On No-Envy and Fair Allocations in General Equilibrium Theory*

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Abstract: This essay revisits and attempts a synthetic consolidation of a subject that was peripheral to general competitive analysis as initially developed by Wald and Arrow-Debreu-McKenzie, but central to social choice theory as developed by Arrow-Harsanyi-Sen and their followers. We provide a retrospective reading that connects to Foley, and even earlier to Steinhaus and Dubins-Spanier; and a prospective one that takes as its point of departure the recent work of Echenique, Fleurbaey, Moulin, Thompson and others that is more oriented to welfare economics, on the one hand, and to matching and network theory on the other. The principal motivation of the work reported here is bring together communities in an exploratory framing that can become the basis for future work of the authors, if not of that of others.

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Contents

[E]conomic theory suggests that the pursuit of equity in the sense of non-envy will lead to some peculiar and unpalatable results. Depending on the user's inclinations, 'equity' can mean almost anything $\cdots$ Although 'equality' is less ambiguous than 'equity', it too has many definitions. [E]galitarian equivalent allocations that are also Pareto optimal exist, even in economies with production.[1] *But this idea is also unworkable; it is simply too airy.*

Feldman (1987)

[S]ince one agent cannot directly consume another agent's leisure, the extension of the concept to production is not immediate. More formally, if one agent's consumption set is not identical with another's, the concept of envy-free allocation is not necessarily well defined. Furthermore, it can be shown that when preferences vary continuously across the population, the only Pareto efficient envy-free allocations are those with equal wealth.[2]

Varian (1987)

1 Introduction

Although there is agreement that inefficient situations should be avoided, some Pareto optimal situations may be intuitively inequitable from the distributional viewpoint. Given the challenge of defining some notion of economic justice or fairness[3], many works attempt to restrict the set of efficient outcomes by requiring properties that, as Pareto optimality, are also ordinal. This is the case of envy-freeness (equity) conditions. The words envy and fair already have a meaning in themselves. The issue is more than semantic since it involves normative aspects. Value judgments are necessary to specify them. Thus, it is hardly surprising that the notions found in the literature are subject to criticism.

Having envy-freeness can be beneficial in certain situations, as it promotes a more harmonious distribution of resources, thereby reducing tensions and conflicts among individuals. However, there are some objections to the envy-free feature. For instance, the equalitarian distribution of resources is envy-free, but it may be inefficient and need not be individually rational. In some situations, a non-envy-free allocation may be substantially more productive

[1]Feldman ascribes this result to Pazner and Schmeidler (1978). The reader is to be warned in that this epigraph, as well as the one following it, rearranges the sentences of the authors taken from their *New Palgrave* entries on 'equity' and 'fairness' respectively.

[2]Varian also writes "A closely related idea is that of an egalitarian equivalent allocation, which is one in which every agent is indifferent between the bundle he holds in that allocation and a bundle in some (hypothetical) equal division allocation."

[3]The notion of fairness was introduced in mathematical studies on the problem of dividing an object among a finite number of individuals so that each is satisfied concerning his preferences with the portion he gets. Dubins and Spanier (1961) and Kuhn (1967) refer to most early works on this subject including that of Steinhaus (1948). Kolm (1972), Phelps (1973), Nozick (1974) and Rawls (1971) are classical treatments that take viewpoints going beyond economics. Also see Phelps (1976) for a framing of the literature in the context of classical welfare economics, and Varian (1976a) for a bibliographic note.

or beneficial, even if it causes some envy among individuals. Moreover, achieving no envy can be difficult or even impossible in certain situations since it does not contemplate the concept of meritocracy or the idea that individuals should be rewarded based on their efforts and contributions. If certain individuals have worked harder or contributed more to the economy, it might not be sensible to allocate resources purely based on eliminating envy, as this could undermine motivation and incentivize free-riding behaviors.

