Impartial Social Rankings: Some Impossibilities

Jorge Alcalde-Unzu*, Dolors Berga†, Riste Gjorgjiev‡

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We model a situation where a set of agents must rank themselves based on their opinions. Each agent submits a message and a function determines the social ranking. We are interested in impartial social ranking functions, that is, those where the message of an agent can not change any social binary comparison of herself with respect to someone else. We obtain impossibility results by additionally considering some classical properties of symmetry across agents.

JEL classification: D71; D70; D63.
Keywords: Impartiality; Social Ranking; Symmetry Properties; Impossibility results.

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1 Introduction

In this paper we study methods to rank a set of agents based on their opinions or messages. Whenever facing this type of collective decision, individuals may be tempted to act strategically in order to get a better result for themselves. In order to deter such manipulations, we study how to aggregate agents' opinions in such a way that the contribution of an agent cannot influence any social judgement of herself with respect to someone else. This property is known as Impartiality.[1]

The idea of impartiality has been studied in other contexts like the division of a good among a set of agents (see de Clippel et al., 2008) and the selection of a winner (see Holzman and Moulin, 2013) or a set of winners (see Tamura and Ohseto, 2013, Kurokawa et al., 2015, and Tamura, 2016). Although the construction of a social ranking is a classical objective in social choice theory, only Kahng et al. (2018), as far as we know, analyze the concept of impartiality in this setting.[2] Our approach is axiomatic and deterministic, contrary to their focus on randomized algorithms. More importantly, they consider a weaker notion of impartiality: while they only require to avoid manipulations where one changes her own position in the social ranking, we also impose that the agents placed above oneself in the social ranking cannot change. Additionally, and on the contrary to the rest of the literature, our approach does not restrict the ways in which agents can express their messages.

Our paper first shows that Impartiality is incompatible with Name Independence, a property of symmetry across agents requiring that the identity of each agent is irrelevant. In fact, we also show that Impartiality is incompatible with another classical property, Candidate Neutrality, requiring symmetry of agents in their role as candidates. The reason of these impossibilities is that Impartiality implies that not all possible ways of ranking the agents should occur, and both Name Independence and Candidate Neutrality requires that the social ranking function should have full range (see the proofs in the Appendix). This is a surprising result since in the context of choosing a single winner, Holzman and Moulin (2013) show that there are impartial rules with full range. Finally, we also present the impossibility of combining, in a non-constant function, the property of Impartiality with Weak Anonymity. A similar result is obtained in Holzman and Moulin's framework (see their Theorem 3) with the classical and stronger anonymity property requiring symmetry in the role of agents as voters.

Another related contribution is Mackenzie (2015). He analyzes the same symmetry properties as ours, but for random impartial mechanisms in Holzman and Moulin's context, and obtains possibility results.[3] Although in our paper we have concentrated on impossibility

[1]Several papers deal with other types of manipulations when choosing a social ranking. See, for example, Bossert and Storken (1992), Sato (2013), Bossert and Sprumont (2014), Amorós (2020), and Fujiwara-Greve et al. (2021).

[2]The literature of peer rating (see Ng and Sun, 2003 and Ohseto, 2012) is also related with our contribution since the outcome of the aggregation of agents' opinions in those papers is a cardinal score for each one, which in fact defines a social ranking. However, they consider a different impartiality axiom.

[3]Recently, Edelman and Por (2021) also analyzes impartial random mechanisms in Holzman and

results, we think that these can guide the search and characterization of impartial social ranking functions using weaker versions of the symmetry axioms analyzed here.

2 The Model

Let $N = \{1, \dots, n\}$ be a finite set of agents. A ranking R is a complete, transitive and antisymmetric binary relation on N and $\mathcal{R}$ is the set of all possible rankings. Given $R \in \mathcal{R}$ and $i \in N$, define $U(R, i) = \{j \in N \setminus \{i\} : j\,R\,i\}$. Let $\mathcal{M}_i$ be the set of possible messages that agent i can declare. Note that $\mathcal{M}_i$ has no predetermined structure. A profile of messages is a list of messages, one of each individual, $m = (m_1, \dots, m_n) \in \times_{i\in N}\mathcal{M}_i$. Some possibilities of $\mathcal{M}_i$ are:

(i) the set of agents, $\mathcal{M}_i = N$, as in the case of nomination rules with self nomination.

