Inflation Targeting under Fiscal Fragility

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Abstract

We model the intertemporal tradeoff between fiscal and monetary policy under inflation targeting. An indebted policymaker chooses public expenditure and inflation. Private agents form expected inflation. The debt level determines target credibility. For an endogenous interval of debt, the fiscal fragility zone (FFZ), there are multiple equilibria, expected inflation is above the announced target, and debt rollover is expensive. Within the FFZ, policymakers should (i) employ fiscal austerity to gradually reduce debt and (ii) increase the inflation target to raise the lower bound of the FFZ. The optimal inflation target is the lowest target whereby the economy exits the FFZ.

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1 Introduction

In a seminal paper, Kydland and Prescott (1977) make a general claim against discretionary policies by arguing that rules are a better way to coordinate expectations. Regarding monetary policy, they conclude that a policymaker "doing what is best, given the current situation, results in an excessive level of inflation, but unemployment is no lower than it would be if inflation (possibly deflation or price stability) were at the socially optimal rate." During the 1980s and 1990s, several countries adopted their prescription, and the inflation targeting regime became the cornerstone of central bank coordination of inflation expectations. However, inflation target ranges are missed frequently in both advanced and emerging economies. Episodes of coordination failures in which inflation expectations suddenly lose their anchor and diverge from the announced targets are common in both country groups.[1] Most of these episodes lack sizable changes in fundamentals that would explain the shift in expectations, raising questions on the limits of inflation targeting to anchor short-term inflation expectations.

We propose a model that rationalizes these observable episodes of coordination failures and self-fulfilling inflation. The heart of our argument is that the economy's fiscal side is fundamental to understanding the capacity of the inflation-targeting regime to coordinate inflation expectations. We model a closed economy in which two types of agents, an altruistic policymaker and private agents, act rationally in an environment with complete information. The policymaker acts jointly as a fiscal authority and central bank, targeting the inflation target by choosing current inflation and financing government expenditures by selling debt. We assume that the policymaker is not perfectly committed to the inflation target and might deviate from it to make fiscal room for spending. Its decision is the solution to the tradeoff between inflating public debt away and keeping inflation on target to avoid the economic costs of deviating. Private agents choose how much debt to hold and form expectations about next-period inflation. Our framework builds on Cole and Kehoe (1996, 2000) and Araujo, Leon, and Santos (2013), expanding their analysis to a monetary policy setting.

In our model, target failures happen when the public debt level exceeds an endogenous threshold limit and enters the fiscal fragility zone (FFZ). When debt is low enough, the interest burden is low, and therefore government spending is high. When debt is above the endogenous cutoff and within the FFZ, the expected inflation rate is higher than the inflation target, generating a higher cost of debt service and lower government spending. If the benefit of abandoning the inflation target exceeds the cost of maintaining it, the policymaker

[1] Roger and Stone (2005) note that targets are often missed (40% in their sample) and sometimes "by substantial amounts and for prolonged periods." Based on our updated data set, used in Appendix B, we conclude that targets are still frequently missed (26%).

will inflate the public debt and increase public spending. Within the FFZ, the policymaker is subject to confidence crises, so the optimal policy may be to gradually reduce the public debt, eliminating confidence crises and supporting the inflation-targeting regime. Moreover, within this region, maintaining higher targets and raising the share of inflation-indexed bonds are valuable tools to anchor expectations and mitigate the risk of inflation overshooting.

The intuition for why a higher inflation target helps to anchor expectations is simple. We show that the amount of partial default available, the difference between the inflation chosen by the policymaker when deviating and the inflation target, decreases at the target level. Higher targets reduce the benefit of a higher inflation level for indebted policymakers, increasing their credibility. Consequently, the endogenous threshold limit that characterizes the FFZ increases with the inflation target level. As a result, the optimal inflation target is the lowest possible such that the economy exits the FFZ. The intuition for adopting indexed bonds is similar. They raise credibility by increasing the cost of deviating from the target.[2]

Our paper provides practical implications for the conduct of monetary policy. It seems naïve to choose a 2% inflation target without considering fiscal fundamentals as countries eventually do. Our model suggests raising the inflation target to help coordinate expectations in the short term. The call for indebted economies to not support low target levels and seek targets compatible with their fundamentals also holds under imperfect information when private agents disagree about inflation forecast (Araujo, Berriel, and Santos, 2016).

Our policy prescriptions reflect some indebted economies' practices as they are more prone to crises and usually have higher inflation targets. Recently, the fiscal limits of inflation targets have been tested as countries increase their debt levels to provide fiscal response against the COVID-19 pandemic and shutdowns.[3] Taylor, Cogan, and Heil (2020) highlight that the US debt level is expected to continue growing, and it should reach 192% in 2050. The fiscal deficits in the US are a structural problem and a challenge to inflation expectation coordination. Sims (2020) argues that the ratio of debt servicing costs to total tax receipts is critical to understanding the temptation to inflate the debt away. Debt services increase with the debt and/or the interest rate. Sims notes that the interest rate is a positive function of the debt-to-GDP ratio with no guarantee that it will be low forever. He argues that a sudden increase in inflation expectations and consequently in the nominal interest rate could trigger

[2]In the Appendix, we test our model predictions using a panel dataset of 20 countries with at least 15 years of inflation targeting. We find evidence that deviations from the target and the probability of overshooting are negatively related to the target level. We also find evidence that deviations from the target are positively related to the debt level.

[3]Hall and Sargent (2022) described the US government's response to the COVID-19 as a "War on COVID-19", comparing the economic policies employed in this period with the two twentieth-century world wars, which significantly increased federal government expenditures and debt. The authors are concerned that the "War on COVID-19" may entail higher inflation rates in the future, following the example of the previous world wars.

an inflation episode, as is true in our model.

Our model also rationalizes the response to the inflationary pressures in Brazil at the end of 2002. In that period, it became clear that the presidential candidate who would win the election could arrive with a new policy framework. As a result, inflation expectations exceeded the upper bound of the target, as seen in Figure 1, indicating a target confidence crisis. In response to rising inflation expectations, Brazilian policymakers twice increased the target for 2003, first at an additional meeting held in June 2002 and again in January 2003. This response by policymakers is in line with the predictions of our model.

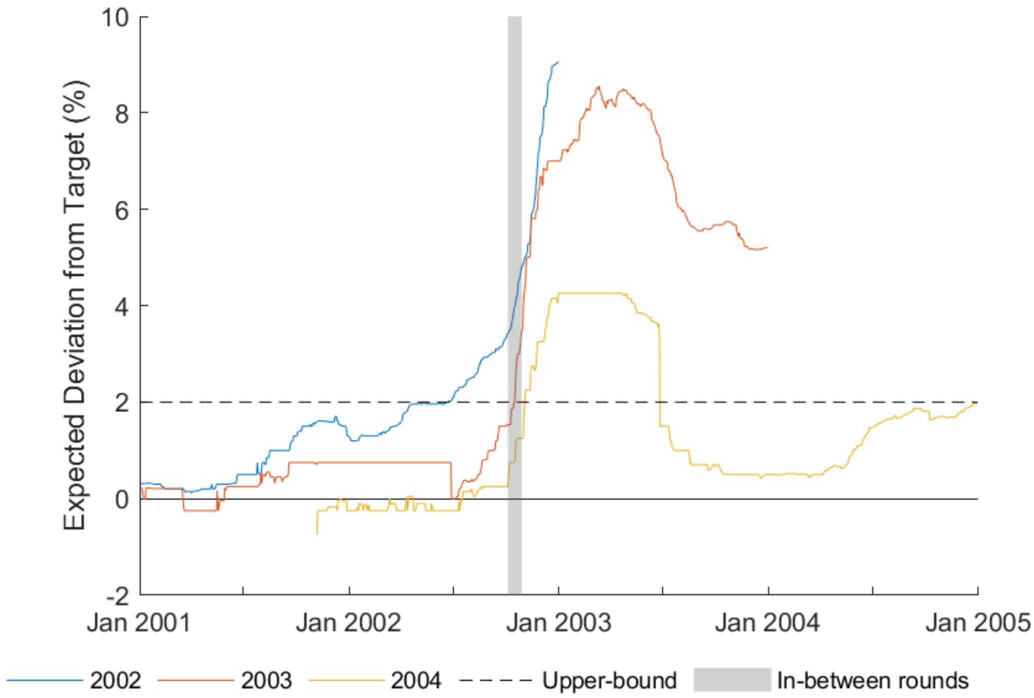

Figure 1: Expectation Crisis in Brazil

This figure shows the inflation expectation crisis that happened in Brazil in 2002. On the y-axis, we plot the expected inflation for the end of the year minus the inflation target for that year. Expected inflation is the mean expected inflation by professional forecasters, collected by the Central Bank of Brazil and available at The Focus – Market Readout. On the x-axis, we plot the date when expected inflation was formed. Until October 2002, expected inflation was within the inflation target bands. However, between rounds of the Presidential election – shaded grey region – inflation expectations overshoot the target's upper bounds at all horizons relevant to the central bank (current year, 1-year ahead, and 2-years ahead).

Novelty: The message on the fiscal limits of monetary policy achievements and the interdependence between fiscal discipline and price stability is amply addressed in the literature. Sargent and Wallace (1981) show the importance of fiscal side to understanding inflation control, followed by Leeper (1991), Sims (1994, 2011), Woodford (1995), Araujo and Leon (2002), Leeper and Leith (2016), Araujo, Berriel, and Santos (2016), and Cochrane (2018). We contribute to this literature with some policy prescriptions for the indebted policymakers in an inflation-targeting regime. Our model formalizes the point examined empirically by Fischer (1983): bond indexing amplifies inflation during crises[4]. This fact raises the cost of deviating from target, making debt indexing an instrument that contributes to exit the crisis zone.

We also innovate by incorporating a policymaker that decides monetary policy strategically, who does not follow a Taylor rule and may use inflation to partially default, into a simple DSGE. This approach closely follows papers on confidence crises in debt markets such as Cole and Kehoe (1996, 2000), Calvo (1988), and Arellano, Mihalache, and Bai (2019). We therefore avoid the inappropriate simplification of considering the central bank as a rule, with no strategic drivers, as in Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007). While other papers explore debt crises and their relation to monetary policy, such as Uribe (2006), Aguiar, Amador, Farhi, and Gopinath (2013), Corsetti and Dedola (2016), Bacchetta, Perazzi, and van Wincoop (2018), and Arellano, Mihalache, and Bai (2019), none of them consider the relation between fiscal and inflation targeting coordination.

Next Sections: In Section 2, we set out the model and derive the recursive form defining the equilibrium. In Section 3, we specify functional forms and parameter values in a quantitative analysis to match the situation in Brazil in 2002. We then analyze the results from our model. In Section 4, we analyze the 2002 confidence crisis in Brazil and the subsequent policy responses. Finally, the last section presents concluding remarks.

2 Model

We consider a closed economy with two types of agents: a policymaker and private agents. Each agent lives infinite periods and forms rational expectations with complete information. The policymaker acts as a combined fiscal and monetary authority, choosing current inflation and selling one-period debt to finance itself. In our setup, the inflation choice reduces to a discrete choice each period of whether to deviate from the target. We assume that the policymaker is altruistic and maximizes private agent welfare. Private agents receive a stream of fixed endowments. In each period, they choose how much debt to hold and form expectations about next-period inflation while considering the exogenous announced inflation target and the current debt level. When multiple equilibria are possible, a sunspot variable determines

[4] According to Fischer (1983), "empirical evidence is that countries with bond indexation were more seriously affected by the oil shock than other countries. . . whether and why bond indexing might play a special role in this regard is a subject for further research."

the equilibrium.

2.1 Basic Setup

Policymaker

We consider an altruistic policymaker who chooses both fiscal and monetary policies to maximize private agents' utility. The policymaker, as a monetary authority, chooses the inflation rate π_t and, as a fiscal authority, the next period's debt D_{t+1}:

$$\max_{\pi_t, D_{t+1}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, g_t) \tag{1}$$

where c_t is private agents' consumption in period t, g_t is government spending on public goods, and β is the intertemporal discount rate $0 < \beta < 1$. Consumption and public goods are nonnegative. We define private agents' utility as a weighted average of linear consumption and government spending utility similar to Cole and Kehoe (2000). The weights are defined by the parameter $\rho \in (0, 1)$ that can be interpreted as a relative preference for consumption:

$$u(c_t, g_t) = \rho c_t + (1 - \rho) v(g_t)$$

where v is a twice-differentiable strictly increasing and strictly concave function of g satisfying $\lim_{g \to 0^+} v(g) = -\infty$.

Linearity in consumption is a strong assumption and deserves further comment. First, it allows us to define the real interest rate as a risk-neutral pricing formula, which approximates the equilibrium ex post real interest rate. It also simplifies the problem by making the debt stationary outside the crisis zone that remains to be defined. Finally, it readily makes the marginal utility of public goods higher than the marginal utility of consumption, since public spending is constrained by debt interest spending and the tax rate, assumed to be fixed.

In each period, the policymaker finances the nonnegative spending g_t and the repayments on previous-period obligations through a fixed tax rate τ[5] on a deterministic endowment e and the issuance of new debt D_{t+1}. We assume that the tax level is not high enough so that total tax revenue exceeds the optimal amount of public spending, which is the level that equates to marginal spending between public and private consumption. Mathematically, we can write this hypothesis as $(1 - \rho) v'(\tau e) \geq \rho$. The government's budget constraint is given

[5]The fixed tax rate hypothesis can be interpreted as a situation in which the policymaker has no additional space to increase taxes to reduce indebtedness without significantly affecting output. This situation is similar to what is observed in a middle-income economy with relatively high tax levels such as Brazil.

by

$$g_t + (1 + r_t)D_t \leq D_{t+1} + \alpha_t \tau e \ , \tag{2}$$

where D_t is the last-period debt.[6] [7]

The fixed endowment is subject to a penalty α_t that depends on the policymaker's choice of inflation. Let π^a denote the exogenously set inflation target. The penalty function α is divided into two components, α^p and α^c. The first component, α^p, depends on the inflation level and reflects the productivity cost of the inflation level on output.[8] We assume a productivity cost of inflation function of the form of

$$\alpha^p(\pi) = (1 - \kappa) + \kappa e^{-\lambda \pi^2} \ ,$$

where $1 - \kappa$ is the lower limit on the inflation cost and λ is a fixed parameter. In this setup, $\alpha(0) = 1$, so the optimal inflation level considering only the productivity cost of inflation is zero. The second component of the penalty function α^c is a permanent fixed cost that affects the economy if the policymaker chooses to deviate from the target, reflecting the effect of a loss of credibility on the economy.[9] This fixed cost is of the form

$$\alpha_t^c = \begin{cases} 0 & \text{if } \pi_t = \pi^a, \alpha_{t-1}^c = 0 \\ -\epsilon & \text{if } \pi_t \neq \pi^a, \alpha_{t-1}^c = 0 \\ \alpha_{t-1} & \text{otherwise} \end{cases}$$

This productivity factor α_t should be understood as a reduced form capturing both the impact of inflation on welfare and output and the cost of deviating from the inflation target on economic activity. Therefore, $\alpha_t = \alpha^p + \alpha^c = \alpha(\pi_t, \pi^a, \alpha_{t-1})$ is a function of current inflation π_t, the inflation target π^a, and its past value α_{t-1} and is differentiable for any $\pi \neq \pi^a$. Finally, we define $\alpha^a = \alpha^p(\pi^a)$, the productivity cost of committing to the inflation target.

