Information Design with Competing Receivers*

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Abstract

We study information design in a model with one sender and several receivers who participate in a general competitive bidding game. The sender chooses a Blackwell experiment which provides the receivers with information about a payoff-relevant state of the world. The receivers then make offers of which the sender accepts at most one. We show that a sender with state-independent preferences optimally provides the same information to all receivers. Moreover, we identify a condition, competitiveness, under which the sender does not benefit from committing ex-ante to an offer-acceptance strategy. With state-dependent preferences, the sender may benefit from providing private information.

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1 Introduction

In many economic environments, a sender provides information to competing receivers. Governments provide information and solicit offers from several companies before assigning the exploitation rights for a natural resource to one of these. In labor markets, workers apply to several potential employers but accept only one job offer. Entrepreneurs pitch their business ideas to different potential investors but may require only a single investor to fund their company.

We analyze optimal information design by a sender who faces competing receivers. In our model, a sender chooses an experiment that provides receivers with information about an unknown payoff-relevant state of the world. Upon observing the information, receivers can each make an offer to the sender. The sender then accepts one of these offers or rejects all of them. Our game, which places little restriction on the available offers or preferences, generalizes the non-discriminatory single-good first-price auction in that a single winning bid is chosen according to the sender's preferences, the winning bid is implemented, and the losing bidders receive their outside options.

We study two research questions. First, does the sender benefit from providing private information to the receivers or is it optimal to provide all information publicly?[1] Second, could the sender benefit from commitment power regarding her choice of a winning offer?

Our first main result, Theorem 1, shows that the provision of public information is optimal for the sender if her preferences are state-independent. For any experiment that may provide the receivers with private information and any associated equilibrium in the competitive-bidding game, we construct a public experiment and an associated equilibrium under which the sender is at least weakly better off. This public experiment garbles the information generated under the original experiment in a way that allows the receivers to infer the designated winning bid. The competitive forces under the original equilibrium ensure that the winning receiver cannot profitably deviate to a bid that the sender finds less favorable under the public experiment. By contrast, there may exist alternative bids which both some bidding receiver and the sender prefer. Thus, the sender may not be able to attain the same outcome as under the original experiment but can ensure a weakly better outcome under the public experiment.

Theorem 1 largely simplifies the sender's information design problem – as it is public, optimal information can be determined using standard tools from the literature (e.g. Kamenica and Gentzkow (2011)'s concavification technique). It can also be seen as a robustness result. If the sender provides different pieces of information to different receivers, the receivers may benefit from communicating their information to each other.

[1] We do not just compare public versus private information provision, but rather allow for fully general information design and ask when the solution induces public or private information.

If all receivers obtain the same information in the first place, the sender obviously does not need to worry about any further transmission of information. Finally, in settings where public distribution of information may be required, for instance for fairness or legal reasons, Theorem 1 says this is of no cost to the sender.

Our second main result, Theorem 2, shows that the sender does not benefit from commitment in her choice of a winning bid if the environment satisfies a *competitiveness* condition. This condition can be understood as the ability of the receivers to closely match – in terms of the sender's payoff – other receivers' offers. Under this condition, competition among receivers is sufficiently strong so that the sender does not gain from increasing competitive pressure on the receivers by committing to some bid acceptance decision. An implication is that the sender's optimal information structure (which is public) remains optimal independently of the extent of commitment power she may potentially have.

The following example of offshore wind power auctions for centrally pre-investigated sites in Germany illustrates the practical relevance of our results.[2] In these auctions, the Federal Network Agency awards the right to build and run an offshore wind park in a specified area in the North Sea. Prior to the auction, the Federal Maritime and Hydrographic Agency conducts several tests on the site of the planned wind park to generate information about wind, seabed, and maritime conditions. This information is then provided to all interested bidders. In a next step, the Federal Network Agency runs a scoring auction in which bidders submit a multidimensional bid consisting of a payment, as well as technological and organizational aspects of their offer. The Federal Network Agency then selects a winning bid. Although the winning bid is determined according to a pre-specified scoring rule, the relevant law explicitly states that the regulator has discretion over how to evaluate qualitative aspects of the bids.[3] Our analysis says that the Federal Network Agency could not benefit from instead providing different pieces of information to different bidders or having less discretion in the choice of a winning bid.

Finally, we consider the case of state-dependent preferences for the sender. Here, the sender may benefit from providing the receivers with private information. In particular, the sender may wish to provide some receivers with information that enables these receivers to adapt their offers to the sender's preferences in order to exert higher competitive pressure on other receivers. We also provide an extended notion of competitiveness under which the provision of public information is still optimal and commitment to a decision rule continues to have no value.

[2]For further details, see `https://www.bundesnetzagentur.de/1007288`.
[3]See §53 (1) WindSeeG.

Related literature Our paper contributes to the literature on information design in first-price auctions as our bidding game generalizes this auction format. Following the seminal contribution of Milgrom and Weber (1982), Bergemann and Pesendorfer (2007) study the optimal design of information and auction rules if the bidders' values are independent and each bidder can only receive information about their own value. They show that it is optimal to induce asymmetric distributions over the bidders' valuations. Thus, as Myerson (1981) suggests, discriminating auction rules are optimal. Bergemann, Brooks and Morris (2017) consider fully general forms of information design in the context of a standard first-price auction. In their Theorem 2, they show that the auctioneer's optimal payoff can be achieved through a public information structure and an efficient outcome of the auction. We extend the optimality of public information and non-discriminating auction rules under fully flexible information design to more general environments. Like Bergemann et al. (2017) our model nests the private and common-value cases, but our generality with respect to sets of available offers and preferences allows for multidimensional bids or budget constraints and a risk averse auctioneer or bidders.

Our paper belongs to the growing literature on information design which analyzes situations in which a sender commits to communicate information to a group of receivers through a Blackwell experiment (Bergemann and Morris, 2013, 2016; Taneva, 2019; Mathevet, Perego and Taneva, 2020).[4] The characterization of optimal information design is challenging in general. Some contributions – such as Alonso and Camara (2016) focus on public experiments in a voting context. Under unanimity voting, Bardhi and Guo (2018) show that public and private persuasion coincide. For other voting rules, they focus on experiments that generate conditionally independent signals. Arieli and Babichenko (2019) show that private information design outperforms public information design in a context where receivers make independent binary decisions. In their context, private and public information design may only be equivalent if receivers are homogeneous. Kolotilin, Mylovanov, Zapechelnyuk and Li (2017) show that public information design is optimal if receivers make independent binary decisions and have private information about their type. We contribute to this literature by showing that public information design performs as well as private information design in many settings with competing receivers. We add the analysis of the role of decision commitment which naturally arises in our setting.

The remainder of this paper is organized as follows. Section 2 introduces the model. Section 3 shows that public experiments are optimal. Section 4 analyzes the role of commitment in choice of winning offer. We discuss our results and extensions in Section 5 and conclude in Section 6.

[4]For the single receiver case, see the seminal contributions by Kamenica and Gentzkow (2011) and Rayo and Segal (2010). Kolotilin (2018) uses a linear programming approach to the single receiver problem, as we do.

2 Model

2.1 Environment

There is one sender and a finite set $I := \{1, \dots, n\}$ of receivers with $n \geq 2$. Each receiver $i \in I$ simultaneously makes an offer $a_i \in A_i$ to the sender. We denote the decision not to make an offer by $a_i^0 \in A_i$ (i.e. receiver i makes a 'null' offer). The sender can accept one of the offers or reject all offers. We denote the decision to reject by a_0^0 and use the notation $A_0 \equiv \{a_0^0\}$ for convenience.[5] Thus, the sender chooses an outcome x from a profile of offers $a \in A \equiv \times_{i=0}^n A_i$.[6] Let $X \equiv \cup_{i=0}^n A_i$ denote the set of all possible outcomes and define the function $\iota : X \to I \cup \{0\}$ as the mapping from each outcome to the receiver who can offer this outcome, i.e., $\iota(x) = i \iff x \in A_i$.

There is a payoff-relevant state of the world $\omega \in \Omega$. This state is initially unknown and all players share a common prior $p \in int(\Delta\Omega)$. The payoff functions of the sender and any receiver $i \in I$ are given by

$$u : X \to \mathbb{R} \quad \text{and} \quad v_i : \Omega \times X \to \mathbb{R}.^7$$

Thus, we assume that the sender's preferences are state-independent.[8] We further assume that all players value their outside option at zero. The sender obtains her outside option whenever she rejects all offers or accepts a null offer. Receivers obtain their outside options whenever they make a null offer, or their offer is not accepted. Thus, we have $u(a_j^0) = v_i(\omega, a_j^0) = 0$ for all $j = 0, 1..., n$, $i \in I$, and $\omega \in \Omega$ and $v_i(\omega, x) = 0$ for all $x \notin A_i$, $i \in I$, and $\omega \in \Omega$. Given a belief $q \in \Delta\Omega$, we denote receiver i's expected payoff from the outcome x by $V_i(q, x) \equiv \sum_\omega q(\omega) v_i(\omega, x)$. Finally, we assume Ω and A_i to be finite sets for all $i \in I$.

2.2 Information Structure

The sender can generate information about the state ω by selecting a Blackwell experiment $\sigma = (S, \mu)$ where S is a set of signal realizations with the product structure $S = S_0 \times S_1 \times \cdots \times S_n$ and the function $\mu : \Omega \to \Delta S$ assigns to each state ω a conditional distribution $\mu(\cdot|\omega)$ over S. The sender observes the signal realization $s_0 \in S_0$ and receiver

[5] Interpret this notation as a passive receiver 0 who always makes the "non-offer" a_0^0.

[6] We sometimes use the notation $a = (a_i, a_{-i})$ and $A = A_i \times A_{-i}$.

[7] For notational convenience we define the auxiliary function $v_0 : \Omega \times X \to \mathbb{R}$ with $v_0(\cdot, \cdot) = 0$.

[8] We discuss the extension to state-dependent sender preferences in Section 5.1.

i observes the signal realization $s_i \in S_i$.[9] We assume all sets S_0, S_1,..., S_n are finite.[10] We denote the set of all such Blackwell experiments by Σ.

An experiment is *public* if it provides the same information to the sender and all receivers, or – formally – if for any $s \in S$, each element s_i of s is a sufficient statistic of the whole vector s. Note that any public experiment is equivalent to some experiment $\sigma^p = (S^p, \mu^p)$ where S^p is a set of signal realizations that is observed by all players and $\{\mu^p(\cdot|\omega)\}_\omega$ is a family of conditional distributions over S^p.[11] We will describe public experiments in this way throughout the paper. The set of all public experiments is Σ^p.

2.3 Base Game

For a given experiment $\sigma = (S, \mu)$, the players play the following Bayesian *game*:

t=0: A signal realization $s \in S$ realizes.

t=1: Each receiver i observes $s_i \in S_i$ and chooses an offer $a_i \in A_i$.

t=2: The sender observes $s_0 \in S_0$ and picks an outcome x from the profile of offers $a \in A$.

We denote a strategy and a belief of receiver i be $\alpha_i : S_i \to \Delta A_i$ and $\rho_i : S_i \to \Delta(\Omega \times S_{-i})$. A strategy and a belief of the sender are given by $\beta : S_0 \times A \to \Delta A$ and $\rho_0 : S_0 \times A \to \Delta(\Omega \times S_{-0})$ where the strategy β satisfies $\beta(s_0, a) \in \Delta a$ for all $s_0 \in S_0$ and $a \in A$, that is, the sender can only accept offers that have been made. We denote by B the set of all such strategies for the sender. We use the equilibrium concept of (weak) perfect Bayesian equilibrium (PBE).[12] We denote by $\mathcal{E}(\sigma)$ the set of all PBEs for a given information structure σ, with a generic element $\varepsilon = ((\beta, \alpha_1, \ldots, \alpha_n), (\rho_0, \ldots, \rho_n))$. In Section 5.2, we extend our results when selecting for PBE that rule out receivers playing 'surely-dominated' strategies which make offers sure to yield negative payoffs.

