Wage Dynamics within Firms

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Abstract

This paper assesses wage setting and wage dynamics in a search and matching framework in which (i) workers and firms on occasion meet multilaterally and (ii) workers can recall previous encounters with firms. Given that firms cannot commit to future wages and that workers cannot commit to not searching on the job, the resulting progression of wages from firms paying just enough to keep their workers yields a compensation structure consistent with well established but difficult to reconcile observations on pay dynamics within jobs at firms. Along with wage tenure effects, serial correlation in wage changes, cohort effects, wage growth are negatively correlated with initial wages.

1 Introduction

How do firms and employees agree wages? The familiar neoclassical frictionless labour market asserts that the law of one price holds and firms pay workers their per period marginal product which is just enough to keep them from leaving.(Lazear and Oyer, 2006)

Evidence from internal labour markets and from worker-firm matched data reveals, however, that the job alone does not determine compensation. Instead, a rich and dynamic picture of pay distributions emerges. In particular,

- similar workers in the same position are not paid the same wage[1]
- job tenure generally has a positive impact on wages although nominal wage cuts occur with regularity.[2]
- serial correlation occurs in wage changes so that there are predictable winners and losers.[3]
- initial labour market conditions matter such that cohorts who earn more on entry maintain their advantage through time - after controlling for composition differences, the progression of a cohort's wage depends in part on the average starting wage.[4]

These observations, especially the last two, are difficult to obtain in competitive labour models, in models of internal labour markets, and in standard search models.

[1]See Mortensen (2003). Baker, Gibbs and Holmstrom (1994b) find a strong individual component to pay determination. Job levels are important to compensation, but there is also substantial individual variation in pay within levels as well as in its growth rate. There are likewise large overlaps in pay across levels. Wage jumps at promotions are much smaller than differences in mean pay across levels.

[2]Elsby and Solon (2019) review the evidence from worker-firm administrative data across multiple countries and find that between 10% and 25% of job stayers experience a year-on-year wage cut. Baker, Gibbs, and Holmstrom (1994a,b); McLaughlin, (1994); and Card and Hyslop, (1997).

[3]Baker, Gibbs, and Holmstrom (1994b); Lillard and Weiss, (1979); and Hause (1980).

[4]Baker, Gibbs, and Holmstrom (1994a,b) find that after controlling for composition differences, the progression of a cohort's wage depends in part on the average starting wage. See also Kahn, (2006); Oyer, (2006); and Oreopoulos, von Wachter, and Heisz, (2006). Although their findings are somewhat different, Beaudry and DiNardo (1991) also report that cohorts matter.

To account for these findings, this paper uses on-the-job search frictions in a market without worker and firm commitments. The matching frictions associated with on-the-job search partially shield firms "from the direct influences of competitive forces in the external market" (Doeringer and Piore, 1985).[5] Given that firms cannot commit to future wages (see Burdett and Coles for wage tenure contracts) and workers cannot commit to not searching on the job, the resulting progression of wages from firms paying just enough to keep their workers yields a compensation structure consistent with the above findings.

Search here, however, differs from conventional 'black-box' random matching frictions. Although numerous studies have shown that the familiar stationary matching specification can generate equilibrium wage dispersion among similar workers, it does not readily yield a sufficiently rich pay pattern of compensation. This incomplete picture may stem from the underlying specification of search frictions rather than from the general search approach. This paper hence adopts an alternative approach, the stock-flow specification, which offers not only a plausible microfoundation for search frictions but also yields a more empirically valid picture of matching dynamics

Stock-flow matching posits two natural as well as relatively novel features of on-the-job search. In particular, workers and firms

i. on occasion encounter each other multilaterally

ii. can also remember past encounters.[6]

The key determinant of compensation is the expected payoff from search. As in Yamiguchi(2010) and Bagger et al. (2007) the outside option evolves but unlike those papers, firms react to the threat of search rather than a trigger - an actual job offer - for renegotiation. Because stock-flow matching in effect builds in duration dependence, the evolving threat of on-the-job search not its realization determines wages. As a result, turnover is less pronounced.

[5]Waldman (2007) reviews the literature on internal labour markets and considers a variety of explanations for wage dynamics based on imperfect information linked to human capital acquisition, job assignment, learning and tournaments. These explanations offer insights but are partial, incomplete explanations.

[6]The matching framework used here is most closely related to the matching models of Taylor (1995), Coles (1999) and Lagos (2000). Emerging empirical evidence indicates this framework has more validity than random matching. See Coles and Smith (1998), Petrongolo and Pissarides (2001), Andrews, Bradley and Upward, (2001), Gregg and Petrongolo (2005), Coles and Petrongolo, (2008), Kuo and Smith (2009).

As the employment progresses, the search option evolves thereby driving the results. Potential competition drives wages but frictions limit its full scope. In a labour market with stock-flow matching, when a seller, i.e. a worker, goes on the market in search of a partner, he or she immediately becomes fully informed about the number of suitable buyers in the stock, i.e. the stock of job vacancies. If lucky, the worker finds several viable options. If the worker is unlucky, the market turns up few or possibly no viable opportunities and only other workers seeking similar work. In the event that no acceptable vacancies exist in the marketplace, the worker must wait alongside other workers to match from the flow of new jobs.

