Does Strategic Shelving of Patents Lead to Market Dominance?

Sougata Poddar♣
Department of Economics
Tippie College of Business, University of Iowa
Iowa City, USA

Yuanzhu Lu
School of Economics and Management
University of Science and Technology Beijing, Beijing, China

May 2025

Abstract

In many markets, we observe scenarios where a firm sometimes pays to acquire new technologies (patents), however, does not use them for its use, i.e., shelves them. By shelving the technology, the technology-acquiring firm can prevent its competitor from using it and thus maintain its strategic advantage in the market. This may create market dominance. In an oligopolistic framework, we show that this happens often when an outside innovator uses exclusive licensing to transfer technology where potential licensees have different efficiency levels of production and have asymmetric absorptive capacities of the transferred technology. However, we also show when this will not happen in the same environment. We find that under fixed fee licensing, when the size of the innovation is not big, technology is shelved, whereas if the innovation is large, it is not shelved. With per-unit royalty licensing, we find interesting non-monotonicity with respect to shelving and no shelving as the size of the innovation increases. We also find out the optimal licensing contract of the innovator in this environment and potential welfare loss due to shelving on the society.

Key Words: Innovator, Cost asymmetry, Absorptive capacity, Licensing, Shelving, Catch-up
JEL Classification: D43, D45, L13

1. Introduction

♣ Corresponding author: University of Iowa, Iowa City, IA, USA, Email: sougata-poddar@uiowa.edu

The market dominance of a few large firms in some sectors is becoming increasingly common these days. In the tech sector, the few big firms that dominate the market are Google (Alphabet), Apple, Facebook (Meta), Microsoft; in the retail sector, Amazon; or in the pharmaceutical sector, Pfizer, Johnson and Johnson are to name a few.[1] Market dominance can happen due to several factors, and some of them are already known and well understood. For example, firms with significant economies of scale, first mover advantage, product proliferation, extensive distribution network, or simply access to better market information can cause market dominance of a firm.

Market dominance through patent system, namely exclusively securing important patents ahead of its competitors can cause it as well. Previously some important studies have been done in this particular area, for example Gilbert and Newbery (1982), Reinganum (1983) [2]. The new element we bring in this study is to show that market dominance can happen because of the specific licensing agreements or contracts between the patent holder (innovator) and the licensee. Thus, it is not just getting the exclusive patent right of a new innovation ahead of its competitor and get some kind of first mover advantage in the market, but we show that the *nature* of the licensing contracts can actually give the licensee a competitive advantage which may result in creating a market dominance. We call this 'strategic shelving' through licensing. This is the new contribution of the paper here.

In industries, we do observe scenarios where a firm sometimes pays to acquire new technologies (patents), however, does not use them for its use, i.e., shelves them. The question is, why a firm would do that when acquiring a new technology (patent) is costly. One of the possible reasons is that by acquiring the new technology (patent) exclusively, the firm can actually prevent its competitor from using the technology by shelving it, and thus maintain its strategic advantage in the market. However, we also ask, will shelving of technology always happen? That is, will it always go only to the

---

[1] In an antitrust trial, Google (although) argues that smart employees explain its success - NY Times, Oct 19, 2023. A battle over Amazon's dominance – NY Times, Sept 27, 2023 - Lina Khan, the F.T.C. chair, takes her first big swing at Amazon.

[2] We will come to an elaborate discussion on this later in the literature review section.

firm who will shelve the technology and stop its competitor from acquiring it, or it can also go to the competitor where the technology is used and not shelved? We ask this question, particularly in the context where the new technology (innovation) comes from an outside innovator and more importantly, where the value of innovation differs across the potential firms. More specifically, we ask how this depends on the nature of the technology licensing contract, i.e., when it is more likely that a technology will be shelved and when it will be actually used. Finally, how does this depend on the cost structures and technology absorbing capacities of the competing firms? These are some of the questions we explore here in a context where the innovator is an outsider, and the firms are potential licensees competing in the same product market. We believe that this direction of research and related issues are not adequately addressed so far in the literature of patent licensing, and we wish to fill that gap here.

Torrisi et al. (2016) employ data from a large-scale survey (Inno S&T) of inventors in Europe, the USA, and Japan who were listed in patent applications filed at the European Patent Office (EPO) with priority years between 2003 and 2005. They find that a substantial share of patents is neither used internally nor used for market transactions, which confirms the importance of strategic patenting and inefficiency in the management of intellectual property.