2.4 The Sender's Problem

The sender's information design problem is to choose an experiment σ which maximizes the sender's expected payoff across all possible PBEs under σ. Denote the probability of

[9]The sender can design information for herself here, but as she has state-independent preferences and will use a sequentially rational strategy to choose a winning offer, she only uses her signal as an endogenous tie-breaking rule. We discuss what happens if the sender does not receive a signal in Section 5.3.

[10]While we require finiteness, we do not put a bound on the number of messages.

[11]Any such experiment σ^p is equivalent to an experiment σ with $S_j = S^p$ for all j, $\mu(s_0 = s_1 = \cdots = s_n = s^p|\omega) = \mu^p(s^p|\omega)$ for all ω, and $\mu(s|\omega) = 0$ for all s with $s_j \neq s_k$ for some j and k.

[12]For a formal definition, see Definition 9.C.3 in Mas-Colell, Whinston and Green (1995).

a profile of offers $a \in A$ for the signal realization $s \in S$ by $\alpha(a|s) \equiv \prod_{i=1}^{n} \alpha_i(a_i|s_i)$. We can then formally define the sender's optimal expected payoff as

$$U^* \equiv \sup_{\sigma \in \Sigma} \sup_{\varepsilon \in \mathcal{E}(\sigma)} \sum_{\omega \in \Omega} \sum_{s \in S} \sum_{a \in A} p(\omega)\mu(s|\omega)\alpha(a|s) \sum_{x \in X} \beta(x|s_0, a)u(x). \tag{1}$$

Thus, we assume that for a given experiment σ, the players coordinate on a sender-optimal equilibrium in the set $\mathcal{E}(\sigma)$. We thereby follow the standard approach in the literature on information design (Bergemann and Morris, 2013, 2016; Taneva, 2019).[13] We say that an *experiment σ is optimal* if the sender attains the optimal payoff U^* under σ for some equilibrium in $\mathcal{E}(\sigma)$. If there is an optimal experiment that is public, we say that *providing private information has no value to the sender.*

It will be helpful to define an *outcome rule* by the mapping $\lambda : \Omega \to \Delta X$ which assigns to each state $\omega \in \Omega$ a conditional distribution over outcomes. Note that any given combination of an experiment σ and strategies for all players induces an outcome rule

$$\lambda(x|\omega) = \sum_{s \in S} \sum_{a \in A} \mu(s|\omega)\alpha(a|s)\beta(x|s_0, a).$$

As the payoffs of all players depend only on the state and the matching outcome, an outcome rule captures all payoff relevant information from the ex-ante perspective. We denote by Λ the set of all outcome rules: $\Lambda \equiv \{\lambda : X \times \Omega \to [0,1] \text{ such that } \lambda(\cdot|\omega) \in \Delta X \text{ for all } \omega \in \Omega\}$.

2.5 Discussion of the model

Our model applies to a wide variety of competitive bidding procedures that share important characteristics of the non-discriminatory single-good first-price auction: a single winning bid is chosen according to the auctioneer's preferences, the winning bid is implemented, and the losing bidders receive their outside option. Our model is general with respect to the players' preferences as well as the offers which the receivers can make in the competitive bidding procedure. The framework allows for risk-aversion, nests private and common value environments, and captures multidimensional bids as in scoring auctions or restricted bid spaces – for instance due to budget constraints or limited commitment and ex-post renegotiation.

[13] Mathevet et al. (2020) study information design under alternative equilibrium selection criteria.

3 Optimality of public experiments

3.1 Main result

Theorem 1. *Providing private information has no value to the sender.*

The optimality of public experiments has two important applied implications. First, there is no cost to transparency. This might be particularly relevant to public institutions – such as the regulator in the offshore wind auctions described in the introduction – who are often subject to freedom of information laws, which force them to provide all of their information to all competitors. Second, the provision of public information is a robust informational policy. If different competitors receive different pieces of information, it can be difficult to prevent them from communicating their information before choosing their offers. This is obviously not an issue if all receivers obtain the same information. In addition, it may be considerably easier for the sender to build up and maintain the reputation underlying the informational commitment if information is provided publicly.

The theorem follows from the observation that for any experiment and associated equilibrium, we can find a public experiment and an equilibrium under which the sender has a weakly higher expected payoff. We now describe the main logic of this result. Take some experiment σ and an associated equilibrium $\varepsilon \in \mathcal{E}(\sigma)$ that jointly induce an outcome rule λ. Can we implement the same outcome rule λ with a public experiment and some equilibrium? A straightforward approach is to use a public experiment which *publicizes* the winning offer under the original experiment and associated equilibrium. Formally, this corresponds to a public experiment $\sigma^p = (S^p, \mu^p)$ where the set of signal realizations is the set of winning offers under λ, i.e., $S^p = supp(\lambda)$, and these are drawn according to the outcome rule, i.e., $\mu^p = \lambda$. Given this public experiment, the key question is whether there exists an associated equilibrium in which for each signal realization $x \in supp(\lambda)$, the receiver $i = \iota(x)$ offers x and all other receivers make weakly worse offers and, thereby, induce the sender to accept x. There are four cases of potential deviations to consider:

(1) Receiver i could deviate to make no offer, i.e. making null offer a_i^0.

(2) Receiver i could deviate to some other offer $a_i \notin \{x, a_i^0\}$ that is weakly worse for the sender than x: $u(a_i) \leq u(x)$.

(3) Receiver i could deviate to some other offer $a_i \notin \{x, a_i^0\}$ that is strictly better for the sender than x: $u(a_i) > u(x)$.

(4) Some receiver $j \neq i$ could deviate to making an offer $a_j \in A_j$ strictly better for the sender than x: $u(a_j) > u(x)$.

Deviations of type (1) induce an *individual rationality* constraint for receiver i. Given the signal realization $s^p = x$, all players hold in equilibrium the posterior belief

$$q_\lambda(\omega|x) \equiv \frac{p(\omega)\lambda(x|\omega)}{\sum_{\omega'} p(\omega')\lambda(x|\omega')}.$$

Receiver i's expected payoff from offering x, $V_i(q_\lambda(x), x)$, therefore equals receiver i's expected payoff conditional on acceptance of his offer x under the experiment σ and equilibrium ε. This expected payoff has to be weakly positive. Otherwise, receiver i would have a strict incentive to deviate to a_i^0 under the original equilibrium ε.

We now turn to deviations of type (2).[14] Consider an offer $a_j \in A_j$ from another receiver $j \neq i$ which is worse for the sender than offer x but better than the deviation offer a_i, i.e., $u(a_j) \in [u(a_i), u(x)]$. If such an offer exists and is made by receiver j, the sender optimally picks a_j and hence the deviation to a_i is not profitable.[15] We say that a_j is a competitive offer to a_i and formalize this notion as follows.

Definition 1. *The offer $x \in A_i$ has a competitive offer at belief $q \in \Delta\Omega$ if there is an offer $a_j \in A_j$, $j \neq i$, such that for any $a_i' \in A_i$ with*

$$V_i(q, a_i') > V_i(q, x) \quad \textit{and} \quad u(a_i') < u(x)$$

we have

$$u(a_i') \leq u(a_j) < u(a_i).$$

In the proof, we show that any offer $x \in supp(\lambda)$ has a competitive offer at the belief $q_\lambda(x)$ and hence, by constructing strategies where these competitive offers are made, no type (2) deviations are profitable. Indeed, if some offer x has no competitive offer at the belief $q_\lambda(x)$, receiver i would have a profitable deviation from x under the original experiment σ and the equilibrium ε. In this case, there is no offer from some receiver j which induces a utility for the sender in the interval $[u(a_i), u(x)]$. Thus, the offers a_i and x are accepted in identical circumstances and generate expected profits of

$$\Pr(accept)V_i(q_\lambda(x), a_i) > \Pr(accept)V_i(q_\lambda(x), x)$$

where $\Pr(accept)$ is their common probability of acceptance.

Finally, consider deviations of type (3) and (4). Note that these are deviations that are in the interest of the sender. While we cannot exclude that such deviations exist, we can use them to improve the sender's expected payoff. In the proof, we show that

[14]The deviations of types (2), (3), and (4) can be understood as *incentive compatibility* constraints.

[15]If $u(a_j) = u(a_i)$, the sender can tie-break in favor of a_j to dissuade a deviation to a_i.

it is always possible to construct an equilibrium that accommodates deviations of types (3) and (4). The resulting outcome rule differs from the outcome rule λ but generates a strictly higher expected payoff for the sender.

We illustrate the construction in the following simple example. Suppose the state is binary with $\Omega = \{1, 2\}$ and prior $p = (\frac{1}{3}, \frac{2}{3})$, there are two receivers with binary offer sets $A_i = \{a_i^0, a_i^1\}$ for $i = 1, 2$, and the payoffs of the players are given by

$$
\begin{aligned}
&u(a_1^1) = 2,\ u(a_2^1) = 1, \\
&v_1(1, a_1^1) = 1,\ v_1(2, a_1^1) = -1, \\
&v_2(1, a_2^1) = 1,\ v_2(2, a_2^1) = -2.
\end{aligned}
$$

Consider now an experiment σ which fully reveals the state to receiver 2 and keeps receiver 1 uninformed. In any equilibrium $\varepsilon \in \mathcal{E}(\sigma)$, receiver 1 always offers a_1^0, receiver 2 offers a_2^1 if $\omega = 1$ and a_2^0 if $\omega = 2$, and the sender accepts a_2^1 if it is offered and picks a_0^0 otherwise.[16] Thus, we obtain an outcome rule $\lambda(x|\omega) = \mathbf{1}(\omega = 1)\delta_{a_2^1} + \mathbf{1}(\omega = 2)\delta_{a_0^0}$. The public experiment which publicizes this outcome rule sends the signal realization $s^p = a_2^1$ if $\omega = 1$ and $s^p = a_0^0$ if $\omega = 2$. In any equilibrium for this public experiment, receiver 1 offers a_1^1 for $s^p = a_2^1$ and this offer is accepted by the sender. Thus, the outcome rule λ cannot be implemented. However, any equilibrium induces a better outcome rule under which the sender accepts the more attractive offer a_1^1 for $\omega = 1$ and the outside option a_0^0 for $\omega = 2$.

The example also illustrates the fact that Theorem 1 is not a revelation principle. There does not exist any public experiment under which the offer a_2^1 is accepted in equilibrium. This is due to the fact that for any common posterior belief, receiver 1 is easier to persuade to offer a_1^1 than receiver 2 is to offer a_2^1. Thus, there exist outcome rules that are only implementable with private experiments. However, these outcome rules do not perform better for the sender than those outcome rules that can be implemented with a public experiment.

3.2 Optimal information

While in general, optimal information structures in games with multiple receivers can be hard to compute, Theorem 1 implies that the optimal public experiment can be determined using the concavification approach of Kamenica and Gentzkow (2011). Given a public belief $q \in \Delta\Omega$, the sender can implement the best offer among those offers that

[16] The sender could equivalently pick a_1^0 or a_2^0 after $\omega = 2$ and the argument would not change.

are individually rational and have a competitive offer at q:

$$X^{ir,c}(q) = \{x \in X : V_{\iota(x)}(q,x) \geq 0 \;\cap x \text{ has competitive offer at } q\}$$

The sender's optimal utility at posterior q is therefore given by $u^*(q) = \max_{x \in X^{ir,c}(q)} u(x)$. Let $\bar{u}(q)$ be the concave closure of $u^*(q)$. We then have $U^* = \int u^*(q) dK(q) = \bar{u}(p)$ where the optimal public experiment induces a distribution K over posterior means.

4 Value of decision commitment

In this section, we study whether the sender may benefit from decision commitment. With decision commitment, the sender selects an experiment σ and a decision strategy β before the receivers make their offers, and the receivers observe these choices. The receivers then play the following base game.

t=0: A signal realization $s \in S$ realizes.

t=1: Each receiver i observes $s_i \in S_i$ and chooses an offer $a_i \in A_i$.

t=2: An outcome x is chosen from the profile of offers $a \in A$, given s_0, according to the strategy β.