In economic theory, envy-freeness originates from Foley (1967), addressing resource allocation and competitive equilibrium for economies with both public and private goods. He suggested that resource distribution among individuals is equitable if no one prefers another agent's bundle to their own. Since then, envy-freeness has been extensively studied as a standard of fairness because this property guarantees that everyone is content with their share and does not envy what others possess.

Considering the problem of distributing total resources in an economy, Varian (1974) refers to envy-free allocations as equitable and defines the concept of *fairness*.[4] An allocation is fair if it is equitable and efficient. He argues that this definition can only be a minimal requirement for fairness since the only facts to be considered are the preferences of the agents and the total amount of goods to be divided. The original position, resulting from an equal division of total resources among agents, can be seen as a hypothetical state that helps analyze the reasons for choosing one allocation rule over another.

Commonly used solutions to the sharing problem, such as the Walrasian equilibrium or the core, depend crucially on the distribution of the endowments among the participants. When endowments are the equal distribution of total resources, the Walrasian allocations are fair, which cannot be ensured with heterogeneous endowments. Feldman and Kirman (1974) proved that an allocation in the core relative to equal division, which is efficient and individually rational, may fail to be envy-free. Therefore, there can be envy in both non-cooperative and cooperative classical solutions. Moreover, Sugden (1984) questions the relevance of fairness and criticizes what he refers to as Varian's theory of fairness, pointing out that it says remarkably little about why these properties are desirable or should be used.

The envy-free original notion has been reformulated in several directions. For instance, Thomson (1982), arguing that what matters to an individual is the average consumption of others and not the distribution of resources among themselves, states that an allocation is *A-envy-free*, *A* standing for anonymous or average, if no agent prefers the average of what everyone else consumes to her bundle. In recent works, Thomson (2025) considers the classical theory

[4] Also see Varian (1975, 1976b). For a critique of Varian, see Sugden (1984) who draws on Pazner (1977). A comprehensive introduction to the work of Pazner remains to be written; for an early treatment, see Hurwicz et al (1985) which reprints six of his papers.

of fair allocations from the viewpoint of replication-invariance. Addressing matching theory, Echenique et al. (2021) define a notion of envy based on participation constraints given by reservation utilities, and Romm et al. (2024) note that providing a formal definition of justified envy in more general environments is not straightforward.

Expanding the problem of fair allocations to further frameworks, such as private ownership economies or production economies is not immediate and involves difficulties, since individuals may contribute differently to the social product. Considering production, Pazner and Schmeidler (1974) show that the concepts of efficiency and the absence of envy can be incompatible and conjecture: *If the labours of at least two individuals command different prices in any technologically efficient production plan, it is possible to define the preferences so that no Pareto-optimal allocation will be envy-free.*[5] Kranich (2020) introduces the concept *resource-envy-freeness* in a production context, where factors must be dedicated to the production and cannot be consumed directly, and remarks that the analysis of resource-envy-free allocations in production is exactly analogous to the analysis of envy-free allocations in exchange. He notes that his model excludes labor and demonstrates the existence of resource-envy-free and efficient allocations, assuming that the aggregate endowment of production factors is commonly owned. Therefore, the difficulties for a valid criterion of envy in economies with production appear in exchange economies with heterogeneous endowments. The initial individual resource diversity can be responsible for legitimate and expected envy.[6]

Avoiding the asymmetries on the endowments in exchange economies, Schmeidler and Vind (1972) define envy-freeness on net trades, showing that any equilibrium net trade is envy-free, and any envy-free net trade added to the equal division of the total endowment will give an envy-free allocation. However, in an economy with unequal endowments, the resulting allocation corresponding with the equilibrium net trade need not be envy-free. On the other hand, following Vind (1971), Varian (1974) proposed an extension of envy-freeness for coalitions. His definition requires that no group of agents envies any other group of the same size. Considering net trades, Gabszewicz (1975) provides another notion of coalitional envy-free outcomes called c-fairness or non-discriminatory allocations (see also Yannelis, 1985).[7] However, c-fairness implies Varian's envy-free only in the case of equal endowments.

