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The Limits of Crowdfunding with Common Values

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The limits of crowdfunding with common values*

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Abstract

This paper establishes the efficiency limits of crowdfunding in the common value case. An entrepreneur sets a threshold and rate of return on investments in a risky project with constant returns to scale. Investors can inspect the project at a cost, before choosing whether to bid to invest. We characterize optimal welfare and profit in the limit as the crowd of investors grows large. With costless information acquisition, crowdfunding asymptotically achieves the first-best because informative bidding by a vanishing fraction of the crowd can ensure almost certain funding success for good projects while excluding bad ones. Costly information precludes this: (a) for intermediate costs, good projects again always succeed in the limit, but bad projects also get funded with a probability that increases linearly in information's cost and decreases in its precision; (b) above a cut-off cost, no information is acquired and crowdfunding adds nothing to standard investment contracts.

Keywords: Information acquisition, information aggregation, wisdom of crowds, asymptotic efficiency, common value.
JEL Classifications: C72, D82, G23, L15.

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1 Introduction

Crowdfunding became a significant channel for entrepreneurial finance soon after online platforms made it easy for entrepreneurs to solicit funds from large crowds of people. In investment-based crowdfunding, the entrepreneur offers a financial return, such as an interest rate or equity share, to those who invest in her project. Critically, she commits to reimburse all funds pledged if they fail to reach her goal or threshold. The idea is that the threshold screens out a greater fraction of bad than good projects so that bid activation conveys good news about project quality. We say that funding success confers a "crowdblessing." This blessing relies on investors pledging more on seeing positive signals. The problem is that, given a high blessing, investors prefer to pledge regardless of their signals, to avoid missing out on the crowdblessed project. So investors may free-ride on creating the blessing. The entrepreneur's challenge is to set her goal and interest rate to attract funds, motivate informative pledging and aggregate the acquired information.

We study an asymptotically large crowd to provide the first sharp characterization of optimal outcomes in a model of crowdfunding with common values and costly inspection. This is important because most theory assumes private values yet quality uncertainty makes common values omnipresent in crowdfunding and fundamental in investment-based crowdfunding. Our main result is that free inspection permits asymptotic efficiency but over-investment is necessary to prevent free-riding when inspection is costly. Specifically, entrepreneurs distort thresholds downwards so that bad projects succeed with a probability proportional to the ratio of inspection cost to signal precision.

Traditionally, entrepreneurs seek finance from banks, angel investors and venture capitalists. These professionals develop expertise and can afford large investment stakes that motivate thorough inspection of promising projects. Crowdfunding platforms allow entrepreneurs to instead post their projects to seek funds from many small investors. Such investors may lack expertise but recent evidence suggests they may jointly access more independent quality signals than do professionals.[1] They may have insight as potential consumers and quick intuitions about trustworthiness, but reading about the project takes time, as does thinking and background research. So we ask how well crowdfunding can foster and harness crowd wisdom with costly as well as free inspection. Our analysis also applies to best effort Initial Public Offerings (IPOs) where the issuer specifies a minimum number of securities that must sell for the IPO to go ahead (see Ritter, 1987). When this threshold is reached, the newly issued shares generate funds for the firm to invest. Our crowdfunding model can characterize the optimal threshold and share price.[2]

[1] Iyer et al. (2016) show how crowdfunders exploit soft information on the debt-based platform, Prosper, to outperform predictions based on hard information such as credit scores used by banks. Mollick and Nanda (2016) find that crowdfunding complements expert decisions for projects on Kickstarter.

[2] Relating to theories of IPO underpricing as compensation of investors for costly inspection (see Ritter and Welch, 2002; Sherman, 1992; Sherman and Titman, 2002; van Bommel, 2002), our results predict that underpricing will disappear as the number of IPO participants grows large.

In our canonical setup, each funder can acquire a single binary signal and can then bid to invest by pledging one unit of money. A bid becomes an investment if funding reaches the entrepreneur's threshold. In this campaign success event, each funder who bids faces a binary return: he loses his unit investment if the entrepreneur's project is bad and he gains the entrepreneur's promised net interest rate if the project is good. The entrepreneur gains the residual return on each unit, which is zero in the bad state. Neglecting dominated strategies, each funder essentially chooses simultaneously between three choices denoted **A** for avoid or not bid, **B** for blind bid and **C** for conditional bid, that is, only bid on seeing a good signal.

To tractably pinpoint the impact of information externalities, we assume constant returns to scale.[3] So the ideal outcome is for all bidders to invest in the entrepreneur's project if it is good and none to invest if it is bad, all without paying positive inspection costs. Even if inspection is free, this ideal is impossible in a finite crowd because of noise in the quality signals but perhaps the ideal can be approached as the crowd grows?

Inducing all bidders to play **C** maximizes information generation. This reveals project quality almost perfectly in the large crowd limit thanks to the strong law of large numbers. Setting the ex post optimal threshold then almost perfectly screens out bad projects and activates all bids on good projects. However, it generates serious inefficiency on good projects by wasting the funds of all bidders who observe false negative signals.

To maintain investment scale, the entrepreneur should instead induce only a small fraction of bidders to play **C**, letting the rest free-ride by playing **B**. In symmetric equilibrium, she needs bidders to mix between playing **B** and **C**. As crowdsize grows, she can make the probability on **C** vanishingly small while maintaining an arbitrarily large number of informative bids. The corresponding ex post optimal threshold again screens quality almost perfectly and now false negative signals waste only a vanishingly small fraction of funds. The expected cost from inspecting is also vanishingly small but can the entrepreneur set interest rates to induce these strategies?

With free inspection, we answer this affirmatively. For any crowdsize, the entrepreneur can set a feasible interest rate that makes bidders indifferent between **B** and **C** and willing to participate. Moreover, as crowdblessing becomes almost perfect, this (net) interest rate goes to zero and the entrepreneur extracts the full, ideal surplus as her profit.

When inspection is costly, this ideal solution is no longer feasible. The interest rate that makes bidders indifferent between **B** and **C** at the ex post optimal threshold becomes so low that bidders prefer to opt out by playing **A**. The entrepreneur is forced to distort her threshold downwards to lower crowdblessing to make bidders willing to inspect. We first prove that bad projects must get funded with a probability at least as high as the cost (per unit) of precision, that is, the inspection cost divided by signal precision. Next,

[3] We discuss the effect of decreasing returns in Section 6, along with other extensions beyond the canonical setup.

in a constructive proof using the central limit theorem, we show how crowdfunding can asymptotically attain the implied upper bound on per capita welfare and profit. The interest rate now necessarily extracts all rent. The asymptotic welfare inefficiency and profit loss relative to the ideal outcome both equal the cost of precision.[4]

Precision can be too costly for crowdfunding to implement any informative equilibrium. At such high costs, crowdfunding offers no advantage over a standard posted-offer contract since nobody ever inspects. By contrast, the first-best achieves the asymptotic ideal outcome for any cost by having a vanishing fraction of bidders inspect.[5]

Related literature

Our two main results speak to classic debates over how effectively markets and votes aggregate information. We describe how our positive and negative results for free and costly information parallel the key insights from those literatures. Then we home in on prior work on crowdfunding. We relate to general mechanism design in the conclusion.

Our efficiency result given free information parallels Grossman's (1976) efficient markets result: pricing mechanisms can then perfectly aggregate dispersed private information. The difference is that crowdfunding sets a fixed price and uses a threshold to adapt investment quantity to aggregate bidding. Similarly, our inefficiency result given costly information mirrors Grossman and Stiglitz's (1980) impossibility result. Just as perfectly informative market prices would remove all incentive for traders to acquire information, a perfect crowdblessing would remove all incentive for inspection.

Wilson (1977) argued that auction theory explicitly captures how market pricing might aggregate dispersed, free information by revealing a true common value in the limit with a large number of bidders. Pesendorfer and Swinkels (1997) show that if signals are bounded, this fully informative pricing requires a double large limit (many units and also many unsatiated bidders). Costly information is more disruptive (Matthews, 1984, see also Persico, 2000). The auction setting differs from crowdfunding since bids compete and a winner's curse replaces our crowdblessing but the broad parallels remain.

Information aggregation has been a central concern in collective decision-making since Condorcet's Jury Theorem. Feddersen and Pesendorfer (1997) prove that strategic voting fully aggregates free, private information about a common value in a large electorate: the limit outcome is the same as when all private information is made common knowledge. In finite committees, Persico (2004) shows that costly information causes inefficient outcomes but he finds no gain from distorting the threshold that aggregates votes, in contrast to our downward distortion result. Allowing richer voting rules, however, Gerardi and Yariv

[4]This result helps explain empirical evidence suggesting the need for well-crafted investment pitches, combined with crowdfunders' personal interests, to ensure low net inspection costs.

[5]Indeed, the entrepreneur can achieve it by ordering bidders to play the symmetric crowdfunding strategies of the second-best with free inspection.

(2008) do derive distinct ex post distortions that serve to raise inspection incentives, similar to our finding.

Within the literature on crowdfunding, we contribute the first tight characterization of outcomes, efficiency and optimal design in the common value setting with endogenous information. Most theoretical papers consider exogenous or free information.

Early work focused on demand-side, private value information. In reward-based crowdfunding, funders are consumers who can advance purchase a new good. Chemla and Tinn (2020), Ellman and Hurkens (2019) and Strausz (2017) share the insight that crowdfunding provides an incentive-compatible test of market demand before the entrepreneur sinks her fixed costs. Crowdfunding aggregates funders' private value information. The ideal threshold gauges demand against fixed costs but in Ellman and Hurkens (2019), a profit-maximizing entrepreneur may distort her threshold upwards to extract rent.[6]

On the supply-side, crowdfunding can aggregate common value information about project quality. Chang (2020) and Cong and Xiao (2024) study profit-maximization with pure common values and free private signals. Chang (2020) considers a continuum of buyers and values. Approximate efficiency is possible via post-production retail sales after a vanishing fraction of crowdfunding purchases reveals project quality. Like us, Cong and Xiao (2024) do not need an after-market. Unlike us, they assume sequential bidding. This creates the potential for learning but also a risk of herding.[7] They find that crowdfunding aggregates information efficiently in the large crowd limit via high thresholds that guarantee quality on successful campaigns and preclude down cascades. Our free information result shows that sequentiality is not needed for asymptotic efficiency.

Only two papers address costly information in this setting. Their models are similar to ours, right down to binary states, binary signals and symmetric equilibria. Hakenes and Schlegel (2014) is particularly close. However, they claim that in all optimal equilibria, all bidders acquire information. Our results show that this much-cited pure strategy result is flawed. We prove that blind-bidding must always predominate in a large crowd. Blind-bidding is necessary to optimize investment scale.[8] Moreover, the probability weight on blind-bidding is often high in finite crowds too (see our companion paper, Ellman and Hurkens, 2024).