A BPE of this game consists of beliefs $(\rho_1, \ldots, \rho_n)$ and strategies $(\alpha_1, \ldots, \alpha_n)$ for the receivers. We denote the set of BPEs of this game by $\bar{\mathcal{E}}(\sigma, \beta)$ with generic element $\bar{\varepsilon}$. In line with our analysis of the case without decision commitment, we assume that the sender can choose her preferred equilibrium from this set. It follows that the sender's optimal payoff under commitment is given by

$$\bar{U} \equiv \sup_{\sigma \in \Sigma, \beta \in B} \sup_{\bar{\varepsilon} \in \bar{\mathcal{E}}(\sigma,\beta)} \sum_{\omega \in \Omega} \sum_{s \in S} \sum_{a \in A} p(\omega)\mu(s|\omega)\alpha(a|s) \sum_{x \in X} \beta(x|s_0, a) u(x).$$

The next lemma paves the way toward a characterization of the sender's optimal payoff under decision commitment.

Lemma 1. *The sender can implement an outcome rule λ under decision commitment if and only if λ is individually rational, that is, $\sum_\omega p(\omega)\lambda(x|\omega)v_{\iota(x)}(\omega, x) \geq 0$ for all $x \in supp(\lambda)$.*

Proof. In the proof of Theorem 1, we establish in the case without decision commitment that for any experiment σ and associated equilibrium $\varepsilon \in \mathcal{E}(\sigma)$, the induced outcome rule λ satisfies $\sum_\omega p(\omega)\lambda(x|\omega)v_{\iota(x)}(\omega, x) \geq 0$ for all $x \in supp(\lambda)$.[17] This ensures that receiver

[17]See the argument following equation 3.

$\iota(x)$ prefers offering x over the non-offer $a^0_{\iota(x)}$. The same requirement needs to be satisfied for any experiment σ, acceptance strategy β and equilibrium $\bar{\varepsilon} \in \bar{\mathcal{E}}(\sigma, \beta)$ in the game with decision commitment. Thus, the sender can implement the outcome rule λ only if λ is individually rational.

To show that the sender can implement any individually rational outcome rule, fix some individually rational outcome rule λ. As in the proof of Theorem 1, we construct the public experiment σ^p with $S^p = supp(\lambda)$ and $\mu^p(x|\omega) = \lambda(x|\omega)$ for any $x \in supp(\lambda)$ and $\omega \in \Omega$. Define the acceptance strategy β^p by $\beta^p(s_0 = x, a) = \delta_x$ if $x \in a$ and $\beta^p(s_0 = x, a) = \delta_{a^0_0}$ otherwise. As the sender commits to rejecting any offer other than x for $s_0 = x$, strategies where receiver $\iota(x)$ makes offer x and all other receivers make null offers a^0_i constitute equilibrium play as x is individually rational for $\iota(x)$. □

Lemma 1 has two interesting implications. The first is that the optimality of public information extends to the setting with decision commitment. Indeed, the proof shows that any individually rational outcome rule can be implemented with a public experiment. Secondly, the lemma allows us to characterize the sender's optimal payoff under decision commitment $\bar{U}$ as the value of the linear program

$$(\bar{P}): \quad \max_{\lambda \in \Lambda} \sum_{\omega \in \Omega} \sum_{x \in X} p(\omega)\lambda(x|\omega)u(x) \quad \text{s.t.} \quad \sum_{\omega \in \Omega} p(\omega)\lambda(x|\omega)v_{\iota(x)}(\omega, x) \geq 0, \; \forall x \in X.$$

This linear program maximizes the sender's expected payoff by choosing from the set of individually rational outcome rules. The value $\bar{U}$ is closely related to the sender's optimal payoff without decision commitment. The only difference is that without decision commitment, the sender faces the additional constraint requiring the existence of competitive offers for each implemented outcome at the respective belief. This observation is suggestive for our following second main result.

Theorem 2. *Decision commitment has no value to the sender if and only if there is an outcome rule $\bar{\lambda}$ which solves the linear program $(\bar{P})$ such that any $x \in supp(\bar{\lambda})$ has a competitive offer at the belief $q_{\bar{\lambda}}(x)$, given by*

$$q_{\bar{\lambda}}(x|\omega) = \frac{p(\omega)\bar{\lambda}(x|\omega)}{\sum_{\omega'} p(\omega')\bar{\lambda}(x|\omega')}.$$

The sufficiency of the theorem's condition is implied by the following argument. Take an outcome rule $\bar{\lambda}$ that is optimal under decision commitment. Suppose any of its outcomes has a competing offer at the belief conditional on the observation of this outcome. Can we implement this outcome rule with a public experiment and an equilibrium under which a projected winning offer is publicized, the projected winner makes the announced offer, and all other receivers make worse offers? As there is always a competitive offer, the winning receiver can be discouraged from making an offer that is worse for the sender. But can we have deviating offers that are better for the sender? The answer is no. Any such deviating offer would need to be individually rational for the respective receiver. But if there a individually rational offers that improve the sender's expected payoff, the initial outcome rule $\bar{\lambda}$ is not optimal, which yields a contradiction.

The necessity argument uses the insight of the proof of Theorem 1, that an optimal outcome rule is only implementable without decision commitment if it is also implementable with the public experiment and equilibrium used in the sufficiency argument above. If any optimal outcome rule has an outcome for which there is no competitive offer at the associated posterior belief, the respective receiver can always profitably deviate by *shading* his offer. Hence, the outcome rule cannot be implemented without decision commitment.

A sufficient condition for decision commitment to have no value is that every offer has a competitive offer at any public belief where the offer is individually rational. This *competitiveness* condition is satisfied if all receivers have equivalent sets of offers from the sender's perspective. Formally, this requires that for any two receivers i and j and any offer $a_i \in A_i$ of receiver i, receiver j has an offer $a_j \in A_j$ which the sender finds equally good, i.e., $u(a_i) = u(a_j)$.

However, there are natural environments that may not be competitive. Take the example of a first-price auction in which bidders face asymmetric budget constraints. Suppose the first bidder has *deep pockets* and can potentially make any bid whereas the second bidder has a tight budget constraint. Then, the second bidder can never exert competitive pressure on those bids of the first bidder that exceed his budget. As a consequence, the sender may benefit from commitment – for example in the form of bid discounts for the second bidder – to incentivize the first bidder to make high bids.

5 Discussion

5.1 Sender state-dependent preferences

We now discuss the case in which the sender's payoff may also depend directly on the state. Thus, the sender has a payoff function of the form

$$u : \Omega \times X \to \mathbb{R}.$$

We assume that the sender's payoff from not accepting any offer is zero, that is, $u(\omega, a_j^0) = 0$ for all $j = 0, 1, \ldots, n$. Denote sender's expected payoff from some offer $x \in X$ given the belief $q \in \Delta\Omega$ by $U(q, x) = \sum_\omega q(\omega)u(\omega, x)$.

We first note that Theorem 1 does not extend to the case of state-dependent sender preferences.

Proposition 1. *With state-dependent preferences, the sender may benefit from providing private information.*

Proof. We prove the result using the following simple example. Suppose the state is binary with $\Omega = \{1, 2\}$ and prior $p = (\frac{1}{3}, \frac{2}{3})$, there are two receivers with offer sets $A_i = \{a_i^0, a_i^1, a_i^2\}$ for $i = 1, 2$, and the payoffs of the players are given by

$$\begin{aligned} &u(\omega, a_1^1) = 10,\ u(\omega, a_1^2) = 1,\ u(\omega, a_2^k) = \mathbf{1}(\omega = k),\\ &v_1(1, a_1^1) = 3,\ v_1(2, a_1^1) = -1,\ v_1(\omega, a_1^2) = 4,\\ &v_2(\omega, a_2^k) = 1. \end{aligned}$$

With a public experiment, all players share the same posterior belief $q = \Pr(\omega = 1)$. Here, receiver 1 can only be incentivized to offer a_1^1 if (i) a_1^1 is individual rational, i.e., $3q + (1 - q)(-1) \geq 0$, and (ii) the offer of receiver 2 exactly matches the state, i.e., $q \in \{0, 1\}$. Conditions (i) and (ii) imply $q = 1$. For all other posterior beliefs, the sender makes a payoff of 1. Thus, the sender's best public experiment maximizes the probability of the posterior $q = 1$, and is therefore the fully revealing experiment which results in an expected payoff of

$$\frac{1}{3} \cdot 10 + \frac{2}{3} \cdot 1 = 4.$$

Consider now the private experiment which provides receiver 1 with no information and fully reveals the state to receiver 2. Given this experiment, there is an equilibrium in which receiver 1 always offers a_1^1, receiver 2's offer matches the state, and the sender accepts the offer a_1^1 if it is made and otherwise accepts the offer of receiver 2. Under this equilibrium, the sender's payoff is 10. Hence, the sender benefits from private information provision. □

In order to discuss the insights of the example in the proof, it is helpful to extend the notion of a competitive offer to state-dependent sender preferences.

Definition 2. *The offer $x \in A_i$ has a competitive offer at belief $q \in \Delta\Omega$ if there is an offer $a_j \in A_j$, $j \neq i$, such that for any $a_i' \in A_i$ with*

$$V_i(q, a_i') > V_i(q, x) \quad \textit{and} \quad U(q, a_i') < U(q, x)$$

we have

$$U(q, a_i') \leq U(q, a_j) < U(q, a_i).$$

In the example, receiver 2 can exert only limited competitive pressure on receiver 1. In particular, the sender has no public experiment which splits the prior into posteriors for which the offer a_1^1 is individually rational for receiver 1 and has a competitive offer from receiver 2. Receiver 2 needs to perfectly learn the state in order to match the sender's state-dependent preferences and exert maximal competitive pressure on receiver 1. However, receiver 1 needs to remain at least partially uninformed to be willing to offer a_1^1. A private experiment can decouple the posterior beliefs of the two receivers and thereby generate the best possible outcome for the sender. It is worth pointing out that the sender does not use the receivers' private information on equilibrium path. The privacy is only used to create a competitive threat that is not used on path.

By contrast, the sender finds it optimal to provide public information even with state-dependent preferences if it is relatively easy to exert competitive pressure on all receivers.

Definition 3. *The environment is competitive if every offer $x \in X$ has a competitive offer at any belief $q \in \Delta\Omega$ for which $V_{\iota(x)}(q, x) \geq 0$.*

Proposition 2. *If the environment is competitive, there is a public experiment which is optimal and commitment has no value.*

We omit the proof of this proposition as it follows exactly the proof of sufficiency for Theorem 2, with minor adaptations to the sender's state-dependent preferences.

In a competitive environment, the sender can exert sufficient competitive pressure on the receivers even when these receivers share the same posterior beliefs. The same competitive pressure also allows the sender to obtain equally good outcomes than those achievable with decision commitment. In particular, the sender has no value of committing to reject certain offers, as the competitive offers make these rejection threats credible.

5.2 Equilibrium selection

We use the equilibrium concept of (weak) perfect Bayesian equilibrium throughout our main analysis. It is well known that players may play weakly dominated strategies under this equilibrium concept. In our context, this means that losing receivers may make offers in equilibrium that, if accepted, would lead to strictly negative payoffs for all states which are possible under this receiver's belief.

In this section, we follow Bergemann et al. (2017) and consider an equilibrium refinement that rules out such surely dominated offers.[18] Formally, an offer $a_i \in A_i$ is *surely dominated* at a belief q if $v_i(\omega, a_i) < 0$ for all $\omega \in supp(q)$. Given an experiment σ, some

[18] As Bergemann et al. (2017) note, this selection is slightly weaker than ruling out weak dominance.

equilibrium $\varepsilon \in \mathcal{E}(\sigma)$ is also an *equilibrium in non-surely dominated strategies* if for each receiver i, the strategy α_i puts probability zero on offers that are surely dominated given the belief ρ_i. We denote the set of such equilibria for the experiment σ by $\mathcal{E}^*(\sigma)$.

