# Should Governments Tax Commodities Uniformly? Theory and Evidence From Brazil*

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**Abstract**

This paper investigates the optimal tax structure under a multi-sector general equilibrium model with tax evasion, monopolistic competition, and the presence of intermediate and final goods. We show that uniform commodity taxation on final goods is no longer optimal and that, as long as tax evasion is not large enough, intermediate goods should not be taxed. Instead, the tax authority should levy uniform effective taxes on final goods and impose lower nominal taxes on sectors which can most evade taxes. We take advantage of a quasi-experiment occurred in Brazil in 1999 to estimate tax compliance elasticities for several sectors and we simulate counterfactual tax policies and evaluate how different tax structures affect the economy.

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# 1 Introduction

Tax evasion and avoidance are serious problems in many countries. According to *Instituto Brasileiro de Planejamento Tributário* (IBPT), tax evasion incurred a cost of R$ 417 billion to the Brazilian government in 2019, equivalent to 15% of all tax revenues. In contrast, in 2002, tax evasion represented a much higher proportion of tax revenues at 32%.

In an economy without tax evasion and under certain conditions (e.g., separable preferences), Atkinson and Stiglitz (1972) show that goods should be taxed at a uniform rate. This is the classical result on uniform commodity taxation,[1] which has also been advocated by policy makers since it is a simple and straightforward tax design which should decrease compliance costs and tax evasion - see the Meade Report (1978). Another important policy lesson in the optimal taxation literature is that transactions between firms should not be taxed, and therefore taxing intermediate goods is not optimal since it distorts the allocation of factors of production between intermediate and final goods decreasing production efficiency (cf., Diamond and Mirrlees, 1971).

In this paper we address the question of the robustness of these policy lessons when the tax authority cannot perfectly monitor economic activities. Should the government tax sectors with different levels of informal activities in the same way? In the presence of tax evasion, should the tax authority rely on intermediate goods taxation?

As Shaw et al. (2010) argue in the Mirlees Review, most of modern optimal tax theory abstract from administrative and compliance costs, as well as tax evasion. However, these may be important features for developing economies. In this paper we investigate the optimal tax design in the presence of tax evasion and when evasion varies by sectors of economic activity. We follow the Ramsey tradition (cf., Ramsey, 1927; Lucas and Stokey, 1983), which determines the optimal tax structure to minimizes economic distortions. Cremer and Gahvari (1993) is the closest paper to ours. They consider an economy with endogenous tax evasion[2] and investigate the optimal commodity taxation problem. They show that under evasion, the uniform taxation is no longer an optimal prescription. We differ from them in the following two main features. First, we consider an economy with intermediate goods and monopolistic competition[3] as in Dixit and Stiglitz (1977). This allows us to investigate whether or not intermediate goods should be taxed in the presence of tax evasion, while they mainly focus on final goods taxation. In addition, we provide clear results on the kind of tax rate differentiation by different sector characteristics (e.g., size, productivity and evasion), which can be easily checked in the data. The concealment technology is given and varies with sectoral features, such as

[1]Regarding distributional issues uniform commodity taxation can also be advocated when there are different instruments for redistribution, such as cash transfers to the poor.

[2]See also Allingham and Sandmo (1972)

[3]See also Cremer and Thisse (1994)

size, productivity and the tax rate.[4]

Evasion may lead to important departs from traditionally recommended tax policies. For example, one of the most accepted results on optimal taxation is the less distortive aspect of direct taxation. Still, in several countries, the share of tax revenue coming from indirect taxation can be relatively high. One explanation might be the difficulty to raise tax revenue directly in comparison to indirect taxes. Boadway et al. (1994) shows that if different taxes have different evasion characteristics, some optimal tax mix of direct and indirect taxation emerges naturally. Cavalcanti and Villamil (2003) show that in the presence of tax evasion the optimal inflation tax is positive and increasing with the size of the informal economy. Emran and Stiglitz (2005) show that a tax reform which eliminate trade taxes and compensate it with a value-added tax might decrease welfare when a large informal sector is present.

The following lessons are learnt. First of all, the introduction of different sectors, monopolistic competition and intermediate goods per se do not change the main optimal taxation prescriptions, i.e., in our framework and under no evasion the uniform commodity taxation is optimal and the government should not rely on intermediate goods taxation. However, in the presence of tax evasion, uniform commodity taxation is no longer an optimal policy. Instead, the tax authority should levy uniform effective taxes and impose lower taxes on sectors which can most evade taxes. In addition, if there are no informal firms optimal taxes on intermediate goods are still zero. However, the presence of a informal sector changes this prescription, with optimal positive taxes on intermediate goods.

This paper is divided into four additional sections besides this introduction. Section 2 presents the economic environment. Section 3 solves the Ramsey problem using the primal approach and derives the main taxation lessons. in Section 4 we describe the quasi-experiment and compute the compliance elasticities for several sectors of the brazilian economy. in Section 5 we implement the model numerically and use the computed elasticities to compute counterfactuals. Section 6 contains concluding remarks

## 2 The model

Consider an one-period real economy with $N \times S$ productive sectors. There are $N \times (S + 1) + 1$ commodities in this economy: $N$ consumption goods, $S$ intermediate goods for each final consumption good and the labor input. There is one representative agent, endowed with one unit of productive time that can be used as leisure or labor in the production of intermediate goods. Government levies sales tax on firms in order to raise an exogenously defined amount of resources, to be disposable.

[4] See Rauch (1991), Amaral and Quintin (2006), Antunes and Cavalcanti (2007) for models in which informal activities arise endogenously.

## 2.1 Representative household

Preferences are defined over consumption $\{C_n\}_{n=1}^{N}$ and the disutility from labor, $l$. We assume that preferences can be represented by a utility function $u : \Re_{+}^{N} \times [0,1] \rightarrow \Re$, given by:

$$U\left(C_1, C_2, \ldots, C_N, l\right) \tag{1}$$

Function $U(\cdot, \cdot, \ldots, \cdot)$ satisfies standard properties. For instance, it is twice continuously differentiable in all arguments, and it is strictly concave in all consumption good, and strictly convex in the disutility from labor. We also assume that preferences are homothetic over the consumption goods, i.e.:

**Assumption 1.** *There are functions $H : \Re_{+}^{N} \rightarrow \Re$, homogenous of degree $k$, and $F : \Re \times [0,1] \rightarrow \Re$ such that:*

$$U\left(C_1, C_2, \ldots, C_N, l\right) = F\left(H\left(C_1, C_2, \ldots, C_N\right), l\right) \tag{2}$$

The representative household owns firms and chooses $\{C_n, l\}_{n=1}^{N}$ in order to maximize (1) subject to:

$$\sum_{n=1}^{N} P_n C_n \leq wl + \sum_{n=1}^{N} \sum_{s=1}^{S} \pi_{n,s} \tag{3}$$

where $\pi_{n,s} = \pi_{n,s}(w, \xi_n, A_{n,s})$ is the profit of intermediate goods firms, which will be detailed in section 2.3.

## 2.2 Consumption Goods

In each sector, $n = 1, \ldots, N$, there is a continuum of firms of measure one. Let $Y_n$ and $\{d_{n,s}\}_{s=1}^{S}$ be output produced and intermediate goods used, respectively, by the representative firm in sector $n$. The technology employed to produce each consumption good is represented by the following CES production function:

$$Y_n = Z_n \left(\sum_{s=1}^{S} d_{n,s}^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}, n = 1, \ldots, N \tag{4}$$

where $Z_n$ is a productive factor and $\theta$ is the elasticity of substitution. Government levies a tax $\tau_n$ on revenue in each sector to finance its spending. However, firms may evade taxes. In such a case, let $\phi^n = \phi^n\left(\tau_n, Z_n\right)$ be the fraction of firm $n$ 's revenue that is declared and hence taxed by the fiscal authority. We assume that $\phi^n$ is decreasing in the tax rate in sector $n$, i.e., $\phi_1^n < 0$, but, it is increasing in the productivity of sector $n, \phi_2^n > 0$. Therefore, firms in high productive sectors have lower probability to evade taxes[5].

[5]Following De Soto et al. (1989), Antunes and Cavalcanti (2007) show that since loan contracts are not well enforced in the informal sector, informal entrepreneurs scale down their size and productivity.

Consumption good producers are price takers and maximize profits. Let the price of consumption good $n$ be $P_n$ and let $p_{n,s}$ be the price of intermediate good $\{n,s\}$. Let $\Pi_n = \Pi_n\left(P_n, \tau_n, \{p_{n,s}\}_{s=1}^{S}\right)$ denotes profits of firms in sector $n$. The profit maximization problem of each firm is:

$$\Pi_n = \max_{\{d_{n,s}\}_{s=1}^{S}} \left\{ (1-\tau_n\phi^n) Z_n P_n \left(\sum_{s=1}^{S} d_{n,s}^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}} - \sum_{s=1}^{S} d_{n,s}p_{n,s}\right\} \tag{5}$$

Under perfect competition ($\Pi_n = 0$), we can show that in equilibrium must hold:

$$(1-\tau_n\phi^n) Z_n P_n = \left(\sum_{s=1}^{S} p_{n,s}^{1-\theta}\right)^{\frac{1}{\theta}} \tag{6}$$

Also, the demand of firm in sector n for the intermediate good $\bar{s}$ is:

$$d_{n,\bar{s}} = \frac{\left(\sum_{s=1}^{S} p_{n,s}^{1-\theta}\right)^{\frac{\theta}{1-\theta}} Y_n}{p_{n,\bar{s}}^{\theta}} = \frac{\Omega_n Y_n}{p_{n,\bar{s}}^{\theta}}, \Omega_n = \left(\sum_{s=1}^{S} p_{n,s}^{1-\theta}\right)^{\frac{\theta}{1-\theta}} \tag{7}$$

Since firms operate under constant return to scale technology and zero profit condition, $Y_n$ is entirely determined by the demand for good $n$, i.e., $C_n$.