Brown and Davies (2020) differ in two key ways. They assume that funders are wealthy and that project scale has an upper bound. This trivializes the problem of investment scale, because high stakes facilitate incentive compatibility and so the entrepreneur optimally raises the individual bidding stake to fully exploit any investment opportunity revealed by aggregate bids. Brown and Davies (2020) study project value maximization.

[6] This permits price-discrimination grounded in differential pivotality motives. With deferred purchases, upward distortion may also reduce fraud (Chemla and Tinn, 2020; Strausz, 2017).

[7] Åstebro et al. (2024) empirically identify early herding that leads to funding of bad projects.

[8] Hakenes and Schlegel (2014) assume a profit-maximizing entrepreneur who knows her project quality but all equilibria involve pooling so an uninformed entrepreneur's optimization is identical.

This differs from our welfare-maximization in neglecting the cost of inspections. So their first-best always involves pure inspection and the key question is whether any interest rate can induce it. They show that for a given crowdsize, the first-best is attainable if information is free or positive but small. They also claim that very large crowds preclude any inspection but this claim is flawed.[9] Our free information result shows that a large crowd permits efficiency even with wealth-constrained funders; this result is robust to an upper bound on scale. We also pinpoint a simple condition (C) for informative equilibria which permits us to develop powerful analytic results, including for positive costs and the realistic case of threshold commitment.

2 Model

The entrepreneur's project. An entrepreneur E has an investment project that yields a binary return. The gross return per unit is $(1+R)$ with $R>0$ if project quality is good, denoted $\omega=\mathcal{G}$, and 0 (net unit return -1) if quality is bad, $\omega=\mathcal{B}$. The prior probability on $\omega=\mathcal{G}$ is μ. E has no private information so tuple (μ,R) characterizes E.

Information and the crowd. E has no money of her own but can solicit finance from a crowd of $N+1$ potential investors, called bidders. Each bidder has one unit of money to invest and can acquire a binary signal $s\in\{G,B\}$ at cost $c\geq 0$. The signals are i.i.d., conditional on the true quality ω, with error rates $\beta>0$ when quality is bad $\mathcal{B}$ and $\gamma>0$ when good $\mathcal{G}$: $P(s_i=B|\mathcal{B})=1-\beta$ and $P(s_i=G|\mathcal{G})=1-\gamma$. The signals are informative: $1-\beta-\gamma>0$. Tuple (N,c,β,γ) characterizes the crowd.

Crowdfunding design. E can raise funds by offering a crowd investment contract or design (n,r) where E sets threshold $n\in\{1,2,\ldots,N+1\}$, interest rate $r\in[0,R]$ and: (1) bidders simultaneously and independently bid one unit or nothing; (2) if the number of realized bids $k<n$, all bids are returned, no money is invested and the game ends; if $k\geq n$, all bids are activated as investments – each of the k unit bidders gives E 1 unit and E invests the k units; (3) investment reveals $\mathcal{B}$ or $\mathcal{G}$ quality; if good $\mathcal{G}$, E pays back $1+r$ to each of the k unit bidders, while if bad $\mathcal{B}$, E pays nothing.

This is the canonical All-or-Nothing (AoN) debt-based crowdfunding design. The campaign is said to "succeed" if funds k reach the threshold n; this *success* implies *activation* of all bids as investments, but does not guarantee a return. Investing the k bids yields gross return $(1+R)k$ if quality is good $\mathcal{G}$ and zero if $\mathcal{B}$. If quality $\mathcal{G}$ is revealed,

[9] With free information, this claim directly contradicts the finite crowd result and for a continuum of bidders, Brown and Davies (2020, Internet Appendix IA1) overlook the fact that a zero interest rate can induce pure inspection. With low positive inspection costs, large crowds do imply excessive rationing if all inspect but some inspection is possible by inducing bidders to mix between abstention and inspection. Brown and Davies (2020) also claim that a winner's curse from relatively greater rationing in the good state complicates implementing the first-best but in fact only rationing in the bad state matters.

E pays out $(1+r)k$ in total, so her total profit is $(R-r)k$ and her per capita profit is $\pi = (R-r)k/(N+1)$. If quality proves to be bad $\mathcal{B}$, E instead gets $\pi = 0$; bidders necessarily bear all the losses as net unit returns of -1. We suppose E either maximizes profit or social welfare. Anticipating how bidders react to any given design (n, r), she implements a profitable or socially efficient equilibrium of the resulting bidding game.

Bidder strategies. We can restrict to three choices, **A**, **B**, **C** where **A** denotes "avoid" (neither inspect nor bid), **B** denotes "blind bid" (bid without inspecting), and **C** denotes "check and conditional bid" (inspect and only bid contingent on observing the signal G). This is valid for three reasons. First, it is clearly dominated to bid contingent on the bad signal B while not bidding after seeing G. Second, it is dominated to spend $c > 0$ and then ignore the observed signal by always or never bidding, independent of the signal. Third, if $c = 0$, inspecting and never bidding is equivalent to **A** and inspecting and always bidding is equivalent to **B**.

We restrict to symmetric equilibria, denoting the possibly mixed strategies by $p = (p_{\mathbf{A}}, p_{\mathbf{B}}, p_{\mathbf{C}})$ on the unit simplex. It is convenient to represent p by the equivalent probabilities, $x_{\mathcal{B}}$ and $x_{\mathcal{G}}$ that a bidder bids in states $\mathcal{B}$ and $\mathcal{G}$. Note that $x_{\mathcal{B}} = p_{\mathbf{B}} + \beta p_{\mathbf{C}}, x_{\mathcal{G}} = p_{\mathbf{B}} + (1-\gamma)p_{\mathbf{C}}$, while $p_{\mathbf{C}} = \frac{x_{\mathcal{G}} - x_{\mathcal{B}}}{1-\beta-\gamma}$ and $p_{\mathbf{B}} = \frac{(1-\gamma)x_{\mathcal{B}} - \beta x_{\mathcal{G}}}{1-\beta-\gamma}$. So $x_{\mathcal{G}} \geq x_{\mathcal{B}}$.

Campaign success rates. Given bidder simultaneity and the conditional independence of signals, conditional on quality ω, any M bidders playing $(x_{\mathcal{B}}, x_{\mathcal{G}})$ generate the Binomial bid distribution $\text{Bin}(M, x_\omega)$; we let $S_m^M(x_\omega)$ denote the probability of m or more bids from these M bidders. Then the campaign success rate given ω is $S_n^{N+1}(x_\omega)$, implying unconditional success rate, $\mu S_n^{N+1}(x_{\mathcal{G}}) + (1-\mu)S_n^{N+1}(x_{\mathcal{B}})$. Similarly, any bidder who bids expects activation given his bid with state-contingent probabilities $S_{n-1}^N(x_{\mathcal{B}}), S_{n-1}^N(x_{\mathcal{G}})$ since success then requires $n-1$ of the other N bidders to bid. We call this last pair, the bad-state and good-state activation rates.

Bidder payoffs. Bidders are risk neutral. Avoidance **A** yields $u(\mathbf{A}) = 0$. Any bidder whose bid is activated by campaign success gains net payoff r if quality $\omega = \mathcal{G}$ and -1 if $\mathcal{B}$. If instead he does not bid or the campaign fails so that his bid is not activated, he gains 0. In addition, he pays sunk cost c if he inspects. Choosing to blind bid **B** implies always bidding. Choosing **C** implies paying c and bidding if his private signal is G, so that, despite not observing ω directly, he bids with probability $1-\gamma$ when $\omega = \mathcal{G}$ and probability β when $\omega = \mathcal{B}$. Each bidder knows the prior quality distribution summarized by μ. A bidder who believes that all others choose strategies $(x_{\mathcal{B}}, x_{\mathcal{G}})$ anticipates that if he bids, he will invest with the bad-state and good-state activation rates $S_{n-1}^N(x_{\mathcal{B}}), S_{n-1}^N(x_{\mathcal{G}})$.

So his respective payoffs from **B** and **C** given $(x_\mathcal{B}, x_\mathcal{G})$ are

$$u(\mathbf{B}; n, r, x_\mathcal{B}, x_\mathcal{G}) = \mu S^N_{n-1}(x_\mathcal{G})r - (1-\mu)S^N_{n-1}(x_\mathcal{B}) \tag{1}$$

$$u(\mathbf{C}; n, r, x_\mathcal{B}, x_\mathcal{G}) = \mu(1-\gamma)S^N_{n-1}(x_\mathcal{G})r - (1-\mu)\beta S^N_{n-1}(x_\mathcal{B}) - c \tag{2}$$

Notice how blind-bidders gain more from increases in r than those who, in choosing **C**, only bid on seeing a good signal G.

Profit and welfare. The expected amount invested *per capita* conditioned on quality ω, given design (n, r) and bidding strategy $(x_\mathcal{B}, x_\mathcal{G})$ is $I^N_n(x_\omega) = x_\omega S^N_{n-1}(x_\omega)$, the probability any given bidder bids times the probability his bid is activated. E's profit is determined by her gain of $R - r$ per unit invested by each bidder who bids when her campaign succeeds and quality is good $\mathcal{G}$. So her *per capita* profit function is

$$\pi(n, r, x_\mathcal{B}, x_\mathcal{G}) = \mu I^N_n(x_\mathcal{G})(R - r) \tag{3}$$

Per capita welfare w equals expected bidder surplus plus entrepreneurial profit π:[10]

$$w(n, x_\mathcal{B}, x_\mathcal{G}) = \mu I^N_n(x_\mathcal{G})R - (1-\mu)I^N_n(x_\mathcal{B}) - c\,\frac{x_\mathcal{G} - x_\mathcal{B}}{1-\beta-\gamma} \tag{4}$$

This incorporates the social gain from investment in the good state $\mathcal{G}$ less social loss from investment in $\mathcal{B}$ and costly inspection $c\,p_\mathbf{C}(x_\mathcal{B}, x_\mathcal{G})$; transfers cancel out.

Timing. We summarize the time ordering, making nature's moves explicit: (1) Nature draws project quality from $\{\mathcal{B}, \mathcal{G}\}$ with no one observing this draw. (2) E sets her design (n, r). (3) Each bidder reacts independently with a choice from $\{\mathbf{A}, \mathbf{B}, \mathbf{C}\}$. (4) These choices determine the number of bids k; if $k \geq n$ (campaign success), E invests all k bids while if $k < n$ (campaign failure), all k bids are reimbursed. (5) For $k \geq n$, project returns are realized and in state $\mathcal{G}$, E pays gross interest $1 + r$ to those who bid.

