# Aggregation, beyond GDP

Marc Fleurbaey

April 20, 2023

**Abstract**

This paper explores the possibility to replace the GDP-shareholder-value nexus, which has been under severe criticism for decades, with a combination of social-welfare compatible objectives at the macroeconomic level and at the firm level. Can GDP be replaced by social welfare and shareholder value be replaced with stakeholder value, in an integrated and consistent fashion? It is shown that money-metric utilities and their aggregation can indeed provide interesting tools to conceive of this integration of micro with macro objectives. But while consistent measures of the contribution of each firm to social welfare can be derived from this analysis, there is little hope to build new national welfare accounts in which micro-level contributions to social level could be simply added up to form social welfare at the macro level.

Keywords: GDP, social welfare, shareholder value, stakeholder value, aggregation.

JEL Classification: D21, D50, D60, I31, L21.

## 1 Introduction

In national accounts, GDP is mostly made of the sum of the value added by the many producing units. These accounts describe how this is distributed as income to households (or retained in firms), and finally how this is spent on consumption or investment. This makes for a very consistent story of a pie being created, then shared, and finally put to good use. This vision of GDP as a national pie motivates its widespread use as an indicator of living standards and economic power in a country.

Measuring production is very useful to gauge economic activity. However, the concept of "value added" has little currency in objectives considered as sound by theorists for productive firms as well as for macroeconomic policy. Truly enough, GDP is often used as a compass by policymakers, but there is a large consensus that GDP is a poor proxy for social welfare. The creators of the modern national system have always warned against interpreting GDP or national income as a measure of welfare, because it measures neither utilities (or utility variations) nor consumer surplus (Vanoli 2002, p. 368). Firms, in turn, generally maximize shareholder value or profit, not value added, and recent discussions have introduced the larger notion of stakeholder value, which brings up again the connection with surplus (Harrison et al. 2019, Magill et al. 2015).

What is wrong with GDP, and what is wrong with shareholder value? Let us first ignore the problems linked to externalities and inequalities, and focus on the fundamental link between value added, profit, and the welfare of the population. GDP measures the economic value of production (net of intermediate goods) and thus misses essential elements that are essential for welfare. In particular, it misses the surplus that consumers draw from consumption, and instead measures the cost to consumers of the goods they consume. Moreover, it ignores the welfare impact of labor on workers, and is computed as if labor had no value or disvalue and was a purely free activity. Consider a household, whose welfare derives from consumer surplus which is, roughly, the difference between the monetary-equivalent subjective value of consumption and its cost, and from worker surplus which measures the difference between earnings and the monetary-equivalent subjective disvalue of work. One could add to this the saving surplus, which measures the difference between the monetary-equivalent surplus of future returns and the amount saved. Compared to that, GDP only records the cost of consumption and the "cost" of savings. In other words, it focuses on two monetary costs, ignores earnings, and ignores all subjective impacts of consumption, labor, and future returns. It tells us how much it costs to households to consume and save, not what they get from this, and treats their labor as free.[1] GDP thus embodies a naive form of productivism, for which labor does not count[2] and consumption is measured by its market value, not by its enjoyable features. Of course, the neglect of externalities and inequalities makes it even more problematic.

Compared with value added, profit appears a less naive measure of performance because it accounts for the cost of labor. But the flaw with shareholder value is simple to identify. The firm is a locus of cooperation between producers and customers, in which every party stands to gain from the venture. Customers get more value than they pay, workers and suppliers of inputs and funding get more income than they require to do the job, and additional surplus remains in the form of profit. There is a total pie generated by the firm, and all participants gain a share of it. Against this background, the shareholder value approach consists in focusing on the share that goes to the equity owners, and treats the other parties in the venture as external agents with whom arm's length relations are limited to market contracts. They are not admitted to the governing bodies of the firm, and treated as cash cows to be milked and costs to be killed. As is well-known from the economics of the monopoly, maximizing one particular share of the pie is likely to be done at the expense of the total pie, and the other parties are squeezed in a wasteful way. Arguably, this does happen in the case of the shareholder value approach to the firm, even if in ideal conditions of perfect competition this should not be the case (Lazear 1995, Fleurbaey and Ponthière 2021).

Both in public debates and in academic publications, there is now an interesting coincidence in calls for going "beyond GDP" at the macro level (see, e.g., Stiglitz et al. 2009, Coyle 2015) and "beyond shareholder value" at the micro level (see, e.g., Harrison et al. 2019, Magill et al. 2015, Kelly 2019, Mayer 2018). But these two lines often remain separate, and the question of linking the two reforms in indicators is seldom raised. This paper seeks how to conceive of the articulation between a social objective at the macro level and business objective at the micro level in a way that is consistent and that relates to well-founded notions of welfare economics. In doing this, it brings together two strands of research that offer elements which will be useful in this paper.

First, focusing on GDP and the macro level, Fleurbaey and Blanchet (2013) review the various approaches to social welfare and how each of them can serve to enrich our measurement of social welfare. In particular, they highlight interesting properties of money-metric utilities, or equivalent incomes, and defend this concept against various criticisms which have been raised by the advocates of more subjective measures of utility. Instead, they argue that equivalent income is a promising well-being representation for social welfare evaluation, including for the incorporation of non-market dimensions of life. There are now multiple applications of this notion for the comparison of living standards across countries (Fleurbaey and Gaulier 2009, Jones and Klenow 2016, Boarini et al. 2022).

Second, for the business objective of the firm, Fleurbaey and Ponthière (2021) study the stakeholder value approach and, following the literature (Harrison et al. 2019, Magill et al. 2015), define it in terms of maximizing the total surplus of all agents who trade with the firm. In addition, they include a measure of the value of externalities for those who do not trade with the firm but are subject to its influence. Their analysis is primarily done in a partial equilibrium approach. Although they extend their partial equilibrium analysis of optimal management rules to a general equilibrium setting, they actually do not examine how the stakeholder value could be defined and measured in a general equilibrium setting. And they do not discuss the issue of adding up the contributions of all firms to make a macroeconomic pie for society as a whole, in the fashion of value added. Interestingly, R. Fouquet

[1] It is chilling to notice how close this approach is to the accounting books of former slave-owners who only had to count how much their slaves' consumption cost them and for whom the slaves' labor had no direct cost.

[2] It is surprising that GDP per hour worked, which gives a better picture of the economic opportunities of a population, thanks to its investments in capital (including human capital) is not a more popular indicator.

(https://www.lse.ac.uk/granthaminstitute/roger-fouquet-research-in-progress/) sets out to measure Net Domestic Consumer Surplus over an extended period of history, adding up the surplus generated for consumers by various industries.

These disconnected strands of research, in light of the national accounting "pie" structure, raise an obvious question. Does there exist a "Holy Grail" measure of economic activity that:

- can serve as a reasonable objective for productive units (like shareholder value or stakeholder value);
- can be aggregated to a macroeconomic level (like value added);
- can, in its aggregate form, serve as a reasonable objective for social welfare (like a social welfare function)?

Traditionally, the notion of surplus is suited to partial equilibrium analysis, whereas money-metric utility, also called equivalent income, is appropriate for general equilibrium analysis. The concepts of compensating variation and equivalent variation, which are computed as variations in equivalent income, provide the classical extension to multiple goods of the traditional notion of the *variation* of the Marshallian surplus, but not its *level*. And the idea of measuring the contribution of the economy to social welfare, in the fashion of a total surplus, is intuitively appealing. Moreover, the existence of a consistent notion of stakeholder value based on a sum of surplus at the firm level (and in partial equilibrium analysis) appears an attractive starting point. One could then hope that equivalent income could be used to adapt the concept of surplus to a general equilibrium framework and be measured in a way that can be aggregated meaningfully. This is what this paper sets out to investigate.

Unless otherwise specified, the setting in which the analysis unfolds is a static one-period situation, which can concretely be imagined to be a year. It is assumed that it makes sense to analyze surplus and well-being in this year independently of what happens in other years.

To give a preview of the results, here is what emerges from this analysis. The concept of surplus *level* can be extended to multiple goods, but is deeply problematic in aggregation over many goods and also quite questionable when aggregated over multiple individuals. But it is possible to define a measure of the contribution of a firm in terms of the total surplus of its stakeholders, as measured in the general equilibrium setting, and to render this contribution consistent with a notion of social welfare, both being expressed in monetary units. However, while the contribution of a particular firm to the total pie makes sense, the total pie is not the sum of the contributions of all the firms. This means that, although it appears possible, and eminently sensible, to define objectives that are consistent at the social level and at the firm level, there is little prospect of devising a parallel form of national accounting that would take a simple additive form as traditional value added accounts. The Holy Grail is elusive, but every decision-maker should be able to figure out what to do and how to play their part in the joint production of social welfare. The GDP-shareholder-value nexus can be replaced, or so will be argued here, by a consistent social-welfare-stakeholder-value approach.