## 2.3 Intermediate Goods

Each intermediate firm $s$ in sector $n$ has monopoly rights in its production of $y_{n,s}$, uses $h_{n,s}$ units of labor and faces demand function given by equation (7)

The technology employed to produce each intermediate good is represented by the following production function:

$$y_{n,s} = A_{n,s} h_{n,s}^{\beta}, \quad n = 1, 2, \ldots, N; s = 1, 2, \ldots, S. \tag{8}$$

$A_{n,s}$ is a labor productive factor associated to firm $s$ in sector $n$ and is assumed to be positive. Also $\beta \in (0,1)$ corresponds to the elasticity of output with respect to labor.

Governments levies a tax $\xi_n$ on revenue of each firm that supply to sector $n$ to finance spending. Firms can, however, evade taxes. Let $\delta^{n,s} = \delta^{n,s}(\xi_n, A_{n,s})$ be the fraction of firm $s$ 's in sector $n$ revenue that is declared and hence taxed by the fiscal authority. Just as in the case of final goods firm, we assume that $\delta^{n,s}$ is decreasing in the tax rate for every intermediate firm supplying to sector $n$, $\delta_1^{n,s} < 0$, but it is increasing in the productivity of firm $s$ in sector $n$, $\delta_2^{n,s} > 0$. The profit maximization problem of each intermediate good firm is:

$$\begin{aligned} &\pi_{n,s}(w, \xi_n, A_{n,s}) = \max_{h_{n,s}} \{p_{n,s}(1-\xi_n\delta^{n,s}) y_{n,s} - w h_{n,s}\} \\ &\text{s.t. } y_{n,s} = \frac{\Omega_n Y_n}{p_{n,s}^{\theta}} \\ &\text{and } y_{n,s} = A_{n,s} h_{n,s}^{\beta} \end{aligned} \tag{9}$$

Or

$$\pi_{n,s} = \max_{h_{n,s}} \left\{ (\Omega_n Y_n)^{\frac{1}{\theta}} (1-\xi_n \delta^{n,s}) A_{n,s}^{\frac{\theta-1}{\theta}} h_{n,s}^{\frac{\theta\beta-\beta}{\theta}-1} - w h_{n,s} \right\} \tag{10}$$

The associated marginal condition[6] for each firm is:

$$\left(\frac{\theta\beta-\beta}{\theta}\right)(\Omega_n Y_n)^{\frac{1}{\theta}} (1-\xi_n \delta^{n,s}) A_{n,s}^{\frac{\theta-1}{\theta}} h_{n,s}^{\frac{\theta\beta-\beta}{\theta}-1} = w \tag{11}$$

Hence firm's profit in optimal is:

$$\pi_{n,s} = \left(\frac{\theta}{\theta\beta-\beta}\right) w h_{n,s} - w h_{n,s} = \left(\frac{\theta-\theta\beta+\beta}{\theta\beta-\beta}\right) w h_{n,s} \tag{12}$$

## 2.4 Government Budget Constraint

The government consumes a basket $\{G_n\}_{n=1}^N$ of final goods and its budget constraint is given by:

$$\sum_{n=1}^N \tau_n \phi^n P_n Y_n + \sum_{n=1}^N \sum_{s=1}^S \xi^n \delta^{n,s} p_{n,s} y_{n,s} = \sum_{n=1}^N P_n G_n \tag{13}$$

## 2.5 Equilibrium

In competitive equilibrium firms producing final goods and households are price takers. Households maximize their utility subject to their budget constraint, firms maximize profits given their technology, labor and all goods markets clear and the government budget constraint is satisfied. Marketing clearing conditions are:

$$\begin{gathered} C_n + G_n = Z_n \left[ \sum_{s=1}^S \left( A_{n,s} l_{n,s}^{\beta} \right)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\partial-1}}, n = 1,2,\ldots,N; \\ h_{n,s} = l_{n,s}, n = 1,2,\ldots,N \text{ and } s = 1,2,\ldots,S; \\ \sum_{n=1}^N \sum_{s=1}^S l_{n,s} = l. \end{gathered} \tag{14}$$

# 3 Ramsey Problem

Following the tradition of Ramsey (1927) we study the problem of choosing the best allocations that are consistent with the competitive equilibrium. The implementability conditions are: (i) the resource constraint for each sector; and (ii) the implementability constraint, given by:

$$\sum_{n=1}^N U_n C_n = -U_l l \tag{15}$$

[6]As in Dixit-Stiglitz (1977), we assume $S$ is sufficiently large to the point that price changes of a single intermediate good do not affect general price index.

Equation (15) corresponds to the household's budget constraint in which we substitute prices and taxes by quantities consistent to the competitive equilibrium. The Ramsey problem (see Lucas and Stokey, 1983) is to choose $\{C_n, l_{n,s}\}_{n=1,s=1}^{N,S}$ to maximize (1) subject to the resource constraint for each sector (equation (14)) and the implementability constraint 15. All proposition proofs can be found in Appendix.

**Proposition 1.** *Suppose that there is no tax evasion. The Ramsey allocation is decentralized with a uniform effective tax rate policy on sectors, i.e.,* $(1-\tau_i)(1-\xi_i) = (1-\tau_j)(1-\xi_j)$ *for every* $i, j \in \{1, 2, \ldots, N\}$.

As in Atkinson and Stiglitz (1972), Proposition 1 states that under no tax evasion the optimal policy is to tax all goods uniformly. However, note that uniformity does not apply to nominal tax rates, but to effective tax rate. Now, Proposition 2 shows that the presence of tax evasion solely in intermediate goods market compels tax authority to equalize nominal tax rates.

**Proposition 2.** *Suppose that only intermediate firms practice some tax evasion. Also, assume that* $A_{j,r} \neq A_{j,\tilde{r}}$*, for some* $\{j, r\}$ *and* $\{j, \tilde{r}\}$*. The Ramsey allocation is decentralized with an uniform tax rate policy on final goods,* $\tau_i = \tau_j$*, and no tax on intermediate goods,* $\xi_i = \xi_j = 0$

The presence of tax evasion on intermediate goods market but not on final goods market throw away the possibility of achieving a decentralized Ramsey allocation through the taxation of intermediate firms' revenue. So, under such scenario, tax reforms should try not only to significantly reduce nominal tax rates on intermediate firms but also to equate nominal tax rates on final goods.

Nevertheless, as shown below, under generalized tax evasion, the prescription to uniformly tax commodity is no longer valid.

**Proposition 3.** *Suppose that every firm can practice some tax evasion. Also, assume that* $Z_i > Z_j$*, for some* $i$ *and* $j$*, and* $A_{j,r} \neq A_{j,\tilde{r}}$*, for some* $\{j, r\}$ *and* $\{j, \tilde{r}\}$*. The Ramsey allocation is decentralized with no taxation on intermediate goods,* $\xi_i = \xi_j = 0$ *and a uniform effective tax rate policy on final good firms,* $\tau_i \phi^i = \tau_j \phi^j$*. Moreover, if the effective tax rate is decreasing for all sectors, then it is optimal to tax heavier the sector with smaller tax evasion.*

Now, if in addition to generalized tax evasion there is also an informal sector, the prescription that intermediate goods should not be taxed changes. That is, in order to achieve the revenue requirement, there must be some level of production inefficiency.

**Proposition 4.** *Suppose there is at least one informal final good firm (that is,* $\phi_n = 0$ *for some* $n$*). Suppose also that there is one intermediate good firm for each final good firm. Then, the Ramsey allocation is decentralized with positive taxation on intermediate goods and a non-uniform tax rate policy on final good firms.*

### 3.1 Alternative Tax Policy on Intermediate Goods

Suppose now that, instead of applying the same tax rate $(\xi_n)$ on every firm that supplies to sector $n$, the government applies taxes $(\xi_s)$ conditional on firm type $s$. It can be imagined as a tax policy in which the authority taxes accordingly to the origination of the goods instead of to their destination.

**Proposition 5.** *Suppose that no firm can evade tax. The Ramsey allocation is decentralized with a uniform tax rate policy on final good firms,* $\tau_i = \tau_j$ *for any* $i, j \in \{1, 2, \ldots, N\}$ *and a uniform tax rate policy on intermediate good firms,* $\xi_r = \xi_v$ *for any* $r, v \in \{1, 2, \ldots, S\}$.

