Ambiguity When Comparing Brands: Caution and Monopolistic Competition*

Mario Pascoa† Jaime Velasco Sanchez‡

March 7, 2025

Abstract

When consumers are not sure how to compare brands, caution creates monopolistic competition. This is addressed by looking at an exchange economy with countably many differentiated goods and analysing the price impact of a coalition of countably many agents withholding their endowments. Such impact persists even when making the coalition smaller and smaller.

Keywords: Monopolistic Competition, Caution, Exchange Economy

JEL Codes: D01, D43, D51

*We want to thank Rodrigo Novinsky for useful comments on a previous version of the present work.
†School of Economics, University of Surrey, Guildford, Surrey, GU2 7XH, United Kingdom. Email: m.pascoa@surrey.ac.uk
‡School of Economics, University of Surrey, Guildford, Surrey, GU2 7XH, United Kingdom. Email: j.velascosanchez@surrey.ac.uk

1 Introduction

Chamberlin [1951] envisioned monopolistic competition arising in a market with a large number of agents supplying differentiated commodities when substitutability is inadequate. It should not be surprising to see caution as a source of inadequate substitutability and the resulting monopolistic competitive behaviour.

In the proposed work, caution prevails when consumers do not know how to weight different brands in their utility functions. The aversion to the ambiguity with respect to these weights makes the consumer care about changes in the average (unweighted utility), as she adds consumption of different brands along an index list. This has some flavour of habit persistence as the consumer fears to consume further brands since it might decrease the utility she has enjoyed so far. We examine if this cautious attitude may give rise to inadequate substitutability, and thus market power, even when having infinitely many goods and consumers, each consumer being endowed with her characteristic commodity.

The market power test is a withholding test inspired in what Ostroy and Zame [1994] did with a continuum of agents and commodities. Their test consisted in having a coalition withholding part of their endowments and looking at the resulting price. Then, they make the measure of the coalition go to zero. An imperfectly competitive outcome prevails if the withholding test fails, in the sense that the sequence of prices of the perturbed economies does not converge back to the equilibrium price of the original economy.

In our countably infinite setting, the sequence of withholding coalitions is formed by removing an increasing set of agents from the original withholding coalition (in the limit, no agents would be withholding any portion of their endowments). Then, we check if the equilibrium prices of the perturbed economy converge to the equilibrium prices of the original economy. We provide two examples where that convergence does not hold, which means failing the withholding test.

To be more precise, the fact that consumers care about the worst case of the moving average utilities makes the preferences over ℓ_∞^+ bundles become Mackey discontinuous. Supporting prices lie in the dual $\left(ba\left(2^{\mathbb{N}}\right)\right)$ but possibly not in the pre-dual space $\left(\ell_1^+\right)$. It is actually the pure charge in the supporting prices (that element not included in ℓ_1^+) which fails to converge in the withholding test.

On a different context, on a countably infinite time horizon setting, Araujo et al. [2019] considered habit persistence with respect to a single good that is being consumed over time. The preferences that we contemplate are formally as theirs. In their work, Arrow-Debreu prices can exhibit a pure charge, making prices particularly sensitive to what happens at limiting dates, and it was shown that the pure charge induces bubbles in assets used for inter-temporal transfers of wealth.

Our result suggests that ambiguity when comparing brands makes consumers cautious and allows for market power of coalitions to persist even when they are as small as we want. We see this outcome in the spirit of Chamberlin [1951]: even though we are in a large numbers environment, being small does not mean being a perfect competitor.

The structure of this document is as follows: First, we present the framework, such as agents' preferences and endowments. After that, we look at equilibrium bundles and prices. Finally, we test the market power of colations withholding endowments.

2 Framework

We consider the commodity space to be ℓ_∞^+, the positive orthant of the space of bounded sequences. Endowments of agent i, w_i, are elements of ℓ_∞^+ too, and prices belong (in the general case) to the dual, $ba\left(2^{\mathbb{N}}\right)$, the space of bounded additive set functions on the natural numbers.

We have countably many agents, i.e. $i \in \mathbb{N}$, which are identified by their own endowment. More precisely, there are as many differentiated merchandises as the natural numbers and there is a one-to-one mapping, M, associating each agent with the commodity she is endowmed with. Formally,

$$
\begin{aligned}
M : \mathbb{N} &\mapsto E \\
i &\rightsquigarrow (0, 0, \ldots, 0, \underbrace{\omega}_{\text{i-th}}, 0, \ldots)
\end{aligned}
$$

where E is the set of such possible endowments.

We depart from standard preferences by assuming that agents are not sure about how to compare brands. This can be seen by taking as baseline utility an additively separable utility, and considering a set C of possible different sequences of weights. The aversion to the ambiguity with respect to these weights makes the consumer evaluate each consumption bundle $x \in \ell_\infty^+$ using the additively separable utility computed with the most penalising sequence of weights $\delta \in C$ for that bundle. Formally:

$$
U^i(x) = \inf_{\delta \in C} \sum_{j \in \mathbb{N}} \delta_j u\left(x_j\right). \tag{1}
$$

Notice that each δ_j is the weight that the consumer gives to the individual utility yielded by x_j units consumed of merchandise j. Rigorously, $C \subset \left\{\delta \in \ell_1^+ \cap B_1(0) : \delta_j \geq \epsilon_j\right\}$ for some sequence ϵ such that $\epsilon_j > 0$, $\forall j \in \mathbb{N}$, with $B_1(0)$ being the unitary ball of ℓ_1. Thus, agents will pick weights $\delta_j \in [\epsilon_j, 1]$ for each $u\left(x_j\right)$ such that the utility of consuming $x \in \ell_\infty^+$ is equal to the infimum of the sum of the individual weighted utilities derived from consuming x_j.

In our more specific case, we will assume that $C : core(v_\epsilon) \cap \hat{C}$, where $\hat{C}$ is the closed convex hull of

$$\left\{(\delta_m)_{m\in\mathbb{N}} : \delta_m(j) = \varsigma_j + \frac{\beta^i}{m} \text{ for } 1 \leq j \leq m,\ \delta_m(j) = \varsigma_j \text{ elsewhere}\right\},$$

in the weak* topology of $ba\left(2^{\mathbb{N}}\right)$ [1]. Notice that under such case, equation (1) can be rewritten as

$$U^i(x) = \sum_j \varsigma_j u(x_j) + \beta^i \inf_j \left(\frac{1}{j}\sum_{k=1}^{j} u(x_k)\right). \tag{2}$$

Equation 2 reflects precautionary behaviour on the preferences of our agents. Not being sure that weights ς are the correct ones, the consumer keeps track of the unweighted averages of utilities as she consumes along the list of commodities.

Notice that in the first summand we have weights ς_j for each $u(x_j)$, in a similar fashion as we had in equation 1. On the second summand, we have another weight for merchandises $j \in \{1, \ldots, m\}$. Such weight is $\frac{\beta^i}{m}$, making $\delta_j = \varsigma_j + \frac{\beta^i}{m}\ \forall j \in \{1, \ldots, m\}$. For the rest of the merchandises in the bundle, i.e. $j \in \mathbb{N} \setminus \{1, \ldots, m\}$, the individual has weights $\delta_m(j) = \varsigma_j$.

It is on this second summand that caution appears: the infimum over the sum represents how the agent is cautious on adding merchandises $j \in \mathbb{N} \setminus \{1, \ldots, m\}$ to her consumption bundle. She is cautious about consuming such goods in the sense of being wary about how consuming these merchandises affects her mean utility, by ckecking what is the worse case scenario given by the infimum of such moving mean utilities. If the infimum happens to be attained at a certain index rather than at a cluster point, then the commodities that are listed beyond that point would not get the extra weight $\frac{\beta^i}{m}$.

These preferences are formally as in one example by Araujo et al. [2019], which was intended to portray habit persistence on consumer preferences in a macroeconomic dynamic

[1]The definition of the core of a capacity can be found in section 6, definition 6.6.

setting. As in their model, we assume that the utility index $u : \mathbb{R}^+ \to \mathbb{R}$ is concave and strictly increasing.

In our framework, instead of having a countably infinite set of dates and a single commodity, we have a countably infinite set of differentiated commodities in a static set up, but the utility functions will be as specified in (2). So, the exchange economy we will be using is defined as

$$\mathcal{E} = \left(\ell_\infty^+, U^i, \omega^i\right)_{i\in\mathbb{N}}, \tag{3}$$

where U^i is as in (2), and endowments as specified above.

3 Equilibrium Allocations and Prices

In this section we state definitions and results on equilibrium prices.

Definition 3.1. An **Arrow-Debreu equilibrium** (AD equilibrium) is defined as a pair (x, π) such that $x \in \ell_\infty^+$ is a feasible allocation, π is a linear functional on $ba\left(2^{\mathbb{N}}\right)$ and, for each i, x^i maximizes U^i in the budget set $\{a \in \ell_\infty^+ : \pi(a - w_i) \leq 0\}$.

As mentioned in Araujo et al. [2019], a supporting price is, up to a scalar multiple, a supergradient of U^i at x^i. Even more, under the conditions mentioned in the previous section and assuming U^i is concave, norm continuous, Mackey upper semi-continuous and such that $U^i(x) > U^i(x')$ whenever $x > x'$, along with assuming that $\sum_{i\in\mathbb{N}} w_i \ggg 0$, there exists an AD equilibrium $(x^i, \pi)_{i\in\mathbb{N}}$ with the price $\pi \in ba\left(2^{\mathbb{N}}\right)$.

Let's have a look now at a characterization of the equilibrium prices. The next result, presented in Araujo et al. [2011], relates the infimum of the equilibrium allocations to the form of the price functional.

Proposition 1. *Given $x \ggg 0$ denote $\underline{x} \equiv \inf x$. Let U be given by (2) with $u : \mathbb{R}^+ \to \mathbb{R}$ concave, increasing and of class $\mathcal{C}^1(0, \infty)$:*

a If $\underline{x}$ is not a cluster point of x, $\partial U(x) \subset \ell_1$. If $\underline{x}$ is attained for infinite indices j, $\partial U(x) \cap \ell_1 \neq \emptyset$ but $\partial U(x) \not\subset \ell_1$.

b If $\underline{x}$ is not attained $\partial U(x) \cap \ell_1 = \emptyset$. Moreover, if $\pi \in \partial U(x)$, then there is a generalized limit LIM *such that, for each $y \in \ell_\infty$,*

$$\pi(y) = \sum_{j=1}^{\infty} \varsigma_j u'(x_j) y_j + \beta^i u'(\underline{x}) \operatorname{LIM}\big(\phi(y)\big).$$

In the previous, $\phi : \ell_\infty^+ \to \ell_\infty^+$ such that $\phi(y)_j = \frac{1}{j}\sum_{k=1}^{j} y_k$, and LIM represents a linear functional taking on each $y \in \ell_\infty$ a value in $[\liminf y, \limsup y]$.

Recall that, by the Yoshida-Hewitt Theorem, we can represent any $\pi \in ba\left(2^{\mathbb{N}}\right)$ as $\pi = \mu + v$, where $\mu \in ca\left(2^{\mathbb{N}}\right)$, the space of countably additive set functions, and $v \in pa\left(2^{\mathbb{N}}\right)$, the space of pure charges. We need to note that, as in Araujo et al. [2019], if U^i is Mackey continuous, then $\partial U^i(x) \subseteq \ell_1$ for $x \ggg 0$. Using the Yoshida-Hewitt decomposition, it can be seen that $\pi(y) = \mu(y) + \nu(y)$, where

$$\mu(y) = \sum_{j=1}^{\infty} \varsigma_j u'(x_j) y_j \text{ (countably additive part) },$$

$$\nu(y) = \beta^i u'(\underline{x}) \operatorname{LIM}\big(\phi(y)\big) \text{ (purely finitely additive part) }.$$

We are mostly interested in the behaviour of the purely finitely additive part of prices. Using lemma 3 in Appendix 6 and the next two lemmas (all of them included in Araujo et al. [2011]), we can have a closer look to that part of the prices.

Lemma 1. *Let B be a finite subset of $\mathbb{N}$. If $\nu \in pch^+$, then $\nu(B) = 0$.*[2]

Lemma 2. *Let $\nu > 0$ be a pure charge such that $\nu(\mathbb{1}) = 1$. Then, $\nu(x) \in [\liminf x, \limsup x]$, for any $x \in \ell_\infty$. In other words, ν is a generalized limit.*[3]

[2] The set pch^+ denotes the positive cone of pure charges on $\left(\mathbb{N}, 2^{\mathbb{N}}\right)$.

[3] The sequence $\mathbb{1}$ is the constant sequence of ones, i.e. $(1, 1, 1, \ldots)$.

In summary, proposition 1 tells us that the price functional has a pure charge when there is at least one agent whose consumption bundle converges to its infimum and that infimum is never attained. The price functional will also have a pure charge when the infimum of the consumption bundle of at least one agent is attained at infinite indices.

