To Own or to Rent?
The Effects of Transaction Taxes on Housing Markets*

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Abstract

Using sales and leasing data, this paper finds three novel effects of a higher property transaction tax: higher buy-to-rent transactions alongside lower buy-to-own transactions, despite both being taxed; lower sales-to-leases and price-to-rent ratios; and longer time-on-the-market. This paper explains these facts by developing a search model with entry of investors and households choosing to own or rent in the presence of credit frictions. A higher transaction tax reduces homeowners' mobility and increases demand for rental properties, which reduces the homeownership rate. The deadweight loss is large at 113% of tax revenue, with more than half of this due to distorting decisions to own or rent.

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KEYWORDS: rental market, buy-to-rent investors, homeownership rate, transaction taxes.

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1 Introduction

Real-estate transaction taxes are a common feature of tax systems around the world. A large and growing literature points to the distorting effects of such taxes on owner occupiers.[1] However, little is known about the implications of transactions taxes for households' tenure choices and landlords' investment decisions, which jointly determine the allocation of properties between the markets for ownership and rentals, and hence the homeownership rate. At least a third of the housing stock is allocated to rental markets, and the homeownership rate is the focus of many policy debates.[2] This paper offers a comprehensive understanding of the impact of transaction taxes on households' decisions along both the intensive margin (moving and transacting) and the extensive margin (owing or renting), and on investors' decisions to buy property.

The paper makes two contributions to the literature. Empirically, it documents the different way buy-to-rent investors respond to a transaction tax compared to owner-occupiers, in spite of the tax applying to both, and the relative effects of the tax on markets for property ownership and rentals as measured by the leases-to-sales and price-to-rent ratios. These facts demonstrate the importance of considering the extensive margin and entry by investors. The paper also establishes that time-on-the-market increases in response to higher transaction taxes, demonstrating the importance of search frictions. Theoretically, to explain these new facts, the paper develops and quantifies a model of housing with both an ownership and a rental market subject to search and credit frictions. The model features housing tenure decisions across the two markets and endogenous moving decisions within the ownership market.

The new empirical evidence comes from using a unique dataset of Multiple Listing Service records on housing sales and leasing transactions for the Greater Toronto Area (GTA) between 2006 and 2018. With observations of leases and rents in addition to sales and prices, the data make it possible to examine both owner-occupied and rental markets and to distinguish purchases made by buy-to-rent investors from those of owner-occupiers.

In 2008, the City of Toronto introduced a new city-level transaction tax, known in Canada as Land Transfer Tax (LTT), at an effective rate of 1.3% of the property price. Importantly, the new tax covers only the City of Toronto but not other parts of the GTA, making it possible to estimate the effects of the tax by comparing housing transactions and homeowner mobility before and after the new LTT across neighbourhoods that are adjacent to but on opposite sides of the city border. The counterfactual is supported by evidence showing that homes on opposite sides of the border are similar in their relevant attributes, and neighbourhoods exhibit similar pre-policy trends for the outcomes of interest. For years spanning the policy change and for

[1] This includes, but is not limited to, Benjamin, Coulson and Yang (1993), Slemrod, Weber and Shan (2017), Kopczuk and Munroe (2015) for the US, Besley, Meads and Surico (2014), Hilber and Lyytikäinen (2017), Best and Kleven (2018) for the UK, Dachis, Duranton and Turner (2012) for Canada, Eerola, Harjunen, Lyytikäinen and Saarimaa (2021), Määttänen and Terviö (2020) for Finland, Fritzsche and Vandrei (2019) for Germany, Davidoff and Leigh (2013) for Australia, Van Ommeren and Van Leuvensteijn (2005) for the Netherlands, Agarwal, Chau, Hu and Wan (2022) for Hong Kong, and Huang, Li and Yang (2021) for Singapore.

[2] See Gabriel and Rosenthal (2015) and Goodman and Mayer (2018).

neighborhoods across the city border, the tax effects on mobility are estimated using monthly data at household level, time-on-the-market and prices with transactions data, and transactions and relative costs across rental and owner-occupied markets with monthly neighbourhood data.

The estimations yield a set of novel facts about the effects of transaction taxes. First, while the LTT reduces overall housing sales and transaction prices in the owner-occupied market, it has the opposite effect on the rental market. The LTT causes the ratio of leases to sales to rise by 23% and the ratio of prices to rents to decline by 4%, suggesting that renting becomes more attractive relative to owning.

Second, for ownership-market transactions, the LTT causes a 10% fall in owner-occupier purchases, but a 9% rise in purchases made by buy-to-rent investors, even though the LTT applies to both. By definition, buy-to-rent investors are those who acquire properties from the ownership market and make them available in the rental market. Thus, the increase in buy-to-rent purchases is consistent with the rise in the ratio of leases to sales. Both indicate a shift towards the rental market, consistent with the recent fall in the homeownership rate in Toronto.[3]

Third, within the ownership market, the LTT reduces homeowners' moving hazard rate by 13% and increases sellers' time-on-the-market by 17%. Based on the pre-policy sample mean, this implies that an average owner-occupier stays in a property for 14 months longer, and a property takes 5 days longer to sell after it is listed.

Together, these findings shed new light on the consequences of transaction taxes. The heterogeneous treatment effects of the LTT on sales versus leases and on home-buyers versus investors indicate that a careful evaluation of transaction taxes must consider flows of property between owner-occupation and the rental market. The estimated time-on-the-market effect suggests that the LTT interacts with search frictions present in the housing market. The longer times taken to sell combined with longer stays by existing homeowners reduce flows both into and out of the stock of houses for sale. Together, these contribute to the decline in transactions by owner occupiers.

The paper develops and calibrates a search model that incorporates the economic forces highlighted by these new findings with the goal of better understanding real-estate transaction taxes. To analyse jointly the ownership and rental markets, the model features households who choose which market to participate in, subject to paying a credit cost to access the market for property ownership. These credit costs represent the costs of mortgage financing or the difficulty of obtaining credit, which are heterogeneous across households. Setting the benefits of homeownership against its costs gives rise to an entry decision on the 'buy' side of the rental market. On the 'sell' side, there is free entry of buy-to-rent investors. The equilibrium homeownership rate is the one consistent with the behaviour of both households and investors.

The estimated LTT effect on time-on-the-market calls for a search-theoretic analysis. The

[3]The homeownership rate, defined as the fraction of properties that are lived in by their owners, is reported by Statistics Canada only at a five-year frequency. In Toronto, it steadily increased from 51% to 54.5% between 1996 and 2006, followed by a gradual decline to 52.3% in 2016.

source of the time taken to sell a property can be broken down into two types of search frictions. First, the time needed for buyers to locate potentially desirable properties to view, which depends on the ratio of buyers to sellers in the market. Second, the need for viewings to reveal the idiosyncratic match quality between a potential buyer and a property.[4] Home-buyers search until they find a property with match quality above a threshold. Given the estimated LTT effect on the moving hazard rate, the model also features endogenous moving: after moving into a property, match quality is subject to occasional idiosyncratic shocks representing life events that make a particular property less well suited to a particular household. After a shock, a household decides whether to move, doing so if match quality is below a threshold.

Along the extensive margin, the model predicts the LTT leads simultaneously to a decrease in purchases by owner-occupiers and an increase in buy-to-rent purchases and leases. The explanation for this hinges on the difference between home-buyers and investors. Owing to the idiosyncratic shocks and the indivisible nature of property, households desire to move between different properties on a number of occasions throughout their lives. Hence, choosing to be an owner-occupier rather than a renter means expecting to pay the LTT every time a new property is purchased. This dissuades some potential home-buyers from incurring the credit cost and entering the ownership market. Since these households must still live somewhere, there is an increase in demand for properties in the rental market.

Investors also face paying the LTT, which reduces the return from purchasing a property. However, a landlord does not need to transact again in the ownership market just because a tenant no longer finds the property suitable and moves out. This implies that investors have less need to transact compared to owner-occupiers who face match-quality shocks.[5] So while the LTT has a direct negative effect on supply in the rental market, this is relatively smaller than the increase in demand for rental properties.

In equilibrium, the LTT causes the price-to-rent ratio to fall by enough to attract more buy-to-rent investors in spite of the tax. These investor purchases of properties from owner-occupiers lead to a decline in the homeownership rate. Buy-to-rent purchases and leases increase, while purchases by owner-occupiers decline, consistent with the empirical evidence.

Turning to the intensive margin, the LTT makes existing owner-occupiers more tolerant of poor match quality, so moving rates decline as households remain in properties for longer on average, consistent with the empirical findings. The indivisibility of housing in the search model means that the moving rate is a proxy for households' renewals of match quality. As match quality with a property has some persistence, households can mitigate the increased tax costs of moving by requiring higher match quality when making a property purchase, thus reducing the need to move again in the future. This greater pickiness of buyers implies longer average time-on-the-market for sellers, and this prediction is also borne out empirically.

[4] The importance of match quality is captured by the number of viewings per transaction. See the evidence from the U.K. and the U.S. presented in Ngai and Sheedy (2020).

[5] Section 4.2 explains why the fact that investors' transactions are a small fraction of total transactions relative to the stock of rental properties implies investors have longer average holding periods than owner-occupiers.

The model's parameters are calibrated to the City of Toronto housing market for the years 2006–2008. Toronto has an active rental market, and the homeownership rate in the city was then around 54%. The model is used to simulate the effects of a 1.3 percentage-point increase in the LTT rate, calibrating to match the estimated LTT effect on homeowners' moving hazard. The model predicts a 17% decrease in transactions by owner occupiers and a 5% increase in buy-to-rent transactions, resulting in a 2.4 percentage points decline in the homeownership rate. The ratio of leases to sales is predicted to rise by 21% and the price-to-rent ratio to decline by 1.4%. Time-on-the-market for sellers goes up by 7.8%. These numbers are broadly consistent in magnitude with the estimated LTT effects that are not directly targeted in the calibration.

While not in the baseline model, households might respond to the LTT by choosing to locate on the other side of the city border to avoid the tax. An extension of the model with an endogenous city population is considered to explore the sensitivity of the results to households having a choice of location, which better matches the empirical strategy of comparing transactions on opposite sides of the city border. However, since houses must be owned or rented by someone in equilibrium, it turns out that the city population does not adjust by much even when there is a free entry condition. Consequently, the results from this exercise are very similar to the baseline case with a fixed population, with the exception of prices, which fall by more when the population is endogenous.

The model spells out two facets of the welfare costs of transaction taxes closely related to the positive predictions. The first is a novel effect on misallocation of properties across the rental and ownership markets through entry of buy-to-rent investors. Intuitively, since owner-occupiers expect to transact more frequently, the same LTT falls more heavily on owner-occupiers than buy-to-rent investors, placing them on uneven footing in respect of the tax. This means the credit cost of the marginal home-buyer must fall, displacing some creditworthy households into the rental market. Transaction taxes therefore distort housing tenure choices.

Second, within the ownership market, there are two consequences for welfare associated with longer time-to-move and longer time-on-the-market. There is a conventional 'lock-in' effect of reduced mobility giving rise to misallocation of properties among owner-occupiers, with match quality falling on average as households move less frequently to renew it. While longer time-on-the-market means that newly matched owner-occupiers enjoy better initial match quality, they also incur costs from more time spent searching.

The implied welfare cost of the higher transaction tax is substantial. The new LTT generates a welfare loss equivalent to 113% of the extra revenue it raises. The distortions to flows between the rental and ownership markets account for a loss equal to 60% of extra revenue raised. Distortions within the rental and ownership markets lead to losses of 14% and 40% of tax revenue respectively. Overall, the presence of the rental market in the analysis accounts for a loss equivalent to 74% of tax revenue, which is around two thirds of the total loss.

There are alternative ways of raising tax revenue from the housing market. The paper first considers a higher tax on buy-to-rent investors that offsets the implicit advantage they derive from a tax system with equal rates. By putting up barriers to entry for investors, it reduces the across-market welfare losses from lower homeownership. However, an important caveat is that increasing the tax on buy-to-rent investors further to raise the homeownership rate would ultimately lead to large welfare costs as uncreditworthy households are displaced into the ownership market because of a lack of rental properties. Deep-pocketed investors play an important role in providing access to housing without everyone needing to pay credit costs.

A second possibility is introducing a housing consumption tax as recently considered by the UK and Australian governments. As with the case of a higher tax on buy-to-rent investors, the use of a housing consumption tax removes the implicit tax advantage of investors over owner-occupiers as all property owners have to pay a tax that is independent of their transaction frequency. This alternative tax has a negligible effect on outcomes both within the ownership market and across the rental and ownership markets, resulting in negligible welfare costs.

The plan of the paper is as follows. Related literature is discussed below. Section 2 presents the data and the estimation of the effects of the LTT in Toronto. Section 3 develops a dual ownership and rental markets model of housing. Section 4 presents the model's qualitative predictions when the transaction tax rises. Section 5 calibrates the model and derives the quantitative effects of the transaction tax and the associated welfare losses due to misallocation across the two markets and distortions within each market.

Related literature In the last two decades, concerns about the costs of real-estate transaction taxes have grown among policymakers and in academic research. Two prominent examples are the 'Henry Review' established by the Australian government and the 'Mirrlees Review' by the UK government. Both reviews found significant costs of transaction taxes owing to reduced mobility and distortions associated with ad valorem taxes. The reviews proposed reforms to replace stamp duty with a land value tax or a tax on housing consumption (Henry, Harmer, Piggott, Ridout and Smith, 2009, Mirrlees, Adam, Besley, Blundell, Bond *et al.*, 2010).

These findings are confirmed by economists studying housing markets using data from Australia, Canada, Finland, Germany, the UK, and the US. The majority of the literature has focused on the effects of transaction taxes on mobility, transaction volumes, or house prices. Among these papers, a few have also computed the welfare costs of transaction taxes per unit of tax revenue raised, such as Dachis, Duranton and Turner (2012) for Canada, Hilber and Lyytikäinen (2017) and Best and Kleven (2018) for the UK, Eerola, Harjunen, Lyytikäinen and Saarimaa (2021) and Määttänen and Terviö (2020) for Finland, Fritzsche and Vandrei (2019) for Germany, and Schmidt (2022) for the Netherlands. These losses are solely due to effects on the intensive margin of fewer transactions and reduced mobility of homeowners. However, as Poterba (1992) noted, "*finding the ultimate behavioral effects requires careful study of how tax parameters affect each household's decision of whether to rent or own as well as the decision of how much housing to consume conditional on tenure.*" The contribution of this paper is to document empirically the effects of transaction taxes on both intensive and extensive margins, and to develop a housing model to quantify the welfare effects of taxes on these two margins.

The use of search-and-matching models to study frictions in the housing market is long established, going back to Wheaton (1990). The papers in the voluminous literature that followed are surveyed by Han and Strange (2015).[6] Among those papers, Lundborg and Skedinger (1999) explicitly study the effects of transaction taxes on search effort in a version of the Wheaton (1990) model. Since they abstract from the rental market and the decision to move, their model cannot be used to analyse the impact on homeownership and mobility. While the majority of housing search models have abstracted from search in the rental market, recent papers by Halket, Pignatti and di Custoza (2015), Ioannides and Zabel (2019), and Bø (2021) explicitly consider search in both ownership and rental markets. Their objectives are different from this paper, focusing instead on issues such as the Beveridge curve in the housing market, and the relationship between price-to-rent ratios and homeownership rates across sub-markets. More importantly, they abstract from the moving decision that is crucial here for both the extensive and intensive margins of adjustment to transaction taxes.

There is also an existing literature that seeks to understand changes in homeownership rates, typically using models without search frictions.[7] A key feature of the analysis here is the general-equilibrium effect of households' tenure choices after the transaction tax on the price-to-rent ratio, which attracts entry of buy-to-rent investors. It is similar in spirit to Sommer and Sullivan (2018), who point to the general-equilibrium effect on homeownership of removing mortgage-interest deductibility through a fall in house prices, which encourages more credit-constrained households to become owner-occupiers as downpayment constraints slacken. This illustrates the importance of a framework where house prices, rents, tenure choices, and entry of investors are all endogenous in general equilibrium.[8] Greenwald and Guren (2021) use a model with these features to investigate the sensitivity of house prices to credit conditions, where key parameters of the model are disciplined by the estimated responses of the price-to-rent ratio and homeownership rate to identified credit shocks.

The empirical strategy of this paper is closest to Dachis, Duranton and Turner (2012) in studying the effects of the 2008 LTT in Toronto. This paper differs in that it examines an array of housing-market outcomes beyond sales prices and volumes, which yields a comprehensive understanding of how housing markets react to transactions taxes, including the market for rental property. By considering a general-equilibrium search model with endogenous moving across and within the ownership and rental markets, this paper finds the welfare loss from transaction taxes is a much larger fraction of the revenue they raise.

Recent works with a related objective to this paper are Kaas, Kocharkov, Preugschat and

[6] For recent examples, see Anenberg and Bayer (2020), Díaz and Jerez (2013), Gabrovski and Ortego-Marti (2019), Guren (2018), Head, Lloyd-Ellis and Sun (2014), Moen, Nenov and Sniekers (2021), Ngai and Tenreyro (2014), Ngai and Sheedy (2020), Piazzesi, Schneider and Stroebel (2020), and Genesove and Han (2012).

[7] For example, Chambers, Garriga and Schlagenhauf (2009), Fisher and Gervais (2011), and Floetotto, Kirker and Stroebel (2016). See Goodman and Mayer (2018) for a survey of the determinants of homeownership rates.

[8] There is an empirical literature that has studied the extent of flows between the rental market and owner-occupation (Glaeser and Gyourko, 2007, Bachmann and Cooper, 2014).

Siassi (2021), Cho, Li and Uren (2021) and Schmidt (2022). They analyse the effects of stamp duty on the homeownership rate and its implications for welfare in models without search frictions. This paper's key advantage is in identifying the differential effects of transaction taxes on buy-to-rent investors and owner-occupiers using micro data on leasing and transaction records. On the theory side, this paper allows for free-entry of buy-to-rent investors in a search model that highlights the indivisible nature of housing. The model rationalizes the empirical finding of opposite effects of transaction taxes on buy-to-rent investors and owner-occupiers.

2 Empirical analysis

2.1 Data

The data on residential real-estate sales and leasing transactions come from Multiple Listing Service (MLS) transaction records for the period 2006–2018 in the Greater Toronto Area (GTA), the fourth largest metropolitan area in North America. Each sale has observations of the property price, the time on the market, the transaction date, and the exact address and neighbourhood. For each lease, the listing date, the lease start date, the monthly rent, the lease term, and the exact address and neighbourhood are observed. For both sales and rental transactions, the MLS data also have detailed property characteristics such as the numbers of bedrooms, washrooms, and kitchens, the lot size (except for condominiums/apartments), the styles of the house and the family room, the basement structure/style, and the heating types/sources.

A key advantage of the analysis comes from the ability to match rental transactions to sales transactions. The MLS is the largest rental listing platform and provides an unusually high coverage of long-term and verifiable rental listings in Toronto. Appendix A.1.1 shows through webscraping and geocoding that MLS rental listings capture over 90% of rental properties in the City of Toronto that were listed on alternatively platforms such as *Toronto Rentals*, the second-most popular rental website serving the GTA since 1995.[9]

Properties that appear in both sales and lease datasets within an 18-month window are identified by their detailed addresses and transaction dates. This is used to generate a novel measure that links the markets for property ownership and rentals. If the sale of a property is followed by its being listed on the rental market between 0 and 18 months after the sale, the purchase is identified as a *buy-to-rent* transaction. Alternatively, if a sale is followed by the property being listed again for sale between 0 and 18 months after the original sale, it is identified as a *buy-to-sell* transaction.[10] The remaining sales transactions are considered to be

[9]Urbanation, a third-party service that independently collects data on rentals in the Greater Toronto Area, estimates that approximately 75–80% of condominium lease activity is captured by MLS data. These estimates are based on examining transacted MLS lease volumes relative to the size and changes in the overall rental stock each year as reported by the Canada Mortgage and Housing Corporation (CMHC). The coverage of sales and leasing activities for single-family houses by the MLS is even larger.

[10]As a robustness check, changing the 18-month threshold to 6, 12, or 24 months does not significantly affect the estimation results.

purchases by owner-occupiers and are designated as *buy-to-own* transactions.

Between 2006 and 2017, the fraction of buy-to-own transactions declines from 89% to 84%, while the fraction of buy-to-rent transactions triples from 4% to 12%.[11] In contrast, buy-to-sell transactions remain stable at around 4% throughout most of the period. Given the small and stable fraction of buy-to-sell transactions, these are excluded from the estimation sample.

Housing-market outcomes are measured at both the market segment and individual transaction levels. A market segment is defined by *property type* $\times$ *community* $\times$ *year* $\times$ *month*. Property types comprise single-family houses, townhouses, condominiums, and apartments. Communities refer to neighbourhoods.[12] For each market segment, housing-market outcome variables are the number of sales, which is broken down into buy-to-own (BTO) and buy-to-rent (BTR) sales, the number of leases, the ratio of the numbers of leases to sales (leases-to-sales ratio), and the average price-to-rent ratio. For individual transactions, outcomes are sales prices and sellers' time-on-the-market. In addition, the number of months since a homeowner purchased a property is precisely observed, regardless of whether the homeowner moves.

Real-estate transaction taxes are common across Canada, where they are known as Land Transfer Tax (LTT). The tax is paid by buyers, and in spite of the name, LTT is applied to the whole property price. Before 2008, residential transactions in the province of Ontario, which includes the whole of the GTA, were subject to a provincial-level land transfer tax, but there was no additional city-level LTT. The City of Toronto experienced a housing boom in the years following 2000 and usually maintained a budget close to balance. Following an unexpected budget shortfall in late 2007, the city council approved a land transfer tax on property transactions within the city that close after 1$^{\text{st}}$ February 2008. The tax revenues were collected to meet municipal workers' demands for higher wages. The institutional background of the LTT is discussed in detail in Dachis, Duranton and Turner (2012).

In the appendix, Table A.1 gives descriptive statistics for the City of Toronto before and after the introduction of the city-level LTT. The rest of the Greater Toronto Area remained with the same provincial-level LTT after February 2008. Table A.2 summarizes the city- and provincial-level LTT schedules. The effective LTT rate is the mean transfer tax as a percentage of the sales price, combining provincial- and city-level taxes, averaged over pre-February-2008 transactions. Using this same set of transactions to control for compositional effects, the effective LTT rate is 1.5% before the February-2008 policy change and 2.8% afterwards. This implies a 1.3 percentage points increase in the effective LTT rate in the City of Toronto.

[11] The rise of buy-to-rent transactions in recent years has been seen in other countries, including the US and Norway (Mills, Molloy and Zarutskie, 2019, Bø, 2021).

[12] There are 296 communities in the GTA, including 140 in the City of Toronto. See www.toronto.ca/city-government/data-research-maps/neighbourhoods-communities/neighbourhood-profiles/.

2.2 Estimating the effects of transaction taxes

The main empirical strategy resembles a variant of the regression discontinuity design in Dachis, Duranton and Turner (2012). While they estimate the six-month effects of the LTT on transaction volumes and sales prices in the market for single-family houses, this paper extends the sample to cover not only a longer time period but also a wider range of residential property types. Most importantly, benefiting from a unique combination of rental and sales data, this paper examines an array of market outcomes above and beyond prices and volumes, which reveals a detailed picture about flows of properties between and within the owner-occupied and rental markets. Between the two markets, the paper makes a new contribution by estimating the effects of the LTT on transaction volumes and costs in the rental market relative to the ownership market. Within the ownership market, the paper enriches previous work by estimating how the LTT affects individual homeowners' moving hazard and the time taken to sell a property.

Figure A.2 illustrates trends in the cross-city-border differences in key housing-market variables before and after the LTT is introduced (2006–2012), restricting the sample to neighbourhoods within five kilometres on either side of the Toronto border.[13] Compared to their nearby suburban neighbours, city residents after the LTT faced lower prices relative to rents, made more leasing transactions relative to sales transactions, and saw fewer buy-to-own but more buy-to-rent transactions. Using transaction-level data, city properties for sale after the LTT spent longer on the market and had lower sales prices. These trends show that the February-2008 introduction of a city-level LTT led to two discrete changes in Toronto's housing market: one at the city border, and the other on the date the city LTT was imposed.

Motivated by these discontinuities, this paper estimates the causal effects of the transaction tax by comparing changes in housing-market outcomes before and after the introduction of the tax in 'treated' city neighbourhoods to changes over the same period in 'untreated' suburban neighbourhoods. The detailed empirical specification is presented in section A.1.3.

For each market outcome, regressions include an indicator for the post-LTT period, an indicator for being in the City of Toronto, a rich set of time-varying housing characteristics (when applicable), along with a broad set of fixed effects: community fixed effects, year fixed effects, month fixed effects, property-type fixed effects (when applicable), and their interactions. These fixed effects flexibly control for housing composition, seasonality, and variation in how different housing-market segments evolve. The key variable of interest is an interaction term named LTT between the city and post-policy period indicator variables. Given that the LTT was implemented for the city of Toronto starting in February 2008, the LTT coefficient captures the impact of the new transaction tax. By allowing for separate time trends for transactions inside

[13] These are obtained from coefficients in regressions of the natural logarithm of the outcome variables on the interactions between years, months, and an indicator for the City of Toronto. Year, month, and City of Toronto fixed effects and other controls are added so that the coefficients represent the percentage differences with respect to the City of Toronto mean for an average community. The data used comprise single-family houses (detached and semi-detached) within 5km of the city border from January 2006 to February 2012.

and outside of the city, the specifications also control for the possibly heterogeneous impact of the financial crisis.[14]

In the baseline estimation, the pre-policy period is January 2006–January 2008 and the post-policy period is February 2008–February 2012. To ensure the housing stock and neighbourhoods are relatively homogeneous, the baseline sample is restricted to properties on the opposite sides of the city border but within 3 or 5 kilometres from the boundary line determining whether the new LTT is applicable. The geography of the sample used for the baseline estimation is depicted in Figure A.3. Importantly, the possibility that housing-market outcome variables make a discrete jump at the border while neighbourhoods continue to change in a smooth manner allows the relationship between the LTT and housing-market outcomes to be isolated. Appendix A.1.4 provides evidence in support of most of the property characteristics not varying significantly across the border, and that the cross-border difference, if any, does not change significantly after the policy. Thus, there are no significant differences in housing-stock composition being picked up by the LTT coefficient.

One legitimate concern is that households may have anticipated the introduction of the new LTT and rushed to make transactions before the cost of buying a property increased. As discussed extensively in Dachis, Duranton and Turner (2012), such anticipation of the 2008 LTT in the Toronto market was quite limited, and would have occurred within three months before the policy change. In light of this, for all specifications, indicators for transactions in the six-month period from November 2007 to April 2008 are included to condition out any run-up in transactions right before the policy change and possible continuation right after it.[15]

2.2.1 Effects across ownership and rental markets

Consider first the estimation of the LTT effects across the ownership and rental markets. The outcomes here are the numbers of leases relative to sales, the price-to-rent ratio, and sales separated into buy-to-own (BTO) and buy-to-rent (BTR) transactions. For these regressions at the market-segment level, the sample is restricted to single-family houses for which the MLS covers almost the universe of transactions in the Greater Toronto Area.

Lease-to-sales and price-to-rent ratios For each market segment, the leases-to-sales ratio is a measure of relative activity in the rental and ownership markets, and the price-to-rent

[14] It is worth noting that the GTA housing market experienced a temporary slowdown triggered by the global financial crisis that started in September 2008, followed by a quick recovery at the beginning of 2009. Unlike the U.S. and some other countries, the slowdown caused by the financial crisis was mild and temporary in Canada (Bordo, Redish and Rockoff, 2015, Haltom, 2013, Walks, 2014).

[15] This strategy for addressing possible anticipation effects is also consistent with Bérard and Trannoy (2018) and Benjamin, Coulson and Yang (1993), both of whom explicitly estimate anticipation effects associated with a real-estate transaction tax. Using French data, the former find that the anticipation effect is limited to one month immediately before the implementation of the tax reform, while post-policy effects last for up to three months. Using data for Philadelphia, the latter find that anticipation effects are very small and limited to two months before the tax change.

Table 1: *Effects of the transaction tax across ownership and rental markets*

Dependent variable	(1)	(2)	(3)	(4)
log (#*Leases*/#*Sales*)	0.234**	0.242***	0.236**	0.264***
	(0.117)	(0.082)	(0.100)	(0.063)
Observations	1355	2660	1782	7730
log (*Price*/*Rent*)	-0.039**	-0.026*	-0.031*	-0.037**
	(0.019)	(0.015)	(0.017)	(0.013)
Observations	1355	2660	1782	7730
log (#*BTO sales*)	-0.101**	-0.097**	-0.087*	-0.122***
	(0.047)	(0.044)	(0.049)	(0.033)
Observations	3736	6363	3811	17190
log (#*BTR sales*)	0.089*	0.099**	0.117**	0.110*
	(0.047)	(0.045)	(0.053)	(0.058)
Observations	531	1031	670	2857
Distance threshold	3km	5km	5km	All
City indicators ±3 m.	Yes	Yes	Yes	Yes
City time trends	Yes	Yes	Yes	Yes
Distance LTT trends		Yes	Yes	Yes
Donut hole			2km	

Notes: Data comprise single-family-house transactions from January 2006 to February 2012. A unit of observation is a market segment defined by *community* × *year* × *month*. Repeat sales transactions taking place within 18 months of one another are discarded. Each cell of the table represents a separate regression of an outcome (specified in the left column) on the LTT interaction dummy. All regressions include a dummy for the post-LTT period, City of Toronto fixed effects, year fixed effects, calendar-month fixed effects, community fixed effects, and their interactions. In the specifications above, the distance threshold is the maximum distance to the Toronto city border for a transaction to be included in the sample. City indicators ±3 m. are six dummy variables for transactions inside the City of Toronto during the last three months of 2007 and the first three of 2008. City time trends indicates the presence of separate time trends for transactions inside and outside the City of Toronto. Distance LTT trend denotes the inclusion of an interaction term between the LTT and a dummy variable for properties between 2.5km and 5km away from the city border in columns (2)–(3) and the interaction between the LTT and the distance from the city border in column (4). Robust standard errors are in parentheses, and *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels.

ratio is a measure of relative cost across the markets. The top panel of Table 1 reports the estimated effects of the LTT on these measures. Column (1), the baseline specification, restricts the sample to 3km on each side of the border. It allows for anticipation effects by including indicators for transactions three months before and after the introduction of the LTT. It further allows for the presence of spatially differentiated time trends on either side of the city border.

The 1.3 percentage-point increase in the effective LTT rate causes a 23% increase in the numbers of leases relative to sales and a 3.9% drop in the price-to-rent ratio. The LTT thus boosts activity in the rental market compared to the ownership market, and raises the rental yield (the inverse of the price-to-rent ratio). Column (2) replicates the baseline regression of column (1) but extends the sample to include all property transactions within 5km of the city border instead of 3km. The coefficients on the lease-to-sales ratio and the price-to-rent ratio remain close to those in column (1).

While discontinuity design is a standard approach to estimate the effects of the tax, it requires two strong assumptions that are worth discussing. The first assumption is that the leases-to-sales and price-to-rent ratios outside the city border are unaffected by the tax change. However, a potential sorting bias is that some buyers may respond to the LTT by switching from making purchases inside the city border to outside, boosting property sales outside the border and hence violating the assumption that the comparison group is unaffected by the tax change. To mitigate this concern, column (3) applies a 'donut approach,' repeating the estimation in column (2) with a distance threshold of 5km, but excluding properties within 2km of each side of the city border. The rationale is that sorting across the border, if it occurs, would most likely happen immediately adjacent to the border. However, the coefficients in column (3) are very close to those in column (2), mitigating this concern.[16] A reason for the robustness of the estimates with respect to sorting is offered in the theoretical framework developed later.

Second, for the regression discontinuity to be an appropriate measure of the LTT effect, the LTT impact must be uniform for all city properties irrespective of their distance to the border. But this will not hold if, for example, people who live further away from the border are more willing to pay the tax because their location demand is less elastic. This concern is addressed in columns (2) and (3) by extending the sample to include properties within 5km of each side of the city border, and by adding an interaction term between exposure to the LTT and a dummy variable for properties between 2.5km and 5km from the border. With this, the LTT effect can differ depending on the distance of a property from downtown. However, the coefficient on the interaction term is small and statistically insignificant in all specifications.[17] More importantly, the coefficients on LTT in both the leases-to-sales and price-to-rent regressions remain consistent across specifications.

Column (4) takes an extreme approach by extending the estimation sample to cover the entire city of Toronto and the adjacent suburban municipalities. This specification estimates the LTT effects on all property transactions within the city. The estimated effects remain close to the baseline specification in magnitude and significance. Moreover, from the interaction term, the distance to the border does change the main LTT effects in any noticeable way.[18]

Given the consistency of the estimates across specifications, column (1) is retained as the main specification from now on. Expanding the geographic coverage allows for more extensive controls and specification checks, but at the cost of adding unobserved heterogeneity and hence

[16] The estimates are again robust if the estimation in column (1) with a distance threshold of 3km is repeated, but excluding properties within 1km of each side of the border.

[17] In column (2), the interaction term's coefficient is 1.6×10^{-5} with standard error of 2.7×10^{-5} in the leases-to-sales regression, and 8.0×10^{-6} with standard error 7.0×10^{-6} in the price-to-rent regression.