**Proposition 6.** *Suppose that only intermediate firms can evade taxes. Also, assume that there exist* $A_{j,r}, A_{j,v}, A_{i,r}, A_{i,v}$, *for some* $j, i \in \{1, 2, \ldots, N\}$ *and* $r, v \in \{1, 2, \ldots, S\}$, *such that* $\frac{\delta^{j,r}}{\delta^{j,v}} \neq \frac{\delta^{i,r}}{\delta^{i,v}}$. *The Ramsey allocation is decentralized with an uniform tax rate policy on final goods,* $\tau_i = \tau_j$, *and no tax on intermediate goods,* $\xi_r = \xi_v = 0$.

**Proposition 7.** *Suppose that every firm can practice some tax evasion. Also, assume that* $Z_i > Z_j$, *for some* $i$ *and* $j$, *and there exist* $A_{j,r}, A_{j,v}, A_{i,r}, A_{i,v}$, *for some* $\{j, i\} \in \{1, 2, \ldots, N\}$ *and* $\{r, v\} \in \{1, 2, \ldots, S\}$, *such that* $\frac{\delta^{j,r}}{\delta^{j,v}} \neq \frac{\delta^{i,r}}{\delta^{i,v}}$. *The Ramsey allocation is decentralized with no taxation on intermediate goods,* $\xi_i = \xi_j = 0$ *and a non-uniform tax rate policy on final good firms,* $\tau_i \neq \tau_j$. *Moreover, if the effective tax rate is decreasing for all sectors, then it is optimal to tax heavier the sector with smaller tax evasion.*

## 4 A quasi-experiment for Brazil

This section aims to estimate tax compliance elasticities for several sectors sector of the Brazilian economy. For this, We take advantage of a change in legislation accurred in 1999 in Brazil, which increased the rate of one tax, the Contribution for Social Purposes (COFINS), but did not increase that of another very similar, the Social Integration Program (PIS). We use this arguably exogenous variation to construct a counterfactual series for COFINS, which allows us to estimate the tax compliance elasticity.

### 4.1 Data and the quasi-experiment

The *Programa de Integração Social* (PIS), the *Programa de Formação do Patrimônio do Servidor Público* (PASEP) and the *Contribuição para o Financiamento da Seguridade Social* (COFINS) are characterized as social contributions, that is, they are taxes that aim to finance social assistance or social security services. All of them have the objective of financing social security as established by Article 195 of the Federal Constitution, with the federal government having the tax competence to collect them within a regime of cumulative incidence at their origin.

The *Programa de Integração Social* (PIS) and the *Programa de Formação do Patrimônio do Servidor Público* (PASEP) were respectively instituted by Supplementary Laws No. 07 and 08 of 1970, while the COFINS was instituted by Supplementary Law No. 70/91. Despite some small changes in scope over the years, the incidence base of both contributions has remained mostly the same, the gross revenue of legal entities.

Currently, the PIS/PASEP and COFINS tax system is quite complex. Both taxes operate under both the cumulative and non-cumulative regime, in addition to some special regimes that could refer to the calculation base or differentiated rates. However, until 2002, PIS and COFINS were calculated only through the cumulative regime. It has been common in laws that regulate taxes for some sectors of economic activity to enjoy exemption or differentiated rates, as in the case of the production and sale of alcohol and financial institutions.

Until 1998, the rates of COFINS and PIS (we will use the term PIS hereinafter as a synonym for the tax contribution to the Program) were, with rare exceptions, respectively 2% and 0.65% of the companies' revenue. However, in February 1999, the COFINS rate was indiscriminately changed to 3% (i.e., a 50% increase in the nominal legal rate). This variation provided a generalized increase in tax revenue (see Figure 1). On the other hand, during the same period, there was no change in the nominal legal rate of PIS-PASEP. Figure (2) shows relative stability in the trajectory of PIS revenue series over the period.

The database used in this study contains tax collection series for COFINS and PIS/PASEP contributions. The data is confidential and was obtained from the Federal Revenue Secretariat of the Ministry of Economy. The series are in current monthly values from January 1998 to December 2001, disaggregated at the class level (4 digits) according to the National Classification of Economic Activities (CNAE) 1.0 of IBGE. The original data form a panel with 564 activity classes and 48 months. However, due to changes in the methodology of economic activity classification adopted after 2000, we chose to restrict ourselves to only 24 months. Therefore, the data is limited from January 1998 to December 1999, when the methodological change occurred. We used the IPCA to adjust the series to the price level of January 1998. In constant values, around R$48.9 billion was collected during the period for COFINS and R$16.8 billion for PIS.

## 4.2 Empirical Strategy

We define the compliance elasticity with respect to the tax rate as follows:

$$\varepsilon_\phi = \frac{d\phi_t}{\phi_t}\frac{\tau_t}{d\tau_t} \tag{16}$$

where $\phi_t$ is the fraction of the taxable base value that the company decides to declare and $\tau_t$ is the nominal tax rate applied.

The first technical problem faced in estimating the value of the compliance elasticity is that the declarable

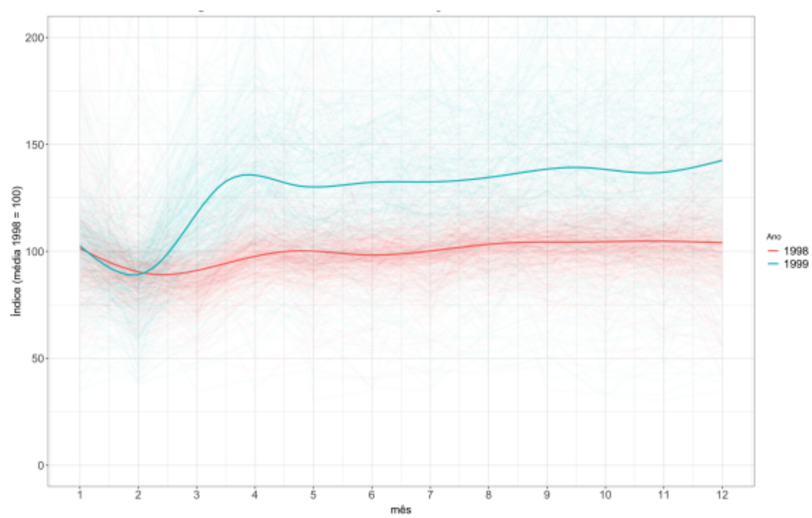


Figure 1: COFINS Monthly Collection Series

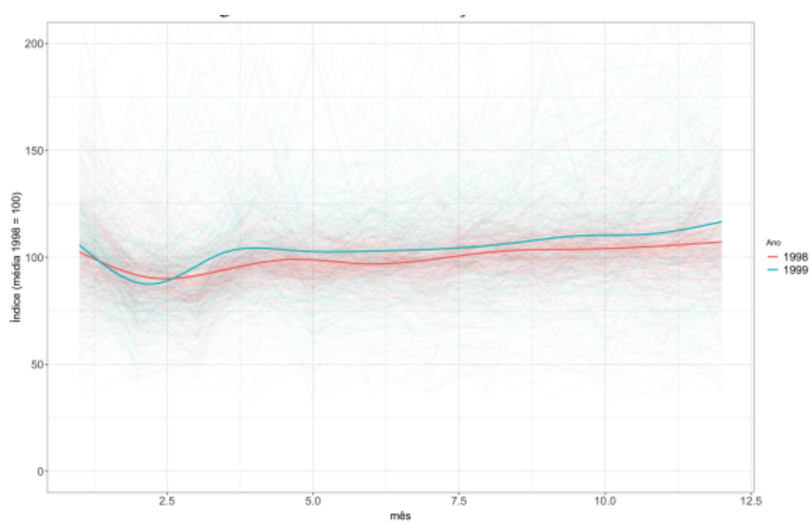


Figure 2: PIS Monthly Collection Series

fraction (or its variation) is not observable in the data. The amount that companies actually declare of their revenues is only known to the companies themselves. In this case, an indirect approach is necessary.

To show how we perform this measurement, let us assume that $R_t^{j,i}$ is the effective collection of the tax $j$ on the turnover of a sector (or company) $i$ at a given time $t$. We define:

$$R_t^{j,i} = Y_t^i \tau_t^{j,i} \phi_t^{j,i} \tag{17}$$

where $Y_t^i$ is the turnover of the sector (or firm) $i$ in period $t$, $\tau_t^{j,i}$ is the tax rate for tax $j$ applied to sector $i$, and therefore, $Y_t^i \tau_t^{j,i}$ is the amount of taxes owed.

For simplicity, let us temporarily suppress the indexers $j, i$ from the notation. In this case, a marginal change in the tax rate leads to the following revenue variation:

$$\frac{\partial R_t}{\partial \tau_t} = \frac{\partial Y_t}{\partial \tau_t} \tau_t \phi_t + Y_t \phi_t + Y_t \tau_t \frac{\partial \phi_t}{\partial \tau_t}$$

It is interesting to note that a possible increase in the tax rate would affect revenue through three effects:

a. **Direct effect:** Increase in effective collection due to the direct reduction of the owed tax rate;

b. **Indirect effect (rescaling):** Change in the firms' revenue scale, as it directly impacts their operational scale decisions;

c. **Indirect effect (evasion):** decrease in effective collection due to the fact that companies are stimulated to increase tax evasion.