Solution concept. We suppose E can implement strategy profile $(x_\mathcal{B}, x_\mathcal{G})$ if this strategy is a Nash equilibrium of the crowdfunding game given design (n, r). This weak implementation concept resolves equilibrium multiplicity by assuming that E can coordinate bidders on her preferred equilibrium. This provides an upper bound on crowdfunding's effectiveness for E. E either maximizes welfare $w(n, x_\mathcal{B}, x_\mathcal{G})$ or profit $\pi(n, r, x_\mathcal{B}, x_\mathcal{G})$ subject to $(x_\mathcal{B}, x_\mathcal{G})$ being a Nash equilibrium given (n, r).

Benchmarks. In the hypothetical case where all actors have free and perfect information about project quality, E maximizes welfare by investing all $N + 1$ units of funds if the project is good and none if it is bad. Any threshold n will do and net rate $r = 0$

[10] We suppress *per capita* qualifications, except in new definitions.

extracts all rent; bidders only bid when quality is good. Since $n = 1$ is equivalent to E posting a standard individual debt contract, "posted offers" suffice here.

Proposition 1. *In the ideal scenario with free and perfect information on project quality, both maximized profit and maximized welfare equal μR: $\overline{\pi} = \overline{w} = \mu R$.*

This provides an immediate upper bound on what E can hope for in the above model of imperfect signals from possibly costly inspection. In our first-best benchmarks, delayed to Section 5 as the derivations build on our second-best analysis, E attains this upper bound asymptotically as crowd size grows large.

The general posted offer benchmark is equally simple if E maximizes welfare: restricting to $n = 1$ removes bidder interactions so E optimally delegates investment decisions by setting $r = R$. This generates $w = \max\{0, w^{\mathbf{B}}, w^{\mathbf{C}}\}$, where substituting $I_1^N(x) = x$ into (4) with $(x_{\mathcal{B}}, x_{\mathcal{G}}) = (0,0), (1,1), (\beta, 1-\gamma)$ for $\mathbf{A}, \mathbf{B}, \mathbf{C}$, we have $w^{\mathbf{A}} = 0$,

$$w^{\mathbf{B}} = \mu R - (1-\mu) \tag{5}$$

$$w^{\mathbf{C}} = \mu R(1-\gamma) - (1-\mu)\beta - c \tag{6}$$

3 Upper bounds for finite crowds

The symmetric strategy $(x_{\mathcal{B}}, x_{\mathcal{G}})$ can be implemented using design (n, r) if and only if $(x_{\mathcal{B}}, x_{\mathcal{G}})$ is a Nash equilibrium given (n, r); this requires that s maximize $u(s; n, r, x_{\mathcal{B}}, x_{\mathcal{G}})$ for every $s \in \{\mathbf{A}, \mathbf{B}, \mathbf{C}\}$ that is chosen with strictly positive probability. We say that an equilibrium is informative if inspection sometimes occurs, $p_{\mathbf{C}} > 0$ or equivalently, $x_{\mathcal{G}} > x_{\mathcal{B}}$. Informativeness requires r to be high enough that $\mathbf{C}$ is individually rational but also low enough to dissuade deviation to $\mathbf{B}$. A high interest rate encourages blind-bidding relative to conditional bidding because conditional bidders gain r less often than blind-bidders by factor $1-\gamma$, the probability of a G signal given $\mathcal{G}$ quality.

The relevant cutoffs from comparing (2) against $u(\mathbf{A}) = 0$ and (1) are

$$r_n^{\mathbf{C}}(x_{\mathcal{B}}, x_{\mathcal{G}}) = \frac{c + (1-\mu)\beta S_{n-1}^N(x_{\mathcal{B}})}{\mu(1-\gamma) S_{n-1}^N(x_{\mathcal{G}})} \tag{7}$$

$$r_n^{\mathbf{CB}}(x_{\mathcal{B}}, x_{\mathcal{G}}) = \frac{(1-\mu)(1-\beta) S_{n-1}^N(x_{\mathcal{B}}) - c}{\mu\gamma S_{n-1}^N(x_{\mathcal{G}})} \tag{8}$$

$r_n^{\mathbf{CB}}(x_{\mathcal{B}}, x_{\mathcal{G}})$ is an upper bound on r for implementing $\mathbf{C}$ and a lower bound on r for implementing $\mathbf{B}$. So an informative equilibrium requires $r_n^{\mathbf{C}} \leq r_n^{\mathbf{CB}}$ at $(x_{\mathcal{B}}, x_{\mathcal{G}})$. This condition simplifies to

$$S_{n-1}^N(x_{\mathcal{B}}) \geq c/\bar{c} \tag{\textbf{C}}$$

where we define

$$\bar{c} = (1-\mu)(1-\beta-\gamma) \tag{9}$$

Proposition 2. *(C) must hold for a design (n, r) to have informative equilibrium, $(x_{\mathcal{B}}, x_{\mathcal{G}})$.*

Condition (C) only depends on $x_{\mathcal{B}}$ and n. If (C) fails, there is no rate r at which design (n, r) implements any $(x_{\mathcal{B}}, x_{\mathcal{G}})$ with $x_{\mathcal{G}} > x_{\mathcal{B}}$. In particular, $S^N_{n-1}(x_{\mathcal{B}}) \leq 1$ and so a cost c exceeding $\bar{c}$ entirely precludes informative equilibria since (C) then fails for all $n, x_{\mathcal{B}}$. For intuition, we rewrite condition (C) as

$$c/\rho \leq (1-\mu)S^N_{n-1}(x_{\mathcal{B}})$$

where $\rho = 1 - \beta - \gamma$ captures signal precision. So c/ρ is the cost of inspecting relative to its informativeness, while the bad-state has prior $1 - \mu$ and bad-state activation rate $S^N_{n-1}(x_{\mathcal{B}})$. Intuitively, a high cost of precise information makes inspection less attractive, while the risk of having a bid activated in the bad state naturally encourages inspection relative to blind-bidding. The good-state activation rate $S^N_{n-1}(x_{\mathcal{G}})$ complements design instrument r so it affects the level but not the ranking of $r^{\mathbf{C}}_n$ and $r^{\mathbf{CB}}_n$.

To implement an uninformative equilibrium, E must set a design (n, r) satisfying $r \geq \max\{r^{\mathbf{B}}_n(x_{\mathcal{B}}, x_{\mathcal{G}}), r^{\mathbf{CB}}_n(x_{\mathcal{B}}, x_{\mathcal{G}})\}$ where $r^{\mathbf{B}}_n(x_{\mathcal{B}}, x_{\mathcal{G}})$ sets (1) equal to 0:

$$r^{\mathbf{B}}_n(x_{\mathcal{B}}, x_{\mathcal{G}}) = \frac{(1-\mu)S^N_{n-1}(x_{\mathcal{B}})}{\mu S^N_{n-1}(x_{\mathcal{G}})} \tag{10}$$

This is the minimal interest rate that makes blind-bidding as good as avoidance. Designs must also respect $r \leq R$, obliged by E's lack of credit. This automatically holds when E maximizes profit and the relaxed problem also suffices when we seek upper bounds.

3.1 Uninformative equilibria

In an uninformative equilibrium, $p_{\mathbf{C}} = 0$ so $x_{\mathcal{B}} = x_{\mathcal{G}}$ for some $0 \leq x_{\mathcal{G}} \leq 1$. The expected investment scale is then state-independent, $I^N_n(x_{\mathcal{B}}) = I^N_n(x_{\mathcal{G}})$, giving welfare

$$w(n, x_{\mathcal{B}}, x_{\mathcal{G}}) = I^N_n(x_{\mathcal{G}})\big[\mu R - (1-\mu)\big] \tag{11}$$

The square-bracketed term equals $w^{\mathbf{B}}$ of (5). If $w^{\mathbf{B}} \leq 0$, **A**, that is, $(x_{\mathcal{B}}, x_{\mathcal{G}}) = (0, 0)$, is an optimal uninformative equilibrium, as $x_{\mathcal{G}} = 0$ minimizes $I^N_n(x_{\mathcal{G}})$ at 0. Pure strategy **A** can always be implemented with $r = 0$ and any n, giving $\pi = w = w^{\mathbf{A}} = 0$.

If $w^{\mathbf{B}} > 0$ and if E can implement **B**, $(x_{\mathcal{B}}, x_{\mathcal{G}}) = (1, 1)$, she maximizes welfare by maximizing $I^N_n(x_{\mathcal{G}})$ at 1 with $x_{\mathcal{G}} = 1$, yielding $w^{\mathbf{B}}$. To implement **B** with design (n, r) requires $r \geq \max\{r^{\mathbf{B}}_n(1, 1), r^{\mathbf{CB}}_n(1, 1)\}$. When $c \geq \bar{c}$, $r^{\mathbf{B}}_n(1, 1) \geq r^{\mathbf{CB}}_n(1, 1)$ so any $r \geq r^{\mathbf{B}}_n(1, 1) = r^{\mathbf{B}}_1 = (1-\mu)/\mu$ can implement $(1, 1)$, giving $w = w^{\mathbf{B}}$. Extracting bidder rent via $r = r^{\mathbf{B}}_1$, E maximizes profit at $\pi = w = w^{\mathbf{B}}$. As $w^{\mathbf{B}} > 0$ implies $r^{\mathbf{B}}_1 < R$, this respects the credit constraint. Denoting uninformative equilibria with superscript U, this proves,

Proposition 3. *For $c \geq \bar{c}$, optimal uninformative equilibria maximize profit and welfare at* $\pi^{\mathrm{U}} = w^{\mathrm{U}} = \max\{0, \mu R - (1-\mu)\} = \max\{0, w^{\mathbf{B}}\}$.

When $c < \bar{c}$, $(1,1)$ may not be implementable but $\max\{0, w^{\mathbf{B}}\}$ is still an upper bound:

Proposition 4. *For $c < \bar{c}$, profit and welfare from uninformative equilibria are both bounded above by* $\overline{\pi}^{\mathrm{U}} = \overline{w}^{\mathrm{U}} = \max\{0, w^{\mathbf{B}}\}$.

These upper bounds prove sufficient in the large crowd setting since we will show that informative equilibria outperform them on $c < \bar{c}$.