[18] In column (4), the coefficient on LTT $\times$ *distance* in the leases-to-sales regression is -1.18×10^{-4} with a standard error of 5.24×10^{-5}. The city of Toronto covers an area of 630.2km^2 with a radius of 14.16km. Within the city, the community with the maximum distance to the border, approximately 18km, is the Waterfront neighborhood. Thus, the LTT effect on the leases-to-sales ratio is much the same throughout the city. The corresponding coefficient in the price-to-rent regression is statistically insignificant and quantitatively irrelevant.

complicating the interpretation of the estimates. This is especially the case for column (4).

Buy-to-own and buy-to-rent transactions Given the relative increase in leasing activity in the city after the LTT is introduced, it is natural to explore the breakdown of sales into buy-to-own and buy-to-rent transactions. Consistent with the literature, total sales volume drops in response to the LTT.[19] But this aggregate effect masks important differences in how owner-occupiers and investors respond to transaction taxes. The second panel of Table 1 shows that the new LTT has opposite effects on buy-to-own (BTO) and buy-to-rent (BTR) transactions, in spite of the same tax rate applying to both. Column (1) shows BTO transactions fall by 10.1%, while BTR transactions rise by 8.9%. These estimates are consistent across specifications.

There are three potential concerns with the finding of opposite LTT effects on BTO and BTR transactions. First, investors and home-buyers may be treated differently in the mortgage market, or with respect to the taxation of capital gains. However, these factors have been conditioned out by the differences-in-differences approach because there is no evidence these treatments differ either across city and suburban real-estate markets, or before and after the LTT is introduced. A second concern is the partial exemptions from LTT given to first-time buyers. Compared to buy-to-rent investors, home-buyers are more likely to be first-time buyers, and hence would benefit more from the partial exemptions. However, this argument points towards the LTT having a more negative effect on BTR than BTO transactions. As this is the opposite of the empirical finding, the direction of the estimated differential LTT effects is robust. Finally, there may be a concern the results are sensitive to the number of months between purchasing and leasing a property used to distinguish BTO and BTR transactions. Table A.4 shows that the results are robust to changing the 18-month threshold to 6, 12, or 24 months.

2.2.2 Effects on mobility, time-on-the-market, and sales prices

The moving hazard rate This section restricts attention to flows within the ownership market and examines first the effects of transaction taxes on individual homeowners' mobility. Unlike many previous studies that use transactions volume to measure mobility, here the data have precise observations of when an individual homeowner puts a property up for sale and when a transaction occurs.

The dynamic pattern of mobility is represented by a moving hazard function: the relationship between the rate at which moving occurs and the number of months since a homeowner purchased a property. This hazard function is estimated using the Kaplan-Meier (KM) method. The KM estimator computes the conditional probability of putting a property up for sale given

[19] Using UK property transactions data, Best and Kleven (2018) find that a temporary one percentage-point cut in the transaction tax rate during the 2008–9 stamp-duty holiday on properties worth between £125,001 and £175,000 led to a 20% increase in transactions. Using German single-family-house sales, Fritzsche and Vandrei (2019) find that a one percentage-point increase in the transaction tax leads to about 7% fewer transactions. Moreover, Dachis, Duranton and Turner (2012) show using postal-code-level data that the LTT caused a 15% decline in sales volume during a six-month window.

the time since the homeowner moved in. Specifically, a unit of observation is each month since a homeowner has bought a property and the event is putting the property up for sale given that this has not occurred so far.[20] The estimated hazard function is shown in Figure A.4. The mean length of time between purchasing a property and listing it for sale is 113 months.

Since the hypothesis of homogeneity of hazard rates over time is not rejected at the 1% level and the estimated hazard function shape is monotonic, the hazard function can be analysed using a Weibull model. The hazard function for homeowner j in a given year-month t is parameterized as

$$\hbar\left(t \mid \mathbf{x}_{jt}, \mathrm{LTT}_{jt}\right) = \varphi t^{\varphi-1} \exp\left(\beta_0 + \mathbf{x}'_{jt}\beta_{\mathbf{x}} + \mathrm{LTT}_{jt}\beta\right) ,$$

where t is time since the homeowner purchased the property, φ is a parameter linked to the gradient of the hazard function, and LTT_{jt} is an indicator for exposure to the new LTT. The vector $\mathbf{x}_{jt}$ is a rich set of controls, including indicators for the post-LTT period and being in the City of Toronto; time-varying property attributes, all interacted with property-type fixed effects; a broad range of fixed effects that flexibly control for the differential evolution of housing-market outcomes across property types and communities; and the price originally paid by the property owner. The original price proxies for non-tax-related moving costs that are positively related to a property's value in both monetary and psychological terms (Hardman and Ioannides, 1995, Han, 2008), meaning households who occupy a property of higher value face higher moving costs even in the absence of transaction taxes. Controlling for the original purchase price enables the LTT effect on residential mobility to be separated from that of other transaction costs.

The estimation results are presented in the top panel of Table 2. With the baseline specification in column (1), the LTT reduces an individual homeowner's moving hazard by 13%. Given the mean length of stay before the tax change is 113 months, this implies homeowners stay in their current home for 14 month longer on average after the LTT. This substantial lock-in effect is consistent with evidence from other countries.[21] The other columns allow for spatially differentiated time trends, substitution across borders, and changes to the city border distance thresholds, respectively. The resulting estimates of the LTT effect are not statistically different from those in column (1). Table A.5 in the appendix shows the results of repeating the estimation for the alternative sample periods 2006–2010 and 2006–2018. The estimated LTT effect remains robust to shorter and longer post-policy periods. The estimated lock-in effect of transaction taxes on residential mobility is not only substantial but also long lasting.

Across all specification, the estimated value of $\log\varphi$ is greater than zero, indicating a moving hazard that increases with time spent living in a property. Furthermore, the effect of the

[20]The estimation sample is extended to cover all property types from this point on given the ability here to control for time-varying house characteristics and homeowner histories.

[21]For example, using data from the Netherlands, Van Ommeren and Van Leuvensteijn (2005) find that a one percentage-point increase in transaction costs as a percentage of property price decreases residential mobility rates by 8.1–12.7%. Using UK data, Hilber and Lyytikäinen (2017) find that a 2 percentage-point increase in stamp duty reduces the annual rate of mobility by 2.6 percentage points.

original purchase price is substantial and significant, suggesting it is important to separate the LTT effect from that of other transaction costs.

Table 2: *Effects of the transaction tax on mobility, time-on-the-market and sales price*

	(1)	(2)	(3)	(4)
	Dependent variable: The event of moving			
LTT	-0.130**	-0.194***	-0.232***	-0.228***
	(0.064)	(0.053)	(0.088)	(0.042)
$\log(\textit{Original purchase price})$	-0.095**	-0.076*	-0.103**	-0.079***
	(0.046)	(0.043)	(0.048)	(0.023)
$\log \varphi$	0.513***	0.523***	0.519***	0.526***
	(0.010)	(0.007)	(0.010)	(0.005)
Observations	1,691,369	2,831,897	1,651,935	5,719,326
	Dependent variable: $\log(\textit{Time-on-the-market})$			
LTT	0.165***	0.163***	0.162***	0.131***
	(0.028)	(0.028)	(0.051)	(0.019)
Observations	20,937	37,397	24,570	185,082
	Dependent variable: $\log(\textit{Sales price})$			
LTT	-0.015**	-0.021***	-0.033**	-0.016***
	(0.007)	(0.006)	(0.011)	(0.003)
Observations	20,937	37,397	24,570	185,082
Distance threshold	3km	5km	5km	All
Property characteristics	Yes	Yes	Yes	Yes
City indicators ± 3 m.	Yes	Yes	Yes	Yes
City time trends	Yes	Yes	Yes	Yes
Distance LTT trends		Yes	Yes	Yes
Donut hole			2km	

Notes: Data comprise all residential property transactions from January 2006 to February 2012. Repeat sales transactions taking place within 18 months of one another are discarded. For the moving hazard estimation, a unit of observation is a homeowner whose property is listed on MLS. Homeowners' times between moves are assumed to follow a Weibull distribution. For time-on-the-market and sales prices, a unit of observation is a transaction recorded on MLS. All regressions include an indicator for the post-LTT period, an indicator for the city of Toronto, property-type fixed effects interacted with a set of time-varying property characteristics, and *year* $\times$ *property type*, *month* $\times$ *community*, *month* $\times$ *property type*, and *community* $\times$ *property type* fixed effects. In the specifications above, the distance threshold is the maximum distance to the Toronto city border for a transaction to be included in the sample. City indicators ± 3 m. are six dummy variables for transactions inside the City of Toronto during the last three months of 2007 and the first three of 2008. City time trends indicates the presence of separate time trends for transactions inside and outside the City of Toronto. Distance LTT trend denotes the inclusion of an interaction term between the LTT and a dummy variable for properties between 2.5km and 5km away from the city border in columns (2)–(3) and the interaction between the LTT and the distance from the city border in column (4). Standard errors clustered by community are in parentheses, and *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels.

Time-on-the-market The substantial lock-in effect estimated above might have implications for home-buyers' search behaviour. In particular, knowing that the LTT increases the length of stay once a transaction is made, buyers would become picker ex ante. This would in turn reduce the speed at which properties are sold. These potential effects of transaction taxes on search behaviour in the housing market have not been examined in the existing literature. This paper sheds light on this point by using transaction-level sales data to estimate the causal effect of the LTT on how long properties remain on the market before being sold.

For a given transaction, time-on-the-market is measured as the number of days between the time when a property was initially listed and the time when a sale was agreed between a buyer and a seller. Using the same regression discontinuity design laid out earlier, the estimated LTT effects on time-on-the-market are reported in the middle panel of Table 2. The baseline column (1) shows that the new LTT leads to a 16.5% increase in time-on-the-market, which is equivalent to five more days based on the pre-LTT sample mean. This estimate is robust to a rich set of controls and alternative specifications as shown in the remaining columns. These findings provide the first evidence on how transaction taxes affect time-on-the-market.

Sales prices Using the same transaction-level sales data, the bottom panel of Table 2 reports the estimated LTT effect on sales prices. Controlling for property characteristics and market conditions, the LTT causes a 1.5% decline in prices. The size of this effect is slightly larger than the new LTT rate, but is not statistically different from it. As shown in Table A.6, the transaction-level sales price LTT effect is consistent with the average sales price effects estimated using market-segment data, and is robust to using a shorter or longer sample period.

3 A dual rental and ownership markets model of housing

This section presents a model to explain the empirical findings of section 2 and to quantify the welfare costs of transaction taxes. The model includes idiosyncratic household-property match quality to understand the effect of the transaction tax on mobility, search frictions to capture the tax effect on time-on-the-market, and credit frictions and an entry decision for investors to capture the differential tax effects across the rental and ownership markets.

There is a city with two housing markets: an ownership market and a rental market. There is a unit measure of ex-ante identical properties and a constant measure ψ of households. Time is continuous, and everyone discounts future payoffs at rate r. Households exit the city exogenously at rate ρ and are replaced by an equal inflow of new households. Investors can enter freely, becoming landlords and renting out properties. Investors simply represent funds invested in real estate — they could be living within the city or elsewhere.

Properties are either up for sale, offered for rent, or not available in either market. They are owned either by those who live in them or by landlords. When not for sale or rent, properties are occupied by a renter or an owner-occupier. Some owners or renters are looking to move, and they choose whether to search in the ownership market or the rental market. Owner-occupiers looking to move put their property up for sale. Landlords choose whether to let or to sell the properties they own. At rate ρ_l, landlords receive a shock forcing them to sell their property, for example, for liquidity reasons.

The measure of buyers in the ownership market is b_o, comprising home-buyers b_h who will live in the property they buy, and investors b_k. The fraction of investors among buyers is denoted by ξ. Those looking to rent are b_l. On the other side of the two markets, properties available for sale are u_o and properties available for rent are u_l. The tightness of market i — the ratio of 'buyers' to 'sellers' — is denoted by θ_i, where $i \in \{o, l\}$ indexes the ownership (o) or rental (l) markets:

$$\xi = \frac{b_k}{b_o} \quad \text{and} \quad \theta_i = \frac{b_i}{u_i}, \quad \text{where} \quad b_o = b_h + b_k\,. \tag{1}$$

Search frictions place limits on meetings between participants in both markets. Meetings are viewings of properties that allow for offers to buy or to rent. Meeting rates are determined by constant-returns-to-scale meeting functions $\Upsilon^i(b_i, u_i)$. The rate $\Upsilon^i(b_i, u_i)/b_i$ at which buyers/renters view properties in market i is denoted by q_i. Constant returns to scale makes q_i a function of market tightness θ_i:

$$q_i = \frac{\Upsilon^i(b_i, u_i)}{b_i} = \Upsilon^i\left(1, \theta_i^{-1}\right), \quad \text{and} \quad \frac{\Upsilon^i(b_i, u_i)}{u_i} = \Upsilon^i(\theta_i, 1) = \theta_i q_i \quad \text{for} \quad i \in \{o, l\}\,. \tag{2}$$

The meeting rate $\Upsilon^i(b_i, u_i)/u_i$ for sellers in market i is $\theta_i q_i$. The meeting function is increasing in both b_i and u_i, hence q_i decreases with θ_i, while $\theta_i q_i$ increases with θ_i. Intuitively, if there are more 'buyers' relative to 'sellers' in a particular market, the meeting rate is lower for those viewing properties but higher for those offering properties for sale or to let.

Owner-occupiers or renters living in a property receive a match-specific flow value ε. At the time of a meeting when a household views a property, match quality ε between the property and the household is drawn from distribution function $G_i(\varepsilon)$ for market i. The distribution of ε could differ across markets, for instance, allowing for a 'warm glow' effect of homeownership where flow values are higher on average. From the perspective of an investor owning a property, all properties are ex-ante identical prior to being viewed by potential tenants or buyers.

Idiosyncratic match quality ε for those living in a property is a persistent variable subject to occasional shocks. These shocks represent life events that make a property less well matched to the occupying household than it originally was. Shocks arrive independently across households and across time at rate a_i, which can differ by housing tenure $i \in \{o, l\}$. For owner-occupiers, the arrival of a shock reduces match quality from ε to $\delta_o \varepsilon$, where $\delta_o < 1$ is a parameter.[22] For renters, match quality ε is reduced to 0 following a shock — effectively $\delta_l = 0$.

Following a shock, owner-occupiers and renters decide whether to move and start searching for another property to live in, with owners putting their current property up for sale. Moving is endogenous and depends on how low match quality has become relative to expectations of match quality in an alternative property, though for renters, moving depends only on the arrival

[22]The model has no shocks that increase match quality, but these would not cause households to move.

of a shock.[23]

Those who decide to move choose to buy or rent their next property by searching in the ownership market or the rental market. Households pay an idiosyncratic credit cost χ when they enter the ownership market for the first time. This can be thought of as household-specific factors affecting the cost or availability of a mortgage, such as credit histories or wealth for downpayments. More specifically, the distribution of χ calibrated using data on loan-to-value ratios and spreads between average mortgage interest rates, mortgage rates for marginal home-buyers, and the risk-free interest rate.

A household's credit cost χ persists over time, but while the household is in the rental market, χ is redrawn with probability γ from a probability distribution $G_m(\chi)$ if the arrival of an exogenous shock causes the household to move — either shocks to match quality or the landlord selling owing to an exit shock. Households exiting the city sell properties they own. When tenants choose to move or exit the city, their landlords decide whether to look for a new tenant or to sell. Households newly entering the city draw an initial value of their credit cost χ from the same distribution $G_m(\chi)$ and decide whether to buy or rent.

3.1 The ownership market

Buyers in the ownership market are either home-buyers or investors. The expected value of owning a property is the same for all investors because they face the same expected rents when their property is let, while home-buyers put different values on properties because of idiosyncratic match quality.

After a buyer has met a seller and viewed a property, revealing the quality of the match to home-buyers, the buyer and seller negotiate a price and a transaction occurs if mutually agreeable. The land transfer tax (LTT) is a proportional tax levied on the transaction price paid by the buyer. Home-buyers and investors face tax rates τ_h and τ_k, which in principle can differ.

The Bellman equation for the value K of being an investor who buys at price P_k is

$$rK = -F_k + q_o\left(U_l - (1+\tau_k)P_k - C_k - K\right) + \dot{K}\,, \tag{3}$$

where $\dot{K}$ is the derivative of the value K with respect to time t (the dependence of variables on time t is not indicated explicitly). There is a flow search cost F_k incurred by investors until they buy, $\tau_k P_k$ is the tax paid on the purchase, and C_k is any other transaction costs that investors pay. Investors meet sellers at rate q_o, and because investors have no idiosyncratic match quality with properties, this is also the rate at which they are able to buy. After buying, investors make properties available for rent and receive the common expected value U_l of being a landlord.

[23] It is possible to extend the model to have $\delta_l > 0$. However, it turns out that endogeneity of moving by renters within the rental market is quantitatively unimportant here, so the model is simplified by assuming $\delta_l = 0$.

The Bellman equation for the value B_o of being a home-buyer is

$$rB_o = -F_h + q_o \int \max\{H(\varepsilon) - C_h - (1+\tau_h)P(\varepsilon) - B_o, 0\}\, \mathrm{d}G_o(\varepsilon) - \rho B_o + \dot{B}_o\,. \tag{4}$$

Buyers make viewings of properties at rate q_o, which reveal match quality ε drawn from distribution $G_o(\varepsilon)$. The value of being an owner-occupier of a property with current match quality ε is $H(\varepsilon)$. After meeting a seller, the home-buyer negotiates a price $P(\varepsilon)$ if a deal is mutually beneficial and moves into the property. This occurs when match quality ε is sufficiently high. Home-buyers incur a flow search cost F_h while looking for properties. If a transaction goes ahead, $\tau_h P(\varepsilon)$ is the tax paid by the home-buyer, and C_h is other transaction costs such as moving costs. Home-buyers, like any other household, exogenously exit the city at rate ρ.

Since properties are ex ante identical, both owner-occupiers and landlords selling their properties have a common expected value U_o, which satisfies the Bellman equation

$$rU_o = -M + \theta_o q_o \Bigg((1-\xi) \int \max\{P(\varepsilon) - C_u - U_o, 0\}\, \mathrm{d}G_o(\varepsilon) + \xi \max\{P_k - C_u - U_o, 0\} \Bigg) + \dot{U}_o\,, \tag{5}$$

where M is the flow cost of maintaining a property paid by all owners, and C_u is a transaction cost paid by sellers. Viewings by buyers occur at rate $\theta_o q_o$, and the probabilities the meeting is with a home-buyer or an investor are the respective fractions $1-\xi$ and ξ of the pool of buyers made up of these two groups. The owner decides whether to sell, receiving price P_k if selling to an investor and $P(\varepsilon)$ if selling to a home-buyer with match quality ε.

The Bellman equation for an owner-occupier's value $H(\varepsilon)$ with current match quality ε is

$$rH(\varepsilon) = \varepsilon - M + a_o\left(\max\{H(\delta_o\varepsilon), B_o + U_o\} - H(\varepsilon)\right) + \rho\left(U_o - H(\varepsilon)\right) + \dot{H}(\varepsilon)\,, \tag{6}$$

where ε is the flow utility derived from occupying a property when match quality is currently ε. Idiosyncratic shocks arrive at rate a_o, reducing match quality to $\delta_o\varepsilon$. The household then decides whether to remain in the property and receive value $H(\delta_o\varepsilon)$, or to move out and become both a seller and a home-buyer, which has a combined value $B_o + U_o$. Moving occurs if match quality $\delta_o\varepsilon$ after a shock has become sufficiently low.

3.2 The rental market

Participants on both sides of the rental market — landlords and potential tenants — are ex ante identical. When a household meets a landlord and views a property, match quality ε is drawn from distribution $G_l(\varepsilon)$. If mutually agreeable, the household moves in and becomes a tenant. There is no commitment and no long-term contract: either the tenant or the landlord can end the relationship at any subsequent time. Rents are determined by ongoing negotiations.

The Bellman equation for the value U_l of a landlord having a property available to let is

$$rU_l = -M + \theta_l q_l \int \max\{L(\varepsilon) + \Pi(\varepsilon) - C_l - U_l, 0\} \mathrm{d}G_l(\varepsilon) + \rho_l(U_o - U_l) + \dot{U}_l \,. \tag{7}$$

The landlord meets households who are potential tenants at rate $\theta_l q_l$. If a tenant with match quality ε moves in, the landlord incurs costs C_l and receives value $L(\varepsilon)$, which includes the ongoing negotiated rents. At the point of agreeing the tenant can move in, there is also negotiation over an initial one-off fee $\Pi(\varepsilon)$ paid by the tenant to the landlord. An exogenous shock with arrival rate ρ_l forces landlords to exit, and those landlords who must sell receive value U_o.

The value of a landlord whose property is currently occupied by a tenant with match quality ε is $L(\varepsilon)$. The Bellman equation for this value function is

$$rL(\varepsilon) = R(\varepsilon) - M - M_l + (a_l + \rho)\,(\max\{U_l, U_o\} - L(\varepsilon)) + \rho_l(U_o - L(\varepsilon)) + \dot{L}(\varepsilon)\,, \tag{8}$$

where $R(\varepsilon)$ is the rent negotiated between landlord and tenant, and M_l is an extra maintenance cost incurred by landlords when properties are let. Idiosyncratic shocks received by tenants cause them to move out of rental properties at rate $a_l + \rho$, either because match quality is reduced to zero or because the household must leave the city. After a tenant moves out, the landlord decides whether to look for another tenant or sell the property, thus receiving the maximum of U_l and U_o.

The value B_l of a household searching for a property to rent satisfies the Bellman equation

$$rB_l = -F_w + q_l \int \max\{W(\varepsilon) - \Pi(\varepsilon) - C_w - B_l, 0\}\, \mathrm{d}G_l(\varepsilon) - \rho B_l + \dot{B}_l\,, \tag{9}$$

where q_l is the rate at which viewings are made, and F_w is the flow cost of searching for a rental property. Viewings reveal match quality ε drawn from a distribution $G_l(\varepsilon)$, and the household becomes a tenant if ε is sufficiently high. If the household moves into a property with match quality ε as a tenant then value $W(\varepsilon)$ is received after paying the initial fee $\Pi(\varepsilon)$ to the landlord and incurring other moving costs C_w. The Bellman equation for the value function $W(\varepsilon)$ is

$$\begin{aligned} rW(\varepsilon) = \varepsilon - R(\varepsilon) + \gamma(a_l + \rho_l)\,(G_m(Z)(B_o - \bar{\chi}) + (1 - G_m(Z))B_l - W(\varepsilon)) \\ + (1-\gamma)(a_l + \rho_l)(B_l - W(\varepsilon)) - \rho W(\varepsilon) + \dot{W}(\varepsilon)\,, \quad \text{with} \quad \bar{\chi} = E[\chi | \chi \leq Z]\,. \end{aligned} \tag{10}$$

The flow utility ε from occupying a rental property is equal to that of an owner-occupied property with the same match quality ε, but the tenant pays rent $R(\varepsilon)$. Rent negotiations ensure landlords and tenants are willing to remain matched until a shock makes it mutually agreeable to terminate the tenancy. Tenancies are brought to an end when households or landlords receive exit shocks with arrival rates ρ and ρ_l, or when shocks with arrival rate a_l reduce tenants' match quality to zero. When moving within the city, a household keeps the same credit cost χ with probability $1-\gamma$, in which case a tenant goes back to the rental market and obtains value B_l.

When a new credit cost χ is drawn, either for tenants who move (with probability γ) or for new entrants to the city, there is a threshold Z for χ below which it is preferable to enter the ownership market and buy a property rather than rent. Doing this has value B_o after the cost χ has been paid.[24] If the cost is too high, a household goes to the rental market. The expected value of a household prior to the realization of χ is an average of $B_o - \bar{\chi}$ and B_l using the probabilities $G_m(Z)$ and $1 - G_m(Z)$ as weights, where $\bar{\chi}$ denotes the expectation of the credit cost χ conditional on actually paying it.

3.3 Stocks and flows across and within the two markets

A property is in any one of four states: for sale (measure u_o), to let (measure u_l), or occupied by an owner or a renter ('occupying' in that the property is currently neither available in the markets for sale or rent). Owner-occupied and renter-occupied properties have measures h_o and h_l, respectively. These measures of the four states sum to the unit measure of all properties:

$$h_o + h_l + u_o + u_l = 1\,. \tag{11}$$

Similarly, the total measure ψ of households is distributed over four possible states: home-buyers (b_h), those looking for a property to rent (b_l), owner-occupiers (h_o), and tenants (h_l). A household occupies at most one property at a time, and households look either to buy or rent if and only if they do not currently occupy a property, so it follows that

$$h_o + h_l + b_h + b_l = \psi\,. \tag{12}$$

The measure of buyers $b_o = b_h + b_k$ in the ownership market includes the measure b_k of those looking to buy as investors. Given free entry of investors, b_k adjusts at all points in time so that the value of entry by further investors is zero:

$$K = 0\,. \tag{13}$$

Entry of first-time home-buyers to the ownership market depends on the threshold Z for the credit cost χ. The marginal new entrant ($\chi = Z$) is indifferent between owing and renting:

$$B_o - Z = B_l\,. \tag{14}$$

Credit costs are drawn from the distribution $G_m(\chi)$ by a fraction γ of tenants who move within the city because of shocks (due to either their own match quality or landlord exit) and all households new to the city. If N_l denotes the flow of tenants who move ($n_l = N_l/h_l$ is the moving rate for tenants h_l), γN_l redraw their credit cost χ. Of those, a fraction $G_m(Z)$ are below the threshold Z and so enter the ownership market as home-buyers. The same applies to the measure $\rho\psi$ of new households who enter the city. The flow of first-time home-buyers is therefore $F = (\gamma n_l h_l + \rho\psi)G_m(Z)$.

Those tenants not drawing a new credit cost when moving (probability $1 - \gamma$), or those

[24] The credit cost χ is modelled as a one-off cost, but that is equivalent in the model here to the present value of a flow credit cost paid for a period of time while a household is an owner-occupier.

whose new credit cost is above Z (probability $1 - G_m(Z)$), search in the rental market for a new property. The flow of owner-occupiers who decide to move is N_o (the moving rate for this group of measure h_o is $n_o = N_o/h_o$), and all enter the ownership market as home-buyers because they have already chosen to pay the credit cost.

Home-buyers b_h and households h_l searching for a rental property exit from this state by either completing a transaction or exiting the city. Viewings are made at rates q_i in the two markets $i \in \{o, l\}$, and the probabilities that the match quality revealed by a viewing is sufficiently high for a mutually agreeable deal with the seller/landlord are π_o and π_l in the two markets. The flows of sales S_h to home-buyers and leases S_l agreed with tenants are

$$S_h = q_o \pi_o b_h \quad \text{and} \quad S_l = q_l \pi_l b_l \,. \tag{15}$$

The laws of motion for the stocks of home-buyers b_h and households looking to rent b_l are thus

$$\dot{b}_h = n_o h_o + (\gamma n_l h_l + \rho \psi) G_m(Z) - (q_o \pi_o + \rho) b_h \,, \quad \text{and} \tag{16}$$

$$\dot{b}_l = (1-\gamma) n_l h_l + (\gamma n_l h_l + \rho \psi)(1 - G_m(Z)) - (q_l \pi_l + \rho) b_l \,. \tag{17}$$

Investors b_k make viewings at rate q_o and are able to transact at this rate because they have no idiosyncratic match quality with properties. The flow of sales to investors is S_k, which added to S_h gives total transactions S_o in the ownership market. The fraction of sales to investors is κ:

$$S_o = S_h + S_k \,, \quad \text{where} \quad S_k = q_o b_k \,, \quad \text{and} \quad \kappa = \frac{S_k}{S_o} = \frac{\xi}{\xi + (1-\xi)\pi_o} \,, \tag{18}$$

where the equation for κ in terms of the fraction of investors ξ follows from (1) and (15).

From the perspectives of sellers and landlords, the transaction rates in the two markets are

$$s_o = \frac{S_o}{u_o} = \theta_o q_o \left(\xi + (1-\xi)\pi_o\right) \,, \quad \text{and} \quad s_l = \frac{S_l}{u_l} = \theta_l q_l \pi_l \,, \tag{19}$$

and hence the laws of motion for properties for sale u_o and to let u_l are

$$\dot{u}_o = (n_o + \rho) h_o + \rho_l (h_l + u_l) - s_o u_o \,, \quad \text{and} \tag{20}$$

$$\dot{u}_l = (a_l + \rho) h_l + \kappa s_o u_o - (s_l + \rho_l) u_l \,. \tag{21}$$

Properties come up for sale if owner-occupiers move within or exit the city, or landlords are hit by an exit shock (irrespective of whether their properties are currently occupied). Properties are offered to let if tenants are hit by a match quality shock or exit the city, or investor purchases make new rental properties available. Properties come off these markets with successful transactions, or in the case of the rental market, if landlords receive an exit shock.

Finally, flows of properties on to and off the two markets imply the following laws of motion for the stocks of owner-occupiers h_o and tenants h_l:

$$\dot{h}_o = (1-\kappa)s_o u_o - (n_o + \rho)h_o\,, \quad \text{and} \tag{22}$$

$$\dot{h}_l = s_l u_l - (n_l + \rho)h_l\,. \tag{23}$$

Flows and stocks in the ownership and rental markets are summarized in Figure A.5 and A.6.

3.4 Functional forms, parameter restrictions, and bargaining protocols

The meeting functions $\Upsilon^i(b_i, u_i)$ for $i \in \{o, l\}$ have Cobb-Douglas functional forms:

$$\Upsilon^i(b_i, u_i) = A_i b_i^{1-\eta_i} u_i^{\eta_i}\,, \quad \text{hence} \quad q_i = A_i \theta_i^{-\eta_i}\,, \tag{24}$$

where A_i is productivity in arranging viewings in market i, and η_i are the elasticities of buyers' and renters' viewing rates with respect to market tightnesses θ_i (see 1 and 2). These parameters can differ across markets. New match qualities ε are drawn from Pareto distributions

$$G_i(\varepsilon) = 1 - \left(\frac{\varepsilon}{\zeta_i}\right)^{-\lambda_i} \quad \text{for } i \in \{o, l\}\,, \tag{25}$$

with ζ_i being the minimum possible draw in market i, and λ_i specifying the distribution shape, in particular, how compressed are realizations of ε towards the minimum. Expected match quality from a viewing in market i is $E_i[\varepsilon] = \zeta_i \lambda_i / (\lambda_i - 1)$ for $\lambda_i > 1$. Draws of the credit cost χ are from a log Normal distribution with mean and standard deviation parameters μ and σ:

$$G_m(\chi) = \Phi\left(\frac{\log \chi - \mu}{\sigma}\right)\,, \quad \text{implying} \quad \bar{\chi} = e^{\mu + \frac{\sigma^2}{2}} \frac{\Phi\left(\frac{\log Z - \mu - \sigma^2}{\sigma}\right)}{\Phi\left(\frac{\log Z - \mu}{\sigma}\right)}\,, \tag{26}$$

where $\Phi(\cdot)$ is the standard Normal CDF, and $\bar{\chi}$ is the expectation of χ conditional on $\chi \leq Z$.

A parameter restriction is imposed so that match-quality shocks in the ownership market are sufficiently large (δ_o is far enough below 1) that some, but not all, owner-occupiers require only one idiosyncratic shock to trigger moving. As has been stated earlier, match-quality shocks to renters are sufficiently large ($\delta_l = 0$) that all tenant moves are exogenous.

The bargaining protocol over the terms of transactions (prices and rents) in all meetings between agents is Nash bargaining. Sellers (whoever they may be) have bargaining power ω_o when selling to a home-buyer, and bargaining power ω_k when selling to an investor. Landlords have bargaining power ω_l in relation to tenants in both their initial meeting and in any subsequent rent negotiations.

3.5 General equilibrium in rental and ownership markets

This section studies the equilibrium allocation of properties and households across the two markets, and the volumes of transactions and their terms (prices and rents) within each market.

3.5.1 Decisions made by owner-occupiers and home-buyers

If the seller of a property meets a home-buyer who draws match quality ε and were to agree to a sale at price $P(\varepsilon)$ then the home-buyer surplus would be $\Sigma_o^h(\varepsilon) = H(\varepsilon) - (1+\tau_h)P(\varepsilon) - C_h - B_o$ and the seller surplus $\Sigma_o^u(\varepsilon) = P(\varepsilon) - C_u - U_o$. The Nash bargaining problem is to choose $P(\varepsilon)$ to maximize $(\Sigma_o^u(\varepsilon))^{\omega_o}\left(\Sigma_o^h(\varepsilon)\right)^{1-\omega_o}$, where the surpluses of both must be non-negative for a transaction to go ahead. The first-order condition is $\Sigma_o^u(\varepsilon)/\Sigma_o^h(\varepsilon) = \omega_o/((1-\omega_o)(1+\tau_h))$, which determines how the joint surplus $\Sigma_o(\varepsilon) = \Sigma_o^h(\varepsilon) + \Sigma_o^u(\varepsilon)$ is to be shared.