The direct effect (first term on the right-hand side of the equation) is quite predictable and depends basically on the variation of the tax rate and the current revenue. For example, if the rate goes up by 10%, revenue will also increase by 10%, since this effect has the existing tax base as its calculation. The second effect (second term on the right-hand side of the equation), however, is sensitive to the magnitude of the tax rate change. That is, if the tax rate rise is marginal (e.g., only a 1% rise in the legal tax rate value), it is possible that few companies have the incentive to start paying taxes. However, a more expressive rise (e.g., a 50% rise in the legal rate) may encourage tax evasion by companies that previously evaded taxes.

The indirect effect due to tax evasion can be interpreted as the decrease in the tax base due to the increase in the tax rate. The estimation of this effect is not as simple as the direct effect described earlier. It is necessary, in this case, to identify how companies in different activity sectors change their decisions to evade or not pay taxes according to changes in the legal rates.

Note that this decision depends on the productive characteristics of each company or sector. It is likely that the level of compliance of each sector depends, for example, on the average size of establishments. If the sector is characterized by large companies and a high capital-labor (or product-labor) ratio (e.g., the oil and natural gas sector), there may be little tax evasion in this sector, as it is more visible to the tax authority. Thus, the decision to evade taxes in this sector should be less sensitive to changes in tax rates. On the other hand, if the sector is characterized by a set of small companies (e.g., clothing and accessories), with a low capital-labor ratio, it is likely that the decision to evade or not depends directly on the level of the rate. From this perspective, it is likely that a reduction in the rate in these sectors will lead to a greater reduction in tax evasion.

If we define the collection and revenue elasticities in relation to the tax rate respectively by

$$\varepsilon_R = \frac{\partial R_t}{\partial \tau_t}\frac{\tau_t}{R_t} \text{ and } \varepsilon_Y = \frac{\partial Y_t}{\partial \tau_t}\frac{\tau_t}{Y_t}, \tag{18}$$

from (17) and some algebraic manipulation, we obtain:

$$\varepsilon_\phi = \varepsilon_R - 1 - \varepsilon_Y. \tag{19}$$

Thus, it is possible to determine the compliance elasticity from the collection and revenue elasticities. We will use this approach to estimate the compliance elasticity. The previous equation can be rewritten as follows:

$$\varepsilon_\phi = \frac{\partial R_t}{\partial \tau_t}\frac{\tau_t}{R_t} - \frac{\partial Y_t}{\partial \tau_t}\frac{\tau_t}{Y_t} - 1.$$

Of the terms that compose the equation above, $\frac{\partial R_t}{\partial \tau_t}$ and $\frac{\partial Y_t}{\partial \tau_t}\frac{1}{Y_t}$ are generally unobservable. Thus, without additional assumptions, it would not be possible to estimate the elasticity of evasion directly.

Firstly, we assume that $\phi_t^{j,i}$ is completely determined by $\tau^{j,i}$ and exogenous characteristics associated with the sector. Additionally, we assume that, at least in the short run (and later, within the scope of the estimation strategy), firms' revenue is invariant to nominal tax rates, $\frac{\partial Y_t^i}{\partial \tau_t^{j,i}} = 0 \; \forall j$. In other words, firms cannot (or do not wish to) promptly modify the size of their operations in response to an increase in tax rates. Thus, we arrive at the following equation:

$$\varepsilon_\phi = \frac{\partial R_t}{\partial \tau_t}\frac{\tau_t}{R_t} - 1.$$

Therefore, in this context, the revenue elasticity would directly determine the elasticity of evasion with respect to the nominal tax rate. Consequently, to calculate the latter, an estimate of the term $\frac{\partial R_t}{\partial \tau_t}$ would suffice.

The discrete version for the calculation of the elasticity of evasion is quite similar, given by:

$$\frac{\Delta R_t}{\Delta \tau_t}\frac{\tau_t}{R_t} = \frac{\hat{R}_t - R_t}{\hat{\tau}_t - \tau_t}\frac{\tau_t}{R_t} \tag{20}$$

Where the terms with hats denote the observed value of the variable after a variation in the tax rate. The following proposition formalizes what we presented earlier, but for the discrete case. That is, it is possible to recover the elasticity of evasion from the revenue elasticity, provided that the counterfactual revenue (in which the value of revenue is invariant to the tax rate) is available.

**Proposition 8.** *Assuming that $\delta$ is the percentage change in the tax rate and that $\bar{R}_t = \hat{Y}_t \hat{\tau}_t \hat{\phi}_t$ is the value of revenue under tax-invariant revenue, then defining $\hat{R}_t = \frac{\hat{Y}_t \hat{\tau}_t \hat{\phi}_t}{(1+\delta)}$ makes it possible to recover the elasticity of evasion with respect to the tax rate.*

To meet the properties required in the previous proposition, we used the event described in Section 4.1 , which closely approximates a natural experiment.

The strategy consists of using the invariance in the PIS tax rate and, therefore, in the evasion fraction, to construct the counterfactual collection of COFINS. For this strategy, as we have seen, we make two assumptions.

The first is that the variation in the COFINS rate did not influence companies' decision on how much to evade the PIS (independence of the evasion fraction of one tax in relation to the rate of other taxes). As we will show below, this would lead the PIS revenue series (adjusted for the rate variation) to become a good predictor for the counterfactual COFINS revenue.

In fact, during the period under analysis, the payments of the contributions due were made in separate collection instruments (*Documento de Arrecadação de Receitas Federais* - DARF). Moreover, companies were not required to discriminate the values of revenues in DARFs. Thus, it was not unlikely that there was some misalignment between the declared fractions of revenue for COFINS and for PIS-PASEP.

The second hypothesis - already raised earlier - is that, during the period around the change in the tax rate, firms were not able to (or did not wish to) adjust their production levels in response to the tax change. That is, the variation in the COFINS tax rate would not have had an impact on firms' production decisions. There are several reasons to believe that the change in the nominal COFINS tax rate would not have exerted a significant influence on the real side of firms. Firstly, the variation (in percentage points) of the COFINS tax rate was relatively small, which would discourage short-term modifications in the operational structure of firms. More importantly, in a short period of months around the date of the tax rate change, legal frictions in Brazil would have limited firms' ability to adjust their production factors to the new reality.

Under these conditions, the identification strategy relies on using a counterfactual revenue estimator that meets the assumptions described in the previous proposition, and the assumption that revenue, as defined by Equation (17), follows the following stochastic process:

$$\begin{aligned} log(R_t) =& log(Y_t) + log(\tau_t) + log(\phi_t) + \epsilon_t \, log(Y_t) = \\ & f(\cdot) + \mu_t \end{aligned} \tag{21}$$

Where $f(\cdot)$ is a deterministic component, and $\epsilon_t$ and $\mu_t$ are independent disturbances with distributions

$N(0, \sigma_{\mu t})$ and $N(0, \sigma_{\epsilon t})$ respectively.

**Proposition 9.** *Let $log(R_t)$ be the stochastic process defined in (21), and $E(\cdot)$ be the expectation operator. Then, the estimator defined by $\hat{E}_\phi = exp\left[E(log\,(R_t)\,|\phi = \overline{\phi}, \tau = \overline{\tau}) - E(log\,(R_t(1+\delta))\,|\phi = \underline{\phi}, \tau = \underline{\tau})\right] -$ 1 identifies the tax comliance elasticity.*

Proposition 9 states that to identify the elasticity in the period, it would suffice to perform a statistical difference of means between periods of the series constructed from the counterfactual revenue estimator.

From now on, we will use the superscript $C$ for variables related to COFINS and the superscript $P$ for variables related to PIS.

Thus, let $t = \bar{T}$ be the moment of the change in the COFINS rate and $\phi_t$ be a variable that takes the value 1 when $t < \bar{T}$ and 0 when $t \geq \bar{T}$. We know in the case of PIS that the effective rate ($\tau_t^P \gamma_t^P$) is invariant over the period, given that both $\gamma_t^P$ - by assumption - and $\tau_t^P$ - by law - did not change

By definition:

$$\begin{aligned} log(A_t^C) - log(A_t^P) &= log\left(\frac{\underline{\tau}^C \underline{\gamma}^C}{\tau^P \gamma^P}\right) + \psi_t, \;\; \text{para } t < \bar{T} \\ log(A_t^C) - log(A_t^P(1+\delta)) &= log\left(\frac{\overline{\tau}^C \overline{\gamma}^C}{(1+\delta)\tau^P \gamma^P}\right) + \psi_t, \;\; \text{para } t >= \bar{T}, \end{aligned} \tag{22}$$

onde $\psi_t = \epsilon_t^C - \epsilon_t^P$.
Defining $\alpha = \left(\frac{\underline{\tau}^C \underline{\gamma}^C}{\tau^P \gamma^P}\right)$ e $\rho = \left(\frac{\overline{\tau}^C \overline{\gamma}^C}{(1+\delta)\tau^P \gamma^P}\right)$, we have:

$$\begin{aligned} log(A_t^C) - log(A_t^P) &= log\,(\alpha) + \psi_t, \;\; \text{para } t < \bar{T} \\ log(A_t^C) - log(A_t^P(1+\delta)) &= log\,(\rho) + \psi_t, \;\; \text{para } t >= \bar{T}. \end{aligned} \tag{23}$$

Let $\beta = \frac{\rho}{\alpha} - 1$. We can estimate $\alpha$ and $\rho$ non-parametrically. Then, $\beta$ satisfies the assumptions of Proposition 1 and, therefore, allows us to recover $\frac{(\hat{\gamma_t^C} - \gamma_t^C)}{\gamma_t^C}$ for COFINS.