In the following proposition, we show that an approximate version of our Theorem 1 holds for generic payoffs of the receivers.[19]

Proposition 3. *Take any experiment σ and equilibrium in non-surely dominated strategies $\varepsilon \in \mathcal{E}^*(\sigma)$ that yield the sender an expected payoff of U. For generic payoffs of the receivers and any $\delta > 0$, there exists a public experiment σ^p and an equilibrium in non-surely dominated strategies $\varepsilon^p \in \mathcal{E}^*(\sigma^p)$ that yield the sender an expected payoff of $U^p > U^* - \delta$.*

Take any experiment σ and associated equilibrium $\varepsilon \in \mathcal{E}^*(\varepsilon)$ in non-surely dominated strategies which induce an outcome rule λ. Consider now the public experiment that publicizes the winning offer. Recall from Theorem 1, that there is an equilibrium in the sender accepts for each $s^p = x$ some offer $a(x) \in X$ with $u(a(x)) \geq u(x)$. The offer $a(x)$ is non-surely dominated at the public belief $q_\lambda(x)$ as it is individually rational. Supporting $a(x)$ in equilibrium requires another receiver to play a competitive offer for $a(x)$ which is non-surely dominated at $q_\lambda(x)$. We show that such an offer must exist at $q_\lambda(x)$ or at a nearby full-support belief. For generic receiver payoffs, individual rationality and incentive compatibility of $a(x)$ can be preserved at nearby beliefs, and hence the sender can arbitrarily approximate the desired distribution of public beliefs.

5.3 No signal for sender

In our main analysis, we allow for Blackwell experiments that provide the sender directly with information about the state of the world. This assumption could be challenged in some applications of our model. For instance, a government agency's ability to infer the costs of constructing a wind park from experiments about seabed and wind conditions may vary with the agency's engineering expertise.

As the sender has state-independent preferences and is sequentially rational in equilibrium, the sender can only use information she directly obtains to break ties. Note that if the sender cannot design a private signal but can observe the realization of any public information provided to receivers, we can still construct the same public experiments and equilibria we do in Theorems 1 and 2 and so the results still apply. If the sender cannot observe any signal realizations herself, in some cases she may not be able to tie-break in favor of the desired winning offer. In this extension, we show that our results extend to

[19] Each receiver's payoff is given by numbers $\{v(\omega, x)\}_{\omega \in \Omega, x \in A_i \setminus \{a_i^0\}}$. We use the notion of genericity with respect to the Lebesque measure on $\mathbb{R}^{n(|A_i|-1)|\Omega|}$ which includes the case of pure common-values.

the case without a sender signal if we allow for cheap talk between the receivers and the sender.[20]

Formally, we say that the *sender has no signal* if she can only choose experiments from the restricted set $\underline{\Sigma} \equiv \{\sigma \in \Sigma : S_0 = \{s_0\}\}$. A receiver-public experiment for this restricted set is a Blackwell experiment in which for each receiver i, the signal realization $s_i \in S_i$ is a sufficient statistic for the full set of signal realizations $s_{-0} \in S_{-0}$. Such a receiver-public experiment can be captured through some experiment $\sigma^p = (S^p, \mu^p)$ whose signal realizations $s^p \in S^p$ are observed by all receivers but not by the sender. We introduce the possibility for cheap-talk by allowing each receiver i to send a message $m_i \in M_i$ in addition to his offer $a_i \in A_i$. We assume that for all receivers, M_i is finite but sufficiently large: $|M_i| \geq |X|$.

Proposition 4. *If the sender has no signal and the receivers can send cheap-talk messages, the sender can attain the same optimal payoff as in the case with a sender signal. Moreover, the sender can optimally use a receiver-public experiment.*

If there are at least three receivers, it is straightforward to see that the sender can obtain the same outcome independently of whether she can observe the public signal realization s^p. In particular, the sender can ask the receivers to report the signal realization through their cheap talk message. If the sender adapts her beliefs according to the signal realization reported by the majority, no individual receiver is pivotal for the sender's beliefs. Thus, there exists an equilibrium in which all receivers report the signal realization truthfully.

The argument for two receivers is more involved. First, it is helpful to note that the public signal realization s^p can only affect the sender's decision if she is indifferent between the receivers' offers. Otherwise, the sender has a strict incentive to pick one offer independently of the signal realization. A potential problem without a sender signal arises if there are three offers $a_1, a_1' \in A_1$, and $a_2 \in A_2$ such that $u(a_1) = u(a_2) < u(a_1')$ and receiver 2 has no offer a_2' with $u(a_2') \in [u(a_1), u(a_1')]$. In this case, the offer a_2 has to serve as competitive offer to a_1 and a_1'. This poses no problem if the public signal realization s^p is observable to the sender who can then always detect any deviation and identify the deviating receiver. By contrast, if the signal realization s^p is only observable to receivers and the receivers make the offers a_1 and a_2, the sender cannot differentiate between a deviation by receiver 1 following the signal realization $s^p = a_1'$ and on-path behavior for $s^p = a_1$. In the proof, we show that this problem can be resolved through cheap talk messages. In particular, we are always in one of the following cases. In the

[20] Alternatively to introducing cheap-talk, our results go through when the sender does not receive a signal but the offer sets A_i are identical – in terms of the sender's payoff – across receivers. That is, for any $i \neq j$ and $a_i \in A_i$, $\exists a_j \in A_j$ with $u(a_j) = u(a_i)$.

first case, receiver 2 finds it optimal to report the signal realizations $s^p = a_1$ and $s^p = a_1'$ truthfully, and therefore the sender is as well off as in the case where she observes the signal directly. In the second case, receiver 2 benefits from misreporting $m_2 = a_1'$ for $s^p = a_1$. This misreport triggers the sender to pick a_2 over a_1 to *punish* receiver 1's alleged deviation from a_1' to a_1. This deviation is strictly profitable for receiver 2 if the offer a_2 is strictly individually rational for receiver 2 at the posterior belief induced by $s^p = a_1$. However, this implies that we can implement a slightly different outcome rule under which a_2 is accepted for $s^p = a_1$ and this outcome rule generates the same expected payoff to the sender as the sender is indifferent between a_1 and a_2.

5.4 Limited sets of experiments

In our formulation of the sender's information design problem, we follow the literature in assuming that the sender can use any Blackwell experiment. In practice, senders may face additional constraints that restrict the sender's ability to generate information, that is, the sender may have to choose from a restricted set of signals $\underline{\Sigma} \subset \Sigma$.

We now want to argue that our main result, Theorem 1, continues to hold with limited sets of experiments if the sender can flexibly coarsen and share information. In the proof of Theorem 1, we show that for any experiment σ and associated equilibrium $\varepsilon \in \mathcal{E}(\sigma)$, there is a public experiment σ^p and an associated equilibrium $\varepsilon^p \in \mathcal{E}(\sigma^p)$ under which the sender obtains a weakly higher payoff than under the original experiment and equilibrium. The public experiment σ^p publicizes the winning offers under the original experiment and equilibrium. Thus, the experiment σ^p does not generate more information than the original experiment σ. In other words, observing the whole profile of signal realizations s of the original experiment σ is more informative than observing σ^p in the sense of Blackwell (1951, 1953). If the sender can freely coarsen and share information between receivers, the public experiment σ^p lies in the set of feasible experiments $\underline{\Sigma}$ as it is obtained from first garbling and then disseminating the information content of the original experiment σ.

6 Conclusion

In this paper, we study information design in general competitive bidding games. We show that public information design is optimal for the sender. Moreover, the sender often does not benefit from having commitment power regarding her offer acceptance decision.

Throughout our analysis, we assume that the sender has complete control over the information structure. In particular, the receivers have no private information at the outset of the relationship. Moreover, we focus on settings in which a single sender faces multiple receivers. We leave extensions of our approach to settings with private information and sender competition for future research.

A Appendix: Omitted proofs

A.1 Proof of Theorem 1

We first provide the following revelation principle that extends standard approaches in the literature to our dynamic settings in which the sender is also player in the base game.

Lemma 2. *For any experiment $\sigma = (S, \mu)$ and associated equilibrium $\varepsilon \in \mathcal{E}(\sigma)$, there is an experiment $\sigma' = (S', \mu')$ and an associated equilibrium $\varepsilon' = ((\beta, \alpha'_1, ..., \alpha'_n), (\rho'_0, ..., \rho'_n)) \in \mathcal{E}(\sigma')$ that generate the same outcome rule $\lambda : \Omega \to \Delta X$ with $S'_i = A_i$ for receivers $i = 1, ..., n$ and $\alpha'_i(a_i) = \delta_{a_i}$.*

Proof. Fix an experiment $\sigma = (S, \mu)$ and an associated equilibrium $\varepsilon \in \mathcal{E}(\sigma)$. Holding the sender's strategy β and the marginal distribution of the sender's signal $\mu_0(s_0|\omega) \equiv \sum_{s_{-0}} \mu(s|\omega)$ fixed, define a static base game between the receivers with the state $\theta \equiv (\omega, s_0)$, the prior belief $f(\theta) \equiv p(\omega)\mu_0(s_0|\omega)$, and the receivers' payoff functions $\gamma_i(\theta, a) \equiv \beta(a_i|a, s_0)v_i(\omega, a_i)$ for all $i \in I$. Define the experiment $\underline{\sigma} = (\underline{S}, \underline{\mu})$ with $\underline{S} = S_{-0}$ and $\underline{\mu} = \{\underline{\mu}(\cdot|\theta)\}$ such that $\underline{\mu}(\underline{s}|\theta) = \mu(s_0, \underline{s}|\omega)/\mu_0(s_0|\omega)$. As $\varepsilon \in \mathcal{E}(\sigma)$, the profile of receivers' strategies and beliefs $((\alpha_1, \ldots, \alpha_n), (\rho_1, \ldots, \rho_n))$ constitutes a Bayes Nash equilibrium of the incomplete information game induced by the static base game and the information structure consisting of the prior f and the experiment $\underline{\sigma}$. Due to the revelation principle in Proposition 2 of Taneva (2019), there exists a direct experiment $\underline{\sigma}' = ((A_1, \ldots, A_n), \underline{\mu}')$ and an associated equilibrium $((\alpha'_1, \ldots, \alpha'_n), (\rho'_1, \ldots, \rho'_n))$ with $\alpha'_i(a_i) = \delta_{a_i}$ for all $i \in I$ which induces an identical distribution over offer profiles. Define the experiment $\sigma' = (S', \mu')$ such that $S' = (S_0, A_1, \ldots, A_n)$ and $\mu'(s_0, a|\omega) = \underline{\mu}'(a|\omega, s_0)\mu_0(s_0|\omega)$. Define the profile of strategies and beliefs $((\beta, \alpha'_1, \ldots, \alpha'_n), (\rho'_0, \rho'_1, \ldots, \rho'_n))$ with $\rho'_0(\omega, a|s_0) = \mu'(\omega, s_0, a)p(\omega)/\sum_{\omega,a} \mu'(\omega, s_0, a)p(\omega)$. Given the experiment σ', all receivers play under this profile best replies given their beliefs and other strategies. Finally, the sender's strategy remains optimal as the receivers' strategies generate the same distribution over offer profiles as in the equilibrium $\varepsilon \in \mathcal{E}(\sigma)$. □

In the remainder of the proof, we show that for any experiment and associated equilibrium, there exist a public experiment and an associated equilibrium which gives the sender at least the same payoff.

Take an arbitrary experiment σ and associated equilibrium $\varepsilon \in \mathcal{E}(\sigma)$ that induce the outcome rule λ. Due to the revelation principle in Lemma 1, it is without loss of generality to assume that $S_i = A_i$ and $\alpha_i(a_i) = \delta_{a_i}$ for all $a_i \in A_i$ and $i \in I$.