In the absence of a transaction tax τ_h, the surplus would be divided according to bargaining powers in line with the usual Nash rule. However, a positive transaction tax rate skews the division in favour of the buyer. Intuitively, owing to the proportional tax, the joint surplus $\Sigma_o(\varepsilon) = H(\varepsilon) - C_h - C_u - B_o - U_o - \tau_h P(\varepsilon)$ is increased by agreeing a lower price, and this lower price increases the buyer's surplus. The resulting split of the surplus is

$$\Sigma_o^h(\varepsilon) = (1-\omega_o^*)\Sigma_o(\varepsilon) \quad \text{and} \quad \Sigma_o^u(\varepsilon) = \omega_o^*\Sigma_o(\varepsilon), \quad \text{where} \quad \omega_o^* \equiv \frac{\omega_o}{1+\tau_h(1-\omega_o)}, \tag{27}$$

and the seller's share ω_o^* is below bargaining power ω_o. The price that delivers the division of the surplus in (27) is $P(\varepsilon) = C_u + U_o + \omega_o^*\Sigma_o(\varepsilon)$, which results in the joint surplus being

$$\Sigma_o(\varepsilon) = \frac{H(\varepsilon) - C_h - B_o - (1+\tau_h)(C_u+U_o)}{1+\tau_h\omega_o^*}. \tag{28}$$

As match quality ε is observable and surplus is transferable, transactions go ahead if $\varepsilon \geq y_o$, where the transaction threshold y_o is the level of match quality where the joint surplus is zero:

$$\Sigma_o(y_o) = 0. \tag{29}$$

Using (25), the proportion π_o of home-buyer viewings that lead to sales and the average transaction price P for home-buyer purchases are

$$\pi_o = \int_{y_o} \mathrm{d}G_o(\varepsilon) = \left(\frac{y_o}{\zeta_o}\right)^{-\lambda_i}, \quad \text{and} \quad P = \frac{1}{\pi_o}\int_{y_o} P(\varepsilon)\mathrm{d}G_o(\varepsilon) = \frac{\omega_o^*\Sigma_o}{\pi_o} + C_u + U_o. \tag{30}$$

Prior to the realization of ε, the ex-ante joint surplus from a home-buyer viewing is

$$\Sigma_o = \int_{y_o} \Sigma_o(\varepsilon)\mathrm{d}G_o(\varepsilon). \tag{31}$$

For existing owner-occupiers, there is a moving decision to be made when a match quality shock is received. Since the value function $H(\varepsilon)$ is increasing in ε, owner-occupiers decide to move if the current level of match quality becomes sufficiently low. The condition for moving is $\varepsilon < x_o$, where the moving threshold x_o is the level of match quality such that the value of continuing to occupy a property equals the sum of the outside options B_o and U_o of being both a buyer and a seller in the ownership market:

$$H(x_o) = B_o + U_o. \tag{32}$$

The condition that some owner-occupiers require only one shock to trigger moving is $\delta_o y_o < x_o$.

The endogenous moving rate n_o is derived from the distribution of match quality over existing owner-occupiers together with the moving threshold x_o. The evolution over time of the distribution of owner-occupiers' match quality depends on idiosyncratic shocks and moving decisions. Surviving matches of households and properties differ along two dimensions, the initial level of match quality, and the number of shocks received since the match formed. By using the Pareto distribution (25) of new match quality, appendix A.2.2 shows that the endogenous moving rate is

$$n_o = a_o - \frac{a_o \zeta_o^{\lambda_o} \delta_o^{\lambda_o} x_o^{-\lambda_o}}{h_o} \int_{\upsilon \to -\infty}^{t} e^{-\left(\rho + a_o(1-\delta_o^{\lambda_o})\right)(t-\upsilon)} (1 - \xi(\upsilon)) \theta_o(\upsilon) q_o(\upsilon) u_o(\upsilon) \mathrm{d}\upsilon \,, \quad (33)$$

where $u_o(t)$ explicitly indicates the dependence of u_o on time t. Given the moving threshold x_o, the moving rate n_o displays history dependence due to persistence in match quality.

3.5.2 Decisions made by landlords and tenants

For landlords and tenants, it is possible to work backwards from ongoing rent negotiations to analyse their behaviour when they first meet during a viewing. Consider a tenant who has already moved into a property with match quality ε, so any transaction and moving costs are sunk. The tenant's surplus from remaining in the property is $\Lambda^w(\varepsilon) = W(\varepsilon) - B_l$, where the outside option is going back to the rental market because the tenant's credit cost χ of becoming a home-buyer does not change unless a shock occurs. The landlord's surplus from keeping the tenant is $\Lambda^l(\varepsilon) = L(\varepsilon) - U_l$, which assumes the outside option of putting the property back on the rental market is better than selling it ($U_l \geq U_o$), as will be confirmed. Both $W(\varepsilon)$ and $L(\varepsilon)$ depend on the rent $R(\varepsilon)$ paid.

The Nash bargaining problem has rent $R(\varepsilon)$ maximize $\left(\Lambda^l(\varepsilon)\right)^{\omega_l} \left(\Lambda^w(\varepsilon)\right)^{1-\omega_l}$, where ω_l is the landlord's bargaining power. There is no commitment to rent payments at any future date. The rent $R(\varepsilon)$ affects the surpluses through $L(\varepsilon)$ and $W(\varepsilon)$ in equations (8) and (10), noting that $\partial L(\varepsilon)/\partial R(\varepsilon) = -\partial W(\varepsilon)/\partial R(\varepsilon)$, so the first-order condition is $\Lambda^l(\varepsilon)/\Lambda^w(\varepsilon) = \omega_l/(1-\omega_l)$. The joint surplus $\Lambda(\varepsilon) = \Lambda^l(\varepsilon) + \Lambda^w(\varepsilon) = W(\varepsilon) + L(\varepsilon) - B_l - U_l$ is therefore divided according to the bargaining powers of the two parties as $\Lambda^l(\varepsilon) = \omega_l \Lambda(\varepsilon)$ and $\Lambda^w(\varepsilon) = (1-\omega_l)\Lambda(\varepsilon)$.

With rents negotiated this way, tenants move out only after a match quality shock, or if leaving the city or their landlord is forced to sell up. Tenants' moving rate n_l within the city is

$$n_l = a_l + \rho_l \,. \quad (34)$$

Now consider a landlord meeting a potential tenant during a viewing that reveals match quality ε. If the landlord agrees the tenant can move in after paying a fee $\Pi(\varepsilon)$ then the two parties incur costs C_l and C_w, respectively.[25] Note that the fee $\Pi(\varepsilon)$ is separate from

[25] The transaction costs C_l and C_w are a type of fixed matching cost, for example, the costs of finding out about the tenant, because they are incurred before bargaining over the rent takes place (see Pissarides, 2009).

the rent $R(\varepsilon)$, which is the subject of ongoing negotiation once the tenant moves in. At this stage, the tenant's surplus is $\Sigma_l^w(\varepsilon) = W(\varepsilon) - \Pi(\varepsilon) - C_w - B_l$ and the landlord's surplus is $\Sigma_l^l(\varepsilon) = L(\varepsilon) + \Pi(\varepsilon) - C_l - U_l$.

If it is mutually agreeable for the tenant to move in (both surpluses are positive) then there is Nash bargaining over the fee $\Pi(\varepsilon)$ with the landlord and tenant having the same bargaining powers ω_l and $1-\omega_l$ as in rent negotiations. The joint surplus $\Sigma_l(\varepsilon) = \Sigma_l^l(\varepsilon) + \Sigma_l^w(\varepsilon)$ is

$$\Sigma_l(\varepsilon) = W(\varepsilon) + L(\varepsilon) - B_l - U_l - C_w - C_l\,, \tag{35}$$

which is divided according to $\Sigma_l^l(\varepsilon) = \omega_l \Sigma_l(\varepsilon)$ and $\Sigma_l^w(\varepsilon) = (1-\omega_l)\Sigma_l(\varepsilon)$. In terms of the surpluses $\Lambda^l(\varepsilon)$ and $\Lambda^w(\varepsilon)$ once the tenant has moved in, the surpluses on meeting can be expressed as $\Sigma_l^l(\varepsilon) = \Lambda^l(\varepsilon) + \Pi(\varepsilon) - C_l$ and $\Sigma_l^w(\varepsilon) = \Lambda^w(\varepsilon) - \Pi(\varepsilon) - C_w$. Since the bargaining problem for new rents is the same as for ongoing rents, the subsequent surplus split is $\Lambda^l(\varepsilon) = \omega_l \Lambda(\varepsilon)$ and $\Lambda^w(\varepsilon) = (1-\omega_l)\Lambda(\varepsilon)$, where $\Sigma_l(\varepsilon) = \Lambda(\varepsilon) - C_l - C_w$, and hence Nash bargaining over the fee $\Pi(\varepsilon)$ yields

$$\Pi(\varepsilon) = \Pi = (1-\omega_l)C_l - \omega_l C_w\,, \tag{36}$$

which is independent of match quality ε. A lease is agreed if $\varepsilon \geq y_l$, where the leasing threshold y_l is the level of match quality ε where the joint surplus $\Sigma_l(\varepsilon)$ from (35) is zero:

$$\Sigma_l(y_l) = 0\,. \tag{37}$$

The proportion π_l of viewings of properties to let that lead to leases and the average rent R are

$$\pi_l = \int_{y_l} \mathrm{d}G_l(\varepsilon) = \left(\frac{y_l}{\zeta_l}\right)^{-\lambda_l}, \quad \text{and} \quad R = \frac{1}{\pi_l}\int_{y_l} R(\varepsilon)\mathrm{d}G_l(\varepsilon)\,. \tag{38}$$

Prior to the realization of ε, the ex-ante expected joint surplus from a rental-market viewing is

$$\Sigma_l = \int_{y_l} \Sigma_l(\varepsilon)\mathrm{d}G_l(\varepsilon)\,. \tag{39}$$

3.5.3 Entry decisions of investors

An investor's surplus from a transaction at price P_k is $\Sigma_k^k = U_l - (1+\tau_k)P_k - C_k - K$ and the seller's surplus is $\Sigma_k^u = P_k - C_u - U_o$. If there are mutual gains from a deal, the price P_k is determined by Nash bargaining, where the seller has bargaining power ω_k when faced with an investor. The joint surplus $\Sigma_k = \Sigma_k^k + \Sigma_k^u$ is split according to $\Sigma_k^u/\Sigma_k^k = \omega_k/((1-\omega_k)(1+\tau_k))$, so the tax τ_k shifts the division of the surplus in favour of the investor:

$$\Sigma_k^k = (1-\omega_k^*)\Sigma_k \quad \text{and} \quad \Sigma_k^u = \omega_k^*\Sigma_k\,, \quad \text{where} \quad \omega_k^* \equiv \frac{\omega_k}{1+\tau_k(1-\omega_k)}\,. \tag{40}$$

Since the joint surplus $\Sigma_k = U_l - C_k - C_u - U_o - K - \tau_k P_k$ is unaffected by considerations of match quality, either all investors are willing to buy or none, so an equilibrium with entry of investors occurs if and only if Σ_k is non-negative. When this is true, investors buy property at the rate q_o they meet sellers, and the price paid by all investors is

$$P_k = C_u + U_o + \omega_k^* \Sigma_k\,. \tag{41}$$

With this price, the joint surplus Σ_k from a meeting between an investor and a seller is

$$\Sigma_k = \frac{U_l - (1+\tau_k)U_o - (1+\tau_k)C_u - C_k}{1+\tau_k\omega_k^*}\,. \tag{42}$$

Note that a non-negative joint surplus Σ_k implies the value of having a property to let is always above the value of having a property for sale ($U_l \geq U_o$). Thus, after purchasing a property, an investor always prefers to keep it rented out.[26] Landlords sell properties only when hit by exit shocks, which arrive at rate ρ_l.

The free-entry condition (13) and the Bellman equation (3) for investors' value K require

$$\Sigma_k = \frac{F_k}{(1-\omega_k^*)q_o}\,, \tag{43}$$

which shows the surplus Σ_k rises with the tightness of the ownership market. Intuitively, the viewing rate q_o decreases when there are more buyers relative to sellers, so investors must be compensated in equilibrium by a higher surplus $(1-\omega_k^*)\Sigma_k$ for them to enter.

3.6 Welfare

Welfare Ω is the sum of the values of all incumbents in the city (homeowners, tenants, landlords, and including owners of unsold houses who have left the city) plus the present values of the payoffs received by those who enter the city. Exit (with value 0) is already accounted for in incumbents' values.

A consistent analysis of welfare requires specifying what the government does with the tax revenue $\Gamma = \tau_h P S_h + \tau_k P_k S_k$ it collects. Revenue is assumed to be spent on public goods of an equal value, or equivalently, on reducing other taxes. The flow benefits of Γ/ψ per person could be added to the Bellman equations of city residents ($H(\varepsilon)$, $W(\varepsilon)$, B_o, and B_l), but rather than changing these equations, equivalently, the present value Ω_τ of the stream of tax revenue Γ is included in welfare Ω. This present value satisfies the Bellman equation $r\Omega_\tau = \Gamma + \dot{\Omega}_\tau$.

The expected payoff of someone entering the city prior to the realization of the credit cost χ is $B_e = (1 - G_m(Z))B_l + G_m(Z)(B_o - \bar{\chi})$, where Z is the credit-cost threshold for entering the ownership market and $\bar{\chi}$ is the average value of χ for those who do so. With a steady population ψ and exit at rate ρ, there are $\rho\psi$ new entrants per unit of time. The expected present value Ω_e of all entrant values satisfies the Bellman equation $r\Omega_e = \rho\psi B_e + \dot{\Omega}_e$.

With H, L, and W denoting the average values of $H(\varepsilon)$, $L(\varepsilon)$, and $W(\varepsilon)$ over the distributions of ε for all surviving matches, welfare is $\Omega = h_o H + h_l(L+W) + b_h B_o + b_l B_l + b_k K +$

[26] In other words, pure 'flippers' — those who buy and sell shortly afterwards — are not present in the model.

$u_oU_o + u_lU_l + \Omega_\tau + \Omega_e$. Appendix A.2.6 shows that welfare Ω satisfies the differential equation

$$r\Omega = h_oQ_h + h_lQ_l - M - h_lM_l - b_hF_h - b_kF_k - b_lF_w - S_o((1-\kappa)C_h + \kappa C_k + C_u) \\ - S_l(C_l + C_w) - (\gamma n_l h_l + \rho\psi)G_m(Z)\bar{\chi} + \dot{\Omega}\,, \quad (44)$$

where Q_h and Q_l denote the average levels of current match quality ε across the h_o owner-occupiers and h_l tenants respectively.[27] Prices and rents drop out from welfare Ω because these are just transfers among market participants. Maintenance costs, flow search costs, non-tax transaction costs, and credit costs are implicitly treated as resource costs that show up as deductions from welfare. This assumes transaction costs reflect the time and resources of market participants and intermediaries that are consumed in completing transactions. Likewise, credit costs, for example, interest-rate spreads on mortgages, are treated as reflecting resources used up by banks. Transaction tax revenue does not appear as a deduction in (44) because it pays for public goods of an equivalent value, or allows other taxes to be reduced while still funding a given amount of public expenditure (of whatever resource cost and utility value).

The average match qualities Q_h and Q_l appearing in the welfare equation (44) are shown in appendix A.2.5 to satisfy the following pair of differential equations:

$$\dot{Q}_h = \frac{(1-\kappa)s_ou_o}{h_o}\left(\frac{\lambda_o}{\lambda_o - 1}y_o - Q_h\right) - (a_o - n_o)\left(Q_h - \frac{\lambda_o}{\lambda_o - 1}x_o\right), \quad \text{and} \quad (45)$$

$$\dot{Q}_l = \frac{s_lu_l}{h_l}\left(\frac{\lambda_l}{\lambda_l - 1}y_l - Q_l\right), \quad (46)$$

which depend on differences between Q_h and Q_l and average new match qualities $\lambda_oy_o/(\lambda_o - 1)$ and $\lambda_ly_l/(\lambda_l - 1)$ in the two markets, and between Q_h and average surviving match quality $\lambda_ox_o/(\lambda_o - 1)$ after match-quality shocks received by owner-occupiers.

3.7 The steady state of the model

For constant tax rates τ_h and τ_k and other parameters, the model predicts the rental and ownership markets converge to a steady state where the fractions of properties and households in various states $(h_o, h_l, u_o, u_l, b_h, b_k)$ are constant over time. This steady state features a constant measure of investors b_k and proportion ξ of buyers they comprise, and constant market tightnesses θ_o and θ_l. The homeownership rate h is defined as the fraction of the population ψ who own a property they occupy h_o or are selling a property they occupied. This is $h = (h_o + (1-\kappa)u_o)/\psi$, where former owner-occupiers selling properties account for a fraction $1-\kappa$ of properties u_o on the market. There is also a steady state for the demographics of owner-occupiers compared to tenants, such as the average age difference α between the groups.

Among those occupying properties, there is a stationary distribution of match quality, which

[27] This assumes all private benefits of owning or renting properties are social benefits. It is possible to envisage other policy distortions that might drive a wedge between private and social benefits such as the tax treatment of owners' implicit rental income or mortgage-interest deductibility.

implies the moving rate n_o in (33) is constant for owner-occupiers moving within the city.[28] Taking account of exit from the city, the expected lengths of occupation of a property by homeowners and tenants are $T_{mo} = 1/(n_o + \rho)$ and $T_{ml} = 1/(n_l + \rho)$ respectively, where the moving rate n_l within the city for tenants is from (34). There is also a steady state for the fraction ϕ of first-time buyers among all purchases by home-buyers.[29]

The average numbers of viewings v_o and v_l needed to sell or lease a property are respectively $v_o = 1/((1-\xi)\pi_o + \xi)$ and $v_l = 1/\pi_l$, and the expected times on the market for properties to sell and lease are $T_{so} = 1/s_o$ and $T_{sl} = 1/s_l$. From the perspective of home-buyers and potential tenants, the expected times taken successfully to find properties are $T_{bh} = 1/(q_o\pi_o)$ and $T_{bl} = 1/(q_l\pi_l)$. On average across buyers in the ownership market, the average time to complete a transaction is $T_{bo} = (1-\kappa)T_{bh} + \kappa T_{bk}$, where $T_{bk} = 1/q_o$ is the expected time taken by investors. This average time can be expressed as $T_{bo} = 1/(q_o(\xi + (1-\xi)\pi_o))$.

These predictions are used to calibrate the model's parameters, allowing quantitative predictions to be made about the effects of transaction taxes and their implications for welfare.

4 Effects of transaction taxes

The effects of higher transaction taxes τ_k, τ_u in the model are governed by the behavioural response of homeowners, renters, and investors, which have implications both within and across the ownership and rental markets. This section lays out the intuition for how the model can account for the empirical findings in section 2. It first discusses the heterogeneous effects of the transaction tax on owner-occupiers and investors, explaining how a higher transaction tax leads to a rise in buy-to-rent transactions and a fall in buy-to-own transactions, even if both are subject to the same transaction tax. It also explains the intuition for how a higher transaction tax leads to longer times between moves and longer times taken to sell within the ownership market, and a lower price-to-rent ratio and homeownership rate.

[28] The following expression for the steady-state moving rate is derived in appendix A.3:

$$n_o = a_o \left(\frac{\rho + a_o\left(1-\delta_o^{\lambda_o}\right) - \rho\delta_o^{\lambda_o}\left(\frac{y_o}{x_o}\right)^{\lambda_o}}{\rho + a_o\left(1-\delta_o^{\lambda_o}\right) + a_o\delta_o^{\lambda_o}\left(\frac{y_o}{x_o}\right)^{\lambda_o}} \right).$$

[29] It is shown in appendix A.5 that the average age difference α between owner-occupiers and tenants and the fraction ϕ of first-time buyers are:

$$\alpha = \left(1 + \frac{\rho}{\rho + n_l + q_l\pi_l}\right)\left(\frac{1}{\rho} - \frac{1}{\rho + \frac{\gamma n_l q_l \pi_l}{\rho + n_l + q_l\pi_l}}\right), \quad \text{and} \quad \phi = \frac{\rho\left(1 + \frac{n_o+\rho}{q_o\pi_o}\right)}{n_o + \rho\left(1 + \frac{n_o+\rho}{q_o\pi_o}\right)}.$$

4.1 Behaviour of households

There are three household behavioural responses to a higher tax rate τ_h. First, as seen in (28), higher τ_h has a direct effect in reducing the ownership-market joint surplus Σ_o because part of the surplus is absorbed by higher taxes. This happens because the costs of transactions both now and when moving again in the future are higher. The fall in the total surplus reduces the value of being a home-buyer,[30] which is seen in the steady-state Bellman equation (4):

$$(r+\rho)B_o = (1-\omega_o^*)q_o\Sigma_o - F_h\,. \tag{47}$$

The fall in the value B_o of being a home-buyer reduces renters' incentive to enter the ownership market given the indifference condition (14), implying a lower equilibrium credit-cost threshold Z. Thus, there are fewer first-time buyers.

Second, a higher tax rate τ_h raises the cost of moving, which makes homeowners more tolerant of worse match quality, meaning a lower moving threshold x_o. This is seen from the homeowner's decision to move in (32). Just as the fall in the total surplus implies a fall in the value of being a home-buyer B_o, it also reduces the value of being a seller U_o, noting that the steady-state Bellman equation of a seller (5) implies

$$rU_o = \theta_o q_o\left(\omega_o^*(1-\xi)\Sigma_o + \omega_k^*\xi\Sigma_k\right) - M\,. \tag{48}$$

The expected value of a seller equals the expected value from selling to a homeowner or an investor. When the share of investors ξ is small, the fall in the total surplus Σ_o is the dominant effect and U_o is lower. Together with the fall in B_o, equation (32) implies a lower moving threshold x_o, which results in longer average times between moves.

Finally, home-buyers become pickier, meaning a higher transaction threshold y_o. Because moving decisions are endogenous and match quality has persistence, home-buyers can reduce the future incidence of moving — and lower the tax they expect to pay — by beginning with better match quality. This intuition is confirmed by equation (29) where the total surplus is an increasing function of y_o (28 shows the value function $H(\varepsilon)$ of a homeowner is an increasing function). A higher tax τ_h reduces the total surplus, requiring a higher y_o for the total surplus to be zero. This higher transaction threshold results in longer average times taken to sell.

All three household behavioural responses to the higher tax rate contribute to the fall in buy-to-own transactions by reducing purchases made by first-time buyers and existing homeowners.

4.2 Behaviour of investors

Similar to the effect on homeowners, the direct effect of a higher tax rate τ_k is to reduce entry of buy-to-rent investors. However, investors as landlords do not have to sell their properties and pay the transaction tax again just because a tenant moves, unlike owner-occupiers who have to

[30] There is a positive effect from higher τ_h as it implies buyers get a larger share of the total surplus (27 shows there is a fall in the effective bargaining power ω_o^* of the seller). This effect is small given that τ_h is small.

buy again and pay the tax every time they want to move. Buy-to-rent investors thus have an implicit tax advantage — even if they face the same tax rates. Intuitively, investors can spread the tax over a longer holding period, reducing the negative direct effect on their entry decision.

It is not possible directly to estimate average holding periods of investors owing to data constraints and the right-censoring problem when the sample is not long enough to observe investors' completed holding periods. However, information on the flow of buy-to-rent transactions and the stock of properties in the rental market can be used to derive the implied average holding period of investors relative to that of owner-occupiers.

The holding period of an investor in the model is the inverse of the exit rate ρ_l. Using the property stock-flow diagram in Figure A.6, a longer investor holding period is an implication of investors' share of transaction flows being smaller than their share of the stock of properties. To see this, equations (23) and (21) imply the stock of properties in the rental market satisfies

$$\dot{h}_l + \dot{u}_l = \kappa s_o u_o - \rho_l (h_l + u_l) \,, \tag{49}$$

which shows that the share of investors transactions κ governs inflows of properties into the rental market, while the exit rate ρ_l governs outflows of properties from the rental market. Using the definition of the homeownership rate $h = h_o + (1-\kappa)u_o$, the law of motion for the stock of owner-occupied properties in (22), and the fact that $\kappa u_o \approx 0$ (the typical stock of houses for sale u_o is around 1–2% of the total stock, and investors' share of transactions κ is about 5%), the steady-state average holding period of an investor relative to a homeowner is

$$\frac{n_o + \rho}{\rho_l} \approx \left(\frac{1-h}{h}\right) \Big/ \left(\frac{\kappa}{1-\kappa}\right) \quad \text{as} \quad \kappa u_o \approx 0 \,. \tag{50}$$

Empirically, the flow of purchases by investors (κ) is much smaller than the stock of properties investors hold ($1-h$). In Toronto before the tax increase, the share of investors' transactions κ was about 5%, whereas the homeownership rate h was 54%. This implies the average holding period of an investor is much longer than the average holding period of an owner-occupier.

An important consequence of these observations is that the direct effect of a transaction tax on investors is smaller than its direct effect on owner-occupiers. An intuition for the workings of the model is shown in Figure 1. The direct effects of a higher transaction tax are reduced entry of investors into the rental market while boosting entry of households, both of which push up the rent-to-price ratio R/P. However, the direct effect is larger for households than for investors, resulting in a fall in the homeownership rate, that is, an increase in $1-h$. Through the free-entry condition, the equilibrium effect of the higher rent-to-price ratio creates incentives for more investors to enter. If the average investor holding period is sufficiently long so that the direct negative effect of the tax on investors is weak and dominated by the positive equilibrium effect, the model implies a rise in buy-to-rent transactions.

4.3 The connections between the model and the empirical findings

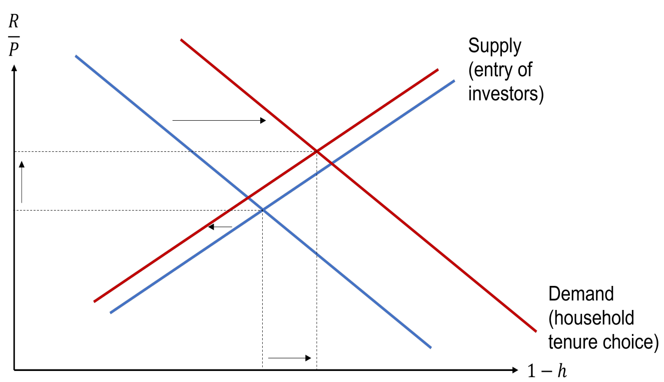
The empirical results in Table 1 reveal the novel heterogeneous effects of transaction taxes across the owner-occupied and rental markets, while those in Table 2 highlight the role of search frictions thorough the tax effect on time-to-sell, and the role of moving decisions through the effect on time-to-move. It is explained above how the renting versus owning decision and the entry decisions of investors together rationalize the heterogeneous tax effects. On the other hand, search frictions related to meeting rates and match quality are crucial for understanding the rise in time-to-sell and time-to-move within the ownership market.

To make quantitative and welfare statements, the next section calibrates the model to match the City of Toronto market before the LTT increase. When implementing rise in LTT in the model, the estimated effect on the monthly moving hazard is chosen as a target. The model's predictions for other untargeted moments then serve as a basis for external validation.

The empirical estimates rely on comparing the City of Toronto to other areas in the GTA before and after the LTT increase. If these two regions are segmented markets, a model focusing on the tax effect in one isolated region with a fixed population would be consistent with how the estimated effects of the LTT inside the city were derived using the empirical difference-in-differences approach across borders, as the aim there is to isolate the impact of LTT inside the city from other changes. On the other hand, if there were movement of people between the two regions then increasing LTT will have an effect on the size of population outside the city.

Theoretically, this effect can be isolated by comparing the baseline case of a fixed population in the model of section 3 to an extension allowing for mobility across regions presented in section 5.4.2. In short, the expected value of entering the city (related to the values of being a homeowner or a renter) will fall with the higher transaction tax. However, if the housing stock

Figure 1: *Intuition for the heterogeneous effects of a higher transaction tax*



Notes: The vertical axis is the rent-to-price ratio R/P, and the horizontal axis is the stock of properties in the rental market $1-h = h_l + u_l + \kappa u_l$, comprising renter-occupied h_l, the existing stock of properties for rent u_l, and new purchases of properties by investors κu_l.

in the city is fixed, since houses must be owned or rented by someone in equilibrium, the value of living inside the city must adjust through a fall in house prices. That means there is a larger fall in house prices predicted by this version of the model compared to the case of segmented markets. Since the population does not change by much in equilibrium, the analysis of quantities and welfare is very similar to the baseline model. Empirical support for this prediction is seen in the findings from Table 1 and Table 2 where the 'donut' specifications yield broadly similar results to the non-'donut' specifications.

5 Quantitative effects of transaction taxes in the model

This section uses the model developed in section 3 to quantify the differential effects of the LTT on owner-occupiers and investors even though the tax rate τ_k on investors is the same as the tax rate τ_h faced by owner-occupiers, both before and after the LTT increase. It also quantifies the increase in leasing activity in the rental market and the lower price-to-rent ratio, the decline in mobility within the ownership market, as well as the rise in time-on-the-market.

The model is calibrated to match features of ownership and rental markets in the City of Toronto before the LTT change. It is then solved using the transaction tax rates prevailing in the city before and after the February 2008 city-level LTT was introduced to derive predictions for the housing-market outcomes studied empirically, as well as for welfare. As explained in section 2, the effective LTT rate rises from 1.5% to 2.8%, an increase of 1.3 percentage points.

5.1 Calibration

The model is calibrated to the City of Toronto housing market in the period January 2006–January 2008 before the LTT change. The tax rates faced by both home-buyers and buy-to-rent investors are set to the effective LTT prior to the change, $\tau_k = \tau_h = 0.015$. The parameters of the model are calibrated to match a list of targets given in Table 3, and the implied parameter values are reported in Table 4. The data sources of all targets are detailed in appendix A.4, and appendix A.5 explains how the calibration procedure works.

In summary, there are three broad sets of targets. The first $(\psi, B_e, \omega_o/\eta_o, \omega_l/\eta_l)$ is directly imposed. The measure of households is chosen to be the same as the measure of properties, that is, $\psi = 1$. Although entry to the city is exogenous in the baseline model, for consistency, the calibration selects parameters where the expected value of entering the city B_e is zero, matching the zero value for those who exit. Finally, the bargaining powers of sellers and landlords are set to be the same as the corresponding elasticities of the meeting functions for the two market, that is, $\omega_o = \eta_o$ and $\omega_l = \eta_l$.

The second set of targets is related to the extensive margin across the ownership and rental markets. These targets are the homeownership rate h, the fraction κ of buy-to-rent purchases among total purchases, the fraction of first-time buyers ϕ, the difference α in the average

Table 3: *Calibration targets*

Targets	Notation	Value
Directly imposed targets		
Equal numbers of households and properties	ψ	1
No incentive for further entry of households into the city	B_e	0
Bargaining powers equal to meeting-function elasticities	$\omega_o/\eta_o = \omega_l/\eta_l$	1
Empirical targets		
Average buy-to-own transaction price	P	\$402k
Effective land transfer tax for all buyers	$\tau_h = \tau_k$	1.5%
Homeownership rate	h	54%
Fraction of purchases made by buy-to-rent investors	κ	5.4%
Fraction of first-time buyers among all home-buyers	ϕ	40%
Difference in average ages of owner-occupiers and renters	α	8.3
Average price-rent ratio for same properties	P_k/R	14.5
Price paid by investors relative to average paid by home-buyers	P_k/P	99%
Non-tax transaction costs of buyers relative to price	$C_h/P = C_k/P_k$	0%
Property maintenance costs relative to price	M/P	2.6%
Landlords' extra maintenance/management costs relative to rent	M_l/R	8%
Seller transaction costs relative to price	C_u/P	4.5%
Landlord transaction costs relative to rent	C_l/R	8.3%
Fraction of landlord transaction costs charged to tenant	Π/C_l	0%
Flow search costs of home-buyers relative to price	F_h/P	3.1%
Flow search costs of investors relative to home-buyers	F_k/F_h	1
Flow search costs of tenants relative to home-buyers	F_w/F_h	1.1
Sellers' average time on the market	T_{so}	0.161
Buyers' average time on the market	T_{bo}	0.206
Landlords' average time on the rental market	T_{sl}	0.066
Average viewings per sale	v_o	20.6
Average viewings per lease	v_l	10.3
Average time between moves for owner-occupiers	T_{mo}	9.25
Average time between moves for tenants	T_{ml}	3.04
Percentage decline of owner-occupier moving rate after new LTT	β	13%
Capitalized credit costs of marginal home-buyer relative to price	Z/P	0.48
Ratio of credit costs of marginal and average home-buyers	$Z/\bar{\chi}$	2.11
Sources of the targets for credit costs		
Risk-free real interest rate	r_f	1.86%
Average real mortgage interest rate	$\bar{r}_c$	4.93%
Real mortgage interest rate of the marginal home-buyer	r_c	6.43%
Initial loan-to-value ratio of first-time buyers	ℓ	80%
Mortgage term	T_c	25

Notes: All time units are in years. See appendix A.4 for data sources and appendix A.5 for the procedure. The targets for Z/P and $Z/\bar{\chi}$ derive from those for r_f, $\bar{r}_c$, r_c, ℓ, and T_c as explained in appendix A.4.

ages of owner-occupiers and renters, investors' price-to-rent ratio P_k/R, and the ratio of prices paid by investors to prices paid by home-buyers P_k/P. The other key targets here are for the capitalized credit costs of marginal home-buyers relative to price Z/P, and the ratio of marginal to average credit costs $Z/\bar{\chi}$. As explained in appendix A.4, these credit-cost targets can themselves be derived from information about mortgage interest rate spreads, the mortgage term, and loan-to-value ratios.