We are interested in economies that have prices as defined in proposition 1, part b. Now, we will explain how to obtain the equilibrium allocations. These should fulfill three conditions:

1. Market clearing (for every merchandise), i.e. $\sum_{i=1}^{\infty} x_j^i = \sum_{1=1}^{\infty} \omega_j^i$,
2. Budget constraint, i.e. $\pi(x^i - \omega^i) = 0;\ \forall i \in \mathbb{N}$,
3. Price up to a scalar multiple belonging to the supergradient, i.e. $\lambda^i \pi \in \partial U^i$.

The last two conditions pertain to the First Order Conditions of the maximization problem. For now, we are going to work with the same preferences for all of the agents, which means they will have the same $u(x^i)$, as well as the same β^i. In section 5.1, we have an example of equilibrium for an economy as described in sections 2 and 3.

4 The Withholding Test

In this section, we describe the Withholding Test. In plain words, its **first part** consists in some agents segregating an amount of their endowments, hoping to have some impact on the price and, therefore, with the intention of increasing the value of their endowments. In order to address and define the problem in proper and rigorous terms, we will introduce some useful definitions.

Let $\mathcal{E} = (\ell_\infty^+, U^i, \omega^i)_{i \in \mathbb{N}}$ be an economy with equilibrium bundles $(x^i)_{i \in \mathbb{N}}$ and prices, for $y \in \ell_\infty^+$,

$$\pi(y) = \sum_{j=1}^{\infty} \varsigma_j u'(x_j) y_j + \beta^i u'(\underline{x}) \,\mathrm{LIM}\,\big(\phi(y)\big),$$

as described in sections 2 and 3. We will give some examples on how to construct withholding coalitions in section 5. In general, we let countably many agents withhold some part c^i of their characteristic endowment. More specifically, denoting by W_k the withholding coalition, we can have

- all agents in W_k withholding a same amount of their endowments, such that $0 < c^i < \inf_{j \in \mathbb{N}} \{\omega_j\}$ $\forall i \in W_k$, with $c^i = c^m$ $\forall i \neq m$, or
- each withholding a different amount of the endowments, such that $0 < c^i < \omega_i^i$ $\forall i \in W_k$.

Let every $i \in \mathbb{N}$ engage in exchange with

$$\mathrm{w}^i = \begin{cases} \omega_j^i & \text{if } j = i \text{ and } j \in \mathbb{N} \setminus W_k, \\ \omega_j^i - c^i & \text{if } j = i \text{ and } j \in W_k, \\ 0 & \text{otherwise.} \end{cases}$$

Based on proposition 1, and as long as the equilibrium allocations do not attain their infimum, the latter will give place to a new economy $\mathcal{E}_{W_k} = (\ell_\infty^+, U^i, \mathrm{w}^i)_{i \in \mathbb{N}}$, with equilibrium bundles $(z^i)_{i \in \mathbb{N}}$ and prices, for $y \in \ell_\infty^+$, that we can write as

$$\pi_{W_k}(y) = \sum_{j \notin W_k} \varsigma_j (u)'(z_j) y_j + \sum_{j \in W_k} \varsigma_j (u)'(z_j) y_j + \beta^i u'(\underline{z}) \operatorname{LIM}\big(\phi(y)\big).$$

The main objective of agents $i \in W_k$ is to increase the appraisal of their original endowments ω^i. On section 5, we work out two examples of economies reaching an equilibrium with withholding. In example 1, finitely many agents do not withhold. In example 2, we have infinitely many agents not withholding, showing that market power does not come from the fact that agents withholding are *more* than those agents not withholding.

Let us describe the **second part** of the Withholding Test. Again, in plain words, we are shrinking coalition W_k, as much as we want, even shrinking it down to disappearing. Thus, we are generating a sequence of economies, $(\mathcal{E}_{W_k})_{k \in \mathbb{N}}$, for which the limiting economy

, $\lim_{k\to\infty} \mathcal{E}_{W_k}$, has no agent in the withholding coalition. Then, we calculate the limit of the equilibrium prices, $\lim_{k\to\infty} \pi_{_{W_k}}$, which shows the impact (or non-impact) of the withholding coalition when it gets as small as one can think of. The question to be answered with the test consists in: *Is the price impact of W_k persistent when the coalition is shrank down to disappearance?*

In summary, our withholding test consists of the next steps:

1. Calculate equilibrium allocation, $\left(x_j^i\right)_{j\in\mathbb{N}}$ $\forall i \in \mathbb{N}$, and prices, π, for our *original* economy $\mathcal{E} = (\ell_\infty^+, U^i, \omega^i)_{i\in\mathbb{N}}$,

2. Let a coalition W_k of countably many agents withhold c^i, with $\inf_{i\in\mathbb{N}}\{\omega_i^i\} > c^i > 0$ (or $0 < c^i < \omega_i^i$), of their endowments,

3. Calculate equilibrium allocation, $\left(z_j^i\right)_{j\in\mathbb{N}}$ $\forall i \in \mathbb{N}$, and prices, $\pi_{_{W_k}}$, for the *perturbed* economy where the coalition W_k of agents is withholding part of their endowments, i.e. $\mathcal{E}_{W_k} = (\ell_\infty^+, U^i, \mathrm{w}^i)_{i\in\mathbb{N}}$,

4. Shrink coalition W_k down as much as we want, even down to disappearing. Thus, generating a sequence of economies $(\mathcal{E}_{W_k})_{k\in\mathbb{N}}$, and calculating the limit of their equilibrium prices, i.e. $\lim_{k\to\infty} \pi_{_{W_k}}$ (in the norm topology of the dual space $ba\left(2^{\mathbb{N}}\right)$),

5. Is the price impact persistent even when the coalition shrank down to disappearance? This is tested by checking if $\pi_{_{W_k}}$ fails to converge to π.

   - If it does not converge, agents in the coalition can actually move prices in their favour, no matter how small the coalition is, by withholding part of their endowments. Thus, we have a monopolistically competitive outcome,

   - If it converges, agents in the coalition cannot actually affect prices in their favour, when the coalition gets arbitrarily small, by withholding part of their endowments. Thus, we have a perfectly competitive outcome.

Before advancing towards the main result, we need to stress the fact that the impact of the withholding falls directly on the generalized limit. Then, it becomes important to notice in which cases do we have a generalized limit in the price functional, based on proposition 1: for now, we are just interested in equilibria such that the infimum of equilibrium allocation, $\underline{x}^i$, is not attained for all $i \in \mathbb{N}$. Both example 1 and example 2 are of this type.

Proposition 2. *Let agents $i \in W_k$ withhold from their endowments an amount c^i, such that $0 < c^i < \inf_{j\in\mathbb{N}}\{\omega_j\}$ $\forall i \in W_k$ (or $0 < c^i < \omega_i^i$ $\forall i \in W_k$). Assume that for every agent i, the infimum $\underline{x}^i$ is not attained for every perturbed economy $\mathcal{E}_{W_k}$, with $k \in \mathbb{N}$.*

- *When the withholding coalition is shrunk down to vanishing, (i.e. $\lim_{k\to\infty} \mathcal{E}_{W_k}$ with $\lim_{k\to\infty} W_k = \emptyset$),*

  1. $\left\|\pi_{W_k} - \pi\right\|_{ba} \not\to 0$ *as* $k \to \infty$, *where* $\left\|\pi_{W_k} - \pi\right\|_{ba} = \sup_{\|y\|_\infty \leq 1} \left|\pi_{W_k}(y) - \pi(y)\right|$,
  2. $\lim_{k\to\infty} \pi_{W_k}(y) - \pi(y) > 0$ *for* $y \in \ell_\infty^+$ *with* $y \ggg 0$.

The proof for proposition 2 is included in appendix 6. The persistent impact of withholding over prices is evident on examples 1 and 2 in section 5.

Finally, let us recall what Chamberlin [1951] considered to be monopolistic competition structure: A market with a large number of firms producing differentiated goods, all of them with some market power. We are considering **market power** as the ability of agents to possibly move equilibrium prices in their favour. Consider $\mathcal{E}$ and the limiting economy $\lim_{k\to\infty} \mathcal{E}_{W_k}$, along with their equilibrium prices and allocations. As described in section 2, both economies have:

- A large amount of agents,
- Differentiated products (since everyone has her own characteristic good),
- Market power of the Withholding Coalition W_k, even when shrunk down to disappearance.

By proposition 2, these qualities make economy $\mathcal{E}$, with equilibrium bundles not attaining their infimum for all $i \in \mathbb{N}$, have a monopolistic competition structure.

5 Examples

5.1 Equilibrium Without Withholding

All agents have the same preferences with $\varsigma_j = \left(\frac{1}{2}\right)^{j-3}\left(\frac{64j+1}{j}\right)^{\frac{-1}{2}}$ and $u(x_j) = \sqrt{x_j}$. Let endowments of agents be

$$\omega^i = \begin{cases} 16\left(\frac{64j+1}{j}\right) & \text{if } j = i \\ 0 & \text{otherwise,} \end{cases}$$

where $\Omega = \left(16\left(\frac{64j+1}{j}\right)\right)_{j\in\mathbb{N}}$. Our candidate for equilibrium allocation is

$$x^i = \left(\left(\frac{1}{2^i}\right) 16\left(\frac{64j+1}{j}\right)\right)_{j\in\mathbb{N}}.$$

For the **third condition** to be achieved and, with that, finding the equilibrium, **prices, up to a scalar multiple, belong to the supergradient, i.e.** $\lambda^i\pi \in \partial U^i$. The equilibrium prices for this economy, for $y \in \ell_\infty^+$, are

$$\begin{aligned}\pi(y) &= \sum_{j=1}^{\infty}\left(\frac{1}{2}\right)^{j-3}\left(\frac{64j+1}{j}\right)^{\frac{-1}{2}}\left(\frac{1}{2}\right)\left(16\left(\frac{1}{2}\right)\left(\frac{64j+1}{j}\right)\right)^{\frac{-1}{2}} y_j \\ &+ \beta^i\left(\frac{1}{2}\right)\left(\frac{1024}{2}\right)^{\frac{-1}{2}} \mathrm{LIM}\left(\phi\left(x^i\right)\right) \\ &= 4\left(\frac{1}{\sqrt{8}}\right)\sum_{j=1}^{\infty}\left(\frac{1}{2}\right)^{j}\left(\frac{64j+1}{j}\right)^{-1} y_j \\ &+ \beta^i\left(\frac{1}{2\sqrt{512}}\right)\mathrm{LIM}\left(\phi\left(y\right)\right).\end{aligned}$$

Now, the λ's for all of the agents are

$$\lambda^i = \frac{\left(16\left(\frac{1}{2^i}\right)\right)^{\frac{-1}{2}} \sum_{j=1}^{\infty} \left(\frac{1}{2}\right)^{j-2} \left(\frac{64j+1}{j}\right)^{-1} y_j + \beta^i \left(\frac{1}{2}\right) \left(\frac{1024}{2^i}\right)^{\frac{-1}{2}} \text{LIM}\left(\phi(y)\right)}{\left(16\left(\frac{1}{2}\right)\right)^{\frac{-1}{2}} \sum_{j=1}^{\infty} \left(\frac{1}{2}\right)^{j-2} \left(\frac{64j+1}{j}\right)^{-1} y_j + \beta^i \left(\frac{1}{2}\right) \left(\frac{1024}{2}\right)^{\frac{-1}{2}} \text{LIM}\left(\phi(y)\right)}$$
$$= \left(\frac{2^i}{2}\right)^{\frac{1}{2}} \quad \forall i \in \mathbb{N},$$

where it is noticeable that λ^i is increasing on the i's. For the **second condition** to be fulfilled, i.e. the **A.D. budget constraint**, is the next we are going to tackle. For it, we need to check that $\pi\left(x^i\right) = \pi\left(\omega^i\right)$, so

$$\pi\left(x^i\right) = 4\left(\frac{1}{\sqrt{8}}\right) \sum_{j=1}^{\infty} \left(\frac{1}{2}\right)^j \left(\frac{64j+1}{j}\right)^{-1} \left(\left(\frac{1}{2^i}\right) 16 \left(\frac{64j+1}{j}\right)\right)$$
$$+ \beta^i \left(\frac{1}{2\sqrt{512}}\right) \left(\frac{1}{2^i}\right) (1024),$$
$$\pi\left(\omega^i\right) = 4\left(\frac{1}{\sqrt{8}}\right) \left(\frac{1}{2}\right)^i \left(\frac{64i+1}{i}\right)^{-1} \left(16\left(\frac{64i+1}{i}\right)\right) = 4\left(\frac{1}{\sqrt{8}}\right)\left(\frac{1}{2}\right)^i (16)$$
$$= \frac{32}{\sqrt{2}}\left(\frac{1}{2^i}\right).$$