(ii) rankings, $\mathcal{M}_i = \mathcal{R}$ in which each agent ranks the possible candidates.

(iii) utility functions, $\mathcal{M}_i = \mathcal{U}$, where $\mathcal{U}$ is the set of functions $u : N \to \mathbb{R}_+$, as in the rating functions, being $\mathbb{R}_+$ the set of non-negative real numbers.

These are cases of common sets of possible messages; i.e., $\mathcal{M}_i = \mathcal{M}_j$ for all $i, j \in N$, but we also allow for personalized sets (e.g., $\mathcal{M}_i = N \setminus \{i\}$, as in Holzman and Moulin, 2013).

Given a permutation σ of N and a message profile m, we denote by $m_\sigma = (m_{\sigma(1)}, \dots, m_{\sigma(n)})$ another message profile where agent i has the message agent $\sigma(i)$ had in m. That is, $\sigma(i)$ is the agent whose message in m is agent i's message in m_σ. Observe that m_σ does not necessarily belong to $\times_{i\in N}\mathcal{M}_i$ when the sets of messages are personalized. Similarly, given a permutation σ of N and a ranking R, we define $\sigma(R)$ as a ranking such that $\sigma(i)\,\sigma(R)\,\sigma(j)$ whenever $i\,R\,j$ for all $i, j \in N$. Finally, given a set of agents $C \subset N$ and a message profile $m \in \times_{i\in N}\mathcal{M}_i$, we denote by m_C and m_{-C} the message subprofiles of the agents of C and $N \setminus C$, respectively. We will denote m_{-i} instead of $m_{-\{i\}}$.

A *social ranking function* is a mapping $f : \times_{i\in N}\mathcal{M}_i \to \mathcal{R}$. We say that f has *full range* if for any $R \in \mathcal{R}$, there exists $m \in \times_{i\in N}\mathcal{M}_i$ such that $f(m) = R$. And f is *constant* if $f(m) = f(m')$ for all $m, m' \in \times_{i\in N}\mathcal{M}_i$.

Impartiality is our main axiom, and it requires that an agent's message could change the ranking of the agents that are in her upper (respectively, lower) contour set between them, but cannot pass an agent from her upper contour set to the lower one, or vice versa.

Definition 1 *A social ranking function f is impartial if for all $i \in N$, all $m \in \times_{i\in N}\mathcal{M}_i$ and all $m'_i \in \mathcal{M}_i$, $U(f(m), i) = U(f(m'_i, m_{-i}), i)$.*

Moulin's context, but without symmetric properties.

3 Results

Social choice theory normally defines two symmetry axioms, anonymity and neutrality, stating respectively that voters and candidates should be treated symmetrically in the aggregation procedure. In our setting voters and candidates are the same. Thus, the idea of symmetry across agents can be explained by a single axiom called Name Independence which states that a social ranking function should not depend on agents' names. Since our framework allows for any structure of the set of agents' possible messages, our version of this property needs to be very general and applicable to all of them. Formally:

Definition 2 *A social ranking function f is Name Independent if for any permutation $\sigma : N \to N$, there is a mapping $\gamma_\sigma : \bigcup_{i\in N} \mathcal{M}_i \to \bigcup_{i\in N} \mathcal{M}_i$ such that*

- $f(m) = \sigma(f(\gamma_\sigma(m_{\sigma(1)}), \ldots, \gamma_\sigma(m_{\sigma(n)})))$ *for all* $m \in \times_{i\in N}\mathcal{M}_i$*; and*
- $\gamma_\sigma(m_{\sigma(i)}) \in \mathcal{M}_i$ *for all* $i \in N$ *and all* $m \in \times_{i\in N}\mathcal{M}_i$*.*