Note that it is not necessary to take a stance on the presence or size of any 'warm glow' effect of homeownership in the calibration. The parameter ζ_l is determined as a residual given the calibrated costs of owning versus renting and the choices of households reflected in the homeownership rate. In particular, a larger average mortgage rate spread over the risk-free rate means the credit costs of becoming an owner-occupier are higher, so for a given homeownership rate and other costs, the 'warm glow' from ownership would need to be larger.

The calibration makes use of credit-cost information not only for an average borrower but also a marginal borrower. Empirically, marginal borrowers are those who do not qualify for loans from major banks and must instead borrow at higher rates from other financial institutions. Together with the average credit cost, this provides information about the shape of the credit-cost distribution over households, in particular, how many households have a credit cost close to the marginal homeowner.

The third set of targets matches search behaviour and costs incurred within the ownership and rental markets. The targets for search behaviour are viewings per sale v_o, viewings per lease v_l, times on the market for buyers T_{bo} and sellers T_{so} in the ownership market, landlords' time on the rental market T_{sl}, and the expected times between moves for homeowners T_{mo} and tenants T_{ml}. The targets for ownership-market costs are homeowners' maintenance cost M, transaction costs excluding taxes for buyers and sellers (C_k, C_h, C_u), and flow search costs of buyers (F_k, F_h). The targets for all of these are given as fractions of the appropriate price P or P_k. The targets for rental-market costs are the extra maintenance costs M_l faced by landlords, landlords' transaction costs C_l, and flow search costs of tenants F_w, all as a fraction of rent R, and the fraction of landlords' transaction costs passed on to tenants. The calibration also matches the model's moving rate response β to the LTT using estimated effect from section 2.

Finally, the units of utility can be normalized so that the model matches the average transaction price P. This means all utility payoffs and costs can be interpreted as dollar equivalents.

5.2 Quantitative effects of transaction taxes

The effects of increasing the transaction tax rates τ_h and τ_k from 1.5% to 2.8% for both home-buyers and investors are reported in Table 5. The steady state of the model is computed for each tax rate using the procedure described in section A.3. The changes in variables across the steady states are reported as log differences for consistency with the econometric estimates of the LTT effects on logarithms of housing-market outcomes from section 2.

Consistent with the econometric evidence and the discussion in section 4, the model predicts that buy-to-own (BTO) and buy-to-rent (BTR) transactions move in opposite directions when

Table 4: *Calibrated parameters*

Parameter description	Notation	Value
Number of households relative to the number of properties	ψ	1
Discount rate for future housing-market payoffs	r	3.3%
Households' exit rate from the city	ρ	4.3%
Investors' exit rate	ρ_l	0.7%
Property maintenance cost	M	10.4
Landlords' extra maintenance/management costs	M_l	2.2
Minimum new match quality in the ownership market	ζ_o	32.1
Minimum new match quality in the rental market	ζ_l	23.4
Home-buyer shape parameter of new match quality distribution	λ_o	30.1
Tenant shape parameter of new match quality distribution	λ_l	33.3
Arrival rate of match quality shocks in the ownership market	a_o	8.1%
Arrival rate of match quality shocks in the rental market	a_l	27.9%
Size of match quality shock in ownership market	δ_o	0.850
Fraction of tenants drawing new credit cost after moving shock	γ	8.3%
Parameter for mean of the distribution of credit costs	μ	5.0
Parameter for standard deviation of the distribution of credit costs	σ	0.67
Transaction costs of buyers excluding taxes	$C_k = C_h$	0
Transaction costs of sellers	C_u	18.1
Transaction costs of landlords	C_l	2.3
Transaction costs of tenants	C_w	0.83
Flow search costs of home-buyers and investors	$F_k = F_h$	12.6
Flow search costs of prospective tenants in the rental market	F_w	13.6
Viewing productivity parameter in the ownership market	A_o	112
Viewing productivity parameter in the rental market	A_l	170
Elasticity of ownership-market meetings with respect to sellers	η_o	0.458
Elasticity of rental-market meetings with respect to landlords	η_l	0.733
Bargaining power of sellers meeting a home-buyer	ω_o	0.458
Bargaining power of sellers meeting an investor	ω_k	0.218
Bargaining power of landlords meeting a prospective tenant	ω_l	0.733

Notes: All time units are in years, and all payoff and cost parameters are measured in thousands of dollars. These parameters exactly match the targets in Table 3 using the calibration procedure from appendix A.5.

the transaction tax rises. Sales to home-buyers fall, while sales to investors rise, despite the two facing the same tax rates. Quantitatively, the model predicts a 17% fall in BTO transactions and a 5% rise in BTR transactions, which captures a substantial part of the observed rise in BTR activity. BTR transactions constitute a relatively small fraction of total transactions, so the overall effect is that total transactions fall. Combined with additional entry of buy-to-rent investors, the ratio of leases to sales increases by 21%, matching closely the observed 23% rise.

These changes across the rental and ownership market imply the homeownership rate falls by around 2.4 percentage points. Data on the homeownership rate in Toronto is not available at the micro level or at high frequencies, so the causal effect of the LTT change cannot be estimated. However, the empirical findings for BTR transactions and leases indicate that the

Table 5: *Simulations of the model following an increase in the transaction tax rate*

Variable	Model prediction	Econometric evidence
Time-to-move for homeowners	13% (matched)	13%
Buy-to-own (BTO) transactions	-17%	-10.1%
Buy-to-rent (BTR) transactions	5.0%	8.9%
Time-to-sell	7.8%	16.5%
Leases-to-sales ratio	21%	23%
Price-to-rent ratio	-1.5%	-3.9%
Average sales price	-1.4%	-2.0%
Homeownership rate	-4.5% (-2.4 p.p.)	-
Transaction tax revenue	44%	-
Effective LTT tax rate	Increased from 1.5% to 2.8% (1.3 p.p.)	

Notes: The responses of variables are reported as log differences. The solution procedure to find the predictions of the model is described in appendix A.3.

homeownership rate would fall after the LTT increase, all else equal.[31]

The predicted average price paid drops by 1.4%, capturing a substantial part of the drop seen empirically. Interestingly, the percentage decline in prices is larger than the 1.3 percentage-point rise in the tax rate.[32] The impact on the average price reflects the expectation that a given property will be subject to the tax each time it is sold, and thus the expected future incidence of the tax is capitalized into property prices. The drop in the average price also drives a 1.5% reduction in the price-to-rent ratio, which reflects the equilibrium effect seen in Figure 1, contributing to the rise in BTR transactions.

The model predicts that the log difference between tax revenue $\Gamma = \tau_h P S_h + \tau_k P_k S_k$ before and after is only 44%, while the log difference of the tax rates is 62% (from 1.5% to 2.8%). This discrepancy is explained by erosion of the tax base: total transactions go down and the average price drops, so the tax base shrinks.

5.3 Welfare effects of transactions taxes

The calibrated model predicts the welfare costs of the LTT are substantial. The new LTT causes welfare to fall by an amount equivalent to 113% of the extra tax revenue it generates. Formally, this measure of welfare loss is the ratio of $r\Delta\Omega$, the change in flow welfare derived from equation (44), to $\Delta\Gamma$, the change in the flow of tax revenue.

The welfare loss arises from distortions across and within ownership and rental markets,

[31] Simply looking at the aggregate data on the homeownership rate in Toronto reveals a rising trend prior to the LTT increase and a flattening out afterwards. The period of stagnation in the homeownership rate coincides with a rising fraction of BTR transactions in the aggregate.

[32] A simple analysis of tax incidence might suggest that prices should change by less than the tax rate because buyers have some bargaining power — see equation (27). That equation also shows a proportional transaction tax reduces the effective bargaining power of sellers, contributing to a lower price.

and both are large.[33] Distortions across the two markets generate a loss equivalent to 60% of the extra tax revenue, which accounts for half of the total loss of 113% of the extra tax revenue. Within the markets, rental- and ownership-market distortions generate losses of 14% and 40% of tax revenue respectively. Overall, the presence of the rental market in the analysis accounts for a welfare loss of 74% of the extra tax revenue beyond the loss within the ownership market itself, or around two thirds of the total loss.

The welfare loss across the two markets results from the drop in the homeownership rate. Some households with low enough credit costs who would otherwise have gained from being owner-occupiers decide to remain renters owing to the extra costs imposed by the transaction tax both now and expected again when they move in the future. The size of this welfare loss largely depends on the distribution of credit costs, which is calibrated using data on mortgage spreads. This is because the credit-cost distribution across households is the relevant source of heterogeneity for the owing-versus-renting decision — everyone shares the same ex-ante expectation of housing utility in the two markets, so there is no lack of substitutability between owner-occupied and rental properties in terms of preferences. The decline in homeownership also adds to the welfare loss through an increase in rental management costs.[34]

Within the ownership market, the welfare loss is mainly due to the fall in match quality, partly offset by lower non-tax transaction costs saved because moving is less frequent. It is also offset by home-buyers being more picky, though that comes at the cost of having to search for longer. Quantitatively, the large size of the welfare loss relates to the indivisibility of housing: households are taxed on the whole value of a property purchase, not the marginal improvement in match quality that comes from moving. The welfare loss within the rental market is much smaller and mainly reflects increased transaction costs from more leases being arranged.

In terms of distributional effects (and ignoring how the extra tax revenue is used), the biggest losers from the LTT are home-buyers who immediately face a higher tax, followed by renters, who lose because it has become harder to transition to being a home-owner, and then existing homeowners who will face the tax when they next move. Least affected are existing landlords who are not directly hit by the tax unless they need to sell.

[33] Using equation (44) in steady state, the change in welfare $\Delta\Omega$ after the tax rise can be decomposed as follows:

$$r\Delta\Omega = (h_o\Delta Q_h - F_h\Delta b_h - C_h\Delta S_h - C_u\Delta S_o) + (h_l\Delta Q_l - F_w\Delta b_l - (C_l + C_w)\Delta S_l) \\ + ((Q_h + \Delta Q_h)\Delta h_o + (Q_l + \Delta Q_l - M_l)\Delta h_l - F_k\Delta b_k - C_k\Delta S_k - \Delta((\gamma n_l h_l + \rho\psi)G_m(Z)\bar{\chi}))\,.$$

The first block of terms result from changes within the ownership market, the second from changes within the rental market, and the third from changes across the two markets.

[34] It is important to note that the model does not imply a monotonic relationship between the homeownership rate and welfare. This can be seen from the final term in the expression for welfare (44), where credit costs associated with increasing homeownership have a negative impact on welfare, all else equal.

5.4 Discussion of the quantitative results

5.4.1 Credit-cost heterogeneity and the size of the tax effects across markets

The extent of the reallocation of households and properties from the ownership market to the rental market after the higher transaction tax depends crucially on the mass of marginal home-buyers prior to the tax increase. These marginal households have a credit cost at the threshold Z. A higher transaction tax lowers the threshold Z, turning these marginal buyers away from the ownership market and leaving them as tenants.

Intuitively, the closer the threshold Z is to the mode of the probability distribution of credit costs χ, the higher is the mass of marginal buyers. Thus, an important empirical target is the mortgage interest rate gap between the marginal and average buyers, which is what determines the probability mass of credit costs near the threshold. As explained in appendix A.4, the gap used in the baseline calibration is 1.5%. This is based on micro-level mortgage data from the Bank of Canada showing that the interest-rate gap between the average borrower and those with low credit scores is around 3% for a typical 5-year mortgage loan. Given that the marginal buyer is likely to be able to pay a lower interest rate after the first five years, the baseline calibration assumes a smaller 1.5% gap to apply to the whole period of mortgage borrowing.

If the gap were increased to 3%, essentially assuming marginal buyers cannot refinance at a better rate after the first five years, the mass of marginal buyers would be smaller. This is depicted in the right panel of Figure 2 compared to the baseline case in the left panel. Increasing the transaction tax lowers the credit threshold Z, but when the mass of marginal buyers is smaller, this has less impact on the number of renters who become buyers.

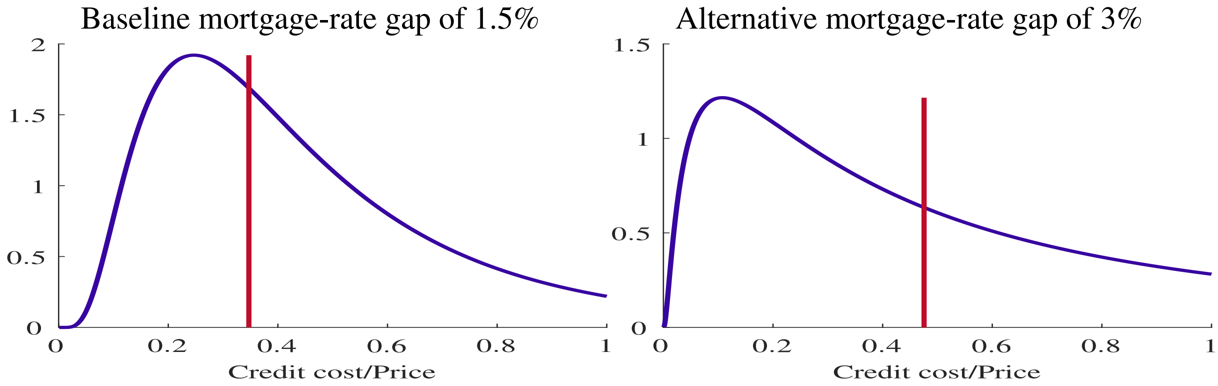
The total welfare cost of the LTT with this alternative calibration is lower at 79% of the extra tax revenue. Distortions across the two markets imply losses of 25% of revenue, distortions within the rental market 5%, and distortions within the ownership market 49%. In this case, the presence of rental market in the analysis accounts for 40% of the total loss. The smaller across-markets loss is due to the smaller 2% predicted increase in buy-to-rent transactions rather than the baseline 5%. The full results can be found in Table A.7. This case provides a lower bound on the quantitative impact of the LTT on owning versus renting and its implications for welfare.

5.4.2 Endogenizing the city population

As well as changing housing tenure, choosing to move less often, and being more picky when buying a property, households could also adjust their location as a way of avoiding the LTT. Studying this margin requires an extension of the model to allow for an endogenous population within the city boundaries.

The simplest change to the model to allow for this is to have a free-entry condition for households as well as investors. Households living in the city continue to exit exogenously at rate ρ, but the flow of households entering the city is endogenous. Inflows continue until

Figure 2: *Distributions of credit costs under different calibrations of mortgage-rate gaps*



Notes: The two panels show the distributions of credit costs implied by different calibrations of the mortgage interest rate gap between marginal and average home-buyers. The red lines show the equilibrium threshold Z/P.

households are indifferent between entering or not:

$$B_e = 0\,, \quad \text{where} \quad B_e = G_m(Z)\,(B_0 - \bar{\chi}) + (1 - G_m(Z))B_l\,. \tag{51}$$

The expected payoff from entering the city is denoted by B_e, where the values conditional on entry (whether a household is a home-buyer or renter) are calculated as before. The consequence of this change to the model is that ψ, the ratio of the number of households in the city to the number of properties, is now an endogenous variable rather than a parameter. This variable is determined by the additional equation (51), but conditional on ψ, all other equations of the model continue to apply. Note that the calibration strategy already imposes $B_e = 0$ in the initial steady state, so this new version of the model is also calibrated in exactly the same way. The only change is that after the LTT rises, the ratio ψ adjusts so that B_e remains zero, and this change in ψ may have implications for how other variables respond.

Performing the same quantitative analysis of the LTT in this version of the model, it turns out that the city population adjusts very little to the tax rise. While the LTT reduces the value of being located with the city boundaries, the city's housing stock must be owned or rented by some household in equilibrium. This causes a larger 3.1% fall in house prices compared to 1.4% in the model with an exogenous population and segmented marketed within and outside the city. However, since the population does not change by much in equilibrium, the analysis of quantities and welfare is very similar to the earlier results. For example, BTO transactions fall by 17.3% instead of 17.0%, and BTR transactions rise by 4.9% instead of 5.0%.

5.5 Alternative ways of raising tax revenue from the housing market

5.5.1 A tax on investors

A key feature of the analysis in this paper is in allowing for free entry of buy-to-rent investors, which helps to understand why the LTT has different effects on BTO and BTR transactions. It also has implications for the distortions created by transaction taxes. Since homeowners are more heavily affected by the same transaction tax rate than investors, a higher tax rate increases distortions in the allocation of housing across the ownership and rental markets.

This novel effect can be isolated by considering a hypothetical tax regime with different tax rates for owner-occupiers and investors. Taking the same increase in τ_h as before, the tax rate τ_k on investors can be raised to such a level where there is no change in the equilibrium homeownership rate. The required change in τ_k for this is from 1.5% to 5.7%. This alternative tax system raises slightly more revenue (up by 52% instead of 44%), but not much more because buy-to-rent investors are a small minority and do not transact frequently on average. Importantly, the welfare loss in this case is considerably smaller, being only 42% of the extra revenue raised instead of 113% with an equal increase in the tax rates τ_h and τ_k.

Intuitively, this exercise shuts down the extensive margin, keeping the homeownership rate unchanged by putting up higher barriers to entry for investors. This offsets the implicit advantage investors receive from not needing to pay the LTT as often as owner-occupiers do when tax rates rise by the same amount. The welfare loss is smaller because the unequal tax rates undo the effects of this distortion.

However, increasing τ_k ever further to raise the homeownership rate would ultimately lead to large welfare costs because uncreditworthy households would be forced into the ownership market owing to a lack of rental properties. This would result in their paying very high borrowing costs, reducing welfare through the final term in (44). Deep-pocketed investors play an important role in providing access to housing without everyone having to incur credit costs.

5.5.2 A tax on housing consumption

A key message of recent policy discussions such as the 'Henry Review' by the Australian government and the 'Mirrlees Review' by the UK government (Henry, Harmer, Piggott, Ridout and Smith, 2009, Mirrlees, Adam, Besley, Blundell, Bond *et al.*, 2010) is the potential reduction in distortions by replacing transaction taxes with taxes on housing consumption, referred to as property taxes in many countries. Such taxes can be represented in the model by an increase in the 'maintenance cost' M of owing a property. This section conducts an experiment by raising M so as to generate the same extra tax revenue as the increase in the LTT. The initial transaction tax is set at 1.5%, and then M is increased so as to generate a 44% rise in tax revenue, which is equivalent to the additional tax revenue raised by the new LTT (see Table 5).

As shown in Table A.8, this alternative method of raising the same tax revenue as the new

LTT has negligible effects on the allocation of properties and households across rental and ownership markets. Its only noticeable effect is lower prices and price-to-rent ratios.

Following the earlier discussion using Figure 1, a higher transaction tax has a smaller negative direct effect on investors because of their relatively longer holding period compared to owner-occupiers — they do not need to sell and buy another property when their tenants move. Using a housing consumption tax instead removes investors' implicit tax advantage over owner-occupiers because all owners have to pay more tax independent of their transaction frequency, implying similar falls in the demand for properties by investors and homeowners. This can be represented using diagram similar to Figure 1 but where the reduction in entry of investors is similar to the increase in entry of renters, resulting in negligible change in the stock of properties $1-h$ in the rental market, but an increase in rent-to-price ratio. The quantitative results show a mild increase in the homeownership rate h of 0.09%, resulting in a mild welfare improvement of 0.02% of extra tax revenue.

6 Conclusions

Using a unique dataset on property sales and leasing transactions, this paper documents two novel effects of a higher transaction tax. First, there is a rise in buy-to-rent transactions and a fall in owner-occupier transactions despite the same tax applying to both. Second, there is a simultaneous fall in the sales-to-leases and price-to-rent ratios.

This paper builds a tractable model with free entry of investors and where households choose renting or owning, with entry to the ownership market incurring a heterogeneous cost of accessing credit. The calibrated model explains the empirical findings and points to a novel welfare cost of transaction taxes. A higher transaction tax distorts the allocation of properties across the two markets by reducing the homeownership rate, as well as distorting the allocation within the ownership market by reducing mobility. The calibrated model implies a substantial welfare loss equivalent to 113% of the increase in tax revenue, with about two thirds due to the analysis allowing for the presence of a rental market.

References

AGARWAL, S., CHAU, K. W., HU, M. R. AND WAN, W. X. (2022), "Tobin tax policy, housing speculation, and property market dynamics", working paper, SSRN. 1

ANENBERG, E. AND BAYER, P. (2020), "Endogenous sources of volatility in housing markets: The joint buyer-seller problem", *International Economic Review*, **61(3)**:1195–1228 (Aug.). 6

BACHMANN, R. AND COOPER, D. (2014), "The ins and arounds in the U.S. housing market", working paper 14–3, Federal Reserve Bank of Boston. 6

BENJAMIN, J. D., COULSON, N. E. AND YANG, S. X. (1993), "Real estate transfer taxes and property values: The Philadelphia story", *Journal of Real Estate Finance and Economics*, **7(2)**:151–157 (Sep.). 1, 10

BÉRARD, G. AND TRANNOY, A. (2018), "The impact of the 2014 increase in the real estate transfer taxes on the French housing market", *Economics and Statistics*, **500-501-502**:179–200 (Oct.). 10

BESLEY, T., MEADS, N. AND SURICO, P. (2014), "The incidence of transaction taxes: Evidence from a stamp duty holiday", *Journal of Public Economics*, **119**:61–70 (Nov.). 1

BEST, M. AND KLEVEN, H. (2018), "Housing market responses to transaction taxes: Evidence from notches and stimulus in the U.K.", *Review of Economic Studies*, **85(1)**:157–193 (Jan.). 1, 5, 13

BØ, E. E. (2021), "Buy to let: The role of rental markets in housing booms", working paper, Oslo Met Housing Lab. 6, 8

BORDO, M. D., REDISH, A. AND ROCKOFF, H. (2015), "Why didn't Canada have a banking crisis in 2008 (or in 1930, or 1907, or ...)", *Economic History Review*, **68(1)**:218–243 (Feb.). 10

CHAMBERS, M., GARRIGA, C. AND SCHLAGENHAUF, D. E. (2009), "Accounting for changes in the homeownership rate", *International Economic Review*, **50(3)**:677–726 (Aug.). 6

CHO, Y., LI, S. M. AND UREN, L. (2021), "Stamping out stamp duty: Property or consumption taxes?", working paper 1/2021, Centre for Applied Macroeconomic Analysis. 7

DACHIS, B., DURANTON, G. AND TURNER, M. A. (2012), "The effects of land transfer taxes on real estate markets: evidence from a natural experiment in Toronto", *Journal of Economic Geography*, **12(2)**:327–354 (Mar.). 1, 5, 6, 8, 9, 10, 13, 46, 47

DAVIDOFF, I. AND LEIGH, A. (2013), "How do stamp duties affect the housing market?", *Economic Record*, **89(286)**:396–410 (Sep.). 1

DÍAZ, A. AND JEREZ, B. (2013), "House prices, sales, and time on the market: A search-theoretic framework", *International Economic Review*, **54**:837–872 (Aug.). 6

EEROLA, E., HARJUNEN, O., LYYTIKÄINEN, T. AND SAARIMAA, T. (2021), "Revisiting the effects of housing transfer taxes", *Journal of Urban Economics*, **124(Jul.)**:103367. 1, 5

FISHER, J. D. M. AND GERVAIS, M. (2011), "Why has home ownership fallen among the young?", *International Economic Review*, **52(3)**:883–912 (Aug.). 6

FLOETOTTO, M., KIRKER, M. AND STROEBEL, J. (2016), "Government intervention in the housing market: Who wins, who loses?", *Journal of Monetary Economics*, **80**:106–123 (Jun.). 6

FRITZSCHE, C. AND VANDREI, L. (2019), "The German real estate transfer tax: Evidence for single-family home transactions", *Regional Science and Urban Economics*, **74**:131–143 (Jan.). 1, 5, 13

GABRIEL, S. A. AND ROSENTHAL, S. S. (2015), "The boom, the bust and the future of homeownership", *Real Estate Economics*, **43(2)**:334–374 (Summer). 1

GABROVSKI, M. AND ORTEGO-MARTI, V. (2019), "The cyclical behavior of the Beveridge curve in the housing market", *Journal of Economic Theory*, **181**:361–381 (May). 6

GENESOVE, D. AND HAN, L. (2012), "Search and matching in the housing market", *Journal of Urban Economics*, **72**:31–45 (July). 6, 67

GLAESER, E. L. AND GYOURKO, J. (2007), "Arbitrage in housing markets", working paper 13704, NBER. 6

GOODMAN, L. S. AND MAYER, C. (2018), "Homeownership and the American Dream", *Journal of Economic Perspectives*, **32(1)**:31–58 (Winter). 1, 6

GREENWALD, D. L. AND GUREN, A. (2021), "Do credit conditions move house prices?", working paper 29391, NBER. 6

GUREN, A. (2018), "House price momentum and strategic complementarity", *Journal of Political Economy*, **126(3)**:1172–1218 (Jun.). 6

HALKET, J., PIGNATTI, M. AND DI CUSTOZA, M. (2015), "Homeownership and the scarcity of rentals", *Journal of Monetary Economics*, **76**:107–123 (Nov.). 6

HALTOM, R. (2013), "Why was Canada exempt from the financial crisis?", *Federal Reserve Bank of Richmond Econ Focus*, **2013(4)**:22–25. 10

HAN, L. (2008), "Hedging house price risk in the presence of lumpy transaction costs", *Journal of Urban Economics*, **64(2)**:270–287 (Sep.). 14

HAN, L. AND STRANGE, W. C. (2015), "The microstructure of housing markets: Search, bargaining, and brokerage", in G. Duranton, J. V. Henderson and W. C. Strange (eds.), *Handbook of Regional And Urban Economics*, vol. 5, North Holland, chap. 13, pp. 813–886. 6

HARDMAN, A. M. AND IOANNIDES, Y. M. (1995), "Moving behavior and the housing market", *Regional Science and Urban Economics*, **25(1)**:21–39 (Feb.). 14

HEAD, A., LLOYD-ELLIS, H. AND SUN, H. (2014), "Search, liquidity, and the dynamics of house prices and construction", *American Economic Review*, **104**:1172–1210 (Apr.). 6

HENRY, K., HARMER, J., PIGGOTT, J., RIDOUT, H. AND SMITH, G. (2009), "Australia's future tax system", Final report, Commonwealth of Australia. 5, 41

HILBER, C. A. L. AND LYYTIKÄINEN, T. (2017), "Transfer taxes and household mobility: Distortion on the housing or labor market?", *Journal of Urban Economics*, **101**:57–73 (Sep.). 1, 5, 14

HUANG, N., LI, H. AND YANG, Y. (2021), "The impact of transaction taxes on the evolution of housing prices", working paper, Nanjing Audit University. 1

IOANNIDES, Y. M. AND ZABEL, J. E. (2019), "Housing and labor market vacancies and Beveridge curves: Theoretical framework and illustrative statistics", in Y. M. Ioannides (ed.), *Recent Developments in the Economics of Housing*, vol. II, Edward Elgar. 6

KAAS, L., KOCHARKOV, G., PREUGSCHAT, E. AND SIASSI, N. (2021), "Low homeownership in Germany — a quantitative exploration", *Journal of the European Economic Association*, **19(1)**:128–164 (Feb.). 6

KOPCZUK, W. AND MUNROE, D. (2015), "Mansion tax: The effect of transfer taxes on the residential real estate market", *American Economic Journal: Economic Policy*, **7(2)**:214–257 (May). 1

LUNDBORG, P. AND SKEDINGER, P. (1999), "Transaction taxes in a search model of the housing market", *Journal of Urban Economics*, **45(2)**:385–399 (Mar.). 6

MÄÄTTÄNEN, N. AND TERVIÖ, M. (2020), "Welfare effects of housing transaction taxes", discussion paper 12551, CEPR. 1, 5

MILLS, J., MOLLOY, R. AND ZARUTSKIE, R. (2019), "Large-scale buy-to-rent investors in the single-family housing market: The emergence of a new asset class", *Real Estate Economics*, **47(2)**:399–430 (Summer). 8

MIRRLEES, J., ADAM, S., BESLEY, T., BLUNDELL, R., BOND, S., CHOTE, R., GAMMIE, M., JOHNSON, P., MYLES, G. AND POTERBA, J. (2010), *Dimensions of Tax Design*, Oxford University Press. 5, 41

MOEN, E. R., NENOV, P. T. AND SNIEKERS, F. (2021), "Buying first or selling first in housing markets", *Journal of the European Economic Association*, **19(1)**:38–81 (Feb.). 6

NGAI, L. R. AND SHEEDY, K. D. (2020), "The decision to move house and aggregate housing-market dynamics", *Journal of the European Economic Association*, **18(5)**:2487–2531 (Oct.). 3, 6

NGAI, L. R. AND TENREYRO, S. (2014), "Hot and cold seasons in the housing market", *American Economic Review*, **104(12)**:3991–4026 (Dec.). 6

PIAZZESI, M., SCHNEIDER, M. AND STROEBEL, J. (2020), "Segmented housing search", *American Economic Review*, **110(3)**:720–59 (Mar.). 6

PISSARIDES, C. A. (2009), "The unemployment volatility puzzle: Is wage stickiness the answer?", *Econometrica*, **77(5)**:1339–1369 (Sep.). 25

POTERBA, J. M. (1992), "Taxation and housing: Old questions, new answers", *American Economic Review*, **82(2)**:237–242 (May). 5

SCHMIDT, D. J. (2022), "Property transfer taxes, residential mobility, and welfare", discussion paper 2022-042, Tinbergen Institute. 5, 7

SLEMROD, J., WEBER, C. AND SHAN, H. (2017), "The behavioral response to housing transfer taxes: Evidence from a notched change in D.C. policy", *Journal of Urban Economics*, **100**:137–153 (Jul.). 1

SOMMER, K. AND SULLIVAN, P. (2018), "Implications of US tax policy for house prices, rents, and homeownership", *American Economic Review*, **108(2)**:241–274 (Feb.). 6

VAN OMMEREN, J. AND VAN LEUVENSTEIJN, M. (2005), "New evidence of the effect of transaction costs on residential mobility", *Journal of Regional Science*, **45(4)**:681–702 (Nov.). 1, 14

WALKS, A. (2014), "Canada's housing bubble story: Mortgage securitization, the state, and the global financial crisis", *International Journal of Urban and Regional Research*, **38(1)**:256–284 (Jan.). 10

WHEATON, W. C. (1990), "Vacancy, search, and prices in a housing market matching model", *Journal of Political Economy*, **98(6)**:1270–1292 (Dec.). 6

A Appendices

A.1 Data, further estimation results, and robustness checks

A.1.1 Evaluating the comprehensiveness of the MLS rental listings data

Since the use of rental listings data in this paper is relatively new to the literature, it is important to examine how comprehensive the Toronto MLS rental listings data are. This section shows through webscraping that MLS data provide an unusually high coverage of long-term and verifiable rental listings in the City of Toronto compared to other online rental platforms. Specifically, MLS data capture over 90% of rental properties listed on the second-most popular rental listing platform in Toronto.

The Multiple Listing Service (MLS) is a database created by the Canadian Real Estate Association (CREA) and used by real-estate professionals to share and access information about properties for sale or lease. It enables cooperation among real-estate agents and brokers, who can pool their listings and share commissions on property transactions. An alternative popular rental listings platform is *Toronto Rentals* (hereafter referred to as TR), which is the second-largest website serving Toronto and the surrounding GTA since 1995.

For the period between 23$^{\text{rd}}$ November 2022 and 23$^{\text{rd}}$ February 2023, all rental listings from the MLS (`realtor.ca`) and from TR (`rentals.ca/toronto`) were webscraped. For each MLS listing, information was collected on the MLS ID, the address (as a string), the listing date, the number of bedrooms, the number of bathrooms, and the asking rent. For each TR listing, the information collected was the address (specified in terms of latitude and longitude), the listing date, the number of bedrooms, the number of bathrooms, and the asking-rent range.

To compare the two scraped datasets, MLS address strings were cleaned and parsed to apply Google Maps AIP to geocode the coordinates of each listing. The MLS listings were then matched with the TR listings by the geocoded address, the number of rooms, and a window around the listing date. Since a property might be listed on one platform first and later on another platform, the comparison was restricted to properties listed on TR between 25$^{\text{th}}$ November and 5$^{\text{th}}$ December 2022. The exercise is to check how many of these listings were also on the MLS during the same or surrounding time period.

Figure A.1 shows a map of the locations of rental listings in the City of Toronto. Yellow dots indicate MLS listings. Grey dots are TR listings that match with listings in the MLS data. Red dots are TR listings that are at least 200 metres away from the closest MLS listing, which is taken as an indicator that these listings were not included in the MLS.

There were 4,432 unique MLS records during the period studied, and the TR dataset includes a total of 3,516 entries. Out of the TR listings, 295 were not matched with MLS records, accounting for approximately 8.4% of the TR data. This fraction is likely to be overestimated because of inaccuracies in the manual matching of MLS listings' coordinates.