Alternatively, we can identify the coefficients above from the following regression:

$$log(A_t^C) - log(A_t^P) - log(1 + I_t \delta) = log\,(\alpha) + log\,(\omega)\, I_t + \psi_t, \tag{24}$$

where $I_t$ is a variable that takes a value of 0 for $t < \bar{T}$ and a value of 1 for $t \geq \bar{T}$. Furthermore, $log\,(\rho) = log\,(\alpha) + log\,(\omega)$. It is easy to show that the parameter of interest in this equation is $\omega$, since $\beta = \omega - 1$. The compliance elasticity can be calculated as $\frac{\omega - 1}{\delta}$. The parameter $\alpha$ can be interpreted as the ratio of effective rates (the fraction of companies' revenue that ends up actually being paid to the tax authority) of the two taxes. Therefore, it represents a relative measure of revenue efficiency given the nominal rates and tax compliance rates. The further this fraction is from 1, the greater the inefficiency in the distribution of rates between the two taxes. Thus, the further the coefficient of equation (24) - the log of this ratio - is from zero, the greater the gains from redistributing rates between the taxes.

We will consider a broader class of processes that includes unobservable random components, invariant over time but specific to each activity class. Note that the only regressor in this model is almost invariant. Thus, we assume that these terms are uncorrelated with the introduced unobservable component. As we are interested in different elasticities, our main goals are to estimate and test the heterogeneity of $w$ in (24). However, as we want to produce robust estimates, we will also evaluate the presence of class-specific error components (random effects) and autocorrelation that could lead to inefficient estimators.

Therefore, we adapted the specification to meet these objectives:

$$
\begin{aligned}
log(A_{it}^{C}) - log(A_{it}^{P}) - log(1 + I_t\delta) =& log\,(\alpha) + log\,(\omega_i)\,I_t + \mu_i + \psi_{it} \\
& ou \\
y_{it} =& log\,(\alpha) + log\,(\omega_i)\,I_t + \mu_i + \psi_{it},
\end{aligned}
\tag{25}
$$

where $\mu_i$ is the unobserved individual effect component for activity class $i$.

In a way, this specification resembles the estimation of a Difference-in-Differences model, where the counterfactual is not entirely observed but artificially constructed. However, here we are not interested in estimating the average treatment effect (or the variation in the tax rate), but rather the individual elasticity or the individual effect.

From this specification, we sought to carry out a series of tests to determine the form and estimators to be used. First, we aimed to identify if the series showed non-stationarity. Then, we moved on to evaluating the error structure of the model, seeking to assess the existence of individual random effects and whether there was serial autocorrelation in the errors (even between groups). Once the error structure was determined, we proceeded to evaluate the heterogeneity of the coefficients related to evasion (or compliance rate). We sought to understand if we could forgo a pooling model in favor of a model with heterogeneous coefficients. Finally, we tested the significance of the coefficients of the activity classes.

It is important to note that this approach partially shields itself from the criticisms pointed out by Bertrand (2004). The main reason is that the estimation does not make use of independent variables and, therefore, does not suffer from the problem of serial correlation in covariates. In addition, we treat serial correlation explicitly, evaluating its presence and considering it in the composition of the error structure of the specification.

## 4.3 Coefficient Estimates

The main interest of this article is to calculate the differences in compliance elasticities between sectors. Thus, the first step is to test for the existence of coefficient heterogeneity, that is, the validity of the pooling

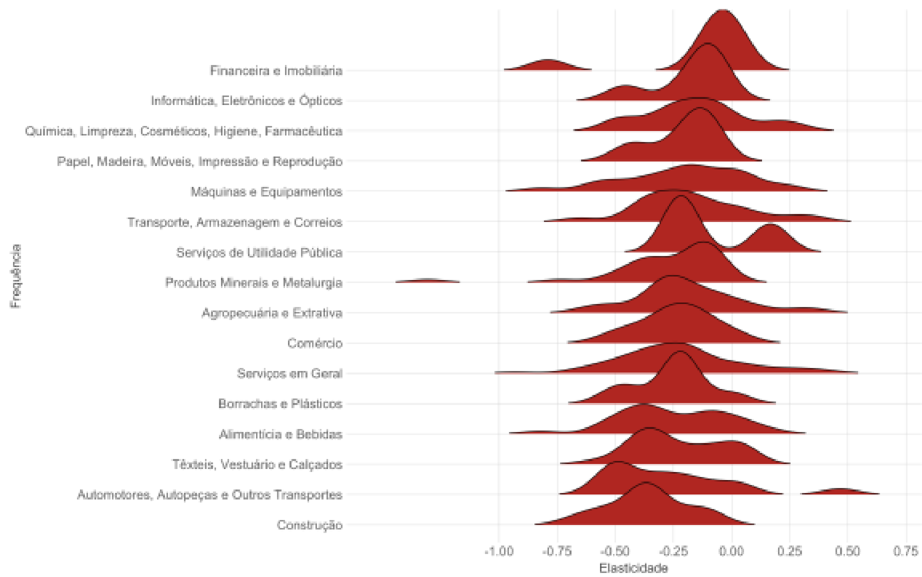


Figure 3: Elasticities frequency by economic activities

specification on the period dummy coefficient. To do so, we apply an F-test of coefficient stability (Chow test), assuming the null hypothesis that all activity classes have the same elasticity. The result indicates that we should reject the hypothesis of homogeneity among the elasticities of the classes (see Table 2)

As we mentioned, despite the series being treated for the presence of outliers, it is likely that some impactful noise still remains. One way to reduce such inconsistencies is to obtain data derived only from the current billing taxation of firms. As we do not have access to such data, we decided to try an aggregation approach, given that excluding classes with non-significant elasticities is not a feasible solution. The fact that it is not possible to distinguish zero elasticity does not mean that it does not exist or should not be considered. Thus, an alternative to reduce volatility is to aggregate classes by segment. However, the practical application of these elasticities does not require such granular levels.

# 5 Numerical Implementation

In this section we the model numerically and perform comparative statics. We evaluate the effects of different tax policies on GDP and welfare, while keeping the same level of government expenditures.

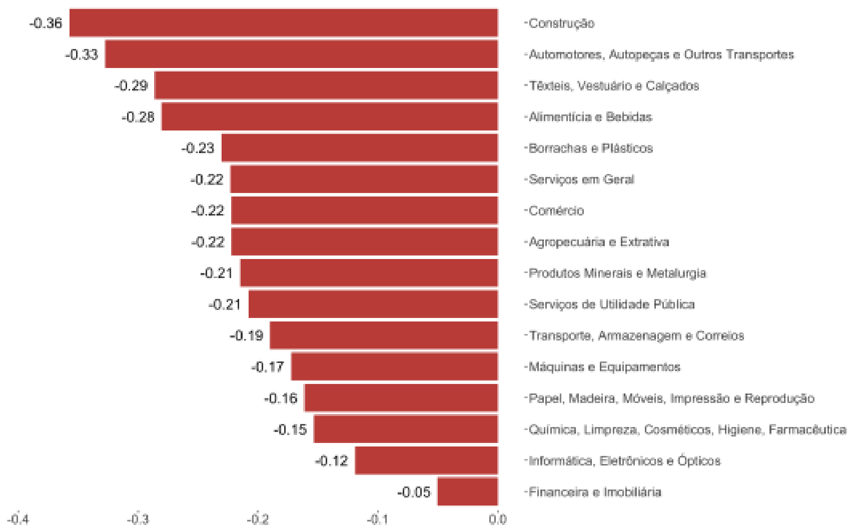


Figure 4: Elasticities by economic activities

## 5.1 Parameterization

First, we choose the utility function to be

$$\mathcal{U} = \eta \sum_{n=1}^{N} \gamma_n \log C_n + (1-\eta)\log(1-l)$$

We set $\eta = 0.33$ so that in equilibrium the labor supply represents one-third of hours worked. Parameter $\beta$ represents the labor income share over GDP and we set $\beta = 0.45$. We use reference values from the literature for the elasticity of substitution. Oberfield and Raval (2014), Redding and Weinstein (2018) and Hobijn and Nechio (2019) find values ranging from 0.75 to 3.22. We set it to $\theta = 1.5$. For the baseline economy tax rates, we use a uniform tax of 20% across sectors, both for final goods and intermediate goods firms. The sectoral values for productivity parameters, share of declared revenues and share of household's income used in each consumption goods are displayed in Table **??**. Also, due to the difficulty to observe the levels of evasion for each sector of the economy, we choose a functional form for the fraction of declared revenues. In particular, we are using $\phi_i = (1-\tau_i)^{\alpha_i \frac{\bar{Z}}{Z_i}}$, where $\bar{Z}$ represents the average productivity. Note that $\phi$ is an increasing function of $\tau$ and a decreasing function of $Z$. We calibrate $\alpha_i$ so that it matches the tax compliance elasticity for each sector.