[6] We restrict our analysis to initial debt levels that leave the policymaker with a nonempty set of feasible choices, $D_t \in [0, D^{max}]$, where D^{max} is high enough. For very high initial debt levels, the policymaker could have no way of satisfying the positive constraint on c and g. To see this, suppose that debt servicing costs are higher than tax revenues, $\left(\frac{1}{\beta} - 1\right) D_0 > \tau e$, which leaves no space for spending. Then, even if the policymaker were to partially default on debt payments, it would still be unable to meet future positive spending restrictions due to the high future debt servicing costs and the inability to use inflationary surprises again.

[7] We also assume a no-Ponzi condition, so the government cannot run-up infinite debt in the long run: $\lim_t \beta^t D_{t+1} = 0$.

[8] See Bailey (1956) and Lucas (2000) for examples of models that find a way for inflation levels to affect welfare and output. Cysne (2009) shows that Bailey's measure provides a measure of the welfare costs of inflation derived from an intertemporal general-equilibrium model, while Campos and Cysne (2018) estimate output costs of inflation for the Brazilian case.

[9] Our approach of assuming an exogenous functional form for the "cost of deviating" is in line with the literature. Exogenous penalty functions are also assumed in self-fulfilling debt crisis models as in Cole and Kehoe (1996, 2000) and in sovereign default models as in Arellano (2008). We interpret the penalty function as a reduced and parsimonious form of capturing the negative impacts of inflation deviation on economic activity.

Given a linear utility in c, we define the ex post real interest rate by:

$$r_t = \frac{1+\pi_t^e}{1+\pi_t}\frac{1}{\beta} - 1 \,, \tag{3}$$

where $\pi_t^e = \mathbb{E}_{t-1}[\pi_t]$ is the expected inflation for period t formed by private agents in period $t-1$ and π_t is current inflation.[10] In this risk-neutral formula for the real interest rate, an agent expects a real return on government bonds of $1/\beta$ from period $t-1$ to period t.

In each period, the policymaker can satisfy the budget constraint by i) adjusting expenditures, ii) issuing new debt D_{t+1}, and iii) partially defaulting on debt through an inflationary surprise $(\pi_t > \pi_t^e)$ and rolling over the remaining debt. When the current inflation rate is equal to the expectation $\pi_t = \pi_t^e$, the ex post real interest rate will equal the inverse of the intertemporal discount rate, $1/\beta$. An inflationary surprise reduces the ex post real interest rate and, consequently, the payments the policymaker makes on its debt. Such a partial default offers additional fiscal room for government spending.

Private Agents

We assume a continuum of infinitely lived private agents who choose consumption and savings to maximize their expected utility:

$$\max_{c_t, d_{t+1}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, g_t) \,, \tag{4}$$

Each period, private agents receive a deterministic endowment e and payments on their bond holdings. The endowment is taxed at a constant rate τ by the government. The private agents' budget constraint is given by:

$$c_t + d_{t+1} \leq (1+r_t)d_t + \alpha_t(1-\tau)e \tag{5}$$

where d_{t+1} represents one-period bonds bought in t and d_t is the previous-period bond hold-

[10] We assume a bounded rationality model in which agents compute the approximate real interest rate r_t, using the risk-neutral approximation (3). This allows us to prove the main theoretical mathematical properties throughout the text while keeping the numerical properties of the computational simulations nearly identical. This is because, as shown in Online Appendix E.1, the risk-neutral real pricing definition in (3) is a close approximation of the equilibrium real interest rate,

$$r_t^{eq} = \frac{1}{\mathbb{E}_{t-1}\left[\frac{1}{1+\pi_t}\right]}\frac{1}{1+\pi_t}\frac{1}{\beta} - 1 \,,$$

that emerges from the first-order condition to the private agents' problem. We confirm the robustness of the approximation in the numerical simulations, as shown in Online Appendix E.2.

ings paying the interest rate $(1 + r_t)$. Private agents also form their inflation expectations π_t^e. The expectations formed will depend on the timing of actions assumed and will be properly defined below.

Discretionary Inflation

We motivate the existence of deviations from the inflation target by modeling an altruistic policymaker who might choose an inflation level higher than the inflation target as a way of transferring resources for increasing public spending.[11] In each period, the policymaker may choose to deviate from the exogenously set inflation target π^a, and private agents understand this when forming their expectations π_t^e. We call the inflation rate chosen by the policymaker when deviating from the target *discretionary inflation*. It is the result of a tradeoff between increasing government spending today against the costs of reducing consumption and the endowment losses due to the costs of deviating from the inflation target. Let π_T^D be the endogenous and optimal level of discretionary inflation chosen at the time T of the deviation.

We assume that once the policymaker deviates from the inflation target, private agents lose confidence in the commitment of the policymaker to the target. Therefore, private agents update the probability of the policymaker deviating in the next period setting it equal to 1. Consequently, after the policymaker deviates, the economy enters a steady state because there is no longer any uncertainty to be resolved. The optimal fiscal policy is to maintain constant debt, such as $D_t = D_{T+1} \; \forall \; t > T$, as shown below in Proposition 3. Finally, the penalty function takes the value $\alpha_T = \alpha(\pi_T)$ when deviating and remains so thereafter. The problem the policymaker resolves when defining the level of discretionary inflation can be written as follows:

$$\max_{\pi, D} u(c_T, g_T) + \frac{\beta}{1-\beta} u(c, g)$$

subject to

$$\begin{aligned} g_T &= \alpha(\pi)\tau e - D_T \frac{1+\pi_T^e}{1+\pi} \frac{1}{\beta} + D \\ g &= \alpha(\pi)\tau e - D\left(\frac{1}{\beta} - 1\right) \\ c_T &= \frac{1+\pi_T^e}{1+\pi} \frac{1}{\beta} D_T - D + \alpha(\pi)(1-\tau)e \\ c &= \left(\frac{1}{\beta} - 1\right) D + \alpha(\pi)(1-\tau)e. \end{aligned} \tag{6}$$

The optimal discretionary inflation level π_T^D is the solution to the problem above given an

[11] We do not model mechanisms of partial default on local currency domestic debt other than inflation, although governments have opted for alternatives such as reduction of principal or lower coupons (Reinhart and Rogoff, 2008).

initial debt level D_T[12]. Given rational expectations, in equilibrium, π_T^D is optimal given π_T^e and vice versa.[13]

The necessary first-order condition for D plus the hypothesis of concavity of u with respect to g readily implies that the policymaker will set a stationary debt level D such that the stationary public consumption equals public consumption at time T, that is, $g_T = g$. Consequently, we have that the optimal stationary debt is equal to

$$D = \frac{1+\pi_T^e}{1+\pi} D_T$$

that is, inflationary surprises ($\pi > \pi_T^e$) reduce steady-state debt, decreasing the debt burden and allowing for higher public consumption both in the short and long-run, which implies a steady-state level of public spending of

$$g = \alpha(\pi)\tau e - \frac{1+\pi_T^e}{1+\pi}\left(\frac{1}{\beta} - 1\right) D_T. \tag{7}$$

We can rewrite the above equation as

$$D_T \frac{1+\pi_T^e}{1+\pi} = \frac{\beta}{1-\beta}\left(\alpha(\pi)\tau e - g\right), \tag{8}$$

which corresponds to the intertemporal budget constraint of the government: the left-hand side is the real value of government debt in period T, which must be equal to the present value of future government surpluses. Choosing the stationary spending level g pins down the inflation level π that satisfies equation (8).[14]

To gain intuition for the optimal choice of g, consider now the first-order condition of the policymaker's problem for choosing discretionary inflation. The first-order condition, already substituting for the steady-state level of debt, is

[12] To avoid unnecessary notation, we drop the time subscript T on inflation and the debt level whenever there is no ambiguity.

[13] We numerically solve this problem by writing it as a fixed point. First, we assume an initial $\pi_{T,0}^e = \pi^a$ and then find the optimal $\pi_{T,1}^D$. We update $\pi_{T,1}^e$ using $\pi_{T,1}^D$ according to the inflation expectation formation process of the private agents that will be explained later. If $\pi_{T,1}^e \neq \pi_{T,0}^e$, the problem is iterated to find the new optimal $\pi_{T,2}^D$ given $\pi_{T,1}^e$. We continue this process until $|\pi_{T,i-1}^e - \pi_{T,i}^e| < \epsilon$, where ϵ is a small number. The existence of a rational expectation of inflation π^e given the optimal discretionary inflation π^D chosen by the policymaker is shown in Online Appendix C.

[14] This is the fiscal theory of the price level (FTPL) restriction emphasized by Cochrane (2022). Thus, our model can be seen as adding to the FTPL by considering an endogenous choice of default and explicitly modeling the role of the inflation target in coordinating inflation in a lower level.

$$[(1-\rho)v'(g)-\rho]\left(\frac{1+\pi_T^e}{(1+\pi)^2}\left(\frac{1}{\beta}-1\right)D_T\right)+(\rho(1-\tau)+(1-\rho)v'(g)\tau)\,\alpha' e=0 \qquad (9)$$

The first term represents the net marginal benefit of allocating spending to public goods through the inflationary surprise, which is positive since by assumption $(1-\rho)v'(g) \geq \rho$ for all feasible g. The second term illustrates the effects of the productivity penalty on the economy, causing lost consumption and government spending.

Timing

Rational expectations govern the strategic interactions between the policymaker and private agents. As in Cole and Kehoe (1996, 2000), multiple self-fulfilling equilibria may occur. Conditional on the debt level, the best response from the policymaker's perspective may depend on the expectations of private agents. If private agents expect a deviation from the target, the best response will be to deviate. If they expect no deviations, the best response will be to keep inflation on target. In this case, we consider an exogenous sunspot variable ζ_t to determine the selection of the equilibrium. The sunspot variable determines which of the possible inflation rates will be the actual inflation rate π_t implemented by the government when there are two equilibrium rates: the inflation target π^a and discretionary inflation π_t^D, which is defined in the next subsection.

At the beginning of each period, uncertainty is resolved through the realization of the sunspot variable ζ_t. The policymaker, considering the sunspot variable previously drawn, chooses how much debt D_{t+1} to sell and the inflation rate π_t, which will either be the target π^a or the discretionary inflation rate π_t^D. Finally, private agents form their expectations about the next period's inflation rate π_{t+1}^e and decide the level of debt d_{t+1}. In summary, the timing of the model is:

- 1^{st} The sunspot variable ζ_t is realized.
- 2^{nd} The policymaker chooses actual inflation π_t, given sunspot ζ_t.
- 3^{rd} The policymaker chooses the next debt level D_{t+1}.
- 4^{th} Private agents form next-period inflation expectations π_{t+1}^e and choose the amount of next-period debt d_{t+1} to hold.

Given this timing, private agents may face uncertainty over which equilibrium will be selected next period when forming their inflation expectations. They will form expectations over the probability of each outcome, considering the exogenous distribution of the sunspot variable that determines the actual inflation rate. Inflation expectations will therefore be $\pi_t^e = f\pi_t^D + (1-f)\pi^a$ where f is the exogenously determined probability of the policymaker deciding to deviate from the inflation target due to an adverse situation, a negative sunspot.

Properties of Discretionary Inflation

Once the timing is defined and we have a formula for expected inflation, we can characterize the discretionary inflation chosen by the policymaker through the following properties:

Proposition 1 ***(Discretionary Inflation is Increasing in Debt)****: If π_T^D is an interior solution to the problem (6), $\alpha''(\pi_T^D) < 0$, and α' is sufficiently bounded such that $\frac{\partial g}{\partial \pi} \geq 0$, then the discretionary inflation level π_T^D is increasing in the initial debt level D_T: $\frac{\partial \pi_T^D}{\partial D_T} > 0$.*

Proof: see Appendix A.1.

Higher debt levels increase interest spending, reducing available funds for public consumption, which causes the policymaker to increase the discretionary inflation level to increase public spending. This is true as long as the penalty function is not too steep, since this could cause a higher inflation level to reduce public consumption.

Proposition 2 ***(Discretionary Inflation Deviation Decreases in the Target Level):*** *Under the same hypothesis of proposition 1, assume that α'' is sufficiently negative for all π_T^D, where $D_T \in [0, D^{max}]$; then, $\frac{\partial \pi_T^D}{\partial \pi^a} < 1$.*

Proof: see Appendix A.1.

If $\frac{\partial \pi_T^D}{\partial \pi^a} < 1$, then the deviation from the target $\pi_T^D - \pi^a$ decreases as the target level π^a rises. The intuition for this result is simple: a higher target induces the policymaker to raise π_T^D to attain the same level of reduction in the interest payment for a given level of debt. However, since the penalty is increasing in inflation and the penalty function is sufficiently concave, raising discretionary inflation causes total output to fall, which in turn reduces tax revenue and therefore spending. The policymaker balances these two effects, reducing deviation from the target.

2.2 Recursive Equilibrium

We define a recursive equilibrium where the policymaker and private agents sequentially choose their actions. At the beginning of each period, the aggregate state $s = (D, \pi^e, \zeta, \alpha_{-1})$

is public since the aggregate debt D, the expected inflation for the current period π^e, the realization of the sunspot variable ζ, and the past penalty α_{-1} have all been determined in the previous period. The policy choices, π and D', the expected inflation for the next period $\pi^{e\prime}$, and the individual debt holdings for the next period d' determine the equilibrium jointly with s. We denote by $\pi(\cdot)$ and $D(\cdot)$ the inflation and debt policy functions, by $r(\cdot)$ the real interest rate function, and by $\pi^e(\cdot)$ the inflation expectation function, all yet to be defined.