There are also short-term rental websites such as *Kijiji* in Toronto. However, listings on these platforms are not included in the analysis for several reasons. First, unlike MLS or TR listings, Kijiji listings are unverified and less reliable, with most of them posted by anonymous users. Second, Kijiji users often forget to remove their listings when they are no longer active, making it questionable in what time window a listing counts as active. Third, Kijiji listings do not provide precise address information and can only be identified at neighbourhood level. Finally, unlike MLS or TR listings, most Kijiji listings are for short-term lets that are distinct from the longer-term rentals in the main analysis.

A.1.2 Descriptive statistics

A.1.3 Empirical specifications

The econometric specification is a variant of the regression discontinuity design developed by Dachis, Duranton and Turner (2012) applied to a broader set of housing-market outcomes.

Let t denote the time before ($t < 0$) or after ($t > 0$) the imposition of the LTT. The time unit used is

Figure A.1: *Rental listings in Toronto between 25th November and 5th December 2022*

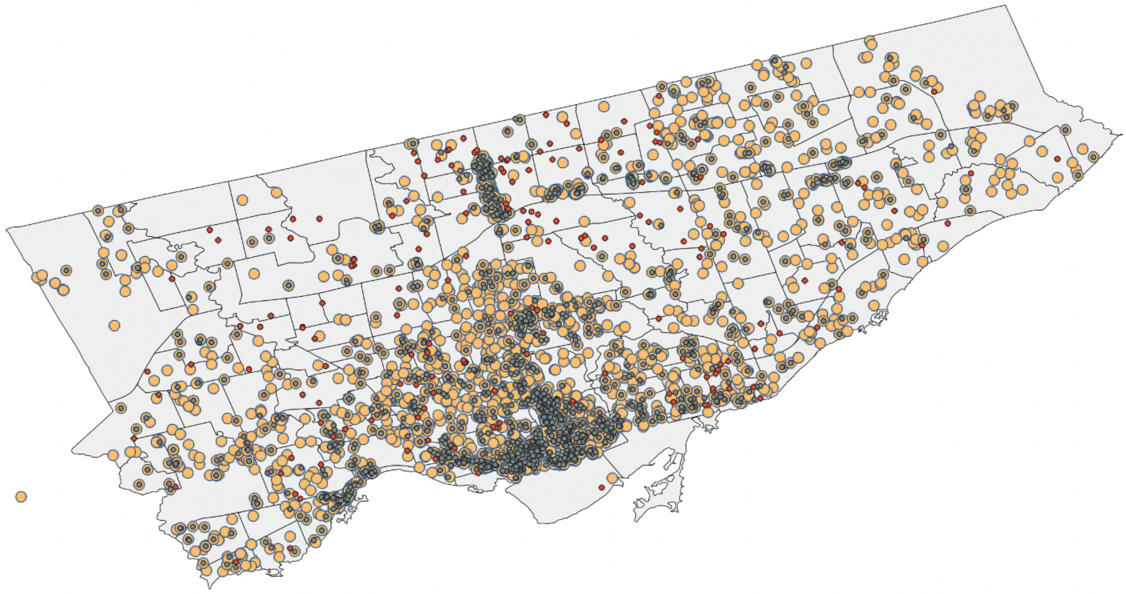


Table A.1: *Descriptive statistics for the City of Toronto municipality*

	Pre-LTT	Post-LTT		Whole sample
	2006:1–2008:1	2008:2–2010:2	2008:2–2012:2	2006:1–2018:2
# BTO sales per year	27,718	23,832	24,621	25,547
# BTR sales per year	1,572	1,685	3,894	2,440
Time on the market (days, mean)	30.5	28.8	27.1	25.4
Time on the market (days, median)	20	18	17	15
Sale price (mean)	401,504	426,363	460,903	555,484
Sale price (median)	318,000	343,000	369,900	419,990
Price-rent ratio (mean)	20.7	20.9	22.2	25.8
Price-rent ratio (median)	16.9	17.9	18.8	21.1

Source: City of Toronto Multiple Listing Service (MLS) residential transaction records (2006–2018).

Table A.2: *Land transfer tax (LTT) rates by property value in the Greater Toronto Area*

City of Toronto (effective from 1st February 2008)		Province of Ontario (effective from 7th May 1997)	
\$0–55,000	0.5%	\$0–55,000	0.5%
\$55,000–400,000	1.0%	\$55,000–250,000	1.0%
\$400,000+	2.0%	\$250,000–400,000	1.5%
		\$400,000+	2.0%

Sources: Municipal Land Transfer Tax, City of Toronto, `http://www.toronto.ca/taxes/mltt.htm`; Provincial Land Transfer Tax, Historical Land Transfer Tax Rates, Province of Ontario. Reproduced from Dachis, Duranton and Turner (2012).
Notes: For the municipal LTT, exemptions are given to first-time buyers for purchases below a value of \$400,000, while for the provincial LTT, the first-time buyer exemption value threshold is \$227,500.

year-months. Let d denote distance from the city border, with $d < 0$ meaning a location in the suburbs and $d > 0$ in the City of Toronto. Let i denote the unit of observation, *community* $\times$ *property type* $\times$ *year* $\times$ *month* in the market-segment regressions, *household* $\times$ *month* in the moving hazard regressions, and a *transaction* in the time-on-the-market and sales price regressions.

Define the following indicator variables based on time and distance:

$$\chi^{\text{POST}} = \begin{cases} 1 & \text{if } t \geq 0 \\ 0 & \text{if } t < 0 \end{cases}, \quad \text{and} \quad \chi^{\text{TO}} = \begin{cases} 1 & \text{if } d \geq 0 \\ 0 & \text{if } d < 0 \end{cases}.$$

The main variable of interest is the LTT dummy $\chi^{\text{TO}} \times \chi^{\text{POST}}$. Let y_{it} denote the outcome of interest, for example, buy-to-own transactions or time-on-the-market. Let $\mathbf{x}_{it}$ denote the vector of property characteristics for unit i at time t. To address anticipation effects that may arise from the announcement of the LTT, define the following dummy variables:

$$\chi^{\tau} = \begin{cases} 1 & \text{if } t = \tau \text{ and } d \leq 0 \\ 0 & \text{otherwise} \end{cases}, \quad \text{for } \tau \in \{-3,-2,-1,0,1,2,3\}.$$

Some regressions include an interaction between the LTT dummy and areas away from the border, e.g, 2km away. To control for these differential effects, define the dummy variables:

$$\chi^{x>\bar{d}} = \begin{cases} 1 & \text{if } t > 0 \text{ and } d \geq \bar{d} \\ 0 & \text{otherwise} \end{cases}, \quad \text{for } \bar{d} > 0.$$

The general model is

$$y_{it} = \lambda \chi^{\text{TO}} \times \chi^{\text{POST}} + \beta_1 \mathbf{x}_{it} + \chi^{\tau} + \nu_t + \delta_i + \varepsilon_{it},$$

where ν_t represents *year* $\times$ *month* fixed effects, δ_i represents *community* fixed effects, and ε_{it} is the error term. Note that because *community* and *year* $\times$ *month* are controlled for, there is no need to include City of Toronto and post-LTT dummies in the equations. In the all-properties sample, *community* $\times$ *property type*, *month* $\times$ *property type*, and *year* $\times$ *property type* fixed effects are also included.

A.1.4 Housing-stock composition

As a check on the assumption that there are no significant housing composition differences potentially picked up by the coefficient on the LTT dummy, columns (1) and (2) of Table A.3 present differences in property characteristics on opposite sides of the city border before the tax rise. To show this, the sample is restricted to the pre-policy period, and each property characteristics is regressed on a border dummy that indicates being inside the city of Toronto, controlling for the usual factors. The border coefficients are statistically insignificant in most cases, and quantitatively small even when statistically significant. This indicates that properties on opposite sides of the border were more or less similar before the new LTT.

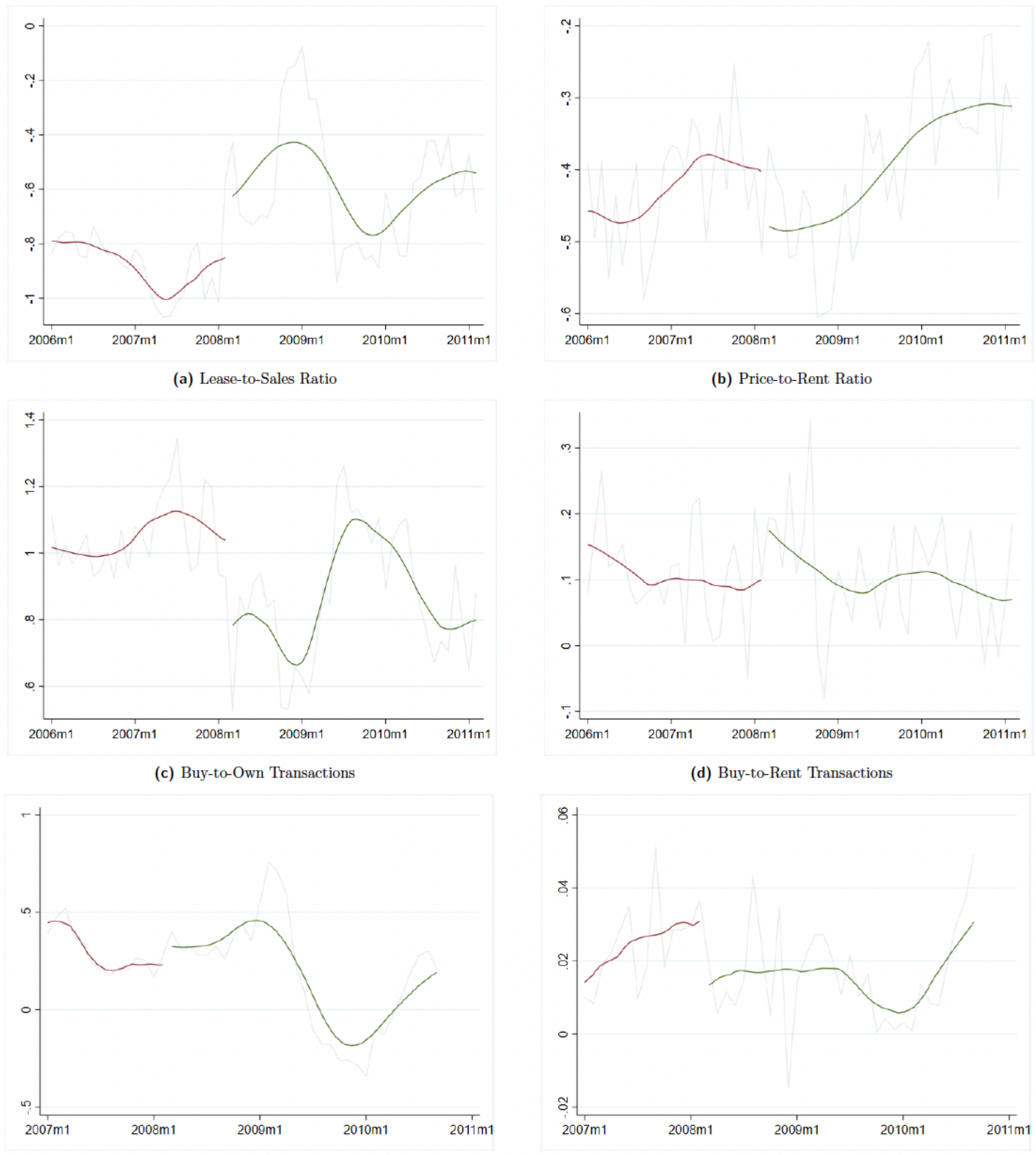
In columns (3) and (4), each property characteristic is further regressed on the LTT dummy that is an interaction of the border dummy and the post-policy dummy, controlling for the usual factors. The LTT dummy coefficients are statistically insignificant in almost all cases. As expected, the cross-border differences in property characteristics, if any, remain stable before and after the new LTT. This ensures the coefficient on the LTT dummy in the main empirical specifications picks up the impact of the new transaction tax, rather than changes in housing-stock composition.

Table A.3: *Comparison of property characteristics across the city border*

Property characteristic	(1)	(2)	(3)	(4)
Heating	0.000490	0.000320	-0.000406	-0.000120
	(0.000394)	(0.000236)	(0.000486)	(0.000329)
Observations	10389	17916	42444	73550
Basement	-0.00498	-0.00831**	-0.00133	0.00234
	(0.00391)	(0.00310)	(0.00458)	(0.00351)
Observations	10389	17916	42444	73550
Family	0.0227	-0.0907***	-0.0478	-0.0145
	(0.0344)	(0.0263)	(0.0381)	(0.0292)
Observations	10347	17834	42444	73548
Fire	0.00368	-0.0229***	-0.00655	-0.000543
	(0.00713)	(0.00562)	(0.00795)	(0.00621)
Observations	10389	17916	42444	73550
Bedrooms	0.00535	0.0157*	0.0138	0.0139
	(0.0105)	(0.00817)	(0.0110)	(0.00870)
Observations	10389	17916	42444	73550
Bathrooms	-0.115***	-0.120***	-0.0229	-0.0200
	(0.0137)	(0.0109)	(0.0157)	(0.0123)
Observations	10389	17916	42444	73550
Rooms	-0.0322	-0.0274	-0.0193	-0.0339*
	(0.0273)	(0.0185)	(0.0232)	(0.0178)
Observations	10389	17916	42444	73550
Lot	-1305.3	-918.3	1051.8	177.8
	(1006.3)	(600.3)	(967.1)	(903.3)
Observations	10389	17916	42444	73550
Distance threshold	3km	5km	3km	5km
LTT sample period	Pre	Pre	All	All

Notes: Data comprise single-family-house transactions from January 2006 to February 2012. A unit of observation is a transaction. In columns (1) and (2), the coefficients are from regressions of a property characteristic on a border dummy that indicates a location is in the City of Toronto. In columns (3) and (4), the coefficients are from regressions of a property characteristic on the LTT dummy that indicates a location in the City of Toronto and in the period after the LTT is introduced. All regressions control for other property characteristics, and year, month, and property-type fixed effects. Regressions for columns (3) and (4) include an indicator for the post-LTT period and an indicator for the City of Toronto. Distance threshold is the maximum distance to the Toronto city border for a transaction to be included in the sample. LTT sample period specifies whether a transaction occurred before or after the new LTT. Standard errors are in parentheses, and *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels.

Figure A.2: *Percentage differences across the City of Toronto border over time*



(a) Lease-to-Sales Ratio

(b) Price-to-Rent Ratio

(c) Buy-to-Own Transactions

(d) Buy-to-Rent Transactions

(e) Time-on-the-market

(f) Sales Price

Notes: The vertical axes show the percentage difference in mean neighbourhood-level outcome variables across the City of Toronto border for the period before and after the introduction of the new LTT. The curves are obtained from kernel-weighted local polynomial regressions of the log of the outcome variables on an interaction between City of Toronto and year-month dummies, controlling for community fixed effects and year-month fixed effects.

Figure A.3: *Geography of the sample used for estimation*

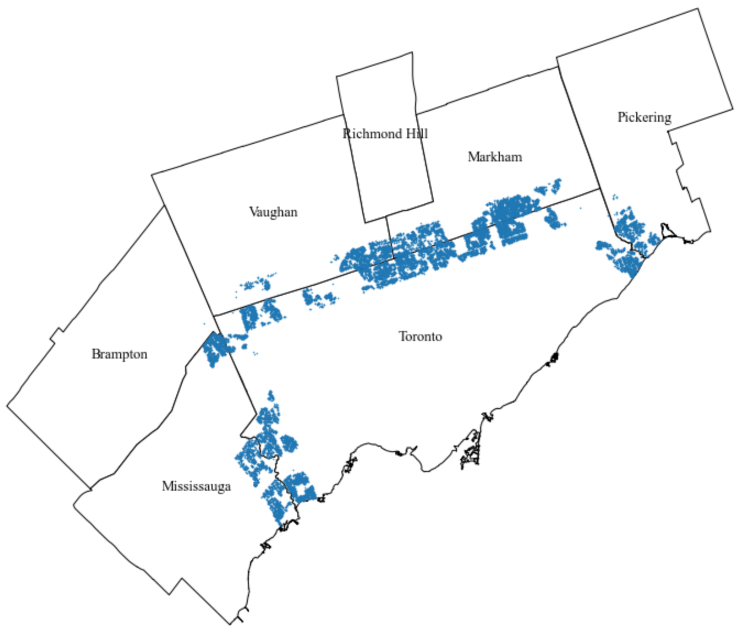
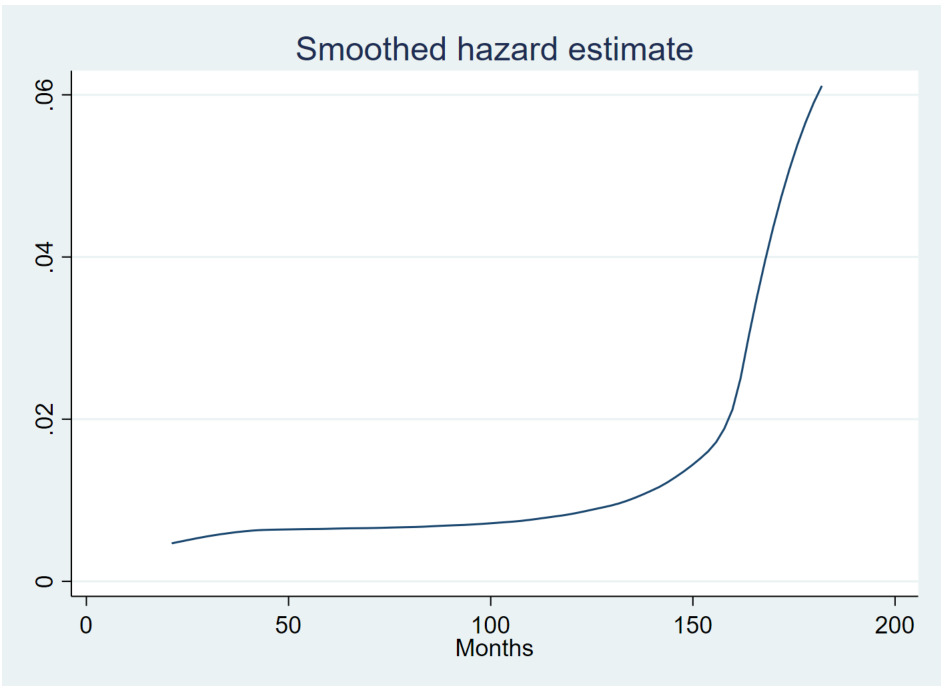


Figure A.4: *Kaplan-Meier estimate of homeowners' moving hazard function*



A.1.5 Robustness checks

Table A.4: *Robustness checks on buy-to-own and buy-to-rent transactions*

Dependent variable	(1)	(2)	(3)	(4)
	6-month cutoff to distinguish BTO and BTR			
log (#*BTO sales*)	-0.115**	-0.135**	-0.117**	-0.158***
	(0.0577)	(0.0433)	(0.0546)	(0.0322)
log (#*BTR sales*)	0.194**	0.200***	0.206***	0.0956**
	(0.0739)	(0.0518)	(0.0612)	(0.0477)
	12-month cutoff to distinguish BTO and BTR			
log (#*BTO sales*)	-0.0835	-0.0972**	-0.0799	-0.128***
	(0.0580)	(0.0438)	(0.0554)	(0.0326)
log (#*BTR sales*)	0.167**	0.144**	0.148**	0.0478
	(0.0637)	(0.0472)	(0.0588)	(0.0431)
	24-month cutoff to distinguish BTO and BTR			
log (#*BTO sales*)	-0.110*	-0.116**	-0.0917	-0.125***
	(0.0592)	(0.0447)	(0.0566)	(0.0333)
log (#*BTR sales*)	0.139**	0.113**	0.114**	0.0298
	(0.0602)	(0.0442)	(0.0526)	(0.0411)
Distance threshold	3km	5km	5km	All
City indicators $\pm$3 m.	Yes	Yes	Yes	Yes
City time trends	Yes	Yes	Yes	Yes
Distance LTT trends		Yes	Yes	Yes
Donut hole			2km	

Notes: See the footnote to Table 1. Standard errors are in parentheses, and *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels.

Table A.5: *Robustness checks on the moving hazard rate*

	(1)	(2)	(3)	(4)	(5)
			Sample period 2006–2010		
LTT	-0.156**	-0.176**	-0.218***	-0.243**	-0.147*
	(0.074)	(0.089)	(0.063)	(0.110)	(0.089)
Observations	1,012,969	682,641	1,690,705	982,110	708,595
			Sample period 2006–2018		
LTT	-0.125**	-0.169**	-0.179***	-0.213**	-0.162***
	(0.061)	(0.074)	(0.048)	(0.071)	(0.047)
Observations	4,327,556	2,927,002	7,306,558	4,296,732	3,009,826
Distance threshold	3km	3km	5km	5km	2km
House characteristics	Yes	Yes	Yes	Yes	Yes
City indicators ± 3 m.	Yes	Yes	Yes	Yes	Yes
City time trends	Yes	Yes	Yes	Yes	Yes
Distance LTT trends			Yes	Yes	
Donut hole		1km		2km	

Notes: A unit of observation is a homeowner whose property is listed on MLS. See the footnote to Table 2. Given that the number of observations within 3km of the border for the 2006–2018 period is 4,327,556, this specification check is not repeated for the entire city owing to the high computational power required. Standard errors clustered by community are in parentheses, and *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels.

Table A.6: *Robustness checks on sales prices at the market-segment level*

Dependent variable	(1)	(2)	(3)	(4)
		Sample period 2006–2010		
$\log(Price)$	-0.0186**	-0.0172***	-0.0122**	-0.0125**
	(0.00610)	(0.00488)	(0.00613)	(0.00442)
Observations	7515	12939	7949	37698
		Sample period 2006–2018		
$\log(Price)$	-0.0200***	-0.0174***	-0.0125**	-0.0155***
	(0.00525)	(0.00418)	(0.00524)	(0.00378)
Observations	11169	19227	11802	55895
Distance threshold	3km	5km	5km	All
City indicators ± 3 m.	Yes	Yes	Yes	Yes
Distance LTT trends			Yes	
Donut hole			2km	

Notes: The estimation sample covers four types of properties: single-family houses, townhouses, condominiums, and apartments. A unit of observation is a market segment defined by *community* × *property type* × *year* × *month*. The dependent variable is the average sales price within each market segment. Each cell of the table represents a separate regression on the LTT interaction dummy. All regressions include a dummy for the post-LTT period, and *city* × *property type*, *year* × *property type*, *month* × *property type*, and *community* × *property type* fixed effects. See the footnote to Table 1. Robust standard errors are in parentheses, and *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels.

Figure A.5: *Household stocks and flows in the model*

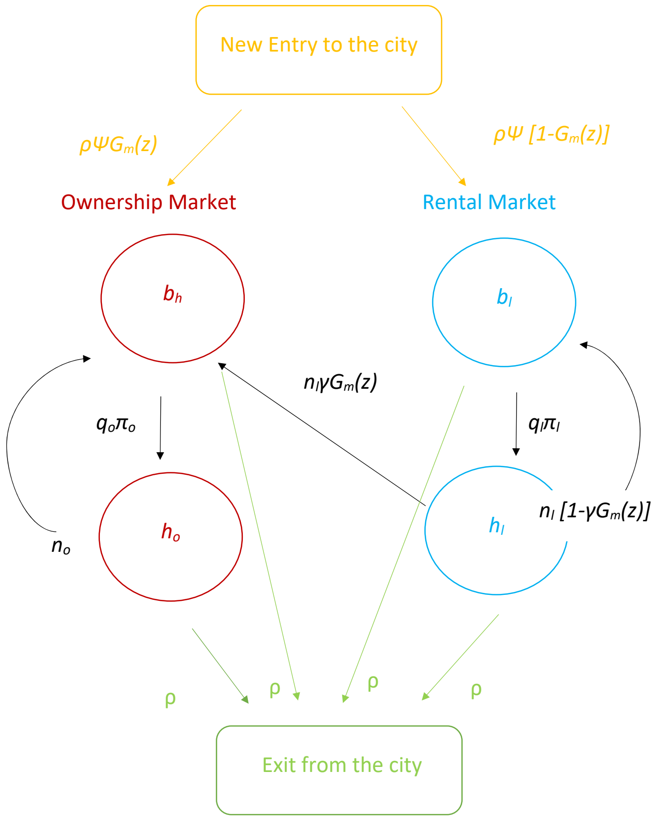
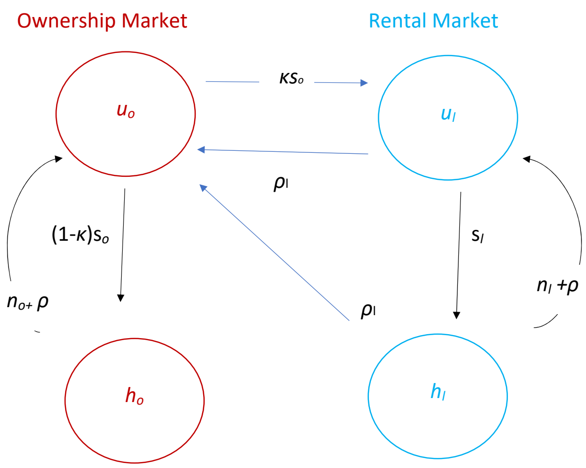


Figure A.6: *Property stocks and flows in the model*



A.2 Deriving the equations of the model

A.2.1 The value functions and thresholds for homeowners and home-buyers

The value function $H(\varepsilon)$ from (6) is increasing in ε. Assuming $\delta_o y_o < x_o$ for all t, by taking ε in a neighbourhood above y_o or any value below, the Bellman equation (6) reduces to the following as $H(\delta_o\varepsilon) < B_o + U_o$:

$$rH(\varepsilon) = \varepsilon - M + a_o(B_o + U_o - H(\varepsilon)) + \rho(U_o - H(\varepsilon)) + \dot{H}(\varepsilon).$$

This simplifies to

$$(r+\rho+a_o)H(\varepsilon) - \dot{H}(\varepsilon) = \varepsilon - M + a_o B_o + (\rho + a_o)U_o, \tag{A.1}$$

and by differentiating both sides with respect to ε in the restricted range:

$$(r+\rho+a_o)H'(\varepsilon) - \dot{H}'(\varepsilon) = 1.$$

For a given ε, this specifies a first-order differential equation in time t for $H'(\varepsilon)$. Since $H'(\varepsilon)$ is not a state variable, there exists a unique stable solution $H'(\varepsilon) = 1/(r+\rho+a_o)$, which is constant over time ($\dot{H}'(\varepsilon) = 0$). As $H'(\varepsilon)$ is independent of ε, integration over match quality ε shows the value function $H(\varepsilon)$ has the form

$$H(\varepsilon) = \underline{H} + \frac{\varepsilon}{r+\rho+a_o}, \quad \text{with} \quad \dot{H}(\varepsilon) = \dot{\underline{H}}, \tag{A.2}$$

where $\underline{H}$ is independent of ε, but may be time varying. This result is valid for ε in a neighbourhood above y_o and all values below. Substituting into (A.1) shows that $\underline{H}$ satisfies

$$(r+\rho+a_o)\underline{H} - \dot{\underline{H}} = a_o B_o + (\rho + a_o)U_o - M. \tag{A.3}$$

Since $x_o < y_o$, equation (32) together with (A.2) implies that

$$x_o = (r+\rho+a_o)(B_o + U_o - \underline{H}). \tag{A.4}$$

The surplus in (28) and the definition of the transaction threshold (29) imply y_o satisfies

$$H(y_o) = H(x_o) + C_h + (1+\tau_h)C_u + \tau_h U_o, \tag{A.5}$$

and combining (A.2) with (A.5) yields

$$y_o = x_o + (r+\rho+a_o)(C_h + (1+\tau_h)C_u + \tau_h U_o). \tag{A.6}$$

The surplus $\Sigma_o(\varepsilon)$ is given in (28) and is divided according to (27). Equation (31) defines the expected surplus Σ_o. The Bellman equation for a buyer (4) can thus be expressed as:

$$(r+\rho)B_o - \dot{B}_o = (1-\omega_o^*)q_o\Sigma_o - F_h. \tag{A.7}$$

The surplus from trade with an investor and its division are given in (40) and (42). Together with the surplus from trade with a home-buyer, the Bellman equation of a seller (5) implies

$$rU_o - \dot{U}_o = \theta_o q_o(\omega_o^*(1-\xi)\Sigma_o + \omega_k^*\xi\Sigma_k) - M. \tag{A.8}$$

Using equations (25), (28), and (29), the expected surplus Σ_o in (31) can be written as

$$\Sigma_o = \int_{y_o}^{\infty} \lambda_o \zeta_o^{\lambda_o} \varepsilon^{-(\lambda_o+1)} \Sigma_o(\varepsilon) \mathrm{d}\varepsilon = \int_{y_o}^{\infty} \frac{\lambda_o \zeta_o^{\lambda_o} \varepsilon^{-(\lambda_o+1)}(H(\varepsilon) - H(y_o))}{1+\tau_h\omega_o^*} \mathrm{d}\varepsilon. \tag{A.9}$$

Defining $\bar{H}(\varepsilon)$ for an arbitrary level of match quality ε and noting the link with Σ_o:

$$\bar{H}(\varepsilon) = \int_{\upsilon=\varepsilon}^{\infty} \lambda_o \varepsilon^{\lambda_o} \upsilon^{-(\lambda_o+1)}(H(\upsilon) - H(\varepsilon))\mathrm{d}\upsilon, \quad \text{where} \quad \Sigma_o = \frac{\zeta_o^{\lambda_o} y_o^{-\lambda_o} \bar{H}(y_o)}{1+\tau_h\omega_o^*}. \tag{A.10}$$

Now restrict attention to ε such that $\delta_o\varepsilon < x_o$, so (6) implies $rH(\varepsilon) = \varepsilon - M + a_o(B_o + U_o - H(\varepsilon)) + \rho(U_o - H(\varepsilon)) + \dot{H}(\varepsilon)$. Since $\delta_o y_o < x_o$, this limits ε to a neighbourhood above y_o and all values below.