Define the public experiment $\sigma^p = (S^p, \mu^p)$ such that $S^p = supp(\lambda)$ and $\mu^p(x|\omega) = \lambda(x|\omega)$ for all $x \in supp(\lambda)$ and $\omega \in \Omega$. Let $q_\lambda(x) \in \Delta\Omega$ denote the posterior public belief after observing $s^p = x$, given by

$$q_\lambda(\omega|x) \equiv \frac{p(\omega)\lambda(x|\omega)}{\sum_{\omega'} p(\omega')\lambda(x|\omega')}. \tag{2}$$

To construct a strategy profile $(\beta^p, \alpha_1^p, \ldots, \alpha_n^p)$ for the public experiment σ^p, we define several objects. Fix $x \in supp(\lambda)$. Let

$$\tilde{A}_{\iota(x)}(x) \equiv \left\{a_{\iota(x)} \in A_{\iota(x)} : V_{\iota(x)}(q_\lambda(x), a_{\iota(x)}) \geq 0 \wedge u(a_{\iota(x)}) \geq u(x)\right\}$$

be the set of offers of receiver $\iota(x)$ that the sender weakly prefers to the offer x and are individually rational for receiver $\iota(x)$ at the belief $q_\lambda(x)$. For any receiver $j \in I$ with $j \neq \iota(x)$, define the set

$$\tilde{A}_j(x) \equiv \left\{a_j \in A_j : V_j(q_\lambda(x), a_j) > 0 \wedge u(a_j) > u(x)\right\}$$

consisting of all offers of receiver j that the sender strictly prefers to x and receiver j strictly prefers to the payoff from outcome $x \notin A_j$ at the belief $q_\lambda(x)$, Given these sets, let

$$\tilde{I}(x) \equiv \underset{\{i \in I : \tilde{A}_i(x) \neq \emptyset\}}{\operatorname{argmax}} \left\{ \max_{a_i \in \tilde{A}_i(x)} u(a_i) \right\}$$

be the set of receivers who can make the most attractive offers to the sender from the set $\{\tilde{A}_i(x)\}_{i=0}^n$.

We first show that $V_{\iota(x)}(q_\lambda(x), x) \geq 0$. Hence the set $\tilde{A}_{\iota(x)}(x)$ always contains x and and $\tilde{I}(x)$ is well defined. Under the equilibrium $\varepsilon \in \mathcal{E}(\sigma)$, following the signal realization $s_{\iota(x)} = x$, receiver $\iota(x)$ prefers to offer x instead of not making any offer. Thus,

$$\sum_{\omega \in \Omega} \sum_{(s_0, a_{-\iota(x)}) \in S_0 \times A_{-\iota(x)}} \rho_{\iota(x)}(\omega, s_0, a_{-\iota(x)}|x)\beta(x|s_0, (x, a_{-\iota(x)}))v_{\iota(x)}(\omega, x) \geq 0. \tag{3}$$

By Bayes' rule, we have in any equilibrium

$$\rho_{\iota(x)}(\omega, s_0, a_{-\iota(x)}|x) = \frac{p(\omega)\mu(s_0, x, a_{-\iota(x)}|\omega)}{\sum_{\omega' \in \Omega} \sum_{(s_0', a_{-\iota(x)}') \in S_0 \times A_{-\iota(x)}} p(\omega')\mu(s_0', x, a_{-\iota(x)}'|\omega')}. \tag{4}$$

Thus, condition (3) can be restated as

$$
\begin{aligned}
&\sum_{\omega\in\Omega} p(\omega) \sum_{(s_0,a_{-\iota(x)})\in S_0\times A_{-\iota(x)}} \mu(s_0,x,a_{-\iota(x)}|\omega)\beta(x|s_0,(x,a_{-\iota(x)}))v_{\iota(x)}(\omega,x)\geq 0\\
\iff &\sum_{\omega\in\Omega} p(\omega) \sum_{(s_0,a)\in S_0\times A} \mu(s_0,a|\omega)\beta(x|s_0,a)v_{\iota(x)}(\omega,x)\geq 0\\
\iff &\sum_{\omega\in\Omega} p(\omega)\lambda(x|\omega)v_{\iota(x)}(\omega,x)\geq 0\\
\iff &V_{\iota(x)}(q_\lambda(x),x)\geq 0
\end{aligned}
$$

where the first step follows from $\beta(x|s_0,a)=0$ for $x\notin a$, the second step follows from the definition of λ, and the last step is implied by the definition of $q_\lambda(x)$.

Thus, $\tilde{I}(x)$ is well defined. Pick some receiver $\tilde{i}(x)\in\tilde{I}(x)$ and let

$$
a_{\tilde{i}(x)}(x)\in \underset{\{a'_{\tilde{i}(x)}\in\tilde{A}_{\tilde{i}(x)}(x):\ u(a'_{\tilde{i}(x)})\geq u(a_j)\ \forall a_j\in\cup_{j\neq\tilde{i}(x)}\tilde{A}_j(x)\}}{\operatorname{argmax}} V_{\tilde{i}(x)}(q_\lambda(x),a'_{\tilde{i}(x)})
$$

be the favorite offer of receiver $\tilde{i}(x)$ among all offers that cannot be improved upon by another receiver j's offer from the set $\tilde{A}_j(x)$. Finally, define for each $j\neq\tilde{i}(x)$ the offer

$$
a^c_j(x)\in \underset{\{a'_j\in A_j:u(a'_j)\leq u(a_{\tilde{i}(x)}(x))\}}{\operatorname{argmax}} u(a'_j)
$$

which is receiver j's most competitive offer not surpassing the offer $a_{\tilde{i}(x)}(x)$.

We can now specify the strategies. For any receiver $i\in I$, let α^p_i be the strategy that sends for each signal $s_p=x$ the offer $a_{\tilde{i}(x)}(x)$ if $i=\tilde{i}(x)$ and $a^c_i(x)$ if $i\neq\tilde{i}(x)$. Define $b(a)=\operatorname{argmax}_{a'_j\in a}u(a'_j)$ and let the sender's strategy β^p be as follows. If $a_{\tilde{i}(x)}(x)\in b(a)$, accept $a_{\tilde{i}(x)}(x)$. If $a_{\tilde{i}(x)}(x)\notin b(a)$, accept some $a_j\in b(a)$ breaking ties first in favor of x whenever $\tilde{i}(x)\neq\iota(x)$, and second (i.e. if $\iota(x)=\tilde{i}(x)$ or $x\notin b(a)$) against offers made by receiver $\tilde{i}(x)$.

Given the public experiment σ^p and the strategy profile $(\beta^p,\alpha^p_1,\ldots,\alpha^p_n)$ inducing the outcome rule λ^p, the sender attains at least as high a payoff as she did under the original experiment and equilibrium σ and ε. This follows from the fact that for each $x\in supp(\lambda)$, $\lambda^p(a_{\tilde{i}(x)}(x)|\omega)=\lambda(x|\omega)$ and $u(a_{\tilde{i}(x)}(x))\geq u(x)$.

It remains to show that the strategy profile $(\beta^p,\alpha^p_1,\ldots,\alpha^p_n)$ and the public beliefs $\{q_\lambda(x)\}_{x\in S^p}$ constitute an equilibrium for the public experiment σ^p. Note first that the sender's strategy β^p is sequentially rational given the other strategies and the beliefs as the strategy always picks an offer from the set of optimal offers $b(a)$.

Next, we turn to the receivers. Fix any signal realization $s^p=x\in S^p$. We check that none of the receivers has an incentive to deviate (as σ^p is public, it suffices to check there are no deviations at each individual signal realization). For notational convenience, for the remainder of the proof let receiver $i = \tilde{i}(x)$ and let $a_i(x) = a_{\tilde{i}(x)}(x)$.

Individual rationality At first, we check that no receiver j strictly benefits from deviating to a_j^0. Receiver i prefers offering $a_i(x)$ over not making any offer if

$$V_i(q_\lambda(x), a_i(x)) \geq 0.$$

This inequality follows directly from the fact that $a_i(x) \in \tilde{A}_i(x)$. It is straightforward to see that any receiver $j \neq i$ does not strictly benefit from deviating to a_j^0 as these receivers have a payoff of zero from offering $a_j^c(x)$ as well.

Incentive compatibility We first show that receiver i does not benefit from offering some $a_i' \neq a_i(x)$. We do this in two cases.

Case 1. Suppose first that $i \neq \iota(x)$. Note that receiver $\iota(x)$ offers the sender at least a payoff of $u(x)$ and that the sender will breaks ties in favor of x whenever i deviates to some $a_i' \neq a_i(x)$ (given $a_i(x)$ will not be available then). Thus, the deviation $a_i' \neq a_i(x)$ can be strictly profitable only if

$$(i)\; V_i(q_\lambda(x), a_i') > V_i(q_\lambda(x), a_i(x)),\; (ii)\; u(a_i') \geq u(a_j^c)\; \forall j \neq i, \text{ and } (iii)\; u(a_i') > u(x).$$

Note that (i) and the individual rationality of $a_i(x)$ imply $V_i(q_\lambda(x), a_i') > 0$; combined with (iii) this implies $a_i' \in \tilde{A}_i(x)$. Recall that due to the definition of $a_i(x)$ it holds that $V_i(q_\lambda(x), a_i(x)) \geq V_i(q_\lambda(x), a_i'')$ for all $a_i'' \in \tilde{A}_i(x)$ which satisfy $u(a_i'') \geq u(a_j')$ for all $a_j' \in \cup_{j\neq i}\tilde{A}_j(x)$. Thus, (i) implies that there exists some $a_j' \in \cup_{j\neq i}\tilde{A}_j(x)$ with $u(a_j') \leq u(a_i(x))$ – due to the definition of $a_i(x)$ – and $u(a_j') > u(a_i')$. By the definition of $a^c_{\iota(a_j')}(x)$, it follows that

$$u(a^c_{\iota(a_j')}(x)) \geq u(a_j') > u(a_i'),$$

which contradicts (ii). Thus, no such strictly profitable deviation a_i' can exist.

Case 2. Next, suppose that $i = \iota(x)$. As the sender breaks ties in favor of receiver i's competitors whenever i deviates from $a_i(x)$ (as $i = \iota(x)$, ties are broken against i even when she offers x), it follows that a deviation a_i' can only be profitable if

$$(i)\; V_i(q_\lambda(x), a_i') > V_i(q_\lambda(x), a_i(x)) \text{ and } (ii)\; u(a_i') > u(a_j^c)\; \forall j \neq i.$$

We consider two subcases. First suppose $a_i' \in \tilde{A}_i(x)$. Condition (i) and the definition of $a_i(x)$ imply that $V_i(q_\lambda(x), a_i') > V_i(q_\lambda(x), a_i(x)) \geq V_i(q_\lambda(x), a_i'')$ for all $a_i'' \in \tilde{A}_i(x)$ that satisfy $u(a_i'') \geq u(a_j')$ for all $a_j' \in \cup_{j \neq i} \tilde{A}_j(x)$. For each $j \neq i$, from the definition of $a_j^c(x)$, we have $u(a_j^c(x)) \geq u(a_j')$ for all $a_j' \in \tilde{A}_j(x)$. Thus, $u(a_i') > u(a_j')$ for all $a_j' \in \cup_{j \neq i} \tilde{A}_j(x)$ due to condition (ii). But this contradicts the definition of $a_i(x)$ as a_i' is instead i's favorite offer in $\tilde{A}_i(x)$ that cannot be surpassed by another receiver's offer in the sets $\tilde{A}_j(x)$; hence a deviation $a_i' \in \tilde{A}_i(x)$ satisfying (i) and (ii) is impossible.