The axiom states that the position of agent i in $f(m)$ should be the same as the position in the social ranking of the agent $\sigma^{-1}(i)$ if we jointly permute the voters with the permutation σ and the messages with the permutation γ_σ. Observe that, meanwhile σ permutes the role of agents as voters, the objective of γ_σ is to permute the role of agents as candidates in the messages. This implication on f is the first point of the definition. Since the sets of possible messages could be personalized in some applications, we also need to impose that each one of the new messages obtained after the permutations belongs to the set of admissible messages of the individual that transmit it in the new message profile; i.e., $\gamma_\sigma(m_{\sigma(j)})$ should belong to $\mathcal{M}_j$. This is exactly the second point of the definition.

Consider two examples, one with personalized messages and the other with a common set of possible messages, of the requirements of Name Independence.

Example 1 Let $N = \{1, 2, 3\}$. Consider that $\mathcal{M}_i = N \setminus \{i\}$, as in Holzman and Moulin (2013). In that case, the natural definition of γ_σ is to make it equal to σ^{-1}. Observe that this guarantees the second point of the definition; that is, $\gamma_\sigma(m_{\sigma(i)}) \in N \setminus i = \mathcal{M}_i$ for all $i \in N$ and all $m \in \times_{i\in N}\mathcal{M}_i$. To analyze the first point of the axiom for this specification of γ_σ, consider the message profile $m = (3, 1, 1)$ and a permutation of the agents such that $\sigma(i) = i + 1$ (module 3). Then, the message profile $(\gamma_\sigma(m_{\sigma(1)}), \gamma_\sigma(m_{\sigma(2)}), \gamma_\sigma(m_{\sigma(3)}))$ is such that agent 1 nominates $\gamma_\sigma(m_{\sigma(1)}) = \gamma_\sigma(m_2) = \gamma_\sigma(1) = \sigma^{-1}(1) = 3$, agent 2 nominates $\gamma_\sigma(m_{\sigma(2)}) = \gamma_\sigma(m_3) = \gamma_\sigma(1) = \sigma^{-1}(1) = 3$ and agent 3 nominates $\gamma_\sigma(m_{\sigma(3)}) = \gamma_\sigma(m_1) = \gamma_\sigma(3) = \sigma^{-1}(3) = 2$. Then, the position of, for instance, agent 1 in a name independent social ranking function with the message profile $m = (3, 1, 1)$ should be the same as the position of agent $\sigma^{-1}(1) = 3$ with the message profile $(\gamma_\sigma(m_{\sigma(1)}), \gamma_\sigma(m_{\sigma(2)}), \gamma_\sigma(m_{\sigma(3)})) = (3, 3, 2)$. That is, if for instance the social ranking in the message profile $m = (3, 1, 1)$ is $1\, f(m)\, 3\, f(m)\, 2$, the social ranking in the message profile $(\gamma_\sigma(m_{\sigma(1)}), \gamma_\sigma(m_{\sigma(2)}), \gamma_\sigma(m_{\sigma(3)})) = (3, 3, 2)$ is

$$3 \quad f(\gamma_\sigma(m_{\sigma(1)}), \gamma_\sigma(m_{\sigma(2)}), \gamma_\sigma(m_{\sigma(3)})) \quad 2 \quad f(\gamma_\sigma(m_{\sigma(1)}), \gamma_\sigma(m_{\sigma(2)}), \gamma_\sigma(m_{\sigma(3)})) \quad 1.$$

Observe that this axiom follows the same idea of the one that Mackenzie (2015) states in his context of choosing a single winner. The difference is that his property states the first point of the implications of our version for a particular specification of γ_σ, and we only require that such implication occurs for some γ_σ satisfying the other point of the property. Note that we cannot be more specific in the structure of γ_σ since we pretend to introduce a definition for any set of possible messages of each agent and not only for this example. As we will see later on, our definition is going to generate impossibility results, thus the adoption of stronger versions of the property will not change them. □