Also looking at the large volume of patent transactions in tech industries,[3] one can presume that a good amount of 'shelving' of the patents may happen, but we also cannot rule out the possibility where the licensed technologies are actually used. So, in this paper, we want to look into things more carefully using a framework of technology

---

[3] Below are some real evidences (source: Hagiu and Yoffie, 2013) which shows to what extent a very high volume of patents is bought and sold in the tech-sector.

- In 2011, a consortium of Apple, Microsoft, and other large firms bought a portfolio of about 6,000 patents from Nextel for $4.5 billion (outbidding Google).
- Google later acquired Motorola Mobile for $12.5 billion, which gave Google a portfolio of over 17,000 patents.
- In 2012, Microsoft bought nearly 1,000 patents from AOL for about $1 billion, and then sold some of the patents to Facebook for $550 million.
-

transfer and patent licensing to understand where a new innovation is used or possibly shelved.

Our paper aims to investigate, from a theoretical perspective, i this particular area of patent licensing in oligopolistic markets. Economic theory has substantially contributed to the understanding of private incentives towards licensing, which has also become central to research & development literature. Following up on that we look into the shelving aspects of new patents and their consequences.

Our main premise of study here is *exclusive* licensing contracts for cost reducing technology from an outside innovator to potential licensees who are competing in the same product market. Note that under exclusive licensing, the innovator chooses a specific licensee to transfer the technology among all the potential licensees. This could be a natural choice in various situations as the process of technology transfer and licensing is often costly.

We consider licensees with *asymmetric absorptive capacities* of the new technology. We explore whether under exclusive licensing, this leads to an outcome where the technology is transferred but not used, that is, shelved. We also explore conditions under which shelving will never happen, and the new technology will be used by the licensee. We also explore if the new technology is licensed to the inefficient firm, whether it can catch up or even leapfrog its efficient rival.

Assume two asymmetric firms/licensees (one efficient and the other inefficient) producing a horizontally differentiated good. There is an outside innovator (independent research lab) who has cost-reducing technology which reduces the marginal costs of the two firms in a non-uniform manner (asymmetric absorptive capacities). We make the extreme assumption that the innovation reduces the marginal cost of the inefficient firm only; the efficient firm does not benefit from it at all.[4]

[4] Note that without this assumption in this framework, the possibility of shelving may not arise as firms will always benefit from the new technology and the new technology will be used. One of the reasons for asymmetric absorptive capacity could be since the efficient firm is already very efficient its scope for cost reduction with the new technology could be very small or negligible whereas the inefficient firm can gain significantly from the new technology. Alternatively, consider an environment where there is high commercialization cost (e.g., Letina et al. (2020)) for the efficient firm for the new cost reducing

In this structure, we find that our results depend on the size of the innovation relative to the initial cost asymmetry of the firms. For example, when we first consider fixed fee licensing, we find that shelving will happen when the size of the innovation is not too big compared to the initial cost asymmetry of the firms. On the other hand, shelving will not happen for relatively large innovations, and this is where the technology goes to the inefficient firm. When we consider per-unit royalty licensing, we find an interesting non-monotonicity with respect to shelving and no shelving as the size of the innovation gets bigger. To this end, we highlight the interplay between the strategic effect of the efficient firm and the cost reduction effect of the inefficient firm.

We also find the optimal licensing contract for the innovator and identify regions of shelving and no shelving in the whole relevant parameter space by examining the licensing contracts we considered here. We believe that this is also a novel finding in the area of technology transfer and patent licensing where we get both shelving and no shelving as equilibrium outcomes.

The rest of the paper is organized as follows. In the next section, we do a brief literature survey. In section 3, we describe our model and the pre-licensing case. In section 4, we do our main analysis on licensing in a spatial model of product differentiation, discuss our results on shelving and no shelving, and equilibrium licensing contracts. In section 5, we do a welfare analysis to see the potential cost of shelving to the society. In section 6, to check the robustness of our main findings the whole licensing analysis is done in the standard model of product differentiation *a la* Singh and Vives (1984). Section 7 concludes.