To define a recursive equilibrium, we work backward on the timing of actions in each period. We start the definition of a recursive equilibrium with private agents because they move last. When forming expectations $\pi^{e\prime}$ at the end of any period, private agents know all their public debt holding d, the aggregate state s, the policymaker's offer of new debt D', current-period inflation π, and the policymaker's optimal policy functions. The following functional equation defines a private agent's value function:

$$
\begin{aligned}
V^{pa}(s,d,\pi,D') = & \max_{c,d'} u(c,g) + \beta \mathbb{E} V^{pa}(s',d',\pi',D'') \\
& \text{subject to} \\
& \qquad c + d' \leq (1 + r(s,\pi))d + \alpha(\pi,\pi^a,\alpha_{-1})(1-\tau)e \\
& \qquad s' = \Big(D', \pi^e(s,d,\pi,D'), \alpha(\pi,\pi^a,\alpha_{-1}), \zeta'\Big) \\
& \qquad \pi' = \pi(s') \\
& \qquad D'' = D(s') \\
& \qquad c \geq 0 \\
& \qquad d' \geq 0
\end{aligned} \tag{10}
$$

in which we assume that private agents cannot sell public debt. The penalty function $\alpha(\cdot)$ is a function of its previous value α_{-1}, the inflation target π^a, and current inflation π.

Each period after the policymaker decides how much debt D' to offer and the inflation rate π, private agents decide how much debt to hold. Let $d'(s,d,\pi,D')$ be their debt policy function. When forming inflation expectations, private agents determine the nominal interest rate for the next period. In the absence of multiple equilibria, they perfectly anticipate π, and the real return is always $1/\beta$. If multiple equilibria are possible, private agents do not know what the policymaker will opt to do.

When forming inflation expectations, private agents consider what the policymaker could do in the next period. Their expectations are defined as $\pi^e(s,d,\pi,D') = \mathbb{E}\pi(s')$, where the expectation is conditional on all information available to the agent at the moment. When forming expectations, the set $(D', \pi^{e\prime}, \alpha) \in s'$ is known to private agents. Hence, the only unknown variable on which private agents form their expectations is the realization of the sunspot variable ζ'. Integrating out the sunspot variable's commonly known distribution, we have

$$\mathbb{E}\pi(s') = \begin{cases} f \times \pi^D(D', \pi^{e\prime}, \alpha) + (1-f) \times \pi^a & \text{if multiple eq.} \\ \pi^D(D', \pi^{e\prime}, \alpha) & \text{if deviating unique eq.} \\ \pi^a & \text{if not deviating unique eq.} \end{cases} \tag{11}$$

where f is the exogenous probability of the adverse equilibrium occurring and $\pi^D(D', \pi^{e\prime}, \alpha)$ is the discretionary inflation chosen by the government when deviating given $(D', \pi^{e\prime}, \alpha) \in s'$.

The policymaker chooses, at the beginning of the period, inflation π and debt issuance D', given state s. The policymaker knows that the next period's debt level affects the private agents' inflation expectations and resolves the following problem:

$$\begin{aligned} V^p(s) = \max_{\pi, D'} u\Big(c(s, d, \pi, D'), g\Big) + \beta \mathbb{E} V^p(s') \\ \text{subject to} \\ g + (1 + r(s,\pi))D \leq D' + \alpha(\pi, \pi^a, \alpha_{-1})\tau e \\ s' = (D', \pi^e(s, d, \pi, D'), \alpha(\pi, \pi^a, \alpha_{-1}), \zeta') \\ g \geq 0 \end{aligned} \tag{12}$$

We can now define a recursive equilibrium for our model economy. An equilibrium is a list of value functions for the representative private agent V^{pa} and for the policymaker V^p; functions $c(\cdot)$ and $d'(\cdot)$ for the private agents' consumption and saving decisions; functions $\pi(\cdot)$ and $D'(\cdot)$ for the policymaker's inflation and debt decisions; an inflation expectation function $\pi^e(\cdot)$; a real interest rate function $r(\cdot)$; and an equation of motion for the aggregate debt level D' such that the following holds:

- Given D' and π, V^{pa} is the value function for the solution to the representative private agents' problem with c, d' and $\pi^{e\prime}$ being the maximizing choices when $d' = D'$.
- Given π^e, V^p is the value function for the solution to the policymaker problem, and both D' and π are the maximizing choices.
- $D'(s)$ equals $d'(s, d, \pi, D')$.

Our definition of an equilibrium is similar to that of Cole and Kehoe (1996) and Cole and Kehoe (2000) and is restricted to a Markov equilibrium. Future conditional plans of the agents can be derived from their policy functions.

2.3 The Fiscal Fragility Zone

The ability of the policymaker to effectively target inflation is restricted by debt levels. Assuming that inflation has always been on target, three different scenarios can be drawn according to the debt level D[15]:

- The *no crisis zone*: D such that $V^p(D, \pi_t = \pi^a, \pi^e = \pi^D) \geq V^p(D, \pi_t = \pi^D, \pi^e = \pi^D) \rightarrow \pi_t = \pi^a = \pi^e$.
- The *fiscal fragility zone*: D such that $\pi \in \{\pi^a, \pi^D\}$ depends on the sunspot.
- The *fiscal dominance zone*: D such that $V^p(D, \pi_t = \pi^D, \pi^e = \pi^a) \geq V^p(D, \pi_t = \pi^a, \pi^e = \pi^a) \rightarrow \pi_t = \pi^D = \pi^e$.

In the first case, the policymaker finds it preferable to keep inflation on target even when private agents believe that it will not. Consequently, only one equilibrium is possible where private agents have faith in the policymaker delivering on the target inflation. Since there is only one optimal choice for the policymaker regardless of the private agents' expectations, the sunspot ζ is disregarded, and the only important variable defining the policymaker's value function V^p is the debt level D. The same holds for the third case when the only equilibrium is the policymaker always deviating from the inflation target.

Whenever the policymaker is in the no-crisis zone or the fiscal dominance zone, it will always choose a stationary debt policy, as shown in Proposition 3:

Proposition 3 ***(Stationary Policy Outside of the Fiscal Fragility Zone):*** *The optimal debt policy chosen by the policymaker outside of the FFZ at period S is stationary, that is, $D_t = D_S$ for all $t \geq S$.*

Proof: see Appendix A.2.

When the policymaker deviates, private agents lose confidence forever in the ability of the government to maintain inflation targeting and, therefore, expect discretionary inflation, $\pi^e = \pi^D$. It is as if the economy enters the fiscal dominance zone. The above proposition justifies why we are allowed to consider only a stationary debt policy in the definition of discretionary inflation (6).

[15]With slight abuse of notation, we denote by $V^p(D, \pi_t, \pi^e)$ the total intertemporal utility attained by the policymaker by choosing inflation level π_t, given debt level D and private agents' expected inflation π^e.

The more interesting scenario is multiple equilibria akin to self-fulfilling target failures. If private agents believe that the target will be delivered, then the policymaker will prefer to do so. On the contrary, in the face of adverse expectations, the policymaker chooses to deviate. In this zone, private agents have doubts about the commitment of the monetary authority to the target. The equilibrium is chosen by the realization of a sunspot, something the government binds its choice to but is unrelated to any observable fundamentals. In the rest of the article, we will interpret it as a deterioration in inflation expectations. That is, inflation expectations go from π_t^e to π_t^D for some reason unrelated to fundamentals.

When government debt is within the FFZ, the expected inflation rate is higher than the inflation target, generating a higher cost of debt service and lower government spending. Thus, the government has an incentive to raise spending, which it can do through inflation or increasing debt. To see this, let us recall the real interest rate on bonds from Equation (3), $r_t = (1 + \pi_t^e)/\beta(1 + \pi_t) - 1$. In the FFZ, inflation expectations will be given by $\pi_t^e = f\pi_t^D + (1 - f)\pi^a$. The real interest rate in the FFZ when the policymaker delivers the target will be given by:

$$1 + r_t = \frac{(1 + f\pi_t^D + (1 - f)\pi^a)}{1 + \pi^a}\frac{1}{\beta} \tag{13}$$

which is higher than the interest rate outside this zone, $1/\beta$, as long as $\pi_t^D > \pi^a$, which will be the case in the FFZ by Proposition 5.

Whether an economy exhibits all of these zones will depend on the credibility cost α^c, as the following proposition shows:

Proposition 4 ***(Existence of the No-crisis and Fiscal Dominance Zones):*** *For a β sufficiently close to 1, we can state the following:*

- *If $\alpha^c = 0$, then every debt level $D \in [0, D^{max}]$ is in the fiscal dominance zone.*
- *For any $\alpha^c < \alpha^a - 1$, there is an interval $[0, D_-]$ in the no-crisis zone, with $D_- > 0$.*
- *There is a sufficiently low $\alpha^c > -1$ such that every $D \in [0, D^{max}]$ is in the no-crisis zone.*

Proof: see Appendix A.3

A zero credibility cost means that it is costless to deviate from the target, and so the policymaker always deviates regardless of debt level D. On the other hand, a credibility cost that is too high means that deviating from the target is never optimal since the high penalty substantially reduces government spending through lower tax revenues.

Proposition 5 shows that whenever the economy is in the FFZ, the policymaker will choose discretionary inflation levels above the inflation target, $\pi_t^D > \pi^a$ as long as there is a marginal utility from government spending that is higher than the marginal utility from private spending. This condition is always satisfied when debt is positive, given the linearity hypothesis for private consumption and strict concavity of public consumption.

Proposition 5 ***(Conditions for Positive Deviation from Target):*** *Suppose that the utility function, penalty, and initial debt level satisfy the stated assumptions and that $\alpha^c < \alpha^a - 1$. Then, in the FFZ, the optimal deviation is always positive.*

Proof: see Appendix A.4.

An altruistic policymaker maximizing private agent welfare may choose to deviate from the inflation target when it has limited fiscal room to finance public spending.

The Inflation Target Coordination Role

The marginal ability of the policymaker to transfer resources through inflation decreases as the target π^a increases, changing the tradeoff determining discretionary inflation. The marginal benefit of discretionary inflation will be reduced given the lower marginal capacity to transfer resources, which is a consequence of Proposition 2.

A policymaker with a higher inflation target will choose a smaller deviation from the target when private agents start doubting the target. Consequently, as deviations decrease, the policymaker will face a lower real interest rate on its bonds in the FFZ. To see this last point, let us examine the implications of the real interest rate in the FFZ. The real interest rate on bonds is given by:

$$r = \frac{1+\pi^e}{1+\pi}\frac{1}{\beta} - 1$$

where $\pi = \pi^a$ and $\pi^e = f\pi^D + (1-f)\pi^a$ in the FFZ. By Proposition 2, we know that $\frac{\partial \pi^D}{\partial \pi^a} < 1$. Therefore, it is also true that $\frac{\partial r}{\partial \pi^a} < 0$,

$$\begin{aligned}
\frac{\partial r}{\partial \pi^a} &= \frac{1}{\beta}\left(\frac{f\frac{\partial \pi^D}{\partial \pi^a} + (1-f)}{1+\pi^a} - \frac{1+f\pi^D+(1-f)\pi^a}{(1+\pi^a)^2}\right) \\
&< \frac{1}{\beta}\left(\frac{1}{1+\pi^a} - \frac{1+f\pi^D+(1-f)\pi^a}{(1+\pi^a)^2}\right) \\
&= \frac{1}{\beta}\left(\frac{f(\pi^a-\pi^D)}{(1+\pi^a)^2}\right) \\
&< 0
\end{aligned}$$

since $\pi^a - \pi^D < 0$.

2.4 Inflation-Indexed Debt

It is not unusual for governments to issue inflation-indexed bonds. We will examine the implications of changing the nature of the bonds. To achieve such indexed bonds within the framework of our model, we change the action timing to give private agents all the needed information to perfectly anticipate policymaker decisions. By allowing private agents to know the realization of the sunspot variable when forming their inflation expectations, bonds will pay a real interest rate $1/\beta$ in all states of nature.

1^{st} The policymaker chooses actual inflation π_t.

2^{nd} The policymaker chooses next debt level D_{t+1}.

3^{rd} The next-period sunspot variable ζ_{t+1} is realized.

4^{th} Private agents form next-period inflation expectations π^e_{t+1} and choose the amount of next-period debt d_{t+1} to hold.

With this new timing, private agents' information sets are given by $(s, d, \pi, D', \zeta') = s'$. Inflation expectations π^e given information set s' will be such that $\pi^e(s') = \pi(s')$ is the policymaker's choice of inflation for the next period.[16] As the policymaker's choices are anticipated, it is no longer possible to transfer resources from private agents in the event of a bad sunspot. In equilibrium, the policymaker would choose $\pi^D = \pi^e$ in the discretionary equilibrium since π^D is optimal given π^e and vice versa. Note that, in equilibrium, discretionary inflation could be different from the announced target $\pi^D \neq \pi^a$. The only way that discretionary inflation could equal the announced target $\pi^D = \pi^a$ would be if the inflation expectation equals the inflation target $\pi^e = \pi^a$. The next section will exploit the differences between indexed and nominal bonds.

3 Quantitative Analysis

In this section, we calibrate the model based on the 2002 confidence crisis in Brazil. The presidential election of 2002 is an interesting case study in that the candidate most likely to win was running on a platform that appeared likely to deteriorate the fiscal situation. Professional forecasters surveyed by the central bank predicted inflation overshooting the target for all horizons. This loss of the credibility of the inflation target in the face of a perceived fiscally fragile situation is the type of event our model is designed to capture.

[16]The different timing only changes problem (6) with respect to how we update inflation expectations π^e.

3.1 Functional Forms and Calibration

Our model is calibrated on a yearly frequency to match the usual time frame targeted by central banks, and almost all parameters correspond to observable values during the 2002 confidence crisis in Brazil. First, we set the inflation target, π^a, to 3.5%, the official target that prevailed in 2002. Second, the discount factor, β, is $1/1.0928$ to match the historical average of the ex post real interest rate between 1996 and 2019.[17] Third, the tax rate on endowments, τ, equals the 2002 general government revenue over GDP, 0.35. Fourth, the exogenous crisis probability, f, matches the country risk captured by the EMBI + Brazil around October 2002. Fifth, we set the endowments, e, to 1.5 so that the public spending marginal utility of the total tax revenues is not lower than the private consumption marginal utility: $(1-\rho)v'(\tau e) \geq \rho$. Finally, we choose a neutral value for consumption preference $\rho = 1/2$ for the baseline exercises. We will use this parameter to obtain some static comparative results later.

The parameter λ of the productivity cost of the inflation penalty is set according to Campos and Cysne (2018)'s estimation of a 0.35% of GDP cost for inflation of 10% in recent Brazilian experience. The fixed cost of deviating ϵ equals 0.002, meaning a permanent 0.2% of GDP penalty for deviating from the target, and it is set to jointly match the gross debt level and the inflation index observed in Brazil in 2002. The crises zone starts at approximately 70% of the debt ratio, the debt observed in 2002. The lower bound for the penalty, κ, is set to 20%. Table 1 summarizes the chosen values. For the calibrated model, we assume that the government spending utility function $v(g) = \log(g)$.