3.2 Informative equilibria

Informative equilibria require (C) to hold so $I_n^N(x_{\mathcal{B}}) = x_{\mathcal{B}} S_{n-1}^N(x_{\mathcal{B}}) \geq x_{\mathcal{B}}\, c/\bar{c}$. As $S_{n-1}^N(x_{\mathcal{G}}) \leq 1$, $I_n^N(x_{\mathcal{G}}) = x_{\mathcal{G}} S_{n-1}^N(x_{\mathcal{G}}) \leq x_{\mathcal{G}}$. Substituting these into (4) gives

$$w(n, x_{\mathcal{B}}, x_{\mathcal{G}}) \leq \ \mu R x_{\mathcal{G}} - (1-\mu)\, x_{\mathcal{B}} \frac{c}{\bar{c}} - c\, \frac{x_{\mathcal{G}} - x_{\mathcal{B}}}{\rho}$$

The $x_{\mathcal{B}}$ terms cancel out and $x_{\mathcal{G}} < 1$, so, using superscript I for informative,

$$w(n, x_{\mathcal{B}}, x_{\mathcal{G}}) \leq x_{\mathcal{G}} \left[\mu R - \frac{c}{\rho}\right] \leq \ \overline{w}^{\mathrm{I}} = \max\{0, \mu R - c/\rho\} \tag{12}$$

and $w(n, x_{\mathcal{B}}, x_{\mathcal{G}}) < \overline{w}^{\mathrm{I}}$ if $\overline{w}^{\mathrm{I}} > 0$. Profit is welfare minus bidder rent so $\pi \leq w$ and $\overline{\pi}^{\mathrm{I}} = \overline{w}^{\mathrm{I}}$.

Proposition 5. *Profit and welfare from informative equilibria are bounded above by* $\overline{\pi}^{\mathrm{I}} = \overline{w}^{\mathrm{I}} = \max\{0, \mu R - c/\rho\}$. *This upper bound* $\overline{w}^{\mathrm{I}} > 0$ *if* $c < \hat{c} = \mu R \rho$.

Informative equilibria are strictly advantageous when $c < \min\{\bar{c}, \hat{c}\}$.[11] The next section shows that E can then asymptotically approach $\overline{w}^{\mathrm{I}}$, so we record two facts.

Lemma 1. *On $c < \min\{\bar{c}, \hat{c}\}$, (a) $\overline{w}^{\mathrm{I}} > \max\{0, w^{\mathbf{B}}, w^{\mathbf{C}}\} \geq \overline{w}^{\mathrm{U}}$; (b) E cannot attain $\overline{w}^{\mathrm{I}}$ in a finite crowd. On $c \geq \min\{\bar{c}, \hat{c}\}$, uninformative equilibria suffice for the optimum.*

Proof. (*a*) $c < \bar{c} \Leftrightarrow c/\rho < (1-\mu)$ so $\overline{w}^{\mathrm{I}} > 0 \Rightarrow \overline{w}^{\mathrm{I}} > w^{\mathbf{B}}$ and $\overline{w}^{\mathrm{I}} - w^{\mathbf{C}} = (\mu R - c/\rho)\gamma + (\bar{c} - c)/\rho > 0$. (*b*) Objectives w, π are continuous functions on a compact space, so a maximum exists for any finite N. E cannot attain $\overline{w}^{\mathrm{I}} > 0$ with $x_{\mathcal{G}} < 1$ as the second inequality of (12) is then strict. Similarly, $x_{\mathcal{G}} = 1$ implies $x_{\mathcal{B}} = 1$ so $w = w(n,1,1) = w^{\mathbf{B}} < \overline{w}^{\mathrm{I}}$ here. On $c \geq \min\{\bar{c}, \hat{c}\}$, $\overline{w}^{\mathrm{I}} \leq 0$ or cannot be implemented; Proposition 3 characterizes E's overall optimized profit and welfare outcomes and crowd size is irrelevant. ■

[11] In the knife-edge case with $c = \bar{c}$, condition (C) requires $n = 1$ so there is no crowdblessing and uninformative equilibria weakly dominate as $w^{\mathbf{C}} = (1-\gamma) w^{\mathbf{B}}$ at $\bar{c}$.

4 Asymptotic results for large crowds

The results of the previous section are all independent of crowd size $N+1$ so they also apply to the large crowd limit. This section establishes the asymptotic limit on what is implementable using informative equilibria when $c < \min\{\bar{c}, \hat{c}\}$. Crowd size matters and we now make it explicit, e.g., replacing $w(n, x_{\mathcal{B}}, x_{\mathcal{G}})$ with $w(n, N, x_{\mathcal{B}}, x_{\mathcal{G}})$.

We will need two standard asymptotic results that apply to any bidder. In equilibrium, each bidder believes that the N other bidders follow strategies $(x_{\mathcal{B}}, x_{\mathcal{G}})$. So, using superscript N to indicate these others, the number X_ω^N of them who bid in state $\omega = \mathcal{B}, \mathcal{G}$ follows a Binomial distribution. In each state, individual bids are independent draws from the same Bernoulli distribution, mean x_ω, variance $\sigma_\omega^2 = x_\omega(1-x_\omega)$. For the zero cost case, it suffices to apply the strong law of large numbers (SLLN) to the sample average (see e.g., Billingsley, 1995, p. 282):

SLLN
$$\bar{X}_\omega^N = \frac{X_\omega^N}{N} \xrightarrow{\text{a.s.}} x_\omega \quad \text{as } N \to \infty \tag{13}$$

For $c > 0$, we further need the central limit theorem (CLT) (Billingsley, 1995, p. 357):

$$Z_\omega^N = \frac{X_\omega^N - N x_\omega}{\sqrt{N x_\omega (1 - x_\omega)}} = \frac{\sqrt{N}\left(\bar{X}_\omega^N - x_\omega\right)}{\sqrt{x_\omega(1-x_\omega)}} = \frac{\sqrt{N}}{\sigma_\omega}\left(\bar{X}_\omega^N - x_\omega\right) \tag{14}$$

converges uniformly to the standard Normal distribution $\mathcal{N}(0,1)$ as $N \to \infty$. So that with Φ denoting the standard Normal CDF (and ϕ denoting its PDF), for any $z \in \mathbb{R}$,

CLT
$$\lim_{N\to\infty} \mathbb{P}\left[Z_\omega^N \le z\right] = \Phi(z) \tag{15}$$

To establish the asymptotic limits of optimal second-best crowdfunding with and without inspection costs, we develop a pair of constructive proofs that follow the same three-step structure. In the first step, we take an informative strategy $(x_{\mathcal{B}}, x_{\mathcal{G}})$ and present a threshold sequence n_N that optimizes the limit values of the state-contingent success rates. Defining $S_\infty(x_\omega) = \lim_{N\to\infty} S_{n_N-1}^N(x_\omega)$ for $\omega = \mathcal{B}, \mathcal{G}$, denoted S_ω for brevity, these are $S_{\mathcal{B}} = 0, S_{\mathcal{G}} = 1$ when $c = 0$ and $S_{\mathcal{B}} = c/\bar{c}, S_{\mathcal{G}} = 1$ in the case where $c > 0$. We also compute the implied welfare. In the second step, we identify a sequence of feasible interest rates that enables E to implement the strategy from step 1 as an equilibrium for each crowd size above some finite value. With this, we compute implied profit. In the third step, we identify how to move the initial strategy $(x_{\mathcal{B}}, x_{\mathcal{G}})$ so that limit welfare and profit converge to their asymptotic optima.

The next two subsections present the formal results and proofs with little further commentary, starting with the case of free inspection.

4.1 Asymptotic optima with free inspection

When inspection costs nothing, step 1 can use a threshold sequence n_N that splits the bid distributions of the bad and good states mid-way.[12] By the SLLN, E asymptotically achieves $S_{\mathcal{B}} = 0$ and $S_{\mathcal{G}} = 1$ for any informative strategy. This guarantees feasibility of the interest rate sequence required for incentive compatibility and a limit rent of zero in step 2. Step 3 delivers the ideal outcome by sending the inspection probability to zero.

Proposition 6. *When inspection costs nothing* ($c = 0$)*, maximized welfare and maximized profit involve informative equilibria that asymptotically generate* $w = \pi = \overline{w} = \mu R$*.*

Proof. The next three steps show how the entrepreneur can implement an equilibrium giving welfare w and profit π arbitrarily close to $\overline{w} = \mu R$.

Step 1. Fix a strategy $(x_{\mathcal{B}}, x_{\mathcal{G}})$ that strictly mixes **B** and **C** by setting $x_{\mathcal{G}} \in (1-\gamma, 1)$ and $x_{\mathcal{B}} : (1 - x_{\mathcal{G}})/(1 - x_{\mathcal{B}}) = \gamma/(1-\beta)$. This ensures that $x_{\mathcal{B}} \in (\beta, 1)$, $p_{\mathbf{A}} = 0$ and $p_{\mathbf{B}}, p_{\mathbf{C}} \in (0, 1)$. Given this strategy, we show that setting threshold

$$n_N = \left\lceil \frac{N}{2}(x_{\mathcal{B}} + x_{\mathcal{G}}) \right\rceil \tag{16}$$

for each N is one way to guarantee that, asympotically as N gets large, campaigns fail for bad projects and succeed for good ones. This claim follows from the SLLN (13) because a bidder knows that if he bids and the project is bad then it will succeed with probability

$$S^N_{n_N-1}(x_{\mathcal{B}}) = \mathbb{P}\left[X^N_{\mathcal{B}} \geq n_N - 1\right] = \mathbb{P}\left[\overline{X}^N_{\mathcal{B}} \geq \nu_N\right]$$

where

$$\nu_N = \frac{n_N - 1}{N} \in \left[\frac{1}{2}(x_{\mathcal{B}} + x_{\mathcal{G}}) - \frac{1}{N}, \frac{1}{2}(x_{\mathcal{B}} + x_{\mathcal{G}})\right)$$

This is the fraction of other bidders that make a given bidder pivotal for activating investment. Since $x_{\mathcal{G}} > x_{\mathcal{B}}$, $\nu_N - x_{\mathcal{B}} \geq \frac{1}{2}(x_{\mathcal{G}} - x_{\mathcal{B}}) - \frac{1}{N} > \frac{x_{\mathcal{G}} - x_{\mathcal{B}}}{4} > 0$ for $N > \frac{4}{x_{\mathcal{G}} - x_{\mathcal{B}}}$. So by (13),

$$\lim_{N\to\infty} S^N_{n_N-1}(x_{\mathcal{B}}) \leq \lim_{N\to\infty} \mathbb{P}\left[\overline{X}^N_{\mathcal{B}} - x_{\mathcal{B}} \geq \frac{x_{\mathcal{G}} - x_{\mathcal{B}}}{4}\right] = 0$$

Similarly, $\nu_N - x_{\mathcal{G}} < \frac{1}{2}(x_{\mathcal{B}} - x_{\mathcal{G}}) < 0$, so when the project is good, by (13),

$$\lim_{N\to\infty} S^N_{n_N-1}(x_{\mathcal{G}}) \geq \lim_{N\to\infty} \mathbb{P}\left[\overline{X}^N_{\mathcal{G}} - x_{\mathcal{G}} \geq \frac{x_{\mathcal{B}} - x_{\mathcal{G}}}{2}\right] = 1$$

This shows that $\lim_{N\to\infty} S^N_{n_N-1}(x_{\mathcal{B}}) = 0$ and $\lim_{N\to\infty} S^N_{n_N-1}(x_{\mathcal{G}}) = 1$. It follows that $I^N_{n_N}(x_{\mathcal{B}}) \to 0$ and $I^N_{n_N}(x_{\mathcal{G}}) \to x_{\mathcal{G}}$ so that by (4) with $c = 0$, letting $w_N(x_{\mathcal{B}}, x_{\mathcal{G}}) =$

[12] Note that designs and equilibria need not be optimal for finite crowds along the sequence.