Example 2 Let $N = \{1, 2, 3\}$. Consider that $\mathcal{M}_i = \mathcal{R}$. In that case, the natural definition of γ_σ would be $\gamma_\sigma(R) = \sigma^{-1}(R)$. Since the set of possible messages is common, the second point of the definition of the axiom is satisfied; that is, $\gamma_\sigma(R_{\sigma(i)}) \in \mathcal{R} = \mathcal{M}_i$ for all $i \in N$ and all $m \in \times_{i \in N} \mathcal{M}_i$. To see the implication of the first point of the axiom for this specification of γ_σ, consider a message profile such that $1\, R_1\, 2\, R_1\, 3$, $1\, R_2\, 3\, R_2\, 2$, and $2\, R_3\, 3\, R_3\, 1$, and a permutation of the agents such that $\sigma(i) = i + 1$ (module 3). Then, the message profile $(\gamma_\sigma(R_{\sigma(1)}), \gamma_\sigma(R_{\sigma(2)}), \gamma_\sigma(R_{\sigma(3)})) = (\gamma_\sigma(R_2), \gamma_\sigma(R_3), \gamma_\sigma(R_1)) = (R'_1, R'_2, R'_3)$ is such that $3\, R'_1\, 2\, R'_1\, 1$, $1\, R'_2\, 2\, R'_2\, 3$ and $3\, R'_3\, 1\, R'_3\, 2$. Then, as in Example 1, the position of for example agent 1 in a name independent social ranking function with the message profile (R_1, R_2, R_3) should be the same as the position of agent $\sigma^{-1}(1) = 3$ with the message profile (R'_1, R'_2, R'_3).

Again, our axiom does not specify that this is exactly the structure of the mapping γ_σ and only requires the existence of some γ_σ satisfying the conditions. □

Another symmetry axiom we are interested in permutes agents' role only as candidates:

Definition 3 *A social ranking function f is Candidate Neutral if for any permutation $\sigma : N \to N$, there is a mapping $\gamma_\sigma : \bigcup_{i \in N} \mathcal{M}_i \to \bigcup_{i \in N} \mathcal{M}_i$ such that*

- $f(m) = \sigma(f(\gamma_\sigma(m_1), \ldots, \gamma_\sigma(m_n)))$ *for all* $m \in \times_{i \in N} \mathcal{M}_i$*; and*
- $\gamma_\sigma(m_i) \in \mathcal{M}_i$ *for all* $i \in N$ *and all* $m_i \in \mathcal{M}_i$.

Candidate Neutrality states that the social ranking position of agent i when the message profile is m should be the same than that of $\sigma^{-1}(i)$ if we permute the messages with the permutation γ_σ. Each one of these symmetry properties, Name Independence and Candidate Neutrality, is too restrictive when combined with Impartiality.

Theorem 1 *There is no social ranking function satisfying Impartiality and either Name Independence or Candidate Neutrality.*

Holzman and Moulin (2013) shows for the problem of choosing a single winner in which the set of possible messages of each agent i is $N \setminus \{i\}$, the incompatibility of Impartiality with the classical property of Anonymity except in the constant rules. The intuition behind this result is the following. Impartiality implies that the message of each agent cannot affect her own outcome, meaning, in Holzman and Moulin (2013)'s context, being or not being selected for the prize. And Anonymity implies that all individual messages are equally relevant for each agent's outcome. Then, it can be deduced that no message can affect the outcome of any agent and the range of any impartial and anonymous function is a singleton. As Theorem 2 shows, this result can be generalized to social ranking functions. In fact, it is possible to state a stronger result. If Impartiality is imposed, instead of requiring that all messages are equally relevant for the outcome of each agent i, as Anonymity does, it has much more sense to require that this is the case for all individual messages except that of i (since Impartiality has imposed that this message is irrelevant for i's outcome). We call this axiom Weak Anonymity. To define it formally, we need to apply a permutation σ to the message profile m that maintains invariant the message of i and to require that the outcome of i with the social ranking function does not change. However, remember that, although $m \in \times_{i \in N} \mathcal{M}_i$, it is possible that $m_\sigma \notin \times_{i \in N} \mathcal{M}_i$ if the sets of messages are personalized. In fact, it could be the case that $\mathcal{M}_j \cap \mathcal{M}_k = \emptyset$ for all $j, k \in N \setminus \{i\}$ and, in that case, the property could not be defined. Therefore, it is necessary to define the property only for some Cartesian products of sets of possible messages in which the message profiles can be permuted. In our result, we consider the case of common sets of possible messages.