Parameter	Value	Meaning	Calibration
β	.915	Discount factor	Ex-post 1996-2019 real interest rate
τ	35%	Tax rate	General gov. revenue in % of GDP
π^a	3.5%	Inflation target	2002 BCB target
f	20%	Crisis prob.	EMBI+ Brazil on 10/2002
e	1.5	Endowment	Expansive gov.
ρ	.50	Pref. for consumpt.	Neutral value
κ	20%	Limit to TFP cost	Brazilian 2002 crisis
ϵ	.002	Fixed cost	Brazilian 2002 crisis
λ	1.77	Welfare cost	Campos and Cysne (2018) estimation for 10% inflation cost

Table 1: Parameters of the Baseline Model

[17] Using inflation-indexed bonds, such as Brazilian Bonds NTN-C or NTN-B, around 2002 would yield similar results.

3.2 Results

An indebted and altruistic policymaker optimally choosing inflation may deviate from the target in the event of an expectation shock. In our calibrated model, the policymaker becomes subject to such shocks after reaching a debt-to-GDP ratio of 70%. Below 70% of the debt ratio, the policymaker always prefers to keep inflation on target. For debt levels exceeding this lower bound, the equilibrium depends on the private agents' expectations, and the policymaker may decide to deviate given a negative sunspot shock. Taking this probability into account, private agents will demand higher nominal interest rates on government bonds once the policymaker exceeds this lower bound debt level. Finally, for debt levels exceeding 90% of GDP, the policymaker will always deviate from the target.

Optimal Fiscal Policy

The policymaker's optimal debt path depends upon the initial value of its debt stock. Outside the FFZ, it prefers to maintain debt levels constant, as shown in Proposition 3. Within the FFZ, it might i) choose fiscal responsibility and run down its debt to avoid the costs of an adverse equilibrium; ii) maintain constant debt levels; or iii) increase its debt to maintain a given spending level. In Figure 2, we plot the next period's debt as a function of current debt. The three possible responses of the policymaker are seen within the FFZ. Those results are similar to those of Cole and Kehoe (1996).

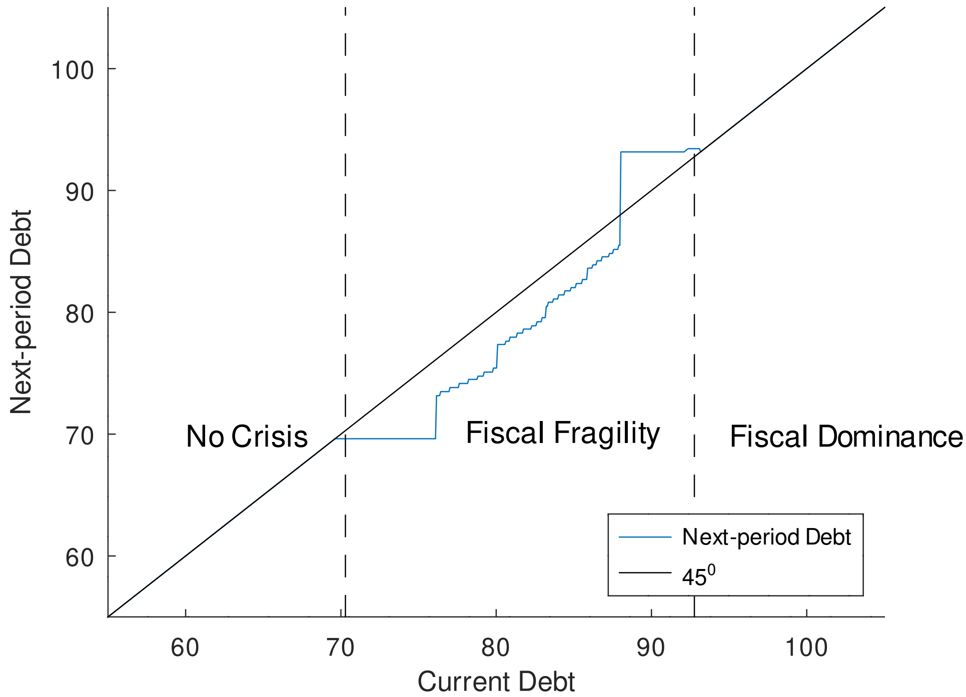

Figure 2: Debt Policy Function

For a moderate initial debt level within the FFZ, the policymaker chooses a fiscally responsible debt path to avoid the expected endowment loss from deviating from the inflation target in the eventuality of adverse inflation expectations. In this region, expected inflation is higher than the target rate, which means that the policymaker faces a higher real interest rate than in the no crisis zone. However, as long as a negative sunspot shock that removes the credibility of the policymaker does not hit the economy, the optimal fiscal policy is to gradually reduce the debt-to-GDP ratio until the economy exits the FFZ. As the policymaker follows this austerity policy, expected inflation gradually declines, reducing the real interest rate burden and making it easier for the economy to exit the FFZ. This can be noted by observing that the slope of the policy function decreases as debt-to-GDP approaches the lower bound of the FFZ. Table 2 presents the expected inflation rates and the corresponding number of periods required to exit the FFZ for different levels of initial debt:

Initial Debt Level	Debt Zone	Expected Inflation	Years to Exit the FFZ
60%	Credibility	3.5%	-
75%	Fiscal Fragility	4.5%	1 year
85%	Fiscal Fragility	4.7%	4 years
100%	Fiscal Dominance	11.5%	-

Table 2: Expected Inflation Rates and the Time at Which the FFZ is Exited in the Calibrated Model for the Brazilian Case

For a high initial debt level and fixed inflation target, the policymaker gradually reduces debt to return to the no-crisis zone. However, it takes a significant number of periods for the policymaker to regain credibility, during which it faces expected inflation rates higher than the target. The optimal policy for stabilizing inflation expectations in an environment of high indebtedness results in higher inflation expectations for a significant amount of time. Gradual exiting as an optimal decision is also present in Cole and Kehoe (1996, 2000) and in Sims (2020). The latter explicitly makes this prescription when claiming that it is not wise to reduce debt quickly by contracting fiscal latitude. This result is in line with many episodes in which countries that experienced sudden increases in their debt-to-GDP ratios needed to stabilize their economies through higher temporary inflationary rates. Hall and Sargent (2022) describe the US post-war experience and attribute an important role in reducing the real value of debt to increases in price levels. They argue that a similar scenario may happen after the rise in the debt-to-GDP ratio that followed the COVID-19 pandemic.

Nevertheless, as the debt level grows, the fiscal room available to the policymaker shrinks due to the increased interest burden. Eventually, it is more desirable to run up debt to maintain spending. This situation happens above debt levels of 88%, as seen in Figure 2. An austerity policy to exit the FFZ is not optimal, and the policymaker eventually suffers an adverse shock and loses credibility. By opting to run up debt, the policymaker will ultimately fail to give the needed fiscal support to the inflation target.

Coordinating Expectations Through the Target

Higher inflation targets may improve the credibility of monetary policy and help coordinate private agents' expectations by increasing the costs of deviating to attain a given inflationary transfer of resources. Private agents use the inflation target to form expectations in the FFZ, and the target functions as a nominal anchor for expectations. In Figure 3, we present the results of a sensitivity analysis of the deviations to changes in the inflation target. We plot $\pi^D - \pi^a$ for three different inflation targets (0%, 3.5%, and 7.5%), keeping the other parameters at their baseline.

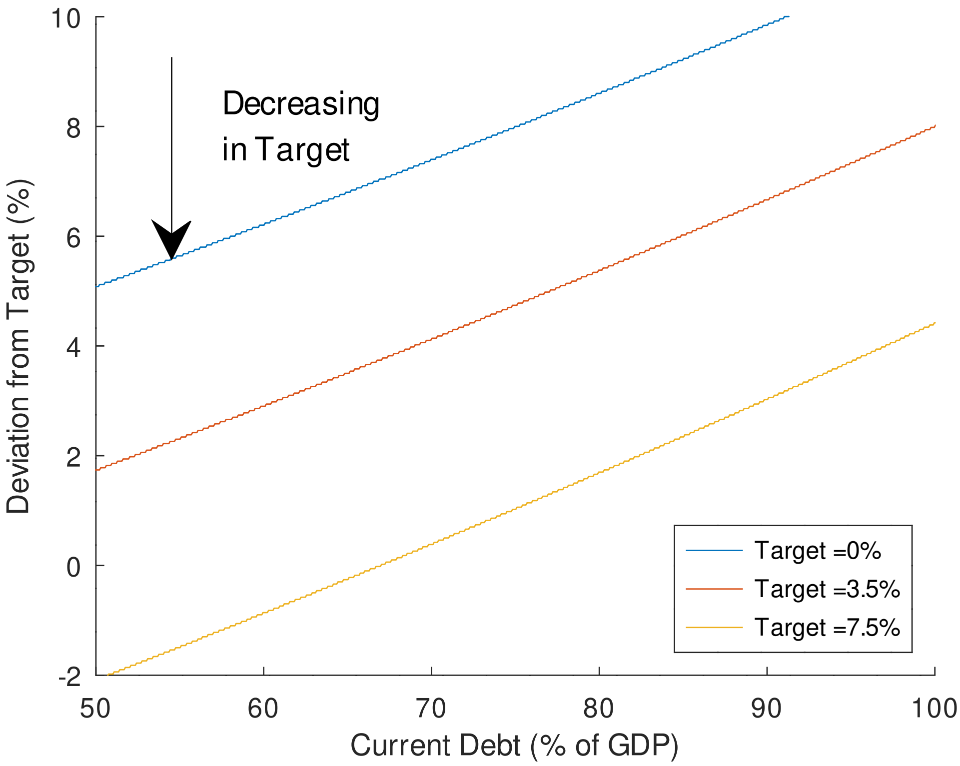

Figure 3: Sensitivity: Deviations to Inflation Target

A higher inflation target improves coordination by the policymaker by reducing the discretionary deviation from the target rate and by reducing the real fiscal burden of debt through a lower real interest rate. First, a higher target rate reduces the marginal capacity of the policymaker to transfer resources, which implies a lower marginal benefit from discretionary inflation. Second, a policymaker with a higher inflation target chooses a smaller deviation from it and, consequently, faces a lower real interest rate on its bonds in the FFZ.

For baseline parameters, deviations $\pi^D - \pi^a$ decrease in the inflation target, reducing the ex post real interest rate in the FFZ. Denote by $\underline{\text{D}}$ the lower bound of the FFZ. For initial debt levels below $\underline{\text{D}}$, the policymaker will have a perfectly credible target, preferring to keep inflation on target regardless of private agent expectations. Above the lower bound, private agents may doubt its commitment. As deviations decrease in the target, it becomes less costly to keep inflation on target for a given debt level. This effect increases the credibility of the inflation target because it remains fully assured up to higher levels of debt, as shown in Figure 4, which plots next-period debt for the different inflation targets.

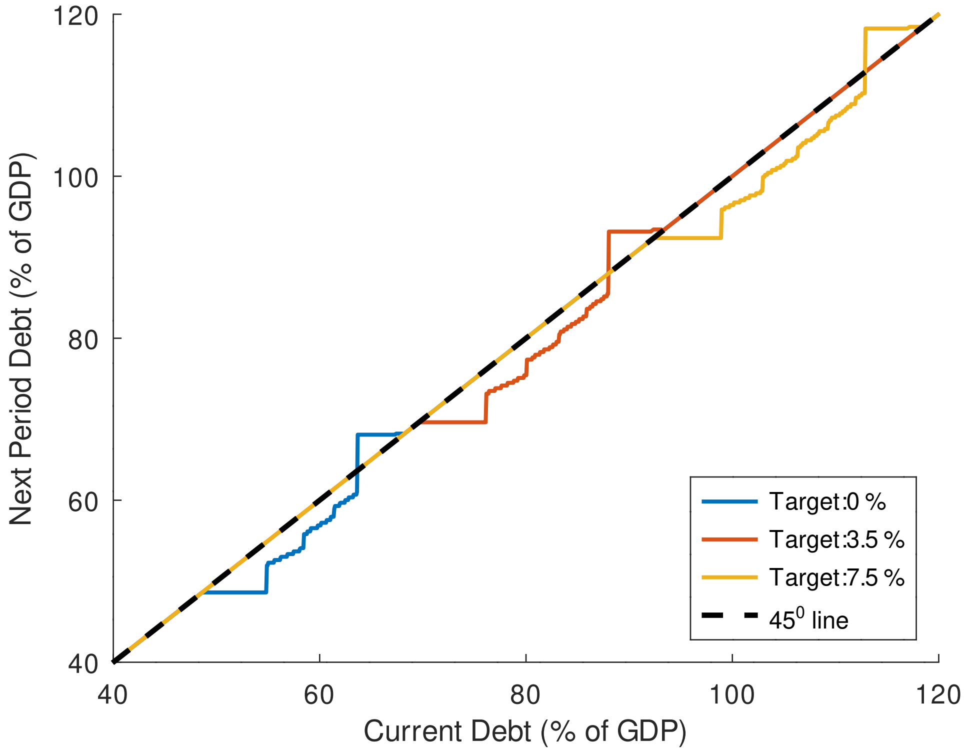

Figure 4: Sensitivity: Inflation Target and the Fiscal Fragility Zone

The lower bound $\underline{\text{D}}$ increases as the target rate raises. This result implies that policymakers should consider current debt levels and fiscal conditions when deciding to decrease the inflation target. This reduction can cause a loss of credibility for the government commitment as it enters the FFZ. While in the FFZ, expected inflation is higher than the target inflation rate, and choosing a low inflation target is costly instead of optimal. This result is in line with Araujo, Berriel, and Santos (2016), where a lower inflation target might reduce the policymaker's coordination ability due to a loss of credibility in its commitment and result in a worse equilibrium outcome.

The above analysis suggests a tradeoff when defining the target inflation rate for an economy with poor fiscal conditions. A lower target means a reduced welfare cost of inflation in the no-crisis zone and reduced discretionary inflation in the fiscal fragility and dominance zones, which is desirable for the policymaker. However, reducing the target also causes lower debt levels to be in the FFZ, which substantially reduces welfare since there is a positive probability that a sunspot shock that causes a permanent credibility loss will hit the economy. By computing the inflation target that maximizes total intertemporal utility for each initial debt level, we see that the optimal target is the lowest target possible such that the current debt level is in the no-crisis zone, as shown in Table 3:

Initial Debt Level	Optimal Inflation Target
60%	2%
70%	4%
80%	5.5%

Table 3: Optimal Inflation Target for Each Debt Level in the Calibrated 2002 Brazilian Case

This result shows that there is a rationale for raising the inflation target in countries in a fragile fiscal situation, even when we consider the existence of costs associated with higher inflation levels. It is optimal for the policymaker to raise the target until the economy exits the FFZ. In this model, since the target is a fixed parameter, the optimal policy outside of the FFZ will be stationary, and it will not be optimal to further reduce the debt level. However, we may conjecture that in a scenario in which the inflation target is treated as a policy instrument, raising the target will be temporary, and further austerity to reduce debt levels even in the no-crisis zone will be optimal to support a lower inflation target in future periods. This is best understood as *gradual disinflation* for economies in a fragile fiscal situation.