Consider wage determination in this set-up with on-the-job search.[7] After job search reveals the number of currently available jobs, all suitable firms bid for the worker's services. If only one option is currently available, the firm offers a monopsony payoff that claims all of the gains to trade for the firm. On the other hand, with more than one firm involved, competitive Bertrand bidding occurs. This time, the worker extracts the gains to trade. At the outset of the employment relationship, wage dispersion obtains and depends on the number of competitive bidders found at that time.

Now suppose that at any time after a firm and worker pair up, the firm can update its offer. In other words, as the firm cannot commit to future wages, a new wage is offered in each instant. The worker can either accept the latest offer or go again to the market to elicit bids. The firm updates its wage offer knowing that as time proceeds, firms and workers come and go and the number of prospective bidders in the market evolves randomly. Job opportunities and competition turnover but the worker and the employer do not directly observe this turnover unless the worker actively engages in on-the-job search. The worker must physically visit the market to learn the actual number of bidders.

This process provides a new source of wage progression with tenure at a firm. Employers who want to avoid bidding with the (anticipated) firms in the market can keep the worker away from the market with a sufficiently high wage offer. Such an offer outbids the evolving threat of on-the-job search, not the actual firms.

No-search wages face two countervailing forces from turnover in the market. Previous bidders gradually leave the market and new options enter the

[7]Taylor (1995) and Coles and Muthoo (1998) examine wages in this set-up without on-the-job search.

market. Outside options therefore can rise or fall depending on this birth and death process. Wages not only differ at the outset, they also evolve in different patterns. For monopsony wages, the unfortunate history (from the worker's perspective) fades and the outside option improves. Low initial wages rise over time. For competitively bid wages, the more favorable history that led to high initial wages fades and eventually a less attractive expectation of the number of new firms matters more. Although wages start at different points and evolve in different patterns, they ultimately converge with long tenures.

Job availability and turnover jointly determine wage dispersion and wage dynamics.

Initial wages and their subsequent progression within a firm combine to create a distribution of wages at a point in time. Although it is difficult to formulate and evaluate an explicit expression for the distribution, numerical methods reveal sensible shapes for a range of parameters. In a homogeneous environment, the cross section of wages is dispersed around an interior mode with prominent tails on both sides. Skewness exists but varies left or right with the underlying fundamentals of the model. The model can also generate reasonable mean-min ratios and thus overcoming the lack of frictional wage dispersion found in standard search models by Hornstein, Krusell and Violante (2007).

The next section describes the general framework and the process governing vacancy turnover. Sections 3 and 4 analyze the worker's and firm's problem and solve for optimal wages. Section 5 and 6 derive wage and employment dynamics. Sections 7 and 8 describe the steady state distribution of wages for homogeneous and heterogeneous workers respectively. Section 9 compares our results with standard sequential search models. The last section concludes.

2 Model

Homogeneous workers and homogeneous firms populate an economy with a small, highly specialized labor market. Both agents are risk neutral, discount the future at rate $r > 0$ and maximize expected lifetime payoffs.

The economy operates over an infinite sequence of discrete time periods of length $dt > 0$. At the start of time ($t = 0$) the economy is empty. As time progresses, new workers individually enter looking for employment at the constant, exogenous Poisson rate $\alpha > 0$. For dt small, $\alpha\, dt$ is the approximate probability that a new worker enters in period t. Likewise, new firms each with a single job or vacancy enter in the same manner and at the same rate but independently of workers. Over time the econmy is therefore balanced with equal expected numbers but at any given point in time there may be either more workers or more firms.

At any point in time, a worker is attached to a particular firm's job if the worker produced output for that firm in the pervious period. If the worker did not produce output for a firm in the previous period, the worker is unattached or equivalently unemployed and actively looking for a job. A firm without an attached worker is a vacancy that is also actively looking to recruit a worker. Unemployed workers receive flow payoff $b\, dt$ per period. Vacant jobs incur the flow cost $c\, dt$. When a worker agrees to produce for a firm, the worker generates output $x\, dt > b\, dt$. To keep the exposition and notation uncluttered, workers and jobs live forever.[8]

In the first stage of each time period (the internal labor market stage), an attached firm offers its worker a wage w in the current period. The worker can then either accept or reject this wage. A worker who accepts the offer receives the wage, generates the per period output and hence remains attached moving on to the next period. A worker who rejects the offer pays a search cost $\xi > 0$ to visit the open labor market. The firm also goes to the open market if the worker rejected its initial offer. If workers in any period try their luck in the labour market, firms become aware of this activity so that there is full observability or perfect monitoring of workers by firms. By rejecting the initial offer, an attached worker in effect chooses to enter the job market and check the posted list of vacancies (if any) from time to time. The worker is said to be searching on the job since (as detailed below) the attached firm remains a feasible employment option.