---

technology and therefore the shelving decision is already endogenized. So, this assumption is a short-cut of such endogenization. We would like to thank one of our referees for providing this apt explanation.
Having said this, Lu and Poddar (2023) consider the case when the efficient firm can also benefit from the cost reducing technology but only under certain condition. In that paper, apart from exclusive licensing, non-exclusive licensing and other issues are analyzed.
Lastly, if one assumes that the new technology only reduces the cost of the efficient firm (the reason could be higher degree of readiness of using a new technology because of higher efficiency), but not the cost of the inefficient firm, so if shelving has to happen, it must be with the inefficient firm. However, it can be shown that shelving never happens here. In other words, willingness to pay for the new technology by the inefficient firm is always lower than that of the efficient firm.

2. Literature Review

When we look at the recent literature on technology transfer, particularly in pharmaceutical and tech industries, we find an interesting story there. It is a story of 'killer acquisitions', where dominant firms acquire inventions without the aim of using the invention or developing it further but only for reducing competition. This is the heart of the argument of what we are referring to as 'strategic shelving' here. Cunningham et al. (2021) using pharmaceutical industry data, show that acquired drug projects are less likely to be developed when they overlap with the acquirer's existing product portfolio, especially when the acquirer's market power is large. Conservative estimates indicate that 5.3 percent to 7.4 percent of acquisitions in their sample are killer acquisitions. Fumagalli et al. (2020) analyze the optimal policy of an antitrust authority towards the acquisitions of potential competitors in a model with financial constraints where the acquirer may decide to shelve the project of the potential entrant. Letina et al. (2020) provides a theory of strategic innovation project choice by incumbents and start-ups and show that prohibiting killer acquisitions strictly reduces the variety of innovation projects, whereas prohibiting other acquisitions only has a weakly negative innovation effect. Norbäck et al. (2020) show that 'acquisitions for sleep' can occur if and only if the quality of a process invention is small; otherwise, the entry profit will be higher than the entry-deterring value.

First of all, the heart of the story of all above mentioned papers is about the acquisition or takeover of the innovative firm or organization by an incumbent firm and its consequences on shelving or further developing the project. Antitrust issues that may arise here due to acquisitions are also addressed and, in some cases, an optimal antitrust policy towards the acquisitions is also explored. Our story is different, and it mainly focuses on the specific licensing arrangements between the innovator and the potential incumbent firm(s) and its impact on shelving without bringing in any story of acquisition. The other difference is that in most of the above-mentioned papers the innovator is an insider, i.e., a competing firm or new entrant in the market, but in our case the innovator is an outsider and is not a participant in the product market competition.

We also note that there is a strand of literature which talks about 'sleeping patents' and its implications. The basic idea of sleeping patents is that a firm may have an incentive to patent new technologies before potential competitors do, but then never bring those patents to the market, i.e., hold 'sleeping patents' (Gilbert and Newbery (1982), Reinganum (1983)). Through sleeping patents, firms more often engage in strategic blocking, namely they prevent competitors from imitating their products and entering the market. Our analysis here is also closely related to this idea where the general theme is strategic patenting of technologies, but we go beyond 'sleeping patents' and look into the specifics of licensing contracts between the patentee and the licensee which may or may not create strategic blocking in the market environment.

The other main difference of our study compared to the above mentioned two papers is that in our paper the innovator is an outsider and hence the nature of licensing contracts between the innovator and the potential licensee matters, whereas in those papers the innovator is an insider, i.e., either an incumbent or a potential entrant, hence the impact of ex-post innovation leads to strategic preemption. In Gilbert and Newbery (1982), innovation is deterministic whereas in Reinganum (1983), the innovation is stochastic. Further, in those papers, there is an element of patent-race between the two firms, whereas in our environment there is no story of patent race as there is only one outside innovator.

3. The Model

Consider two firms, firm A and firm B located in a linear city represented by a unit interval $[0,1]$ (*a la* Hotelling (1929)). Firm A is located at 0 whereas firm B is located at 1, that is, at the two extremes of the linear city. Both firms produce homogenous goods with constant but different marginal costs of production and compete in prices. We assume that consumers are uniformly distributed over the interval $[0,1]$. Each consumer purchases exactly one unit of the good either from firm A $(price\ p_A)$ or firm B $(price\ p_B)$. $v > 0$ denotes gross utility of a consumer derived from the good. The transportation cost per unit of distance is t and it is borne by the consumers. [5]

The utility function of a consumer located at x is given by:

$$U = v - p_A - tx \qquad \text{if buys from firm A}$$

$$= v - p_B - t(1-x) \qquad \text{if buys from firm B}$$

We assume that the market is fully covered and the total demand is normalized to 1. The demand functions for firm A and firm B can be calculated as:

$$Q_A = \frac{1}{2} + \frac{p_B - p_A}{2t} \qquad \text{if } p_B - p_A \in (-t, t)$$

$$= \quad 0 \qquad \text{if} \quad p_B - p_A \leq -t$$

$$= \quad 1 \qquad \text{if} \quad p_B - p_A \geq t$$

and $Q_B = 1 - Q_A$.