Inflation-Indexed Debt

Indexed debt was defined by taking away the uncertainty about which equilibrium would be selected next period and, consequently, revealing the sunspot variable to private agents. As a result, private agents are able to correctly anticipate inflation and obtain a constant real interest rate on their bond holdings. We show that indexed debt, so defined, comes with higher inflation.

Recall that we find discretionary inflation by solving for the discretionary policymaker's optimal inflation given expectations; that is, given π^e, we find the optimal π^D. The difference between the two timing assumptions is in the formation of inflation expectations. In the FFZ with nominal debt, the inflation expectation π^e is equal to $f\pi^D+(1-f)\pi^a$. Agents form expectations accounting for the probability of the policymaker delivering the target. With indexed debt, the inflation expectation π^e is equal to π^D. Agents do not form expectations accounting for the probability of the policymaker delivering the target. Intuitively, the policymaker attempts to transfer resources. However, it is unable to use inflation to partially default when subjected to a negative expectations shock since private agents adapt their expectations. This dynamic leads to higher levels of discretionary inflation. The optimal inflation chosen by the policymaker when its debt stock is higher nominal versus inflation-indexed is depicted in Figure 5.

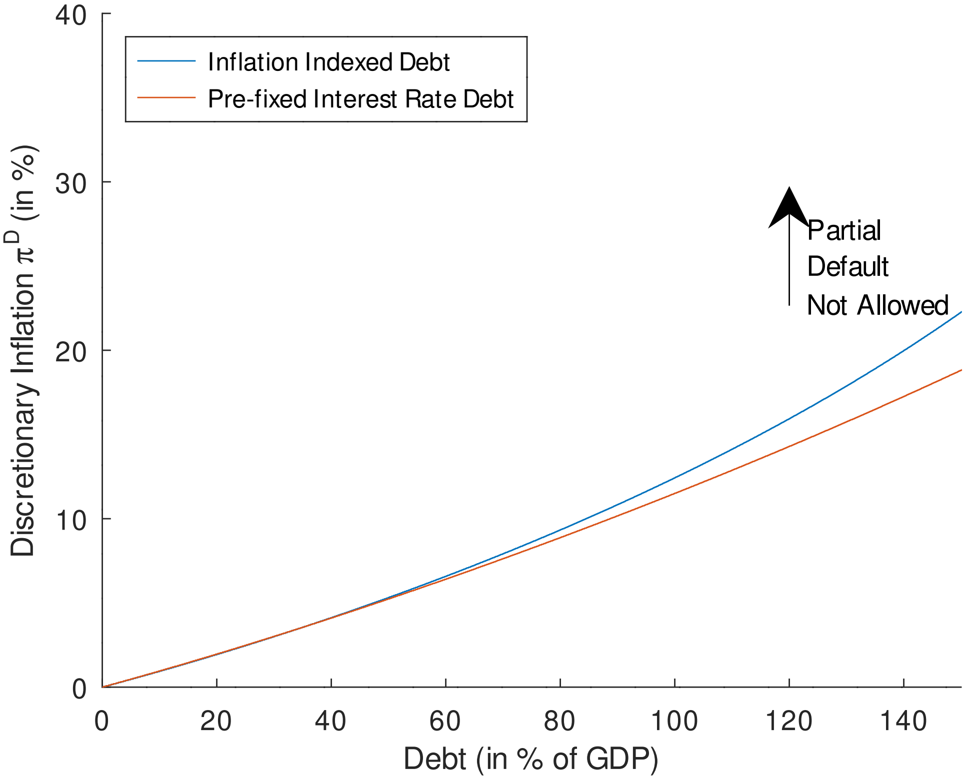

Figure 5: Discretionary Inflation

The higher discretionary inflation resulting from this timing may change the credibility of the inflation target for an initial debt stock, as the cost of maintaining the target increases in discretionary inflation under adverse expectations. Debt levels D in the no-crisis zone support the inflation target with certainty. Higher levels of discretionary inflation imply a higher penalty when deviating, and given that with indexed debt we have that $\pi^e = \pi^D$, the stationary level of public spending is lower when deviating, since the policymaker is not able to reduce interest spending. As a consequence, there is a higher penalty when deviating from the target when the government is financed by indexed bonds. This makes it more beneficial for the policymaker to commit to the target and avoid the crisis zone. As a result, the lower bound of the FFZ $\underline{\mathrm{D}}$ increases when debt is indexed, indicating a higher credibility of the target because a greater set of initial debt levels fully supports it.

Preference for Spending

A shock to preferences can connect our model to the situation observed in Brazil during the 2002 confidence crisis. Suppose that policymaker preferences shift toward giving more weight to public spending. Decreasing ρ would be tantamount to increasing the weight of public spending. This shift changes marginal utilities and the optimal allocation of resources, increasing the share going to public spending. The altruistic policymaker chooses higher discretionary inflation levels.

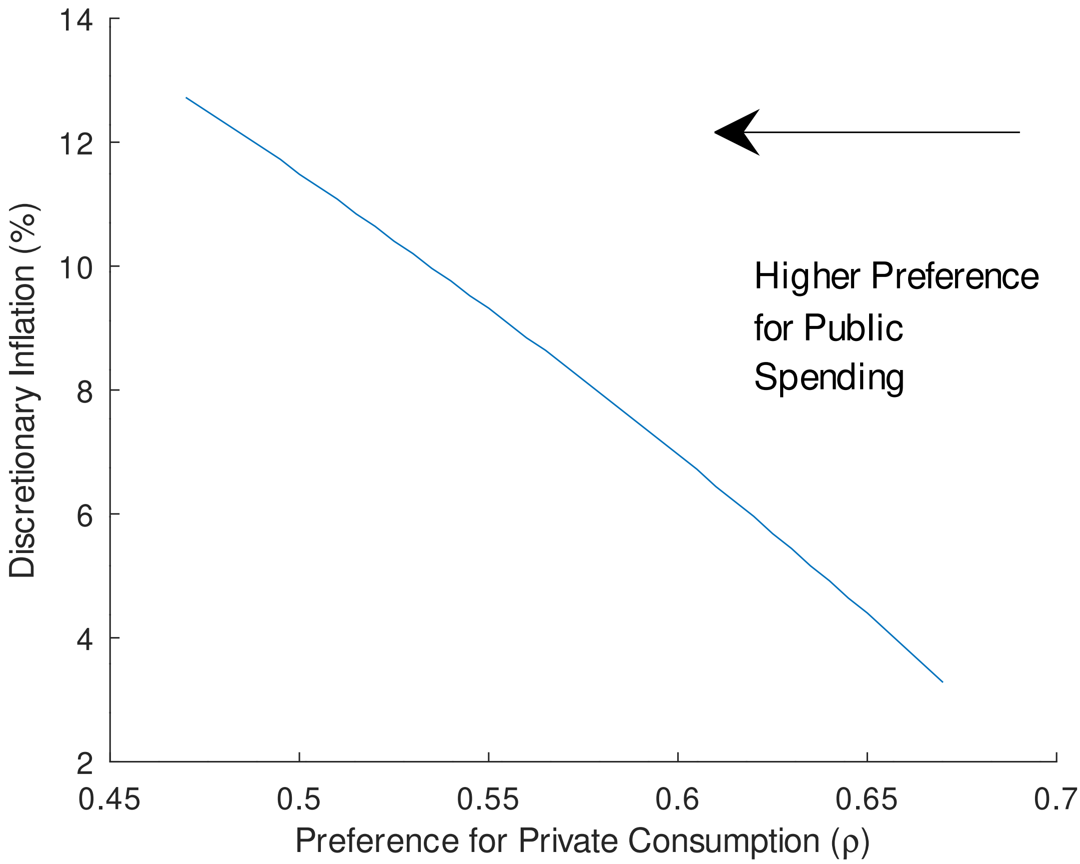

Figure 6: Sensitivity: Preference for Public Spending when $D = 100\%$

Given a debt level, a relatively higher preference for public spending increases the level of discretionary inflation. For initial debt, a preference shock could push the policymaker into the FFZ. A sufficiently large shock to ρ could result in the loss of credibility of the target under adverse expectations. Private agents would adapt their inflation expectations. A non-null probability assigned to an adverse event would increase expectations compared to a scenario where the target is perfectly assured. Such a preference shock explains how expectations can suddenly overshoot the target, as happened in Brazil in 2002.

4 2002 Confidence Crisis in Brazil

In 2002 and 2003, Brazilian policymakers faced inflationary pressures when it became clear that the left-wing presidential candidate would win. The perception was that his victory would mean implementing a new policy framework that could undermine the previous inflation reduction. Consequently, inflation expectations overshot the target's upper bounds at all horizons relevant to the central bank, as shown in Figure 1. We map this event in our model as a shock to the preference for spending in the parameter ρ. Sensitivity analysis in Section 3.2 shows that, for a given initial debt stock, the target could lose credibility after a preference shock. By favoring more public spending, the policymaker could become vulnerable to adverse shocks that would make it deviate from the inflation target. Private agents taking this probability into account when forming expectations would increase their forecasts of future inflation, precisely as observed in 2002.

In response to rising inflation expectations, the outgoing and new administrations took several steps. First, to coordinate inflation expectations in the short run, they increased the target for 2003 in an additional meeting held in June 2002 and unofficially again in January 2003. Second, during 2003, public debt reduction sustained responsible macroeconomic policies. Ultimately, inflation expectations converged back to the target. These policy responses closely mirror the prescriptions suggested by our model. We will consider each of these policies in further detail.

Fiscal Policy

After the 2002 election, the government gradually reduced the gross public debt. The gross debt declined from nearly 80% of GDP in 2002 to nearly 70% in 2004. Furthermore, the government continued to run primary surpluses to meet its debt obligations in a signal of fiscal responsibility. The primary surplus increased from 2.16% of GDP in 2001 to 2.70% in 2004. From the perspective of our model, such fiscal policy is compatible with the policymaker attempting to exit the FFZ and give the needed fiscal support to its inflation target.

Inflation Target

Before the October elections, the 2003 target was exceptionally revised upward from a previously announced 3.25% to 4%. Similarly, the upper and lower bounds widened from -/+ 2% to 2.5%. In January 2003, the Ministry of Finance sent a letter stating that the adjusted target would be 8.5% in 2003 and 5.5% in 2004. The latter was confirmed by the National Monetary Committee as the inflation target for 2004 in June 2003, as we can see in Table 4. From the perspective of our model, an indebted policymaker with a higher inflation target might be more credible. The higher and more credible inflation target serves as a nominal anchor, making private agents readjust their inflation expectations.

Year	Date When Set	Target	Bounds
2002	28/6/2000	3.50	2.0
2003	28/6/2001	3.25	2.0
	27/6/2002	4.00	2.5
	21/1/2003	8.50	
2004	27/6/2002	3.75	2.5
	21/1/2003	5.50	
	25/6/2003	5.50	2.5
2005	25/6/2003	4.50	2.5

Table 4: Brazil – Official Inflation Targets

5 Remarks

High public debt allows for inflation due to target coordination failure, depressing private consumption and GDP. We propose a model to describe the intertemporal tradeoff between fiscal and monetary policy when forward-looking and rational private agents finance an altruistic policymaker. Indebted policymakers have a limited budget and are subject to expectation shocks forcing them to accept a higher interest rate with inflation on the preannounced target or accept higher inflation. Our results endorse fiscal austerity to gradually lower the public debt to prevent coordination failure and self-confirmed inflation. However, if the debt is high, the policymaker should avoid an excessively low inflation target, while raising the target may restore credibility and allow policymakers to exit the crisis zone.

In recurrent episodes of emerging-market crises, high public debt is associated with the difficulty of achieving low inflation. High public debt also led to high inflation in advanced economies after the world wars, and it may do so again after the COVID-19 pandemic. In these cases, we suggest a set of tools based on our model that was successfully implemented in Brazil during the 2002 confidence crisis and can be used by central bankers who might face doubts about their credibility to sustain an inflation target: higher target levels and inflation indexed bonds.

Our paper has valuable normative policy implications: optimal disinflation must be gradual and accompanied by fiscal improvement (reducing the debt level). Our results also reinforces the conventional wisdom on the failure of fixed exchange rate systems as their adoption represents a sudden and inflexible disinflationary process.

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A Proofs

A.1 Characterization of Discretionary Inflation

A.1.1 Discretionary Inflation is Increasing in the Debt Level

Suppose that π_T^D is an interior solution to the discretionary inflation problem. Recall that $\pi_T^e = f\pi_T^D + (1-f)\pi^a$, where f is the probability of a negative sunspot shock. To obtain $\partial\pi_T^D/\partial D_T$, we differentiate the first-order condition (9) with respect to the initial debt level D_T to obtain:

$$\begin{aligned}\frac{\partial(9)}{\partial\pi_T^D}\frac{\partial\pi_T^D}{\partial D_T} =& -(1-\rho)v''(g)\frac{\partial g}{\partial D_T}\left[\frac{1+\pi_T^e}{(1+\pi_T^D)^2}\left(\frac{1}{\beta}-1\right)D_T + \alpha'\tau e\right] \\ & - [(1-\rho)v'(g)-\rho]\frac{1+\pi_T^e}{(1+\pi_T^D)^2}\left(\frac{1}{\beta}-1\right)\end{aligned} \tag{14}$$

where

$$\frac{\partial g}{\partial D_T} = -\frac{1+\pi_T^e}{1+\pi_T^D}\left(\frac{1}{\beta}-1\right) < 0$$

and

$$\begin{aligned}\frac{\partial(9)}{\partial\pi_T^D} =& (1-\rho)v''(g)\frac{\partial g}{\partial\pi}\left[\frac{\partial g}{\partial\pi} + \frac{f}{(1+\pi_T^D)^2}\left(\frac{1}{\beta}-1\right)D_T\right] \\ & - [(1-\rho)v'(g)-\rho]\left\{\frac{f}{(1+\pi_T^D)^2} + \frac{2(1-f)(1+\pi^a)}{(1+\pi_T^D)^3}\right\}\left(\frac{1}{\beta}-1\right)D_T \\ & + [\rho(1-\tau)+(1-\rho)v'(g)\tau]\,\alpha''e\end{aligned} \tag{15}$$

is the derivative of the FOC with respect to discretionary inflation π_T^D, where

$$\frac{\partial g}{\partial\pi} = \alpha'\tau e + \frac{(1-f)(1+\pi^a)}{(1+\pi)^2}\left(\frac{1}{\beta}-1\right)D_T.$$

To prove that $\partial\pi_T^D/\partial D_T > 0$, we show that the term on the right-hand side of (14) and the term in (15) are both negative.