Following the stock-flow matching approach (see Smith, 2020, for an overview) information about the availability of firms and workers in the

[8]Job destruction shocks can be incorporated (death and discounting are related) but some caution would be needed. A familiar approach specifies that workers become unemployed whereas firms leave the market following a job destruction shock. Given equal and exogenous arrival rates, this specification would lead the number of workers growing unboundedly higher than the number of firms. Endogenous firm entry would, of course, remedy this difficulty. We abstract from this situation. Unlike standard matching models (e.g. Pissarides, 2001) we do not endogenize the number of vacancy/firms in the economy. We will, however, still consider below what happens to worker in the event of a job loss

open market stage is centralized. Unemployed entrant workers as well as offer-rejecting workers register their availability at a job centre, on a website, or on some other established platform as soon as they enter the market looking for partners. Vacant entrant jobs and rejected firms similarly post the availability of their employment opportunities. The firm maintains this listing until the job attaches or hires a worker.

Agents in the centralized marketplace are perfectly informed about all available trading opportunities that they have registered there. When any worker enters the marketplace, he or she immediately observes the number of vacancies in the market as well as any other workers in the job centre. After the worker checks the list of posted vacancies, there are no frictions or delays in processing the information. All information regarding the viability of a position is immediately made clear and common knowledge at the job centre.

As such, there are no impediments to trade after entry, either as new born entrants or as formerly attached agents. Given the number workers and jobs in the marketplace, a complete information, competitive auction occurs. Each worker receives a wage offer from each firm, including the all just rejected firms who can update their wage offers. Workers indicate acceptable offers from the bidding firms. Wage offers are for only the current period. Firms cannot commit to future wage payments in their wage offers. Workers likewise cannot commit to withholding future search for other employers. The bids and the acceptance decisions thus correspond to initial wages although all decision making is based on expectations of future behaviour.

LATER: The process of pairing workers and acceptable jobs offers proceeds from the short side. Arbitrarily chosen agents on the short side select one of the willing or acceptable long side potential partners until all short side agents have had a chance to pair up. Unsatisfied (i.e. unmatched) workers and firms remain behind as unemployed workers and vacant firms who wait for further trading opportunities.

When an attached firm makes its (initial internal) offer and when the worker decides to accept or reject the attached firms offer, they are both unaware of the entry of worker and of jobs since they last visited the market place. In particular, since the date they became first attached or since the last time the worker searched on the job, whichever is shortest, the Poisson arrival processes govern their beliefs about what other agents they expect to encounter in the job center. Thus the firm and worker beliefs in the stages before on the job search are based on the workers and firms that were there at the last visit and the duration since its last visit. Since workers enter at Poisson rate α, workers and firms share the belief that the probability of i new entrants over a duration τ since the last auction is given by

$$\pi_i(\tau) = \frac{e^{-\beta\tau}(\beta\tau)^i}{i!} \tag{1}$$

A symmetric belief applies to the number of new jobs that entered.

As visiting the market reveals this information completely and the worker updates its beliefs accordingly. For simplicity, unemployed workers pay no search costs to visit the market.

An entrant who encounters more than one potential partner holds a full information, first price Bertrand auction.

The relevant state of the market can be summarized by an integer, N, denoting the known participants from the auction that last attached the worker with the firm, a duration $\tau \geq 0$ denoting the time since that auction.

3 No On-the-Job Search

Suppose there are $B > 0$ firms bidding for $S > 0$ workers in the open market auction. Since both agents are homogeneous, the outcome is well understood. The opitmal strategy for a worker is a reservation wage strategy - the worker will accept any offer that yields a discounted expected payoff greater than or equal to the continuation value of waiting as unemployed worker in the job market. Firms will likewise have a threshold bid strategy - the payoff to having a worker accept its offer must be less than or equal to the value of being a vacancy. These continuation payoffs depend on the expected number of traders in future periods.

Given these strategies, the short side of the market determines the outcome of the auction. If $B > S$, firms bid up to their threshold bids. Given there are gains to trade, these bids exceed, the reservation wages and workers are willing to accept these bids. All workers are hired leaving $B - S$ vacancies indifferent between waiting and having hired. On the other hand, if $S > B$, foirms offer the worker's reservation wage. The worker accepts leaving $S - B$ unemployed.

More details of the protocol for allocating workers to firms are unnecessary. The result is the standard Bertrand outcome leaving the long side indifferent between being attached or unattached. It is important to note, however, that the payoffs to going forward are contingent on net agent entry, in this case the difference between B and S, not the actual levels.

WE CAN ASSESS THE PAYOFFS GIVEN 1 AGENT ON THE OTHER SIDE. CONSIDER ONLY PURE STRATEGY EQUILIBRIA. IF A WORKER SEARCHES NEXT PERIOD IT GETS $bdt + (1 + rdt)\xi dt + V$. UNEMPLOYED GETS SAME WITHOUT SEARCH COST. ALTERNATIVELY A WORKER WHO DOESN'T SEARCH NEXT PERIOD LEAVES $B - S$.

Suppose for now that there is no on-the-job search or equivalently that search costs ξ are prohibitively high. Coles and Muthoo (1998) demonstrate that in this framework without impediments to trade there is a unique Markov equilibrium in which exchange occurs immediately. Given that immediate trade takes place,

Let

$$N_t = Stock\ of\ workers\ who\ entered - Stock\ of\ vacancies\ that\ entered \tag{2}$$

denote the history at date t of net agent entry. Since attached workers and firms either both go do are both absent from the central market place, N_t represents <u>either</u>

- the number of traders in competition on the same side of the market

<u>or</u>

- the number of potential partners on the other side.