Using (32):

$$r(H(\upsilon)-H(\varepsilon)) = (\upsilon-\varepsilon)+a_o\left(\max\{H(\delta_o\upsilon),H(x_o)\}-H(\upsilon)\right)-a_o(H(x_o)-H(\varepsilon)) \\ -\rho(H(\upsilon)-H(\varepsilon))+(\dot{H}(\upsilon)-\dot{H}(\varepsilon)),$$

which holds for any $\upsilon \geq \varepsilon$. This simplifies to

$$(r+\rho+a_o)(H(\upsilon)-H(\varepsilon))-(\dot{H}(\upsilon)-\dot{H}(\varepsilon)) = (\upsilon-\varepsilon)+a_o\max\{H(\delta_o\upsilon)-H(x_o),0\},$$

and multiplying both sides by $\lambda_o\varepsilon^{\lambda_o}\upsilon^{-(\lambda_o+1)}$, integrating over υ, and using (A.10):

$$(r+\rho+a_o)\bar{H}(\varepsilon)-\dot{\bar{H}}(\varepsilon) = \int_{\upsilon=\varepsilon}^{\infty}\lambda_o\varepsilon^{\lambda_o}\upsilon^{-(\lambda_o+1)}\left((\upsilon-\varepsilon)+a_o\max\{H(\delta_o\upsilon)-H(x_o),0\}\right)\mathrm{d}\upsilon\,, \quad \text{(A.11)}$$

where the time derivative of $\bar{H}(\varepsilon)$ is obtained from (A.10):

$$\dot{\bar{H}}(\varepsilon) = \int_{\upsilon=\varepsilon}^{\infty}\lambda_o\varepsilon^{\lambda_o}\upsilon^{-(\lambda_o+1)}(\dot{H}(\upsilon)-\dot{H}(\varepsilon))\mathrm{d}\upsilon\,.$$

In (A.11), the term in $(\upsilon-\varepsilon)$ integrates to $\varepsilon/(\lambda_o-1)$ using the formula for the mean of a Pareto distribution. The second term is zero for $\upsilon < x_o/\delta_o$ because $H(\delta_o\upsilon)$ is increasing in υ. Hence, equation (A.11) becomes

$$(r+\rho+a_o)\bar{H}(\varepsilon)-\dot{\bar{H}}(\varepsilon) = \frac{\varepsilon}{\lambda_o-1}+a_o\varepsilon^{\lambda_o}\int_{\upsilon=x_o/\delta_o}^{\infty}\lambda_o\upsilon^{-(\lambda_o+1)}(H(\delta_o\upsilon)-H(x_o))\mathrm{d}\upsilon\,,$$

and with the change of variable $j=\delta_o\upsilon$ in the second integral, this can be written as

$$(r+\rho+a_o)\bar{H}(\varepsilon)-\dot{\bar{H}}(\varepsilon) = \frac{\varepsilon}{\lambda_o-1}+a_o\delta_o^{\lambda_o}\varepsilon^{\lambda_o}\int_{j=x_o}^{\infty}\lambda_o j^{-(\lambda_o+1)}(H(j)-H(x_o))\mathrm{d}j\,. \quad \text{(A.12)}$$

Make the following definition of a new variable X_o:

$$X_o(t) = (\lambda_o-1)\left(r+\rho+a_o(1-\delta_o^{\lambda_p})\right)\int_{\upsilon=t}^{\infty}(r+\rho+a_o)e^{-(r+\rho+a_o)(\upsilon-t)}\Bigg(\\ \int_{\varepsilon=x_o}^{\infty}\lambda_o\varepsilon^{-(\lambda_o+1)}(H(\varepsilon,\upsilon)-H(x_o,\upsilon))\mathrm{d}\varepsilon\Bigg)\mathrm{d}\upsilon\,. \quad \text{(A.13)}$$

By differentiating with respect to time t this variable must satisfy the differential equation

$$(r+\rho+a_o)X_o-\dot{X}_o = (\lambda_o-1)(r+\rho+a_o)\left(r+\rho+a_o(1-\delta_o^{\lambda_p})\right)x_o^{-\lambda_o}\bar{H}(x_o) \\ = (\lambda_o-1)(r+\rho+a_o)\left(r+\rho+a_o(1-\delta_o^{\lambda_p})\right)\int_{\varepsilon=x_o}^{\infty}\lambda_o\varepsilon^{-(\lambda_o+1)}(H(\varepsilon)-H(x_o))\mathrm{d}\varepsilon\,, \quad \text{(A.14)}$$

which uses the definition of $\bar{H}(\varepsilon)$ in (A.10). Substituting into equation (A.12):

$$(r+\rho+a_o)\bar{H}(\varepsilon)-\dot{\bar{H}}(\varepsilon) = \frac{1}{\lambda_o-1}\left(\varepsilon+\frac{a_o\delta_o^{\lambda_o}\varepsilon^{\lambda_o}\left((r+\rho+a_o)X_o-\dot{X}_o\right)}{(r+\rho+a_o)\left(r+\rho+a_o(1-\delta_o^{\lambda_p})\right)}\right),$$

and by collecting terms this can be written as

$$(r+\rho+a_o)\left(\bar{H}(\varepsilon)-\frac{a_o\delta_o^{\lambda_o}\varepsilon^{\lambda_o}}{(\lambda_o-1)(r+\rho+a_o)\left(r+\rho+a_o(1-\delta_o^{\lambda_p})\right)}X_o\right) \\ -\left(\dot{\bar{H}}(\varepsilon)-\frac{a_o\delta_o^{\lambda_o}\varepsilon^{\lambda_o}}{(\lambda_o-1)(r+\rho+a_o)\left(r+\rho+a_o(1-\delta_o^{\lambda_p})\right)}\dot{X}_o\right) = \frac{\varepsilon}{\lambda_o-1}\,.$$

Since the right-hand side is time invariant and none of the variables is predetermined, it follows for each fixed ε there is a unique stable solution for $\bar{H}(\varepsilon)-a_o\delta_o^{\lambda_o}\varepsilon^{\lambda_o}X_o/((\lambda_o-1)(r+\rho+a_o)(r+\rho+a_o(1-\delta_o^{\lambda_p})))$ that is time invariant and equal to $\varepsilon/((\lambda_o-1)(r+\rho+a_o))$. This demonstrates that for any given

ε in a neighbourhood above y_o or any value below it, the function $\bar{H}(\varepsilon)$ is given by

$$\bar{H}(\varepsilon) = \frac{1}{(\lambda_o - 1)(r+\rho+a_o)} \left(\varepsilon + \frac{a_o \delta_o^{\lambda_o} \varepsilon^{\lambda_o}}{r+\rho+a_o(1-\delta_o^{\lambda_p})} X_o \right) . \tag{A.15}$$

Evaluating (A.15) at $\varepsilon = x_o$ and multiplying by $(\lambda_o - 1)(r+\rho+a_o)(r+\rho+a_o(1-\delta_o^{\lambda_p}))x_o^{-\lambda_o}$:

$$(\lambda_o - 1)(r+\rho+a_o)\left(r+\rho+a_o(1-\delta_o^{\lambda_p})\right) x_o^{-\lambda_o} \bar{H}(x_o) = \left(r+\rho+a_o(1-\delta_o^{\lambda_p})\right) x_o^{1-\lambda_o} + a_o \delta_o^{\lambda_o} X_o ,$$

and substituting into (A.14) shows that X_o satisfies an equation in the moving threshold x_o:

$$\left(r+\rho+a_o(1-\delta_o^{\lambda_p})\right) X_o - \dot{X}_o = \left(r+\rho+a_o(1-\delta_o^{\lambda_p})\right) x_o^{1-\lambda_o} . \tag{A.16}$$

Finally, evaluating (A.15) at $\varepsilon = y_o$ and substituting into (A.10):

$$\Sigma_o = \frac{\zeta_o^{\lambda_o}}{(1+\tau_h \omega_o^*)(\lambda_o - 1)(r+\rho+a_o)} \left(y_o^{1-\lambda_o} + \frac{a_o \delta_o^{\lambda_o}}{r+\rho+a_o(1-\delta_o^{\lambda_o})} X_o \right) . \tag{A.17}$$

In summary, (A.3), (A.4), (A.6), (A.7), (A.8), (A.16), and (A.17) form a system of differential equations in y_o, x_o, X_o, Σ_o, $\underline{H}$, B_o, and U_o, which take as given Σ_k, q_o and ξ.

A.2.2 The moving rate in the ownership market

The flow of owner-occupiers who move within the city is denoted by N_o, and the moving rate is $n_o = N_o/h_o$. The group of existing homeowners h_o is made up of matches that formed at various points in the past and have survived to the present. Moving requires that homeowners receive an idiosyncratic shock, which has arrival rate a_o independent of history. A measure $a_o h_o$ of households thus decide whether to move.

All matches began as a viewing with some initial match quality ε. Using (1), the flow of viewings v_h done by home-buyers in the ownership market at a point in time is

$$v_h = q_o b_h = (1-\xi)\theta_o q_o u_o . \tag{A.18}$$

Initial match quality drawn in viewings is from a Pareto(ζ_o, λ_o) distribution (see 25). This match quality distribution has been truncated when transaction decisions were made and possibly when subsequent idiosyncratic shocks have occurred. Consider a group of surviving homeowners where initial match quality has been previously truncated at $\underline{\varepsilon}$. This group constitutes a fraction $\zeta_o^{\lambda_o} \underline{\varepsilon}^{-\lambda_o}$ of the initial measure of viewings, and the distribution of ε conditional on survival is Pareto$(\underline{\varepsilon}, \lambda_o)$. Among this group, consider those whose current match quality is a multiple Δ of original match quality ε, where Δ is equal to δ_o raised to the power of the number of past shocks received.

Now consider a new idiosyncratic shock. Current match quality becomes $\varepsilon' = \delta_o \Delta \varepsilon$ in terms of initial match quality ε. Moving is optimal if $\varepsilon' < x_o$, so only those with initial match quality $\varepsilon \geq x_o/(\delta_o \Delta)$ survive. Since $\delta_o < 1$ and $\delta_o y_o < x_o$, there is a range of variation in thresholds y_o and x_o that ensures $x_o/(\delta_o \Delta) > \underline{\varepsilon}$. Given the Pareto distribution, the proportion of the surviving group that does not move after the new shock is $\underline{\varepsilon}^{\lambda_o}(x_o/(\delta_o \Delta))^{-\lambda_o} = x_o^{-\lambda_o} \delta_o^{\lambda_o} \Delta^{\lambda_o} \underline{\varepsilon}^{\lambda_o}$. Since that surviving group is a fraction $\zeta_o^{\lambda_o} \underline{\varepsilon}^{-\lambda_o}$ of the original set of viewings, those that do not move after the new shock are a fraction $x_o^{-\lambda_o} \delta_o^{\lambda_o} \Delta^{\lambda_o} \underline{\varepsilon}^{\lambda_o} \times \zeta_o^{\lambda_o} \underline{\varepsilon}^{-\lambda_o} = (\zeta_o^{\lambda_o} x_o^{-\lambda_o} \delta_o^{\lambda_o}) \times \Delta^{\lambda_o}$ of that set of viewings. This is independent of any past truncation thresholds $\underline{\varepsilon}$ owing to the properties of the Pareto distribution.

The measure of the group choosing not to move after a new shock does depend on the total accumulated size Δ of past idiosyncratic shocks. Let Ξ_o be the integral of Δ^{λ_o} over the measure of current and past viewings done by households who have not yet exited the city. Since the size of the group choosing not to move is a common multiple $\zeta_o^{\lambda_o} x_o^{-\lambda_o} \delta_o^{\lambda_o}$ of Δ^{λ_o}, the measure of those choosing not to move after a new shock is $a_o \zeta_o^{\lambda_o} x_o^{-\lambda_o} \delta_o^{\lambda_o} \Xi_o$. Therefore, the size of the group of movers is

$$N_o = a_o h_o - a_o \zeta_o^{\lambda_o} x_o^{-\lambda_o} \delta_o^{\lambda_o} \Xi_o . \tag{A.19}$$

Since the arrival of idiosyncratic shocks is independent of history, a fraction a_o of the group used to define Ξ_o have Δ^{λ_o} reduced to $\delta_o^{\lambda_o}\Delta^{\lambda_o}$. Exit from the group occurs at rate ρ, and new viewings occur that start from $\Delta^{\lambda_o} = 1$ with measure v_h from (A.18). The equation for Ξ_o is

$$\dot{\Xi}_o = v_h + a_o(\delta_o^{\lambda_o}\Xi_o - \Xi_o) - \rho\Xi_o\,. \tag{A.20}$$

Define the following weighted average of current and past levels of home-buyer viewings v_h:

$$\bar{v}_h(t) = \int_{\upsilon\to-\infty}^{t} (\rho + a_o(1-\delta_o^{\lambda_o}))e^{-(\rho+a_o(1-\delta_o^{\lambda_o}))(t-\upsilon)} v_h(\upsilon)\mathrm{d}\upsilon\,,$$

and note that it satisfies the differential equation

$$\dot{\bar{v}}_h + (\rho + a_o(1-\delta_o^{\lambda_o}))\bar{v}_h = (\rho + a_o(1-\delta_o^{\lambda_o}))v_h\,. \tag{A.21}$$

A comparison of (A.20) and (A.21) shows that $\Xi_o = \bar{v}_h/(\rho + a_o(1-\delta_o^{\lambda_o}))$, and substituting this into (A.19) yields an equation for the moving rate $n_o = N_o/h_o$:

$$n_o = a_o - \frac{a_o\zeta_o^{\lambda_o}\delta_o^{\lambda_o}x_o^{-\lambda_o}\bar{v}_h}{(\rho + a_o(1-\delta_o^{\lambda_o}))h_o}\,. \tag{A.22}$$

Using the formula for $\bar{v}_h(t)$ and (A.18), this confirms equation (33) for the moving rate n_o.

A.2.3 The threshold and value functions in the rental market

By adding the Bellman equations (8) and (10) for the landlord and tenant value functions:

$$\begin{aligned} r(L(\varepsilon)+W(\varepsilon)) = \varepsilon - M - M_l + (\rho + a_l)(U_l - L(\varepsilon)) + \rho_l(U_o - L(\varepsilon)) + (1-\gamma)n_l(B_l - W(\varepsilon)) \\ + \gamma n_l(G_m(Z)(B_o - \bar{\chi}) + (1-G_m(Z))B_l - W(\varepsilon)) - \rho W(\varepsilon) + \dot{L}(\varepsilon) + \dot{W}(\varepsilon)\,. \end{aligned}$$

Letting $J(\varepsilon) = L(\varepsilon) + W(\varepsilon)$ denote the joint value, this can be rearranged and simplified, noting $B_o - B_l = Z$ and $n_l = a_l + \rho_l$ from (14) and (34):

$$(r+\rho+n_l)J(\varepsilon) = \varepsilon - M - M_l + (\rho + a_l)U_l + \rho_l U_o + n_l B_l + \gamma n_l G_m(Z)(Z - \bar{\chi}) + \dot{J}(\varepsilon)\,. \tag{A.23}$$

Differentiating with respect to ε leads to the differential equation

$$(r+\rho+n_l)J'(\varepsilon) = 1 + \dot{J}'(\varepsilon)\,.$$

This equation has a unique non-explosive solution for $J'(\varepsilon)$ for any given value of ε:

$$J'(\varepsilon) = \frac{1}{r+\rho+n_l}\,.$$

This time-invariant solution ($\dot{J}'(\varepsilon) = 0$) implies the solution for $J(\varepsilon)$ takes the following form:

$$J(\varepsilon) = \underline{J} + \frac{\varepsilon}{r+\rho+n_l}\,, \tag{A.24}$$

where $\underline{J}$ can be time varying in general. Substituting back into (A.23) and noting $\dot{J}(\varepsilon) = \dot{\underline{J}}$ shows that $\underline{J}$ satisfies the differential equation

$$(r+\rho+n_l)\underline{J} = n_l B_l + (\rho + a_l)U_l + \rho_l U_o - M - M_l + \gamma n_l G_m(Z)(Z - \bar{\chi}) + \dot{\underline{J}}\,. \tag{A.25}$$

The joint rental surplus from (35) is linked to $J(\varepsilon)$ by

$$\Sigma_l(\varepsilon) = J(\varepsilon) - C_l - C_w - B_l - U_l\,, \tag{A.26}$$

and together with (A.24), the definition of the rental transaction threshold y_l in (37) implies

$$y_l = (r+\rho+n_l)(B_l + U_l - \underline{J} + C_l + C_w)\,. \tag{A.27}$$

Using (37), (A.24), and (A.26), it follows that $\Sigma_l(\varepsilon) = (\varepsilon - y_l)/(r+\rho+a_l)$. The Pareto distribution in (25) then implies the expected rental surplus from (39) is

$$\Sigma_l = \frac{\zeta_l^{\lambda_l} y_l^{1-\lambda_l}}{(\lambda_l - 1)(r+\rho+n_l)}\,. \tag{A.28}$$

Using $\Sigma_l^l(\varepsilon) = L(\varepsilon) + \Pi(\varepsilon) - C_l - U_l = \omega_l \Sigma_l(\varepsilon)$, (35), and (39), equation (7) for U_l becomes

$$(r+\rho_l)U_l - \dot{U}_l = \omega_l \theta_l q_l \Sigma_l - M + \rho_l U_o \,. \tag{A.29}$$

Similarly, with $\Sigma_l^w(\varepsilon) = W(\varepsilon) - \Pi(\varepsilon) - C_w - B_l = (1-\omega_l)\Sigma_l$, equation (9) for B_l becomes

$$(r+\rho)B_l - \dot{B}_l = (1-\omega_l) q_l \Sigma_l - F_w \,. \tag{A.30}$$

The credit cost threshold Z satisfies (14). In summary, equations (A.25), (A.27), (A.28), (A.29), (A.30), and (14) determine y_l, Z, Σ_l, $\underline{J}$, B_l, and U_l.

The Bellman equation (8) can be written as follows:

$$(r+\rho+n_l)(L(\varepsilon) - U_l) = R(\varepsilon) - M - M_l - (r+\rho_l)U_l + \rho_l U_o + \dot{L}(\varepsilon) \,,$$

and substituting from (A.29) implies that rents $R(\varepsilon)$ are

$$R(\varepsilon) = M_l + \omega_l \theta_l q_l \Sigma_l + (r+\rho+n_l)(L(\varepsilon) - U_l) - (\dot{L}(\varepsilon) - \dot{U}_l) \,.$$

Since $\Lambda^l(\varepsilon) = L(\varepsilon) - U_l$ and $\dot{L}(\varepsilon) - \dot{U}_l = \dot{\Lambda}^l(\varepsilon)$, the surplus division $\Lambda^l(\varepsilon) = \omega_l \Lambda(\varepsilon)$ implies

$$R(\varepsilon) = M_l + \omega_l \theta_l q_l \Sigma_l + \omega_l \left((r+\rho+n_l)\Lambda(\varepsilon) - \dot{\Lambda}(\varepsilon) \right) \,.$$

By substituting from (35) and noting $\Sigma_l(\varepsilon) = \Lambda(\varepsilon) - (C_l + C_w)$, the equation for rents becomes

$$R(\varepsilon) = M_l + \omega_l (r+\rho+n_l)(C_w + C_l) + \omega_l \theta_l q_l \Sigma_l + \omega_l \left((r+\rho+n_l)\Sigma_l(\varepsilon) - \dot{\Sigma}_l(\varepsilon) \right) \,.$$

Noting $\dot{\Sigma}_l(\varepsilon) = -\dot{y}_l/(r+\rho+n_l)$ for all ε, and using the definition of average rents R from (38):

$$R = M_l + \omega_l (r+\rho+n_l)(C_w + C_l) + \omega_l \theta_l q_l \Sigma_l + \omega_l \left((r+\rho+n_l)\frac{\Sigma_l}{\pi_l} + \frac{\dot{y}_l}{r+\rho+n_l} \right) ,$$

which can be written as

$$R = M_l + \omega_l (r+\rho+n_l)(C_l + C_w) + \omega_l (r+\rho+n_l+\theta_l q_l \pi_l)\frac{\Sigma_l}{\pi_l} + \frac{\omega_l}{r+\rho+n_l}\dot{y}_l \,. \tag{A.31}$$

A.2.4 The relationship between market tightness across the two markets

The total measures of properties in (11) and households in (12) together with the definitions of the fraction of investors and market tightnesses from (1) imply

$$((1-\xi)\,\theta_o - 1)\,u_o + (\theta_l - 1)\,u_l = \psi - 1 \,, \tag{A.32}$$

which yields a relationship between the market tightnesses θ_o and θ_l across the two markets given stocks of properties for sale u_o and properties for rent u_l, and the fraction ξ of investors.

A.2.5 Average match quality and the average value functions

Let Ψ_h denote the integral of ε over all current owner-occupiers. There is a flow of $v_h \pi_o$ of new owner-occupier matches, and using (1), (18), and (A.18), the size of this flow can be expressed as $(1-\kappa)s_o u_o$. Since the transaction threshold is y_o, the Pareto distribution (25) implies the average value of ε in these new matches is $\lambda_o y_o/(\lambda_o - 1)$, so these new matches add to Ψ_h at rate $(1-\kappa)s_o u_o \lambda_o y_o/(\lambda_o - 1)$ over time.

Matches are destroyed (sending the contribution to Ψ_h to zero) if households exit the city or match-quality shocks arrive and households choose to move. Households exit the city at rate ρ, reducing Ψ_h by $\rho\Psi_h$. Match-quality shocks arrive randomly at rate a_o for the measure h_o of owner-occupiers, leading to a flow N_o of movers out of the group $a_o h_o$ receiving a shock, which reduces the contribution to Ψ_h of those N_o to zero. For the group of size $a_o h_o - N_o$ that receives a shock but does not move, the conditional distribution of surviving match quality ε is truncated at x_o, which is a Pareto distribution with shape parameter λ_o across all cohorts within that group. The mean of the truncated distribution is therefore

$\lambda_o x_o/(\lambda_o - 1)$. Putting together all these effects on Ψ_h, the following differential equation must hold:

$$\dot{\Psi}_h = (1-\kappa)s_o u_o \frac{\lambda_o y_o}{\lambda_o - 1} + \left(N_o \times 0 + (a_o h_o - N_o) \times \frac{\lambda_o x_o}{\lambda_o - 1} - a_o \Psi_h\right) - \rho \Psi_h .$$

Average match quality among owner-occupiers is $Q_h = \Psi_h/h_o$, thus $\dot{Q}_h = \dot{\Psi}_h/h_o - (\dot{h}_o/h_o)Q_h = \dot{\Psi}_h/h_o - (((1-\kappa)s_o u_o/h_o) - (n_o + \rho))Q_h$, where the second equation uses the differential equation for h_o in (22). Together with the equation for $\dot{\Psi}_h$ above and the definition of the moving rate $n_o = N_o/h_o$, average match quality Q_h must satisfy the differential equation (45).

Let Ψ_l denote the equivalent summation of surviving match quality in the rental market. Rental viewings occur at rate $v_l = q_l b_l$, and with leasing threshold y_l for match quality, these add to Ψ_l at rate $q_l \pi_l \lambda_l y_l b_l/(\lambda_l - 1)$ over time. Using (1) and (19), the flow increment to Ψ_l is $s_l u_l \lambda_l y_l/(\lambda_l - 1)$. Matches are destroyed if households exit the city (rate ρ), landlords must sell up (rate ρ_l), or match quality falls to zero (rate a_l). The differential equation for Ψ_l is thus $\dot{\Psi}_l = s_l u_l(\lambda_l y_l/(\lambda_l - 1)) - (a_l + \rho_l + \rho)\Psi_l$. Average match quality for tenants is $Q_l = \Psi_l/h_l$, hence $\dot{Q}_l = (\dot{\Psi}_l/h_l) - (\dot{h}_l/h_l)Q_l$, and by substituting $\dot{h}_l/h_l = (s_l u_l/h_l) - (n_l + \rho)$ from (23), the differential equation for Q_l is (46), which uses $n_l = a_l + \rho_l$ from (34).

Let $G_h(\varepsilon)$ denote the distribution function of current match quality ε. The average value $H(\varepsilon)$ across all h_o matches and the integral of these values are denoted by H and Θ:

$$H = \int_\varepsilon H(\varepsilon)\mathrm{d}G_h(\varepsilon), \quad \text{and } \Theta = h_o H = \int_\varepsilon H(\varepsilon)\varsigma(\varepsilon)\mathrm{d}\varepsilon, \quad \text{where } \varsigma(\varepsilon) = h_o G_h'(\varepsilon).$$

Differentiating Θ with respect to time implies $\dot{\Theta} = \int_\varepsilon \left(\dot{H}(\varepsilon)\varsigma(\varepsilon) + H(\varepsilon)\dot{\varsigma}(\varepsilon)\right)\mathrm{d}\varepsilon$ and hence

$$r\Theta - \dot{\Theta} = \int_\varepsilon \left(rH(\varepsilon) - \dot{H}(\varepsilon)\right)\varsigma(\varepsilon)\mathrm{d}\varepsilon - \int_\varepsilon H(\varepsilon)\dot{\varsigma}(\varepsilon)\mathrm{d}\varepsilon . \tag{A.33}$$

Shocks scaling down match quality ε to $\delta_o\varepsilon$ occur with arrival rate a_o, which triggers moving if match quality falls below x_o. There is also exogenous exit from the city at rate ρ. New matches form at rate S_h and begin with ε having distribution function $G_o(\varepsilon)/\pi_o$ for $\varepsilon \geq y_o$, where $\pi_o = 1 - G_o(y_o)$. The dynamics of the density function $\varsigma(\varepsilon) = h_o G_h'(\varepsilon)$ describing the distribution of ε across all matches are thus:

$$\dot{\varsigma}(\varepsilon) = \begin{cases} -(a_o + \rho)\varsigma(\varepsilon) & \text{if } \varepsilon < x_o \\ a_o \delta_o^{-1}\varsigma(\delta_o^{-1}\varepsilon) - (a_o + \rho)\varsigma(\varepsilon) & \text{if } x_o \leq \varepsilon < y_o \\ (S_h/\pi_o)G_o'(\varepsilon) + a_o\delta_o^{-1}\varsigma(\delta_o^{-1}\varepsilon) - (a_o + \rho)\varsigma(\varepsilon) & \text{if } y_o \leq \varepsilon \end{cases} .$$

It follows that

$$\begin{aligned} \int_\varepsilon H(\varepsilon)\dot{\varsigma}(\varepsilon)\mathrm{d}\varepsilon &= \frac{S_h}{\pi_o}\int_{\varepsilon = y_o} H(\varepsilon)\mathrm{d}G_o(\varepsilon) + \frac{a_o}{\delta_o}\int_{\varepsilon = x_o} H(\varepsilon)\varsigma\left(\frac{\varepsilon}{\delta_o}\right)\mathrm{d}\varepsilon - (a_o + \rho)h_o H \\ &= q_o b_h \int_{\varepsilon = y_o} H(\varepsilon)\mathrm{d}G_o(\varepsilon) + a_o \int_{\varepsilon = x_o/\delta_o} H(\delta_o\varepsilon)\varsigma(\varepsilon)\mathrm{d}\varepsilon - (a_o + \rho)h_o H , \end{aligned} \tag{A.34}$$

which uses $S_h = q_o \pi_o b_h$ and a change of variable $\varepsilon' = \varepsilon/\delta_o$ in the second term. Using the Bellman equation (6) for $H(\varepsilon)$ and the definitions of H and Q_h:

$$\begin{aligned} \int_\varepsilon \left(rH(\varepsilon) - \dot{H}(\varepsilon)\right)\varsigma(\varepsilon)\mathrm{d}\varepsilon &= \int_\varepsilon (\varepsilon - M)\mathrm{d}\varsigma(\varepsilon) + a_o \int_{\varepsilon = x_o/\delta_o}^\infty H(\delta_o\varepsilon)\varsigma(\varepsilon)\mathrm{d}\varepsilon + a_o(B_o + U_o)\int_{\varepsilon=0}^{x_o/\delta_o} \varsigma(\varepsilon)\mathrm{d}\varepsilon \\ - a_o \int_\varepsilon H(\varepsilon)\varsigma(\varepsilon)\mathrm{d}\varepsilon + \rho\int_\varepsilon (U_o - H(\varepsilon))\varsigma(\varepsilon)\mathrm{d}\varepsilon &= (Q_h - M)h_o + a_o \int_{\varepsilon = x_o/\delta_o}^\infty H(\delta_o\varepsilon)\varsigma(\varepsilon)\mathrm{d}\varepsilon \\ &\quad + n_o(B_o + U_o)h_o - a_o H h_o + \rho(U_o - H)h_o , \quad \text{where } \int_{\varepsilon=0}^{x_o/\delta_o} a_o\varsigma(\varepsilon) = n_o h_o . \end{aligned} \tag{A.35}$$

The final equation links the number of moves $n_o h_o$ within the city to the integral of $a_o\varsigma(\varepsilon)$ up to $\varepsilon =$

x_o/δ_o. Substituting equations (A.34) and (A.35) into (A.33):

$$r\Theta - \dot{\Theta} = (Q_h - M)h_o + n_o(B_o + U_o) + \rho U_o - q_o b_h \int_{\varepsilon=y_o} H(\varepsilon)\mathrm{d}G_o(\varepsilon)\,.$$

Since $H = \Theta/h_o$ implies $\dot{H} = \dot{\Theta}/h_o - H\dot{h}_o/h_o$, the equation above and (22) for $\dot{h}_o$ imply H satisfies the following equation, noting $(1-\kappa)s_o u_o = \pi_o q_o b_h$:

$$rH = Q_h - M - n_o(H - B_o - U_o) - \rho(H - U_o) - \frac{\pi_o q_o b_h}{h_o}\left(\frac{1}{\pi_o}\int_{\varepsilon=y_o} H(\varepsilon)\mathrm{d}G_o(\varepsilon) - H\right) + \dot{H}\,. \quad \text{(A.36)}$$

Let L, W, and $\bar{R}$ be the average values of $L(\varepsilon)$, $W(\varepsilon)$, and $R(\varepsilon)$ across the distribution of match quality ε for all surviving matches in the rental market. The same method used to derive (A.36) can be applied to show the equivalent for L of the Bellman equation (8) for $L(\varepsilon)$ is:

$$rL = \bar{R} - M - M_l - (a_l + \rho)(L - U_l) - \rho_l(L - U_o) - \frac{\pi_l q_l b_l}{h_l}\left(\frac{1}{\pi_l}\int_{\varepsilon=y_l} L(\varepsilon)\mathrm{d}G_l(\varepsilon) - L\right) + \dot{L}\,, \quad \text{(A.37)}$$

and the equivalent of (10) in terms of W is as follows, where Q_l is average rental match quality:

$$rW = Q_l - \bar{R} - (1-\gamma)n_l(W - B_l) - \gamma n_l\left(W - G_m(Z)(B_o - \bar{\chi}) - (1 - G_m(Z))B_l\right) - \rho W - \frac{\pi_l q_l b_l}{h_l}\left(\frac{1}{\pi_l}\int_{\varepsilon=y_l} W(\varepsilon)\mathrm{d}G_l(\varepsilon) - W\right) + \dot{W}\,. \quad \text{(A.38)}$$

A.2.6 Welfare

With H, L, and W denoting the average values of $H(\varepsilon)$, $L(\varepsilon)$, and $W(\varepsilon)$ over the distributions of all surviving matches, aggregate welfare Ω is defined as follows:

$$\Omega = h_o H + h_l(L + W) + b_h B_o + b_l B_l + b_k K + u_o U_o + u_l U_l + \Omega_\tau + \Omega_e\,, \quad \text{(A.39)}$$

where Ω_τ is the present value of the stream of tax revenue $\Gamma = \tau_h P S_h + \tau_k P_k S_k$, and Ω_e is the expected present values of new entrants to the city. Differentiating Ω with respect to t:

$$r\Omega = h_o(rH - \dot{H}) + h_l(rL - \dot{L}) + h_l(rW - \dot{W}) + b_k(rK - \dot{K}) + b_h(rB_o - \dot{B}_l) + b_l(rB_l - \dot{B}_l) + u_o(rU_o - \dot{U}_o) + u_l(rU_l - \dot{U}_l) + (r\Omega_\tau - \dot{\Omega}_\tau) + (r\Omega_e - \dot{\Omega}_e) - H\dot{h}_o - (L + W)\dot{h}_l - B_o\dot{b}_h - B_l\dot{b}_l - K\dot{b}_k - U_o\dot{u}_o - U_l\dot{u}_l + \dot{\Omega}\,.$$

Substituting Bellman equations (3), (4), (5), (7), (9), (A.36), (A.37), (A.38), $r\Omega_\tau = \tau_h P S_h + \tau_k P_k S_k + \dot{\Omega}_\tau$, $r\Omega_e = \rho\psi((1 - G_m(Z))B_l + G_m(Z)(B_o - \bar{\chi})) + \dot{\Omega}_e$, and laws of motion (16), (17), (20), (21), (22), and

(23):

$$
\begin{aligned}
r\Omega = {} & h_o\left(Q_h - M + n_o(B_o + U_o - H) + \rho(U_o - H) - \frac{\pi_o q_o b_h}{h_o}\left(\frac{1}{\pi_o}\int_{\varepsilon=y_o} H(\varepsilon)\mathrm{d}G_o(\varepsilon) - H\right)\right) \\
& + h_l\left(\bar{R} - M - M_l + (a_l + \rho)(U_l - L) + \rho_l(U_o - L) - \frac{\pi_l q_l b_l}{h_l}\left(\frac{1}{\pi_l}\int_{\varepsilon=y_l} L(\varepsilon)\mathrm{d}G_l(\varepsilon) - L\right)\right) \\
& + h_l\bigg(Q_l - \bar{R} + (1-\gamma)n_l(B_l - W) + \gamma n_l\left(G_m(Z)(B_o - \bar{\chi}) + (1 - G_m(Z))B_l - W\right) \\
& - \rho W - \frac{\pi_l q_l b_l}{h_l}\left(\frac{1}{\pi_l}\int_{\varepsilon=y_l} W(\varepsilon)\mathrm{d}G_l(\varepsilon) - W\right)\bigg) + b_k\left(-F_k + q_o U_l - q_o(1+\tau_k)P_k - q_o C_k - q_o K\right) \\
& + b_h\left(-F_h + q_o\int_{\varepsilon=y_o} H(\varepsilon)\mathrm{d}G_o(\varepsilon) - q_o\pi_o(1+\tau_h)P - q_o\pi_o C_h - q_o\pi_o B_o - \rho B_o\right) \\
& + b_l\left(-F_w + q_l\int_{\varepsilon=y_l} W(\varepsilon)\mathrm{d}G_o(\varepsilon) - q_l\pi_l\Pi - q_l\pi_l C_w - q_l\pi_l B_l - \rho B_l\right) + (\tau_h P S_h + \tau_k P_k S_k) \\
& + u_o\left(-M + \theta_o q_o(1-\xi)\pi_o P - \theta_o q_o(1-\xi)\pi_o C_u - \theta_o q_o(1-\xi)\pi_o U_o + q_o\theta_o\xi P_k - q_o\theta_o\xi C_u - q_o\theta_o\xi U_o\right) \\
& + u_l\left(-M + \theta_l q_l\int_{\varepsilon=y_l} L(\varepsilon)\mathrm{d}G_o(\varepsilon) + \theta_l q_l\pi_l\Pi - \theta_l q_l\pi_l C_l - \theta_l q_l\pi_l U_l + \rho_l U_o - \rho_l U_l\right) \\
& - H\left((1-\kappa)s_o u_o - (n_o + \rho)h_o\right) - (L + W)\left(s_l u_l - (n_l + \rho)h_l\right) - U_o\left((n_o + \rho)h_o + \rho_l(h_l + u_l) - s_o u_o\right) \\
& - U_l\left((a_l + \rho)h_l + \kappa s_o u_o - (s_l + \rho)u_l\right) - B_o\left(n_o h_o + (\gamma n_l h_l + \rho\psi)G_m(Z) - (q_o\pi_o + \rho)b_h\right) \\
& - B_l\left((1-\gamma)n_l h_l + (\gamma n_l h_l + \rho\psi)(1 - G_m(Z)) - (q_l\pi_l + \rho)b_l\right) - K\dot{b}_k \\
& + \rho\psi\left((1 - G_m(Z))B_l + G_m(Z)(B_o - \bar{\chi})\right) + \dot{\Omega}\,. \qquad \text{(A.40)}
\end{aligned}
$$

Observe that all the value functions on the right-hand side cancel out, reflecting transitions of particular individuals between states. For H, note $(1-\kappa)s_o u_o = \pi_o q_o b_h$; for L and W, note $s_l u_l = \pi_l q_l b_l$ and $n_l = a_l + \rho_l$; for the integral over $L(\varepsilon)$, $\theta_l q_l u_l = q_l b_l$; for U_o, $\theta_o q_o((1-\xi)\pi_o + \xi) = s_o$; for U_l, $\theta_l q_l \pi_l = s_l$; and $K = 0$ because of the free-entry condition (A.51)

Next, observe that payments of rent $\bar{R}$ and initial tenancy fees Π cancel out from (A.40) (noting $b_l = \theta_l u_l$), as do house prices P and P_k (noting $\theta_o(1-\xi)u_o = b_h$ and $\theta_o\xi u_o = b_k$). This is because such payments are simply transfers among individuals that net out. The same is true for prices inclusive of tax (noting $S_h = q_o\pi_o b_h$ and $S_k = q_o b_k$) because of the assumption that tax revenue is used to provide public goods.