First, it is straightforward that

$$\frac{1+\pi_T^e}{(1+\pi_T^D)^2}\left(\frac{1}{\beta}-1\right)D_T + \alpha'\tau e = \frac{\partial g}{\partial\pi} + \frac{f}{(1+\pi_T^D)^2}\left(\frac{1}{\beta}-1\right)D_T \geq 0$$

for all D_T. Additionally, $v'' < 0$ and $[(1-\rho)v'(g)-\rho] > 0$ for any g by assumption. This means that the first two terms in (14) are nonpositive, and the second term is negative for every D_T.

Now, in (15), we can analyze term-by-term to check that, as long as α' is not too steep such that $\frac{\partial g}{\partial \pi} \geq 0$, every term is non-positive for every D_T, with the last term always being strictly negative since we assume that $\alpha'' < 0$. This concludes the proof.

A.1.2 Discretionary Inflation Deviation is Decreasing in the Inflation Target

To prove this proposition, we again assume that π_T^D is an interior solution to the maximization problem and differentiate the FOC (9), now with respect to π^a, to obtain:

$$\begin{aligned}\frac{\partial(9)}{\partial \pi_T^D}\frac{\partial \pi_T^D}{\partial \pi^a} &= -(1-\rho)v''(g)\frac{\partial g}{\partial \pi^a}\left(\frac{1+\pi_T^e}{(1+\pi_T^D)^2}\left(\frac{1}{\beta}-1\right)D_T + \alpha'\tau e\right) \\ &\quad -\left[(1-\rho)v'(g)-\rho\right]\frac{1-f}{(1+\pi_T^D)^2}\left(\frac{1}{\beta}-1\right)D_T. \end{aligned} \tag{16}$$

We can immediately see that the right-hand side of Equation (16) is negative by the same analysis done in the previous proof. We want to show that $\partial \pi_T^D/\partial \pi^a < 1$. Rewrite Equation (16) as

$$(\text{term } 1)\frac{\partial \pi_T^D}{\partial \pi^a} = \text{term } 2.$$

We need to show that term 2 - term 1 > 0, since this implies that term $2/$term 1< 1. Note that

$$\begin{aligned}\text{term 2} - \text{term 1} = &-(1-\rho)v''(g)\left(\frac{\partial g}{\partial \pi^a}+\frac{\partial g}{\partial \pi}\right)\left(\frac{1+\pi_T^e}{(1+\pi_T^D)^2}\left(\frac{1}{\beta}-1\right)D_T+\alpha'\tau e\right) \\ &-\left[(1-\rho)v'(g)-\rho\right]\left\{\frac{1-2f}{(1+\pi_T^D)^2}-\frac{2(1-f)(1+\pi^a)}{(1+\pi_T^D)^3}\right\}\left(\frac{1}{\beta}-1\right)D_T \\ &-\left[\rho(1-\tau)+(1-\rho)v'(g)\tau\right]\alpha'' e. \end{aligned} \tag{17}$$

We can check that the first and second terms in (17) are zero when $D_T = 0$, while $\alpha'' < 0$ for all π_T^D, by assumption, so that (17) is positive. Now, the second term is positive as long as $\pi_T^D \leq 1$, while the first term depends on the sign of $\frac{\partial g}{\partial \pi^a} + \frac{\partial g}{\partial \pi}$, which is negative for $\pi_T^D \geq \pi^a$. Thus, if α'' is sufficiently negative that the last term dominates the first when $\pi_T^D \geq \pi^a$ and the first and the second if $\pi_T^D > 1$ for some D_T, then we have the desired inequality and the proof is complete.

A.2 Optimal Debt Policy Outside the Fiscal Fragility Zone

Outside of the FFZ, there is only a unique inflation equilibrium, making it perfectly anticipated. The policymaker's problem can be reduced to the following:

$$\max_{D_{t+1}} \sum_{t \geq S} \beta^t u(c_t, g_t)$$
$$\text{s.t. } c_t = \frac{1}{\beta} D_t + \alpha(1-\tau)e - D_{t+1}$$
$$g_t = D_{t+1} + \alpha\tau e - \frac{1}{\beta} D_t$$

The first-order condition (FOC) for D_{t+1} yields:

$$(1-\rho)v'(g_t) - \rho = (1-\rho)v'(g_{t+1}) - \rho$$

which implies $v'(g_t) = v'(g_{t+1})$ for all t. Given that v strictly concave in g, we must have $g_{t+1} = g_t$. Replacing g_t and g_{t+1} by the government budget equation, iterating forward and taking limits, we obtain:

$$\lim_{t\to\infty} D_{S+t} = \sum_{i=1}^{\infty} \left(\frac{1}{\beta}\right)^i (D_{S+1} - D_S) + D_{S+1}. \tag{18}$$

Suppose that $D_{S+1} \neq D_S$; then, the policymaker will either run up infinite debt or credit. Equation (18) also implies that

$$\lim_{t\to\infty} \beta^{S+t-1} D_{S+t} = \beta^S \sum_{i=0}^{\infty} \beta^i (D_{S+1} - D_S) + \lim_{t\to\infty} \beta^{S+t-1} D_{S+1} = \frac{\beta^S}{1-\beta}(D_{S+1} - D_S).$$

The no-Ponzi condition for this problem states that

$$\lim_{t\to\infty} \beta^t D_{t+1} = 0,$$

so that if $D_{S+1} \neq D_S$, this condition is violated. This means that the only optimal trajectory for debt outside of the FFZ is the stationary state such that $D_t = D_S = D_{S+1}$ for all t.

A.3 Existence of the No-crisis and Fiscal Dominance Zones

To prove the first statement, assume that $\alpha^c = 0$. We want to prove that $V^p(D_T, \pi_T = \pi_T^D, \pi_T^e = \pi^a) \geq V^p(D_T, \pi_T = \pi^a, \pi_T^e = \pi^a)$, for every D_T, which means that the policymaker finds it optimal to deviate even if the private sector expects the target. The right-hand side of the inequality above is the total intertemporal utility attained when committing to the inflation target, which is $\frac{1}{1-\beta}u(c^a, g^a)$, where $g^a = \alpha(\pi^a)\tau e - \left(\frac{1}{\beta} - 1\right) D_T$, and $c^a = \alpha(\pi^a)e - g^a$.

The left-hand side of the inequality is the solution to the deviation problem (6) when $\pi^e_T = \pi^a$. Since we assumed that $\alpha^c = 0$, the solution to the deviation problem is greater or equal than $\frac{1}{1-\beta}u(c^a, g^a)$, since this is obtained simply by setting $D = D_T$ and $\pi = \pi^a$ in the maximization problem. This proves the first statement.

To prove the second statement we need to show that there exists a debt level D_{-} such that $V^p(D, \pi_t = \pi^a, \pi^e = \pi^D) \geq V^p(D, \pi_t = \pi^D, \pi^e = \pi^D)$ for all $D \in [0, D_{-}]$. For an arbitrary debt level D, we can consider the utility attained by the stationary policy of committing to the target when the private sector expects the deviation π^D and compare it to the total utility of deviating from the target. D is will be in the no-crisis zone if

$$\begin{aligned} &\frac{1}{1-\beta}u\left(\alpha^a(1-\tau)e + \left(\frac{1+\pi^D}{1+\pi^a}\frac{1}{\beta} - 1\right)D, \alpha^a\tau e - \left(\frac{1+\pi^D}{1+\pi^a}\frac{1}{\beta} - 1\right)D\right) \\ &\quad \geq \frac{1}{1-\beta}u\left(\alpha(\pi^D)(1-\tau)e + \left(\frac{1}{\beta} - 1\right)D, \alpha(\pi^D)\tau e - \left(\frac{1}{\beta} - 1\right)D\right). \end{aligned} \tag{19}$$

If $\pi^D < \pi^a$, we know that D is in the no-crisis zone by Proposition 5. It is straightforward from the FOC (9) that $\pi^0 = 0$, and since we proved in Proposition 1 that π^D is increasing in D, we know that all D values in an interval $[0, D^*)$ are in the no-crisis zone for some D^*, such that $\pi^{D^*} = \pi^a$. For $D = D^*$, the inequality (19) becomes

$$\begin{aligned} &\frac{1}{1-\beta}u\left(\alpha^a(1-\tau)e + \left(\frac{1}{\beta} - 1\right)D^*, \alpha^a\tau e - \left(\frac{1}{\beta} - 1\right)D^*\right) \\ &> \frac{1}{1-\beta}u\left(\alpha(\pi^a)(1-\tau)e + \left(\frac{1}{\beta} - 1\right)D^*, \alpha(\pi^a)\tau e - \left(\frac{1}{\beta} - 1\right)D^*\right), \end{aligned} \tag{20}$$

where the strict inequality is true by monotonicity since $\alpha(\pi^a) = \alpha^a + \alpha^c < \alpha^a$, by assumption. By continuity, we can conclude that the strict inequality (19) is valid for an interval $[0, D_{-}]$, with $D_{-} > D^*$, which gives our stated result.

To prove the last statement, we need to show that for a sufficiently high cost α^c, the condition (19) is valid for all D. We can rewrite this condition, using the mean value theorem, as

$$
\begin{aligned}
&[\rho(1-\tau)+(1-\rho)v'(g_\theta)\tau]\,(\alpha^a-\alpha(\pi^D))e \\
&\quad\geq [(1-\rho)v'(g_\theta)-\rho]\left(\frac{\pi^D-\pi^a}{1+\pi^a}\frac{1}{\beta}D\right),
\end{aligned}
\tag{21}
$$

where g_θ is a value between the spending levels in the right-hand and left-hand side in (19). By Proposition 5, we know that if the fixed cost is high enough and $\pi^D<\pi^a$, D is in the no-crisis zone, and we only need to consider the case where $\pi^D\geq\pi^a$. For (21) to be valid for all D, it is sufficient that the condition

$$
(\alpha^a-\alpha(\pi^D))\tau e \geq \frac{\pi^D-\pi^a}{1+\pi^a}\frac{1}{\beta}D
\tag{22}
$$

be valid for all D. Since the right-hand side of Equation (22) is total interest spending when the government commits to the target, and public spending is always nonnegative, we have that

$$
\frac{\pi^D-\pi^a}{1+\pi^a}\frac{1}{\beta}D \leq \tau\alpha^a e-\left(\frac{1}{\beta}-1\right)D \leq (\alpha^a-\alpha(\pi^D))\tau e
$$

as long as the condition

$$
\left(\frac{1}{\beta}-1\right)D \geq \alpha(\pi^D)\tau e
$$

is satisfied for all D. However, since we only need to consider $D\geq D_-$, it suffices to take a fixed cost α^c sufficiently negative that

$$
\left(\frac{1}{\beta}-1\right)D_- \geq \alpha(\pi^D)\tau e
$$

is valid, and the proof is complete for all D.

A.4 Above-Target Discretionary Inflation

The intuition for the proof is simple: we show that if it is the case that $\pi^D<\pi^a$, then there exists a feasible policy that does not deviate from the target and attains a higher intertemporal utility then deviating, even if private agents believe that the policymaker will deviate from target. This implies that the economy is in the no-crisis zone. Therefore, whenever the economy is in the FFZ, deviation from target must be positive.

For a given debt level D, assume that $\pi^D<\pi^a$, that is, the discretionary inflation rate is lower than the target rate. We want to prove that in this case, $V^p(D,\pi_t=\pi^a,\pi^e=\pi^D)\geq V^p(D,\pi_t=\pi^D,\pi^e=\pi^D)$, that is, the policymaker follows the inflation target even when the private agents expect the policymaker to deviate. Let T be the time of deviation, and assume that private agents expect the policymaker to deviate; then, according to Equation (6), total government spending both in period T and in the stationary long-run will be equal to

$$g^D = \alpha(\pi^D)\tau e - \left(\frac{1}{\beta} - 1\right) D, \tag{23}$$

since agents expect the deviation.

As an alternative, the government can choose the feasible path of not deviating from the target and following a stationary spending policy:

$$g^a = \tau\alpha^a e - \left(\frac{1+\pi^D}{1+\pi^a}\frac{1}{\beta}\right) D. \tag{24}$$

Since $\pi^D < \pi^a$, we have that $g^D < g^a$, as

$$g^a - g^D = (\alpha^a - \alpha(\pi^D))\tau e + \frac{\pi^a - \pi^D}{1+\pi^a}\frac{1}{\beta}D > 0.$$

To compare the total intertemporal utility of the two policies, we only need to compare which one of the allocations achieves a higher utility in any period, since they are stationary allocations. Let $c^D = \alpha(\pi^D)e - g^D$ and $c^a = \alpha^a e - g^a$ be the market-clearing private consumption in each scenario. By the concavity of the utility function, we have that

$$\begin{aligned} u(c^D, g^D) - u(c^a, g^a) \leq& \rho(c^D - c^a) + (1-\rho)v'(g^a)(g^D - g^a) \\ =& \rho(\alpha(\pi^D) - \alpha^a)e + ((1-\rho)v'(g^a) - \rho)(g^D - g^a) < 0 \end{aligned} \tag{25}$$

since, by the assumption, we have that $(1-\rho)v'(g^a) - \rho \geq 0$.

Now, since there is a feasible policy trajectory in which the policymaker follows the inflation target and its intertemporal utility is greater than that attained by deviating to the inflation rate π^D, this means that the optimal policy chosen by the policymaker when following the target must also attain a higher utility than the attained by deviating, that is, $V^p(D, \pi_t = \pi^a, \pi^e = \pi^D) \geq V^p(D, \pi_t = \pi^D, \pi^e = \pi^D)$. However, this means that the policymaker chooses to follow the target even when private agents expect it to deviate, so that debt level D is in the no-crisis zone.

B Empirical Results

The calibrated model leads to the conclusions that i) the size of the deviation could be reduced by increasing the target and reducing debt and ii) the probability of overshooting the target would increase with debt and decrease with higher target levels. The present section investigates whether there is empirical evidence for the predictions based on our model. We construct a dataset that includes 20 countries with at least 15 years of inflation targeting[18] covering the period from 2000 to 2019. Targets are those reported by the respective central banks that were manually collected from each central bank web page. Inflation and gross debt and revenue to GDP statistics are from the IMF. With regard to inflation, end-of-year consumer price inflation is the target benchmark. Some general statistics are reported in Table 5. The variables present both inter- and intracountry variability. In the case of CPI targets, 55% of our sample changed the target at least once. Most of the changes are in middle-income countries.[19]

	Debt/GDP	Revenue/GDP	CPI EOY	CPI target
Average	45.2	32.9	3.9	3.2
Min	13.4	16.4	1.5	1.5
Max	80.8	56.1	15.4	8.2

Table 5: Data Description

Real effective exchange rate (Reer) and GDP gap estimates enter robustness checks. When Reer statistics were not available from the IMF, other sources were accessed.[20] GDP gap estimates are constructed using quarterly seasonally adjusted GDP volume statistics from the IMF. When not available, the unadjusted equivalents are seasonally adjusted with the Arima X-11 procedure.[21] The quarterly GDP gap statistics are obtained applying an HP filter with a smoothing parameter of 1600. To mitigate the endpoint bias of the filter at the beginning of each series, we estimate the gap for the longer 1996Q1 - 2020Q1 period. Finally, the yearly GDP gap is defined as the average gap over the relevant period.