If $N_t \in \mathbb{N}^+ = \{1, 2, 3, ...\}$, there is at least one unemployed worker on the long side of the market waiting for a firm to enter the market with a vacancy.

Bids and accepted offers depend on this state of the market. Given immediate trade occurs, three relevant cases arise in this setting for a firm with an open vacancy that is about to enter the job center:

- No viable workers are waiting in which case the vacancy remains open after entry.
- The market has exactly one worker available so that entry of this firm triggers immediate employment. Let

$$\bar{V} + b/r = [s(x - b + c) + b]/r$$

and

$$\bar{\Pi} - c/r = [(1-s)(x-b+c) - c]/r$$

represent the exogenous shares where $s \in [0,1]$ in this scenario.[9]

- Two or more workers are available in which case the workers competitively bid against each other for the new job opening which also results in immediate employment.

Similar cases apply for when a worker enters.

Once a worker and firm join together in employment, the joint payoff to this infinitely-lived match is x/r. The bidding process determines the allocation of this surplus as offers of employment share the value of the match. To find the respective shares in each state, consider first the case in which two or more workers are waiting to bid against each other when a firm arrives. Note that when a firm enters the market at date t, the state changes so that $N_t = N_{t-dt} - 1$. If at date $t - dt$ there are at least two available workers waiting, then at date t when bidding takes place, the entering vacancy has $N_{t-dt} = N_t + 1$ bidders. Thus, for this case, accounting for the change in state at firm entry, we have $N_t \in \mathbb{N}^+ = \{1, 2, 3, ...\}$.

Bertrand bidding implies that the firm captures the entire gains to trade when there are two or more workers available. Agents on the long side of the market - in this case workers - compete with each other and bid up to their own reservation value at which point they are indifferent between working and waiting. As a trading surplus exists, these bids exceed the other side's - i.e. the firm's - reservation payoff. The short side trader selects the highest bid, or more generally selects randomly among the set of identical, highest bids from the indifferent long side traders.

To derive the reservation payoffs, let $V(N_t)$ denote the expected payoff for a worker waiting on the long side of the market who has $N_t - 1$ other workers competing for employment. Since there are no jobs currently available as this worker waits for jobs to appear, standard dynamic techniques imply

$$V(N_t) = \frac{1}{1+rdt}\left[b\,dt + \alpha dt\, V(N_t+1) + \alpha dt\, V(N_t-1) + (1-2\alpha)\, V(N_t)\right] \quad N_t \in N^+$$

[9] It is possible to endogenize these shares using a number of approaches. For example alternating offer bargaining as in Coles and Muthoo (1998) splits net match surplus. Alternatively without competition in the auction, the firm could act monopsonistically for this particular worker and make a take-it-or-leave-it offer to the worker. In this case the firm extracts the entire gains to trade thereby making the worker indifferent between employment and unemployment.

The worker receives net flow payments b while waiting. During a short interval of duration dt, a competing worker arrives with probability $\alpha\,dt$ and increases the number of available workers by one. With the same probability a firm arrives during this interval and the workers bid for this job. Bertrand competition raises offers until the worker is indifferent between employment and waiting with one less competitor. In the case there are no other bidders when a firm arrives and the worker gets $V(0) = \bar{V} + b/r$.

The solution to the difference equation is given by

$$V(N_t) = \lambda^{N_t}\bar{V} + \frac{b}{r} \qquad N_t \in \mathbb{N}^+$$

where

$$\lambda = \frac{r + 2\alpha - (r^2 + 4r\alpha)^{1/2}}{2\alpha}$$

As noted, when bidding takes place (at t in state N_t), workers will offer up to the point where they are indifferent between working and waiting, implying that the payoff at the start of employment given initial state N_t equals the payoff to waiting. Let $E(N_t, 0)$ denote the expected payoff to employment at the start of the relationship (the duration of employment is zero). It follows that

$$E(N_t, 0) = V(N_t)$$

and splitting of the match surplus implies that the firms share is given by

$$\Pi(N_t) = \frac{x}{r} - V(N_t) = \frac{x}{r} - \lambda^{N_t}\bar{V} - \frac{b}{r} \qquad N_t \in \mathbb{N}^+$$

To find the respective shares with two or more firms bidding, now consider the case in which firms with a vacancy are waiting on the long side of the market for a worker to enter, that is $N_t \in \mathbb{N}^- = \{-1, -2, -3, ...\}$. Applying the same logic, the payoff for these firms while they wait is given by

$$\Pi(N_t) = \frac{1}{1 + rdt}\left[-c\,dt + \alpha dt\,\Pi(N_t + 1) + \alpha dt\,\Pi(N_t - 1) + (1 - 2\alpha)\,\Pi(N_t)\right]$$

The solution to the difference equations is given by

$$\Pi(N_t) = \lambda^{-N_t}\bar{\Pi} - \frac{c}{r} \qquad N_t \in \mathbb{N}^-$$

As the match produces x/r, the splitting of the match payoff implies that

$$V(N_t) = E(N_t, 0) = \frac{x}{r} - \Pi(N_t) = \frac{x}{r} - \lambda^{-N_t}\bar{\Pi} + \frac{c}{r} \qquad N_t \in \mathbb{N}^-$$

4 On the Job Search

Without on-the-job search, only match shares from a permanent match are known. A wide variety of compensation schemes can deliver these shares so wages are indeterminate at this stage. Allowing (less costly) on-the-job search during the match helps to pin down wages.