Suppose now that there exists some $a_i' \notin \tilde{A}_i(x)$ which satisfies conditions (i) and (ii). Any $a_i' \notin \tilde{A}_i(x)$ which respects condition (i) needs to satisfy $u(a_i') < u(x)$. The offer a_i' can then only satisfy condition (ii) if there does not exist some $a_j' \in \cup_{j \neq i} A_j$ with $u(a_j') \in [u(a_i'), u(x)]$ (if such an a_j' existed, then as $u(a_i(x)) \geq u(x)$, we would have that $u(a_{\iota(j')}^c) \geq u(a_j') \geq u(a_i')$). We show now that this leads to a contradiction with x being offered by receiver $i = \iota(x)$ under the original experiment σ and associated equilibrium ε. Following the signal realization $s_i = x$ of σ, receiver i finds it optimal to offer x instead of a_i' if

$$\sum_{\omega \in \Omega} \sum_{(s_0, a_{-i}) \in S_0 \times A_{-i}} \rho_i(\omega, s_0, a_{-i}|x) \beta(x|s_0, (x, a_{-i})) v_i(\omega, x)$$
$$\geq \sum_{\omega \in \Omega} \sum_{(s_0, a_{-i}) \in S_0 \times A_{-i}} \rho_i(\omega, s_0, a_{-i}|x) \beta(a_i'|s_0, (a_i', a_{-i})) v_i(\omega, a_i').$$

Recall that equilibrium beliefs satisfy equation (4). Moreover, note that $\beta(x|s_0, (x, a_{-i})) = \beta(a_i'|s_0, (a_i', a_{-i}))$ as $u(a_i') < u(x)$ and there does not exist some offer from other receivers which generates a sender utility in $[u(a_i'), u(x)]$. Thus, the inequality can be written as

$$\sum_{\omega \in \Omega} p(\omega) \sum_{(s_0, a_{-i}) \in S_0 \times A_{-i}} \mu(\omega, s_0, a_{-i}|x) \beta(x|s_0, (x, a_{-i})) v_i(\omega, x)$$
$$\geq \sum_{\omega \in \Omega} p(\omega) \sum_{(s_0, a_{-i}) \in S_0 \times A_{-i}} \mu(\omega, s_0, a_{-i}|x) \beta(x|s_0, (x, a_{-i})) v_i(\omega, a_i')$$

which is – due to $\beta(x|s_0, a) = 0$ for $x \notin a$ – equivalent to

$$\sum_{\omega \in \Omega} p(\omega) \lambda(x|\omega) v_i(\omega, x) \geq \sum_{\omega \in \Omega} p(\omega) \lambda(x|\omega) v_i(\omega, a_i') \iff V_i(q_\lambda(x), x) \geq V_i(q_\lambda(x), a_i').$$

As $V_i(q_\lambda(x), a_i(x)) \geq V_i(q_\lambda(x), x)$ by the definition of $a_i(x)$, we obtain a contradiction to condition (i).

We finally check whether some receiver $j \neq i$ has an incentive to deviate to offering $a_j' \neq a_j^c(x)$. Such a deviation can only be profitable if it generates a strictly positive payoff and is accepted; this requires (given tie-breaking in favor of $a_i(x)$):

$$(i)\ V_j(q_\lambda(x), a_j') > 0 \text{ and } (ii)\ u(a_j') > u(a_i(x)).$$

Suppose that some receiver j has an offer a'_j which satisfies (i) and (ii). As $u(a_i(x)) \geq u(x)$, it follows that $a'_j \in \tilde{A}_j(x)$. However, this implies by condition (ii) that there exists some $a'_k \in \cup_{k \neq i} \tilde{A}_k(x)$ with $u(a'_k) > u(a_i(x))$. Thus, $i \notin \tilde{I}(x)$, which is a contradiction. $\square$

A.2 Proof of Theorem 2

We first prove sufficiency. Suppose there exists an outcome rule $\bar{\lambda}$ that solves $(\bar{P})$ such that any $x \in supp(\bar{\lambda})$ has a competitive offer at the public belief $q_{\bar{\lambda}}(x)$. We show that the outcome rule $\bar{\lambda}$ can be implemented without decision commitment. Consider the public experiment $\bar{\sigma}^p$ given by $\bar{S}^p = supp(\bar{\lambda})$ and $\mu^p(x|\omega) = \bar{\lambda}(x|\omega)$ for any $x \in supp(\bar{\lambda})$. Let the strategies $(\bar{\alpha}_1, \dots, \bar{\alpha}_n)$ for the receivers and $\bar{\beta}$ for the sender prescribe the following behavior for some signal realization $\bar{s}^p = x \in supp(\bar{\lambda})$: receiver $\iota(x)$ offers x, some receiver $j \neq \iota(x)$ who has a competitive offer $a^c_j \in A_j$ to x at the public belief $q_{\bar{\lambda}}(x)$ offers a^c_j, any other receiver $k \neq j, \iota(x)$ offers a^0_k, and the sender picks for any profile of offers a an offer from the optimal set $b(a)$, breaking ties in favor of x if $x \in a$ and in favor of a^c_j if $a^c_j \in a$ and $x \notin a$.

We prove that the combination of the strategies $(\bar{\beta}, \bar{\alpha}_1, \dots, \bar{\alpha}_n)$ and the public beliefs $\{q_{\bar{\lambda}}(x)\}_{x \in supp(\bar{\lambda})}$ lies in the set of equilibria $\mathcal{E}(\bar{\sigma}^p)$.

Take any $\bar{s}^p = x \in supp(\bar{\lambda})$. It is obvious that each receiver $k \neq \iota(x)$ weakly prefers following their strategy rather than deviating to a^0_k as both options give a payoff of zero. Receiver $\iota(x)$ weakly prefers offering x to $a^0_{\iota(x)}$ as the expected payoff from the former is weakly positive due to

$$V_{\iota(x)}(q_{\bar{\lambda}}(x), x) \geq 0 \iff \sum_{\omega \in \Omega} p(\omega)\bar{\lambda}(x|\omega) v_{\iota(x)}(\omega, x) \geq 0.$$

Moreover, receiver $\iota(x)$ never benefits from offering some $a_{\iota(x)} \neq x$ with $u(a_{\iota(x)}) \leq u(x)$ as this would result in the sender accepting the competing offer a^c_j with certainty. Any other receiver $k \neq \iota(x)$ would also not benefit from offering any $a_k \in A_k$ with $u(a_k) \leq u(x)$ as this would also surely result in x being accepted by the sender. By contrast, some receiver $i \in I$ would benefit from offering some $x' \neq x$ with $u(x') > u(x)$ if $V_{\iota(x')}(q_{\bar{\lambda}}(x), x') > V_{\iota(x')}(q_{\bar{\lambda}}(x), x)$. However, the existence of such a deviation x' contradicts the optimality of $\bar{\lambda}$ in $(\bar{P})$. To see this, define the outcome rule λ' such that $\lambda'(y|\omega) = \bar{\lambda}(y|\omega)$ for all ω and $y \neq x, x'$, $\lambda'(x|\omega) = 0$ for all ω, and $\lambda'(x'|\omega) = \bar{\lambda}(x'|\omega) + \bar{\lambda}(x|\omega)$ for all ω. The outcome rule λ' satisfies the constraint of $(\bar{P})$ as $\sum_\omega p(\omega)\lambda'(y|\omega) v_{\iota(y)}(\omega, y) =$

$\sum_\omega p(\omega)\bar{\lambda}(y|\omega)v_{\iota(y)}(\omega, y) \geq 0$ for all $y \neq x, x'$ and

$$\sum_{\omega\in\Omega} p(\omega)\lambda'(x'|\omega)v_{\iota(x)}(\omega, x') = \sum_{\omega\in\Omega} p(\omega)\bar{\lambda}(x'|\omega)v_{\iota(x)}(\omega, x') + \sum_{\omega\in\Omega} p(\omega)\bar{\lambda}(x|\omega)v_{\iota(x)}(\omega, x') \geq 0$$

as the first term on the right-hand side of the equation is weakly positive by individual rationality of $\bar{\lambda}$ and

$$V_{\iota(x')}(x', q_{\bar{\lambda}}(x)) > V_{\iota(x')}(x, q_{\bar{\lambda}}(x)) \geq 0 \implies \sum_{\omega\in\Omega} p(\omega)\bar{\lambda}(x'|\omega)v_{\iota(x')}(\omega, x') \geq 0.$$

The sender's expected payoff under λ' is strictly higher than under $\bar{\lambda}$ as

$$\sum_{\omega\in\Omega}\sum_{y\in X} p(\omega)\lambda'(y|\omega)u(y) = \sum_{\omega\in\Omega}\sum_{y\in X} p(\omega)\bar{\lambda}(y|\omega)u(y) + \sum_{\omega\in\Omega} p(\omega)\bar{\lambda}(x|\omega)(u(x') - u(x))$$

with $u(x') > u(x)$. Thus, $\bar{\lambda}$ is not a solution to $(\bar{P})$, which is a contradiction.

It remains to prove necessity. Suppose that for any solution $\bar{\lambda}$ of the linear program $(\bar{P})$ there is some outcome $\bar{x} \in supp(\bar{\lambda})$ which does not have a competitive offer at the belief $q_{\bar{\lambda}}(\bar{x})$. We know from the proof of Theorem 1 that any such outcome rule $\bar{\lambda}$ can be implemented without decision commitment only if it can be implemented with the public experiment $\bar{\sigma}^p = (supp(\bar{\lambda}), \bar{\lambda})$ specified above and an equilibrium $\varepsilon \in \mathcal{E}(\bar{\sigma}^p)$ under which receiver $\iota(x)$ offers x for $\bar{s}^p = x$ and any other receiver $k \neq \iota(x)$ offers some $a_k \in A_k$ with $u(a_k) \leq u(x)$. For the signal realization $\bar{s}^p = \bar{x}$, there is by our assumption no competitive offer to $\bar{x}$ at the public belief $q_{\bar{\lambda}}(\bar{x})$. Thus, receiver $\iota(\bar{x})$ has some other offer $a_{\iota(\bar{x})}$ which satisfies $u(a_{\iota(\bar{x})}) > u(a_k)$ for all offers $a_k \in X \setminus A_{\iota(\bar{x})}$ with $u(a_k) \leq u(x)$ and $V_{\iota(\bar{x})}(q_{\bar{\lambda}}(\bar{x}), a_{\iota(\bar{x})}) > V_{\iota(\bar{x})}(q_{\bar{\lambda}}(\bar{x}), \bar{x})$. Thus, receiver $\iota(\bar{x})$ has a strict incentive to deviate to offering $a_{\iota(\bar{x})}$. □

A.3 Proof of Proposition 3

Fix some (generic) receiver payoffs and take an arbitrary experiment σ and associated equilibrium $\varepsilon \in \mathcal{E}^*(\sigma)$ in non-surely dominated strategies that induce the outcome rule λ. Let $U = \sum_{\omega\in\Omega}\sum_{x\in X} p(\omega)\lambda(x|\omega)u(x)$ be the sender's payoff from (σ, ε). Fix some $\delta > 0$. We will construct a public experiment and an associated equilibrium in non-surely dominated strategies which yield the sender an expected payoff $U^p > U - \delta$.

The following technical lemma will be useful in showing that if an offer satisfies some individual rationality and incentive compatibility constraints at some belief, the same holds true for nearby beliefs, including some that have full support on Ω. This will allow us to find non-surely dominated competitive offers at nearby beliefs.

Lemma 3. *For any $a_i \in A_i \setminus \{a_i^0\}$ and $A_i' \subseteq A_i \setminus \{a_i\}$, define the set of beliefs*

$$Q(a_i, A_i') = \{q \in \Delta\Omega : V_i(q, a_i) \geq 0 \quad \textit{and} \quad V_i(q, a_i) \geq V_i(q, a_i') \ \forall a_i' \in A_i'\}.$$

For generic payoffs of receiver i, the set $Q(a_i, A_i')$ is either empty or is convex and contains some $q' \in Q(a_i, A_i')$ with $q'(\omega) > 0$ for all $\omega \in \Omega$.