### 5.2 Counterfactuals

The optimal tax rule as described in Proposition 3 states that $\tau_i \phi^i = \tau_j \phi^j$ and that the intermediate goods are not taxed. So if we change sector's $i$ tax rate from $\tau_i$ to $\tau_i' = \tau_i + \Delta\tau_i$, the corresponding level of declared revenue by firm $i$ will be

$$\phi_i' = \phi_i \Big(1 + \varepsilon_i \frac{\Delta\tau_i}{\tau_i}\Big),$$

where $\varepsilon_i$ is the elasticity of $\phi_i$ with respect to $\tau_i$. So in the optimal we must have

$$(\tau_i + \Delta\tau_i)\phi_i \Big(1 + \varepsilon_i \frac{\Delta\tau_i}{\tau_i}\Big) = (\tau_j + \Delta\tau_j)\phi_j \Big(1 + \varepsilon_j \frac{\Delta\tau_j}{\tau_j}\Big), \text{ for all } i, j.$$

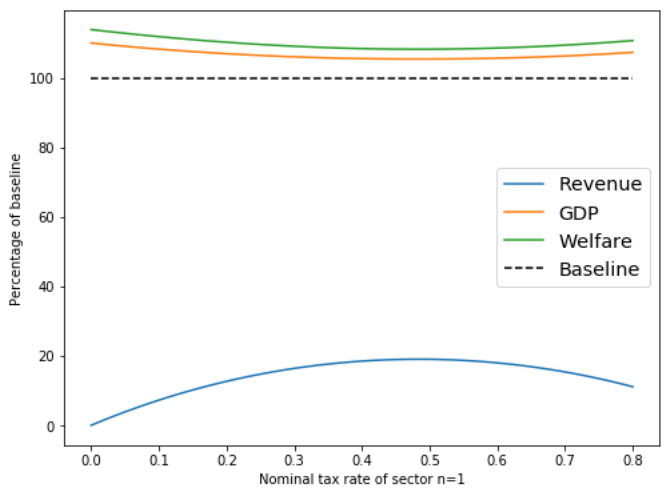


Figure 5: Optimal tax policy

Note that this means that there is a degree of freedom in the choice of tax levels. That is, we can choose how much to change in the nominal tax rate of sector $i$, and the remaining taxes are determined. This policy is shown in Figure 5, for different levels of nominal taxes chosen for sector 1 (Agriculture). Note that we can choose this sector to change the nominal tax rate without loss of generality. The vertical axis measures the counterfactual variables as percentages of the baseline variables, so the dotted line represents the baseline economy. Note that for all levels of final goods taxes, we have an increase both in GDP and welfare, although the government's revenues fall sharply, which means that the government should look for other sources in order to finance its spending's. At the maximum of the revenues, which is about 20% of the baseline, we have an increase of 5% of GDP and 8% of welfare, while the nominal tax for sector 1 is 48% but the effective tax is only about 10%.

Despite the benefits of the optimal tax policy reflected in figure 5, the magnitude of the losses in revenues suggest that this policy is unfeasible. Since these losses come from cutting intermediate goods taxes, we

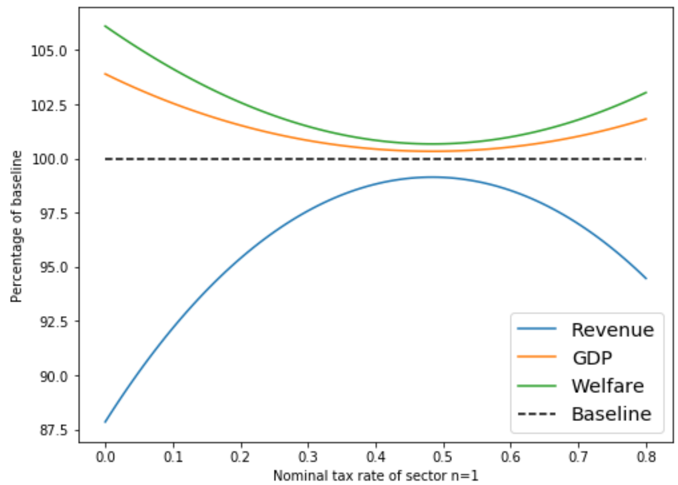


Figure 6: Optimal tax policy on final goods. Intermediate goods taxes are unchanged

implement alternative policies that change this prescription. First, we keep the intermediate goods taxes unchanged, this is, a 20% nominal tax rate, while we change the final goods taxes as we did before. This is shown in Figure 6. Note that in this policy there are still losses in the tax revenues, but they are sgnificantly smaller. At the maximum point the revenues are 99.13% of the baseline,while there is a 0.32% increase in GDP and 0.66% in welfare. Again the nominal tax levels are 48% for sector 1 (Agriculture) and the effective tax is about 10%.

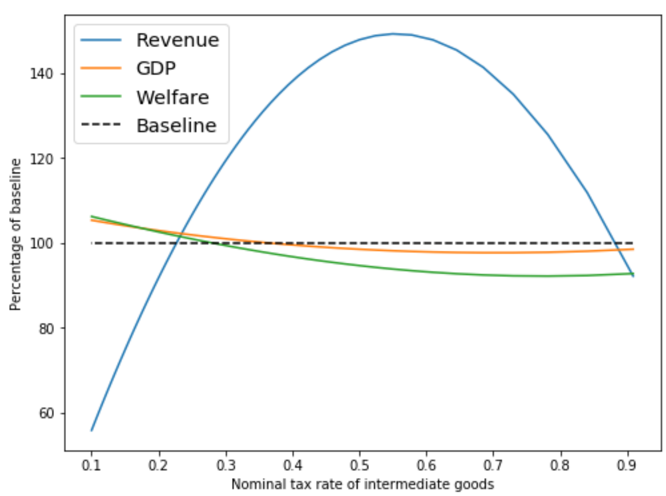


Figure 7: Optimal tax policy on final goods. Intermediate goods taxes are unchanged

Second, we impose a policy in which we keep the effective tax on (final goods) sector 1 unchanged, and we vary the intermediate goods taxes. Note that in this policy the nominal taxes of the sectors are modified accordingly to first part of proposition 3. This policy is shown in Figure 7. Note that there is a range of intermediate goods taxes in which we have a Pareto improvement: GDP and welfare increase while the government is able to finance its consumption. At the point that the revenue is constant relative to the baseline, we have a 2.26% increase in GDP and 1.55% of welfare.

## 6 Concluding Remarks

Classic tax evasion literature pay little or no attention to developing countries context of endemic tax evasion and how firms intrinsic characteristics would affect tax evasion. Assuming that levying commodities are necessary, we exploit whether the opportunity to tax circumvention would affect traditional policy recommendations on commodities and intermediate goods taxation. In other words, we assess whether homogeneous tax rates on commodities and zero tax rate on intermediate goods rules still applies. We also evaluate whether different collection systems would modify such scenarios.

In order to do so, we followed Ramsey (1927) approach to optimal taxation and, under a monopolistic competition model (Dixit and Stiglitz, 1977), allowed for the possibility of indirect tax evasion by firm in both final and intermediate goods sectors. Concealment technology was set in such a way that firm innate features, such as productivity, affects the probability of detection.

Our results show when there is no evasion, uniform nominal tax rates recommendation still applies to commodities taxation. However, in the presence of tax evasion, uniform nominal taxes rates are no longer an optimal policy. We derived optimal taxation conditions that dictates that tax authority should levy homogeneous effective taxes rates (and heterogeneous nominal tax rates) on final consumption goods. In addition, optimal taxes on intermediate goods are still zero even when intermediate firms are allowed to conceal revenues. Finally, optimal tax rules do not depend on which collection systems the tax authority sets up.

Optimal taxation rules under generalized tax evasion suggest that effective taxes rates, rather then nominal taxes rates, should be uniform. However it would require the government to know in advance how every final good evasion respond to taxes, i.e., $\phi^j(\tau_j, Z_j)$ for every $j$. As firms characteristics evolve along lifetime, taxes rates would have to adapt regularly to obey to derived rule. Since such policy can not be realistic implemented, government could rely on feasible convergent approaches. For example, could carefully choose a set of final goods to which it is be possible to diminish tax rates and simultaneously augment tax compliance. For such a set it would be wise to charge alleviated tax rates.

This paper did not addressed the question of how the presence of more complex utility functions structures would affect results. Also, it would be of great utility to explore how $\phi$ would endogenously emerge and to implement a more sophisticated strategy for the estimation of the parameters.