$w(n_N, N, x_{\mathcal{B}}, x_{\mathcal{G}})$, welfare converges to

$$w_\infty(x_{\mathcal{B}}, x_{\mathcal{G}}) = \lim_{N\to\infty} w_N(x_{\mathcal{B}}, x_{\mathcal{G}}) = \lim_{N\to\infty} w(n_N, N, x_{\mathcal{B}}, x_{\mathcal{G}}) = \mu x_{\mathcal{G}} R \tag{17}$$

Step 2. We now verify that for any large enough N, E can implement $(x_{\mathcal{B}}, x_{\mathcal{G}})$ as an equilibrium with crowdfunding design (n_N, r_N) where n_N is given by (16) and rate r_N is given by setting $c = 0$ in (8): $r_N = r^{\mathbf{CB}}_{n_N}(N, x_{\mathcal{B}}, x_{\mathcal{G}}) = \frac{(1-\mu)(1-\beta)S^N_{n_N-1}(x_{\mathcal{B}})}{\mu\gamma S^N_{n_N-1}(x_{\mathcal{G}})}$. As $N \to \infty$, this rate $r_N \to 0$ from above, so for large enough N, $r_N \in [0, R]$, as required by E's credit constraint. Since $c = 0$, condition (**C**) holds automatically and since $\frac{1-\beta}{\gamma} > 1 > \frac{\beta}{1-\gamma}$, $r_N = r^{\mathbf{CB}}_{n_N} > r^{\mathbf{B}}_{n_N} > r^{\mathbf{C}}_{n_N}$ at $(x_{\mathcal{B}}, x_{\mathcal{G}})$, from (7), (8) and (10). So bidders are willing to mix on **B** and **C**, ensuring that $(x_{\mathcal{B}}, x_{\mathcal{G}})$ is an equilibrium. In addition, as $r_N \to 0$, bidder rent converges to zero and $\pi_\infty(x_{\mathcal{B}}, x_{\mathcal{G}}) = w_\infty(x_{\mathcal{B}}, x_{\mathcal{G}})$.

Step 3. Setting $x_{\mathcal{G}}$ arbitrarily close to 1 in step 1 brings the limit welfare $\mu x_{\mathcal{G}} R$ arbitrarily close to μR. Letting $w_\infty = \lim_{x_{\mathcal{G}}\to 1} w_\infty(x_{\mathcal{B}}, x_{\mathcal{G}})$, we conclude that $w_\infty = \overline{w} = \mu R$. By step 2, we also have $\pi_\infty = \overline{w}$. ■

4.2 Asymptotic optima with costly inspection

Now we turn to the harder challenge: implementation with costly inspection. For a given threshold, E again ensures incentive compatibility by setting $r_N = r^{\mathbf{CB}}_{n_N}$ but $c > 0$ obliges her to lower the threshold n_N to satisfy condition (**C**). Thanks to this reduced threshold n_N, it turns out that $r_N = r^{\mathbf{CB}}_{n_N}$ again ensures participation despite the positive cost. While condition (**C**) now blocks E from achieving $S_{\mathcal{B}} = 0$, we show that she can maintain $S_{\mathcal{G}} = 1$ and minimize investment in the bad state subject to (**C**) by asymptotically implementing $S_{\mathcal{B}} = c/\bar{c}$. In this way, E achieves the upper bound of Proposition 5.

Proposition 7. *For $0 < c < \bar{c}$, welfare and profit from informative equilibria asymptotically implement $w = \pi = \overline{w}^{\mathrm{I}} = \max\{0, \mu R - c/\rho\}$ whenever $\overline{w}^{\mathrm{I}} > 0$.*

Proof. The following three steps show that the entrepreneur can implement an equilibrium giving welfare w and profit π arbitrarily close to $\overline{w}^{\mathrm{I}} > 0$. Recall that $\rho = 1 - \beta - \gamma$ so that if $\overline{w}^{\mathrm{I}} > 0$, $\overline{w}^{\mathrm{I}} = \mu R - c/\rho$.

Step 1. Exactly as in the proof of Proposition 6, we consider a strategy $(x_{\mathcal{B}}, x_{\mathcal{G}})$ that mixes strictly between **B** and **C**, but must now set the thresholds n_N differently. For any N, we set n_N equal to the unique $n \in \{1, 2, \ldots, N+1\}$ satisfying

$$S^N_n(x_{\mathcal{B}}) < c/\bar{c} \leq S^N_{n-1}(x_{\mathcal{B}}) \tag{18}$$

This is well-defined because at $n = 1$, $S^N_{n-1}(x_{\mathcal{B}}) = 1 > c/\bar{c}$ since $c < \bar{c}$ and at $n = N+1$, $S^N_n(x_{\mathcal{B}}) = 0 < c/\bar{c}$ since $c > 0$. Equivalently, n_N is the largest threshold at which condition (**C**) holds at $x_{\mathcal{B}}$.

To study asymptotic success rates in each state, we again let $\nu_N = (n_N - 1)/N$, the fractional threshold on other bidders that makes a given bidder pivotal given threshold sequence n_N. As when $c = 0$, we can prove that $S^N_{n_N-1}(x_\mathcal{G}) \to 1$ as $N \to \infty$ but $c > 0$ precludes hoping for $S^N_{n_N-1}(x_\mathcal{B}) \to 0$. Nonetheless, using the CLT (15), we can prove that this threshold sequence achieves $S^N_{n_N-1}(x_\mathcal{B}) \to c/\bar{c}$. That is, it minimizes undesirable activation of bids by ensuring that (**C**) holds asymptotically with equality. We prove these results via a series of 3 claims using also the SLLN (13) and identities,

$$S^N_{n_N-1}(x_\mathcal{B}) = \mathbb{P}\left[X^N_\mathcal{B} \geq n_N - 1\right] = \mathbb{P}\left[\overline{X}^N_\mathcal{B} \geq \nu_N\right] = \mathbb{P}\left[Z^N_\mathcal{B} \geq \frac{\sqrt{N}}{\sigma_\mathcal{B}}\left(\nu_N - x_\mathcal{B}\right)\right] \tag{19}$$

$$S^N_{n_N}(x_\mathcal{B}) = \mathbb{P}\left[X^N_\mathcal{B} \geq n_N\right] = \mathbb{P}\left[\overline{X}^N_\mathcal{B} \geq \nu_N + \frac{1}{N}\right] \tag{20}$$

$$S^N_{n_N-1}(x_\mathcal{G}) = \mathbb{P}\left[\overline{X}^N_\mathcal{G} \geq \nu_N\right] \tag{21}$$

Claim 1. $\lim_{N\to\infty} \nu_N = x_\mathcal{B}$.

Proof. Suppose that the claim is false. Since $1 \leq n_N \leq N+1$, $\nu_N \in [0,1]$ and so ν_N must have a subsequence that converges to some $x'_\mathcal{B} \neq x_\mathcal{B}$.

Suppose $x'_\mathcal{B} > x_\mathcal{B}$. Without loss, we suppose the sequence itself converges: $\nu_N \to x'_\mathcal{B}$. So for sufficiently large N, $\nu_N > (x'_\mathcal{B} + x_\mathcal{B})/2 > x_\mathcal{B}$. Applying (13) gives

$$\lim_{N\to\infty} S^N_{n_N-1}(x_\mathcal{B}) = \lim_{N\to\infty} \mathbb{P}\left[\overline{X}^N_\mathcal{B} \geq \nu_N\right] \leq \lim_{N\to\infty} \mathbb{P}\left[\overline{X}^N_\mathcal{B} \geq \frac{x'_\mathcal{B} + x_\mathcal{B}}{2}\right] = 0$$

which contradicts the fact that $S^N_{n_N-1}(x_\mathcal{B}) \geq c/\bar{c} > 0$.

Suppose instead that $x'_\mathcal{B} < x_\mathcal{B}$. By similar arguments, for sufficiently large N, $\nu_N + \frac{1}{N} < (x'_\mathcal{B} + x_\mathcal{B})/2 < x_\mathcal{B}$. Applying (13) gives

$$\lim_{N\to\infty} S^N_{n_N}(x_\mathcal{B}) = \lim_{N\to\infty} \mathbb{P}\left[\overline{X}^N_\mathcal{B} \geq \nu_N + \frac{1}{N}\right] \geq \lim_{N\to\infty} \mathbb{P}\left[\overline{X}^N_\mathcal{B} > \frac{x'_\mathcal{B} + x_\mathcal{B}}{2}\right] = 1$$

which contradicts the fact that $S^N_{n_N}(x_\mathcal{B}) < c/\bar{c} < 1$. □

Claim 2. $\lim_{N\to\infty} S^N_{n_N-1}(x_\mathcal{B}) = c/\bar{c}$.

Proof. For any $y \in \mathbb{R}$ and $\delta > 0$, by the CLT (15), we have

$$\lim_{N\to\infty} \mathbb{P}\left[y \leq Z^N_\mathcal{B} < y + \delta\right] = \Phi\left(y+\delta\right) - \Phi\left(y\right) \leq \delta\phi(0)$$

The convergence is uniform, so letting $\delta = 1/(\sigma_{\mathcal{B}}\sqrt{N})$ and $y = \sqrt{N}\,(\nu_N - x_{\mathcal{B}})\,/\sigma_{\mathcal{B}}$,

$$\begin{aligned}
\lim_{N\to\infty} S^N_{n_N-1}(x_{\mathcal{B}}) - S^N_{n_N}(x_{\mathcal{B}}) &= \lim_{N\to\infty} \mathbb{P}\left[\nu_N \le \overline{X}^N_{\mathcal{B}} < \nu_N + \frac{1}{N}\right] \\
&= \lim_{N\to\infty} \mathbb{P}\left[\frac{\sqrt{N}}{\sigma_{\mathcal{B}}}\,(\nu_N - x_{\mathcal{B}}) \le Z^N_{\mathcal{B}} < \frac{\sqrt{N}}{\sigma_{\mathcal{B}}}\left(\nu_N + \frac{1}{N} - x_{\mathcal{B}}\right)\right] \\
&\le \lim_{N\to\infty} \frac{\phi(0)}{\sigma_{\mathcal{B}}\sqrt{N}} = 0
\end{aligned}$$

So by (18), $S^N_{n_N-1}(x_{\mathcal{B}}) \to c/\bar{c}$. □

Claim 3. $\lim_{N\to\infty} S^N_{n_N-1}(x_{\mathcal{G}}) = 1$.