Notice that $\text{LIM}\left(\phi\left(x^i\right)\right) = \left(\frac{1}{2^i}\right) 1024$ since it is the infimum of x^i. Another issue to notice is that the pure charge part of $\pi\left(\omega^i\right)$ vanished since its limit is 0 $\forall i \in \mathbb{N}$. Coming back to the A.D. budget constraint,

$$4\left(\frac{1}{\sqrt{8}}\right)\left(\frac{1}{2^i}\right)(16)\sum_{j=1}^{\infty}\left(\frac{1}{2}\right)^j + \beta^i\left(\frac{1024}{2\sqrt{512}}\right)\left(\frac{1}{2^i}\right) = \frac{32}{\sqrt{2}}\left(\frac{1}{2^i}\right)$$
$$\Longleftrightarrow$$
$$2\left(\frac{1}{\sqrt{8}}\right)\left(\frac{1}{2^i}\right)(16)\sum_{j=2}^{\infty}\left(\frac{1}{2}\right)^{j-1} + \beta^i\left(\frac{1024}{2\sqrt{512}}\right)\left(\frac{1}{2^i}\right) = \frac{32}{\sqrt{2}}\left(\frac{1}{2^i}\right),$$

yielding

$$\beta^i\left(\frac{512}{\sqrt{512}}\right) = \left(\frac{2}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right)(16)$$
$$\Rightarrow \beta^i = \frac{16\sqrt{512}}{512\sqrt{2}} = \frac{16(16)}{512} = \frac{1}{2}.$$

Thus, $\beta^i = \frac{1}{2}$ for the A.D. budget constraint to hold. Finally, for the **first condition**, i.e. **Market Clearing**, we need to check that

$$\sum_{i=1}^{\infty} \omega_j^i = \sum_{i=1}^{\infty} x_j^i \quad \forall j \in \mathbb{N}.$$

The previous equation translates into

$$16\left(\frac{64j+1}{j}\right) = \sum_{i=1}^{\infty}\left(\frac{1}{2^i}\right)16\left(\frac{64j+1}{j}\right) \quad \forall j \in \mathbb{N}$$
$$\Rightarrow 16\left(\frac{64j+1}{j}\right) = 16\left(\frac{64j+1}{j}\right) \quad \forall j \in \mathbb{N}.$$

Therefore, allocation $(x^i)_{i\in\mathbb{N}}$, along with $\beta^i = \frac{1}{2}\ \forall i \in \mathbb{N}$, and prices for $y \in \ell_\infty^+$

$$\pi(y) = 4\left(\frac{1}{\sqrt{8}}\right)\sum_{j=1}^{\infty}\left(\frac{1}{2}\right)^j\left(\frac{64j+1}{j}\right)^{-1} y_j$$
$$+ \beta^i\left(\frac{1}{2\sqrt{512}}\right)\mathrm{LIM}\left(\phi\left(y\right)\right),$$

are an equilibrium for the economy $\mathcal{E} = (\ell_\infty^+, U^i, \omega^i)_{i\in\mathbb{N}}$. Notice that the equilibrium attained in the example would not change if we took another generalized limit such as $\mathrm{LIM}\left(\varphi(y)\right)$ with $\varphi: \ \ell_\infty^+ \to \ell_\infty^+$ and

$$\varphi(y)_j = \frac{1}{j}\sum_{k=1}^{j} y_{2k}.$$

5.2 Building Coalitions

Let us explain how coalitions are built. We have two cases for the sequences: on the first one, we build a coalition leaving finitely many agents outside of it. On the second case, we are leaving countably many agents out of the coalition.

- In example 1, assume that agents in the tail are those withholding. Therefore the sequence $(e_l)_{l\in\mathbb{N}}$ takes the form

$$(e_1, e_2, e_3, e_4, \ldots) = (m, m+1, m+2, m+3, \ldots),$$

where agent $m \in \mathbb{N}$ is the first agent to withhold part of her endowments.

- In example 2, assume that agents indexed by an even number are those withholding. Therefore the sequence $(e_l)_{l\in\mathbb{N}}$ takes the form

$$(e_1, e_2, e_3, e_4, \ldots) = (2, 4, 6, 8, \ldots).$$

5.3 When a Tail of Agents is Withholding

5.3.1 Perturbed Equilibrium

Example 1. In the perturbed economy $\mathcal{E}_{W_k}$, agents indexed by $i > m + k - 2$ withhold part of their endowments, with $k \in \mathbb{N}$. Notice that the withholding coalition is $W_k = \{m+k-1, m+k, m+k+1, \ldots\}$. Now, we are getting the analysis of a whole other economy, which we are calling $\mathcal{E}_{W_k}$. Thus, assuming they withhold $12\left(\frac{64j+1}{j}\right)$ units of their endowment, the endowments of this economy are

$$\mathrm{w}^i = \begin{cases} 16\left(\frac{64j+1}{j}\right) & \text{if } j = i \text{ and } j \notin W_k, \\ 4\left(\frac{64j+1}{j}\right) & \text{if } j = i \text{ and } j \in W_k, \\ 0 & \text{otherwise.} \end{cases}$$

Our candidate to be an equilibrium allocation for $i \in \mathbb{N}$ is

$$z^i = \begin{cases} \left(\frac{1}{2^i}\right) 16\left(\frac{64j+1}{j}\right) & \text{if } j \notin W_k, \\ \left(\frac{1}{2^i}\right) 4\left(\frac{64j+1}{j}\right) & \text{if } j \in W_k. \end{cases}$$

Recall that, for the **third condition** to be achieved, **prices belong to the supergradient up to a scalar multiple, i.e.** $\lambda^i \pi \in \partial U^i$. In this example, we are going to use the generalized limit $\mathrm{LIM}\,(\phi(y))$ for the prices. Thus, the equilibrium prices for economy $\mathcal{E}_{W_k}$ become, for $y \in \ell_\infty^+$,

$$\pi_{W_k}(y) = \sum_{j \notin W_k} \left(\frac{1}{2}\right)^{j-3} \left(\frac{64j+1}{j}\right)^{\frac{-1}{2}} \left(\frac{1}{2}\right) \left(\left(\frac{1}{2}\right) 16 \left(\frac{64j+1}{j}\right)\right)^{\frac{-1}{2}} y_j$$
$$+ \sum_{j \in W_k} \left(\frac{1}{2}\right)^{j-3} \left(\frac{64j+1}{j}\right)^{\frac{-1}{2}} \left(\frac{1}{2}\right) \left(\left(\frac{1}{2}\right) 4 \left(\frac{64j+1}{j}\right)\right)^{\frac{-1}{2}} y_j$$
$$+ \beta^i \left(\frac{1}{2}\right) (2(64))^{\frac{-1}{2}} \operatorname{LIM}\left(\phi(y)\right),$$

yielding

$$\pi_{W_k}(y) = 4\left(\frac{1}{\sqrt{8}}\right) \sum_{j \notin W_k} \left(\frac{1}{2}\right)^{j} \left(\frac{64j+1}{j}\right)^{-1} y_j$$
$$+ 4\left(\frac{1}{\sqrt{2}}\right) \sum_{j \in W_k} \left(\frac{1}{2}\right)^{j} \left(\frac{64j+1}{j}\right)^{-1} y_j + \beta^i \left(\frac{1}{16\sqrt{2}}\right) \operatorname{LIM}\left(\phi(y)\right).$$

Now, the λ's for all of the agents are gotten by knowing that $\lambda^i \partial U^1 = \partial U^i$. Thus,

$$\lambda^i \left(4\left(\frac{1}{\sqrt{8}}\right) \sum_{j \notin W_k} \left(\frac{1}{2}\right)^{j} \left(\frac{64j+1}{j}\right)^{-1} y_j + 4\left(\frac{1}{\sqrt{2}}\right) \sum_{j \in W_k} \left(\frac{1}{2}\right)^{j} \left(\frac{64j+1}{j}\right)^{-1} y_j \right.$$
$$\left. + \beta^i \left(\frac{1}{2}\right) \left(64\left(\frac{4}{2}\right)\right)^{\frac{-1}{2}} \operatorname{LIM}\left(\phi(y)\right)\right)$$
$$= 4\left(\left(\frac{1}{2^i}\right) 16\right)^{\frac{-1}{2}} \sum_{j \notin W_k} \left(\frac{1}{2}\right)^{j} \left(\frac{64j+1}{j}\right)^{-1} y_j +$$
$$4\left(\left(\frac{1}{2^i}\right) \frac{32}{8}\right)^{\frac{-1}{2}} \sum_{j \in W_k} \left(\frac{1}{2}\right)^{j} \left(\frac{64j+1}{j}\right)^{-1} y_j + \beta^i \left(\frac{1}{2}\right) \left(64\left(\frac{4}{2^i}\right)\right)^{\frac{-1}{2}} \operatorname{LIM}\left(\phi(y)\right),$$

yielding

$$\lambda^i \sqrt{2} \left(\sum_{j \notin W_k} \left(\frac{1}{2}\right)^{j} \left(\frac{64j+1}{j}\right)^{-1} y_j + 2 \sum_{j \in W_k} \left(\frac{1}{2}\right)^{j} \left(\frac{64j+1}{j}\right)^{-1} y_j \right.$$
$$\left. + \beta^i \left(\frac{1}{2}\right) (64(4))^{\frac{-1}{2}} \operatorname{LIM}\left(\phi(y)\right)\right)$$
$$= \left(\frac{1}{2^i}\right)^{\frac{-1}{2}} \sum_{j \notin W_k} \left(\frac{1}{2}\right)^{j} \left(\frac{64j+1}{j}\right)^{-1} y_j +$$
$$\left(\frac{1}{2^i}\right)^{\frac{-1}{2}} 2 \sum_{j \in W_k} \left(\frac{1}{2}\right)^{j} \left(\frac{64j+1}{j}\right)^{-1} y_j + \beta^i \left(\frac{1}{2^i}\right)^{\frac{-1}{2}} \left(\frac{1}{2}\right) (64(4))^{\frac{-1}{2}} \operatorname{LIM}\left(\phi(y)\right).$$

Thus,

$$\lambda^i = \left(\left(\frac{1}{2^i}\right)^{\frac{-1}{2}} \left(\sum_{j\notin W_k} \left(\frac{1}{2}\right)^j \left(\frac{64j+1}{j}\right)^{-1} y_j + 2\sum_{j\in W_k} \left(\frac{1}{2}\right)^j \left(\frac{64j+1}{j}\right)^{-1} y_j\right.\right.$$
$$\left.\left. + \beta^i \left(\frac{1}{2}\right) (64\,(4))^{\frac{-1}{2}} \, \mathrm{LIM}\left(\phi(y)\right)\right)\right)$$
$$\left(\sqrt{2}\left(\sum_{j\notin W_k} \left(\frac{1}{2}\right)^j \left(\frac{64j+1}{j}\right)^{-1} y_j + 2\sum_{j\in W_k} \left(\frac{1}{2}\right)^j \left(\frac{64j+1}{j}\right)^{-1} y_j\right.\right.$$
$$\left.\left. + \beta^i \left(\frac{1}{2}\right) (64\,(4))^{\frac{-1}{2}} \, \mathrm{LIM}\left(\phi(y)\right)\right)\right)^{-1}$$
$$= \left(\frac{2^i}{2}\right)^{\frac{1}{2}}.$$

Again, as in the original economy in subsection 5.1, it is noticeable that λ^i is increasing on the i's. For the **second condition** to be fulfilled, i.e. the **A.D. budget constraint**, first we need to use the same β^i that we found to be applicable in the original economy. Second, we need to check that $\pi_{_{W_k}}\left(z^i\right) = \pi_{_{W_k}}\left(\mathrm{w}^i\right)$, for two cases of agents: Those withholding, i.e. $i \in W_k$, and those which are not withholding, i.e. $i \notin W_k$. For those $i \notin W_k$, we have

$$\pi_{_{W_k}}\left(z^i\right) = 4\left(\frac{1}{\sqrt{8}}\right)\sum_{j\notin W_k}\left(\frac{1}{2}\right)^j\left(\frac{64j+1}{j}\right)^{-1}\left(\left(\frac{1}{2^i}\right)16\left(\frac{64j+1}{j}\right)\right)$$
$$+ 4\left(\frac{1}{\sqrt{2}}\right)\sum_{j\in W_k}\left(\frac{1}{2}\right)^j\left(\frac{64j+1}{j}\right)^{-1}\left(\left(\frac{1}{2^i}\right)4\left(\frac{64j+1}{j}\right)\right)$$
$$+ \beta^i\left(\frac{1}{16\sqrt{2}}\right)\left(\left(\frac{1}{2^i}\right)4(64)\right)$$
$$= 4\left(\frac{1}{\sqrt{8}}\right)\left(\left(\frac{1}{2^i}\right)16\right)\sum_{j\notin W_k}\left(\frac{1}{2}\right)^j$$
$$+ 4\left(\frac{1}{\sqrt{2}}\right)\left(\left(\frac{1}{2^i}\right)4\right)\sum_{j\in W_k}\left(\frac{1}{2}\right)^j + \beta^i\left(\frac{1}{16\sqrt{2}}\right)\left(\frac{1}{2^i}\right)(4(64))$$
$$= \frac{32}{\sqrt{2}}\left(\frac{1}{2^i}\right)\sum_{j\notin W_k}\left(\frac{1}{2}\right)^j + \frac{16}{\sqrt{2}}\left(\frac{1}{2^i}\right)\sum_{j\in W_k}\left(\frac{1}{2}\right)^j + \frac{16}{\sqrt{2}}\left(\frac{1}{2^i}\right)\beta^i,$$
$$\pi_{_{W_k}}\left(\mathrm{w}^i\right) = 4\left(\frac{1}{\sqrt{8}}\right)\left(\frac{1}{2}\right)^i\left(\frac{64i+1}{i}\right)^{-1}\left(16\left(\frac{64i+1}{i}\right)\right) = \frac{32}{\sqrt{2}}\left(\frac{1}{2}\right)^i.$$