The paper is organized as follows. The next section revisits the standard argument that is often used, after Samuelson (1956), to link national income to social welfare. This argument generally neglects labor, but once labor is introduced, the link between firm-level accounts and national accounts appears more clearly, delivering a surprising rebuttal of GDP and a clear alternative. In section 3 the main notations and bridges between partial and general equilibrium concepts are introduced. The aggregation over individuals and the bridge between individual and social welfare is tackled in section 4. Section 5 introduces the notion of surplus from transactions in order to measure the contribution of a particular firm rather than the contribution of a particular good to welfare. Building on these preliminaries, a definition of the contribution of a firm to social welfare is proposed in section 6. Section 7 extends the analysis to non-market dimensions of well-being and externalities generated by firms. Section 8 concludes the paper.

## 2 From GDP to profit to surplus

Firms maximize profit whereas nations focus on national income. The conventional reading of this is that firms, in a capitalist economy, are dominated by shareholders and thus focus on a narrow conception of the surplus they generate, whereas national government cannot ignore the share of workers in the economy and therefore logically look at the sum of profits and wages. The problem with this reasoning is that it would not be reasonable for firms to maximize their value added. In a literal sense, something "does not add up" in this line of thought.

The simple message of this section is the following: Maximizing value added is a meaningful objective neither at the micro nor at the macro level. The variation of welfare from one period to another can be approximated by the variation of total profits in volume, not by the variation of total value added in volume.

Consider a social welfare function

$$W\left(U_i\left(c_i, l_i\right); i = 1, ..., n\right)$$

and a market economy in which every individual $i$ maximizes $U_i\left(c_i, l_i\right)$ under the constraint $pc_i = wl_i + I_i$, where $c_i$ denotes a consumption bundle, $l_i$ a bundle of labor services, $p$ and $w$ the corresponding prices and wages, and $I_i$ unearned income. Developing a computation made in Fleurbaey and Blanchet (2013) following Samuelson (1956), one has

$$\begin{aligned} dW &= \sum_i \frac{\partial W}{\partial U_i}\left(\sum_k \frac{\partial U_i}{\partial c_{ik}} dc_{ik} + \sum_h \frac{\partial U_i}{\partial l_{ih}} dl_{ih}\right) \\ &= \sum_i \frac{\partial W}{\partial U_i}\lambda_i\left(pdc_i - wdl_i\right) \end{aligned}$$

where $\lambda_i$ is the Lagrange multiplier of the budget constraint for individual $i$, and is therefore equal to $i$'s marginal utility of money. In the case of an optimal distribution of resources, all the marginal social values of money ($\frac{\partial W}{\partial U_i}\lambda_i$) are equal, and then $dW$ is proportional to

$$\sum_i \left(pdc_i - wdl_i\right). \tag{1}$$

Compare this with the volume variation of the sum of profits over all firms $j$ producing $q_j$ with labor $l_l$ and intermediate goods $x_j$:

$$\sum_j \left(p\left(dq_j - dx_j\right) - wdl_l\right). \tag{2}$$

At the market equilibrium,

$$\sum_j q_j = \sum_i c_i + \sum_j x_j \text{ and } \sum_j l_j = \sum_i l_i,$$

which implies that the two expressions (1) and (2) are equal.

This reasoning shows that this approximation argument does not link social welfare to national income, but to national profit. This makes a lot of intuitive sense when one observes that at the individual level, the budget constraint for economic transactions reads $pc_i - wl_i = I_i$, where $I_i$ is the real constraint for the individual trading-off leisure and consumption. Individual welfare being linked to $pc_i - wl_i$, it is normal that under certain conditions about the distribution, social welfare is linked to the sum of these expressions.

However, it would of course be a mistake to identify profit and social welfare without precaution, because the argument is not about the variation of profit in value, and not even about a deflated form of this variation. The expressions (1) and (2) depict a volume variation at fixed prices.

Moreover, it can be argued that profit is too narrow an objective for firms and that socially responsible firms instead maximize the total value to stakeholders. Importantly, this total value is not the same as the value added. It is the sum of surpluses of all the stakeholders, not the sum of their market incomes. For consumers, their expenditures contibute negatively to the mesure of their surplus, whereas for workers their wages contribute positively. In both cases, the other central part of the computation of surplus is willingness to pay for the consumers and willingess to accept for the workers. The analysis of total stakeholder value in a partial equilibrium setting was made in Fleurbaey and Ponthière (2023) and this need not be further developed here.

It remains to rigorously articulate the link between stakeholder value at the micro level of the firm and social welfare at the macro level of the nation. This is the purpose of this paper. A key intuitive point that helps reconcile the approximation of welfare in terms of profit, presented in this section, and the definition of welfare in terms of surplus, studied in the sequel, is that prices and wages correspond to marginal values of the consumers' willingness to pay and the workers' willingness to accept. Thus, when looking at a small variation, prices and wages provide good approximations of the marginal effect of a change in quantities on welfare.

# 3 From one good to multiple commodities

This section introduces the basic notions of surplus in the context of multiple commodities. It also reviews particular cases (quasi-linear preferences) and issues that complicate interpersonal comparisons, such as unequal access to the labor market or variable preferences.

## 3.1 General case

Consider a household whose utility is $U(x)$, where $x$ is a $\ell$-vector of real numbers depicting commodities bought ($x_k > 0$) or sold ($x_k < 0$). Let $p$ denote the price vector and $I$ the unearned income (or wealth) of the household. The usual concepts of consumer theory are denoted and defined as follows, assuming that demand is always unique, and using the same notation for functions with different domains but identical images, as they cannot be confused when their arguments differ:

- Marshallian demand: $x(p,I) = \arg\max\{U(x)\,|px \leq I\}$.
- Indirect utility: $U(p,I) = U(x(p,I)) = \max\{U(x)\,|px \leq I\}$.
- Expenditure function: $e(p,u) = \min\{px|U(x) \geq u\}$.
- Hicksian demand: $h(p,u) = \arg\min\{px|U(x) \geq u\}$.
- Direct equivalent income: $e(p_0,x) = e(p_0,U(x))$.
- (Indirect) equivalent income: $e(p_0,p,I) = e(p_0,U(x(p,I)))$.

Recall that $e(p,p,I) = I$. In the definition of equivalent income, $p_0$ is a reference price vector. The choice of this reference is discussed in section 2.6. In most of this paper, the current price will be used as the reference, but when prices change it may be important to keep a constant reference price for consistency purposes. This is a classical problem in cost-benefit analysis (Boadway 2016).

Consumer surplus can be defined over a subset $K$ of commodities as the money-equivalent benefit of having access to these commodities. To formally define it, one needs to introduce the demand that would arise when barred from these commodities. Let $x_K = (x_k)_{k\in K}$.

- Marshallian demand when $K$ is off limits: $x^{\overline{K}}(p,I) = \arg\max\{U(x)\,|px \leq I, x_K = 0\}$.
- Consumer surplus from $K$: $S^K(p,I) = I - e\left(p, x^{\overline{K}}(p,I)\right)$.

So, the consumer surplus from accessing $K$ is the difference in equivalent income between the unconstrained situation and the constrained situation in which $K$ is off limits. Observe that $x^{\overline{K}}(p,I)$ is independent of $p_K$. But one also has $x^{\overline{K}}(p,I) = x((\infty_K, p_{-K}), I)$, i.e., $x^{\overline{K}}(p,I)$ is the demand that would occur if commodities from $K$ had an infinite price (or a zero price when selling the commodity—the symbol $\infty$ is used to capture both cases). Moreover, one always has $e(p, x(p, I)) = I$. This implies that, singling out a commodity $k$ that is bought rather than sold, one has

$$e\left(\left(\infty_k, p_{-k}\right), x^{\overline{k}}(p, I)\right) = e\left(\left(\infty_k, p_{-k}\right), x^{\overline{k}}\left(\left(\infty, p_{-k}\right), I\right)\right) = I$$

and thus one can relate the surplus on a single good to the integral below Hicksian demand, when Shephard's Lemma[3] can be applied:

$$\begin{aligned} S^k(p, I) &= e\left(\left(\infty_k, p_{-k}\right), x^{\overline{k}}(p, I)\right) - e\left(p, x^{\overline{k}}(p, I)\right) \\ &= \int_{p_k}^{+\infty} h\left(\left(t, p_{-k}\right), U\left(x^{\overline{k}}(p, I)\right)\right) dt. \end{aligned}$$

In consumer theory, it is traditional to distinguish the compensating variation from the equivalent variation, when prices or income vary:

- Compensating variation: $e(p, p, I) - e(p, p', I')$.
- Equivalent variation: $e(p', p, I) - e(p', p', I')$.