Finally, we stress that the presence of envy in outcomes, following Foley's (1967) or Varian's (1974) notion, may come from an unequal initial distribution of resources. Thus, envy becomes

[5] Envy-freeness may also be incompatible with efficiency in economies with differential information; see De Clippel (2008) and his references.

[6] In the context of production under increasing returns to scale, Vohra (1992) brings additional difficulties to bear on the subject.

[7] In contrast to Schmeidler and Vind (1972), Gabszewicz's definition does not require a preference order on the set of net trades.

a signal of inequalities. In this respect, we remind Rawls's (1971) claim: *If we resent our having less than others, it must be because we think that their being better off is the result of unjust institutions. Those who express resentment must be prepared to show why certain institutions are unjust or how others have injured them.*[8] Over the years, extensive research has focused on envy-free and fair allocations in different scenarios, including distributing goods among a group of individuals, cake-cutting, housing allocation, and welfare functions issues. One finds a recent growing interest in employing machine learning and artificial intelligence techniques addressing the issue of envy-freeness and developing practical algorithms and mechanisms for real-world applications.

In this note, we consider pure exchange economies, where agents are characterized by their consumption set, preferences, and asymmetric private endowments. Focusing on the problem of resource allocation, we note that while inefficiency must be avoided, Varian's envy-freeness is not a sensible property for solving the sharing problem in unequal societies. Borrowing the term *justified* from matching theory, we propose that envy is justified when it is limited to comparable individuals, meaning they have similar opportunities or endowments. In this way, we define the set of justified-envy-free allocations. This notion allows us to meet the net trades and outcomes approaches concerning individual and coalitional envy-freeness. Moreover, our approach leads to a new notion of core, which we refer to as the envy-free core, that lies between the set of c-fair or non-discriminatory allocations and the core, overcoming the difficulty pointed out by Feldman and Kirman (1974).

The remainder of this note is structured as follows. Section 2 revisits the concept of envy-free outcomes and introduces our definition of justified envy. Section 3 presents a form of envy-freeness based on the priority order established by a price vector. Sections 4 and 5 show that under our justified envy notion, it is equivalent to define envy in terms of net trades or allocations, individually and coalitionally. Furthermore, the justified envy-free outcomes give rise to the concept of the *envy-free core*.

2 Revisiting the envy-free notion

Consider an economy $\mathcal{E}$ with a set $N = \{1, \ldots, n\}$ of consumers who trade ℓ commodities. Each consumer i is endowed with $\omega_i \in \mathbb{R}^\ell_+$ and has a preference relation $\succcurlyeq_i$ on $\mathbb{R}^\ell_+$. Let $W = \sum_{i=1}^n \omega_i$ be the total endowments. A feasible allocation x assings a bundle $x_i \in \mathbb{R}^\ell_+$ to each individual $i \in N$, such that $\sum_{i=1}^n x_i \leqslant W$.

Following Foley (1967), given a feasible allocation x, the consumer i envies j if $x_j \succ_i x_i$. An allocation is envy-free if there is no envy. Considering the problem of dividing a fixed amount

[8]See page 467 in Rawls (1999), and more generally Sections 80 and 81.

of goods among a fixed number of agents, Varian (1974) calls equitable to envy-free allocations and defines fairness as envy-free and efficient outcomes.