Notice that $\text{LIM}\left(\phi\left(z^i\right)\right) = \left(\frac{1}{2^i}\right)4(64)$ since it is the infimum of z^i. Another issue to notice is that the pure charge part of $\pi_{_{W_k}}\left(\mathrm{w}^i\right)$ vanished since its limit is 0 $\forall i \in \mathbb{N}$. Coming back to the A.D. budget constraint,

$$\frac{32}{\sqrt{2}}\left(\frac{1}{2^i}\right)\sum_{j\notin W_k}\left(\frac{1}{2}\right)^j + \frac{16}{\sqrt{2}}\left(\frac{1}{2^i}\right)\sum_{j\in W_k}\left(\frac{1}{2}\right)^j + \frac{16}{\sqrt{2}}\left(\frac{1}{2^i}\right)\beta^i = \frac{32}{\sqrt{2}}\left(\frac{1}{2^i}\right)$$

$$\Longleftrightarrow$$

$$16\beta^i = 32 - 32\sum_{j=1}^{m+k-2}\left(\frac{1}{2}\right)^j - 16\sum_{j=m+k-1}^{\infty}\left(\frac{1}{2}\right)^j,$$

yielding

$$\beta^i = 2 - 2\sum_{j=1}^{m+k-2}\left(\frac{1}{2}\right)^j - 2\sum_{j=m+k-1}^{\infty}\left(\frac{1}{2}\right)^{j+1}.$$

Now, adding a proper zero to the last term of the RHS, i.e. $-\sum_{j=1}^{\infty}\left(2^j\left(\frac{1}{2}\right)^{m+k+j-2} - 2^{j+1}\left(\frac{1}{2}\right)^{m+k+j-1}\right)$, we get

$$\begin{aligned}
&-\sum_{j=1}^{\infty}\left(2^j\left(\frac{1}{2}\right)^{m+k+j-2} - 2^{j+1}\left(\frac{1}{2}\right)^{m+k+j-1}\right) - 2\left(\frac{1}{2}\right)^{m+k-1} - 2\left(\frac{1}{2}\right)^{m+k} \\
&- \ldots = -\left(2\left(\frac{1}{2}\right)^{m+k-1} - 4\left(\frac{1}{2}\right)^{m+k}\right) - \left(4\left(\frac{1}{2}\right)^{m+k} - 8\left(\frac{1}{2}\right)^{m+k+1}\right) \\
&- \ldots - 2\left(\frac{1}{2}\right)^{m+k-1} - 2\left(\frac{1}{2}\right)^{m+k} - 2\left(\frac{1}{2}\right)^{m+k+1} - 2\left(\frac{1}{2}\right)^{m+k+2} - \ldots \\
=& -2\left(\frac{1}{2}\right)^{m+k-1} + \left(4\left(\frac{1}{2}\right)^{m+k} - 4\left(\frac{1}{2}\right)^{m+k}\right) - 2\left(\frac{1}{2}\right)^{m+k} \\
&+ \left(8\left(\frac{1}{2}\right)^{m+k+1} - 8\left(\frac{1}{2}\right)^{m+k+1}\right) - 2\left(\frac{1}{2}\right)^{m+k+1} + \ldots \\
=& -2\left(\frac{1}{2}\right)^{m+k-1} - 2\left(\frac{1}{2}\right)^{m+k} - 2\left(\frac{1}{2}\right)^{m+k+1} - \ldots = -2\sum_{j=m+k-1}^{\infty}\left(\frac{1}{2}\right)^j.
\end{aligned}$$

Thus,

$$\beta^i = 2 - 2\sum_{j=1}^{m+k-2}\left(\frac{1}{2}\right)^j - 2\sum_{j=m+k-1}^{\infty}\left(\frac{1}{2}\right)^{j+1}$$
$$- \sum_{j=1}^{\infty}\left(2^j\left(\frac{1}{2}\right)^{m+k+j-2} - 2^{j+1}\left(\frac{1}{2}\right)^{m+k+j-1}\right)$$
$$= 2 - 2\sum_{j=1}^{\infty}\left(\frac{1}{2}\right)^j = 1 - 2\sum_{j=2}^{\infty}\left(\frac{1}{2}\right)^j.$$

Then, again, we need to add a proper zero, i.e. $\sum_{j=0}^{\infty}\left(2^j\left(\frac{1}{2}\right)^{j+1} - 2^{j+1}\left(\frac{1}{2}\right)^{j+2}\right)$. So

$$\beta^i = 1 - 2\sum_{j=2}^{\infty}\left(\frac{1}{2}\right)^j + \sum_{j=0}^{\infty}\left(2^j\left(\frac{1}{2}\right)^{j+1} - 2^{j+1}\left(\frac{1}{2}\right)^{j+2}\right)$$
$$= 1 - \frac{1}{2} - \frac{1}{4} - \frac{1}{8} - \ldots + \left(\frac{1}{2} - \frac{2}{4}\right) + \left(\frac{2}{4} - \frac{4}{8}\right) + \left(\frac{4}{8} - \frac{8}{16}\right) + \ldots$$
$$= 1 + \left(\frac{1}{2} - \frac{1}{2}\right) - \frac{1}{4} + \left(\frac{2}{4} - \frac{2}{4}\right) - \frac{1}{8} + \left(\frac{4}{8} - \frac{4}{8}\right) - \frac{1}{16} + \ldots$$
$$= 1 - \frac{1}{4} - \frac{1}{8} - \frac{1}{16} - \ldots = 1 - \sum_{j=1}^{\infty}\left(\frac{1}{2}\right)^{j+1} = \frac{1}{2}.$$

Working on the A.D. budget constraint for agents $i \in W_k$, we have that

$$\pi_{W_k}\left(\mathrm{w}^i\right) = 4\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)^i\left(\frac{64i+1}{i}\right)^{-1}\left(4\left(\frac{64i+1}{i}\right)\right) = \frac{16}{\sqrt{2}}\left(\frac{1}{2}\right)^i.$$

Thus, $\pi_{W_k}\left(z^i\right) = \pi_{W_k}\left(\mathrm{w}^i\right)$ turns into

$$\frac{32}{\sqrt{2}}\left(\frac{1}{2^i}\right)\sum_{j\notin W_k}\left(\frac{1}{2}\right)^j + \frac{16}{\sqrt{2}}\left(\frac{1}{2^i}\right)\sum_{j\in W_k}\left(\frac{1}{2}\right)^j + \frac{16}{\sqrt{2}}\left(\frac{1}{2^i}\right)\beta^i = \frac{16}{\sqrt{2}}\left(\frac{1}{2^i}\right)$$
$$\Longleftrightarrow$$
$$16\beta^i = 16 - 32\sum_{j=1}^{m+k-2}\left(\frac{1}{2}\right)^j - 16\sum_{j=m+k-1}^{\infty}\left(\frac{1}{2}\right)^j,$$

pushing us towards

$$\beta^i = 1 - 2\sum_{j=1}^{m+k-2}\left(\frac{1}{2}\right)^j - 2\sum_{j=m+k-1}^{\infty}\left(\frac{1}{2}\right)^{j+1}.$$

Now, adding a proper zero to the last term of the RHS, i.e. $-\sum_{j=1}^{\infty}\left(2^j\left(\frac{1}{2}\right)^{m+k+j-2}-2^{j+1}\left(\frac{1}{2}\right)^{m+k+j-1}\right)$, we get

$$\begin{aligned}\beta^i &= 1-2\sum_{j=1}^{m+k-2}\left(\frac{1}{2}\right)^j-2\sum_{j=m+k-1}^{\infty}\left(\frac{1}{2}\right)^{j+1}\\ &-\sum_{j=1}^{\infty}\left(2^j\left(\frac{1}{2}\right)^{m+k+j-2}-2^{j+1}\left(\frac{1}{2}\right)^{m+k+j-1}\right)\\ &=1-2\sum_{j=1}^{\infty}\left(\frac{1}{2}\right)^j.\end{aligned}$$

Then, again, we need to add a proper zero, i.e. $\sum_{j=0}^{\infty}\left(2^j\left(\frac{1}{2}\right)^j-2^{j+1}\left(\frac{1}{2}\right)^{j+1}\right)$. So

$$\begin{aligned}\beta_i &= 1-2\sum_{j=1}^{\infty}\left(\frac{1}{2}\right)^j+\sum_{j=0}^{\infty}\left(2^j\left(\frac{1}{2}\right)^j-2^{j+1}\left(\frac{1}{2}\right)^{j+1}\right)\\ &=1-1-\frac{1}{2}-\frac{1}{4}-\ldots+\left(1-\frac{2}{2}\right)+\left(\frac{2}{2}-\frac{4}{4}\right)+\left(\frac{4}{4}-\frac{8}{8}\right)+\ldots\\ &=1+\left(\frac{2}{2}-\frac{2}{2}\right)-\frac{1}{2}+\left(\frac{4}{4}-\frac{4}{4}\right)-\frac{1}{4}+\left(\frac{8}{8}-\frac{8}{8}\right)-\frac{1}{8}+\ldots\\ &=1-\frac{1}{2}-\frac{1}{4}-\frac{1}{8}-\ldots=1-\sum_{j=1}^{\infty}\left(\frac{1}{2}\right)^j.\end{aligned}$$

Finally, adding another appropriate zero, i.e. $\sum_{j=0}^{\infty}\left(2^j\left(\frac{1}{2}\right)^{j+1}-2^{j+1}\left(\frac{1}{2}\right)^{j+2}\right)$, we get

$$\begin{aligned}\beta_i &= 1-\sum_{j=1}^{\infty}\left(\frac{1}{2}\right)^j+\sum_{j=0}^{\infty}\left(2^j\left(\frac{1}{2}\right)^{j+1}-2^{j+1}\left(\frac{1}{2}\right)^{j+2}\right)\\ &=1-\sum_{j=1}^{\infty}\left(\frac{1}{2}\right)^{j+1}=\frac{1}{2}.\end{aligned}$$

Therefore, we have showed that the second condition holds. Finally, for the **first condition**, i.e. **Market Clearing**, we need to check that

$$\sum_{i=1}^{\infty} z_j^i=\sum_{i=1}^{\infty}\mathrm{w}_j^i \quad \forall j\in\mathbb{N}.$$

Thus, for those $j\notin W_k$:

$$\sum_{i=1}^{\infty} z_j^i = \sum_{i=1}^{\infty} \mathrm{w}_j^i \quad \forall j \notin W_k$$

$$\Rightarrow \sum_{i=1}^{\infty} \left(\frac{1}{2^i}\right) 16 \left(\frac{64j+1}{j}\right) = 16 \left(\frac{64j+1}{j}\right) \quad \forall j \notin W_k$$

$$\Rightarrow 16 \left(\frac{64j+1}{j}\right) = 16 \left(\frac{64j+1}{j}\right) \quad \forall j \notin W_k.$$

For those $j \in W_k$:

$$\sum_{i=1}^{\infty} z_j^i = \sum_{i=1}^{\infty} \mathrm{w}_j^i \quad \forall j \in W_k$$

$$\Rightarrow \sum_{i=1}^{\infty} \left(\frac{1}{2^i}\right) 4 \left(\frac{64j+1}{j}\right) = 16 \left(\frac{64j+1}{j}\right) - 12 \left(\frac{64j+1}{j}\right) \quad \forall j \in W_k$$

$$\Rightarrow 4 \left(\frac{64j+1}{j}\right) = 4 \left(\frac{64j+1}{j}\right) \quad \forall j \in W_k.$$

Therefore, allocation $(z^i)_{i\in\mathbb{N}}$, along with $\beta^i = \frac{1}{2}\ \forall i \in \mathbb{N}$, and prices

$$\pi_{W_k}(y) = 4\left(\frac{1}{\sqrt{8}}\right) \sum_{j \notin W_k} \left(\frac{1}{2}\right)^j \left(\frac{64j+1}{j}\right)^{-1} y_j + 4\left(\frac{1}{\sqrt{2}}\right) \sum_{j \in W_k} \left(\frac{1}{2}\right)^j \left(\frac{64j+1}{j}\right)^{-1} y_j + \beta^i \left(\frac{1}{16\sqrt{2}}\right) \mathrm{LIM}\left(\phi(y)\right).$$

are an equilibrium for the economy $\mathcal{E}_{W_k} = (\ell_\infty^+, U^i, \mathrm{w}^i)_{i\in\mathbb{N}}$.