The surplus as defined above can be written as a compensating variation

$$e(p, p, I) - e\left(p, \left(\infty_K, p_{-K}\right), I\right).$$

The corresponding equivalent variation is

$$e\left(\left(\infty_K, p_{-K}\right), p, I\right) - e\left(\left(\infty_K, p_{-K}\right), \left(\infty_K, p_{-K}\right), I\right),$$

and is less amenable to considering all commodities jointly, as explained below.

Let $L = \{1, ..., \ell\}$ be the set of all commodities and examine the notion of total surplus from all commodities. By definition, $x^{\overline{L}}(p, I) = 0$. In the case when all goods are bought, one has $e\left(p, x^{\overline{L}}(p, I)\right) = 0$, which implies that $S^L(p, I) = I$. Total surplus over all commodities is simply the level of unearned income. Since $e(p, p, I) = I$, the two notions of surplus and equivalent income coincide in this case.

However, how sensible is the notion of surplus applied to all commodities? This surplus refers to a baseline defined as complete autarky, which is quite artificial since this situation is generally not viable in a developed market economy. Moreover, the equivalent-variation formulation of the surplus for all commodities does not even make much sense: it also takes complete autarky as the baseline, and then asks how much monetary gain in autarky (or with all prices being infinitely high) would be as worthy as market access at current prices. This question has no meaningful answer, since money without access to goods has no value.

When some commodities are sold, one typically has $e(p, U(0)) < 0$, because one can start with a debt, repay it (and more) by selling goods, buy some other goods to compensate for this loss, and reach the utility $U(0)$. In this case, $S^L(p, I) = I - e(p, U(0))$ is greater than $I$. In contrast, one still has $e(p, p, I) = I$. A difference between total surplus and equivalent income thus emerges. What should we make of it? Interestingly, the usual measure of economic standing, i.e., total income (earned and unearned), clearly overestimates welfare, as it neglects the disutility of selling commodities (in particular labor), as emphasized in the introduction. Does equivalent income, in contrast, underestimate it because it focuses on unearned income and apparently neglects the surplus from selling commodities? In fact, equivalent income, by definition, computes the level of unearned income that one would need in order to reach current satisfaction, with free access to the labor market. It treats selling labor symmetrically

[3] According to Shephard's Lemma, $\frac{\partial e}{\partial p_k}(p, u) = h_k(p, u)$.

with buying commodities. This appears perfectly acceptable when the choice of buying and selling is up to the household, and what determines the opportunities is primarily unearned income. However, inequalities in skills make this approach insufficiently responsive to opportunities: not everyone is able to sell the services of a CEO or a movie star. This issue is examined in section 2.3.

## 3.2 Quasi-linear additive case

The quasi-linear additive case is special not just by the function form of utility, i.e., $U(x,z) = \sum_{k=1}^{\ell} v_k(x_k) + z$, where each $v_k$ is strictly concave and $z$ is the numeraire, but also because it is always implicitly assumed that there is no lower bound on the value of $z$. Thus, the consumption set, i.e., the set of possible bundles, generally differs from the general case introduced in section 2.1.

This implies that the surplus on all commodities, including $z$, is always greater than $I$, as it equals $I$ plus the surplus reaped from the non-$z$ commodities.

To simplify notations, assume $v_k(0) = 0$. Then, when $k$ is bought and $v_k$ is continuously differentiable, one has $h_k(p,u) = v_k'^{-1}(p_k)$ and thus, for $K \subset L$,

$$\begin{aligned} S^K(p,I) &= \sum_{k\in K} \int_{p_k}^{+\infty} v_k'^{-1}(t)\,dt \\ &= \sum_{k\in K} (v_k(x_k) - p_k x_k). \end{aligned}$$

For commodities that are sold rather than bought, a similar computation yields the same term, but with $p_k x_k < v_k(x_k) < 0$ instead of $v_k(x_k) > p_k x_k > 0$. One computes that

$$\begin{aligned} e(p,u) &= u - \sum_{k\in L} (v_k(x_k(p_k)) - p_k x_k(p_k)) \\ &= u - S^L(p,I), \end{aligned}$$

so that $e(p,0) < 0$ when $S^L(p,I) > 0$, meaning that, even if the agent starts with a debt (negative $I$), $z$ can be further sold in order to buy commodities and reach the utility $U(0,0)$.

The total surplus, including from $z$, is then

$$\begin{aligned} S^{L\cup\{z\}}(p,I) &= I - e(p,0) \\ &= I + \sum_{k\in L} (v_k(x_k) - p_k x_k) \\ &= U(x,z) - U(0,0). \end{aligned}$$

Thus, one sees that the quasi-linear case offers three equivalent ways to compute the total surplus: 1) difference in equivalent income between the optimal bundle and autarky; 2) unearned income augmented of the surplus on the non-$z$ commodities; 3) utility difference between the optimal bundle and autarky. In the general case, the first two approaches do not measure surplus.

A valuable property of the quasi-linear case is the additivity of surpluses over subsets of commodities. Namely, for $K, K' \in L$, $K \cap K' = \emptyset$,

$$S^{K\cup K'}(p,I) = S^K(p,I) + S^{K'}(p,I).$$

This property is generally not satisfied otherwise, which makes it impossble to interpret the total surplus over several commodities as the sum of surpluses over these commodities.

### 3.3 Unequal access to the labor market

Total surplus, in the presence of labor, is greater than $I$ because $e(p, U(0)) < 0$. When individuals are unequally skilled, the absolute value of $e(p, U(0))$ tends, other things equal, to increase with the wage rate the individual obtains on the labor market. In this way, $S^L(p, I) = I - e(p, U(0))$ captures wage inequalities across individuals through its second term. This is odd, because $e(p, U(0))$ is referring to a special situation: the debt in unearned income that would suffice if the individual had to pay it through working and ended up consuming just enough to compensate the disutility of working, barely reaching the utility of autarky. Skilled individuals can more easily repay the debt and can therefore shoulder a larger debt, which ultimately makes their total surplus greater.

Equivalent income, in contrast, does not refer to such an artificial autarkic situation. But, when using $p_0 = p$ as the reference price, it is not meaningfully comparable across individuals when they do not have access to the same markets, because it is then equal to $I$ independently of the individual's wage.

There are several ways to develop equivalent income measures that take account of such inequalities (see in particular Fleurbaey and Maniquet 2011, 2018). A simple one consists of focusing on working time, and one can then consider that different individuals face different prices for their time. Equivalent income $e(p_0, p, I)$ can then involve a single price level in $p_0$ for time, whereas the same component in $p$ differs across individuals. A salient reference price level in fairness theory is zero, implying that equivalent income is then equal to the level of unearned income that would suffice for the individual if work was free and not paid. Another salient value is the minimum wage. Adopting a low level for this reference price of time has the virtue of making the people who are particularly averse to work (e.g., because they only have access to unpleasant jobs) appear particularly disadvantaged.

Another method consists in treating different labor services as different commodities, and treating the segments of the labor market to which the individual does not have access as featuring a zero wage for this individual. One can then compute equivalent income $e(p_0, p, I)$ where $p_0$ takes the same (low) value for all segments of the labor market. This is equivalent to the previous approach under some assumptions. However, if individuals would really like to practice certain professions, this approach may underestimate their disadvantage because it assumes that they could practice their favorite activity at the reference wage.

An important consequence of these observations is that taking the current price $p$ as the reference in applications, as done in many of the results of this paper, is problematic because it implicitly condones the inequalities in skills and access to various labor markets.

### 3.4 Variable prices

In the general case, surplus is not comparable across situations which differ in the price vector $p$. The expression $I - e(p, U(0))$ can, for instance, boil down to $I$ in the case when all goods are bought, and obviously $I$ cannot be used without some deflator for prices. Even in the quasi-linear case, in which the expression $U(x, z) - U(0, 0)$ appears to be a direct measurement in utils, the dependence on prices is contained in the presence of the numeraire $z$ in which utility is expressed. If the price of the numeraire itself varies, the formula is not comparable. However, dividing the surplus by the price of the numeraire (when it departs from unity) can make the measure comparable across allocations with different price vectors.

In contrast to the surplus, equivalent income is built for the case of variable prices, by taking a fixed reference price which must be used throughout all situations that have to be compared when prices vary. A limitation of this approach is that when prices vary a lot, the reference price may be quite different from the current prices, making the computation of equivalent income rely on distant extrapolation of preferences.

Another limitation of equivalent income (and the whole set of measures considered so far) is that it depends on the set of commodities remaining unchanged. When new commodities arrive, some reference price should be adopted for them, but then equivalent income is no longer comparable with the measures made before the arrival of the new commodities.