3.1 The Pre-Licensing Game

First, we examine the case where the outside innovator is not there, and the two asymmetric firms A and B are competing in the market. We assume that firm A is more efficient than firm B, implying $c_A \leq c_B$. Let us define $\delta = c_B - c_A \geq 0$ to capture the cost difference.

Assumption 1: $\delta < 3t$. We need this so that the less efficient firm's equilibrium quantity is positive.

The pre-licensing equilibrium prices, demands and profits can be given as:

[5] Hotelling framework of spatial competition allows us to specifically address those markets where the consumer demand is inelastic. Many markets including cell phones, electronic gadgets, computers, home appliances, cars etc. fall in the category.
To see the broader implications of the results and to check the robustness of our findings, later we also do the complete analysis using the standard general product differentiation model *a la* Singh and Vives (1984) where the demand is elastic.

$$p_A = \frac{1}{3}(3t + 2c_A + c_B) = c_A + \frac{1}{3}(3t + \delta),$$

$$p_B = \frac{1}{3}(3t + c_A + 2c_B) = c_B + \frac{1}{3}(3t - \delta),$$

$$Q_A = \frac{1}{6t}(3t - c_A + c_B) = \frac{1}{6t}(3t + \delta),$$

$$Q_B = \frac{1}{6t}(3t + c_A - c_B) = \frac{1}{6t}(3t - \delta),$$

$$\pi_A = \frac{1}{18t}(3t - c_A + c_B)^2 = \frac{1}{18t}(3t + \delta)^2,$$

$$\pi_B = \frac{1}{18t}(3t + c_A - c_B)^2 = \frac{1}{18t}(3t - \delta)^2.$$

4. The Licensing Game

Now we have the outside innovator who offers exclusive license of the cost reducing innovation of size $\epsilon > 0$. The innovation if licensed helps reduce the per-unit marginal cost of the inefficient firm B only by ϵ. That is, the innovation only benefits the inefficient firm B and has no value to the efficient firm A as far as cost reduction is concerned.

The timing of the licensing game is given as follows:

Stage 1: The outside innovator offers a licensing scheme to firm A. Firm A accepts or rejects the offer. If firm A rejects, then the innovator offers a possibly different licensing scheme to firm B. Firm B accepts or rejects the offer.[6]

Stage 2: Knowing which firm accepts the licensing contract, both firms compete in prices and products are sold to consumers.

Assumption 2: $\epsilon < 3t + \delta$. We need this to ensure both firms' equilibrium

[6] The innovator will not offer a licensing scheme to firm B first since the threat of licensing to firm A after firm B's rejection is useless given that the innovation does not benefit firm A at all. So, we do not consider the case in which the outsider innovator offers a licensing scheme to firm B first.

quantities are positive in the licensing game.

Assumption 3: $c_B > 3t + \delta$. We need this so that the cost-reducing innovation can be fully utilized.

We also look into the following two aspects after licensing takes place.

Catch-Up: We say catch-up occurs if after licensing in equilibrium the *effective* cost gap between the firms is reduced.[7]

Leapfrog: If the inefficient firm becomes efficient (i.e., becomes the low cost firm) after licensing in equilibrium.

Now we look into the licensing game with specific licensing contracts. We use backward induction to find the best licensing scheme for the innovator.

4.1 Fixed Fee

Suppose that firm A rejects the innovator's offer. Then the innovator will offer an attractive licensing scheme so that firm B will accept it. If firm B gets the technology by paying the fixed fee F_B, then it will accept if $F_B \le \frac{1}{18t}(3t - \delta + \epsilon)^2 - \frac{1}{18t}(3t - \delta)^2 = \frac{\epsilon}{18t}\{6t - 2\delta + \epsilon\} = g_B$, and firm A's profit will be $\frac{1}{18t}(3t + \delta - \epsilon)^2$. Clearly, the innovator will set $F_B = g_B$.