5.3.2 Price Impact

When shrinking down to disappearance the withholding coalition, $\lim_{k\to\infty} \mathcal{E}_{W_k}$ prices are, for $y \in \ell_\infty^+$,

$$\lim_{k\to\infty} \pi_{W_k}(y) = 4\left(\frac{1}{\sqrt{8}}\right) \sum_{j=1}^{\infty} \left(\frac{1}{2}\right)^j \left(\frac{64j+1}{j}\right)^{-1} y_j + \beta^i \left(\frac{1}{16\sqrt{2}}\right) \mathrm{LIM}\left(\phi(y)\right) = \pi_{W_\infty}.$$

Notice that, if there was no price impact, $\pi_{W_\infty}(y) - \pi(y) = 0$, where $\pi(y)$ are the prices in subsection 5.1. However,

$$\pi_{W_\infty}(y) - \pi(y) = \beta^i \left(\frac{1}{16\sqrt{2}} - \frac{1}{2\sqrt{512}} \right) \text{LIM}\left(\phi\left(y\right)\right)$$
$$= \beta^i \left(\frac{1}{32\sqrt{2}} \right) \text{LIM}\left(\phi\left(y\right)\right).$$

The norm $\beta^i u'(\underline{x})$ of the pure charge is larger for the limiting price $\lim_{k\to\infty} \pi_{W_k}(y)$ than for the price $\pi(y)$ of the unperturbed economy, since marginal utility u' is decreasing and the infimum $\underline{x}$ is (persistently) lower in the perturbed economies. This is the reason why $\lim_{k\to\infty} \pi_{W_k}(y)$ does not converge to $\pi(y)$ in the norm topology of $ba\left(2^{\mathbb{N}}\right)$.

5.3.3 Incentive Compatibility

Recall that utility of agent $m+k-1 \in \mathbb{N}$, for $y \in \ell_\infty^+$, is given by

$$U^{m+k-1}(y) = \sum_{j=1}^{\infty} \left(\frac{1}{2}\right)^{j-3} \left(\frac{64j+1}{j}\right)^{\frac{-1}{2}} u(y_j) + \beta^{m+k-1} \inf_j \left(\frac{1}{j} \sum_{h=1}^{j} u(x_h) \right).$$

So, calculating the indirect utility for agent $m+k-1 \in \mathbb{N}$ by consuming the equilibrium bundle allocated to her within economy $\mathcal{E}$, we have

$$\begin{aligned}
U^{m+k-1}\left(x^{m+k-1}\right) &= \sum_{j=1}^{\infty} \left(\frac{1}{2}\right)^{j-3} \left(\frac{64j+1}{j}\right)^{\frac{-1}{2}} u\left(x_j^{m+k-1}\right) \\
&\quad + \beta^{m+k-1} \inf_j \left(\frac{1}{j} \sum_{h=1}^{j} u\left(x_h^{m+k-1}\right) \right) \\
&= \sum_{j=1}^{\infty} \left(\frac{1}{2}\right)^{j-3} \left(\frac{64j+1}{j}\right)^{\frac{-1}{2}} \sqrt{\left(\frac{1}{2^{m+k-1}}\right) 16 \left(\frac{64j+1}{j}\right)} \\
&\quad + \beta^{m+k-1} \inf_j \left(\frac{1}{j} \sum_{h=1}^{j} \sqrt{\left(\frac{1}{2^{m+k-1}}\right) 16 \left(\frac{64h+1}{h}\right)} \right) \\
&= \sqrt{\left(\frac{1}{2}\right)^{m+k-1}} 8(4) \sum_{j=1}^{\infty} \left(\frac{1}{2}\right)^{j} + \beta^{m+k-1} \sqrt{\left(\frac{1}{2}\right)^{m+k-1}} (4(8)) \\
&= \sqrt{\left(\frac{1}{2}\right)^{m+k-1}} 8(4) \left(1 + \frac{1}{2}\right) = \sqrt{\left(\frac{1}{2}\right)^{m+k-1}} 8(6) = 48\sqrt{\left(\frac{1}{2}\right)^{m+k-1}}
\end{aligned}$$

.

Notice that agent $m+k-1 \in W_k$, which is the "*first agent*" to withhold within the withholding coalition inside of the perturbed economy $\mathcal{E}_{W_k}$, will have in her possession the next bundle, which is a sequence whose terms are given by

$$q_j^{m+k-1} = \begin{cases} \left(\frac{1}{2^{m+k-1}}\right) 16 \left(\frac{64j+1}{j}\right) & \text{if } j \notin W_k, \\ \left(\frac{1}{2^{m+k-1}}\right) 4 \left(\frac{64j+1}{j}\right) + 12 \left(\frac{64j+1}{j}\right) & \text{if } j = m+k-1 \in W_k, \\ \left(\frac{1}{2^{m+k-1}}\right) 4 \left(\frac{64j+1}{j}\right) & \text{if } j \neq m+k-1 \in W_k. \end{cases}$$

So, calculating the indirect utility of agent $m+k-1 \in \mathbb{N}$ by consuming $\left(q_j^{m+k-1}\right)_{j\in\mathbb{N}}$, we have

$$\begin{aligned} U^{m+k-1}\left(q^{m+k-1}\right) &= \sum_{j=1}^{\infty} \left(\frac{1}{2}\right)^{j-3} \left(\frac{64j+1}{j}\right)^{\frac{-1}{2}} u\left(q_j^{m+k-1}\right) \\ &+ \beta^{m+k-1} \inf_j \left(\frac{1}{j}\sum_{h=1}^{j} u\left(q_h^{m+k-1}\right)\right) \\ &= \sum_{j=1}^{m+k-2} \left(\frac{1}{2}\right)^{j-3} \left(\frac{64j+1}{j}\right)^{\frac{-1}{2}} \sqrt{\left(\frac{1}{2^{m+k-1}}\right) 16 \left(\frac{64j+1}{j}\right)} \\ &+ \left(\frac{1}{2}\right)^{m+k-1-3} \left(\frac{64(m+k-1)+1}{m+k-1}\right)^{\frac{-1}{2}} \\ &\sqrt{\left(\frac{1}{2^{m+k-1}}\right) 4 \left(\frac{64(m+k-1)+1}{m+k-1}\right) + 12 \left(\frac{64(m+k-1)+1}{m+k-1}\right)} \\ &+ \sum_{j=m+k}^{\infty} \left(\frac{1}{2}\right)^{j-3} \left(\frac{64j+1}{j}\right)^{\frac{-1}{2}} \sqrt{\left(\frac{1}{2^{m+k-1}}\right) 4 \left(\frac{64j+1}{j}\right)} \\ &+ \left(\frac{1}{2}\right) \inf_j \left(\frac{1}{j}\sum_{h=1}^{j} u\left(q_h^{m+k-1}\right)\right) \\ &= 8\sqrt{\left(\frac{1}{2^{m+k-1}}\right)(4)} \sum_{j=1}^{m+k-2} \left(\frac{1}{2}\right)^{j} + 8\left(\frac{1}{2}\right)^{m+k-1} \sqrt{\left(\frac{1}{2}\right)^{m+k-1} 4 + 12} \\ &+ 8\sqrt{\left(\frac{1}{2^{m+k-1}}\right)(2)} \sum_{j=m+k}^{\infty} \left(\frac{1}{2}\right)^{j} + \left(\frac{1}{2}\right) \sqrt{\left(\frac{1}{2^{m+k-1}}\right)(2(8))} \end{aligned}$$

$$= 8(2)\sqrt{\left(\frac{1}{2^{m+k-1}}\right)}\left(\sum_{j=1}^{m+k-2}\left(\frac{1}{2}\right)^{j+1} + \sum_{j=m+k}^{\infty}\left(\frac{1}{2}\right)^{j} + \frac{1}{2}\right)$$
$$+ 8\left(\frac{1}{2}\right)^{m+k-1}\sqrt{\left(\frac{1}{2}\right)^{m+k-1}4 + 12}$$
$$= 8(2)\sqrt{\left(\frac{1}{2^{m+k-1}}\right)}\left(\sum_{j=1}^{\infty}\left(\frac{1}{2}\right)^{j}\right) + 8\left(\frac{1}{2}\right)^{m+k-1}\sqrt{\left(\frac{1}{2}\right)^{m+k-1}4 + 12}.$$

We can add two adequate zeros in brackets on the first summand of the RHS, i.e., adding $a = \sum_{j=1}^{\infty} 2^{j-1}\left(\frac{1}{2}\right)^{j-1} - 2^{j}\left(\frac{1}{2}\right)^{j}$ and $b = \sum_{j=1}^{\infty} 2^{j}\left(\frac{1}{2}\right)^{j-1} - 2^{j+1}\left(\frac{1}{2}\right)^{j}$ to the term $\sum_{j=1}^{\infty}\left(\frac{1}{2}\right)^{j}$. This is

$$U^{m+k-1}\left(q^{m+k-1}\right) = 8(2)\sqrt{\left(\frac{1}{2^{m+k-1}}\right)}\left(\sum_{j=1}^{\infty}\left(\frac{1}{2}\right)^{j}\right) + 8\left(\frac{1}{2}\right)^{m+k-1}\sqrt{\left(\frac{1}{2}\right)^{m+k-1}4 + 12}$$
$$= 8(2)\sqrt{\left(\frac{1}{2^{m+k-1}}\right)}\left(\sum_{j=1}^{\infty}\left(\frac{1}{2}\right)^{j} + a + b\right)$$
$$+ 8\left(\frac{1}{2}\right)^{m+k-1}\sqrt{\left(\frac{1}{2}\right)^{m+k-1}4 + 12}$$
$$= 8(2)\sqrt{\left(\frac{1}{2^{m+k-1}}\right)}\left(\sum_{j=1}^{\infty}\left(\frac{1}{2}\right)^{j-2}\right) + 8\left(\frac{1}{2}\right)^{m+k-1}\sqrt{\left(\frac{1}{2}\right)^{m+k-1}4 + 12}$$
$$= 8(2)\sqrt{\left(\frac{1}{2^{m+k-1}}\right)}(4) + 8\left(\frac{1}{2}\right)^{m+k-1}\sqrt{\left(\frac{1}{2}\right)^{m+k-1}4 + 12}$$
$$= 8(8)\sqrt{\left(\frac{1}{2^{m+k-1}}\right)} + 8\left(\frac{1}{2}\right)^{m+k-1}\sqrt{\left(\frac{1}{2}\right)^{m+k-1}4 + 12}.$$

It is now evident that $U^{m+k-1}\left(q^{m+k-1}\right) > U^{m+k-1}\left(x^{m+k-1}\right)$, generating enough incentives for agents in W_k to withhold 75% of their endowments and consume them.

5.4 When Both Withholders and Non-withholders are Infinitely Many

Now, agents indexed by even numbers are withholding part of their endowments. Even though there are commodities indexed by odd numbers that are as close as possible to those being withheld, there is still inadequate substitutability to allow the withholding coalition to have market power no matter its size. This effect persists in the limit.