Proof. We now apply the SLLN to the fraction $\overline{X}^N_{\mathcal{G}}$ of others' bids in the good state $\mathcal{G}$. Since $\nu_N \to x_{\mathcal{B}}$ by Claim 1, $\nu_N < \frac{x_{\mathcal{B}}+x_{\mathcal{G}}}{2} < x_{\mathcal{G}}$ for large enough N. So again, by (13),

$$\lim_{N\to\infty} S^N_{n_N-1}(x_{\mathcal{G}}) \ge \lim_{N\to\infty} \mathbb{P}\left[\overline{X}^N_{\mathcal{G}} \ge \frac{x_{\mathcal{B}}+x_{\mathcal{G}}}{2}\right] = 1$$

and we conclude that $\lim_{N\to\infty} S^N_{n_N-1}(x_{\mathcal{G}}) = 1$. □

By Claims 2 and 3, $I^N_{n_N}(x_{\mathcal{G}}) \to x_{\mathcal{G}}$ and $I^N_{n_N}(x_{\mathcal{B}}) \to x_{\mathcal{B}}\, c/\bar{c}$, so welfare (4) converges to

$$\begin{aligned}
w_\infty(x_{\mathcal{B}}, x_{\mathcal{G}}) = \lim_{N\to\infty} w(n_N, N, x_{\mathcal{B}}, x_{\mathcal{G}}) &= \mu x_{\mathcal{G}} R - (1-\mu) x_{\mathcal{B}} \frac{c}{\bar{c}} - c\, \frac{x_{\mathcal{G}} - x_{\mathcal{B}}}{\rho} \\
&= x_{\mathcal{G}} \left[\mu R - \frac{c}{\rho}\right] \\
&= x_{\mathcal{G}}\ \overline{w}^{\mathrm{I}} \qquad (22)
\end{aligned}$$

The $x_{\mathcal{B}}$ term cancels out nicely because $\bar{c} = (1-\mu)\rho$.

Step 2. To verify the feasibility of implementation of $(x_{\mathcal{B}}, x_{\mathcal{G}})$ for large enough N using the new threshold sequence n_N defined by (18), we set rate r_N to satisfy (8):

$$r_N = r^{\mathbf{CB}}_{n_N}(N, x_{\mathcal{B}}, x_{\mathcal{G}}) = \frac{(1-\mu)(1-\beta) S^N_{n_N-1}(x_{\mathcal{B}}) - c}{\mu\gamma S^N_{n_N-1}(x_{\mathcal{G}})}$$

Using Claims 2 and 3, as $N \to \infty$,

$$r_N \to \frac{(1-\mu)(1-\beta)c/\bar{c} - c}{\mu\gamma} = \frac{c(1-\beta-\rho)}{\mu\rho\gamma} = \frac{c}{\mu\rho}$$

Now $c/\mu\rho < R$ since $\overline{w}^{\mathrm{I}} > 0$. So for large enough N, $r_N \in [0, R]$, as required by E's credit constraint. To verify equilibrium, first note that condition (**C**) obviously holds by construction – see (18). Second, substituting $c \le \bar{c}\, S^N_{n_N-1}(x_{\mathcal{B}})$ into (8) and (7), simplifying and comparing with (10) shows that $r_N = r^{\mathbf{CB}}_{n_N} \ge r^{\mathbf{B}}_{n_N} \ge r^{\mathbf{C}}_{n_N}$ at $(x_{\mathcal{B}}, x_{\mathcal{G}})$ so bidders are willing to mix according to the equilibrium defined by $(x_{\mathcal{B}}, x_{\mathcal{G}})$.

Step 3. Setting $x_{\mathcal{G}}$ arbitrarily close to 1 in step 1 brings the limit welfare of (22), $w_\infty(x_{\mathcal{B}}, x_{\mathcal{G}}) = x_{\mathcal{G}}\,\overline{w}^{\mathrm{I}}$, arbitrarily close to $\overline{w}^{\mathrm{I}}$ so that $w_\infty = \lim_{x_{\mathcal{G}}\to 1} w_\infty(x_{\mathcal{B}}, x_{\mathcal{G}}) = \overline{w}^{\mathrm{I}}$.

Finally, just as $r_N \to c/\mu\rho$ in step 2, substituting $\lim_{N\to\infty} S^N_{n_N-1}(x_{\mathcal{B}}) = c/\bar{c}$ into (7) and (10) readily shows that $r^{\mathbf{B}}_{n_N}(N, x_{\mathcal{B}}, x_{\mathcal{G}}), r^{\mathbf{C}}_{n_N}(N, x_{\mathcal{B}}, x_{\mathcal{G}})$ also converge on $c/\mu\rho$. It follows that bidder rent converges to 0 at $(x_{\mathcal{B}}, x_{\mathcal{G}})$ and so $\pi_\infty = w_\infty$. Raising $x_{\mathcal{G}}$ to approach 1 therefore delivers π_∞ arbitrarily close to $\overline{w}^{\mathrm{I}} = \mu R - c/\rho$. ∎

This positive cost result might appear to but does not embed the zero cost result. The threshold constructions are necessarily distinct. The asymptotically optimized outcomes are nonetheless continuous in cost c at $c = 0$ because $S_{\mathcal{B}} = 0$ in Proposition 6 and $S_{\mathcal{B}} = c/\bar{c}$ converges to 0 as c goes to 0 in Proposition 7.

4.3 The overall asymptotic optimum

To characterize the asymptotic optimum over both equilibrium types, we let

$$\pi(N) = \max_{n,r,x_{\mathcal{B}},x_{\mathcal{G}}} \pi(n, N, r, x_{\mathcal{B}}, x_{\mathcal{G}}) \text{ s.t. } (x_{\mathcal{B}}, x_{\mathcal{G}}) \text{ is a Nash equilibrium for } (n, r) \text{ given } N$$

denote the optimized profit from crowdfunding for crowd size $N + 1$. We define $w(N)$ analogously. Since any equilibrium is either informative or uninformative, taking the maximum of the upper bounds from Propositions 3 to 5 reveals that $\pi(N) \le w(N) \le \max\{\overline{w}^{\mathrm{U}}, \overline{w}^{\mathrm{I}}\} = \max\{0, w^{\mathbf{B}}, \mu R - c/\rho\}$. Since independent of crowd size, this upper bound holds for arbitrarily large crowds. We now show that it is asymptotically tight.

Theorem 1. *The optimized welfare and optimized profit from crowdfunding with an asymptotically large crowd satisfy*

$$\lim_{N\to\infty} \pi(N) = \lim_{N\to\infty} w(N) = \max\{0, \mu R - (1-\mu), \mu R - c/\rho\} = \max\{w^{\mathbf{A}}, w^{\mathbf{B}}, \overline{w}^{\mathrm{I}}\}$$

As suggested by the last expression, when the maximum equals:
(a) $w^{\mathbf{A}} = 0$, E optimally implements **A**
(b) $w^{\mathbf{B}} > 0$, E optimally implements **B**
(c) $\overline{w}^{\mathrm{I}} > \max\{0, w^{\mathbf{B}}\}$, E optimally implements a mix of **B** *and* **C** *to approach the limit.*

Proof. It remains to show how E reaches the maximum.
(a) If the maximum is zero, E can optimally implement **A** since **A** is implementable for any N and gives $\pi(N) = w(N) = 0$, so $\lim_{N\to\infty} \pi(N) = \lim_{N\to\infty} w(N) = w^{\mathbf{A}} = 0$.
(b) If the maximum is $w^{\mathbf{B}} = \mu R - (1-\mu) > 0$, then $(1-\mu) \le c/\rho$, or equivalently, $c \ge \bar{c}$, so E can implement **B** paying zero rent for any crowd size $N + 1$ by Proposition 3. This yields $\pi = w = w^{\mathbf{B}}$ so $\lim_{N\to\infty} \pi(N) = \lim_{N\to\infty} w(N) = w^{\mathbf{B}}$.
(c) Finally, if $\overline{w}^{\mathrm{I}} > 0$ and $\mu R - c/\rho > \mu R - (1-\mu)$, then $c < \bar{c}$, and by Proposition 6

(for $c = 0$) and by Proposition 7 (for $c > 0$), E can optimally attain $\overline{w}^{\mathrm{I}}$ via a sequence of informative equilibria that mix on **B** and **C**. ■

This theorem's proof and Lemma 1(a) show that optimized profit and welfare from an asymptotically large crowd depend on inspection cost c as follows:[13]

Corollary 1. *(a) For $c < \min\{\bar{c}, \hat{c}\}$, asymptotically optimal crowdfunding uses informative equilibria to maximize profit and welfare at $\overline{\pi}^{\mathrm{I}} = \overline{w}^{\mathrm{I}} = \mu R - c/\rho$.*

(b) If $c \in (\hat{c}, \bar{c})$, nobody inspects and optimal crowdfunding is equivalent to posted offers, yielding trivial uninformative equilibria with $\pi^{\mathrm{U}} = w^{\mathrm{U}} = 0$.

(c) For $c > \bar{c}$, optimal crowdfunding is again equivalent to posted offers, yielding uninformative equilibria that maximize profit and welfare at $\pi^{\mathrm{U}} = w^{\mathrm{U}} = \max\{0, w^{\mathbf{B}}\}$.

The central lesson is that crowdfunding with an asymptotically large crowd delivers strict gains over posted offers when the inspection cost satisfies case (a)'s two necessary conditions: $c < \bar{c}$ and $c < \hat{c}$. By Proposition 2, the first allows E to implement informative equilibria with a non-trivial threshold $n > 1$ so that the crowd plays a role. The second allows this to generate a positive surplus by bounding the excess bad state activation required by condition (**C**) since $S_{\mathcal{B}} = c/\bar{c}$ is then below $\mu R/(1-\mu)$.

The intuition for why optimal crowdfunding dominates posted offers in the large crowd limit, given moderate costs, is that crowdblessing generates strict positive externalities. In the limit, good-state activation is undistorted at $S_{\mathcal{G}} = 1$ and bad-state activation $S_{\mathcal{B}} < 1$ since $c < \bar{c}$. So the crowdfunding threshold strictly improves on posted offers with blind bids that are always activated; the per capita burden of inspection costs becomes negligible as does missing out by inspectors with bad signals. Compared to posted offers that induce inspection, crowdfunding blessing can similarly reduce wasteful investment and optimal crowdfunding does even better.

