1980). On the aggregation of information in competitive markets. *Journal of Economic Theory*, *22*(3), 477–498.

Holmström, B., & Tirole, J. (1997). Financial intermediation, loanable funds, and the real sector. *Quarterly Journal of Economics*, *112*(3), 663–691.

Kyle, A. S. (1989). Informed speculation with imperfect competition. *Review of Economic Studies*, *56*(3), 317–355.

Levine, R. (1997). Financial development and economic growth: Views and agenda. *Journal of Economic Literature*, *35*(2), 688–726.

Levine, R. (2005). Finance and growth: Theory and evidence. In P. Aghion & S. N. Durlauf (Eds.), *Handbook of economic growth.* Amsterdam: North-Holland.

Medio, A., & Raines, B. (2006). Backward dynamics in economics. The inverse limit approach. *Journal of Economic Dynamics and Control*, *31*(5), 1633–1671.

Myerson, R. B. (1979). Incentive compatibility and the bargaining problem. *Econometrica*, *47*(1), 61–73.

Pagano, M. (1993). Financial markets and growth: An overview. *European Economic Review*, *37*(2-3), 613–622.

Smith, R. T. (1998). Banking competition and macroeconomic performance. *Journal of Money, Credit and Banking*, *30*(4), 793–815.

Solow, R. M. (1956). A contribution to the theory of economic growth. *Quarterly Journal of Economics*, *70*(1), 65–94.

Williamson, S. D. (1987). Financial intermediation, business failures, and real business cycles. *Journal of Political Economy*, *95*(6), 1196–1216.