Proof. We prove inductively that $Q(a_i, A_i')$, when nonempty, is a convex polytope with dimension $|\Omega| - 1$. As it is a subset of the $|\Omega| - 1$-dimensional simplex $\Delta\Omega$, the set $Q(a_i, A_i')$ then contains some q' with $q'(\omega) > 0$ for all $\omega \in \Omega$.

Let $A_i' = \{a_i^1, ..., a_i^K\}$. Define the inequalities

$$\sum_{\omega \in \Omega} v_i(\omega, a_i) q(\omega) \geq 0 \quad \text{(IR)}$$

and for each $k = 0, 1, ..., K$:

$$\sum_{\omega \in \Omega} (v_i(\omega, a_i) - v_i(\omega, a_i^k)) q(\omega) \geq 0 \quad \text{(IC-k)}.$$

Suppose $K = 0$. If $Q(a_i, A_i')$ is nonempty, some $q \in \Delta\Omega$ satisfies (IR), and as for generic receiver payoffs $v_i(\omega, a_i) \neq 0$ for all $\omega \in \Omega$ and $a_i \neq a_i^0$, we have $v_i(\omega', a_i) > 0$ for some $\omega' \in \Omega$. Continuity of $V_i(q, a_i)$ ensures all $q \in \Delta\Omega$ in a neighborhood of the degenerate belief on state ω' satisfy (IR). Hence $Q(a_i, A_i')$ has dimension $|\Omega| - 1$. It is also a convex polytope as it is bounded and given by the intersection of the half-spaces $\{q(\omega) \geq 0\}_{\omega \in \Omega}$ and $\sum_{\omega \in \Omega} q(\omega) = 1$, – which define $\Delta\Omega$ – and (IR).

We now make the inductive step for $K > 0$. Suppose for any $a_i \neq a_i^0$ and sets $A_i' \subseteq A_i \setminus \{a_i\}$ of size up to $K - 1$, $Q(a_i, A_i')$ is either empty or a convex polytope of dimension $|\Omega| - 1$. If $Q(a_i, A_i' \setminus \{a_i^K\})$ is empty then $Q(a_i, A_i')$ is empty as well. Otherwise, $Q(a_i, A_i' \setminus \{a_i^K\})$ is a nonempty convex polytope with dimension $|\Omega| - 1$; $Q(a_i, A_i' \setminus \{a_i^K\})$ can then be represented by $|\Omega|$ vertices $q^1, ..., q^{|\Omega|} \in \Delta\Omega$. The set $Q(a_, A_i')$ is the intersection of $Q(a_i, A_i' \setminus \{a_i^K\})$ and $q \in \mathbb{R}^{|\Omega|}$ satisfying (IC-K) (a half-space). If the hyperplane HIC-K

$$\{q \in \mathbb{R}^{|\Omega|} : \sum_{\omega \in \Omega} (v_i(\omega, a_i) - v_i(\omega, a_i^K)) q(\omega) = 0\}$$

does not intersect $Q(a_i, A_i' \setminus \{a_i^K\})$, then $Q(a_i, A_i')$ is either empty or is equal to $Q(a_i, A_i' \setminus \{a_i^K\})$ and we are done. If hyperplane HIC-K intersects the interior of $Q(a_i, A_i' \setminus \{a_i^K\})$, it partitions $Q(a_i, A_i' \setminus \{a_i^K\})$ into two $|\Omega| - 1$-dimensional convex polytopes, one of which is $Q(a_i, A'_i)$ in which case we are done as well. The remaining case is that the hyperplane HIC-K intersects the boundary but not the interior of $Q(a_i, A'_i \setminus \{a_i^K\})$. This case is non-generic as it requires HIC-K to contain some vertex of $Q(a_i, A'_i \setminus \{a_i^K\})$, i.e. for some $q \in \{q^1, ..., q^{|\Omega|}\}$, $(v_i(\omega, a_i) - v_i(\omega, a_i^K))q(\omega) = 0$.[21] Only a (Lebesque) measure zero of receiver i's payoffs from offer a_i^K, $\{v_i(\omega, a_i^K)\}_{\omega\in\Omega}$, satisfy this linear equation. □

Define the public experiment $\sigma^p = (\mu^p, S^p)$ as in the proof of Theorem 1 by $S^p = supp(\lambda)$ and $\mu^p(x|\omega) = \lambda(x|\omega)$ for $x \in supp(\lambda)$. For $s^p = x \in supp(\lambda)$, the posterior belief is $q_\lambda(x)$ as defined in Equation (2). We now use the previous lemma to show the following result.

Lemma 4. *Fix any $d > 0$. For each $x \in supp(\lambda)$, there exists a belief $q_\lambda^d(x)$ such that: (1) $|q_\lambda^d(x) - q_\lambda(x)| < d$, and (2) conditional on public belief $q_\lambda^d(x)$ there exist non-surely dominated offers constituting equilibrium play and yielding the sender a payoff of at least $u(x)$.*

Proof. Fix some $x \in supp(\lambda)$. Following the proof of Theorem 1, define the quantities $\tilde{A}_{\tilde{\iota}(x)}$, $\tilde{A}_j(x)$ for all $j \neq \iota(x)$, $\tilde{I}(x)$, $\tilde{i}(x)$ and $a_{\tilde{i}(x)}(x)$. We prove the result in two cases.
Case 1. Suppose $\tilde{A}_j(x)$ is nonempty for some $j \neq \iota(x)$. For each $j \neq \tilde{i}(x)$, define $a_j^c(x) = a_j^0$ if $\tilde{A}_j(x) = \emptyset$ and let otherwise

$$a_j^c(x) \in \underset{\{a'_j \in \tilde{A}_j(x): u(a'_j) \leq u(a_{\tilde{i}(x)}(x))\}}{\operatorname{argmax}} u(a'_j).$$

Consider the following strategies. The sender plays any sequentially rational strategy breaking ties in favor of $a_{\tilde{i}(x)}(x)$ at $s^p = x$ and against receiver $\tilde{i}(x)$ when $a_{\tilde{i}(x)}(x)$ is not available. Receiver $\tilde{i}(x)$ offers $a_{\tilde{i}(x)}(x)$ and any receiver $j \neq \tilde{i}(x)$ offers $a_j^c(x)$. Note first that each offer made is non-surely dominated at $q_\lambda(x)$. The offer made by a receiver j is either in $\tilde{A}_j(x)$ – yielding by the definition of $\tilde{A}_j(x)$ weakly positive expected payoff for $j = \tilde{i}(x)$ and strictly positive payoff for $j \neq \tilde{i}(x)$ – or is the null offer. Second, note that these strategies constitute equilibrium strategies for the public experiment σ^p. By definition of $a_{\tilde{i}(x)}(x)$, for any offer $a'_{\tilde{i}(x)} \in A_{\tilde{i}(x)}$ with $V_{\tilde{i}(x)}(q_\lambda(x), a'_{\tilde{i}(x)}) > V_{\tilde{i}(x)}(q_\lambda(x), a_{\tilde{i}(x)}(x))$, there exists an offer $a_j \in \cup_{j\neq\tilde{i}(x)}\tilde{A}_j(x)$ with $u(a_j) \geq u(a'_{\tilde{i}(x)})$. Some such a_j is played according to the prescribed strategies, and so receiver $\tilde{i}(x)$ has no profitable deviation. For any receiver $j \neq \tilde{i}(x)$, any offer $a_j \in A_j$ with $V_j(q_\lambda(x), a_j) > 0$ has $u(a_j) \leq u(a_{\tilde{i}(x)}(x))$. Hence, she has no profitable deviations given the sender's tie-breaking. It follows that we can set $q_\lambda^d(x) = q_\lambda(x)$.

[21] To intersect the boundary and not the interior, as HIC-K is a hyperplane is must contain a face (of some dimension) of $Q(a_i, A'_i \setminus \{a_i^K\})$, or just touch a vertex of it.

Case 2. Suppose $\tilde{A}_j(x)$ is empty for all $j \neq \iota(x)$. Then, as $\tilde{A}_{\iota(x)}$ is nonempty ($x \in \tilde{A}_{\iota(x)}$ by proof of Theorem 1), $\tilde{i}(x) = \iota(x)$. By the proof of Theorem 1 (see 'Incentive Compatibility' Case 2), x has a competitive offer $c(x) \in \cup_{j \neq \iota(x)} A_j$ at $q_\lambda(x)$ which is played with positive probability under (σ, ε). As ε is an equilibrium in non-surely dominated strategies, $v_{\iota(c(x))}(\omega^c, c(x)) \geq 0$ for some $\omega^c \in \Omega$ (or else $c(x)$ is surely dominated at all beliefs).

By definition of $a_{\tilde{i}(x)}$, $V_{\iota(x)}(q_\lambda(x), a_{\tilde{i}(x)}) \geq 0$. Let $A'_{\iota(x)} = \{a'_{\iota(x)} \in A_{\iota(x)} \setminus \{a_{\tilde{i}(x)}\} : u(a'_{\iota(x)}) > u(c(x))\}$. Again by definition of $a_{\tilde{i}(x)}$, we have $V_{\iota(x)}(q_\lambda(x), a_{\tilde{i}(x)}) \geq V_i(q_\lambda(x), a'_{\iota(x)})$ for all $a'_{\iota(x)} \in A'_{\iota(x)}$. Hence $Q(a_{\tilde{i}(x)}, A'_{\iota(x)})$ is nonempty and by Lemma 3 is convex and contains some $q' \in \Delta\Omega$ with full support on Ω. Picking some $w \in (0,1)$ close to enough to 1, we can define the belief $q^d_\lambda(x) \equiv w q_\lambda(x) + (1-w)q'$ which satisfies $q^d_\lambda(x) \in Q(a_{\tilde{i}(x)}, A'_{\iota(x)})$ and $|q^d_\lambda(x) - q_\lambda(x)| < d$. For all $\omega \in \Omega$, $q^d_\lambda(\omega|x) > 0$. Consider strategies where at a signal realization inducing public belief $q^d_\lambda(x)$: (1) the sender plays any sequentially rational strategy breaking ties in favor of $a_{\tilde{i}(x)}$ when available and against receiver $\iota(x)$ otherwise; (2) receiver $\iota(x)$ offers $a_{\tilde{i}(x)}$, (3) receiver $\iota(c(x))$ offers $c(x)$, (4) all other receivers j offer a^0_j. These strategies constitute equilibrium play in non-surely dominated strategies at $q^d_\lambda(x)$. Receiver $\iota(x)$ gets weakly positive utility from $a_{\tilde{i}(x)}$ and will not be accepted if she makes any offer yielding the sender weakly less than utility $u(c(x))$. As $q^d_\lambda(x) \in Q(a_{\tilde{i}(x)}, A'_{\iota(x)})$, there is no other offer she would like to deviate to. As $u(a_{\tilde{i}(x)}) \geq u(x)$ and the set $\tilde{A}_j(x)$ is empty for each $j \neq \iota(x)$, no other receiver has a profitable deviation to an offer that would be accepted. As $q^d_\lambda(\omega^c|x) > 0$, $c(x)$ is non-surely dominated at this belief. Finally, $u(a_{\tilde{i}(x)}) \geq u(x)$. Thus, the sender attains weakly higher utility than $u(x)$. □

By Proposition 1 in Kamenica and Gentzkow (2011), any public experiment can be equivalently represented by the Bayes-plausible distribution of public posterior beliefs it induces at each signal realization. Thus, the public experiment σ^p is equivalent to a distribution $\nu^p \in \Delta(supp(\lambda))$ over the beliefs $\{q_\lambda(x)\}_{x \in supp(\lambda)}$ with $\nu^p(x) \equiv \sum_\omega p(\omega)\mu^p(x|\omega)$. For any $d > 0$, we will now construct a public experiment $\sigma^d = (S^d, \mu^d)$, which generates beliefs $q^d_\lambda(x)$ with respective probabilities approaching $\nu^p(x)$ as $d \to 0$.