We highlight two points. $\overline{w}^{\mathrm{I}}$ depends on β and γ only through the compound measure of precision ρ. So ρ is a sufficient statistic for the optimal asymptotic outcomes, even though the specific decomposition between the two types of error is important in finite crowds. Large crowds raise the benefit from informative crowdfunding in that, for any finite crowd, by Lemma 1(b), there is a larger crowd with a greater crowdfunding surplus. When instead condition (**C**) is prohibitive, posted offers are optimal. These induce non-trivial outcomes (blind-bidding) so long as bidders are optimistic enough, but there is no role for the crowd. So crowd size only matters in the moderate cost case where optimal equilibria are informative.

[13] Writing $\overline{w}^{\mathrm{I}} = \mu R(1 - c/\hat{c})$ and $w^{\mathbf{B}} = \mu R(1 - \bar{c}/\hat{c})$ neatly clarifies how case (a) suffices for $\overline{w}^{\mathrm{I}}$ to be maximal and why $w^{\mathbf{B}} < 0$, so that $w^{\mathrm{U}} \leq 0$, when case (b) is non-empty.

5 First-best

As noted above, the ideal outcome for entrepreneur E and the bidders is for all bidders to invest when the state is good, for none to invest when it is bad and for nobody to pay any inspection costs. This would give $w = \overline{w} = \mu R$. Of course, some inspection is vital for adapting investment to the state, so a positive cost $c > 0$ certainly prevents the finite crowd first-best from achieving this ideal; imperfect signals also preclude the ideal outcome with $c = 0$ for finite N. Nonetheless, as the crowd grows large, by the strong law (SLLN), a vanishing fraction of inspecting bidders suffices to reveal the state almost perfectly and E asymptotically achieves ideal investment outcome at negligible cost in the first-best, for any cost c.

Formally, E's first-best problem is to optimize her objective when free from incentive constraints, though still subject to standard ex ante bidder participation constraints so first-best profit-maximization is meaningful. She can tell any subset of bidders to inspect and report their signals honestly and can instruct bidders to invest as a function of all the reported signals. Optimally she chooses a number $M \leq N + 1$ to inspect and then instructs all $N+1$ to invest in her project if the number of good signals exceeds a threshold m (derived in Ellman and Hurkens (2024)).

The "CF-first-best" constrains E to assign strategies within the above crowdfunding (CF) game. E can partly replicate any finite first-best by setting threshold $n = N + 1 + m - M$ and telling M bidders to play $\mathbf{C}$ and $N + 1 - M$ to blind bid. Blind bidders plus those who inspect and see a good signal then invest exactly as in the unconstrained first-best (for any given M realized signals), but those who inspect and see a bad signal no longer invest, independent of the crowd's aggregate information. Crowdfunding wastes funds because bidders communicate their bad signals by not bidding.

The "SCF-first-best" further constrains E to choose Symmetric CF strategies, so E optimizes (n, r) and $(x_{\mathcal{B}}, x_{\mathcal{G}})$ in the CF game with incentive constraints removed. This adds two further possible inefficiencies. First, the number of bidders who actually inspect becomes stochastic. Second, the crowdfunding protocol cannot take into account the realized number of inspections, since a blind bidder is indistinguishable from a bidder who observes a good signal.

Despite these possible inefficiencies, Proposition 8 shows that all three first-best variants obtain the ideal welfare and profit outcomes in the large crowd limit. So neither the CF nor even the SCF constraint impairs the first-best. To state the results, we let $w^*_\infty, w^{CF*}_\infty, w^{SCF*}_\infty$ respectively denote the unconstrained, CF- and SCF-constrained first-best welfare limits when E maximizes welfare. Similarly, $\pi^*_\infty, \pi^{CF*}_\infty, \pi^{SCF*}_\infty$ denote a profit-maximizer's limit profits, while subscript N denotes the finite crowd variants.

5.1 Asymptotic first-best solutions

The SCF-constrained first-best follows from the proof for the free inspection second-best Proposition 6, simply omitting the need for equilibrium in step 2. The construction used there to achieve $S_{\mathcal{B}} = 0$ and $S_{\mathcal{G}} = 1$ is feasible even with a positive inspection cost, because E can ignore incentive constraints and can, for large enough N, adapt the interest rate r_N to guarantee participation, while extracting all rent. Since step 3 sends the inspection probability converging to zero, positive costs have negligible impact and E achieves the ideal profit and welfare outcomes.

Proposition 8. *The SCF-first-best asymptotically achieves ideal profit and welfare* $\pi_\infty^{SCF*} = w_\infty^{SCF*} = \overline{w} = \mu R$ *for any finite cost* c.

Proof of Proposition 8. The construction used in the proof of Proposition 6 works with the following minimal changes. In step 1, we retain n_N given by (16) but to allow for positive cost, since $p_{\mathbf{C}} = \frac{x_{\mathcal{G}} - x_{\mathcal{B}}}{\rho}$, (17) adjusts to

$$w_\infty(x_{\mathcal{B}}, x_{\mathcal{G}}) = \mu x_{\mathcal{G}} R - c\,\frac{x_{\mathcal{G}} - x_{\mathcal{B}}}{\rho} \tag{23}$$

We choose $x_{\mathcal{G}}$ close enough to 1 in step 1 to ensure that $w_\infty(x_{\mathcal{B}}, x_{\mathcal{G}}) > 0$.

In step 2, E can now neglect incentive constraints but we need to verify that the interest rate needed for ex ante bidder participation is feasible with $c > 0$ and to assess its impact on profit-maximizers. Bidders are risk neutral and any bidder's payoff from playing $(x_{\mathcal{B}}, x_{\mathcal{G}})$, when all other N bidders do the same, increases linearly in r. The payoff is strictly negative when $r = 0$ and equals welfare $w(n_N, N, x_{\mathcal{B}}, x_{\mathcal{G}})$ when $r = R$, since profit is then zero. This welfare is strictly positive for sufficiently large N as $\lim_{N\to\infty} w(n_N, N, x_{\mathcal{B}}, x_{\mathcal{G}}) = w_\infty(x_{\mathcal{B}}, x_{\mathcal{G}}) > 0$ by the choice of $(x_{\mathcal{B}}, x_{\mathcal{G}})$ in step 1. So there exists a unique $r_N \in (0, R)$ that gives zero bidder surplus and $\pi_N(x_{\mathcal{B}}, x_{\mathcal{G}}) = w_N(x_{\mathcal{B}}, x_{\mathcal{G}})$.

In step 3, we again let $x_{\mathcal{G}} - x_{\mathcal{B}} \to 0$. The cost term in (23) goes to zero so that since $x_{\mathcal{G}} \to 1$ still, E again achieves $\pi_\infty = w_\infty = \overline{w}$. ■

The unconstrained first-best result, Proposition 9, follows directly:[14]

Proposition 9. *In the first-best, as the crowd grows large, maximized profit and maximized welfare converge to their optimized full information values:* $\pi_\infty^* = w_\infty^* = \overline{w} = \mu R$.

Proof of Proposition 9. Note that $w_N^{SCF*} \leq w_N^* \leq \overline{w}$ since the ideal outcome provides a clear upper bound and the SCF constraint cannot facilitate optimization. $w_\infty^* = \overline{w}$ follows immediately from Proposition 8 which shows that $\lim_{N\to\infty} w_N^{SCF*} = w_\infty^{SCF*} = \overline{w}$. The exact same proof applies for profit. ■

In the limit, the CF restriction does not impede E for the same reason that the direct per capita cost of inspecting is trivial: welfare and profit place negligible weight on the

[14]By an identical argument, $\pi_\infty^{CF*} = w_\infty^{CF*} = \overline{w} = \mu R$.

bidders who inspect and therefore on the subset of those who miss out on investing in the good state via a private bad signal. The reason symmetry in SCF does not impede either is that the SLLN fully mitigates stochastics in the number of signals collected just as it did for stochastics in the numbers of collected signals that are good and bad.

Since the benchmark limits all achieve the ideal outcome $\overline{w} = \mu R$, we can attribute the second-best deviation from this ideal to incentive constraints. $\overline{w}^{\mathrm{I}} = \mu R = \overline{w}$ at $c = 0$ so incentive compatibility has no efficiency cost. As we discuss below, when information is free, bidders have no conflicting interests so it is intuitive that bidders should be able to coordinate on an efficient outcome without need for a benevolent planner with coercive power. When information is costly, while direct costs can essentially be alleviated to zero in the limit, wasteful activation of investments in the bad state is vital to induce information gathering.

6 Concluding discussion

This paper characterizes the optimal design and value of crowdfunding when project quality is a common value and entrepreneurs raise funds from an asymptotically large crowd of investors. Informative crowdfunding is feasible whenever the precision cost c/ρ lies below the bad project base rate $1 - \mu$. Good projects then almost always get funded but the entrepreneur's optimal threshold for aggregating bidder information is distorted downwards to ensure that some bidders inspect. This distortion leads to funding success of bad projects in proportion to the cost of precision. Under prior optimism (project viability given base rates), informative crowdfunding strictly raises profit and welfare relative to posted offer contracts. Under a pessimistic prior, strict gains require the precision cost to lie below the ideal return μR. In both cases, the first-best achieves the ideal return so the second-best welfare loss exactly equals the cost of precision.

We solved a canonical model. Below, we explain how extending beyond its simplifications adds further insights. Higher stakes relax the incentive problem and reduce the required threshold distortion, whereas decreasing returns push in the opposite direction. Continuous signals and private values do not affect the core insight but offer promising avenues for endogenizing parameters in empirical work. Our strongest result, that of the costly inspection case, is robust to communication but communication renders the crowdfunding mechanism superfluous when inspection is free. We also briefly discuss the potential value and difficulties in studying sequential play, the relevance and irrelevance of asymmetric strategies and the costs and benefits of crowdfunding's salient features relative to more general mechanisms.

Stake size. In our model, each bidder has one unit of funds and entrepreneurs impose unit stakes.[15] If instead bidders have κ units of money available to invest, E optimally adjusts her design to set a monetary threshold of $n\kappa$ instead of n. By risk neutrality and constant returns, bidders then bid κ units of funds rather than 1 unit. This scales up all payoffs except the cost term, so that the entrepreneur's objective function and implementation problem are equivalent to the original ones with c reduced to c/κ. Higher stakes relax condition (**C**) and facilitate informative equilibria. They reduce the second-best welfare distortion in per capita, per unit of funds terms: once κ exceeds $c/\min\{\bar{c},\hat{c}\}$, further stake increases raise the optimized per unit welfare $\overline{w}^{\mathrm{I}}(\kappa) = \mu R - c/\kappa\rho$. Extending beyond the constant returns case modifies this result, as we now discuss.