Let us compare the normalized surplus and the equivalent income in the quasi-linear case, in which $U(x,z) = \frac{\sum_{k=1}^{\ell} v_k(x_k)+z}{p_z}$ after normalization by the price of the numeraire. One has:

$$S^{L\cup\{z\}} = \frac{I + \sum_{k\in L}(v_k(x_k) - p_k x_k)}{p_z} = U(x,z) - U(0,0) = \frac{I - e(p,0)}{p_z},$$

$$e(p_0,p,I) = \frac{p_{0z}}{p_z}\left(I + \sum_{k\in L}(v_k(x_k(p_k)) - p_k x_k(p_k))\right) - \sum_{k\in L}(v_k(x_k(p_{0k})) - p_k x_k(p_{0k})).$$

This suggests that a key difference lies in the dependence of the surplus on current prices, as opposed to reference prices for equivalent income. One could consider computing the surplus $e(p,p,I) - e(p,U(0))$ with a reference price as in equivalent income, in the following way:

$$e(p_0,p,I) - e(p_0,U(0)) = \frac{p_{0z}}{p_z}\left(I + \sum_{k\in L}(v_k(x_k(p_k)) - p_k x_k(p_k))\right).$$

If the reference price $p_{0z} = 1$, the two formulas for the surplus coincide, and the link with equivalent income is clear.

### 3.5 Variable preferences

If an individual changes preferences from one situation to another (e.g., from one year to another), can we make comparisons between the two situations? In fact, assessing two situations for a single individual with two different preference relations is formally exactly identical as assessing two individual situations involving two different individuals with different preferences. Intrapersonal comparisons with variable preferences should not be harder than interpersonal comparisons.

Admittedly, interpersonal comparisons are considered difficult and full of ethical issues, but both surplus and equivalent income are designed to be comparable across different individuals or different preferences. Indeed, they are measured in monetary units, and represent the equivalent gain (for the surplus) or level (for equivalent income) for an individual who would have exactly the required level of unearned income in the relevant situations, at well-chosen reference prices. In brief, if interpersonal comparisons in wealth or income are deemed generally relevant for ethical purposes, for instance because they represent the degree of affluence of individuals and their opportunities to lead their lives, then these measures are worthy of consideration.

### 3.6 Choosing the reference prices

The vector $p_0$ plays a crucial role in the computation of equivalent income, or of a refinement of the surplus that extends to variable prices such as $e(p_0,p,I) - e(p_0,U(0))$. Theoretical considerations developed in Fleurbaey and Blanchet (2013) suggest taking a price vector that supports the Scitovsky set[4] for the whole allocation that includes all the specific allocations over which comparisons is to be make (e.g., the world allocation when comparisons are made across countries, or the intertemporal allocation when comparison is made over years), but this is impractical when the whole allocation is not known, for instance when it includes the uncertain future.

[4] The Scitovsky set is defined for an allocation $(x_1,...,x_N)$ over a population of $N$ individuals as the set of total resources which could be distributed to provide individuals with their utility at the contemplated allocation:

$$\left\{\sum_{i=1}^{N} y_i \middle| \forall i, U_i(y_i) \geq U_i(x_i)\right\}.$$

However, some qualitative ideas can be retained from the theoretical approach. Taking a price vector that is not too far from the price vectors in the contemplated allocations is useful in order to avoid relying on distant preferences. Some averaging of various possible price references, as commonly done for purchasing-power-parity price indexes, can be justified on this ground.

Chaining is another related possibility, in order to deal with time series. In theory, it is problematic as it can lead to inconsistent evaluations (such as cyclical rankings of allocations). But in practice, this may provide a reasonable compromise between theoretical grounding and practical applicability.

In conclusion to this section, one can retain the main following insights:

1. The surplus and the equivalent income are closely related, since for given prices, the former is computed as a difference in the latter, between the current situation and a baseline. In the quasi-linear case, the surplus has additional interpretations as augmented income and as utility gain from the baseline (measured in the numeraire).

2. The surplus is problematic for several reasons. First, when computed jointly for all commodities, it refers to a very artificial autarky situation. Second, outside the quasi-linear case, the surplus is not well equipped to deal with wage inequalities and price variations.

3. In contrast, the equivalent income is a direct measure that does not take autarky as a baseline and is naturally designed to deal with wage inequalities and price variations, through a suitable choice of reference prices and wages.

4. Both of these measures can handle variable preferences (and interpersonal comparisons) in a defensible way, if comparisons in terms of resources are deemed relevant.

5. But none of them is designed to address changes in the composition of existing commodities. It is possible, retrospectively, to re-compute these measures with the union of the sets of commodities that have been available at all times, but this can only be done ex post, and must involve assumptions about preferences for commodities that were not known at the time.

# 4 From one to several individuals

Since equivalent income is directly comparable across individuals and handles price variations and unequal wages, it can be used as an index of well-being for a social welfare function

$$W\left(e_i\left(p_0, x_i\right); i = 1, ..., N\right),$$

where $i$ is the individual's index and $N$ the number of individuals. Such a social welfare is given axiomatic foundations in Bosmans et al. (2018).

The simplest social welfare function is the sum

$$\sum_i e_i\left(p_0, x_i\right).$$

This function ignores inequalities. Actually, it spans the Pareto-efficient allocations when one varies the reference prices $p_0$ and maximizes it. For a given allocation $(x_1, ..., x_N)$, it provides the smallest value at $p_0$ of all the bundles belonging to the Scitovsky set (defined in fn (4)), a set that is exclusively focused on efficiency, as one can check that:

$$\sum_i e_i\left(p_0, x_i\right) = \min\left\{p_0 \sum_i y_i | \forall i, U_i\left(y_i\right) \geq U_i\left(x_i\right)\right\}.$$

Consider an arbitrary social welfare function defined in terms of the weighted sum of some utility representations of the individuals' preferences, and let use the indirect utilities:

$$\sum_i \alpha_i U_i(p, I_i).$$

The marginal social value of money for $i$ is equal to $\alpha_i \frac{\partial U_i}{\partial I_i}$. Then, this social welfare function can be approximated, to the first order, by the social welfare function

$$\sum_i \beta_i e_i(p_0, p, I_i),$$

where the weights $\beta_i$ are related to the weights $\alpha_i$ in the following way:

$$\beta_i = \frac{\alpha_i \frac{\partial U_i}{\partial I_i}}{\frac{\partial e_i}{\partial I_i}} = \frac{\alpha_i}{\frac{\partial e_i}{\partial U_i}(p_0, U_i)}.$$

In the special case in which $p_0 = p$, one simply has $\beta_i = \alpha_i \frac{\partial U_i}{\partial I_i}$, because $\frac{\partial e_i}{\partial I_i} = 1$ in this case. It is natural that the weights on equivalent income should be closely related to the marginal social value of money $\alpha_i \frac{\partial U_i}{\partial I_i}$ for the various individuals.

# 5 Valuing variations in the allocation

Before seeking a measure of a particular firm's contribution to social welfare, it is worth studying the consequences on social welfare of changes in production plans and prices that firms might undertake.

Consider a market economy with production, in which each household $i$ has a net transaction $x_i$ subject to a budget constraint $px_i \leq I_i = \sum_j s_{ij}\pi_j$, where $s_{ij}$ is the share of equity held by household $i$ in firm $j$, while $\pi_j$ is firm $j$'s profit. Each firm $j$ has a production plan $y_j \in \mathbb{R}^\ell$ (with $y_{jk} > 0$ for outputs, $y_{jk} < 0$ for inputs) and profit $\pi_j = py_j$, and is submitted to a technological constraint $y_j \in \mathcal{Y}_j$. An allocation is feasible if $\sum_i x_i = \sum_j y_j$, and this equality should hold at a market equilibrium. This model is compatible with firms having some market power, which can be modelled by making the firm have monopoly or monopsony over the goods for which it has market power. We will assume that households are price-takers. Let $x = (x_1, ..., x_N)$ and $y = (y_1, ..., y_J)$ denote the allocations of households' and firms' bundles.

What is the contribution to social welfare of a change in the allocation $dx, dy$ such that $\sum_i dx_i = \sum_j dy_j$, associated with a change in prices $dp$? Let us first focus on the impact on the simple social welfare indicator $\sum_i e_i(p, x_i)$, taking the initial price vector $p$ as the reference.

The impact of $dx$ is equal to

$$\sum_k \sum_i \frac{\partial e_i}{\partial x_{ik}} dx_{ik} = p \sum_i dx_i = p \sum_j dy_j$$

because $\frac{\partial e_i}{\partial x_{ik}}(p, x_i) = p_k$ for all $i$ when $x_i = x_i(p, I_i)$. Indeed, in this case, $e_i(p, x_i) = e_i(p, U_i(p, I_i))$ so that

$$\frac{\partial e_i}{\partial x_{ik}}(p, x_i) = \frac{\partial e_i}{\partial U_i}(p, U_i(p, I_i)) \frac{\partial U_i}{\partial I_i}(p, I_i) p_k,$$

with $\frac{\partial e_i}{\partial U_i}(p, U_i(p, I_i)) \frac{\partial U_i}{\partial I_i}(p, I_i) = 1$ because the expenditure function and the indirect utility are inverse functions of one another with respect to utility/income.