Suppose that during employment, a worker can decide to visit the job centre in search of a better offer. If on-the-job search occurs, the outcome is common knowledge - both the worker and the firm become informed about the number of available employment opportunities for the worker in the job centre.[10] Given the number of viable opportunities found in the job centre, a new auction results and the new or re-negotiated wage depends on the number of vacancies in the job centre at that moment of on-the-job search.

Consider an employed worker weighing up the option of on-the-job search who

- was hired at the job center when the state of the market at that time was N_t
- believes that all other employed workers do not search.
- last visited the job centre a duration $\tau \geq 0$ ago

Normalizing the hiring date so that $t = 0$, the expected payoff to job search while employed is

$$W(N_0, \tau) = -\xi + \sum_{k=-\infty}^{\infty} E(k, 0) f(k; N_0, \tau)$$

where $f(k; N_0, \tau)$ is the probability of state k given a duration τ since initial state N_0. The (Poisson) number of workers less the (Poisson) number of

[10] Common knowledge rules out the possibility that a worker visits the job centre and calls for an auction only if conditions are favorable. As demonstrated below the firm can infer worker behavior from its wage offer.

vacancies follows a Skellam distribution for N_t (Irwin, 1937, Skellam 1946)[11]

$$f(k;0,t) = \Pr(N_t = k, k > 0) = e^{-2\alpha t}\sum_{j=N}^{\infty}(\alpha t)^{2j-N}\frac{j!}{(j-N)!}$$

The laws of motion for f

$$f(N_\tau;N_0,\tau+d\tau) = (1-2\alpha d\tau)\, f(N_\tau;N_0,\tau)+\alpha d\tau\, f(N_\tau+1;N_0,\tau)-\alpha d\tau\, f(N_\tau-1;N_0,\tau)$$

imply

$$\underline{\dot{f}}(N_\tau;N_0,\tau+d\tau) = \alpha f(N_\tau+1;N_0,\tau) - 2\alpha f(N_\tau;N_0,\tau) + \alpha f(N_\tau-1;N_0,\tau)$$

It follows that the payoff to search while employed is given by

$$W(N_0,\tau) = -\xi+\sum_{j=1}^{\infty}\lambda^k f(j;N_0,\tau)\bar{V}-\sum_{i=0}^{\infty}\lambda^i f(-i;N_0,\tau)\bar{\Pi}+F(0)(x-b+c)/r+b/r$$

which evolves according to

$$\dot{W}(N_0,\tau) = \frac{\alpha(1-\lambda)^2}{\lambda}\sum_{j=1}^{\infty}\lambda^j[\bar{V}f(j;N_0,\tau)-\bar{\Pi}f(-j;N_0,\tau)]+\alpha(1-\lambda)f(0)[\bar{V}-\bar{\Pi}]$$

5 Firms

The worker's payoff to not going to the market at any point in time depends on the wage offer at the time. Suppose a firm offers the instantaneous wage $\hat{w}(N_t,\tau)\,d\tau$ (for the current interval of duration $d\tau$) to its existing worker who last visited the job centre τ periods ago at which time it was in state N_t. Let $E(N_t,\tau)$ denote the expected payoff of employment at the point in time $t+\tau$. If the worker accepts the current wage and decides not to search at this point in time (again normalizing $t=0$), it follows that the worker's expect payoff is

$$E(N_0,\tau) = \hat{w}(N_0,\tau)d\tau+\frac{1}{1+rd\tau}\max\{E(N_0,\tau+d\tau),W(N_0,\tau+d\tau)\}+O(d\tau^2).$$

[11] This probability can also be expressed for any k using a modified Bessel function of the first kind,

$$f(k;0,t) = \Pr(N_t = k) = e^{-2\alpha t}I_k(2\alpha t)$$

where $I_k(z) = I_{|k|}(z)$.

Manipulating and letting $d\tau$ become small gives

$$rE(N_0, \tau) = \hat{w}(N_0, \tau) + \max\{\dot{E}(N_0, \tau), \dot{W}(N_0, \tau). \quad (3)$$

Likewise, if the worker accepts the wage offer $\hat{w}(N_0, \tau)$, the firm's payoff is given by

$$\Pi(N_0, \tau) = [x - \hat{w}(N_0, \tau)]d\tau + \frac{1}{1 + rd\tau}\Pi(N_0, \tau + d\tau, k) + O(d\tau^2),$$

A firm can clearly offer a sufficiently high wage such that $E(N_0, \tau) \geq W(N_0, \tau)$. In this case, because search is common knowledge, the firm effectively bribes the worker to not to visit the job centre at duration τ. Moreover, if the firm chooses to make such a no-search offer, the firm would optimally offer the lowest possible wage that satisfies this criteria so that the $E = W$ constraint binds. Given (N_0, τ), a worker's no-search wage implies

$$E(N_0, \tau) = W(N_0, \tau).$$

Let $w(N_0, \tau)$ denote the lowest wage that makes the worker willing to not visit the job centre.