Definition 4 *Suppose that $\mathcal{M}_i = \mathcal{M}_j$ for all $i, j \in N$. A social ranking function f is Weak Anonymous if for all $i \in N$, all $m \in \times_{i \in N} \mathcal{M}_i$, and all permutations σ of N with $\sigma(i) = i$, $U(f(m), i) = U(f(m_\sigma), i)$.*

The combination of this axiom with Impartiality also leads to constant social ranking functions.[4]

Theorem 2 *Suppose that $\mathcal{M}_i = \mathcal{M}_j$ for all $i, j \in N$. Any Impartial and Weakly Anonymous social ranking function is constant.*

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[4]Impartiality is independent of each one of the symmetry axioms. Examples are available upon request.

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Appendix

Proof of Theorem 1

Define $U(f) = \{i \in N : \exists m \in \times_{i\in N}\mathcal{M}_i$ such that $U(f(m), i) = \emptyset\}$ and $L(f) = \{i \in N : \exists m \in \times_{i\in N}\mathcal{M}_i$ such that $U(f(m), i) = N \setminus \{i\}\}$. That is, $U(f)$ (respectively, $L(f)$) is the set of agents that are in the first (respectively, last) position in the social ranking for at least one profile of messages. The proof is structured in two lemmas, whose results are incompatible and thus prove the impossibility.

Lemma 1 *Let f be an impartial social ranking function. Then, $U(f) \cap L(f) = \emptyset$.*

Proof: Suppose by contradiction that there is an agent $i \in U(f) \cap L(f)$. Let $m = (m_1, \ldots, m_n)$ and $m' = (m'_1, \ldots, m'_n)$ be such that $U(f(m), i) = \emptyset$ and $U(f(m'), i) = N \setminus \{i\}$. Starting at m, we construct a sequence of message profiles such that each message profile is obtained changing from the previous message profile one agent's message from the one in m to the one in m' in the following order: we select in each step the agent who is ranked highest in the outcome by f in the last message profile (among those agents whose message has not still changed). Observe that this sequence ends at message profile m'.

Since $U(f(m),i) = \emptyset$, the first two profiles of the sequence are m and (m'_i, m_{-i}). Since $U(f(m),i) = \emptyset$, we obtain by Impartiality that $U(f(m'_i, m_{-i}), i) = \emptyset$. Then, we now change the message of agent $j \in N$ such that $U(f(m'_i, m_{-i}), j) = \{i\}$. That is, the third profile of the sequence is $(m'_{\{i,j\}}, m_{-\{i,j\}})$. Since $U(f(m'_i, m_{-i}), j) = \{i\}$, we obtain by Impartiality that $U(f(m'_{\{i,j\}}, m_{-\{i,j\}}), j) = \{i\}$. We now change the message of agent k such that $U(f(m'_{\{i,j\}}, m_{-\{i,j\}}), k) = \{i,j\}$ and, then, the next profile of the sequence is $(m'_{\{i,j,k\}}, m_{-\{i,j,k\}})$. Since $U(f(m'_{\{i,j\}}, m_{-\{i,j\}}), k) = \{i,j\}$, applying Impartiality again we have that $U(f(m'_{\{i,j,k\}}, m_{-\{i,j,k\}}), k) = \{i,j\}$. Continuing this procedure we will obtain that $i \in U(f(m'), l)$ for some $l \in N$, which contradicts that $U(f(m'), i) = N \setminus \{i\}$. □