As for the impact of $dp$, one computes the double impact on $\sum_i e_i(p, U_i(p, I_i))$ that goes through the price vector in $U_i(p, I_i)$ and the income effect through $I_i = \sum_j s_{ij} p y_j$:

$$\sum_i \sum_k \frac{\partial e_i}{\partial U_i} \frac{\partial U_i}{\partial p_k} dp_k + \sum_i \frac{\partial e_i}{\partial I_i} \sum_j s_{ij} y_j dp$$

By Roy's identity, $\frac{\partial e_i}{\partial U_i} \frac{\partial U_i}{\partial p_k} = -x_{ik}$, thus the first term is $-dp \sum_i x_i$. As $\frac{\partial e_i}{\partial I_i} = 1$, the second term is $dp \sum_j dy_j$. By the condition $\sum_i dx_i = \sum_j dy_j$, the two effects cancel out, and the impact of a change $dp$ is null. A change in price is only redistributing resources across households, and this is a matter of indifference for the social welfare function $\sum_i e_i(p, x_i)$.

Although this analysis only sheds light on marginal changes to the allocation, it provides some insights into how the productive sector, and each firm, can maximize their contribution, as stated in the following proposition.

**Proposition** A change in the allocation $(dx, dy, dp)$ induces

$$d \sum_i e_i(p, x_i) = p \sum_j dy_j,$$

i.e., is valued by the change in total profit at current prices. Thus, any particular firm that changes its production plan so that $pdy_j > 0$ contributes positively to social welfare measured by $\sum_i e_i(p, x_i)$.

This result is in line with results in Magill et al. (2015) and Fleurbaey and Ponthière (2021) according to which firms that maximize the surplus of their stakeholders should not make use of their market power and should maximize profit at the prevailing prices.

Let us now introduce social welfare weights into $\sum_i \alpha_i e_i(p, x_i)$, which amounts to postulating that the distribution of resources is not socially optimal. One computes:

$$\sum_k \sum_i \alpha_i \frac{\partial e_i}{\partial x_{ik}} dx_{ik} = \sum_i \alpha_i p dx_i$$

and

$$\sum_i \sum_k \alpha_i \frac{\partial e_i}{\partial U_i} \frac{\partial U_i}{\partial p_k} dp_k + \sum_i \alpha_i \frac{\partial e_i}{\partial I_i} \sum_j s_{ij} y_j dp = \left( -\sum_i \alpha_i x_i + \sum_j \hat{s}_j y_j \right) dp,$$

where $\hat{s}_j = \sum_i \alpha_i s_{ij}$. As appears intuitive, a firm altering its plan $y_j$ will improve social welfare the more its serves households with high social priority, and when it changes its prices, this depends on whether the goods are consumed by high-priority consumers (and labor is performed by high-priority workers) and whether its shareholders are high-priority households.

Finally, let us consider the case in which the reference $p_0 \neq p$, which is bound to be the case for sensible ways of dealing with labor market inequalities. One then computes

$$\begin{aligned} \frac{\partial e_i}{\partial x_{ik}}(p_0, x_i) &= \frac{\partial e_i}{\partial U_i}(p_0, U_i(p, I_i)) \frac{\partial U_i}{\partial I_i}(p, I_i) p_k \\ &= \frac{\frac{\partial e_i}{\partial U_i}(p_0, U_i(p, I_i))}{\frac{\partial e_i}{\partial U_i}(p, U_i(p, I_i))} p_k, \end{aligned}$$

so that

$$\sum_k \sum_i \frac{\partial e_i}{\partial x_{ik}} dx_{ik} = \sum_i \frac{\frac{\partial e_i}{\partial U_i}(p_0, U_i(p, I_i))}{\frac{\partial e_i}{\partial U_i}(p, U_i(p, I_i))} p dx_i,$$

and the correction term appearing in front of $pdx_i$ is unlikely to differ much from 1 when $p_0$ differs from $p$ mainly on the wage components, and income effects associated with labor are not too large. In particular, in the quasi-linear case, $\frac{\partial e_i}{\partial U_i}$ is always equal to the price of the numeraire in the reference price.

## 6 Surplus from transactions

In order to define and measure the contribution a firm makes to social welfare, one must reason in terms of access of stakeholders to transactions rather than commodities, because a given agent may have access to a commodity through several firms in competition, and may benefit from a firm through several commodities (as well as from working in it). This requires a slight amendment to the notions of surplus introduced so far, in order to define surplus from a transaction, as distinct from surplus from particular commodities.

Let us write the bundle $x$ as the result of particular transactions $t_m \in \mathbb{R}^\ell$: $x = \sum_m t_m$, where $t_m$ is an individual transaction, which can be two-way (buying or selling) or one-way (pure transfer). In this context, it is now important to take one component of $x$ to represent monetary assets, recalling that the analysis here is meant to deal with a small time period like a year. For clarity, this component will be labelled $z$, as in the quasi-linear model, whereas $x$ will be confined to the other commodities. The monetary asset serves to represent transactions where a transfer of a commodity is compensated by a transfer of money. These are indeed the most common transactions in a market economy.

The surplus from a set of transactions $T$ is

$$S^T(p, I) = I - e\left(p, z + p\sum_{t\in T} t, x - \sum_{t\in T} t\right),$$

where the baseline is computed by cancelling the transactions from set $T$ and cancelling the monetary payments associated with them. The surplus over all transactions is the same as the surplus over all commodities. But additivity of surpluses from transactions is even harder to obtain than for surpluses from commodities. Indeed, even in the case of quasi-linear additive utility, due to the non-linearity of the $v_k$ functions in the expression $U(x, z) = \sum_{k=1}^{\ell} v_k(x_k) + z$, generally, for $T \cap T' = \emptyset$,

$$S^{T\cup T'}(p, I) \neq S^T(p, I) + S^{T'}(p, I).$$

Let $t_{ij}$ denote the total transaction between individual $i$ and firm $j$. In order to compute the whole benefit for $i$ of interacting with $j$, one must also include the dividend received. Thus, the surplus for $i$ from this transaction and the dividend received is:

$$e_i(p, z_i, x_i) - e_i(p, z_i + pt_{ij} - s_{ij}\pi_j, x_i - t_{ij}).$$

It is more intuitive to represent the operations in the first term than in negative fashion in the second term, so it is convenient to rewrite this expression as

$$e_i(p, \bar{z}_i - pt_{ij} + s_{ij}\pi_j, \bar{x}_i + t_{ij}) - e_i(p, \bar{z}_i, \bar{x}_i),$$

where the baseline bundle is defined by $\bar{z}_i = z_i + pt_{ij} - s_{ij}\pi_j, \bar{x}_i = x_i - t_{ij}$.

These notions of surplus are not a mere extension of the original notion but a real shift, because the original notion takes as the baseline the scenario in which the individual is barred from a market but free to adjust the choice of commodities as a consequence. Here in contrast, the baseline is the scenario in which the transaction vanishes and no adjustment is made to $x$, while the individual is simply reimbursed for the cancelled expense. This makes a big difference when preferences are not quasi-linear. Consider the example of a consumer buying cane sugar rather than white sugar because of a small preference for the former. If access to cane sugar was denied, the original notion of surplus would let the consumer substitute white sugar (a different commodity), implying that the surplus from access to cane sugar can appear very small even if the surplus from access to sugar in general is large. With the new notion, removing the expenditure of cane sugar directly reveals the large surplus from sugar because substitution with white sugar is not allowed. This issue, however, does not arise with additively separable preferences, especially with quasi-linear preferences $U(x,z) = \sum_{k=1}^{\ell} v_k(x_k) + z$, for which the optimal quantity of any commodity does not depend on the quantities of other commodities.

As firms also trade among themselves, it is important to define the surplus another firm gets from its interactions with firm $j$. Here is one natural approach, in line with the definitions for households. Let the production plan of a firm be decomposed into outputs $q$ and inputs $x$, so that one can write it as $y = (q,x)$ with $q \geq 0, x \leq 0$. When a firm buys inputs $t < 0$, its surplus from the transaction can be computed as the difference in profit between the current profit and what it could make without the inputs $t$:

$$py - \max_{q':(q',x-t)\in\mathcal{Y}} p(q', x-t),$$

which assumes that readjusting outputs is possible, but not inputs. Similarly, when it sells outputs $t > 0$, its surplus is the difference in profit from what it could make without these sales:

$$py - \max_{x':(q-t,x')\in\mathcal{Y}} p(q-t, x').$$

This convention of assuming that the firm can adjust one side of its production (inputs or outputs) and not the other side may not be the only possible one, and one could make different assumptions about what the firm is allowed to adjust. What appears necessary is that the counterfactual plan performed after removal of the transaction with firm $j$ should remain feasible according to the technology constraint $\mathcal{Y}$.