Restricting the worker to not search, the firm chooses the wage $w(N_0, \tau)$ to maximize Π subject to an incentive compatibility constraint that induces the worker to not search on the job $E(N_0, \tau) \geq W(N_0, \tau)$

$$\begin{aligned} \Pi(N_0, \tau) &= \min_w [x - w]d\tau + \frac{1}{1 + rd\tau}\Pi(N_0, \tau + d\tau, k) \\ s.t.\ E(N_0, \tau) &= W(N_0, \tau) \end{aligned}$$

This formulation embeds the restriction that the firm is not able to commit to future wages. The constraint is built into the Bellman formulation.

$$r\Pi(N_0, \tau) = x - w(N_0, \tau) + \dot{\Pi}(N_0, \tau)$$

By construction, the payoff to the firm of paying the no search wage is

$$\frac{x}{r} - E(N_0, \tau) = \frac{x}{r} - W(N_0, \tau).$$

In contrast, any wage offer below the no-search threshold $w(N_0, \tau)$ triggers a visit to the job center where all information is revealed. The payoff to a firm inducing the worker to search is determined after the worker pays the search cost. Hence this search payoff to the firm equals

$$\frac{x}{r} - W(N_0, \tau) - \xi$$

It is more profitable for the firm to avoid the outcome of worker on-the-job search. The argument applies for any τ hence the jointly optimal outcome is a relationship that avoids incurring search costs.

Given there is a positive cost of visiting the job centre, it is efficient for the worker and the firm to save the search cost and split the match benefits within the current employment match at any given τ. In this economy, on-the-job search is a wasteful, rent seeking activity. Once a match is formed, search does not generate any further gains to trade or match specific rents. The participation constraints of both agents bind at the same wage. The market does not fundamentally change when the worker visits the job centre. There are no new opportunities generated by a visit - existing opportunities are merely realized. Search does not change the expected gains to trade at any given point in time, it just reallocates the division of these benefits. Since workers and firms share the same risk neutral, intratemporal preferences, and since all firms are identical, there is no potential role for meaningful on-the-job search.

5.1 Wages Dynamics

A firm that does not face direct competition in the job centre ($N_0 \in \mathbb{N}^+$) offers the worker an expected payoff that makes the worker indifferent from accepting employment or staying/becoming unemployed $E(N_0, 0) = W(N_0)$. As the worker is always indifferent between accepting the wage offer and search,

$$rE(N_0, \tau) = w(N_0, \tau) + \dot{E}(N_0, \tau)$$

which implies

$$w(N_0, \tau) = rW(N_0, \tau) - \dot{W}(N_0, \tau)$$

Plugging in from above and using $r\lambda = \alpha(1-\lambda)^2$ gives

$$\begin{aligned} w(N_0, \tau) &= \frac{\alpha(1-\lambda)f(0; N_0, \tau)}{\lambda}[\lambda\bar{V} - \bar{\Pi}] + F(0; N_0, \tau)(x - b + c) + b - r\xi \\ &= \frac{rf(0; N_0, \tau)}{1-\lambda}[\lambda\bar{V} - \bar{\Pi}] + F(0; N_0, \tau)(x - b + c) + b - r\xi \end{aligned}$$

$$= \left[\frac{-f(0;N_0,\tau)}{(1-\lambda)}[1-s(1+\lambda)] + F(0;N_0,\tau)\right](x-b+c)+b-r\xi$$

which describes wages that firms pay their workers after duration τ given that there were N_0 other firms at the hiring stage competing for the worker. At any time τ, wages are positively related to the probability the current employer would find competition for the worker's services ($F(0)$) which evolves over time with the duration of employment. The worker is prepared to accept a lower wage and avoid re-negotiating the terms of employment when there is a higher probability that the employer can become a monopsonist.

Initial Wages

Initial wages take three values. If $N_0 = 0$, then $f(0;0,0) = F(0;0;0) = 1$ and

$$w(0,0) = \frac{s-\lambda(1-s)}{1-\lambda}(x-b+c)+b-r\xi$$

For $s = 1/2$, $w(0;0,0) = (x+b+c)/2 - r\xi$.

Starting wages when there are excess workers competing for the job ($N_0 \in N^+$) implies $f(0;N_0,0) = F(0;N_0,0) = 0$. When workers face competition, they compete down to their flow payment in unemployment

$$w(0;N_0,0)_{|N_0\in\mathbb{N}^+} = b - r\xi.$$

Conversely, workers who receive offers from competing vacancies ($N_0 \in N^-$) implies $f(0) = 0$ and $F(0) = 1$ and hence

$$w(0;N_0,0)_{|N_0\in\mathbb{N}^-} = x + c - r\xi.$$

Workers in this case are initially paid their marginal product (plus the firms search costs) when there is more than one bidder. (The firms search costs are an artifact of not dropping out of the market. If the firm received flow benefits rather than costs, the firm would not pay above marginal costs by the same logic that workers receive b.)