[18]The countries in the sample are Australia, Brazil, Canada, Chile, Colombia, the Czech Republic, Iceland, Indonesia, Israel, Mexico, New Zealand, Norway, Peru, the Philippines, Poland, South Africa, Sweden, Thailand, Turkey, and the United Kingdom.

[19]We used the World Bank classification.

[20]BIS for Peru, Indonesia, and Turkey. Bank of Thailand for Thailand.

[21]This was the case for Peru and Turkey.

Deviations from the Target

The FOC of the discretionary inflation problem from 6 relates the deviation of inflation $\pi_{i,t}$ from the inflation target $\pi^a_{i,t}$ to observable and latent variables for each country i. We estimate the following model,[22]

$$\pi_{i,t} - \pi^a_{i,t} = \beta_1 \text{revenue}_{i,t} + \beta_2 \text{debt}_{i,t} + \beta_3 \pi^a_{i,t} + \beta_4 \text{revenue}_{i,t} * \text{debt}_{i,t} + c_i + u_{i,t} \tag{26}$$

where the idiosyncratic error $u_{i,t}$ satisfies $\mathbb{E}(u_{i,t}|X_{i,1}, ..., X_{i,T}, c_i) = 0,\ t = 1, ..., T$ with $X_{i,t}$ being a vector of the observable regressors at time t for country i. The variables and parameters of the model are mapped into both observed series and latent variables. We map the model variables D, τe, and π^a to gross debt (%GDP), revenue (%GDP), and the inflation target. The unobservable variables e, f, c_1, and c_2 are mapped into a country fixed effect c_i that captures the time-constant individual heterogeneity between countries. We use a fixed effect estimator because it seems reasonable to assume that their choices of debt, revenue and inflation target are related to the unobserved characteristics of each country c_i. In other words, we cannot assume $\mathbb{E}(X_{i,t}c_i) = 0\ \forall t$ as required for a random effect estimator.[23]

In terms of interpretation, the net impact of debt should be positive. Given higher levels of debt, the policymaker will have more incentive for discretionary inflation. Furthermore, discretionary inflation increases in debt. Hence, the deviation to increase in debt levels as the policymaker will be more likely to deviate and will choose higher discretionary inflation when doing so. Given an interaction term in (26), one would have to examine the joint impact captured by β_2 and β_4 for a given level of revenue to GDP. We also expect the coefficient on the inflation target to be negative because the policymaker could help coordinate private agents' expectations by adopting a more credible (higher) inflation target in given situations. Were inflation perfectly anchored, changing the target would not result in changes in expected deviation. In other words, the coefficient β_3 would equal zero. Finally, higher revenue means that the policymaker has more fiscal room for spending. This room decreases the incentives to transfer resources through discretionary inflation, leading to a negative net impact of revenue. Given the interaction term between debt and revenue, the joint impact captured by β_1 and β_4 should be negative for a given level of debt.

Estimation I in Table 6 is the basic model from (26). The remaining estimations, II-V, are robustness checks.

In estimation I, deviations from the target are on average negatively related to the target

[22] In Online Appendix D we show how (26) is related to the FOC of the discretionary inflation problem from (9).

[23] A Hausman test between a fixed and random effect estimator similarly suggests the use of the former.

	I	II	III	IV	V
Revenue	0.171^{**}	0.098	0.063	0.125^{*}	0.087
	(0.076)	(0.076)	(0.078)	(0.070)	(0.072)
Debt	0.069^{*}	0.074^{**}	0.073^{**}	0.062^{**}	0.058^{*}
	(0.035)	(0.034)	(0.034)	(0.031)	(0.032)
Debt * Revenue/100	-0.194^{*}	-0.168^{*}	-0.149	-0.163^{*}	-0.136
	(0.099)	(0.096)	(0.096)	(0.088)	(0.089)
Target	-0.403^{***}	-0.458^{***}	-0.441^{***}	-0.360^{***}	-0.342^{***}
	(0.063)	(0.062)	(0.062)	(0.059)	(0.058)
GDP Gap			0.363^{***}		0.342^{***}
			(0.102)		(0.095)
Reer YoY				-13.956^{***}	-13.645^{***}
				(1.648)	(1.653)
FE	Country	Country & Time	Country & Time	Country & Time	Country & Time
R^2	0.290	0.408	0.433	0.515	0.537
Num. obs.	382	382	374	372	364

Note: *p<0.1; **p<0.05; ***p<0.01

Table 6: Results – Deviations from the Inflation Target

level. In the case of perfectly anchored inflation, the coefficient should not be significantly different from zero. We also have a positive coefficient on debt and a negative coefficient for the interaction term between debt and revenues. This can be interpreted as higher debt implying higher deviations for countries with limited revenues. For revenues no higher than 35% of GDP, the net impact of debt is positive. This result applies to the middle-income countries in our sample. The result goes in the direction of what the theoretical model predicted, as both the probability of deviating and deviations from the target are positively related to debt levels. On average, countries with higher debt levels have higher deviations from their inflation target.

The coefficient on revenue is positive in all settings although not always significant. Given the interaction term with debt, the net impact of revenue is positive up to debt levels of 88%, above the maximum in our sample. Hence, the impact of higher revenue is to increase deviations from the inflation target. Although this goes against what was expected from the theoretical model, one could argue that higher revenue could be correlated with preferences for public spending that in turn could lead to inflationary pressure.

The results remain after accounting for different types of shocks and variables usually associated with inflation dynamics. In estimation II, we include a time fixed effect to account for global shocks such as commodity prices. In our sample, 2008 stands out, as many countries overshot their inflation targets after the financial crisis. The time dummies are meant to take such global comovements in inflation into account. Estimation III also includes shocks to the real effective exchange rate. Estimation IV adds the impact of deviations from potential GDP on inflation.

Probability of Overshooting the Target

The policymaker overshoots the inflation target when end-of-year inflation exceeds the upper bound of the target.[24] In the theoretical model, the policymaker had more incentive to overshoot the target when it had limited fiscal space due to high debt servicing cost. We estimate a similar equation to (26) but with regard to the probability of overshooting the target:

$$I_{\pi_{i,t}>\overline{\pi}^A_{i,t}} = \beta_1 \text{revenue}_{i,t} + \beta_2 \text{debt}_{i,t} + \beta_3 \text{target}_{i,t} + \beta_4 \text{revenue}_{i,t} * \text{debt}_{i,t} + c_i + u_{i,t} \tag{27}$$

where $\overline{\pi}^A_{i,t}$ is the upper bound of the inflation target for country i at time t. The indicator $I_{\pi_{i,t}>\overline{\pi}^A_{i,t}} = 1$ when inflation $\pi_{i,t}$ overshoots the upper bound of the inflation target $\overline{\pi}^A_{i,t}$. The idiosyncratic error $u_{i,t}$ satisfies $\mathbb{E}(u_{i,t}|X_{i,1},...,X_{i,T},c_i) = 0,\ t = 1,...,T$. The probability of overshooting the target will then be a logistic function:

$$Pr(I_{\pi_{i,t}>\overline{\pi}^A_{i,t}} = 1|X_{i,t}, c_i) = \frac{1}{1+e^{-X'_{i,t}\beta - c_i}}, \quad t = 1,...,T \tag{28}$$

The expected results and dynamics are quite similar to those in the previous section with an expected net positive impact of debt, negative impact of the inflation target, and negative impact of revenue on the probability of overshooting the target. Each year in the sample, at least two countries overshoot their respective inflation target. The years 2007 and 2008 stand out, as over half of the countries overshoot their inflation target. A time dummy is likely to capture this effect. Additionally, virtually all countries except two overshoot their target at least once, with some countries such as Turkey close to being serial overshooters. Overall, middle-income countries overshoot the target more often than high-income countries. Nevertheless, high-income countries overshoot the target 39 times.

The first column of Table 7 is the baseline model, while the remaining columns represent robustness checks similar in spirit to the previous section. When considering the net impact of debt on the probability of overshooting the target, the coefficients have similar signs to the previous estimates with regard to deviations from the target. Estimation I has the most restrictive condition for a net positive effect of debt. For revenues over 30% of GDP, the net effect of debt stops being positive. Not all middle-income countries in our sample have revenue below this level. However, the effects are not statistically significant in any of the settings.

The net impact of revenue remains positive for debt levels in the sample, not in the

[24] Some countries adopt pointwise targets instead of tolerance bounds. This is for instance the case for the UK and Norway. In such cases, we used the average upper tolerance limit from the rest of the sample (1.2%).

	I	II	III	IV	V
Revenue	0.145	0.108	0.084	0.115	0.082
	(0.091)	(0.105)	(0.109)	(0.110)	(0.113)
Debt	0.034	0.055	0.053	0.050	0.042
	(0.044)	(0.052)	(0.053)	(0.053)	(0.054)
Debt*Revenue/100	-0.114	-0.107	-0.088	-0.121	-0.085
	(0.125)	(0.149)	(0.151)	(0.151)	(0.154)
Target	-0.624^{**}	-1.242^{***}	-1.207^{***}	-0.990^{**}	-0.936^{**}
	(0.263)	(0.376)	(0.376)	(0.390)	(0.386)
GDP Gap			0.206		0.218
			(0.158)		(0.167)
Reer YoY				-10.493^{***}	-10.262^{***}
				(3.062)	(3.101)
Num. obs.	377	377	369	368	360
Log Likelihood	−178.526	−151.367	−149.281	−139.619	−137.954

Note: *p<0.1; **p<0.05; ***p<0.01

Table 7: Results – Probability of Overshooting the Target

same direction as predicted by the theoretical model. The predicted negative impact of debt is based on increased fiscal room provided by higher revenue, which decreases the incentive to use discretionary inflation to transfer resources away from private agents' debt. However, another channel is possible. Revenue might be correlated with some other factors such as a higher preference for government spending, which would increase incentives to use inflation for transfer of resources. This channel could explain our results.

The probability of overshooting the target is negatively related to the target level and significant at the 5% level in all settings. Our interpretation is that some countries might have inflation targets that are too low, making it more likely to overshoot the target more often. Those counties could improve their ability to keep inflation on target by adopting higher targets. The results remain little changed when including shocks to exchange rates, the output gap, or a time dummy. Changes in the real effective exchange rate seem to be an important factor in causing policymakers to overshoot their inflation target. The output gap is not significant.

C Online Appendix: Solution to Discretionary Inflation

Proposition 6 *Let the utility function $u(c, g)$ and the penalty function $\alpha(\pi)$ be such that they satisfy the already stated assumptions. If the universe of possible inflation choices is defined on the compact set $[0, \overline{\pi}]$ where $\overline{\pi} > 0$ is some upper limit, then there exists a discretionary inflation level π^D such that π^D is optimal given private agents' inflation expectations π^e and vice versa.*

Proof: To prove that there exists a discretionary inflation level π^D such that π^D is optimal given π^e, and vice versa, we will use Brouwer's fixed point theorem. Since we are only interested in the universe of limited inflation, we state that $\pi^D \in [0, \overline{\pi}]$ where $\overline{\pi} > 0$ is an upper limit for the possible inflation levels. Let $\pi : [0, \overline{\pi}] \to [0, \overline{\pi}]$ be the function mapping private agents' expectations into the policymaker's inflation choice as defined by the discretionary inflation problem in Equation 6.

Let us now define the auxiliary function $\tilde{\pi}(\pi^D) := \pi(f\pi^D + (1-f)\pi^a) = \pi(\pi^e)$. Since $\tilde{\pi} : [0, \overline{\pi}] \to [0, \overline{\pi}]$ maps a compact interval on $\mathbb{R}$ into itself, we only need to prove that it is continuous to use Brouwer's theorem for the existence of a fixed point.

First, by assumption, we know that the penalty function $\alpha : [0, \overline{\pi}] \to (0, 1)$ mapping discretionary inflation into total factor productivity is continuous, assuming that the policymaker already chose to deviate from the target. Hence, the consumption choice will also be continuous. The same holds for government spending.

Second, the utility function $u : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}$ mapping government spending and private consumption into a utility scale is also continuous by assumption.

Combining the mapping of discretionary inflation $[0, \overline{\pi}]$ into consumption and spending $\mathbb{R}_+ \times \mathbb{R}_+$ and the mapping of consumption and spending $\mathbb{R}_+ \times \mathbb{R}_+$ into a utility scale $\mathbb{R}$, it is clear that the mapping of discretionary inflation $[0, \overline{\pi}]$ into a utility scale $\mathbb{R}$ will also be continuous. Finally, given that the argmax operator, mapping $[0, \overline{\pi}]$ into $[0, \overline{\pi}]$, maintains those properties, we have that $\tilde{\pi} : [0, \overline{\pi}] \to [0, \overline{\pi}]$ is continuous, which is what we sought to demonstrate.

D Online appendix: Testing the FOC

The FOC of the discretionary inflation problem from Equation 9 when assuming linear utility in consumption is given by:

$$[(1-\rho)v'(g)-\rho]\left(\frac{1+\pi_T^e}{(1+\pi)^2}\left(\frac{1}{\beta}-1\right)D_T\right)+(\rho(1-\tau)+(1-\rho)v'(g)\tau)\,\alpha' e=0.$$

π_T^e are the private agents' expectations at time T, π_T^D is the optimal discretionary inflation chosen by the policymaker when deviating from the target at time T, D_T is the level of debt, τe the policymaker's revenues and α' is the marginal productivity shock when deviating.