Given $\mathcal{E}$, let us define the auxiliar economy $\mathcal{E}^*$ which coincides with $\mathcal{E}$, except that endowments of every agent is $\frac{W}{n}$. Let us assume that the economy $\mathcal{E}$ has Walrasian equilibrium, and then $\mathcal{E}^*$ has an equilibrium (p^*, x^*). Now, x^* is an efficient allocation in $\mathcal{E}^*$ and also in $\mathcal{E}$ (efficient allocation depends on the aggregate resources but not on its distribution). To show that x^* is envy-free in the economy $\mathcal{E}$, let assume that there are consumers i, j such that $x_j^* \succ_i x_i^*$. This contradicts that x^* is an equilibrium allocation in $\mathcal{E}^*$ since, in this economy, the budget set is the same for every agent. Indeed, if $w_i \geqslant w_j$, then in any allocation decentralized by a price system p, individual i cannot envy j, since $p \cdot w_i \geqslant p \cdot w_j$.

The allocation x^*, showing the general existence of envy-free allocations, belongs to the core of $\mathcal{E}^*$. However, it may not be individually rational (hence not in the core) of the original economy $\mathcal{E}$. Thus, this result highlights that Varian's fairness does not imply individual rationality. To see this point, consider the economy with identical weakly monotone[9] preferences. If there is an agent i such that $\omega_i \gg \frac{W}{n}$, then the equalitarian allocation which assigns $\frac{W}{n}$ to every consumer, is not individually rational.

Varian's fairness may be unsuitable in scenarios with heterogeneous endowments. If $w_i \geqslant w_j, w_i \neq w_j$, in any allocation decentralized by a price system p, individual i cannot envy j. However, if $p \gg 0$ and both have the same monotonic preferences $\succcurlyeq$, j envies i. Indeed, let x_j the bundle selected by j, we have $p \cdot x_j = p \cdot w_j < p \cdot w_i$. There are bundles $z \geqslant x_j, z \neq x_j$ with $p \cdot z \leqslant p \cdot w_i$ and, by monotonicity, $z \succ x_j$, then as i can choose z, we have $x_i \succcurlyeq z \succ x_j$.

Therefore, Varian's fair allocations may exclude classical solutions to the sharing problem, highlighting reasons for revisiting the concept. Envy-freeness is a valuable property when the envy in question is justified. If individuals feel they are not receiving an adequate share compared to others in similar circumstances, envy is deemed justified. For instance, if two agents have similar qualifications or resources, and one receives a significantly better outcome than the other, the latter may feel justified in their envy.

Attempting to justify envy, based on individuals' initial resources, one could say:

(i) Individual i justifiably envies j if $w_i \sim_i w_j$ (which occurs in the particular case when both have the same endowment) and $x_j \succ_i x_i$.

(ii) Given an allocation x, the agent i envies j if $\omega_i \succcurlyeq_i \omega_j$ and $x_j \succ_i x_i$. That is, i feels a justified envy towards j in the assignment x because, $x_j \succ_i x_i$ when initially $w_i \succcurlyeq_i \omega_j$.

We remark that if a consumer i envies j following notion (i), then i envies j according to (ii).

[9]A preference $\succcurlyeq$ is weakly monotone if $x \gg y$ implies $x \succ y$, and is monotone if $x > y, x \neq y$ implies $x \succ y$.

Let us consider an economy with two agents, 1 and 2, and two goods, x and y. Endowments are $\omega_1 = (1,0), \omega_2 = (0,1)$. Preferences are represented by the utility functions $U_1(x,y) = xy, U_2(x,y) = x^2y$. The Walrasian equilibrium is given by the price vector $(1,3/4)$ and the allocation α assigning $\alpha_1 = (1/2, 2/3)$ to consumer 1 and $\alpha_2 = (1/2, 1/3)$ to consumer 2. Following Varian, agent 2 envies 1 at equilibrium, since $U_2(\alpha_1) > U_2(\alpha_2)$. This envy is justified with definitions (i) and (ii) because $U_2(\omega_2) = U_2(\omega_1)$.