Suppose that the innovator offers an attractive licensing scheme to firm A so that firm A will accept it. If firm A gets the technology by paying the fixed fee F_A, then it will accept as long as $F_A \le \frac{1}{18t}(3t + \delta)^2 - \frac{1}{18t}(3t + \delta - \epsilon)^2 = \frac{\epsilon}{18t}\{6t + 2\delta - \epsilon\} = g_A$. Clearly, the innovator will set $F_A = g_A$.

Comparing g_A and g_B yields the result in the following proposition.

Proposition 1

Under exclusive fixed fee licensing,

---

[7] See also Badia 2019 on technology licensing under Cournot competition on the aspect of catching up.

(i) *If $\epsilon < 2\delta$, then the efficient firm gets the license and shelves; no catch-up. The licensing contract is given by* $g_A = \frac{\epsilon(6t+2\delta-\epsilon)}{18t}$.

(ii) *If $2\delta < \epsilon < 3t + \delta$, then the less efficient firm gets the license and there is no shelving. Here catch-up happens with leapfrogging. The licensing contract is given by* $g_B = \frac{\epsilon(6t-2\delta+\epsilon)}{18t}$.

The following diagram helps us to understand when the technology will be shelved and when it will not be shelved in the whole (relevant) parameter space.

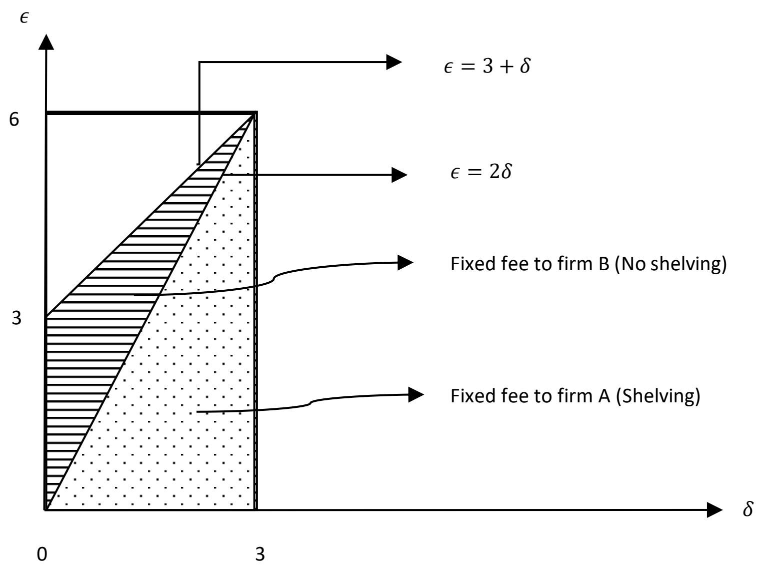


Figure 1: Shelving and No Shelving under Optimal Fixed Fee (Set t =1)

Note that it is only when the cost-reducing innovation is relatively large compared to the initial cost asymmetry, the less efficient firm will win the license. The less efficient firm can benefit from the innovation by so much that the efficient firm finds it too costly to prevent its rival to get the license. Otherwise, the technology goes to the efficient firm and is shelved.

4.2 Royalty

We can use the same method as the one in subsection 4.1 to find the optimal exclusive royalty licensing contract, which is stated in the following proposition.

Proposition 2

(i) *If* $(12\sqrt{2}-15)t \approx 1.971t < \delta < 3t$ *and* $\epsilon_1 < \epsilon < \epsilon_2$, *where* $\epsilon_1 = \frac{3t+\delta-\sqrt{-63t^2+\delta^2+30t\delta}}{2}$ and $\epsilon_2 = \frac{3t+\delta+\sqrt{-63t^2+\delta^2+30t\delta}}{2}$, *an exclusive royalty licensing to firm A is better than to firm B for the innovator. The licensing contract is given by* $r_A^* = \frac{\delta+\epsilon-3t}{2}$*. Shelving happens and there is catch-up since firm A's effective marginal cost increases but no leapfrogging.*