Example 2. Let us now assume that agents $i \in E = \{2n : n \in \mathbb{N}\}$ withhold some of their endowments, while agents $i \in O = \{2n+1 : n \in \mathbb{N}\}$ do not. Notice that the withholding coalition is $W_k = \{2k, 2(k+1), 2(k+2), \ldots\}$. We are calling $\mathcal{E}_{W_k}$ to this new economy. Thus, assuming they withhold $15\left(\frac{64j+1}{j}\right)$ units of them, the endowments of this economy are

$$\mathrm{w}^i = \begin{cases} 16\left(\frac{64j+1}{j}\right) & \text{if } j = i \text{ and } j \notin W_k, \\ 1\left(\frac{64j+1}{j}\right) & \text{if } j = i \text{ and } j \in W_k, \\ 0 & \text{otherwise.} \end{cases}$$

Our candidate to be an equilibrium allocation for $i \in \mathbb{N}$ is

$$z^i = \begin{cases} \left(\frac{1}{2^i}\right) 16\left(\frac{64j+1}{j}\right) & \text{if } j \notin W_k, \\ \left(\frac{1}{2^i}\right) 1\left(\frac{64j+1}{j}\right) & \text{if } j \in W_k. \end{cases}$$

Recall that, for the **third condition** to be achieved, **prices belong to the supergradient up to a scalar multiple, i.e.** $\lambda^i \pi \in \partial U^i$. In this example, we are going to use the generalized limit $\mathrm{LIM}(\varphi(y))$ for the prices. Thus, the equilibrium prices for economy $\mathcal{E}_{W_k}$ become

$$\begin{aligned} \pi_{W_k}(y) = & \sum_{j \notin W_k} \left(\frac{1}{2}\right)^{j-3} \left(\frac{64j+1}{j}\right)^{\frac{-1}{2}} \left(\frac{1}{2}\right) \left(\left(\frac{1}{2}\right) 16 \left(\frac{64j+1}{j}\right)\right)^{\frac{-1}{2}} y_j \\ & + \sum_{j \in W_k} \left(\frac{1}{2}\right)^{j-3} \left(\frac{64j+1}{j}\right)^{\frac{-1}{2}} \left(\frac{1}{2}\right) \left(\left(\frac{1}{2}\right) 1 \left(\frac{64j+1}{j}\right)\right)^{\frac{-1}{2}} y_j \\ & + \beta^i \left(\frac{1}{2}\right) \left(\frac{64}{2}\right)^{\frac{-1}{2}} \mathrm{LIM}\left(\varphi(y)\right), \end{aligned}$$

yielding

$$\pi_{W_k}(y) = 4\left(\frac{1}{\sqrt{8}}\right)\sum_{j\notin W_k}\left(\frac{1}{2}\right)^j\left(\frac{64j+1}{j}\right)^{-1}y_j$$
$$+ 4\sqrt{2}\sum_{j\in W_k}\left(\frac{1}{2}\right)^j\left(\frac{64j+1}{j}\right)^{-1}y_j + \beta^i\left(\frac{1}{8\sqrt{2}}\right)\mathrm{LIM}\left(\varphi(y)\right).$$

Now, the λ's for all of the agents are gotten by knowing that $\lambda^i\partial U^1 = \partial U^i$. Thus,

$$\lambda^i\left(\sqrt{2}\sum_{j\notin W_k}\left(\frac{1}{2}\right)^j\left(\frac{64j+1}{j}\right)^{-1}y_j + 4\sqrt{2}\sum_{j\in W_k}\left(\frac{1}{2}\right)^j\left(\frac{64j+1}{j}\right)^{-1}y_j\right.$$
$$\left. + \beta^i\left(\frac{1}{8\sqrt{2}}\right)\mathrm{LIM}\left(\varphi(y)\right)\right)$$
$$= \left(\frac{1}{2^i}\right)^{\frac{-1}{2}}\sum_{j\notin W_k}\left(\frac{1}{2}\right)^j\left(\frac{64j+1}{j}\right)^{-1}y_j +$$
$$4\left(\frac{1}{2^i}\right)^{\frac{-1}{2}}\sum_{j\in W_k}\left(\frac{1}{2}\right)^j\left(\frac{64j+1}{j}\right)^{-1}y_j + \beta^i\left(\frac{\sqrt{2^i}}{16}\right)\mathrm{LIM}\left(\varphi(y)\right),$$

yielding

$$\lambda^i\sqrt{2}\left(\sum_{j\notin W_k}\left(\frac{1}{2}\right)^j\left(\frac{64j+1}{j}\right)^{-1}y_j + 4\sum_{j\in W_k}\left(\frac{1}{2}\right)^j\left(\frac{64j+1}{j}\right)^{-1}y_j\right.$$
$$\left. + \beta^i\left(\frac{1}{16}\right)\mathrm{LIM}\left(\varphi(y)\right)\right)$$
$$= \left(\frac{1}{2^i}\right)^{\frac{-1}{2}}\left(\sum_{j\notin W_k}\left(\frac{1}{2}\right)^j\left(\frac{64j+1}{j}\right)^{-1}y_j +\right.$$
$$\left. 4\sum_{j\in W_k}\left(\frac{1}{2}\right)^j\left(\frac{64j+1}{j}\right)^{-1}y_j + \beta^i\left(\frac{1}{16}\right)\mathrm{LIM}\left(\varphi(y)\right)\right).$$

Thus,

$$
\begin{aligned}
\lambda^i = &\left(\left(\frac{1}{2^i}\right)^{\frac{-1}{2}}\left(\sum_{j\notin W_k}\left(\frac{1}{2}\right)^j\left(\frac{64j+1}{j}\right)^{-1} y_j + 4\sum_{j\in W_k}\left(\frac{1}{2}\right)^j\left(\frac{64j+1}{j}\right)^{-1} y_j\right.\right.\\
&\left.\left. +\ \beta^i\left(\frac{1}{16}\right)\mathrm{LIM}\left(\varphi(y)\right)\right)\right)\\
&\left(\sqrt{2}\left(\sum_{j\notin W_k}\left(\frac{1}{2}\right)^j\left(\frac{64j+1}{j}\right)^{-1} y_j + 4\sum_{j\in W_k}\left(\frac{1}{2}\right)^j\left(\frac{64j+1}{j}\right)^{-1} y_j\right.\right.\\
&\left.\left. +\ +\beta^i\left(\frac{1}{16}\right)\mathrm{LIM}\left(\phi(y)\right)\right)\right)^{-1}\\
&= \left(\frac{2^i}{2}\right)^{\frac{1}{2}}.
\end{aligned}
$$

Again, as in the original economy in subsection 5.1, it is noticeable that λ^i is increasing on the i's. For the **second condition** to be fulfilled, i.e. the **A.D. budget constraint**, first we need to use the same β^i that we found to be applicable in the original economy. Second, we need to check that $\pi_{W_k}(z^i) = \pi_{W_k}(\mathrm{w}^i)$, for two cases of agents: those withholding, i.e. $i \in W_k$, and those which are not withholding, i.e. $i \notin W_k$. For those $i \notin W_k$, I have

$$
\begin{aligned}
\pi_{W_k}(z^i) = &\ 4\left(\frac{1}{\sqrt{8}}\right)\sum_{j\notin W_k}\left(\frac{1}{2}\right)^j\left(\frac{64j+1}{j}\right)^{-1}\left(\left(\frac{1}{2^i}\right)16\left(\frac{64j+1}{j}\right)\right)\\
&+4\sqrt{2}\sum_{j\in W_k}\left(\frac{1}{2}\right)^j\left(\frac{64j+1}{j}\right)^{-1}\left(\left(\frac{1}{2^i}\right)1\left(\frac{64j+1}{j}\right)\right)\\
&+\beta^i\left(\frac{1}{8\sqrt{2}}\right)\left(\left(\frac{1}{2^i}\right)64\right)\\
&=4\left(\frac{1}{\sqrt{8}}\right)\left(\left(\frac{1}{2^i}\right)16\right)\sum_{j\notin W_k}\left(\frac{1}{2}\right)^j\\
&+4\sqrt{2}\left(\left(\frac{1}{2^i}\right)1\right)\sum_{j\in W_k}\left(\frac{1}{2}\right)^j+\beta^i\left(\frac{1}{8\sqrt{2}}\right)\left(\left(\frac{1}{2^i}\right)64\right)\\
&=\frac{32}{\sqrt{2}}\left(\frac{1}{2^i}\right)\sum_{j\notin W_k}\left(\frac{1}{2}\right)^j+4\sqrt{2}\left(\frac{1}{2^i}\right)\sum_{j\in W_k}\left(\frac{1}{2}\right)^j+\left(4\sqrt{2}\right)\beta^i\left(\frac{1}{2^i}\right),\\
\pi_{W_k}(\mathrm{w}^i) = &\ 4\left(\frac{1}{\sqrt{8}}\right)\left(\frac{1}{2}\right)^i\left(\frac{64i+1}{i}\right)^{-1}\left(16\left(\frac{64i+1}{i}\right)\right)=\frac{32}{\sqrt{2}}\left(\frac{1}{2}\right)^i.
\end{aligned}
$$

Notice that $\mathrm{LIM}\left(\varphi\left(z^i\right)\right) = \left(\frac{1}{2^i}\right)(1)(64)$ since it is the infimum of the average (over the even indexes) of z^i. Another issue to notice is that the pure charge part of $\pi_{W_k}(\mathrm{w}^i)$ vanished since its limit is 0 $\forall i \in \mathbb{N}$. Coming back to the A.D. budget constraint,

$$\frac{32}{\sqrt{2}}\left(\frac{1}{2^i}\right)\sum_{j\notin W_k}\left(\frac{1}{2}\right)^j + 4\sqrt{2}\left(\frac{1}{2^i}\right)\sum_{j\in W_k}\left(\frac{1}{2}\right)^j + \left(4\sqrt{2}\right)\beta^i\left(\frac{1}{2^i}\right) = \frac{32}{\sqrt{2}}\left(\frac{1}{2^i}\right)$$

$$\Longleftrightarrow$$

$$4\sqrt{2}\beta^i = \frac{32}{\sqrt{2}} - \frac{32}{\sqrt{2}}\left(\sum_{j=1}^{\infty}\left(\frac{1}{2}\right)^{2j-1} + \sum_{j=1}^{k-1}\left(\frac{1}{2}\right)^{2j}\right) - \sum_{j=k}^{\infty}\left(\frac{1}{2}\right)^{2j},$$

yielding

$$\beta^i = 4 - 4\left(\sum_{j=1}^{\infty}\left(\frac{1}{2}\right)^{2j-1} + \sum_{j=1}^{k-1}\left(\frac{1}{2}\right)^{2j}\right) - 4\sum_{j=k}^{\infty}\left(\frac{1}{2}\right)^{2(j+1)}.$$

Now, adding a proper zero to the last term of the RHS, i.e. $-\sum_{j=1}^{\infty}\left(4^j\left(\frac{1}{2}\right)^{2(k+j-1)} - 4^{j+1}\left(\frac{1}{2}\right)^{2(k+j)}\right)$, I get

$$\begin{aligned}
&-\sum_{j=1}^{\infty}\left(4^j\left(\frac{1}{2}\right)^{2(k+j-1)} - 4^{j+1}\left(\frac{1}{2}\right)^{2(k+j)}\right) - 4\left(\frac{1}{2}\right)^{2(k+1)} - 4\left(\frac{1}{2}\right)^{2(k+2)} - \ldots \\
=& -\left(4\left(\frac{1}{2}\right)^{2k} - 4^2\left(\frac{1}{2}\right)^{2(k+1)}\right) - \left(4^2\left(\frac{1}{2}\right)^{2(k+1)} - 4^3\left(\frac{1}{2}\right)^{2(k+2)}\right) - \ldots \\
&- 4\left(\frac{1}{2}\right)^{2(k+1)} - 4\left(\frac{1}{2}\right)^{2(k+2)} - 4\left(\frac{1}{2}\right)^{2(k+3)} - 4\left(\frac{1}{2}\right)^{2(k+4)} - \ldots \\
=& -4\left(\frac{1}{2}\right)^{2k} + \left(4^2\left(\frac{1}{2}\right)^{2(k+1)} - 4^2\left(\frac{1}{2}\right)^{2(k+1)}\right) - 4\left(\frac{1}{2}\right)^{2(k+1)} \\
&+ \left(4^3\left(\frac{1}{2}\right)^{2(k+2)} - 4^3\left(\frac{1}{2}\right)^{2(k+2)}\right) - 4\left(\frac{1}{2}\right)^{2(k+2)} + \ldots \\
=& -4\left(\frac{1}{2}\right)^{2k} - 4\left(\frac{1}{2}\right)^{2(k+1)} - 4\left(\frac{1}{2}\right)^{2(k+2)} - \ldots = -4\sum_{j=k}^{\infty}\left(\frac{1}{2}\right)^{2j}.
\end{aligned}$$

Thus,

$$
\begin{aligned}
\beta^i &= 4 - 4\left(\sum_{j=1}^{\infty}\left(\frac{1}{2}\right)^{2j-1} + \sum_{j=1}^{k-1}\left(\frac{1}{2}\right)^{2j}\right) - 4\sum_{j=k}^{\infty}\left(\frac{1}{2}\right)^{2(j+1)} \\
&- \sum_{j=1}^{\infty}\left(4^j\left(\frac{1}{2}\right)^{2(k+j-1)} - 4^{j+1}\left(\frac{1}{2}\right)^{2(k+j)}\right) \\
&= 4 - 4\left(\sum_{j=1}^{\infty}\left(\frac{1}{2}\right)^{2j-1} + \sum_{j=1}^{k-1}\left(\frac{1}{2}\right)^{2j}\right) - 4\sum_{j=k}^{\infty}\left(\frac{1}{2}\right)^{2j}.
\end{aligned}
$$