# 7 The total contribution of a firm to the economy

Equipped with the definitions provided in the previous section, one can compute the total contribution of a firm to households and other firms as:

$$\begin{aligned} TC_j = &\sum_{i=1}^{N} \left[e_i(p, z_i, x_i) - e_i(p, z_i + pt_{ij} - s_{ij}\pi_j, x_i - t_{ij})\right] \\ &+ \sum_{l\neq j} \left[py_l - \max_{q':(q',x_l-t_{lj})\in Y_l} p(q', x_l - t_{lj})\right] \\ &+ \sum_{l\neq j} \left[py_l - \max_{x':(q_l-t_{ljl},x')\in Y_l} p(q_l - t_{lj}, x')\right] \end{aligned}$$

where the $l$ index runs over all other firms, and $\pi_j = py_j = p\left(\sum_{i=1}^{N} t_{ij} - \sum_{l\neq j} t_{lj}\right)$. In dual fashion, this can also be written in terms of positive contribution to a baseline as follows:

$$\begin{aligned} TC_j = &\sum_{i} \left[e_i(p, \bar{z}_i - pt_{ij} + s_{ij}\pi_j, \bar{x}_i + t_{ij}) - e_i(p, \bar{z}_i, \bar{x}_i)\right] \\ &+ \sum_{l} \left[\max_{q':(q',\bar{x}_l+t_{lj})\in Y_l} p(q', \bar{x}_l + t_{lj}) - p(\bar{q}_l, \bar{x}_l)\right] \end{aligned}$$

$$+\sum_l \left[ \max_{x':(\bar{q}_l+t_{ljl},x')\in Y_l} p\left(\bar{q}_l+t_{lj},x'\right) - p\left(\bar{q}_l,\bar{x}_l\right)\right].$$

This expression (in its two equivalent formulations) relies on current prices as references, but one could also adopt different reference prices in the computation of the household surplus, especially with the goal of taking account of unequal skills.

Let us now examine how a change in a firm's strategy impacts the value of its total contribution. Consider a change $dt_{ij}$ first. One computes, for the case in which $s_{ij}=0$:

$$\frac{\partial}{\partial t_{ijk}} e_i\left(p,\bar{z}_i - pt_{ij},\bar{x}_i + t_{ij}\right) = -\frac{\partial e_i}{\partial z_i}p_k + \frac{\partial e_i}{\partial x_{ik}} = 0$$

because $\frac{\partial e_i}{\partial z_i}=1$ and $\frac{\partial e_i}{\partial x_{ik}}=p_k$, which means that $dt_{ij}$ has zero direct impact on $i$—this comes from the envelope theorem. Likewise, the impact of $dt_{lj}$ on firm $l$'s profit is equal to zero by the envelope theorem. Therefore, the only impact of $dt_{ij},dt_{lj}$ comes through the change in $\pi_j = py_j$, i.e. through $pdt_{ij},pdt_{lj}$ because

$$\sum_i \frac{\partial e_i}{\partial z_i} s_{ij} d\pi_j = d\pi_j.$$

Now consider a change in prices controlled by firm $j$, say $dp_k$ (while keeping the same reference price in equivalent incomes). On households' side, this has the following impact:

$$\sum_i \frac{\partial e_i}{\partial z_i}\left(-t_{ijk} + s_{ij}y_{jk}\right)dp_k,$$

whereas on the firms' side, the impact is equal to

$$\sum_l t_{ljk}dp_k.$$

Summing up, one obtains a total impact equal to

$$\left(y_{jk} - \sum_i t_{ijk} + \sum_l t_{ljk}\right)dp_k = 0,$$

once again in line with the results of section 4. Finally, we have obtained the following proposition.

**Proposition** A change in firm $j$'s trades and prices $\left(\left(dt_{ij}\right)_{i=1,\ldots,N},\left(dt_{lj}\right)_{l\neq j},dp\right)$ induces

$$dTC_j = pdy_j,$$

i.e., is measured by the change in its profit at current prices.

This analysis has provided us with a definition of the level of the contribution of the firm, $TC_j$, as well as a clear way to measure the variation of $TC_j$ under small changes in the firm's strategy. While measuring the level of $TC_j$ may be difficult because it involves subjective information about households and technology information about other firms, the assessment of $dTC_j$ is much easier because it only involves information at the disposal of firm $j$. The consistency between this formula and the formula obtained for the impact on social welfare in Proposition (5) is quite important, as it means that the firm pursuing the maximization of $TC_j$ with thereby optimally contribute to social welfare measured by the sum of equivalent incomes.

The discussion of the case in which the reference price vector in equivalent incomes $p_0$ differs from current prices (and especially wages), the discussion of section 4 applies with little modification. The level of $TC_j$ now involves the following terms for households:

$$e_i\left(p_0,\bar{z}_i-pt_{ij}+s_{ij}\pi_j,\bar{x}_i+t_{ij}\right)-e_i\left(p_0,\bar{z}_i,\bar{x}_i\right),$$

and one still has the envelope result

$$\frac{\partial}{\partial t_{ijk}}e_i\left(p_0,\bar{z}_i-pt_{ij},\bar{x}_i+t_{ij}\right)=\frac{\frac{\partial e_i}{\partial U_i}\left(p_0,U_i\left(p,I_i\right)\right)}{\frac{\partial e_i}{\partial U_i}\left(p,U_i\left(p,I_i\right)\right)}\left(-p_k+p_k\right)=0,$$

whereas

$$\sum_i\frac{\partial e_i}{\partial z_i}s_{ij}d\pi_j=\sum_i\frac{\frac{\partial e_i}{\partial U_i}\left(p_0,U_i\left(p,I_i\right)\right)}{\frac{\partial e_i}{\partial U_i}\left(p,U_i\left(p,I_i\right)\right)}s_{ij}d\pi_j\simeq d\pi_j$$

provided that $\frac{\frac{\partial e_i}{\partial U_i}(p_0,U_i(p,I_i))}{\frac{\partial e_i}{\partial U_i}(p,U_i(p,I_i))}\simeq 1$ for all $i$.

The introduction of weights into the measurement of social welfare can be incorporated in the measurement of $TC_j$, taking account not only of the social weight of households trading with $j$, but also of the social weight of the shareholders of firms that trade with $j$. This yields the following expression:

$$\begin{aligned}TC_j=&\sum_i\alpha_i\left[e_i\left(p,\bar{z}_i-pt_{ij}+s_{ij}\pi_j,\bar{x}_i+t_{ij}\right)-e_i\left(p,\bar{z}_i,\bar{x}_i\right)\right]\\&+\sum_l\alpha_l\left[\max_{q':(q',\bar{x}_l+t_{lj})\in Y_l}p\left(q',\bar{x}_l+t_{lj}\right)-p\left(\bar{q}_l,\bar{x}_l\right)\right]\\&+\sum_l\sum_i\alpha_l\left[\max_{x':(\bar{q}_l+t_{ljl},x')\in Y_l}p\left(\bar{q}_l+t_{lj},x'\right)-p\left(\bar{q}_l,\bar{x}_l\right)\right],\end{aligned}$$

where $\alpha_l=\sum_i\alpha_i s_{il}$ can be interpreted as the average social weight of firm $l$'s shareholders. In the assessment of $dTC_j$, the envelope theorem arguments remain unaltered, and the overall impact of a change $\left(\left(dt_{ij}\right)_{i=1,\ldots,N},\left(dt_{lj}\right)_{l\neq j},dp\right)$ depends on the distributional impact of dividends and price impacts as follows:

$$dTC_j=\alpha_j pdy_j+\left(\sum_i\left(\alpha_j-\alpha_i\right)t_{ij}-\sum_l\left(\alpha_j-\alpha_l\right)t_{lj}\right)dp,$$

where the second term, which transparently displays the differences in weights between firm $j$'s shareholders and the other stakeholders affected by its prices, is obtained through the following steps, for each good $k$:

$$\begin{aligned}\sum_i\alpha_i\frac{\partial e_i}{\partial z_i}\left(-t_{ijk}+s_{ij}y_{jk}\right)+\sum_l\alpha_l t_{ljk}&=\alpha_j y_{jk}-\sum_i\alpha_i t_{ijk}+\sum_l\alpha_l t_{ljk}\\&=\alpha_j\left(\sum_i t_{ijk}-\sum_l t_{ljk}\right)-\sum_i\alpha_i t_{ijk}+\sum_l\alpha_l t_{ljk}\\&=\sum_i\left(\alpha_j-\alpha_i\right)t_{ijk}-\sum_l\left(\alpha_j-\alpha_l\right)t_{ljk}.\end{aligned}$$

One of the questions raised in the introduction is whether the measure of a firm's contribution can be aggregated over firms in a way that is consistent with a notion of social welfare. In other words, can one take $\sum_j TC_j$ as the measure of the contribution of the productive sector to social welfare? This would require among other things that the sum

$$\sum_i\sum_j\left[e_i\left(p,\bar{z}_i-pt_{ij}+s_{ij}\pi_j,\bar{x}_i+t_{ij}\right)-e_i\left(p,\bar{z}_i,\bar{x}_i\right)\right]$$

be meaningful. Unfortunately, this is similar to the problem encountered with summing up surpluses obtained from different transactions. This will generally not provide a sensible measure of social welfare. An additional hurdle for summing up $TC_j$ is that it includes part of the profit of the firms which trade with $j$, but also the full profit of $j$ distributed to shareholders, so that adding up this over all firms would double count profits.