Decreasing returns. Assuming constant returns to scale implies that bidders only affect each other via the impact of their bids on campaign success. Constant returns are appropriate for reward-based crowdfunding where the product has an unknown but common value for consumers and production can easily be scaled up; initial increasing returns can essentially be neglected given that we study a large crowd. On the other hand, decreasing returns to scale are more realistic when an entrepreneur sells equity shares in her firm or seeks a fixed-size loan. Decreasing returns cause problems because they limit the expected stake per investor and this exacerbates inspection incentives, as explained just above.[16] This is innocuous when information is free but with strictly positive cost, decreasing returns can affect optimal outcomes. It may be optimal to limit crowd size by inducing some bidders to choose "avoid" (**A**).[17] Nonetheless, our costly inspection results do hold approximately with mild decreasing returns. In particular, if returns are constant up to some cutoff, as this cutoff grows large, our large crowd solution applies, independent of the relative rates of divergence.[18]

Continuous signals. Our model assumes binary signals with exogenous false positive and false negative rates β and γ but the insights also apply with continuous signals (bounded or unbounded). Equilibria where bidders mix between blind-bidding and inspecting are still optimal but the inspection choice involves an endogenous signal cutoff. Given an increasing likelihood ratio,[19] an informed bidder bids whenever his signal exceeds a cutoff $\bar{s}$ since higher signals carry better news about project quality.[20] The cutoff depends on rate r and, via crowdblessing, on the threshold, other bidders' mixing and

[15]The limited stake size, normalized to one, is motivated by wealth constraints and the U.S. 2012 JOBS Act (Title 17, Ch.II §227.100) that allows startups to raise funds directly from unaccredited investors via online platforms but limits their total financial stakes to a small percentage of annual income or net worth. For details, see www.ecfr.gov (last amended, 23/1/2025, accessed 25/1/2025).

[16]A winner's curse from greater rationing in the good state does not affect the key incentive constraint.

[17]In practice, entrepreneurs can simply approach fewer investors, IPO underwriters can close equity offerings and P2P borrowers close loan campaigns once fully subscribed.

[18]Per capita welfare expressions rescale by the ratio of maximum scale to crowdsize if less than unity.

[19]That is, $\phi_{\mathcal{G}}(s)/\phi_{\mathcal{B}}(s)$ is increasing in s where $\phi_{\omega}(s)$ denotes the pdf in state $\omega = \mathcal{B}, \mathcal{G}$.

[20]Our unit bid assumption prevents bidders from communicating more than binary information about their signal, but only the need for inspection incentives impedes efficient information aggregation.

cutoffs. Equilibrium conditions endogenize the cutoff and determine $\beta(\bar{s})$ and $\gamma(\bar{s})$.

Notice that $\beta(\bar{s})$ falls while $\gamma(\bar{s})$ rises with $\bar{s}$. This supports our choice to differentiate the two types of error. More importantly, crowdfunding design can be tailored to raise precision $\rho(\bar{s})$ and hence per capita welfare $\mu R - c/\rho(\bar{s})$. We conjecture that this requires leaving a rent to bidders, so profit-maximizing entrepreneurs would choose designs that induce lower information precision.

Private values. Our pure common value treatment spotlights crowdblessing in isolation but the insights readily extend to hybrid settings that also feature private values. This is important for extending beyond investment-based to donation- and reward-based crowdfunding where funders value quality but also vary in their tastes for a given good or reward. Such heterogeneity affects inspection incentives; adapting our results is straightforward provided bidders know the distribution of private tastes.

More subtly, it may be costly to learn about private as well as common values. While precise results depend on the inspection technology, our distortion insight remains fundamental. Indeed, downward threshold distortion is even needed when quality signals are effectively free as side-benefits of bidders always inspecting to learn their private taste (e.g., listening to a musical demo reveals the quality as well as type of music). The reason is that compensating for the private inspection cost requires a price discount and discounting encourages quality-blind bidding. So crowdblessing still requires the distortion central to this paper. As a result, these extensions adapt the cost term of Proposition 7 instead of expanding applicability of the Proposition 6 efficiency result.

Sequential play. Our model assumes that investors move simultaneously. In practice, most crowdfunding campaigns accept bids over several weeks and platforms publicise cumulative funds. Bidders can then update beliefs based on earlier bidding. This complicates the analysis of incentives for costly information acquisition. With free information, as noted above, Cong and Xiao (2024) do show that sequential play can achieve the same efficiency in large crowds as we derive for simultaneous moves. However, introducing costly inspection would doubtless create costs and benefits of sequentiality. Even random and imperfectly observed arrivals complicate substantially (see Herrera and Hörner, 2013).[21] Both extensions are beyond the scope of this paper.

Asymmetric equilibria. As in the majority of theoretical papers on crowdfunding, we focus on symmetric equilibria. Symmetry simplifies both notation and analysis, as each bidder holds the same expectations about others' strategic behavior. It is an appropriate assumption when analyzing an anonymous crowd, as when crowd size is uncertain (see Myerson, 1998). Moreover, when information is free, the restriction to symmetric equilibria is innocuous, as Proposition 6 shows.

[21] They have no threshold. Deb et al. (2024) and Ellman and Fabi (2022) study crowdfunding with random arrivals and costly bidding, as from inspection costs, but only for private values.

Symmetry's impact with a positive cost is less obvious. Asymmetric equilibria that assign a finite number of bidders to inspect sometimes raise welfare.[22] However, in such equilibria all other bidders free-ride and receive a rent. By contrast, the symmetric optimum avoids paying any rent. So we confidently conjecture that symmetry is optimal for a profit-maximizing entrepreneur even when asymmetry is possible.

Communication. In the optimal equilibria of our crowdfunding game, each bidder who inspects effectively communicates a good or bad signal via the decision to bid or not bid. This fails to distinguish bidders with good signals from blind-bidders and it requires those with bad signals to miss out on crowdblessed projects. So there seems to be scope for unrestricted communication after inspection and before bidding to raise crowdfunding efficiency. This is true for a finite crowd, but large crowds trivialize the implied inefficiencies from both concerns, limiting the scope for cheap talk benefits.[23] It is true that *after* sufficient inspection, truthful cheap talk would reveal project quality, enabling bidders to invest efficiently, but revealing quality to all would remove all incentives to inspect.[24] Cheap talk cannot evade the underlying incentive problem. Indeed, we conjecture that cheap talk has no impact on the asymptotic optimal crowdfunding outcome.

By contrast, cheap talk does have a significant impact on what is possible *without* crowdfunding in the free inspection case: it asymptotically permits the ideal outcome for any posted offer $r > 0$.[25] The reason is simple. Thanks to constant returns and the pure common value assumption, bidder preferences are perfectly aligned in the absence of information acquisition costs. So bidders have no motive to hide information from one another, nor even from a profit-maximizing entrepreneur once she has posted her offer r.

With costly acquisition, while cheap talk can still enhance posted offer outcomes, they remain less efficient than crowdfunding in our large crowd setting. In sum, when bidders can share their signals, crowdfunding has no added value if information is free but is valuable if information is costly. So our costly information results are more important than the less surprising, free information result of Proposition 6.

More general mechanisms. Crowdfunding platforms adopt simple, transparent, fair and readily understood rules which they can credibly enforce. These features make participation palatable to non-expert investors but our results raise two questions: Are they the underlying cause of the welfare loss that we have characterized? Can the rules be adjusted to raise efficiency without becoming too unattractive to bidders?

[22] A similar asymmetry is implicit in Gerardi and Yariv's (2008) upper bound on committee size.

[23] Cheap talk does no harm as babbling equilibria always support the original crowdfunding outcomes.

[24] With free information, comprehensive and truthful cheap talk *is* an equilibrium, but then cheap talk adds nothing since crowdfunding already asymptotically attains the ideal outcome. Similarly, Vickrey-Clarke-Groves mechanisms are ex post efficient and overall efficient with free information (despite value correlation, c.f., McLean and Postlewaite, 2015) but induce heavy free-riding when inspection is costly.

[25] Moreover, cheap talk would render any crowdfunding threshold irrelevant, just as deliberation removes differences between non-unanimous voting rules in Gerardi and Yariv (2007).

To see how adapting crowdfunding might help, consider letting the entrepreneur keep funds without investing them when they exceed the optimal crowdfunding threshold but fall short of an extra threshold set at the ex post efficient level. This maintains information acquisition incentives and removes ex post investment inefficiency. So it achieves the first-best in the limit on the informative crowdfunding cost range, $c \leq \bar{c}$. However, this breaks the All-or-Nothing (AoN) property of fully refunding all bids on failed crowdfunding campaigns. Partial refunding may lead to undesirable behavioral reactions or perverse incentives not contemplated in the canonical model. In a follow-up study, we assess how those effects may explain the popularity of AoN crowdfunding.

There, we also study what general mechanism design can achieve. Cremer and McLean (1985, 1988) suggest that using arbitrarily complex message spaces and action and transfer rules may permit full surplus extraction given the signal correlations induced by a common value. Limited liability can be a problem and these studies neglect endogenous information which can also obstruct efficiency and rent extraction, as shown in e.g., Bikhchandani and Obara (2017); Bergemann et al. (2009). Nevertheless, in our setting, general mechanisms *do* permit efficiency and rent extraction for any information cost and despite limited liability. So it is only the triple combination of limited liability, costly information *and* crowdfunding restrictions that precludes the asymptotic first-best.

As with the partial refund variant, this theoretical solution has potential downsides. Aside from greater complexity, optimal static mechanisms necessarily create interim rent heterogeneity that can generate distrust or conflict. Imposing anonymity alleviates this but the resulting mechanism only works on an intermediate cost range and remains susceptible to collusion and negative behavioral reactions. Ultimately, empirical evidence is needed. Experimental tests of AoN crowdfunding versus variants with partial refunds, as already studied in private value, public good settings, could readily assess behavioral reactions in the common value setting.

Empirical research on crowdfunding has grown rapidly alongside expanding datasets (c.f., footnotes 1 and 6). This raises the potential to test our core theoretical predictions. Ingenuity will be needed but a key prediction is clear and sharp: specifically for large crowds, inspection costs will be positively associated with false positive crowdblessing, that is with campaign success of bad projects. Combined with empirical and experimental tests, we hope our theory can guide policies to support crowdfunding's growth without overly exposing inexpert investors to the risks that underlie current regulations (see footnote 14).

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