Consider now that both agents have the same preferences represented by $U(x,y) = xy$, and endowments are $\hat{\omega}_1 = (2,1)$, and $\hat{\omega}_2 = (1,1)$. Then, the equilibrium price vector is $(1,3/2)$, and the Walrasian allocation is given by $\hat{\alpha}_1 = (7/4, 7/6)$ and $\hat{\alpha}_2 = (5/4, 5/6)$. We deduce that $\hat{\alpha}$ is not Varian envy-free since $U(\hat{\alpha}_1) > U(\hat{\alpha}_2)$. However, this envy is justified neither with the notion (i) nor (ii) since $U(\hat{\omega}_1) > U(\hat{\omega}_2)$. That is, in this example, the Walrasian equilibrium is justified envy-free with any of the previous definitions. On the other hand, to show that (i) and (ii) differ, let us consider the allocation that assigns the bundle $(1/2, 1/2)$ to agent 1 and $(5/2, 3/2)$ to agent 2. In this case, the individual 1 envies 2, but the envy is justified only according to (ii).

The examples show that Walrasian allocations may present envy with the definitions (i) and (ii) and that both notions are different and also differ from the original one by Varian.

Next, arguing that envy can be only justified when comparing individuals with the same endowments, we state the following definition.

Definition. Justified envy. Consider an allocation x in the economy $\mathcal{E}$. We say that agent i envies j if $w_i = w_j$ and $x_j \succ_i x_i$. That is, envy is only justified for agents with the same initial resources.

Note that the Walrasian allocations are justified envy-free. Assuming fully informed agents with monotonic preferences (they can forecast equilibrium prices), Walrasian allocations are envy-free with definitions (i) and (ii). Indeed, let (p^*, x^*) be an equilibrium. If agents can forecast the price p^* and $x_j^* \succ_i x_i^*$ then $p^* \cdot w_j > p^* \cdot w_i$ (otherwise i could chooses x_j^*), consequently, i would prefer ω_j to ω_i.

Feldman and Kirman (1974) proved that an allocation in the core relative to equal division (therefore individually rational and Pareto dominating equal division) may fail to be Varian fair. Thus, this result shows that allocations in the core are not necessarily envy-free with any of the above definitions.

3 Justified envy and priority orders

In matching theory, the notion of justified envy has been introduced by Abdulkadiroğlu and Sönmez (2003) within the context of school choice problems. In this framework, envy is justified when a student prefers the assigned school to another student over whom she has priority. It is important to note that this concept relies on a specific priority order. Romm, Roth, and Shorrer (2024) remark that formally defining justified envy in more general environments (with arbitrary preferences, feasibility constraints, and contracts) is not straightforward. They highlight that it is not always clear who is prioritized over whom, which makes it difficult to determine what type of envy is "justified."

We claim that, in exchange economies settings, price systems provide a natural priority order. A price vector associates a value to each commodity bundle, determining the income available to each consumer, given their initial resources. Prices provide an order of the consumers' budget sets, defined by the market value of their endowments.

Given a price vector $p \in \mathbb{R}^\ell_+$, we say that the individual i p-justifiably envies j at the allocation x if $x_j \succ_i x_i$ and $p \cdot \omega_j \leqslant p \cdot \omega_i$. An allocation is p-envy-free if there is no p-justifed envy.[10] If at price p the bundle x_i maximizes $\succcurlyeq_i$ for every i belonging to a coalition S, then there is no p-justified envy among the members of S. In particular, if (p, x) is a Walrasian equilibrium for the economy $\mathcal{E}$, then x is p-envy-free. Moreover, by the second welfare theorem, if $x \gg 0$ is an efficient allocation, then x can be decentralized by a price system in the economy where the endowments are given by x. We conclude that this decentralization mechanism makes x a p-envy-free allocation.