With value functions, rents, and prices cancelling out from (A.40), the Bellman equation for welfare Ω is (44), where the coefficient of M comes from noting $h_o + h_l + u_o + u_l = 1$ and the coefficients on transaction costs come from $S_h = q_o\pi_o b_h$, $S_k = q_o b_k$, $S_o = S_h + S_k = q_o\theta_o((1-\xi)\pi_o + \xi)u_o$, and $S_l = \theta_l q_l \pi_l u_l = q_l \pi_l b_l$.

A.3 Existence of a steady state and the solution method

Equations for a steady state In a steady state where $\dot{B}_o = 0$ and $\dot{U}_o = 0$, the Bellman equations (A.7) and (A.8) become

$$(r+\rho)B_o = -F_h + (1-\omega_o^*)q_o\Sigma_o\,, \quad \text{and} \qquad \text{(A.41)}$$

$$rU_o = \theta_o q_o((1-\xi)\omega_o^*\Sigma_o + \xi\omega_k^*\Sigma_k) - M\,. \qquad \text{(A.42)}$$

Substituting from (A.8) into (A.6):

$$y_o = x_o + (r + \rho + a_o)\left(C_h + C_u + \tau_h\left(C_u - \frac{M}{r} + \frac{\theta_o q_o((1-\xi)\omega_o^*\Sigma_o + \xi\omega_k^*\Sigma_k)}{r}\right)\right)\,. \qquad \text{(A.43)}$$

The joint surplus $\Sigma_k = F_k/(1-\omega_k^*)q_o$ from selling to an investor comes from equation (43). In a steady state with $\dot{\underline{H}} = 0$, (A.3) implies that $(r + \rho + a_o)\underline{H} = a_o B_o + (\rho + a_o)U_o - M$. Substituting into (A.4) implies $x_o = (r+\rho+a_o)(B_o+U_o) - a_oB_o - (\rho+a_o)U_o + M$ and hence

$$x_o = M + (r+\rho)B_o + rU_o\,.$$

Then substituting the values of B_o and U_o from (A.41) and (A.42) yields

$$x_o + F_h = (1-\omega_o^* + (1-\xi)\omega_o^*\theta_o)\,q_o\Sigma_o + \theta_o q_o \xi\,\omega_k^*\Sigma_k\,. \tag{A.44}$$

With $\dot{X}_o = 0$ in steady state, equation (A.16) shows that $X_o = x_o^{1-\lambda_o}$. Substitution into (A.17) implies the expected joint surplus is

$$\Sigma_o = \frac{\zeta_o^{\lambda_o}}{(r+\rho+a_o)(\lambda_o-1)(1+\tau_h\omega_o^*)}\left(y_o^{1-\lambda_o} + \frac{a_o\delta_o^{\lambda_o}x_o^{1-\lambda_o}}{r+\rho+a_o\left(1-\delta_o^{\lambda_o}\right)}\right). \tag{A.45}$$

The average transaction price P from (30) can be written as follows by using (A.42) for U_o:

$$P = \left(\frac{r+\theta_oq_o(1-\xi)\pi_o}{r}\right)\left(\frac{\omega_o^*\Sigma_o}{\pi_o}\right) + \frac{\theta_oq_o\xi\,\omega_k^*\Sigma_k}{r} + C_u - \frac{M}{r}\,. \tag{A.46}$$

With $\dot{B}_l = 0$ and $\dot{U}_l = 0$, the Bellman equations (A.29) and (A.30) become

$$rB_l = -F_w + (1-\omega_l)q_l\Sigma_l - \rho B_l\,, \quad \text{and} \tag{A.47}$$

$$(r+\rho_l)U_l = \omega_l\theta_lq_l\Sigma_l - M + \rho_lU_o\,. \tag{A.48}$$

In steady state, $\dot{\underline{J}} = 0$, which yields $(r+\rho+n_l)\underline{J} = n_lB_l + (\rho+a_l)U_l + \rho_lU_o - M - M_l + \gamma n_lG_m(Z)(Z-\bar{\chi})$ using (A.25). Substituting into (A.27) and using $n_l = a_l + \rho_l$ implies

$$y_l = M + M_l + (r+\rho)B_l + (r+\rho_l)U_l - \rho_lU_o + (r+\rho+n_l)(C_l+C_w) - \gamma n_lG_m(Z)(Z-\bar{\chi})\,,$$

and by using (A.47) and (A.48) this becomes

$$y_l = M_l - F_w + (r+n_l+\rho)(C_w+C_l) - \gamma n_lG_m(Z)(Z-\bar{\chi}) + (1-\omega_l+\omega_l\theta_l)q_l\Sigma_l\,. \tag{A.49}$$

The rent equation (A.31) in steady state is

$$R = M_l + \omega_l(r+\rho+n_l)(C_l+C_w) + \omega_l(r+\rho+n_l+\theta_lq_l\pi_l)\frac{\Sigma_l}{\pi_l}\,. \tag{A.50}$$

Multiplying both sides of (42) by $r+\rho_l$ and substituting for $(r+\rho_l)U_l$ from (A.48) leads to

$$\omega_l\theta_lq_l\Sigma_l = M - \rho_lU_o + (r+\rho_l)(1+\tau_k)U_o + (r+\rho_l)\left((1+\tau_k)C_u + C_k + (1+\tau_k\omega_k^*)\Sigma_k\right).$$

Using $(r+\rho_l)(1+\tau_k)U_o - \rho_lU_o = (1+\tau_k(1+(\rho_l/r))rU_o$ and substituting from (A.42) implies:

$$\begin{aligned}\omega_l\theta_lq_l\Sigma_l = {} & \left(1+\tau_k\left(1+\frac{\rho_l}{r}\right)\right)\theta_oq_o\left((1-\xi)\omega_o^*\Sigma_o + \xi\,\omega_k^*\Sigma_k\right) \\ & + (r+\rho_l)\left((1+\tau_k)C_u + C_k + (1+\tau_k\omega_k^*)\Sigma_k\right) - \tau_k\left(1+\frac{\rho_l}{r}\right)M\,.\end{aligned} \tag{A.51}$$

By substituting B_o and B_l from (A.41) and (A.47) into (14):

$$(1-\omega_o^*)q_o\Sigma_o - (1-\omega_l)q_l\Sigma_l = (r+\rho)Z + F_h - F_w\,. \tag{A.52}$$

The price paid by investors in equilibrium is obtained from (41) and (A.42):

$$P_k = C_u + \frac{\theta_oq_o((1-\xi)\omega_o^*\Sigma_o + \xi\,\omega_k^*\Sigma_k) - M}{r} + \omega_k^*\Sigma_k\,. \tag{A.53}$$

Imposing a steady state $\dot{Q}_h = 0$ and $\dot{Q}_l = 0$ in the match quality equations (45) and (46):

$$Q_h = \frac{\lambda_o}{\lambda_o-1}\left(\frac{n_o+\rho}{a_o+\rho}y_o + \frac{a_o-n_o}{a_o+\rho}x_o\right), \quad \text{and} \quad Q_l = \frac{\lambda_l}{\lambda_l-1}y_l\,,$$

which also make use of $\dot{h}_o = 0$, $\dot{h}_l = 0$, and (22) and (23).

The solution method is to reduce the problem to a numerical search over the fraction ξ of investors among buyers and ownership-market tightness θ_o to find the roots of two equations representing equilibrium in the ownership and rental markets.

Ownership-market transaction threshold Conditional on ξ and θ_o, within this search, there is also a numerical search to find the transaction threshold y_o in the ownership market. Equation (43) implies $q_o\Sigma_k = F_k/(1-\omega_k^*)$ and equation (A.44) implies $q_o\Sigma_o = (x_o+F_h-\xi\theta_o q_o\omega_k^*\Sigma_k)/(1-\omega_o^*+(1-\xi)\omega_o^*\theta_o)$. Together:

$$\theta_o q_o\left((1-\xi)\omega_o^*\Sigma_o+\xi\omega_k^*\Sigma_k\right) = \frac{\omega_o^*\theta_o}{1-\omega_o^*+(1-\xi)\omega_o^*\theta_o}\left((1-\xi)(x_o+F_h)+\xi\frac{(1-\omega_o^*)\omega_k^*}{\omega_o^*(1-\omega_k^*)}F_k\right).$$

Taking a value of y_o, the moving threshold x_o must satisfy (A.43), and substituting the expression above yields a linear equation for x_o that can be solved in terms of y_o:

$$x_o = \frac{y_o-(r+\rho+a_o)\left(C_h+(1+\tau_h)C_u-\tau_h\frac{M}{r}+\tau_h\frac{\theta_o\omega_o^*}{1-\omega_o^*+(1-\xi)\omega_o^*\theta_o}\left(\frac{(1-\xi)F_h}{r}+\frac{\xi(1-\omega_o^*)\omega_k^*F_k}{\omega_o^*(1-\omega_k^*)r}\right)\right)}{1+\tau_h\left(\frac{(1-\xi)\omega_o^*\theta_o}{1-\omega_o^*+(1-\xi)\omega_o^*\theta_o}\right)\left(\frac{r+\rho+a_o}{r}\right)}. \quad \text{(A.54)}$$

Now combine equations (43), (A.44), (A.45), and substitute $q_o = A_o\theta_o^{-\eta_o}$ from (24):

$$x_o+F_h-\frac{(1-\omega_o^*+(1-\xi)\omega_o^*\theta_o)A_o\theta_o^{-\eta_o}\zeta_o^{\lambda_o}}{(1+\tau_h\omega_o^*)(r+\rho+a_o)(\lambda_o-1)}\left(y_o^{1-\lambda_o}+\frac{a_o\delta_o^{\lambda_o}x_o^{1-\lambda_o}}{r+\rho+a_o(1-\delta_o^{\lambda_o})}\right)-\frac{\xi\theta_o\omega_k^*F_k}{1-\omega_k^*}=0. \quad \text{(A.55)}$$

Observe that the left-hand side of (A.55) is strictly increasing in x_o and y_o. As the value of x_o implied by (A.54) is strictly increasing in y_o, it follows that any solution of (A.54) and (A.55) for x_o and y_o is unique. Since the left-hand side of (A.55) is sure to be positive for large y_o and x_o, existence of a solution can be confirmed by checking whether the left-hand side is negative at $y_o = \zeta_o$, the minimum value of y_o.

Ownership-market variables Once y_o is found, the transaction probability in the ownership market conditional on a viewing is $\pi_o = (\zeta_o/y_o)^{\lambda_o}$. This yields κ from (18) given the value of ξ. Moreover, given that $q_o = A_o\theta_o^{-\eta_o}$ is known conditional on θ_o, the sales rate s_o is found using (19). The moving threshold x_o is obtained from (A.54), and it can be verified whether $\delta_o y_o < x_o$ is satisfied. The surplus Σ_o is found by substituting the thresholds into (A.45), and $\Sigma_k = F_k/((1-\omega_k^*)q_o)$ comes from (43).

A steady state has $\dot{u}_o = 0$ and $\dot{h}_o = 0$, so (20) and (22) require

$$s_o u_o = (n_o+\rho)h_o+\rho_l(h_l+u_l), \quad \text{and} \quad \text{(A.56)}$$

$$(1-\kappa)s_o u_o = (n_o+\rho)h_o. \quad \text{(A.57)}$$

Since (11) implies $h_l+u_l = 1-h_o-u_o$, dividing both sides of (A.56) by $\rho_l > 0$ and substituting for h_l+u_l implies $u_o+h_o-((n_o+\rho)/\rho_l)h_o+(s_o/\rho_l)u_o = 1$. Equation (A.57) implies $h_o = ((1-\kappa)s_o/(n_o+\rho))u_o$, and substituting into the previous equation for u_o and solving:

$$u_o = \frac{1}{1+\frac{(1-\kappa)s_o}{n_o+\rho}+\frac{\kappa s_o}{\rho_l}}, \quad \text{and} \quad h_o = \frac{(1-\kappa)s_o}{n_o+\rho}u_o. \quad \text{(A.58)}$$

This yields the homeownership rate h from the formula given in section 3.7.

Evaluating the moving rate equation (33) at a steady state and substituting $\zeta_o^{\lambda_o} = \pi_o y_o^{\lambda_o}$:

$$n_o = a_o - \frac{a_o\delta_o^{\lambda_o}\left(\frac{y_o}{x_o}\right)^{\lambda_o}}{\rho+a_o\left(1-\delta_o^{\lambda_o}\right)}\frac{(1-\xi)\theta_o q_o\pi_o u_o}{h_o}.$$

Equations (18) and (19) imply that $(1-\xi)\theta_o q_o\pi_o = (1-\kappa)s_o$, and hence using (A.57), $(1-\xi)\theta_o q_o\pi_o u_o/h_o = n_o+\rho$. Substituting this into the above yields an equation in n_o, which has the solution given in footnote 28.

Rental-market variables The moving rate $n_l = a_l + \rho_l$ in the rental market is given by parameters according to (34). Conditional on θ_o and ξ, there is also a numerical search for the transaction threshold y_l in the rental market. Given a value of y_l, the implied transaction probability from (38) is $\pi_l = (\zeta_l/y_l)^{\lambda_l}$. Using the formula (A.28) for the rental-market surplus:

$$\Sigma_l = \frac{\pi_l y_l}{(\lambda_l - 1)(r + \rho + n_l)} \, .$$

Observe that $\omega_l \theta_l q_l \Sigma_l = \omega_l y_l s_l / ((\lambda_l - 1)(r + \rho + n_l))$, where $s_l = \theta_l q_l \pi_l$ is the letting rate from (19). By using this to substitute for the left-hand side of (A.51), the letting rate implied by y_l is

$$s_l = \frac{(\lambda_l - 1)(r + \rho + n_l)}{\omega_l y_l} \Bigg(\left(1 + \tau_k \left(1 + \frac{\rho_l}{r}\right)\right) \theta_o q_o \left((1 - \xi)\omega_o^* \Sigma_o + \xi \omega_k^* \Sigma_k\right) \\ + (r + \rho_l)\left((1 + \tau_k) C_u + C_k + (1 + \tau_k \omega_k^*)\Sigma_k\right) - \tau_k \left(1 + \frac{\rho_l}{r}\right) M \Bigg) , \quad \text{(A.59)}$$

where the surpluses Σ_o and Σ_k are obtained when the ownership-market variables are found. Equation (2) gives the meeting rate $q_l = A_l \theta_l^{-\eta_l}$, and hence the letting rate $s_l = \theta_l q_l \pi_l$ satisfies $s_l = A_l \pi_l \theta_l^{1-\eta_l}$. The implied market tightness in the rental market is

$$\theta_l = \left(\frac{s_l}{A_l \pi_l}\right)^{\frac{1}{1-\eta_l}} , \quad \text{(A.60)}$$

and this also gives $q_l = A_l \theta_l^{-\eta_l}$.

In steady state, $\dot{u}_l = 0$ and $\dot{h}_l = 0$, hence equations (21) and (23) require

$$(s_l + \rho_l) u_l = (a_l + \rho) h_l + \kappa s_o u_o \, , \quad \text{and} \quad \text{(A.61)}$$

$$s_l u_l = (n_l + \rho) h_l \, . \quad \text{(A.62)}$$

Equations (11) and (A.62) imply $h_l + u_l = 1 - h_o - u_o$ and $h_l = (s_l/(n_l + \rho)) u_l$. Combining these two equations and using the known values of h_o and u_o:

$$u_l = \frac{1 - h_o - u_o}{1 + \frac{s_l}{n_l + \rho}} , \quad \text{and} \quad h_l = \frac{s_l}{n_l + \rho} u_l \, . \quad \text{(A.63)}$$

Since (11) holds and (A.56), (A.57), and (A.62) are imposed, (A.61) holds automatically.

The steady state also has $\dot{b}_h = 0$ and $\dot{b}_l = 0$, which means that the following must hold:

$$(q_o \pi_o + \rho) b_h = n_o h_o + (\gamma n_l h_l + \rho \psi) G_m(Z) , \quad \text{and} \quad \text{(A.64)}$$

$$(q_l \pi_l + \rho) b_l = (1 - \gamma) n_l h_l + (\gamma n_l h_l + \rho \psi)(1 - G_m(Z)) \, . \quad \text{(A.65)}$$

Since (1) implies $b_h = (1 - \xi)\theta_o u_o$, which is known, the value of $G_m(Z)$ is obtained by rearranging (A.64):

$$G_m(Z) = \frac{(\rho + q_o \pi_o)(1 - \xi)\theta_o u_o - n_o h_o}{\gamma n_l h_l + \rho \psi} ,$$

and it can be checked that $G_m(Z)$ is a well defined probability. Given that (12) will hold along with (A.57), (A.62), and (A.64), equation (A.65) is satisfied automatically. The threshold Z is obtained by inverting equation (26) with the known probability $G_m(Z)$:

$$Z = e^{\mu + \sigma \Phi^{-1}(G_m(Z))} ,$$

and the average credit cost $\bar{\chi}$ follows immediately from (26) using Z. Finally, with all these variables known conditional on y_l, the value of y_l itself can be found by searching for a solution of equation (A.49). It can be checked whether the solution satisfies $y_l > \zeta_l$.

Criteria for the fraction of investors and market tightness Finally, two equations are needed to pin down the fraction of investors among buyers and ownership-market tightness. Conditional on each pair of values of ξ and θ_o, the steps above show how θ_l, u_o, and u_l can be calculated. With these, the first criterion to be checked is equation (A.32). The second criterion is the indifference threshold condition (A.52), where q_o, q_l, Σ_o, Σ_l, and Z can be obtained as above for given ξ and θ_o. Searching over values of ξ and θ_o that satisfy these two criteria, the equilibrium is found.

Moving hazard function in the ownership market Let $\varkappa(T)$ denote the steady-state survival function of matches in the ownership market. This gives the fraction of matches that remain in existence after T years have elapsed. Assume the transaction and moving thresholds y_o and x_o remain constant over time.

In order for a match to survive for T years, first, the household must not leave the city during that time. With constant exit rate ρ, this has probability $e^{-\rho T}$. Second, the household must choose to remain after any shocks to idiosyncratic match quality have occurred. These shocks arrive independently at rate a_o, so the number of shocks j that occur over a period of time T has a Poisson$(a_o T)$ distribution. The probability of exactly j shocks is $e^{-a_o T}(a_o T)^j/j!$ for $j = 0, 1, 2, \ldots$.

If initial match quality is ε, after j shocks, match quality is now equal to $\varepsilon' = \delta_o^j \varepsilon$. The household chooses not to move house if $\varepsilon' \geq x_o$, which is equivalent to $\varepsilon \geq x_o/\delta_o^j$ in terms of initial match quality ε (and if this condition holds for some j then it also holds for any smaller j because $\delta_o < 1$ and x_o remains constant over time). New match quality has a Pareto(y_o, λ_o) distribution, so the probability that $\varepsilon \geq x_o/\delta_o^j$ is $((x_o/\delta_o^j)/y_o)^{-\lambda_o}$. This is well defined if $x_o/\delta_o^j > y_o$, which is true for all $j \geq 1$ because $\delta_o y_o < x_o$. With zero shocks ($j = 0$), households remain in the same property unless they leave the city.

The fraction of households who remain in the same property for T years is therefore

$$\varkappa(T) = e^{-\rho T}\left(e^{-a_o T} + \sum_{j=1}^{\infty} e^{-a_o T}\frac{(a_o T)^j}{j!}\left(\frac{x_o/\delta_o^j}{y_o}\right)^{-\lambda_o}\right) = e^{-(a_o+\rho)T}\left(1 + \left(\frac{y_o}{x_o}\right)^{\lambda_o}\sum_{j=1}^{\infty}\frac{(a_o\delta_o^{\lambda_o}T)^j}{j!}\right)$$
$$= e^{-(a_o+\rho)T}\left(1 + \left(\frac{y_o}{x_o}\right)^{\lambda_o}\left(e^{a_o\delta_o^{\lambda_o}T} - 1\right)\right) = \left(\frac{y_o}{x_o}\right)^{\lambda_o} e^{-\left(a_o(1-\delta_o^{\lambda_o})+\rho\right)T} - \left(\left(\frac{y_o}{x_o}\right)^{\lambda_o} - 1\right)e^{-(a_o+\rho)T}.$$

The implied hazard function is given by $\hbar(T) = -\varkappa'(T)/\varkappa(T)$, which follows immediately:

$$\hbar(T) = \frac{\left(a_o(1-\delta_o^{\lambda_o})+\rho\right)\left(\frac{y_o}{x_o}\right)^{\lambda_o} e^{-\left(a_o(1-\delta_o^{\lambda_o})+\rho\right)T} - (a_o+\rho)\left(\left(\frac{y_o}{x_o}\right)^{\lambda_o} - 1\right)e^{-(a_o+\rho)T}}{\left(\frac{y_o}{x_o}\right)^{\lambda_o} e^{-\left(a_o(1-\delta_o^{\lambda_o})+\rho\right)T} - \left(\left(\frac{y_o}{x_o}\right)^{\lambda_o} - 1\right)e^{-(a_o+\rho)T}}.$$

The density function of the probability distribution of moving times T is given by $\hbar(T)\varkappa(T)$, and hence the expected moving time is the integral under the survival function, $T_{mo} = \int_{T=0}^{\infty}\varkappa(T)\mathrm{d}T$. In the cross-section of households at a point in time, the distribution of time spent in the same property has density function $\varkappa(T)/T_{mo}$, and the implied average hazard rate is $\int_{T=0}^{\infty}\hbar(T)(\varkappa(T)/T_{mo})\mathrm{d}T = 1/T_{mo} = n_o + \rho$.

A.4 Calibration targets

In Toronto, the land transfer tax is the main transaction cost paid by buyers of property. The effective LTT rate is 1.5% in the pre-policy period (January 2006–January 2008), so $\tau_h = \tau_k = 0.015$. The parameters of the model are chosen to match the City of Toronto housing market in the pre-policy period. The average sale price from Table A.1 is \$402,000 during this period.

Non-tax transaction costs in the ownership market Apart from land transfer tax, buyers may pay a home inspection cost of about \$500, but this is very small relative to average house prices. So it is assumed buyers pay no transaction costs other than LTT, that is, $C_h = C_k = 0$.

From the side of sellers of property, the primary cost is the real-estate agent commission. Using Multiple Listing Service sales data, the average commission rate is about 4.5% of price. There are some other costs such as legal fees of around \$1,000, but these are negligible in comparison. Sellers may sometimes spend roughly \$2,500 on staging, but the seller's agent might cover this expense as part of their commission, so not all sellers pay for staging out of their own pocket. Thus, C_u is set to be 4.5% of the average house price.

Maintenance costs The maintenance cost M as a homeowner is set so that it is 2.6% of the average property price. This cost is made up of a 2% physical maintenance cost and a 0.6% property tax in Toronto. The extra maintenance cost of being a landlord, M_l, is set to be 8% of average rent. This cost includes two parts: approximately 5–7% that the landlord uses to hire a property manager, and approximately 1% that the landlord uses to pay for services such as taking out garbage, shovelling snow, and salting the walkways.

Transaction costs in the rental market In Toronto, landlords typically pay one month's rent to real-estate agents to lease their properties. So C_l is set to be $1/12$ of average annual rent. Tenants in Toronto do not typically pay a monetary transaction cost when renting a property, so the tenancy fee Π is set to zero.

Flows within each housing market Flows within the two housing markets are related to the average time between moves, times on the market, and viewings per sale and lease. Information on time-to-move, time-to-sell, and time-to-lease is derived from Toronto MLS data on sales and rental transactions during the pre-policy period. Estimates of the moving hazard function imply that homeowners move after $T_{mo} = 9.25$ years on average The average duration of stay for a tenant is 1,109 days, so $T_{ml} = 3.04$ years. Average time-to-sell for homeowners is 30.5 days and average time-to-rent is 18.7 days. During this period, the fraction of withdrawals from for-sale listings is 48% and from for-lease listings is 22%. In light of these withdrawals, the targets are $T_{so} = (30.5/365)/(1-0.48)$ and $T_{sl} = (18.7/365)/(1-0.22)$. This adjustment is made because time-on-the-market in the data is calculated from the final successful listing without accounting for earlier unsuccessful attempts, so true time-on-the-market is longer.

Data on buyers' time-on-the-market and viewings per sale and per lease are not available for Toronto. Using the 'Profile of Buyers and Sellers' survey collected by NAR in the United States, Genesove and Han (2012) report that for the period 2006–2009 the ratio of average time-to-buy to average time-to-sell is 1.28, and the average number of homes viewed by buyers is 10.7. Using this information, the targets used are $T_{bo} = 1.28 \times T_{so}$ and $v_o = 10.7/(1-0.48)$, where the latter adjusts viewings per sale to account for the withdrawal rate seen in Toronto. The idea is that viewings of properties that have been withdrawn from the market are not counted, so actual viewings are larger than reported viewings in the final successful listing. There is no data on the number of properties that renters view on average. According to an industry expert, renters view fewer properties than buyers, so the target adopted is half the number of viewings per sale ($v_l = v_o/2$).

Flow search costs There are no direct estimates of the flow costs of searching F_h, F_k, and F_w. The approach taken here is to base an estimate of search costs on the opportunity cost of time spent searching. More specifically, for buyers in the ownership market (the same for home-buyers and investors), assume one property viewing entails the loss of half a day's income, so the value of $F_h = F_k$ can be calibrated by adding up the costs of making the expected number of viewings. With viewings per sale equal to the average number of viewings made by a buyer, the total search cost is equated to $0.5 \times v_o \times (Y/365)$, where Y denotes average annual income. Thus, the calibration sets $T_{bo}F_h = 0.5 v_o Y/365$, and dividing both sides by PT_{bo} implies $F_h/P = 0.5 \times (1/365)(Y/P)(v_o/T_{bo})$. Taking the median household-level income from Statistics Canada implies a price-to-income ratio of $P/Y = 5.6$ in Toronto in 2007. Given the value of v_o/T_{bo}, the implied buyer's flow search cost, $F_h = F_k$, is 3.1% of the average price.

The same logic is applied to the flow search costs of tenants, where it is assumed that viewing a rental property takes half the time needed to view a property to buy. Thus, the ratio of tenants' and home-buyers' flow search costs F_w/F_h is set to $0.5 \times (v_l/v_o) \times (T_{bo}/T_{bl})$.

Household tenure and entry of investors Based on the 2006 City of Toronto Profile Report, the homeownership rate is $h = 54\%$, the average age of homeowners is 53.3, and the average age of tenants is 45.0. Hence the target for the difference between the average ages of homeowners and renters is $\alpha = 8.3$. There is no survey that specifically captures the proportion of first-time buyers in Toronto. The Canadian Association of Accredited Mortgage Professionals (now called Mortgage Professionals Canada) undertook a survey in 2015 finding that the fraction is as high as 45% of purchases, which is consistent with the 44% found in the 2018 Canadian Household Survey for the Greater Toronto Area. On the other hand, data from the Canada Mortgage and Housing Corporation suggests the fraction of first-time buyers is about a third. Based on this information, the calibration target is $\phi = 0.4$.

Using Toronto MLS data on sales and rental transactions, the fraction of purchases by buy-to-rent investors is 5.4% during the pre-policy period, so $\kappa = 0.054$. The price-to-rent ratio for the same property is 14.5 in 2007, and the ratio of average prices paid by investors to prices paid by home-buyers is 0.99. Hence, $P_k/R = 14.5$ and $P_k/P = 0.99$ are used as targets.

Credit costs The credit cost χ of becoming a homeowner is computed from a comparison of the mortgage rate r_c the household would face relative to the risk-free interest rate r_f on government bonds. The interest rates r_c and r_f are real interest rates. There is a spread between them due to unmodelled financial frictions. The risk-free real rate r_f used to discount future cashflows need not be the same as the discount rate r applied to future utility flows from owning property (allowing for an unmodelled housing risk premium between r and r_f). Assume all these interest rates are expected to remain constant over the mortgage term.

Suppose a household buys a property at price P at date $t = 0$ by taking out a mortgage with loan-to-value ratio ℓ. Assume the mortgage has term T_c and a constant real repayment I over its term. Let $D(t)$ denote the outstanding mortgage balance at date t, which has initial condition $D(0) = \ell P$ and terminal condition $D(T_c) = 0$. The mortgage balance evolves over time according to the differential equation:

$$\dot{D}(t) = r_c D(t) - I \quad \text{and hence} \quad \frac{\mathrm{d}(e^{-r_c t}D(t))}{\mathrm{d}t} = -Ie^{-r_c t}.$$

Solving this differential equation using the initial condition $D(0) = \ell P$ implies:

$$D(t) = e^{r_c t}\ell P - \frac{I}{r_c}(e^{r_c t} - 1).$$

The terminal condition $D(T_c) = 0$ requires that the constant real repayment I satisfies:

$$I = \frac{r_c \ell P}{1 - e^{-r_c T_c}}.$$

In the model, homeowners exit at rate ρ, in which case it is assumed they repay their mortgage in full (using the proceeds from selling their property). Hence, there is a probability $e^{-\rho t}$ that the date-t repayment I will be made, and a probability $\rho e^{-\rho t}$ that the whole balance $D(t)$ is repaid at date t. The credit cost χ is the present value of the expected stream of repayments discounted at rate r_f minus the amount borrowed (which would equal the present value of the repayments if $r_c = r_f$ in the absence of credit-market imperfections):

$$\chi = \int_{t=0}^{T_c} e^{-r_f t} e^{-\rho t} I \mathrm{d}t + \int_{t=0}^{T_c} e^{-r_f t} e^{-\rho t} \rho D(t) \mathrm{d}t - \ell P.$$

To derive an explicit formula for χ, first observe that

$$\int_{t=0}^{T_c} e^{-r_f t} e^{-\rho t} \mathrm{d}t = \frac{1 - e^{-(r_f+\rho)T_c}}{r_f + \rho} \quad \text{and} \quad \int_{t=0}^{T_c} e^{-r_f t} e^{-\rho t} e^{r_c t} \mathrm{d}t = \frac{1 - e^{-(r_f+\rho-r_c)T_c}}{r_f + \rho - r_c}.$$

Together with the formulas for $D(t)$ and I, the credit cost can thus be written as follows:

$$
\begin{aligned}
\chi &= \frac{\left(I+\frac{\rho I}{r_c}\right)}{(r_f+\rho)}(1-e^{-(r_f+\rho)T_c}) + \frac{\rho\left(\ell P - \frac{I}{r_c}\right)}{(r_f+\rho-r_c)}(1-e^{-(r_f+\rho-r_c)T_c}) - \ell P \\
&= \left(\frac{(r_c+\rho)(1-e^{-(r_f+\rho)T_c})}{(r_f+\rho)(1-e^{-r_cT_c})} + \frac{\rho\left(1-\frac{1}{1-e^{-r_cT_c}}\right)(1-e^{-(r_f+\rho-r_c)T_c})}{(r_f+\rho-r_c)} - 1\right)\ell P \\
&= \frac{\left((r_c+\rho)(1-e^{-(r_f+\rho)T_c}) - \frac{\rho(r_f+\rho)}{r_f+\rho-r_c}(e^{-r_cT_c}-e^{-(r_f+\rho)T_c}) - (r_f+\rho)(1-e^{-r_cT_c})\right)\ell P}{(r_f+\rho)(1-e^{-r_cT_c})} \\
&= \frac{\left((r_c-r_f) + \frac{\rho(r_f+\rho)-(r_c+\rho)(r_f+\rho-r_c)}{r_f+\rho-r_c}e^{-(r_f+\rho)T_c} - \frac{(r_f+\rho)(r_f+\rho-r_c)-\rho(r_f+\rho)}{r_f+\rho-r_c}e^{-r_cT_c}\right)\ell P}{(r_f+\rho)(1-e^{-r_cT_c})},
\end{aligned}
$$

and dividing both sides by price P and simplifying:

$$
\frac{\chi}{P} = \left(1 + \frac{r_c}{r_f+\rho-r_c}e^{-(r_f+\rho)T_c} - \frac{r_f+\rho}{r_f+\rho-r_c}e^{-r_cT_c}\right)\frac{(r_c-r_f)\ell}{(r_f+\rho)(1-e^{-r_cT_c})}.
$$

This equation is used to determine calibration targets for the marginal credit cost Z relative to the average property price P, and for the marginal credit cost Z relative to the average credit cost $\bar{\chi}$ conditional on becoming a homeowner.