Now, adding a proper zero, i.e. $3 - 3$, we arrive to

$$
\begin{aligned}
\beta^i &= 4 - 4\left(\sum_{j=1}^{\infty}\left(\frac{1}{2}\right)^{2j-1} + \sum_{j=1}^{k-1}\left(\frac{1}{2}\right)^{2j}\right) - 4\sum_{j=k}^{\infty}\left(\frac{1}{2}\right)^{2j} + 3 - 3 \\
&= 1 - 4\left(\sum_{j=1}^{\infty}\left(\frac{1}{2}\right)^{2j-1} + \sum_{j=1}^{k-1}\left(\frac{1}{2}\right)^{2j}\right) - 4\sum_{j=k}^{\infty}\left(\frac{1}{2}\right)^{2j} + 3 \\
&= 1 - 4\left(\sum_{j=1}^{\infty}\left(\frac{1}{2}\right)^{2j-1} + \sum_{j=1}^{k-1}\left(\frac{1}{2}\right)^{2j}\right) - 4\sum_{j=k}^{\infty}\left(\frac{1}{2}\right)^{2j} + 3\sum_{j=1}^{\infty}\left(\frac{1}{2}\right)^{j},
\end{aligned}
$$

and since

$$
\sum_{j=1}^{\infty}\left(\frac{1}{2}\right)^{2j-1} + \sum_{j=1}^{k-1}\left(\frac{1}{2}\right)^{2j} + \sum_{j=k}^{\infty}\left(\frac{1}{2}\right)^{2j} = \sum_{j=1}^{\infty}\left(\frac{1}{2}\right)^{j},
$$

we get

$$
\beta^i = 1 - 4\sum_{j=1}^{\infty}\left(\frac{1}{2}\right)^{j} + 3\sum_{j=1}^{\infty}\left(\frac{1}{2}\right)^{j} = 1 - \sum_{j=1}^{\infty}\left(\frac{1}{2}\right)^{j}.
$$

Finally, adding another appropriate zero, i.e. $\sum_{j=0}^{\infty}\left(2^j\left(\frac{1}{2}\right)^{j+1} - 2^{j+1}\left(\frac{1}{2}\right)^{j+2}\right)$, I get

$$
\begin{aligned}
\beta^i &= 1 - \sum_{j=1}^{\infty}\left(\frac{1}{2}\right)^{j} + \sum_{j=0}^{\infty}\left(2^j\left(\frac{1}{2}\right)^{j+1} - 2^{j+1}\left(\frac{1}{2}\right)^{j+2}\right) \\
&= 1 - \sum_{j=1}^{\infty}\left(\frac{1}{2}\right)^{j+1} = \frac{1}{2}.
\end{aligned}
$$

Working on the A.D. budget constraint for agents $i \in W_k$, we have that

$$
\pi_{W_k}\left(\mathrm{w}^i\right) = 4\sqrt{2}\left(\frac{1}{2}\right)^{i}\left(\frac{64i+1}{i}\right)^{-1}\left(1\left(\frac{64i+1}{i}\right)\right) = 4\sqrt{2}\left(\frac{1}{2}\right)^{i}.
$$

Thus, $\pi_{_{W_k}}\left(z^i\right) = \pi_{_{W_k}}\left(\mathrm{w}^i\right)$ turns into

$$\frac{32}{\sqrt{2}}\left(\frac{1}{2^i}\right)\sum_{j\notin W_k}\left(\frac{1}{2}\right)^j + 4\sqrt{2}\left(\frac{1}{2^i}\right)\sum_{j\in W_k}\left(\frac{1}{2}\right)^j + 4\sqrt{2}\left(\frac{1}{2^i}\right)\beta^i = 4\sqrt{2}\left(\frac{1}{2}\right)^i$$

$$\Longleftrightarrow$$

$$4\sqrt{2}\beta^i = 4\sqrt{2} - \frac{32}{\sqrt{2}}\left(\sum_{j=1}^{\infty}\left(\frac{1}{2}\right)^{2j-1} + \sum_{j=1}^{k-1}\left(\frac{1}{2}\right)^{2j}\right) - 4\sqrt{2}\sum_{j=k}^{\infty}\left(\frac{1}{2}\right)^{2j},$$

pushing us towards

$$\beta^i = 1 - 4\left(\sum_{j=1}^{\infty}\left(\frac{1}{2}\right)^{2j-1} + \sum_{j=1}^{k-1}\left(\frac{1}{2}\right)^{2j}\right) - 4\sum_{j=k}^{\infty}\left(\frac{1}{2}\right)^{2(j+1)}.$$

Now, adding a proper zero to the last term of the RHS, i.e. $-\sum_{j=1}^{\infty}\left(4^j\left(\frac{1}{2}\right)^{2(k+j-1)} - 4^{j+1}\left(\frac{1}{2}\right)^{2(k+j)}\right)$, I get

$$\begin{aligned}\beta^i &= 1 - 4\left(\sum_{j=1}^{\infty}\left(\frac{1}{2}\right)^{2j-1} + \sum_{j=1}^{k-1}\left(\frac{1}{2}\right)^{2j}\right) - 4\sum_{j=k}^{\infty}\left(\frac{1}{2}\right)^{2j}\\ &= 1 - 4\sum_{j=1}^{\infty}\left(\frac{1}{2}\right)^{j}.\end{aligned}$$

Using the same technique as previously, we need to add three adequate zeros to the RHS of the last equation. These zeros are:

$$\begin{aligned}&-\sum_{j=1}^{\infty}\left(2^{j+1}\left(\frac{1}{2}\right)^{j} - 2^{j+2}\left(\frac{1}{2}\right)^{j+1}\right),\\ &-\sum_{j=1}^{\infty}\left(2^{j+1}\left(\frac{1}{2}\right)^{j+1} - 2^{j+2}\left(\frac{1}{2}\right)^{j+2}\right),\\ &-\sum_{j=1}^{\infty}\left(2^{j+1}\left(\frac{1}{2}\right)^{j+2} - 2^{j+2}\left(\frac{1}{2}\right)^{j+3}\right).\end{aligned}$$

Consequently,

$$
\begin{aligned}
\beta^i = 1 - 4\sum_{j=1}^{\infty}\left(\frac{1}{2}\right)^j &- \sum_{j=1}^{\infty}\left(2^{j+1}\left(\frac{1}{2}\right)^j - 2^{j+2}\left(\frac{1}{2}\right)^{j+1}\right)\\
&- \sum_{j=1}^{\infty}\left(2^{j+1}\left(\frac{1}{2}\right)^{j+1} - 2^{j+2}\left(\frac{1}{2}\right)^{j+2}\right)\\
&- \sum_{j=1}^{\infty}\left(2^{j+1}\left(\frac{1}{2}\right)^{j+2} - 2^{j+2}\left(\frac{1}{2}\right)^{j+3}\right)\\
&= 1 - 4\sum_{j=4}^{\infty}\left(\frac{1}{2}\right)^j = 1 - 4\left(\frac{1}{8}\right) = \frac{1}{2}.
\end{aligned}
$$

Therefore, we have showed that the second condition holds. Finally, for the **first condition**, i.e. **Market Clearing**, I need to check that

$$\sum_{i=1}^{\infty} z_j^i = \sum_{i=1}^{\infty} \mathrm{w}_j^i \quad \forall j \in \mathbb{N}.$$

Thus, for those $j \notin W_k$:

$$
\begin{aligned}
\sum_{i=1}^{\infty} z_j^i &= \sum_{i=1}^{\infty} \mathrm{w}_j^i \quad \forall j \notin W_k\\
\Rightarrow \sum_{i=1}^{\infty}\left(\frac{1}{2^i}\right)16\left(\frac{64j+1}{j}\right) &= 16\left(\frac{64j+1}{j}\right) \quad \forall j \notin W_k\\
\Rightarrow 16\left(\frac{64j+1}{j}\right) &= 16\left(\frac{64j+1}{j}\right) \quad \forall j \notin W_k.
\end{aligned}
$$

For those $j \in W_k$:

$$
\begin{aligned}
\sum_{i=1}^{\infty} z_j^i &= \sum_{i=1}^{\infty} \mathrm{w}_j^i \quad \forall j \in W_k\\
\Rightarrow \sum_{i=1}^{\infty}\left(\frac{1}{2^{i+3}}\right)8\left(\frac{64j+1}{j}\right) &= 16\left(\frac{64j+1}{j}\right) - 15\left(\frac{64j+1}{j}\right) \quad \forall j \in W_k\\
\Rightarrow 1\left(\frac{64j+1}{j}\right) &= 1\left(\frac{64j+1}{j}\right) \quad \forall j \in W_k.
\end{aligned}
$$

Therefore, allocation $(z^i)_{i\in\mathbb{N}}$, along with $\beta^i = \frac{1}{2}$ $\forall i \in \mathbb{N}$, and prices, for $y \in \ell_\infty^+$,

$$\pi_{W_k}(y) = 4\left(\frac{1}{\sqrt{8}}\right)\sum_{j\notin W_k}\left(\frac{1}{2}\right)^j\left(\frac{64j+1}{j}\right)^{-1} y_j$$
$$+ 4\sqrt{2}\sum_{j\in W_k}\left(\frac{1}{2}\right)^j\left(\frac{64j+1}{j}\right)^{-1} y_j + \beta^i\left(\frac{1}{8\sqrt{2}}\right)\mathrm{LIM}\left(\varphi(y)\right).$$

are an equilibrium for the economy $\mathcal{E}_{W_k} = (\ell_\infty^+, U^i, \mathrm{w}^i)_{i\in\mathbb{N}}$.

Thus, when shrinking down to disappearance the withholding coalition, $\lim_{k\to\infty}\mathcal{E}_{W_k}$, prices are, for $y \in \ell_\infty^+$,

$$\lim_{k\to\infty}\pi_{W_k}(y) = 4\left(\frac{1}{\sqrt{8}}\right)\sum_{j=1}^{\infty}\left(\frac{1}{2}\right)^j\left(\frac{64j+1}{j}\right)^{-1} y_j$$
$$+ \beta^i\left(\frac{1}{8\sqrt{2}}\right)\mathrm{LIM}\left(\varphi(y)\right) = \pi_{W_\infty}.$$

Notice that, if there was no price impact, $\pi_{W_\infty}(y) - \pi(y) = 0$, where $\pi(y)$ are the prices for the economy described in subsection 5.1. However,

$$\pi_{W_\infty}(y) - \pi(y) = \beta^i\left(\frac{1}{8\sqrt{2}} - \frac{1}{2\sqrt{512}}\right)\mathrm{LIM}\left(\varphi(y)\right)$$
$$= \beta^i\left(\frac{3}{32\sqrt{2}}\right)\mathrm{LIM}\left(\varphi(y)\right).$$

Therefore, $\lim_{k\to\infty}\pi_{W_\infty}(y)$ does not converge to $\pi(y)$ in the norm topology of $ba\left(2^{\mathbb{N}}\right)$.

6 Conclusion

We illustrated how an aversion to ambiguity in the comparison of different brands can make agents cautious and keep track on how the mean utility evolves as they consume along the commodities list. This was shown to allow for withholding coalitions to have a market power which actually does not vanish when in the limit no agent is withholding.

A limiting price impact for this withholding exercise occurs precisely when Arrow-Debreu prices are in the dual $\left(ba\left(2^{\mathbb{N}}\right)\right)$ but not in the pre-dual (ℓ_1^+). In fact, the price impact of the withholding coalition involves also the pure charge part of the prices (given by a generalized limit), and this is the part that does not converge back to the price of the original economy.

One might think that the impact of the withholding coalition on prices resides directly on the fact that such coalition is made up of countably many agents, leaving a finite amount of agents not withholding. However, as shown in example 2, we can have countably many agents not withholding and still have a persistent effect on prices.

There are some directions over which the research can improve: having different preferences (allowing agents to have different β^i) and adding a production sector might enrich the environment and the analysis of the present work.

References

C. D. Aliprantis and K. C. Border. *Infinite Dimensional Analysis: A Hitchhiker's Guide*. Springer Berlin Heidelberg, 3rd edition edition, 2006.

A. Araujo, R. Novinski, and M. R. Páscoa. General equilibrium, wariness and efficient bubbles. *Journal of Economic Theory*, 146(3):785 – 811, 2011.

A. Araujo, J. P. Gama, R. Novinski, and M. R. Páscoa. Endogenous discounting, wariness, and efficient capital taxation. *Journal of Economic Theory*, 183:520 – 545, 2019.

E. H. Chamberlin. Monopolistic competition revisited. *Economica*, 18(72):343 – 362, 1951.

J. M. Ostroy and W. R. Zame. Nonatomic economies and the boundaries of perfect competition. *Econometrica: Journal of the Econometric Society*, 62(3):593 – 633, May 1994.

K. Yosida and E. Hewitt. Finitely additive measures. *Transactions of the American Mathematical Society*, 72(1):46–66, 1952.

Appendix A. Auxiliary Definitions and Lemmas

Mainly, definitions come from Aliprantis and Border's "Infinite Dimensional Analysis: A Hitchhiker's Guide" Aliprantis and Border [2006]. Lemmas will mainly come from Araujo et al. [2011] and Araujo et al. [2019].

Definition 6.1. Consider the measure space $\left(\mathbb{N}, 2^{\mathbb{N}}, \mu\right)$ where $2^{\mathbb{N}}$ is the power set of the naturals, μ is the counting measure and $p \in \mathbb{N}$. Given that $\int_E f d\mu = \sum_{n\in\mathbb{N}} f(n)\ \forall E \subset \mathbb{N}$, we define

$$\ell_p(\mu) = \left\{ f : \mathbb{N} \to \mathbb{R} : \sum_{n=1}^{\infty} |f(n)|^p < +\infty \right\},$$

identifying its elements with the sequences $(x_n)_{n\in\mathbb{N}}$ such that $\sum_{n=1}^{\infty} |x_n|^p < +\infty$. **The space** ℓ_p over $\mathbb{R}$ is normed with $\|x\|_p = (\sum_{n=1}^{\infty} |x_n|^p)^{\frac{1}{p}}$.