4 Fair net trades *vs.* fair allocations

In exchange economies, there is a one-to-one correspondence between net trades and allocations. In the economy $\mathcal{E}$, each allocation $x = (x_1, \ldots, x_n) \in \mathbb{R}^{\ell n}_+$ defines a net trade $z_i(x_i) = x_i - \omega_i \in \mathbb{R}^\ell$ for each agent $i \in N$. A net trade $z = (z_1, \ldots, z_n) \in \mathbb{R}^{\ell n}$ is feasible when $\sum_{i=1}^n z_i \leqslant 0$ and $x_i(z_i) = \omega_i + z_i \in \mathbb{R}^\ell_+$ for every i. The allocation x is feasible if so is the net trade $z(x)$. Furthermore, each preference relation $\succcurlyeq_i$ on the consumption set $\mathbb{R}^\ell_+$ defines a preference relation $\widehat{\succcurlyeq}_i$ on the i's agent net trade set $Z_i = \{z_i \in \mathbb{R}^\ell | \omega_i + z_i \in \mathbb{R}^\ell_+\}$, and vice versa, by the rule: $a \,\widehat{\succ}_i\, b$ if and only if $x_i(a) = \omega_i + a \succ_i \omega_i + b = x_i(b)$.

Schmeidler and Vind (1972) consider for each i a preference $\widehat{\succcurlyeq}_i$ on the set Z_i and define envy on net trades. Individual i envies j at the net trade z if $\omega_i + z_j \geqslant 0$ and $z_j \widehat{\succ}_i z_i$. A net trade is envy-free if there are no agents i, j such that i envies j. A net trade z is competitive if there is a price system p such that for every consumer i the following conditions hold: *(i)* $p \cdot z_i = 0$ and *(ii)* $p \cdot h > p \cdot z_i$ for every $h \in Z_i$ such that $h \,\widehat{\succ}_i\, z_i$. Any competitive net trade z is envy-free; otherwise, there is a consumer j who is envied by a consumer i, and then $p \cdot z_j > p \cdot z_i$ at the corresponding competitive prices p, which is a contradiction with the previous property *(i)*.

[10] Note that if there is justified envy in an allocation, then there is also p-justified envy for every price p.

We remark that a net trade may be envy-free, and one finds envy in the resulting allocation. Note that x^* is a Walrasian allocation if only if $z^* = x^* - \omega$ is a Walrasian net trade. Walrasian net trades are envy-free, although this is not always true for every Walrasian allocation.[11] Note that there is no envy between agents who have the same equilibrium wealth.

Consider the notion of justified envy that we have proposed: an allocation is envy-free if there is no envy for agents with the same endowments. We deduce that a net trade z is justified envy-free if so is the allocation $x(z)$ and, reciprocally, the allocation x is justified envy-free if so is the net trade $z(x)$.

Net trades can be understood as a mechanism of exchange where the resulting allocations are the outcomes. Fair trades can undermine fair allocations; in other words, a fair mechanism may lead to unfair outcomes. Feldman and Kirman show (1974) that fairness is not preserved by competitive trades, trades to the core, or even fair trades. It is so because the original envy-free notion, known as equity, is not affected by inequalities in the distribution of endowments. Our justified envy notion helps to overcome this misleading issue.

5 Coalitions and fairness

The envy-freeness criterion, which compares individual bundles or trades, has been extended to coalitions.

Varian (1974) proposed an extension of envy-freeness called coalition fairness (or group no-envy), which requires that no group of agents envies any other group of the same size.[12] There is coalitional envy at an allocation x if there are two coalitions A and B, such that the number of members in B is no more than in A, and an allocation y with the following properties:

- $y_i \succ_i x_i$ for every agent $i \in A$, and
- $\sum_{i \in A} y_i = \sum_{i \in B} x_i$.

If the conditions above hold, A envies B. Note that if the individual i envies j at x, then $A = \{i\}$ envies $B = \{j\}$ with the previous definition. Therefore, coalitional fairness implies ennvy-free.

Gabszewicz (1975) states that an allocation x is c-fair in the economy $\mathcal{E}$ if there is no disjoint coalitions A and B and an assignment y such that:

- $y_i \succ_i x_i$ for every agent $i \in A$, and
- $\sum_{i \in A} (y_i - \omega_i) = \sum_{i \in B} (x_i - \omega_i)$.