A mortgage term of 25 years ($T_c = 25$) and an average loan-to-value ratio of 80% ($\ell = 0.8$) are assumed. Focusing on interest rates fixed for five years as a typical mortgage product, the 5-year conventional mortgage rate from Statistics Canada was 7.07% in 2007. Given an inflation rate of 2.14%, the implied real mortgage rate is 4.93% for an average homeowner. Since the average mortgage cost is based on 5-year fixed rates, the equivalent risk-free rate comes from 5-year government bonds. These had a yield of 4% in 2007, so the real risk-free rate is 1.86%.

Information on different mortgage rates is then used to compute credit costs for a marginal buyer. Based on micro-level mortgage data from the Bank of Canada, the average contract mortgage rate during 2017–2018 was around 3.11%. Borrowers with low credit scores who did not qualify for loans from major banks could obtain mortgages from trust companies or private lenders at mortgage rates of around 6.15%, suggesting a mortgage rate gap of 3% between the marginal and average home-buyer.

But households faced with a high mortgage rate when they first buy a house do not necessarily continue with that rate for the whole time they are mortgage borrowers. They can build up equity and improve their credit score, and thus obtain a mortgage rate closer to the average when they refinance. The baseline calibration assumes that a marginal home-buyer is able to close half of the initial gap with the average home-buyer over the whole term of the mortgage loan. This translates into a mortgage gap of 1.5%, implying the real mortgage rate for the marginal buyer is 6.43%.

In summary, $z = Z/P$ is derived from the formula for χ/P using $T_c = 25$, $\ell = 0.8$, $r_f = 1.86\%$, $r_c = 6.43\%$ (marginal), and the value of ρ obtained from the calibration method. The target for $Z/\bar{\chi}$ is derived by taking the ratio of χ/P for $r_c = 6.43\%$ (marginal) and $\bar{r}_c = 4.93\%$ (average), with the other terms being the same.

A.5 Calibration method

This section describes how to find the set of parameters exactly matching the calibration targets.

Fraction of investors among buyers Combining equation (18) and the formula for v_o from section 3.7, the fraction of purchases made by investors is $\kappa = \xi v_o$, where ξ is the fraction of investors among buyers (see 1) and v_o is average viewings per sale. Given empirical targets for κ and v_o, the required fraction ξ is

$$\xi = \frac{\kappa}{\nu_o}. \tag{A.66}$$

Transaction probabilities and selling and letting rates Using the formulas for ν_o, ν_l, T_{so}, and T_{sl} from section 3.7 and the value of ξ from (A.66), the targets for ν_o, ν_l, T_{so}, T_{sl} give:

$$\pi_o = \frac{\nu_o^{-1} - \xi}{1-\xi} \quad , \quad \pi_l = \frac{1}{\nu_l} \quad \text{and} \quad s_o = \frac{1}{T_{so}} \quad , \quad s_l = \frac{1}{T_{sl}}. \tag{A.67}$$

Uses of the housing stock The formulas for T_{so}, T_{sl}, T_{mo}, and T_{ml} from section 3.7 and (A.57) and (A.62) imply $u_o = (T_{so}/((1-\kappa)T_{mo}))h_o$ and $u_l = (T_{sl}/T_{ml})h_l$. The homeownership rate h defined in section 3.7 satisfies $h_o + (1-\kappa)u_o = \psi h$, and substituting for u_o in terms of h_o yields $(1+T_{so}/T_{mo})h_o = \psi h$. This is solved for h_o in terms of targets for h, ψ, and the time to move and time on the market. Similarly, by substituting for u_l, $h_l + u_l = (1+T_{sl}/T_{ml})h_l$, and (11) implies $h_l + u_l = 1-(h_o+u_o)$. Putting these equations together yields h_o, u_o, h_l, and u_l:

$$h_o = \frac{\psi h}{1+\frac{T_{so}}{T_{mo}}} \quad , \quad u_o = \frac{T_{so}}{(1-\kappa)T_{mo}}h_o \quad , \quad h_l = \frac{1-(h_o+u_o)}{1+\frac{T_{sl}}{T_{ml}}} \quad , \quad u_l = \frac{T_{sl}}{T_{ml}}h_l. \tag{A.68}$$

Exit rate of investors Using (A.56), $s_o u_o = (n_o+\rho)h_o + \rho_l(h_l+u_l)$, and solving for ρ_l yields $\rho_l = (s_o u_o - (n_o+\rho)h_o)/(h_l+u_l)$. With $(1-\kappa)s_o u_o = (n_o+\rho)h_o$ from (A.57):

$$\rho_l = \frac{\kappa s_o u_o}{h_l+u_l}, \tag{A.69}$$

which can be calculated using (A.67) and (A.68).

Market tightness Using equation (19) and the formulas for T_{so} and T_{bo} in section 3.7, it follows that $T_{bo} = \theta_o T_{so}$, so θ_o can be deduced from targets for T_{bo} and T_{so}. The definitions in (1) imply that $b_h = (1-\xi)\theta_o u_o$ and $b_l = \theta_l u_l$, and hence equation (12) can be solved for θ_l by substituting for b_o and b_l:

$$\theta_o = \frac{T_{bo}}{T_{so}}, \quad \theta_l = \frac{\psi - h_o - h_l - (1-\xi)\theta_o u_o}{u_l}, \quad \text{and} \quad T_{bl} = \theta_l T_{sl}, \tag{A.70}$$

where the final equation gives the value of T_{bl} using (19), θ_l, and T_{sl}, which cannot be chosen freely given the other targets. With these variables known, the viewing rates for home-buyers and renters follow from the formulas given in section 3.7:

$$q_o = \frac{\nu_o}{T_{bo}}, \quad q_l = \frac{\nu_l}{T_{bl}}, \quad \text{and} \quad T_{bh} = \left(\frac{1-\xi}{1-\kappa}\right)T_{bo}, \tag{A.71}$$

where the final equation is the time-to-buy T_{bh} from section 3.7 for home-buyers implied by the other targets using (18).

Transitions to homeownership The fraction of first-time buyers among home-buyers is ϕ. Using the law of motion for home-buyers (16), the value of ϕ in a steady state with $\dot{b}_h = 0$ is

$$\phi = \frac{(\gamma n_l h_l + \rho\psi)G_m(Z)}{n_o h_o + (\gamma n_l h_l + \rho\psi)G_m(Z)} = \frac{(q_o\pi_o+\rho)b_h - n_o h_o}{(q_o\pi_o+\rho)b_h}.$$

This can be calculated from the ratio of inflows of buyers because all home-buyers transact at the same rate conditional on entering the stock b_h. The second expression for ϕ follows because b_h is a steady state. In steady state, (A.56) implies $(n_o+\rho)h_o = (1-\kappa)s_o u_o$, and (1), (18), and (19) imply $(1-\kappa)s_o u_o = q_o\pi_o b_h$. Dividing numerator and denominator of the expression for ϕ by h_o and substituting

$q_o\pi_o b_h/h_o = n_o+\rho$:

$$\phi = \frac{\left(1+\frac{\rho}{q_o\pi_o}\right)(n_o+\rho)-n_o}{\left(1+\frac{\rho}{q_o\pi_o}\right)(n_o+\rho)}.$$

Rearranging yields the formula for ϕ in footnote 29, and this can be written in terms of the time to move T_{mo} and home-buyers' time on the market T_{bh} using the expressions from section 3.7:

$$\phi = \frac{\rho\left(1+\frac{T_{bh}}{T_{mo}}\right)}{\frac{1}{T_{mo}}+\rho\frac{T_{bh}}{T_{mo}}}.$$

This can be rearranged to give the value of ρ in terms of ϕ and other known targets, and with this, the implied value of n_o can also be found from $n_o = (1/T_{mo})-\rho$:

$$\rho = \frac{\phi}{T_{mo}+(1-\phi)T_{bh}}, \quad \text{and} \quad n_o = \frac{(1-\phi)(T_{mo}+T_{bh})}{T_{mo}(T_{mo}+(1-\phi)T_{bh})}. \tag{A.72}$$

Taking ρ from (A.72) and using the formula for T_{ml} yields $n_l = T_{ml}^{-1}-\rho$, and it can be checked whether this is positive. With (34) and ρ_l from (A.69), the parameter $a_l = n_l-\rho_l$ is obtained.

Let g_{ho}, g_{hl}, g_{bh}, and g_{bl} be the average ages of the household heads of those in h_o, h_l, b_h, and b_l, and g_h and g_l the average ages of those in h_o+b_h and h_l+b_l. The calibration target for the difference in the average ages of homeowners and renters is $\alpha = g_h-g_l$. Furthermore, let g_e and g_f denote the average age of new entrants to the city and first-time buyers respectively. Taking the group in h_o+b_h, exit occurs at rate ρ with first-time buyers of measure $\rho(h_o+b_h)$ arriving in steady state. The differential equation for the average age is thus $\dot{g}_h = 1-\rho g_h+\rho g_f$. A steady-state age distribution therefore has $g_h = g_f+\rho^{-1}$. It is convenient to consider all average ages relative to average age at first entry to the city, which are denoted by $\alpha_h = g_h-g_e$, $\alpha_l = g_l-g_e$, and similarly for the other groups. The definition of α and the average homeowner versus first-time buyer age difference imply:

$$\alpha = \alpha_h-\alpha_l, \quad \text{and} \quad \alpha_h = \alpha_f+\rho^{-1}. \tag{A.73}$$

Now consider the group h_l. There is exit at rate $n_l+\rho$ and entry $q_l\pi_l b_l/h_l = (n_l+\rho)$ from b_l as a proportion of the group h_l (see A.62 with $q_l\pi_l b_l = s_l u_l$), where the average age at entry is g_{bl}. Thus, $1 = (n_l+\rho)(g_{hl}-g_{bl})$ and hence:

$$\alpha_{hl} = \alpha_{bl}+(n_l+\rho)^{-1}. \tag{A.74}$$

Since $g_l = (h_l/(h_l+b_l))g_{hl}+(b_l/(h_l+b_l))g_{bl}$ by definition, it follows that $g_{hl}-g_l = (b_l/(h_l+b_l))(g_{hl}-g_{bl})$, and by using (A.74) and the formula for T_{ml} from section 3.7:

$$\alpha_{hl} = \alpha_l+\frac{b_l}{h_l+b_l}T_{ml}. \tag{A.75}$$

For the group b_l, given (A.65), there are outflows at rate $q_l\pi_l+\rho$, and inflows of proportion $\rho\psi(1-G_m(Z))/b_l$ from outside the city (average age g_e) and of proportion $(1-\gamma G_m(Z))n_l h_l/b_l$ from h_l (average age g_{hl}), hence:

$$1+\frac{\rho\psi(1-G_m(Z))}{b_l}g_e+\frac{n_l(1-\gamma G_m(Z))h_l}{b_l}g_{hl} = (q_l\pi_l+\rho)g_{bl}.$$

Using $\rho\psi(1-G_m(Z)) = (q_l\pi_l+\rho)b_l-(1-\gamma G_m(Z))n_l h_l$ from (A.65), this equation becomes $b_l = (1-\gamma G_m(Z))n_l h_l\alpha_{hl} = (q_l\pi_l+\rho)b_l\alpha_{bl}$. Substituting (A.74) and using (A.65) again leads to $\rho\psi(1-G_m(Z))\alpha_{hl} = b_l+(q_l\pi_l+\rho)b_l T_{ml}$. With $\theta_l = b_l/u_l$, $s_l = \theta_l q_l\pi_l$, and $s_l u_l = h_l/T_{ml}$ from (A.61), it follows that $(q_l\pi_l+\rho)b_l T_{ml} = (h_l/T_{ml})T_{ml}+\rho b_l T_{ml} = h_l+\rho b_l T_{ml}$, and by putting these equations together:

$$\alpha_{hl} = \frac{(h_l+b_l)+\rho b_l T_{ml}}{\rho\psi(1-G_m(Z))}. \tag{A.76}$$

Finally, consider the ages of first-time buyers. Using (16), a fraction $\gamma n_l h_l G_m(Z)/((\gamma n_l h_l+\rho\psi)G_m(Z))$

come from h_l, and a fraction $\rho\psi G_m(Z)/((\gamma n_l h_l+\rho\psi)G_m(Z))$ are new entrants to the city. Therefore, $g_f=(\gamma n_l h_l/(\gamma n_l h_l+\rho\psi))g_{hl}+(\rho\psi/(\gamma n_l h_l+\rho\psi))g_e$, and hence:

$$\alpha_f=\alpha_{hl}-\frac{\rho\psi}{\gamma n_l h_l+\rho\psi}\alpha_{hl}=\alpha_{hl}-\frac{(h_l+b_l)+\rho b_l T_{ml}}{(\gamma n_l h_l+\rho\psi)(1-G_m(Z))}, \tag{A.77}$$

where the second expression substitutes from (A.76). Using (A.61) and (A.65) again to write $(\gamma n_l h_l+\rho\psi)(1-G_m(Z))=(q_l\pi_l+\rho)b_l-(1-\gamma)n_l h_l=s_l u_l+\rho b_l-n_l(1-\gamma)h_l=(n_l+\rho)h_l+\rho b_l-(1-\gamma)n_l h_l=\rho(h_l+b_l)+\gamma n_l h_l$. Substituting this and (A.75) into (A.77):

$$\alpha_f=\alpha_l+\frac{b_l T_{ml}}{h_l+b_l}-\frac{(h_l+b_l)+\rho b_l T_{ml}}{\rho(h_l+b_l)+\gamma n_l h_l}=\alpha_l+\frac{T_{bl}T_{ml}}{T_{ml}+T_{bl}}-\frac{1+\rho\frac{T_{bl}T_{ml}}{T_{ml}+T_{bl}}}{\rho+\gamma n_l\frac{T_{ml}}{T_{ml}+T_{bl}}}, \tag{A.78}$$

where the second expression makes use of (A.68) and (A.70). Combining this formula with the two equations in (A.73) and simplifying yields the difference in average ages:

$$\alpha=\left(1+\rho\frac{T_{ml}T_{bl}}{T_{ml}+T_{bl}}\right)\left(\frac{1}{\rho}-\frac{1}{\rho+\gamma n_l\frac{T_{ml}}{T_{ml}+T_{bl}}}\right).$$

This is confirms the expression for α in footnote 29 with reference to the formulas given in section 3.7. Since ρ is known from earlier, this gives an equation for γ in terms of the targets:

$$\gamma=\frac{\alpha\rho^2(T_{ml}+T_{bl})^2}{((1-\alpha\rho)(T_{ml}+T_{bl})+\rho T_{bl}T_{ml})n_l T_{ml}}. \tag{A.79}$$

Furthermore, the targets pin down the value of $G_m(Z)$. Since $(\gamma n_l h_l+\rho\psi)(1-G_m(Z))=\rho(h_l+b_l)+\gamma n_l h_l$ as shown above, the value of $G_m(Z)$ must satisfy:

$$G_m(Z)=\frac{\rho\psi-\rho(h_l+b_l)}{\gamma n_l h_l+\rho\psi}, \tag{A.80}$$

and all the terms in this expression are known.

Discount rate and bargaining powers The methodology here is to search over values of the discount rate r to solve one equation. Conditional on r, the bargaining powers ω_o, ω_k, and ω_l can be found as follows.

Dividing both sides of the price equation (A.46) by P and rearranging yields:

$$\frac{\omega_o^*\Sigma_o}{\pi_o P}=\frac{(1-c_u)r+m-\theta_o q_o\xi\frac{\omega_k^*\Sigma_k}{P}}{r+\theta_o q_o(1-\xi)\pi_o}, \tag{A.81}$$

where $c_u=C_u/P$ and $m=M/P$ are known targets. Using equations (A.46) and (A.53), it follows that $P-P_k=(\omega_o^*\Sigma_o/\pi_o)-\omega_k^*\Sigma_k$, and hence $p_k=P_k/P$ satisfies:

$$1-p_k=\frac{\omega_o^*\Sigma_o}{\pi_o P}-\frac{\omega_k^*\Sigma_k}{P}, \quad \text{with} \quad \frac{\omega_k^*\Sigma_k}{P}=\frac{f_{kh}f_h}{q_o}\frac{\omega_k^*}{1-\omega_k^*}, \tag{A.82}$$

where the expression for $\omega_k^*\Sigma_k/P$ comes from (43) and the definitions of the targets $f_h=F_h/P$ and $f_{kh}=F_k/F_h$. Substituting for $\omega_o^*\Sigma_o/(\pi_o P)$ from (A.81) in the first equation of (A.82) implies $(r+\theta_o q_o(\xi+(1-\xi)\pi_o))(\omega_k^*\Sigma_k/P)=(1-c_u)r+m+(p_k-1)(r+\theta_o q_o(1-\xi)\pi_o)$, and then using the second part of (A.82):

$$\frac{\omega_k^*}{1-\omega_k^*}=\frac{q_o}{f_{kh}f_h}\frac{(1-c_u)r+m+(p_k-1)(r+\theta_o q_o(1-\xi)\pi_o)}{r+\theta_o q_o(\xi+(1-\xi)\pi_o)}. \tag{A.83}$$

This can be calculated using r, the targets, and other variables known so far. Since (40) implies $\omega_k^*/(1-\omega_k^*)=(\omega_k/(1-\omega_k))/(1+\tau_k)$, the implied seller bargaining power when facing an investor is $\omega_k=(\omega_k^*/(1-\omega_k^*))/((1/(1+\tau_k))+(\omega_k^*/(1-\omega_k^*)))$.

Using equation (36) for the equilibrium tenancy fee Π and the definition of the target $c_{wl}=\Pi/C_l$, it follows that the sum of the rental transaction costs $C_l + C_w$ is

$$C_l + C_w = \left(\frac{1-c_{wl}}{\omega_l}\right) C_l \,. \tag{A.84}$$

Dividing both sides of the rent equation (A.50) by R, substituting for $C_l + C_w$ using the equation above, and rearranging yields:

$$\frac{\omega_l \Sigma_l}{\pi_l R} = \frac{1 - m_l - (r + n_l + \rho)(1-c_{wl})c_l}{r + n_l + \rho + \theta_l q_l \pi_l} = \frac{1 - m_l - (r + T_{ml}^{-1})(1-c_{wl})c_l}{r + T_{ml}^{-1} + T_{sl}^{-1}} \,, \tag{A.85}$$

where $m_l = M_l/R$ and $c_l = C_l/R$ are known targets, and the second equation uses $T_{ml} = 1/(n_o + \rho)$ and $T_{sl} = 1/s_l = 1/(\theta_l q_l \pi_l)$. The value function of a new entrant to the city is $B_e = (1 - G_m(Z))B_l + G_m(Z)(B_o - \bar{\chi})$, which can be written as $B_e = B_l + G_m(Z)(Z - \bar{\chi})$ using equation (14) for the threshold cost Z. Solving equation (A.47) for the renter value function B_l, substituting into the equation for B_e and dividing both sides by P:

$$\frac{B_e}{P} = \frac{(1-\omega_l)\frac{q_l \Sigma_l}{P} - \frac{F_w}{P}}{r+\rho} + G_m(Z)\left(1 - \frac{\bar{\chi}}{Z}\right)\frac{Z}{P} = \frac{\frac{p_k}{p_r T_{sl}}\left(\frac{\omega_l \Sigma_l}{\pi_l R}\right)\left(\frac{1-\omega_l}{\omega_l}\right) - f_{wh} f_h}{r+\rho} + G_m(Z)\left(1 - \frac{\bar{\chi}}{Z}\right) z \,,$$

which is stated in terms of targets $p_k = P_k/P$, $p_r = P_k/R$, $f_{wh} = F_w/F_h$, and $z = Z/P$. Letting $b_e = B_e/P$ denote the target for entrants' payoff, this equation can be solved for $\omega_l/(1-\omega_l)$:

$$\frac{\omega_l}{1-\omega_l} = \frac{\frac{p_k}{p_r T_{sl}}\left(\frac{\omega_l \Sigma_l}{\pi_l R}\right)}{f_{wh} f_h - (r+\rho)\left(z\left(1 - \frac{\bar{\chi}}{Z}\right) G_m(Z) - b_e\right)} \,. \tag{A.86}$$

This can be calculated using r, the targets, and the known value of $\omega_l \Sigma_l/(\pi_l R)$ from (A.85). The bargaining power of a landlord is thus $\omega_l = (\omega_l/(1-\omega_l))/(1 + \omega_l/(1-\omega_l)))$.

With ω_k and ω_k^* known conditional on r, substituting (A.82) into (A.81) yields:

$$\frac{\omega_o^* \Sigma_o}{\pi_o P} = \frac{(1-c_u)r + m - \theta_o \xi f_{kh} f_h \frac{\omega_k^*}{1-\omega_k^*}}{r + \theta_o q_o (1-\xi)\pi_o} \,, \tag{A.87}$$

which is known given the targets conditional on r. Dividing the marginal first-time buyer indifference condition (A.52) by price P yields $((1-\omega_o^*)/\omega_o^*) q_o \pi_o (\omega_o^* \Sigma_o/(\pi_o P)) = ((1-\omega_l)/\omega_l)(q_l \pi_l p_k/p_r)(\omega_l \Sigma_l/(\pi_l R)) + (r+\rho)z + f_h - f_{wh} f_h$. Noting that $(1-\omega_o^*)/\omega_o^* = (1+\tau_h)(1-\omega_o)/\omega_o$ from (27) and $T_{bl} = 1/(q_l \pi_l)$, this equation can be solved for $\omega_o/(1-\omega_o)$:

$$\frac{\omega_o}{1-\omega_o} = \frac{\frac{(1+\tau_h)}{T_{bh}}\left(\frac{\omega_o^* \Sigma_o}{\pi_o P}\right)}{\frac{p_k}{p_r T_{bl}}\left(\frac{1-\omega_l}{\omega_l}\right)\left(\frac{\omega_l \Sigma_l}{\pi_l R}\right) + (r+\rho)z + (1-f_{wh})f_h} \,. \tag{A.88}$$

This expression can be evaluated using (A.85), (A.86), and (A.87). Hence, sellers' bargaining power when faced with a home-buyer is given by $\omega_o = (\omega_o/(1-\omega_o))/(1 + (\omega_o/(1-\omega_o)))$.

Next, taking the free entry condition (A.51) and dividing both sides by P:

$$\begin{aligned}\frac{\theta_l q_l \pi_l p_k}{p_r}\left(\frac{\omega_l \Sigma_l}{\pi_l R}\right) &= \left(1 + \tau_k\left(1 + \frac{\rho_l}{r}\right)\right)\theta_o q_o \left((1-\xi)\pi_o\left(\frac{\omega_o^* \Sigma_o}{\pi_o P}\right) + \xi\left(\frac{\omega_k^* \Sigma_k}{P}\right)\right) \\ &\quad + (r+\rho)\left((1+\tau_k)c_u + c_k p_k + (1 + \tau_k \omega_k^*)\frac{\Sigma_k}{P}\right) - \tau_k\left(1 + \frac{\rho_l}{r}\right) m \,,\end{aligned}$$

noting the definition $c_k = C_k/P_k$. Substituting for Σ_k/P using (A.82) and solving for $p_r = P_k/R$:

$$p_r = p_k\theta_l q_l \pi_l \left(\frac{\omega_l \Sigma_l}{\pi_l R}\right)\left(\left(1+\tau_k\left(1+\frac{\rho_l}{r}\right)\right)\theta_o q_o(1-\xi)\pi_o \frac{\omega_o^* \Sigma_o}{\pi_o P} + (r+\rho)\left((1+\tau_k)c_u + c_k p_k\right)\right.$$
$$\left. + \left(r+\rho+\left(1+\tau_k\left(1+\frac{\rho_l}{r}\right)\right)\frac{\theta_o q_o \xi \omega_k}{1+\tau_k}\right)\frac{f_{kh} f_h}{(1-\omega_k)q_o} - \tau_k\left(1+\frac{\rho_l}{r}\right)m\right)^{-1}, \quad \text{(A.89)}$$

which uses $\omega_k^*/(1-\omega_k^*) = (\omega_k/(1-\omega_k))/(1+\tau_k)$ and $(1+\tau_k\omega_k^*)/\omega_k^* = (1+\tau_k)/\omega_k$. The formula for p_r depends on known calibration targets and r, and as p_r is itself a target, equation (A.89) can be solved numerically to determine the discount rate r.

Meeting functions With ω_o and ω_l known, the meeting function elasticities η_o and η_l are derived from the calibration targets for ω_o/η_o and ω_l/η_l. Since market tightnesses θ_o and θ_l are determined in (A.70) and the viewing rates in (A.71), the meeting function productivity parameters A_o and A_l are those satisfying (24):

$$A_o = q_o\theta_o^{\eta_o} \quad \text{and} \quad A_l = q_l\theta_l^{\eta_k}.$$

Ownership-market match-quality distribution and idiosyncratic shocks A new variable β_o is introduced at this stage, which is defined as follows:

$$\beta_o = \frac{\lambda_o a_o \delta_o^{\lambda_o}\left(\frac{y_o}{x_o}\right)^{\lambda_o}}{\rho + a_o\left(1-\delta_o^{\lambda_o}\right)}. \quad \text{(A.90)}$$

Suppose there is a target value of β_o alongside other targets. At the final stage, econometric evidence on the response β of moving rates to the LTT change is used to determine β_o.

There is a numerical procedure to determine the arrival rate a_o of idiosyncratic shocks. The formula in footnote 28 implies the steady-state moving rate n_o can be written in terms of a_o, λ_o, ρ and β_o from (A.90):

$$n_o = \frac{a_o - \rho\frac{\beta_o}{\lambda_o}}{1+\frac{\beta_o}{\lambda_o}}.$$

Conditional on a value of a_o, the value of λ_o is found by solving this equation:

$$\lambda_o = \frac{(n_o+\rho)\beta_o}{a_o - n_o}, \quad \text{(A.91)}$$

using the provisional target for β_o and the values of ρ and n_o from (A.72). Next, take equation (A.44) and divide both sides by P. By making use of the second equation in (A.82):

$$\frac{x_o}{P} = \frac{(1-\omega_o^* + (1-\xi)\omega_o^*\theta_o)q_o\pi_o}{\omega_o^*}\left(\frac{\omega_o^*\Sigma_o}{\pi_o P}\right) + \frac{\omega_k^*}{(1-\omega_k^*)}\xi\theta_o f_{kh} f_h - f_h. \quad \text{(A.92)}$$

Similarly, dividing both sides of (A.43) by P:

$$\frac{y_o}{P} = \frac{x_o}{P} + (r+\rho+a_o)\left(\frac{\tau_h}{r}\left((1-\xi)\theta_o q_o \pi_o \frac{\omega_o^*\Sigma_o}{\pi_o P} + \xi\theta_o f_{kh} f_h \frac{\omega_k^*}{(1-\omega_k^*)}\right)\right.$$
$$\left. + c_h + (1+\tau_h)c_u - \tau_h\frac{m}{r}\right), \quad \text{(A.93)}$$

where (A.82) has been used again. Together, (A.92) and (A.93) give $y_o/x_o = (y_o/P)/(x_o/P)$ in terms of a_o and the calibration targets. With λ_o from (A.91) and y_o/x_o, equation (A.90) can be rearranged to solve for the idiosyncratic shock size parameter δ_o:

$$\delta_o = \left(\frac{\left(1+\frac{\rho}{a_o}\right)\beta_o}{\beta_o + \lambda_o \left(\frac{y_o}{x_o}\right)^{\lambda_o}} \right)^{\frac{1}{\lambda_o}} .$$

With the target for P and y_o/P known from (A.93), the value of $y_o = (y_o/P)P$ is deduced. Using $\pi_o = (\zeta_o/y_o)^{\lambda_o}$ from (30), it follows that $\zeta_o = y_o \pi_o^{1/\lambda_o}$, so ζ_o is known given y_o, λ_o, and π_o from (A.67). While both payoffs and costs can be scaled without loss of generality, the target for P provides a normalization that determines ζ_o. The cost parameters $C_h = c_h P$, $C_u = c_u P$, $C_k = c_k p_k P$, $F_h = f_h P$, $F_k = f_{kh} F_h$, and $M = mP$ follow from P and the other targets.

The value of P together with (A.92) determines x_o. Furthermore, Σ_o follows from the known value of $\omega_o^* \Sigma_o/(\pi_o P)$ in (A.87) and ω_o^* and π_o. Since these variables are all computed conditional on a conjecture for a_o, the value of a_o is verified by a numerical search to check whether the equation for Σ_o in (A.45) holds. The requirement $\delta_o y_o < x_o$ is verified at this stage.

Distribution of credit costs The value of $G_m(Z)$ has been determined in (A.80). Using (26), the marginal credit cost Z and the parameters μ and σ of the probability distribution satisfy:

$$\frac{\log Z - \mu}{\sigma} = \Phi^{-1}(G_m(Z)) .$$

Using (26) to obtain an equation for $\log \bar{\chi}$ and subtracting this from $\log Z = \mu + \sigma\Phi^{-1}(G_m(Z))$:

$$\log\left(\frac{Z}{\bar{\chi}}\right) = \log G_m(Z) - \log \Phi\left(\Phi^{-1}(G_m(Z)) - \sigma\right) + \sigma\Phi^{-1}(G_m(Z)) - \frac{\sigma^2}{2} ,$$

noting that μ cancels out. Using the known value of $G_m(Z)$ and the target for $Z/\bar{\chi}$, this equation can be solved numerically to find the standard deviation parameter σ. Note that $Z = zP$ using the known value of P and the target for $z = Z/P$. Together with σ solving the equation above, the value of the mean parameter is $\mu = \log Z - \sigma\Phi^{-1}(G_m(Z))$. The implied value of $\bar{\chi}$ follows from Z and the target $Z/\bar{\chi}$.

Rental-market parameters Given P, the target for p_k determines the price paid by investors $P_k = p_k P$, and the target for p_r determines the average rent $R = P_k/p_r$. The targets for m_l, c_l, and c_w then imply values of the cost parameters $M_l = m_l R$, and $C_l = c_l R$. The target for f_{wh} gives $F_w = f_{wh} F_h$ using the value of F_h obtained earlier. Using (A.84), $C_w = ((1-c_{wl})/\omega_l - 1)C_l$, which can be calculated using the target c_{wl} and the known values of ω_l and C_l.

With π_l known from (A.67), the value of Σ_l can be deduced from (A.87) using the values of R and ω_l. Equation (A.49) then implies $y_l = M_l - F_w + (r + n_l + \rho)(C_l + C_w) - \gamma n_l G(Z)(Z - \bar{\chi}) + (1 - \omega_l + \omega_l \theta_l) q_l \Sigma_l$. Since $\pi_l = (\zeta_l/y_l)^{\lambda_l}$ from (38), equation (A.28) can be rearranged to solve for λ_l in terms of y_l, π_l, and Σ_l:

$$\lambda_l = 1 + \frac{\pi_l y_l}{(r + \rho + n_l)\Sigma_l} .$$

Knowing λ_l allows the parameter ζ_l to be deduced from the equation for π_l as $\zeta_l = y_l \pi_l^{1/\lambda_l}$.

Response of the moving rate to the land transfer tax Conditional on a value of β_o from (A.90), all other targets have been matched. A numerical search over β_o is then used to match the model's predicted response β of the moving rate to the LTT with the econometric estimate.

A.6 Additional quantitative results

Table A.7: *Tax effects with 3% gap between average and marginal mortgage rates*

Variable	Model prediction	Econometric evidence
Time-to-move for homeowners	13% (matched)	13%
Buy-to-own (BTO) transactions	−15%	−10.1%
Buy-to-rent (BTR) transactions	1.9%	8.9%
Time-to-sell	8.6%	16.5%
Leases-to-sales ratio	15%	23%
Price-to-rent ratio	−1.8%	−3.9%
Average sales price	−1.9%	−2.0%
Homeownership rate	−1.6% (−0.9 p.p.)	-
Transaction tax revenue	46%	-
Effective LTT tax rate	Increased from 1.5% to 2.8% (1.3 p.p.)	

Notes: This table reports the simulation results for a rise in the transaction tax rate when the gap between the average mortgage and the marginal mortgage interest rate is calibrated to be 3%. The responses of variables are reported as log differences.

Table A.8: *Increase in housing consumption tax versus increase in transaction tax*

Variable	Higher transaction tax	Housing consumption tax
Time-to-move for homeowners	13%	−0.18%
Buy-to-own (BTO) transactions	−17%	0.26%
Buy-to-rent (BTR) transactions	5.0%	−0.10%
Time-to-sell	7.8%	−0.12%
Leases-to-sales ratio	21%	−0.34%
Price-to-rent ratio	−1.5%	−1.58%
Average sales price	−1.4%	−1.57%
Homeownership rate	−4.5%	0.09%
Welfare loss as a fraction of tax revenue	113%	−0.02%
Decomposition of welfare cost		
Across-market welfare loss	60%	−0.013%
Within-ownership market welfare loss	40%	−0.002%
Within-rental-market welfare loss	14%	−0.003%

Notes: This table compares the simulation results of a rise in the housing consumption tax (through M) with the baseline results of a rise in the transaction tax reported in Table 5. The initial transaction tax is set at 1.5% in both cases, and the increase in tax in each case yields a 44% increase in tax revenue. The responses of variables are reported as log differences.