Definition 6.2. Consider the measure space $\left(\mathbb{N}, 2^{\mathbb{N}}, \mu\right)$. We define

$$\ell_\infty(\mu) = \left\{ f : \mathbb{N} \to \mathbb{R} : \sup_{n\in\mathbb{N}} |f(n)| < +\infty \right\}.$$

identifying it as the sequence space whose elements are the bounded sequences. **The space** ℓ_∞ over $\mathbb{R}$ is normed with $\|x\|_\infty = \sup_{n\in\mathbb{N}} |x_n|$, and $(x_n)_{n\in\mathbb{N}} \in \ell_\infty$.

Definition 6.3. The collection of all signed charges having bounded variation over $\mathbb{N}$, denoted $ba\left(2^{\mathbb{N}}\right)$, is called **the space of charges**. It is an ordered space with the ordering $\geq$ defined setwise, $\mu \geq v$ if $\mu(A) \geq v(A)\ \forall A \in 2^{\mathbb{N}}, \forall \mu, v \in ba(2^{\mathbb{N}})$ and with the norm being $\|\mu\|_{ba} = V_\mu = |\mu|(\mathbb{N})$ taking

$$V_\mu = \sup \left\{ \sum_{i=1}^{n} |\mu(A_i)| : \{A_1, \ldots, A_n\} \text{ is a partition of } \mathbb{N} \right\}.$$

Definition 6.4. A capacity is a function $v : 2^{\mathbb{N}} \to \mathbb{R}$ such that $v(\emptyset) = 0$ and $v(A) \leq v(B)$ whenever $A \subseteq B$.

Definition 6.5. A capacity v is convex when $v(A \cup B) + v(A \cap B) \geq v(a) + v(B)$ for all $A, B \subset \mathbb{N}$.

Definition 6.6. The set $core(v)$ is defined as $\{\eta \in ba : \eta \geq v,\ \eta(\mathbb{N} = 1)\}$.

Definition 6.7. Given a dual pair $\langle X, X' \rangle$ and a concave function f on X, we say that $x' \in X'$ is a **supergradient of f at x** if it satisfies the following supergradient inequality

$$f(y) \leq f(x) + x'(y-x),\ \forall y \in X.$$

We refer to the collection of supergradients as **superdifferential**, denoted by $\partial f(x)$.

Definition 6.8. Define the one-sided directional derivatives $d^+ f(x) : X \to \mathbb{R}$ and $d^- f(x) : X \to \mathbb{R}$ at x in direction $v \in \ell_\infty$ as

$$d^+ f(x)(v) = \lim_{\lambda \downarrow 0} \frac{f(x+\lambda v) - f(x)}{\lambda},$$
$$d^- f(x)(v) = \lim_{\lambda \uparrow 0} \frac{f(x+\lambda v) - f(x)}{\lambda}.$$

Definition 6.9. Let X be a set, and let $\mathcal{A}$ be a σ-algebra on X. A function μ whose domain is the σ-algebra $\mathcal{A}$ and whose values belong to the extended half-line $[0, +\infty]$ is said to be countably additive if it satisfies

$$\mu\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} \mu\left(A_i\right),$$

for each infinite sequence $\{A_i\}_{i \in \mathbb{N}}$ of disjoint sets that belong to $\mathcal{A}$. A measure (or a countably additive measure) on $\mathcal{A}$ is a function $\mu : \mathcal{A} \to [0, \infty]$ that satisfies $\mu(\emptyset) = 0$ and is countably additive.

Definition 6.10. Let $\mathcal{A}$ be an algebra (not necessarily a σ-algebra) on the set X. A function μ whose domain is $\mathcal{A}$ and whose values belong to $[0, +\infty]$ is finitely additive if it satisfies

$$\mu\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} \mu\left(A_i\right),$$

for each finite sequence $A_1, A_2, \ldots, A_n$ of disjoint sets that belong to $\mathcal{A}$. A finitely additive measure on the algebra $\mathcal{A}$ is a function $\mu : \mathcal{A} \to [0, \infty]$ that satisfies $\mu(\emptyset) = 0$ and is finitely additive.

Definition 6.11. A finitely additive measure ψ is dominated by τ (denoted by $\psi \leq \tau$) if for every subset $S \subset \mathcal{A}$, we have $\psi(S) \leq \tau(S)$.

Definition 6.12. The finitely additive measure τ is said to be purely finitely additive if the inequalities $0 \leq \psi \leq \tau$ with ψ countably additive imply that $\psi = 0$.

Lemma 3. *Let* $\mathbf{1}(n)$ *the real sequence whose first* n *elements are zero and the remaining ones are equal to* 1*. If* v *is a pure positive charge in* $\left(\mathbb{N}, 2^{\mathbb{N}}\right)$*, then* $\frac{v(x)}{\|v\|_{ba}} \in [\liminf x, \limsup x], \forall x \in \ell_\infty$*. If for some* $\mu \in ca$*,* $\mu + v \in \partial U^i(x)$*, then* $\|v\|_{ba} \in \left[\lim_n d^+ U^i\big(x; \mathbf{1}(n)\big), \lim_n d^- U^i\big(x; \mathbf{1}(n)\big)\right]$*.*

Next, we have one of Yosida and Hewitt theorems included in Yosida and Hewitt [1952]. To understand their notation, notice the next facts:

- X is an abstract set (on our case, $\mathbb{N}$),
- $\mathcal{M}$ is a family of subsets of X closed under the formation of finite unions and of complements (on our case $2^{\mathbb{N}}$),
- $\overline{\mathcal{M}}$ is the smallest family of sets containing $\mathcal{M}$ and closed under the formation of countable unions and of complements (on our case $2^{\mathbb{N}}$),
- ϕ is a finitely additive measure on $\mathcal{M}$ (on our case ν),
- $\Phi(X, \mathcal{M})$ is the set of all measures for a fixed X and $\mathcal{M}$.

Theorem 4. *Let $\mathcal{M} = \overline{\mathcal{M}}$. Then if π is purely finitely additive and not less than 0 and ψ is countably additive and not less than* 0*, there exists a decreasing sequence $B_1, B_2, \ldots, B_n, \ldots$ of elements of $\mathcal{M}$ such that* $\lim_{n\to\infty} \psi(B_n) = 0$ *and* $\pi(B_n) = \pi(X)(n = 1, 2, 3, \ldots)$*. Conversely, if $\phi \in \Phi$ and the above conditions hold for all countably additive Ψ, then ϕ is purely finitely additive.*

Theorem 5. *Let τ be a measure defined over the measure space (X, S).*

1. *If $(E_n)_{n\in\mathbb{N}}$ is an increasing sequence of elements in S, then*

$$\tau\left(\bigcup_{n=1}^{\infty} E_n\right) = \lim_{n\to\infty} \tau(E_n).$$

2. *If $(F_n)_{n\in\mathbb{N}}$ is a decreasing sequence of elements in S, then*

$$\tau\left(\bigcap_{n=1}^{\infty} F_n\right) \leq \lim_{n\to\infty} \tau(F_n).$$

   If also $\tau(F_k) < +\infty$ for some $k \in \mathbb{N}$, then equality is attained.

Appendix B. Proofs

Proof of Proposition 2. Define $\mathcal{E} = (\ell_\infty^+, U^i, \omega^i)_{i\in\mathbb{N}}$ with equilibrium allocations $(x^i)_{i\in\mathbb{N}}$ and prices, for $y \in \ell_\infty^+$,

$$\pi(y) = \sum_{j=1}^{\infty} \varsigma_j (u)'(x_j^q) y_j + \beta^q u'(\underline{x}^q) \operatorname{LIM}\big(\phi(y)\big),$$

for a given $q \in \mathbb{N}$, as described in sections 2 and 3. Now, let countably many agents withhold some part of their characteristic endowment. This is, let every $i \in \mathbb{N}$ engage in exchange with

$$\mathrm{w}^i = \begin{cases} \omega_j^i & \text{if } j = i \text{ and } j \notin W_k, \\ \omega_j^i - c^i & \text{if } j = i \text{ and } j \in W_k, \\ 0 & \text{otherwise,} \end{cases}$$

giving place to $\mathcal{E}_{W_k} = (\ell_\infty^+, U^i, \mathrm{w}^i)_{i \in \mathbb{N}}$, with a withholding coalition $W_k = \{w_l : l \geq k \in \mathbb{N}\}$, equilibrium allocations $(z^i)_{i \in \mathbb{N}}$ and prices, for $y \in \ell_\infty^+$, that we can write as

$$\pi_{_{W_k}}(y) = \sum_{j \notin W_k} \varsigma_j (u)'(z_j^q) y_j + \sum_{j \in W_k} \varsigma_j (u)'(z_j^q) y_j + \beta^q u'(\underline{z}^q) \operatorname{LIM}\big(\phi(y)\big),$$

for a given $q \in \mathbb{N}$, since for every $k \in \mathbb{N}$, $\underline{z}$ is not attained $\forall i \in \mathbb{N}$. As showed in Araujo et al. [2011], equilibria for all such economies exist. All of them fulfil the three conditions described in section 3. In particular, they fulfil market clearing for every $j \in \mathbb{N}$. So, for goods $j \in W_k$, we have that

$$\sum_{i=1}^{\infty} z_j^i = \omega_j^i - c^j = \sum_{i=1}^{\infty} x_j^i - c^j \;\Rightarrow \sum_{i=1}^{\infty} \left(x_j^i - z_j^i\right) = c^j.$$

From the previous expression, we can conclude that there exists at least one agent $n \in W_k$ such that $x_j^n > z_j^n$. Notice that this is true for every $j \in W_k$.

Consider the set

$$B_k = \left\{ z_j^n : x_j^n > z_j^n \ \forall j \in W_k \right\}$$

and take $\inf\limits_{j \in W_k} B_k$. Since coalition W_k is made of countably many agents and, by construction of B_k, there exists an agent, $q* \in \mathbb{N}$ such that $\inf\limits_{j \in W_k} B_k = \underline{z}^{q*}$. Thus, without loss of generality, I can pin such an agent $q* = q$ to calculate prices in economies $\mathcal{E}$ and $\mathcal{E}_{W_k}$ $\forall k \in \mathbb{N}$, i.e.

$$\pi(y) = \sum_{j=1}^{\infty} \varsigma_j (u)' \left(x_j^{q*}\right) y_j + \beta^i u' \left(\underline{x}^{q*}\right) \operatorname{LIM}\big(\phi(y)\big),$$

$$\pi_{_{W_k}}(y) = \sum_{j \notin W_k} \varsigma_j (u)' \left(z_j^{q*}\right) y_j + \sum_{j \in W_k} \varsigma_j (u)' \left(z_j^{q*}\right) y_j + \beta^i u' \left(\underline{z}^{q*}\right) \operatorname{LIM}\big(\phi(y)\big).$$

Now, shrinking the coalition down to disappearance, prices in the limiting economy $\lim_{k\to\infty} \mathcal{E}_{W_k}$ become

$$\lim_{k\to\infty} \pi_{_{W_k}}(y) = \sum_{j=1}^{\infty} \varsigma_j (u)' \left(x_j^{q*}\right) y_j + \beta^i u' \left(\underline{z}^{q*}\right) \mathrm{LIM}\left(\phi(y)\right).$$

There are three important facts in the calculation of the limit:

- $u' \left(\underline{z}^{q*}\right)$ is a constant, and would not change, regardless W_k being shrunk up to disappearance,
- $\sum_{j\notin W_k} \varsigma_j (u)' \left(z_j^{q*}\right) y_j + \sum_{j\in W_k} \varsigma_j (u)' \left(z_j^{q*}\right) y_j$ turned into $\sum_{j=1}^{\infty} \varsigma_j (u)' \left(x_j^{q*}\right) y_j$ since there is no more withholding in the limiting economy $\lim_{k\to\infty} \mathcal{E}_{W_k}$,
- As mentioned in section 2, u is concave and strictly increasing, making u' to be decreasing. The previous implies that $u' \left(\underline{z}^{q*}\right) > u' \left(\underline{x}^{q*}\right)$, since $\inf_{j\in W_k} B_k = \underline{z}^{q*}$.

Finally, for $y \in \ell_\infty^+$ with $y \ggg 0$,

$$\lim_{k\to\infty} \pi_{_{W_k}}(y) - \pi(y) = \beta^i \left(u' \left(\underline{z}^{q*}\right) - u' \left(\underline{x}^{q*}\right)\right) \mathrm{LIM}\left(\phi\left(y\right)\right) > 0,$$

showing that $\lim_{k\to\infty} \pi_{_{W_k}}(y) > \pi(y)$, i.e. prices in the limiting economy $\lim_{k\to\infty} \mathcal{E}_{W_k}$ are higher than those in $\mathcal{E}$. □