[11] See the previous examples.

[12] Varian points out that this definition is due to Vind (1971).

Yannelis refers to a c-fair allocation as nondiscriminatory. Note that none of the concepts above need an ordering $\widehat{\succcurlyeq}$ on the set of individual net trades. However, the feasibility of y is described in terms of commodity bundles (outcomes) in the first definition and in terms of net trades (exchange process) in the second.

In contrast to Varian's coalitional fairness, in an economy with unequal endowments, any c-fair allocation is in the core, and envy-freeness is not guaranteed. We remark that c-fairness implies justified envy-free. To see this, let x be an allocation and assume there are two agents i, j with the same endowments such that i envies j, i.e., $\omega_i = \omega_j$ and $x_j \succ_i x_i$. Taking $A = \{i\}$ and $B = \{j\}$, and $y_i = x_j$ in the above definition, we deduce that x is not c-fair. Therefore, if an allocation is c-fair, no individual can envy another with the same initial resources.

The natural extension of our notion of justified envy to coalitions restricts comparisons to groups of consumers with equal aggregate resources. Coalitional fairness becomes equivalent by considering net trades or outcomes whenever envy is justified for coalitions whose total endowments are the same. In short, coalitional justified envy is equivalent in term of net trades or allocations.

On the other hand, the set of c-fair or nondiscriminatory allocations contains the Walrasian outcomes and is included in the core, with both inclusions being strict. (See Gabsezwicz, 1975, and Yannelis, 1985).

To finish this section, we propose another cooperative solution.

Definition. Envy-free core. A feasible allocation belongs to the envy-free core if it is not blocked by any coalition and there is no envy among individuals with the same endowments.

We have stated that if an allocation is c-fair, no individual can envy another with the same initial resources. Thus, the set of c-fair allocations is included in the envy-free core, that is included in the core. Next we show that both inclusions are strict. For this, we adapt an example from Gabszewicz (1975).

Consider the exchange economy with two commodities x, y and three consumers with identical preferences given by the utility function $u(x, y) = x^{\frac{1}{2}} + y^{\frac{1}{2}}$. The initial endowments are $\omega_1 = (0, 8), \omega_2 = (4, 0)$, and $\omega_3 = (4, 0)$. The allocation $x_1 = (5.5, 5.5), x_2 = (1.5, 1.5)$, and $x_3 = (1, 1)$ is in the core, but it is not justified envy-free since 3 justified envies 2. Then, the envy-free core is strictly contained in the core.

If the endowments of agent 2 are 3 are $\hat{\omega}_2 = (4.1, 0)$ and $\hat{\omega}_3 = (3.9, 0)$, the previous allocation $x = ((5.5, 5.5), (1.5, 1.5), (1, 1))$ is still in the core of the economy with the modified endowments, and it is justified envy-free because the individual endowments are different, but it is not c-fair. Note that $u(1.3, 1.5) > u(1, 1)$ and the coalition $\{3\}$ obtains $y = (1.3, 1.5)$ via the net trade of coalition $\{2\}$, since $(1.3, 1.5) - (3.9, 0) = x_2 - \hat{\omega}_2 = (1.5, 1.5) - (4.1, 0)$. Therefore, the set of c-fair allocations is strictly included in the envy-free core, concluding that our cooperative solution is less demanding than c-fairness.

References

Abdulkadiroğlu, A., Sönmez, T. (2003). School choice: a mechanism design approach. American Economic Review 93(3), 729–747.

De Clippel, G. (2008). Equity, envy and efficiency under asymmetric information. Economics Letters 99, 265–267.

Dubins, L. E., Spanier, E. H. (1961). How to cut a cake fairly. The American Mathematical Monthly 68(1), 1–17.

Echenique, F., Miralles, A., Zhang, J. (2021). Fairness and efficiency for allocations with participation constraints. Journal of Economic Theory 195, 105274.