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# Appendix

## Proofs of propositions

First, let's derive equation (15). From the first order conditions of the consumer problem, $\lambda U_n = P_n$ and $\lambda U_l = -w$, where $\lambda$ is the Lagrange multiplier. Moreover, in equilibrium $\sum_{n,s} \pi_{n,s} = \rho wl$. Then, in an interior soultion

$$\begin{aligned}&\sum_{n=1}^{N} P_n C_n - wl - \sum_{n=1}^{N}\sum_{s=1}^{S} \pi_{n,s} \\ =&\sum_{n=1}^{N} \lambda U_n C_n + \lambda U_l l(1+\rho) = 0.\end{aligned}$$

Replacing $U_n = F_H H_n$ and using the fact that $H$ is homothetic, we have

$$F_H kH + F_l l(1+\rho) = 0$$

**Proposition 1** Suppose that there is no tax evasion (i.e., $\phi^n(\tau_n, Z_n) = \delta^{n,s}(\xi_n, A_{n,s}) = 1$ for all $\{n,s\}$). The Ramsey allocation is decentralized with an uniform effective tax rate policy on sectors, i.e., $(1-\tau_i)(1-\xi_i) = (1-\tau_j)(1-\xi_j)$ for every $i, j$ in $1, 2, \ldots, N$.

*Proof.* Let $\lambda_j$ and $\psi$ be the Lagrange multipliers for the resource constraint of sector $j$, and for the implementability condition (15). The first-order condition of the Ramsey Problem with respect to $C_j$ is

$$\frac{\lambda_j}{H_j} = F_H + \psi\left(F_{HH}kH + F_H k + l(1+\rho)F_{lH}\right) \quad j = 1, 2 \ldots, N. \tag{26}$$

Hence:

$$\begin{aligned}\frac{\lambda_i}{\lambda_j} &= \frac{F_H H_i}{F_H H_j} \\ &= \frac{U_i}{U_j}\end{aligned} \tag{27}$$

for any $i, j \in 1, 2 \ldots, N$. The first-order conditions with respect to $l_{i,v}$ and $l_{j,r}$ imply that

$$\frac{\lambda_j}{\lambda_i} = \frac{Y_i^{\frac{1}{\theta}} y_{i,v}^{\frac{\theta-1}{\theta}} l_{i,v}^{-1}}{Y_j^{\frac{1}{\theta}} y_{j,r}^{\frac{\theta-1}{\theta}} l_{j,r}^{-1}} \tag{28}$$

Therefore

$$\frac{U_j}{U_i} = \frac{Y_i^{\frac{1}{\theta}} y_{i,v}^{\frac{\theta-1}{\theta}} l_{i,v}^{-1}}{Y_j^{\frac{1}{\theta}} y_{j,r}^{\frac{\theta-1}{\theta}} l_{j,r}^{-1}} \tag{29}$$

In the competitive equilibrium we have that

$$\frac{U_j}{U_i} = \frac{(1-\tau_i)(1-\xi_i)\, Y_i^{\frac{1}{\theta}} y_{i,v}^{\frac{\theta-1}{\theta}} l_{i,v}^{-1}}{(1-\tau_j)(1-\xi_j)\, Y_j^{\frac{1}{\theta}} y_{j,r}^{\frac{\theta-1}{\theta}} l_{j,r}^{-1}} \tag{30}$$

The last two equations imply that the Ramsey allocation can be decentralized with an uniform effective tax policy, i.e., $(1-\tau_i)(1-\xi_i) = (1-\tau_j)(1-\xi_j)$. ■

**Proposition 2** Suppose that only intermediate firms practice some tax evasion, $\{\phi^n(\tau_n, Z_n) = 1\}_n = 1^N$ and $\{\delta^{n,s}(\xi_n, A_{n,s}) < 1\}_{n,s=1}^{N,S}$. Also, assume that $A_{j,r} \neq A_{j,\tilde{r}}$, for some $\{j, r\}$ and $\{j, \tilde{r}\}$. The Ramsey allocation is decentralized with an uniform tax rate policy on final goods, $\tau_i = \tau_j$, and no tax on intermediate goods, $\xi_i = \xi_j = 0$

*Proof.* Let $\lambda_j$ and $\psi$ be the Lagrange multipliers for the resource constraint of sector $j$, and for the implementability condition (15). The first-order conditions of the Ramsey Problem with respect to $C_i$ and $C_j$ jointly with the first-order conditions with respect to $l_{i,v}$ and $l_{j,r}$ imply that

$$\frac{U_j}{U_i} = \frac{Y_i^{\frac{1}{\theta}} y_{i,v}^{\frac{\theta-1}{\theta}} l_{i,v}^{-1}}{Y_j^{\frac{1}{\theta}} y_{j,r}^{\frac{\theta-1}{\theta}} l_{j,r}^{-1}} \tag{31}$$

In the competitive equilibrium we have that

$$\frac{U_j}{U_i} = \frac{(1-\tau_i)\left(1-\xi_i\delta^{i,v}\right) Y_i^{\frac{1}{\theta}} y_{i,v}^{\frac{\theta-1}{\theta}} l_{i,v}^{-1}}{(1-\tau_j)\left(1-\xi_j\delta^{j,r}\right) Y_j^{\frac{1}{\theta}} y_{j,r}^{\frac{\theta-1}{\theta}} l_{j,r}^{-1}} \tag{32}$$

The last two equations imply that

$$(1-\tau_j)\left(1-\xi_j\delta^{j,r}\right) = (1-\tau_j)\left(1-\xi_j\delta^{j,\bar{r}}\right) \tag{33}$$

Since $A_{j,r} \neq A_{j,\bar{r}}$, it follows that $\xi_j = 0$. Together first order conditions and (22) imply that the Ramsey allocation can be decentralized only through an uniform tax rate on final goods firms, i.e., $\tau_i = \tau_j$. ■

**Proposition 3** Suppose that every firm can practice some tax evasion, $\{\phi^n(\tau_n, Z_n) < 1\}_{n=1}^N$ and $\{\delta^{n,s}(\xi_n, A_{n,s}) < 1\}_{n,s=1}^{N,S}$. Also, assume that $Z_i > Z_j$, for some $i$ and $j$, and $A_{j,r} \neq A_{j,\bar{r}}$, for some $\{j, r\}$ and $\{j, \tilde{r}\}$. The Ramsey allocation is decentralized with no taxation on intermediate goods, $\xi_i = \xi_j = 0$ and a non-uniform tax rate policy on final good firms, $\tau_i \neq \tau_j$. Moreover, i. If $\left\{f^j(\tau, A_j) = \phi^j(\tau, A_j)\tau\right\}_{j=1}^N$ and $\left\{f'^j(\tau) < 0\right\}_{j=1}^N$, then $\tau_j < \tau_i$ for $\phi^j(\tau) < \phi^i(\tau)$. ii. If $\left\{f^j(\tau) = \phi^j(\tau)\tau\right\}_{j=1}^N$ and $\left\{f'^j(\tau) > 0\right\}_{j=1}^N$, then $\tau_j > \tau_i$ for $\phi^j(\tau) < \phi^i(\tau)$.

*Proof.* Just like previous proofs, let $\lambda_j$ and $\psi$ be the Lagrange multipliers for the resource constraint of sector $j$, and for the implementability condition (15). Once again, the first-order conditions of the Ramsey Problem with respect to $C_i$ and $C_j$ jointly with the first-order conditions with respect to $l_{i,v}$ and $l_{j,r}$ imply that

$$\frac{U_j}{U_i} = \frac{Y_i^{\frac{1}{\theta}} y_{i,v}^{\frac{\theta-1}{\theta}} l_{i,v}^{-1}}{Y_j^{\frac{1}{\theta}} y_{j,r}^{\frac{\theta-1}{\theta}} l_{j,r}^{-1}} \tag{34}$$

In competitive equilibrium

$$\frac{U_j}{U_i} = \frac{\left(1-\tau_i\phi^i\right)\left(1-\xi_i\delta^{i,v}\right) Y_i^{\frac{1}{\theta}} y_{i,v}^{\frac{\theta-1}{\theta}} l_{i,v}^{-1}}{\left(1-\tau_j\phi^j\right)\left(1-\xi_j\delta^{j,r}\right) Y_j^{\frac{1}{\theta}} y_{j,r}^{\frac{\theta-1}{\theta}} l_{j,r}^{-1}} \tag{35}$$

Together, equations (24) and (25) imply:

$$\left(1-\tau_i\phi^i\right)\left(1-\xi_i\delta^{i,v}\right)=\left(1-\tau_j\phi^j\right)\left(1-\xi_j\delta^{j,r}\right) \tag{36}$$

For $\{j,r\}$ and $\{j,\tilde{r}\}$ we have $\xi_j\delta^{j,\tilde{r}}=\xi_j\delta^{j,r}$. Since $A_{j,r}\neq A_{j,\tilde{r}}$ it follows that $\xi_j=0$ Together with (26) it implies

$$\tau_i\phi^i=\tau_j\phi^j$$

If $\left\{f'^j(\tau)<0\right\}_{j=1}^N$ and $\phi^j(\tau)<\phi^i(\tau)$, then $\tau_j<\tau_i$. Similarly if $\left\{f'^j(\tau)>0\right\}_{j=1}^N$ and $\phi^j(\tau)<\phi^i(\tau)$, then $\tau_j>\tau_i$. ■

**Proposition 4** Just like the previous proof, the following equation still holds

$$\left(1-\tau_i\phi^i\right)\left(1-\xi_i\delta^{i,v}\right)=\left(1-\tau_j\phi^j\right)\left(1-\xi_j\delta^{j,r}\right) \tag{37}$$

If there is a informal final good firm, then

$$1-\xi_i\delta^{i,v}=\left(1-\tau_j\phi^j\right)\left(1-\xi_j\delta^{j,r}\right) \tag{38}$$

**Proposition 5** Suppose that no firm can evade tax, $\{\phi^n(\tau_n,Z_n)=1\}_{n=1}^N$ and $\{\delta^{n,s}(\xi_s,A_{n,s})=1\}_{n,s=1}^{N,S}$. The Ramsey allocation is decentralized with

1. a uniform tax rate policy on final good firms, $\tau_i=\tau_j$ for any $i,j\in\{1,2,\ldots,N\}$.