One very special case in which the aggregation would work is when utilities are quasi-linear and additive, households trade every commodity with a single firm (i.e., bread from a particular baker, meat from a particular butcher, and so on), and when firms do not trade among themselves. In this case, $\sum_j TC_j$ would equal total household surplus from commodity trade augmented by total profit.

All in all, it is imperative to consider $TC_j$ as the contribution that the firm makes to social welfare from a *special baseline* that includes everything else except the firm's presence. This is a meaningful notion, and it is very convenient that the firm can maximize it and thereby contribute to a sensible notion of social welfare. But this cannot be added up over all firms in general in a way that would be consistent with a sensible notion of social welfare.

# 8 Extension to non-market dimensions and externalities

In this section, we examine how to incorporate additional dimensions of well-being into the concepts of surplus and equivalent income. By definition, the surplus is the equivalent monetary gain from access to a particular market or a particular set of markets. It is therefore designed for the market sphere. But one can extend it to compute the equivalent monetary gain from enjoying some non-market aspects of life, such as public goods. As the surplus is defined with equivalent income, the extension is linked to the extension of equivalent income to non-market dimensions.

Let $b = (b_t)_{t=1,\dots,T}$ denote the vector of non-market quality of life attributes. One can first extend all basic notions as follows:

- Direct utility: $U(x,b)$
- Marshallian demand: $x(p,I,b) = \arg\max\{U(x,b)\,|px \leq I\}$.
- Indirect utility: $U(p,I,b) = U(x(p,I,b),b) = \max\{U(x,b)\,|px \leq I\}$.
- Expenditure function: $e(p,b,u) = \min\{px|U(x,b) \geq u\}$.
- Hicksian demand: $h(p,b,u) = \arg\min\{px|U(x,b) \geq u\}$.
- Direct equivalent income: $e(p_0,b_0,x,b) = e(p_0,b_0,U(x,b))$.
- (Indirect) equivalent income: $e(p_0,b_0,p,I,b) = e(p_0,b_0,U(p,I,b))$.

Note how a reference $q_0$ appears in the definition of equivalent income: it measures how much income would be needed to enjoy the current level of satisfaction if the prices *and non-market life* were at the reference level. It is important to introduce this reference $q_0$ in order to make equivalent income comparable across situations with different non-market attributes, just as one uses a reference $p_0$ in order to make it comparable across situations with different prices.

In order to define the surplus, one can proceed with the following definitions:

- Marshallian demand when $K$ is off limits: $x^{\overline{K}}(p,I,b) = \arg\max\{U(x,b)\,|px \leq I, x_K = 0\}$.
- Consumer surplus from $K$: $S^K(p,I,b) = I - e\left(p,b,x^{\overline{K}}(p,I,b),b\right)$.

Then, one can compute the surplus from access to commodities from the $K$ subset and from enjoying $b$ rather than some benchmark $b^*$:

- Consumer surplus from $K, q$: $S^{K,q}(p, I, b) = I - e\left(p, q, x^{\overline{K}}(p, I, b^*), b^*\right)$.

Importantly, the benchmark $b^*$ can be a desirable target that is better than the current $q$, so that one could talk about consumer deficit rather than surplus, with respect to $b$.

Let us focus on the equivalent income measure, which is better suited to comparisons across situations with varying prices and quality of life. One critical issue is the choice of the benchmark $b_0$. The implications of this choice are rather clear. If a modest level of $b_0$ (worse than $b$) is adopted, those who care a lot about non-market quality of life are deemed better off, other things equal, whereas the opposite occurs if an ambitious level is taken (better than $b$). It appears that the most intuitive choice involves taking for $b_0$ a level that is considered "normal." Even if normality may be culturally contingent, it provides natural comparisons. For instance, if good health is considered normal, and is taken as the reference, then people who have mediocre health are deemed worse off when they care more about health, other things equal, which is quite appealing. If a pristine natural environment is considered normal, and is taken as the reference, then people living in a degraded natural environment are deemed worse off when they care more about the environment, other things equal.

The fact that different individuals may have different preferences over non-market life creates an additional hurdle. One way to deal with it is to actually individualize $q_0$, and take the best value that the individual would like to have in the contemplated situation, as suggested in Fleurbaey and Blanchet (2013). This is done with the following definition of equivalent income:

- Direct equivalent income: $e^*(b_0, x, b) = \min_{b_0} e(p_0, b_0, U(x, b))$.
- (Indirect) equivalent income: $e^*(p_0, p, I, b) = \min_{b_0} e(p_0, b_0, U(p, I, b))$.

This approach takes the reference to be the best for the individual, rather than the normal level according to cultural standards. Reviewing a list of potential examples such as health or the environment suggests that the normal and the best are often aligned. But consider the case of population density. Some like crowded surroundings, other like quiet places, and there is no cultural "normal" that covers rural and urban areas jointly. It makes sense to consider that those who do not live in their preferred rural or urban setting are worse off when they care more about it, other things equal. This appears much more sensible than taking, say, the urban environment as the reference and judging those who care a lot of population density to be better off when they are lucky to enjoy their preferred rural place than when they are lucky to enjoy their preferred urban place.

Let us consider the surplus of households being impacted by firm $j$'s plan $y_j$ through quality of life, written as a function of $y_j$ among other things:

$$\sum_i \left[e_i\left(p, b, \bar{z}_i - pt_{ij} + s_{ij}\pi_j, \bar{x}_i + t_{ij}, b_i(y_j)\right) - e_i\left(p, q, \bar{z}_i, \bar{x}_i, \bar{b}\right)\right].$$

We will ignore inter-firm externalities for simplicity. It is then easy to derive the following extension of proposition (7), in which a Pigouvian correction to prices appears:[5]

**Proposition** A change in firm $j$'s trades and prices $\left((dt_{ij})_{i=1,\dots,N}, (dt_{lj})_{l\neq j}, dp\right)$ induces

$$dTC_j = pdy_j + \sum_{i,t,k} \frac{\partial e_i}{\partial b_{it}} \frac{\partial b_{it}}{\partial y_{jk}} dy_{jk} = (p - \tau_j)\, dy_j,$$

where $\tau_{jk} = -\sum_{i,t} \frac{\partial e_i}{\partial b_{it}} \frac{\partial b_{it}}{\partial y_{jk}}$.

[5]This correction is consistent with general equilibrium results obtained by Fleurbaey and Ponthière (2021) for efficient allocations.

When equivalent income is defined taking $p, b$ as the reference, then $\tau_{jk} = -\sum_{i,t} \frac{\partial e_i}{\partial b_{it}} \frac{\partial b_{it}}{\partial y_{jk}}$ corresponds to the total willingness to pay to avoid $y_{jk}$. Indeed, one has

$$\frac{\partial U / \partial b_t}{\partial U / \partial I} = \frac{\partial U}{\partial b_t} \frac{\partial e}{\partial U} = \frac{\partial e}{\partial b_t} \left(p, b, p, I, (b_t)_{t=1,\ldots,T}\right),$$

because $\frac{\partial e}{\partial U} \frac{\partial U}{\partial I} = 1$ for $e(p, b, U(p, I, b)) \equiv I$.

For other references, the interpretation is less simple but the formula in the proposition remains valid. For instance, for the equivalent income $e^*$, one obtains that willingness to pay is corrected by a factor similar to what was obtained in section 4:

$$\frac{\partial e^*}{\partial b_t} = \frac{\partial e}{\partial U} \left(p_0, b_0^*, U(x, b)\right) \frac{\partial U}{\partial b_t} = \frac{\frac{\partial e}{\partial U} \left(p_0, b_0^*, U(x, b)\right)}{\frac{\partial e}{\partial U} \left(p, b, U(x, b)\right)} \frac{\frac{\partial U}{\partial b_t}}{\frac{\partial U}{\partial I}},$$

where $b_0^* = \arg\min_{b_0} e\left(p_0, b_0, U(x, b)\right)$.