Feldman, A. (1987). Equity. In: The New Palgrave Dictionary of Economics. Palgrave Macmillan, London.

Feldman, A., Kirman, A. (1974). Fairness and Envy. American Economic Review 64(6), 995–1005.

Foley, D. (1967). Resource allocation and the public sector. Yale Econ Essays 7, 45–98.

Gabszewicz, J. (1975). Coalitional fairness of allocations in pure exchange economies. Econometrica 43(4), 661–68.

Hurwicz, L., Schmeidler, D., Sonnenschein, H. (eds.) (1985). Social goals and social organization: Essays in memory of Elisha Pazner. Cambridge University Press.

Kolm, S. (1972). Justice et Équité. Paris: Editions du Centre National de la Recherche Scientifique, 1972. English Edition, MIT Press, (1988)

Kranich, L. (2020). Resource-envy-free and efficient allocations: A new solution for production economies with dedicated factors. Journal of Mathematical Economics 89, 1–7.

Kuhn, H. W. (1967). On games of fair division. In Essays in Mathematical Economics, in Honor of Oskar Morgenstern, Princeton: Princeton University Press.

Nozick, R. (1974). Anarchy, state, and utopia. Basic Books, New York.

Pazner, E. A. (1977). Pitfalls in the theory of fairness. Journal of Economic Theory 14, 458-466.

Pazner, E.A., Schmeidler, D. (1974). A Difficulty in the Concept of Fairness. The Review of Economic Studies 41 (3), 441–443.

Pazner, E.A., Schmeidler, D. (1978). Egalitarian equivalent allocations: A new concept of economic equity. Quarterly Journal of Economics 92(4), 671–687.

Phelps, E. (1973). Economic justice. Penguin, Middlesex.

Phelps, E. (1976). Recent developments in welfare economics: justice et équité. In M. Intrilligator, Frontiers of Quantitative Economics. Vol. I, North Holland, Amsterdam.

Rawls, J. (1999). A theory of justice. Revised Edition. Harvard University Press, Cambridge, MA. Original edition 1971.

Romm, A., Roth, A.E., Shorrer, R.I. (2024). Stability vs. no justified envy. Games and Economic Behavior 148, 357–366.

Schmeidler, D., Vind, K. (1972). Fair net trades. Econometrica 40(4), 637–42.

Steinhaus, H. (1948). The problem of fair division. Econometrica 16, 1014.

Sugden, R. (1984). Is fairness good? A critique of Varian's theory of fairness. Noûs 18(3), 505–511.

Thomson, W. (1982). An informationally efficient equity criterion. Journal of Public Economics 18(2), 243–263.

Thomson, W. (2025). Agent-wise–replication invariance, the Walrasian solution and the uniform rule. Economic Theory 79(2), 357–377

Varian, H.R. (1974). Equity, envy, and efficiency. Journal of Economic Theory 9, 63–91.

Varian, H.R. (1975). Distributive justice, welfare economics, and the theory of fairness. Philosophy and Public Affairs 4(3), 223-247.

Varian, H.R. (1976a). On the history of concepts of fairness. Journal of Economic Theory 13(3), 486-487.

Varian, H.R. (1976b). Two problems in the theory of fairness, Journal of Public Economics 5(3-4), 249-260.

Varian, H.R. (1987). Fairness. In: The New Palgrave Dictionary of Economics. Palgrave Macmillan, London.

Vind, K. (1971). Lecture notes for Economics 288, Stanford University.

Vohra, R. (1992). Equity and efficiency in non-convex economies. Social Choice and Welfare 9(2), 185-202.

Yannelis, N.C. (1985). Value and fairness. In: Aliprantis, C.D., Burkinshaw, O., Rothman, N.J. (eds) Advances in Equilibrium Theory. Lecture Notes in Economics and Mathematical Systems, vol 244. Springer, Berlin, Heidelberg.