Wages Over Time

Differentiation establishes that wage progression satisfies

$$\begin{aligned}\dot{w}(N_0,\tau) &= \frac{\alpha(1-\lambda)\dot{f}(0;N_0,\tau)}{\lambda}[\lambda\bar{V}-\bar{\Pi}] + \dot{F}(0;N_0,\tau)](x-b+c)\\ &= \left[\frac{\dot{f}(0;N_0,\tau)}{1-\lambda}[s\lambda-(1-s)] + \dot{F}(0;N_0,\tau)\right](x-b+c)\end{aligned}$$

If $\bar{V} = \bar{\Pi}$, (i.e. $s = 1/2$)

$$\dot{w}(N_0, \tau) = -[\dot{f}(0; N_0, \tau)/2 - \dot{F}(0; N_0, \tau)](x - b + c)$$

which is positive (negative) for $N_0 < 0$ ($N_0 > 0$).

As

$$F(0) = f(0) + \sum_{i=-\infty}^{-1} f(i)$$

it can also be established that

$$\begin{aligned} w(N_0, \tau) &= rW(N_0, \tau) - \dot{W}(N_0, \tau) \\ &= b + \{F(0) - [\alpha\lambda s - \alpha(1-s)/\lambda + \alpha]f(0)/r\}(x - b + c) \\ &= b + \{[r - \alpha\lambda s - \alpha(1-s)/\lambda + \alpha]f(0) + rF(-1)\}(x - b + c)/r \end{aligned}$$

As

$$\begin{aligned} & r - \alpha\lambda s - \alpha(1-s)/\lambda + \alpha \\ = \; & [r\lambda - \alpha\lambda^2 s - \alpha(1-s) + \alpha\lambda]/\lambda \\ = \; & \alpha[(1-\lambda)^2 - \lambda^2 s - (1-s) + \lambda]/\lambda \\ = \; & \alpha[s(1-\lambda^2) + \lambda(1-\lambda)]/\lambda \\ = \; & \alpha(1-\lambda)[s(1+\lambda) + \lambda]/\lambda > 0 \end{aligned}$$

It follows that

$$\dot{w}(N_0, \tau) = \{\alpha(1-\lambda)[s(1+\lambda) + \lambda]\dot{f}(0)/\lambda + r\dot{F}(-1)\}(x - b + c\}/r$$

Wages can increase of decrease depending on the initial market conditions. The symmetry of the Skellam distribution (given equal arrival rates), implies that wages for $N \in \mathbb{N}^+$ are decreasing over time ($\dot{w} < 0$), are constant for $N_0 = 0$ and are increasing for $N_0 \in \mathbb{N}^-$. When $N_\tau \in \mathbb{N}^-$, the firm is initially in a monopsonistic position on the short side of the market. As turnover occurs in the job center, the probability that the current employer remains a monopsonist decreases over time. The outside option of the worker therefore improves and the firm increases its wage offer to avoid the worker visiting the job centre. On the other hand, for $N_\tau \in \mathbb{N}^+$, the same turnover increases the likelihood over time that the current employer could become a monopsonist. The worker's outside option and hence $w(N_0, \tau)$ decreases with τ, as the threat of any potential loss brought about by an induced visit to the job centre increases over time.

Although wages start and evolve very differently from different states N_t, all wages limit to the same value as the employment spell becomes long:

$$\lim_{t \to \infty} w(N_0, \tau) = (x + b + c)/2 \tag{4}$$

which can differ from initial wages when $N_0 = 0$. The Skellam distribution flattens out over time with variance $2\alpha\tau$. In the limit as $\tau \to \infty$, all wages converge as the history of initial conditions recedes. The Skellam process governing vacancy turnover implies that the distribution of N_t converges to a unique distribution with a mean zero and variance equal to ∞ as $t \to \infty$. The history of the initial state fades (including $N_0 = 0$) over time so that eventually all workers face the same prospects in the job centre. Since the effect of N_0 is only transitory, wages converge to a unique wage, $w(N_0, \infty)$ which increases with the expected number of vacancies in the job centre and converges to

MOVE OR DELETE If the worker comes away from a visit to the job centre either unemployed or hired monopsonistically, then $N_t \in \mathbb{N}$. If a competitive worker auction occurs (an auction with more than one bidder), there were $N_t + 1 \geq 2$ bidders. (Recall the state is defined to include both the firm and the worker before they agree employment.) One worker won the auction and the other N_t remain behind. END

5.2 Numerical example

Initial wages take on three possible values. Seven parameters $(\alpha, x, b, c, \xi, r, s)$ then govern the subsequent wage progression as described by $w(N_0, \tau)$ for duration τ and initial hiring state N_0. The focal parameter (in addition to N_0) is the rate of churning in the market - α. This Poisson arrival rate of agents on both sides of the market determines the evolution of the Skellam distribution at the core of the wage determination process. The baseline specification used below is that the probability of an arrival of at least one worker in a given month equals one half so that $\alpha = 0.6931$.

Workers and firms inhabit a small, isolated market to highlight the essential mechanics of the model. An observed statistical market will typically contain numerous such entities. As observed statistics are broader than the model, the quantitative approach adopted here replicates and aggregates the model across a number of small markets to match up against familiar statistics. The objective is not to generate accurate quantitative predictions nor to precisely measure the model. The model abstracts from prominent features that are likely to affect entry as well as pay and thereby alter the alignment of the model with data. This evaluation should thus be viewed as a demonstration that under some fairly generic parameterizations, the model can qualitatively deliver compensation patterns within specific jobs that are consistent with established regularities in the data.