In conclusion of this section, it appears that incorporating non-market quality of life and externalities is possible and gives a fuller picture of the contribution of firms to social welfare. Moreover, the variation $dTC_j$ is once again relatively easy to conceptualize and measure, and involves a Pigouvian correction to market prices that is very much in line with the polluter pays principle.

# 9 Conclusions

Here are the key insights that come out of this analysis.

1. The surplus suffers from the fact that it is useful only "in the small," but not "in the large." More specifically, it is a meaningful notion for the measure of benefit from a particular transaction or from access to a particular commodity. But it is not additive, so that total surplus is not equal to the sum of surpluses from particular transactions or particular commodities. And total surplus, for individuals, takes as the baseline a situation of autarky that is very artificial and generally not viable.

2. In a general equilibrium setting, surplus is defined on the basis of equivalent income. And equivalent income provides a sufficient measure of comprehensive economic advantage that avoids the reference to autarky. It can be a useful ingredient in a social welfare function.

3. The surplus is however useful to measure the contribution of a particular firm to the economy, and the total surplus generated by a firm can serve as a good concept of the objective it should pursue (i.e., stakeholder value rather than shareholder value). One can see the surplus generated by a firm as its marginal contribution to society. A firm pursuing this objective will optimally contribute to social welfare. It is a sensible measure even if the surpluses generated by all the firms do not add up to a meaningful quantity.

4. While the total surplus generated by a particular firm is hard to empirically measure, the variations induced by the firm's changes in strategies can be assessed with information that is more readily available to the firm (except for the Pigouvian correction for externalities which requires an estimation of societal willingness to pay).

Coming back to the "Holy Grail" question, it appears that the notions of surplus and equivalent income serve different purposes. The surplus *can* serve as a reasonable objective for productive units (esp. to represent stakeholder value); but it *cannot* meaningfully be aggregated to a macroeconomic level and thus it cannot serve as an ingredient to a reasonable objective for social welfare. In contrast, the equivalent income, which appears in the computation of the surplus in a general equilibrium setting, can provide the relevant part of the surplus objective of firms in the stakeholder approach, and can be aggregated to a macroeconomic level. While empirical computations of social indicators based on equivalent incomes can be pursued (as in Jones and Klenow 2016, Boarini et al. 2022), there is little hope of constructing an additive form of national accounts of equivalent incomes in which the contribution of firms would add up to a total pie, in similar fashion as the accounting of value added for the computation of GDP. This impossibility appears a minor problem, since the key factor in reforming reporting and practices is to develop sensible indicators at the macro and at the micro level, and to make sure that microeconomic agents pursuing the micro indicators will optimally contribute to the macro indicator. That such macro and micro indicators are mathematically related in a more complex way than a simple sum should not matter much. It has been standard in social welfare analysis to consider that social welfare is not a simple sum of individual well-being, under inequality aversion.

Talking about inequality aversion, it should be noted that a direct sum of surpluses or equivalent incomes ignores inequalities and it might be better, at least as the societal level, to rely on a weighted sum where the worse off individuals receive a suitable degree of priority. It may be harder for private firms to rely on similar weights but there is no logical inconsistency in their adopting strategies that deviate from total surplus maximization and take special care, including via their market power, of the worse off in society. However, their action in favor of a better distribution would generally be less effective than public fiscal tools, unless they have specific information about beneficiaries that government agencies would lack.

Externalities affecting non-market quality of life can have their effects registered in the level of equivalent incomes adjusted for non-market dimensions of life, as defined in section 7. Adjusting responsible surplus-maximization for externalities involves a Pigouvian correction to market prices by the firm and estimation of societal willingness to pay for such externalities. While this correction complicates matters significantly for the firm, it relies on concepts from cost-benefit analysis that are rather standard.

To sum up, the key message of this paper is that we appear to be ready to replace the GDP-shareholder-value nexus by a social-welfare-stakeholder-value approach in which equivalent income serves to measure individual advantage in the social welfare function and firms pursue the maximization of the total surplus they generate for all their stakeholders, where this surplus is also measured as the gain in equivalent income (for households) and profit (for firms) that stakeholders reap from their interaction with the firm. If social welfare and total surplus at the firm level are both measured in terms of simple sums, this means that the focus is on efficiency and that the distribution of resources is considered to be socially optimal. If it is not optimal, weights can be introduced at both levels. Note that the introduction of weights allows for a wide range of social welfare approaches, including ones in which equivalent incomes are not considered the best measure for interpersonal comparisons of well-being.

This social-welfare-stakeholder-value approach is therefore very flexible and can accommodate many approaches about interpersonal comparisons, inequalities and externalities. What remains to be done to make it is easily applicable is to develop methods to implement practical measurements at the societal and the firm level. Measures of social welfare have already been performed, as already mentioned. For the firm level, it is not clear that measuring total surplus has more than academic interest, since the firm can assess its management by relying on rules that primarily rely on marginal values and the assessment of changes in its strategies. Fleurbaey and Ponthière (2021) develop a list of management rules that firms can pursue to maximize their total contribution, and such recipes for good practices may be all that firms really need. Similar rules have been proposed in the literature (e.g., Mayer 2018, Heath 2014). One should hope that this field will promptly develop and converge toward a set of management guidelines that can serve harmonizing the goals of the business world with social welfare at large.

**Acknowledgments**: This paper has benefited from interactions with many colleagues in the Economic Measurement Chair, including the workshop “Beyond and Around GDP,” and from the audience’s reactions at the 2022 SSCW Congress.

## References


[1] Boadway R. 2016, "Cost-benefit analysis," in M.D. Adler, M. Fleurbaey eds., *Oxford Handbook of Well-Being and Public Policy*, Oxford University Press.

[2] Boarini R., M. Fleurbaey, F. Murtin, P. Schreyer 2022, "Well-being during the great recession: new evidence from a measure of multi-dimensional living standards with heterogeneous preferences," *Scandinavian Journal of Economics* 124: 104–138.

[3] Bosmans K., K. Decancq, E. Ooghe 2018, "Who's afraid of aggregating money metrics?", *Theoretical Economics* 13: 467–484.

[4] Coyle D. 2015, *GDP: A Brief but Affectionate History*, Princeton University Press.

[5] Fleurbaey M., D. Blanchet 2013, *Beyond GDP. Measuring Welfare and Assessing Sustainability*, New York: Oxford University Press.

[6] Fleurbaey M., G. Gaulier 2009, "International comparisons of living standards by equivalent incomes," *Scandinavian Journal of Economics* 111: 597–624.

[7] Fleurbaey M., F. Maniquet 2011, *A Theory of Fairness and Social Welfare*, Cambridge University Press.

[8] Fleurbaey M., F. Maniquet 2018, "Optimal income taxation theory and principles of fairness," *Journal of Economic Literature* 56: 1029–1079.

[9] Fleurbaey M., G. Ponthière 2021, "The stakeholder corporation and social welfare," available at SSRN: https://ssrn.com/abstract=3900451.

[10] Harrison J.S., J.B. Barney, R.E. Freeman, R.A. Phillips (eds.) 2019, *The Cambridge Handbook of Stakeholder Theory*, Cambridge: Cambridge University Press.

[11] Heath J. 2014, *Morality, Competition, and the Firm. The Market Failures Approach to Business Ethics*, Oxford: Oxford University Press.

[12] Jones C.I., P.J. Klenow 2016, "Beyond GDP: welfare across countries and time," *American Economic Review* 106: 2426–2457.

[13] Kelly C. 2019, "Repurposing our economies – and our businesses," *Global Solutions Journal*, 4: 33–63.

[14] Lazear E.P. 1995, *Personnel Economics*, Cambridge, Mass.: MIT Press.

[15] Magill M., M. Quinzii, J.C. Rochet 2015, "A theory of the stakeholder corporation," *Econometrica* 83: 1685–1725.

[16] Mayer C. 2018, *Prosperity: Better Business Makes the Greater Good*, Oxford: Oxford University Press.

[17] Samuelson P. 1956, "Social Indifference Curves," *Quarterly Journal of Economics* 70: 1–22.

[18] Stiglitz J.E., A.K. Sen, J.P. Fitoussi 2009, *Report by the Commission on the Measurement of Economic Performance and Social Progress.* https://ec.europa.eu/eurostat/documents/8131721/8131772/Stiglitz-Sen-Fitoussi-Commission-report.pdf

[19] Vanoli A. 2002, *Une histoire de la comptabilité nationale*, Paris: La Découverte.