The equation can be rewritten as:

$$D_T\left((1-\rho)v'(g)-\rho\right)\left(\frac{1+\pi_T^e}{(1+\pi_T^D)^2}\right)=\tau e\alpha'\frac{\beta}{1-\beta}\left[\frac{1-\tau}{\tau}\rho+(1-\rho)v'(g)\right]$$

Taking logs, we obtain:

$$\begin{aligned}&d+\log((1-\rho)v'(g)-\rho)+\log(1+\pi_T^e)-2\log(1+\pi_T^D)\\&=\log(\tau e)+\log(\alpha')+\log\left(\frac{\beta}{1-\beta}\left[\frac{1-\tau}{\tau}\rho+(1-\rho)v'(g)\right]\right)\end{aligned}$$

where $d=\log(D_T)$. Replacing expectations with $\pi_T^e=f\pi_T^D+(1-f)\pi^a$ and using the approximation for $\log(1+x)\simeq x$ for small x, we have $\log(1+\pi_T^e)-2\log(1+\pi_T^D)=(2-f)(\pi^a-\pi_T^D)-\pi^a$. Hence:

$$\pi_T^D-\pi^a=-\frac{\log(\tau e)}{2-f}+\frac{d}{2-f}-\frac{\pi^a}{2-f}-\frac{c}{2-f}$$

Where $c=\log\left(\frac{\beta}{1-\beta}\left[\frac{1-\tau}{\tau}\rho+(1-\rho)v'(g)\right]\right)$ will also capture effects of debt levels d and revenue τe through the marginal utility of government spending. This unfortunately makes the coefficients less straightforward to interpret without any prior calibration and initial conditions. We propose to model the relationship for country i as follows:

$$\pi_{i,t}-\pi_{i,t}^a=\beta_{0,i}+\beta_1\text{revenue}_{i,t}+\beta_2\text{debt}_{i,t}+\beta_3\text{target}_{i,t}+\beta_4\text{revenue}_{i,t}*\text{debt}_{i,t}+u_{i,t}$$

where the interaction term between revenue and public debt is meant to capture the dynamics of the marginal utility of government spending at time t. The idiosyncratic error term $u_{i,t}$ satisfies $\mathbb{E}(u_{i,t}|X_{i,1},...,X_{i,T},c_i) = 0, t = 1,...,T$. Coefficient $\beta_{0,i}$ captures a country fixed effect, while all other coefficients are common to all countries.

E Online appendix: Approximation of the Real Interest Rate

E.1 Definition of the Real Interest Rate

The FOC of the utility maximization problem for the consumer gives the following ex post equilibrium real interest rate

$$1 + r_t^{eq} = \frac{1}{\mathbb{E}\left[\frac{1}{1+\pi_t}\right]} \frac{1}{1+\pi_t} \frac{1}{\beta}$$

which differs from the ex post real interest rate defined in the text of

$$1 + r_t = \frac{1+\pi_t^e}{1+\pi_t} \frac{1}{\beta}.$$

We show that the definition used in the model is a good approximation of the equilibrium ex post real interest rate, in the sense that the difference between them is negligible as long as the inflation rate is not far from zero. To see this, consider the following Taylor expansion of the function $\frac{1}{1+\pi_t}$ around the expected inflation rate π_t^e:

$$\frac{1}{1+\pi_t} = \frac{1}{1+\pi_t^e} - \frac{1}{(1+\pi_t^e)^2}(\pi_t - \pi_t^e) + \sum_{j=2}^{\infty} (-1)^{j+1} \frac{1}{(1+\pi_t^e)^{j+1}} \frac{(\pi_t - \pi_t^e)^j}{j!}.$$

Taking expectations and considering that the random variable π_t has bounded support, we obtain

$$\mathbb{E}\frac{1}{1+\pi_t} = \frac{1}{1+\pi_t^e} + \sum_{j=2}^{\infty} (-1)^{j+1} \frac{1}{(1+\pi_t^e)^{j+1}} \frac{m_j(\pi_t)}{j!}$$

where $m_j(\pi_t)$ is the j-th moment of the random variable π_t. Now, let ϵ_t be the maximum absolute value the random variable $\pi_t - \pi_t^e$ assumes, which is either $\pi_t^e - \pi^a$ or $\pi_t^D - \pi_t^e$, where π_t^D is the discretionary inflation rate chosen by the policy maker. It is trivial that $|m_j(\pi_t)| < (\epsilon_t)^j$, and if we consider only parameter specifications such that $\epsilon_t < 0.5$, we can use the inequality $e^x - 1 - x < x^2$ for $x < 0.5$ to conclude that

$$\mathbb{E}\frac{1}{1+\pi_t} - \frac{1}{1+\pi_t^e} \leq \frac{(\epsilon_t)^2}{(1+\pi_t^e)^3},$$

which is the same as

$$(1+\pi_t^e)\mathbb{E}\frac{1}{1+\pi_t} = 1 + \xi_t$$

where ξ_t is the estimation error, which is bounded by $\left(\frac{\epsilon_t}{1+\pi_t^e}\right)^2$.

We can now estimate the difference between the equilibrium real ex post interest rate and the definition used in the text by

$$\frac{1+r_t^{eq}}{1+r_t} = 1+\xi_t$$

and by taking logs this relation approximates to $r_t^{eq} - r_t = \xi_t$. The error ξ_t is smaller than the square of the maximum deviation from expected inflation, which will be numerically close to zero in any reasonable calibration of the model.

E.2 Robustness: Numerical Results with Equilibrium Real Interest Rate

We present below the numerical exercise of Section 3 replicated using the equilibrium real interest rate r_t^{eq} as defined in the text. The results are nearly identical. Moreover, note that the definition of the no-crisis, fiscal fragility and fiscal dominance zones is independent of the definition in use since they are defined considering a certainty scenario in which expected inflation is equal either to π^D or π^a, so that in both cases we have $r^{eq} = r$.

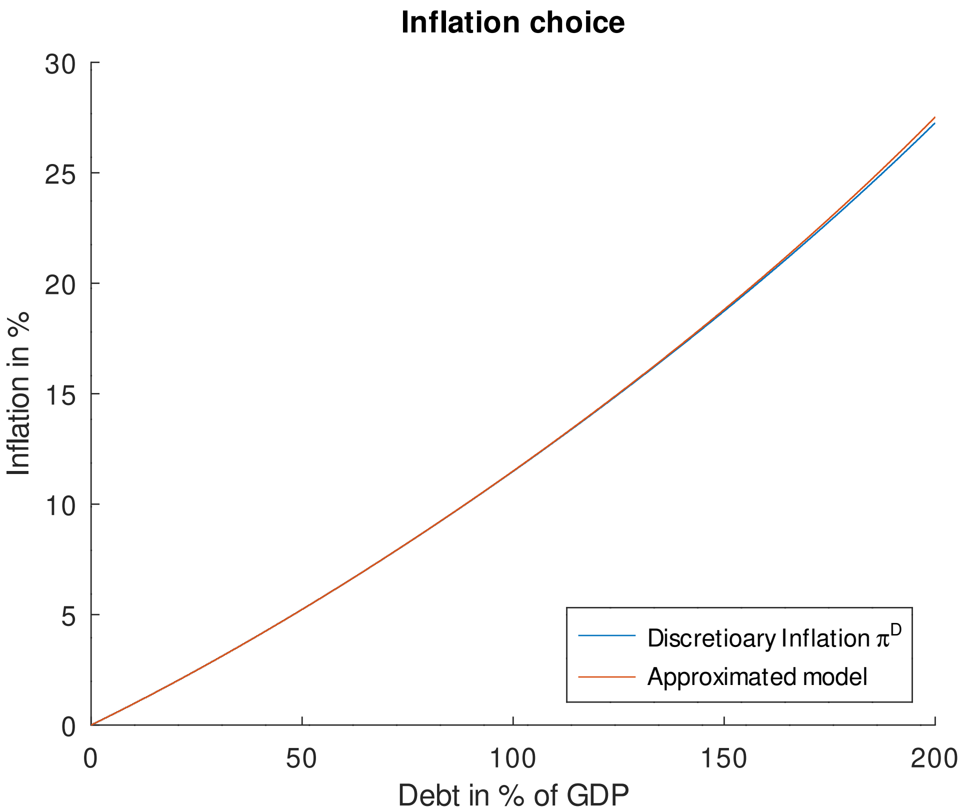

Figure 7: Discretionary Inflation

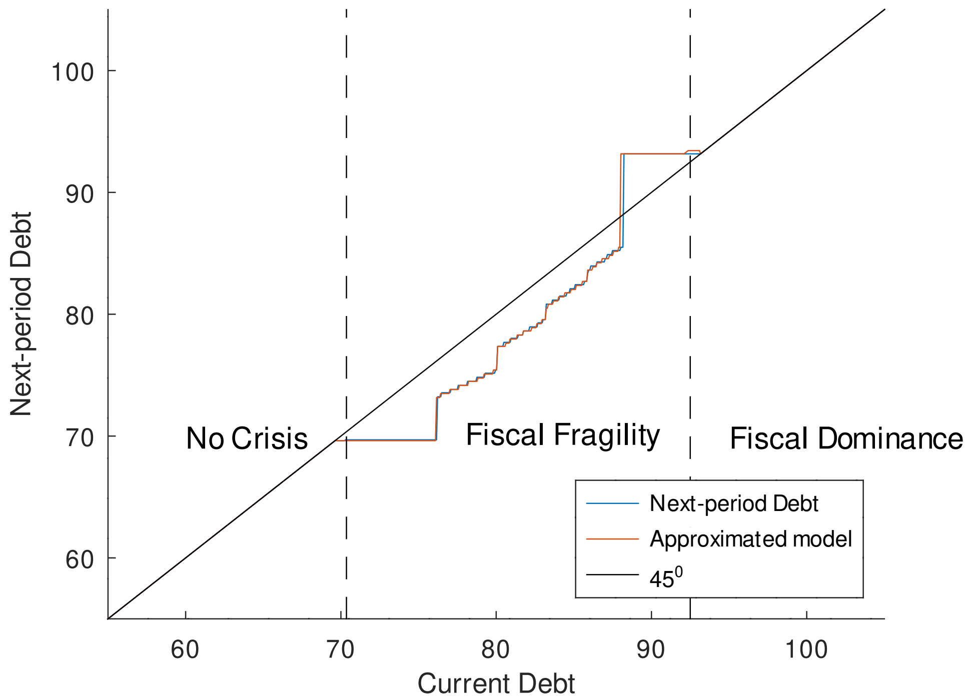

Figure 8: Debt Policy Function

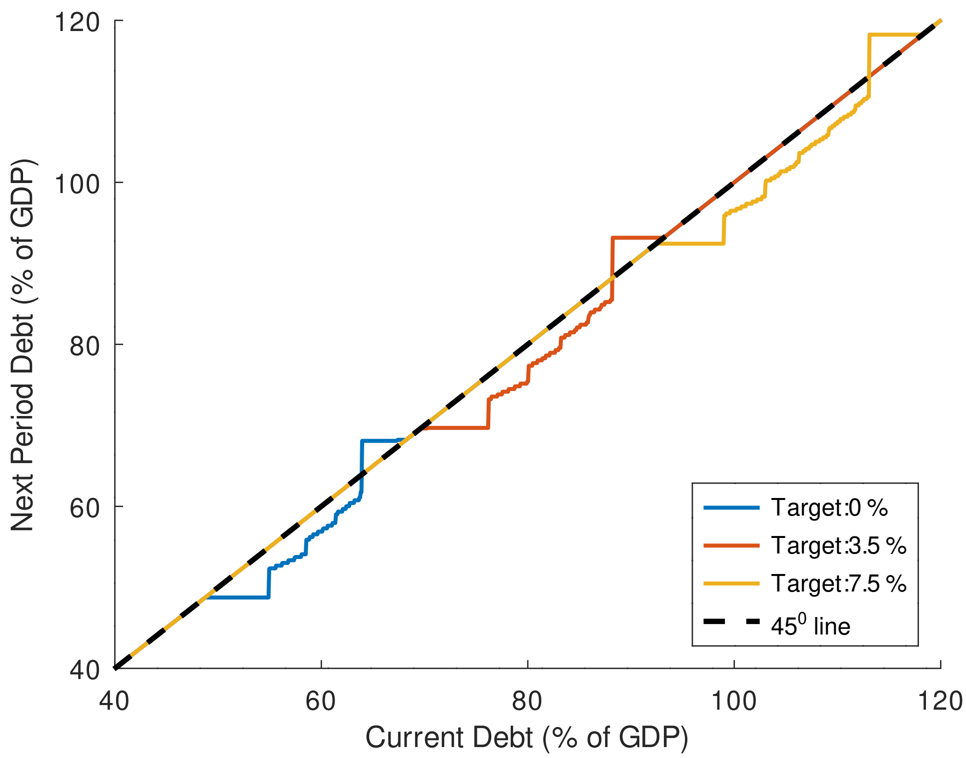

Figure 9: Sensitivity to Target

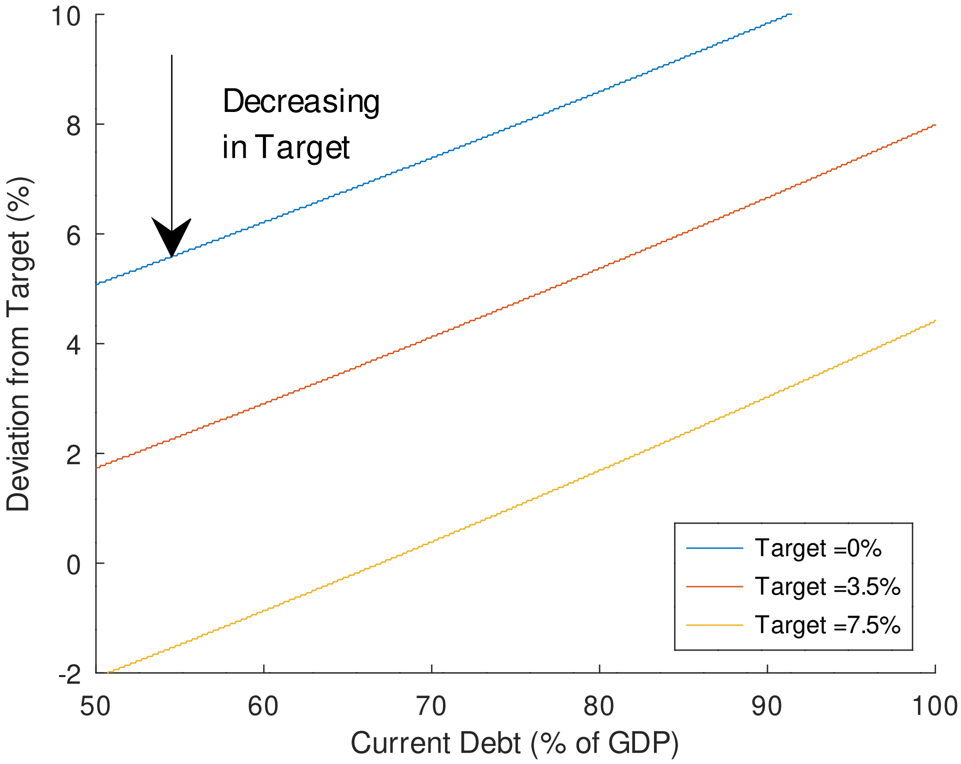

Figure 10: Deviation Sensitivity to Target