Wages and turnover

To first gauge the impact of the arrival rate parameter α, Figures 1 and 2 plot wage progressions for two values of α and various initial conditions N_0. The high α used for Figure 1 is twice the baseline case, the low value used for Figure 2 is one half the baseline value. The remaining six parameters (x, b, c, ξ, r, s) given in Table 1 are standard values. The time period is taken to be one month.

Table 1. Parameter Values

Parameter	Value
x	1
b	.20
c	.10
ξ	0
r	0.0042
s	0.5

As evidence from comparing Figure 1 with Figure 2, persistence falls as turnover increasesthat is as the model more closely approximates a competitive setting. The high α turnover plot in Figure 1 converges faster than the low α plot in Figure 2. However, persistence lasts even for the high value α. For $N_0 = -1$, after ten years the wage remains more than 1.3% above the convergent $N_0 = 0$ wage. For $N_0 = 2, 5, 10$ these figures are $2.7\%, 6.7\%, 13.3\%$. The impact of the initial conditions ultimately fades at a decreasing rate but this rate is increasing very early on in the employment spell. The pattern for a given starting point changes in short order from convex to concave for $N_0 \in Z^+$ and from concave to convex for $N_0 \in Z^-$. None the less, wage changes are predictable, both positive and negative, serially correlated and persistent, which all conform with the evidence noted above.

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Wage dispersion

To relate the variety of wages found in the model to observed market-wide regularities, the market is repeatedly simulated. As there is no ergodic steady state in the model - the Skellam distribution of N_t given N_0 depends on t - the market in the model starts with no employment (no matched pairs or wages) for a given N_0. Simulated entry of workers and firms occurs over 120 periods or ten years. Repeating the exercise over 250 markets all with the same N_0 yields a panel of wages for employed workers as well as information on unemployment and vacancies. Cross section wages from the last period are computed using all of the employer-employee pairs that formed during this period. These wages are hence conditional on the initial N_t at hiring and the subsequent duration of employment.

The unconditional wage dispersion in the simulated wages depends critically on the specification of N_0. Evidence suggests that at any point in time there are more unemployed workers than jobs which clashes directly with the symmetric $N_0 = 0$ specification. Roughly equal numbers of vacancies and firms result when starting from this situation. The symmetry from $N_0 = 0$ likewise generates equal numbers (in expectation) of above and below average wages all converging at the same pace hence a symmetrically shaped distribution of wages that does not correspond well with observed wage distributions. Perhaps more fundamentally, this symmetric specification of this sort also generates no correlation between tenure and wages.

These limitations of the stylized model may be linked to omitted factors. Firms and workers differ even if they need to arrive over time in equal numbers to avoid having the market become one sided. Specifying distinct differences for entry significantly complicates the analysis and is left for future work. We proxy these factors by setting $N_0 > 0$. To deliver more suitable outcomes, the simulations specify that markets initially begin with more workers than firms - $N_0 = 5$. In the long run there will be some markets which become $N > 0$ but on average markets will tend to have more workers than jobs.

Figure 3 presents the simulated distribution of wages in the final period given the initial setting.[12] As expected the wage distribution is left-skewed.

[12] The associated U-V ratio equals 2.14 and the ratio of the mean wage to the minimum wage is 2.78. The unemployment rate is just below ten percent at 9.76%.

Wage rises are more much more prevalent than wage cuts although these cuts do occur.

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<u>Tenure effect</u>

Regressing the cross sectional wage in the last period on tenure and initial hiring conditionals N_t yield the tenure effects. In particular, regressing logged wages on logged tenure and (unlogged) N_t at the time of hiring yields

$$\ln(w) = -0.860 + 0.082[0.0051 * ln(Tenure) - 0.041[0.0002] * N \qquad R^2 = 0.875$$

with standard errors in square brackets. Tenure on average raises wages.

Note that in this regression the measure for initial condition N at hiring is very precise for the individual. Although several authors document that initial conditions matter and are persistent, the measures of job competition used are far more general than in the above regression. Local unemployment rates for example are broad measures whereas N is very particular to the individuals circumstances. Finding the appropriate benchmark in the model is unclear but rerunning the regression without any such measure does not substantially alter the tenure coefficient estimate or its significance.

6 Conclusion

How responsive are wages to changes in the external labor market? This paper delivers

- wage dispersion and wage growth dispersion including wage cuts
- persistence - serial correlation in wages that creates predictable winners and losers
- initial conditions matter - which can be broadly viewed as cohort effects

REFERENCES Initial conditions (Macro perspective) - see Martins, Solon and Thomas / Carniero, Guimaraes and Portugal / Oreopoulos et al / Oyer /Haefke Sonntag and van Rens / Beadry and DiNardo

Permanent versus transitory shock literature - the rate of churning α (linked to the extent of the market) affects the persistence of the shock; local conditions N_0 determine the inital magnitude

Fallick and Fleischman (2004), Nagypal (2005, 2008) report that only a small fraction (less than 5 percent) of employed workers are actively searching. Jolivet, Postel-Vinay and Robin (2006) find that "relative to involuntary mobility (reallocation shocks and lay-offs), voluntary mobility is a rather rare event" in many European countries and in the US.