As $p \in int(\Delta\Omega)$, there exists $r > 0$ such that for any $q \in \mathbb{R}^{|\Omega|}$ with $\sum_{\omega \in \Omega} q(\omega) = 1$ and $|p - q| \leq r$, we have $q \in \Delta\Omega$, i.e., there is a closed ball around p in $\Delta\Omega$. Fix some such r. For any $d > 0$, construct beliefs $\{q^d_\lambda(x)\}_{x \in supp(\lambda)}$ following Lemma 4. Let $p^d(\cdot) \equiv \sum_{x \in supp[\lambda]} \nu^p(x) q^d_\lambda(\cdot|x)$. Note that $p^d \in \Delta\Omega$ as it is a convex combination of $\{q^d_\lambda(x)\}_{x \in supp[\lambda]}$. For $d \to 0$, we have0 $p^d \to p$ as each $q^d_\lambda(x) \to q_\lambda(x)$ and $\sum_{x \in supp[\lambda]} \nu^p(\cdot|x) q_\lambda(x) = p(\cdot)$ by Bayes-plausibility of σ^p. Let $p^{-d} \equiv p + r\frac{p-p^d}{|p-p^d|}$. We have $p^{-d} \in \Delta\Omega$ as $|p - p^{-d}| = r$ and $\sum_{\omega \in \Omega} p^{-d}(\omega) = 1$. Note that p lies on a line between p^d and p^{-d}, so there exists a unique $m^d \in [0,1]$ with $m^d p^d + (1-m^d)p^{-d} = p$. As $d \to 0$, $m^d \to 1$ due to $p^d \to p$.

We construct σ^d to induce a Bayes-plausible distribution over posteriors in the set $\{q^d_\lambda(x)\}_{x\in supp[\lambda]} \cup \{p^{-d}\}$. The experiment induces belief $q^d_\lambda(x)$ for $x \in supp[\lambda]$ with probability $m^d\nu^p(x)$ and induces belief p^{-d} with probability $1 - m^d$. At each public belief $q^d_\lambda(x)$, define ε^d by strategies following the construction in Lemma 4. Conditional on public belief p^{-d}, let ε^d prescribe any non-surely dominated equilibrium strategies (an equilibrium exists, as there are finite actions and perfect information). Note that here the sender must obtain a weakly positive payoff. Note that $\sigma^d \in \mathcal{E}^*(\sigma^d)$ and that the senders' expected payoff under $(\sigma^d, \varepsilon^d)$ satisfies

$$U^d \geq \sum_{x\in supp[\lambda]} m^d\nu^d(x)u(a_{\tilde{i}(x)}) \geq \sum_{x\in supp[\lambda]} m^d\nu^d(x)u(x) = m^dU.$$

Picking small enough d, $U^d > U - \delta$ as when $d \to 0$, $m^d \to 1$. □

A.4 Proof of Proposition 4

We focus on the case of two receivers. The case with three or more receivers is straightforward and described in the main text. To simplify the exposition, we make two mild assumptions. First, assume that $u(a_i) \geq 0$ for all $a_i \in A_i$ and $i = 1, 2$. Second, suppose that $u(a_i) \neq u(a'_i)$ for all $a_i, a'_i \in A_i$ with $a_i \neq a'_i$. We also order the offer sets $A_i = \{a^0_i, a^1_i, \dots, a^{K_i}_i\}$ such that $u(a^k_i) < u(a^{k+1}_i)$ for all $k = 0, \dots, K_i - 1$ and $i = 1, 2$. Finally, suppose $u(a^1_1) \leq u(a^1_2)$ without loss of generality.

We first construct a strictly ordered set $\vec{X}$ consisting of the possible outcomes in the set X. In particular, let $\vec{X} = \{x_0 = a^0_1, x_1 = a^0_2, x_2 = a^1_1, \dots\}$ and then add outcomes from X according to increasing sender utility. If the last added element is $x_k \in A_i$ and the next highest utility is generated by two offers $x \in A_j$ and $x' \in A_i$, such that $u(x) = u(x')$, set $x_{k+1} = x$ and $x_{k+2} = x'$.

Fix now a combination of a public experiment σ^p and an equilibrium $\varepsilon \in \mathcal{E}(\sigma^p)$ as described in the proof of Theorem 1 that induces the outcome rule λ and generates the optimal expected payoff U^* with a sender signal.

Next, we assign to each $x_k \in \vec{X} \cap supp(\lambda)$ a competitive offer. For $x_k \in A_i$, let $c(x_k) = x_l$ such that $\ell = \max\{\ell' : \ell' < k \wedge x_{\ell'} \in A_j\}$, i.e. x_ℓ is the closest less attractive offer made by the other player.

Next, we construct an outcome rule λ' which induces the same expected payoff to the sender as λ. Identify all $a_i \in supp(\lambda)$ with $a_j = c(a_i)$ such that i) $u(a_i) = u(a_j)$, ii) $a_j \in supp(\lambda)$ or $\exists a'_i \in supp(\lambda) \cap A_i$ with $c(a'_i) = a_j$, and iii) $V_j(q_\lambda(a_i), a_j) > 0$. We then define the new outcome rule λ' such that for each a_i with $a_j = c(a_i)$ satisfying i), ii), and iii), we have $\lambda'(a_i|\omega) = 0$ and $\lambda'(a_j|\omega) = \lambda(a_i|\omega) + \lambda(a_j|\omega)$, and for all other

$a_i \in supp(\lambda)$, we set $\lambda(a_i|\omega) = \lambda'(a_i|\omega)$. Note that λ' induces the same expected payoff as λ as it shifts probability mass only between offers a_i and a_j with $u(a_i) = u(a_j)$. For each $a_i = x_k \in supp(\lambda')$ with $x_k \notin supp(\lambda)$, define as above $c(x_k) = x_l$ such that $\ell = \max\{\ell' : \ell' < k \wedge x_{\ell'} \in A_j\}$.

We aim to implement the outcome rule λ'. To this purpose, we first define the receiver-public experiment $\sigma' = (S', \mu')$ with $S' = supp(\lambda')$ and $\mu' = \lambda'$. We aim to construct an equilibrium ε' for σ' consisting of strategies of the receivers and the sender. To simplify the exposition, we set the sets of cheap-talk messages M_i to $M_i = X$. For a given signal realization $s' = a_i \in supp(\lambda')$ of σ', the strategy α_i' of receiver i prescribes the receiver to offer a_i and send the message $m_i = a_i$, while the strategy of receiver j prescribes the offer $a_j = c(a_i)$ and the message $m_j = a_i$. To specify the sender's strategy β', we define the set of pure strategies for receiver i that are *consistent* with the outcome rule λ' and the receivers' strategies α_1' and α_2'. A pure strategy (a_i, m_i) is consistent if $a_i \in supp(\lambda')$ and $m_i \in \{a_i, a_j\}$ with $a_i = c(a_j)$, or if $a_i \notin supp(\lambda')$ and $m_i = a_j$ for $a_i = c(a_j)$. All other pure strategies are inconsistent. For given offers and messages (a_i, m_i, a_j, m_j) with $u(a_i) > u(a_j)$, the sender picks a_i. For (a_i, m_i, a_j, m_j) with $u(a_i) = u(a_j)$, we have by construction $a_j = c(a_i)$ and let the sender choose according to the following rules. If both strategies (a_i, m_i) and (a_j, m_j) are consistent and $m_i = m_j = a_i$, then choose a_i. If both strategies (a_i, m_i) and (a_j, m_j) are consistent and $m_i \neq m_j$, then choose a_j. If only one strategy is consistent, accept the offer of the receiver who played consistently. If both strategies are inconsistent, pick any offer from $\{a_i, a_j\}$. Note that this strategy is sequentially rational for the sender.

Fix a signal realization $s' = a_i \in supp(\lambda')$. We study whether the receivers have an incentive to deviate from the strategies $(a_i, m_i = a_i)$ and $(a_j = c(a_i), m_j = a_i)$. Consider first deviations by receiver i (j) to $a_i' \neq a_i$ $(a_j' \neq a_j)$ with $u(a_i') \neq u(a_j)$ $(u(a_j') \neq u(a_i))$. Any deviation by receiver $h \in \{i, j\}$ to some a_h' with $u(a_h') > u(a_i)$ cannot be profitable. If it was, a_h' would be implementable for the belief $q_{\lambda'}(a_i)$, resulting in an improvement of the expected payoff to some value strictly above U^*, which yields a contradiction. Any deviating offer a_h' with $u(a_h') < u(a_j)$ is rejected and therefore trivially unprofitable. Similarly, a deviation by receiver j to a_j' with $u(a_j') < u(a_i)$ is rejected and unprofitable. If $s' = a_i$ is equivalent to the signal realization $s^p = a_i$ under our original experiment σ^p, a deviation by receiver i to a_i' with $u(a_j) < u(a_i') < u(a_i)$ is unprofitable as this deviation was also available under the equilibrium $\varepsilon^p \in \mathcal{E}(\sigma^p)$. If $s' = a_i$ is not equivalent to the signal realization $s^p = a_i$ under our original experiment σ^p, then there does not exist a deviation a_i' with $u(a_j) < u(a_i') < u(a_i)$ as otherwise, a_i could not have been the competitive offer to the 'deleted' offer a_j by construction of $\vec{X}$ and the function $c(\cdot)$ under the original outcome rule λ.

Next consider deviations a_i' (a_j') by receiver i (j) such that $u(a_i') = u(a_j)$ ($u(a_j') = u(a_i)$). Suppose at first that $a_i' \neq a_i$ ($a_j' \neq a_j$). If $a_i' \notin supp(\lambda')$, the offer a_i' is always rejected as it is inconsistent, and the deviation is therefore not profitable. For $a_i' \in supp(\lambda')$, we have either i) $a_i' = c(a_j)$ or ii) $a_j = c(a_i')$. Any inconsistent deviation results in rejection and is therefore not profitable. In case i), consistency requires $m_i = a_j$, Thus, $m_i \neq m_j = a_i$, and a_j is picked according to the sender's strategy. In case ii), consistency requires $m_i = a_i'$. Thus, $m_i \neq m_j = a_i$, and again a_j is selected according to the sender's strategy. Thus, no such deviation is profitable for receiver i. Consider now receiver j. If $a_j' \notin supp(\lambda')$, the deviating offer a_j' is inconsistent and therefore unprofitable. If $a_j' \in supp(\lambda')$, we have $a_i = c(a_j')$ by the definition of $c(\cdot)$. Inconsistent deviations are clearly unprofitable. The only consistent deviation therefore features $m_j = a_j' \neq m_i = a_i$. Thus, a_i is picked according to the sender's strategy, and the deviation a_j' is unprofitable.

Finally, we consider deviations only in the cheap-talk message, i.e., $a_i' = a_i$ ($a_j' = a_j$). If $u(a_i) > u(a_j)$, such deviations are clearly inconsequential and therefore unprofitable. Thus, we focus on $u(a_i) = u(a_j)$. Here again, receiver i has clearly no benefit from deviating only in the message as this does not affect the outcome. For receiver j, there are two ways how to induce the sender into accepting $a_j' = a_j$ with a deviation in the message. For $a_j \in supp(\lambda')$, this is achieved by $m_j = a_j$ according to the sender's strategy. For $a_j \notin supp(\lambda)$, the same outcome can be obtained if there exists another offer a_i' of receiver i which has $u(a_i') > u(a_i)$ and $a_j = c(a_i')$. Note now that $u(a_i) = u(a_j)$ is only possible for signal realizations $s' = a_i$ that have an equivalent signal realization $s^p = a_i$ under the original experiment σ^p. This in turn implies that any deviation in the message that induces the sender to accept a_j needs to satisfy $V_j(q_\lambda(a_i), a_j) \leq 0$ by the construction of λ'. Thus, also these deviations are unprofitable and $(a_i, m_i = a_i, a_j = c(a_i), m_j = a_i)$ constitutes equilibrium play for $s' = a_i$ given the sender's strategy. □

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