2. a uniform tax rate policy on intermediate good firms, $\xi_r=\xi_v$ for any $r,v\in\{1,2,\ldots,S\}$.

*Proof.* From previous proofs we know that from Ramsey and competitive equilibria we must have

$$(1-\tau_i)(1-\xi_v)=(1-\tau_j)(1-\xi_r)\ \text{ for any } i,j\in\{1,2,\ldots,N\};\ \text{and } v,r\in\{1,2,\ldots,S\} \tag{39}$$

It is easy to show that $\tau_i=\tau_j$ for any $i,j\in\{1,2,\ldots,N\}$ and $\xi_r=\xi_v$ for any $r,v\in\{1,2,\ldots,S\}$. ■

**Proposition 6** Suppose that only intermediate firms can evade tax, $\{\phi^n(\tau_n,Z_n)=1\}_{n=1}^N$ and $\{\delta^{n,s}(\xi_s,A_{n,s})<1\}_{n,s=1}^{N,S}$. Also, assume that there exist $A_{j,r},A_{j,v},A_{i,r},A_{i,v}$, for some $\{j,i\}\in\{1,2,\ldots,N\}$ and $\{r,v\}\in\{1,2,\ldots,S\}$, such that $\frac{\delta^{j,r}(\xi_r,A_{j,r})}{\delta_{r,}(\xi_r,A_{j,v})}\neq\frac{\delta^{4,r}(\xi_r,A_{i,r})}{\delta^{ir,v}(\xi_v,A_{i,v})}$. Once again, the Ramsey allocation is decentralized with an uniform tax rate policy on final goods, $\tau_i=\tau_j$, and no tax on intermediate goods, $\xi_r=\xi_v=0$.

*Proof.* It follows from previous proofs that in Ramsay allocation we must have:

$$(1-\tau_i)\left(1-\xi_v\delta^{i,v}(\xi_v,A_{i,v})\right)=(1-\tau_j)\left(1-\xi_r\delta^{j,r}(\xi_r,A_{j,r})\right) \tag{40}$$

Suppose that tax authority sets $\xi_r\neq0$ and $\xi_v\neq0$. For any two sectors $j,i\in\{1,2,\ldots,N\}$ it would require:

$$\frac{\xi_r}{\xi_v}=\frac{\delta^{j,r}(\xi_r,A_{j,r})}{\delta^{j,v}(\xi_v,A_{j,v})}=\frac{\delta^{i,r}(\xi_r,A_{i,r})}{\delta^{i,v}(\xi_v,A_{i,v})}.$$

It is easy to prove that it is not possible simultaneously to have $\xi_r = 0$ and $\xi_v \neq 0$. Since $\xi_r = \xi_v = 0$, it implies that $\tau_i = \tau_j$. ■

**Proposition 7** Suppose that every firm can practice some tax evasion, $\{\phi^n(\tau_n, Z_n) < 1\}_{n=1}^{N}$ and $\{\delta^{n,s}(\xi_s, A_{n,s})$ $1\}_{n,s=1.}^{N,S}$ Also, assume that $Z_i > Z_j$, for some $i$ and $j$, and there exist $A_{j,r}, A_{j,v}, A_{i,r}, A_{i,v}$, for some $\{j,i\} \in \{1,2,\ldots,N\}$ and $\{r,v\} \in \{1,2,\ldots,S\}$, such that $\frac{\delta^{j,r}(\xi_r, A_{j,r})}{\delta, v}(\xi_r, A_{j,v}) \neq \frac{\delta^{(,r}(\xi_r, A_{i,r})}{\delta^{r,}(\xi_v, A_{i,v})}$. The Ramsey allocation is decentralized with no taxation on intermediate goods, $\xi_i = \xi_j = 0$ and a non-uniform tax rate policy on final good firms, $\tau_i \neq \tau_j$. Moreover,

1. If $\left\{f^j(\tau, A_j) = \phi^j(\tau, A_j)\tau\right\}_{j=1}^{N}$ and $\left\{f'^j(\tau) < 0\right\}_{j=1}^{N}$, then $\tau_j < \tau_i$ for $\phi^j(\tau) < \phi^i(\tau)$.
2. If $\left\{f^j(\tau) = \phi^j(\tau)\tau\right\}_{j=1}^{N}$ and $\left\{f'^j(\tau) > 0\right\}_{j=1}^{N}$, then $\tau_j > \tau_i$ for $\phi^j(\tau) < \phi^i(\tau)$.

*Proof.* Just similar to previous proofs, in Ramsey allocations must hold:

$$\left(1 - \tau_i\phi^i(\tau_i, Z_i)\right)\left(1 - \xi_v\delta^{i,v}(\xi_v, A_{i,v})\right) = \left(1 - \tau_j\phi^j(\tau_j, Z_j)\right)\left(1 - \xi_r\delta^{j,r}(\xi_r, A_{j,r})\right) \quad (41)$$

It follows from Proposition 5's proof that $\xi_s = 0$ for every $s \in \{1,2,\ldots,S\}$. Which means:

$$\tau_i\phi^i(\tau_i, Z_i) = \tau_j\phi^j(\tau_j, Z_j)$$

If $\left\{f'^j(\tau) < 0\right\}_{j=1}^{N}$ and $\phi^j(\tau) < \phi^i(\tau)$, then $\tau_j < \tau_i$. Similarly if $\left\{f'^j(\tau) > 0\right\}_{j=1}^{N}$ and $\phi^j(\tau) < \phi^i(\tau)$, then $\tau_j > \tau_i$.

Item 1 (and 2) suggests that if the effective tax rate, $\phi(\tau)\tau$, is decreasing (increasing) with the tax rate $\tau$, then it is optimal to tax heavier the sector with smaller (larger) tax evasion. ■

**Proposition 8** Assuming that $\delta$ is the percentage change in the tax rate and that $\bar{R}_t = \hat{Y}_t\hat{\tau}_t\hat{\phi}_t$ is the value of revenue under tax-invariant revenue, then defining $\hat{R}_t = \frac{\hat{Y}_t\hat{\tau}_t\hat{\phi}_t}{(1+\delta)}$ makes it possible to recover the elasticity of evasion with respect to the tax rate.

*Proof.* The invariance of revenue in collection implies that $Y_t = \hat{Y}_t$. Substituting $\hat{A}_t$ in the definition of elasticity (equation 20) and using the fact that $\tau_t = (1+\delta)\hat{\tau}_t$, we have:

$$\frac{\Delta A_t}{\Delta \tau_t}\frac{\tau_t}{A_t} = \frac{\frac{Y_t\hat{\tau}t\hat{\gamma}t}{(1+\delta)} - Y_t\tau_t\gamma_t}{\hat{\tau}_t - \tau_t}\frac{\tau_t}{A_t} = \frac{(\hat{\gamma}_t - \gamma_t)}{\gamma_t}\frac{\tau_t}{(\hat{\tau}_t - \tau_t)} \quad (42)$$

Which is exactly the value of interest. ■

**Proposition**9 Let $log(A_t)$ be the stochastic process defined in (21), and $E(\cdot)$ be the expectation operator. Then, the estimator defined by $\hat{E}_C = exp^{\left[E(log(A_t)|\gamma=\overline{\gamma},\tau=\overline{\tau}) - E(log(A_t(1+\delta))|\gamma=\underline{\gamma},\tau=\underline{\tau})\right]} - 1$ identifies the evasion elasticity.

*Proof.* Substituting 21 into the estimator:

$$\begin{aligned} log(\hat{E}_C + 1) =& (log\,(A_t)\,|\gamma = \overline{\gamma}, \tau = \overline{\tau}) - (log\,(A_t(1+\delta))\,|\gamma = \underline{\gamma}, \tau = \underline{\tau}) \\ =& f(\cdot) + log(\overline{\tau}) + log(\overline{\gamma}) - f(\cdot) - log(\underline{\gamma}) - log(\underline{\tau}(1+\delta)) \\ =& log(\overline{\tau}) + log(\overline{\gamma}) - log(\underline{\gamma}) - log(\underline{\tau}(1+\delta)). \end{aligned}$$

Since $\underline{\tau}(1+\delta) = \overline{\tau}$, we have:

$$\hat{E}_C = \frac{\overline{\gamma} - \underline{\gamma}}{\underline{\gamma}}. \tag{43}$$

■