Lemma 2 *Let f be a social ranking function satisfying Name Independence or Candidate Neutrality. Then, f has full range.*

Proof: We do the proof for Name Independence (for Candidate Neutrality is similar). Consider any $R \in \mathcal{R}$. Take any $m \in \times_{i \in N} M_i$ and construct a permutation σ of N such that for all $i \in N$, $|\{k \in N : k\, R\, i\}| = |\{k \in N : k\, f(m)\, \sigma(i)\}|$. By construction, $f(m) = \sigma(R)$. By Name Independence, there exists a mapping γ_σ such that $\gamma_\sigma(m_{\sigma(i)}) \in \mathcal{M}_i$ for all $i \in N$ and $f(m) = \sigma(f(\gamma_\sigma(m_{\sigma(1)}), \ldots, \gamma_\sigma(m_{\sigma(n)})))$. Then, we have that $R = f(\gamma_\sigma(m_{\sigma(1)}), \ldots, \gamma_\sigma(m_{\sigma(n)}))$ and, thus, R belongs to the range of f. □

Proof of Theorem 2

Observe first that all constant social ranking functions satisfy Impartiality and Weak Anonymity. Suppose now by contradiction that there is f satisfying these axioms that is not constant. Then, there exist $m, m' \in \times_{i \in N} \mathcal{M}_i$ such that $f(m) \neq f(m')$. Starting at m, construct a sequence of message profiles such that in each step of the sequence the message of an agent $i \in N$ is changed from m_i to m'_i such that the sequence ends at m'. Consider the last message profile of the sequence in which f selects $f(m)$, denoted by (m'_A, m_{-A}), and the next message profile of the sequence, $(m'_{A \cup \{j\}}, m_{-(A \cup \{j\})})$. Then, $f(m'_A, m_{-A}) \neq f(m'_{A \cup \{j\}}, m_{-(A \cup \{j\})})$. Therefore, there are two agents $k, k' \in N$ such that $k \in U(f(m'_A, m_{-A}), k')$ and $k \notin U(f(m'_{A \cup \{j\}}, m_{-(A \cup \{j\})}), k')$. Consider the permutation σ of N such that $\sigma(j) = k$, $\sigma(k) = j$ and $\sigma(i) = i$ for all $i \in N \setminus \{j,k\}$. Then, construct the message profiles $(m'_A, m_{-A})_\sigma$ and $(m'_{A \cup \{j\}}, m_{-(A \cup \{j\})})_\sigma$. By Weak Anonymity we deduce that $U(f((m'_A, m_{-A})_\sigma), k') = U(f(m'_A, m_{-A}), k')$ and that $U(f((m'_{A \cup \{j\}}, m_{-(A \cup \{j\})})_\sigma), k') = U(f(m'_{A \cup \{j\}}, m_{-(A \cup \{j\})}), k')$. Then, we get $k \in U(f((m'_A, m_{-A})_\sigma), k')$ and $k \notin U(f((m'_{A \cup \{j\}}, m_{-(A \cup \{j\})})_\sigma), k')$. Equivalently, $k' \notin U(f((m'_A, m_{-A})_\sigma), k)$ and $k' \in U(f((m'_{A \cup \{j\}}, m_{-(A \cup \{j\})})_\sigma), k)$. Observe that the message profiles $(m'_A, m_{-A})_\sigma$ and $(m'_{A \cup \{j\}}, m_{-(A \cup \{j\})})_\sigma$ only differ in the message of agent k. Then, Impartiality implies that $U(f((m'_A, m_{-A})_\sigma), k) = U(f((m'_{A \cup \{j\}}, m_{-(A \cup \{j\})})_\sigma), k)$. We have arrived at a contradiction and, thus, the theorem is proved.