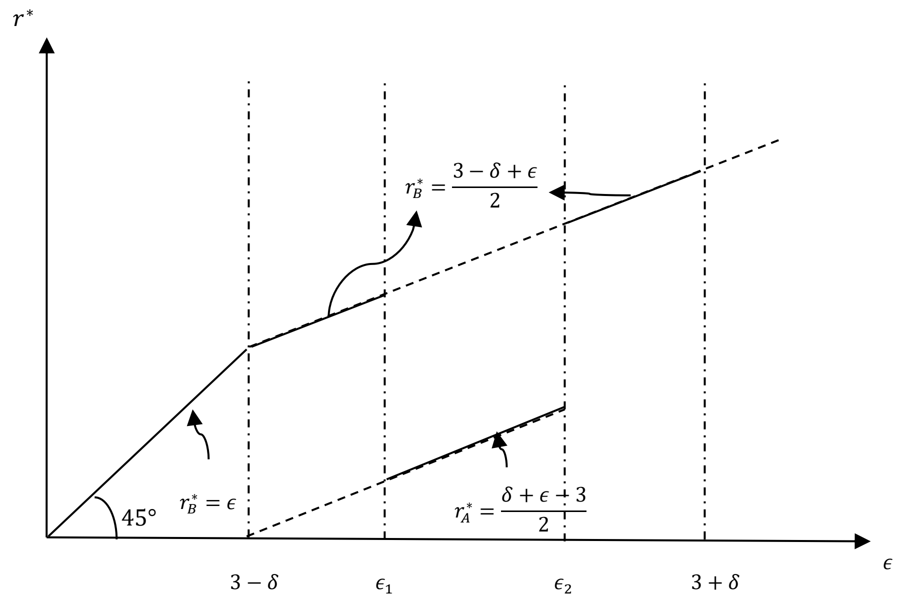
(ii) *In all the other situations, an exclusive royalty licensing to firm B is better for the innovator. License is not shelved.*

(a) When $0 < \epsilon \leq 3t - \delta$*, i.e., when the size of the innovation is small, the licensing contract is given by* $r_B^* = \epsilon$*. No catch-up here.*

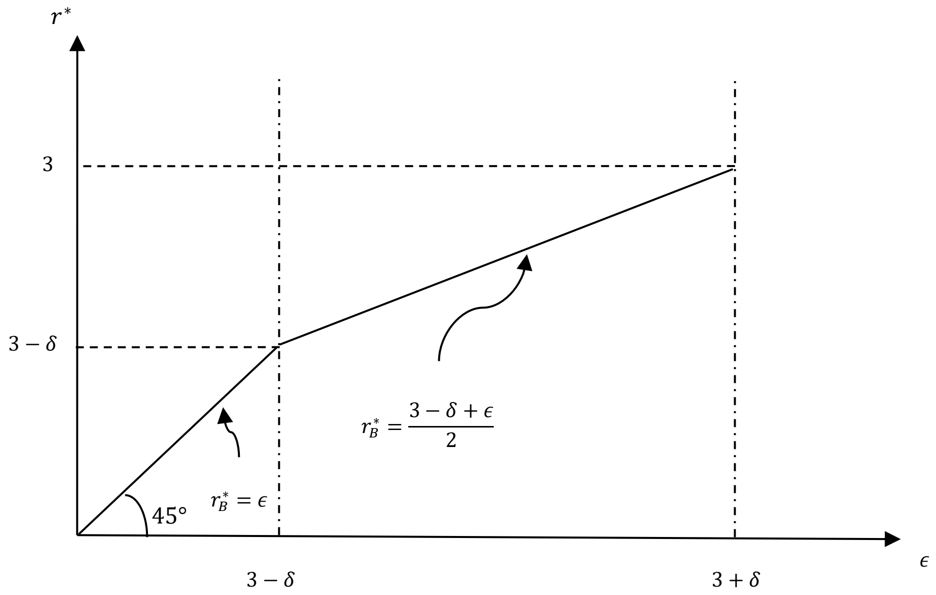
(b) When $3t - \delta < \epsilon < 3t + \delta$*, i.e., when the size of the innovation is large (except within the range* $\epsilon_1 < \epsilon < \epsilon_2$*), the licensing contract is given by* $r_B^* = \frac{3t-\delta+\epsilon}{2}$*. We have catch-up here, but no leapfrogging for any degree of innovation.*

Proof: See Appendix A.

The following diagrams help us to understand how the optimal royalty contract plays out and looks like in the whole (relevant) parameter space.



(a) When $1.971 < \delta < 3$ (Set $t = 1$)



(b) When $0 < \delta < 1.971$ (Set $t = 1$)

Figure 2: Tracking Optimal Royalty

We can track the optimal royalty contract and non-monotonicity on shelving here as shown above. In Figure 2(a), first the technology goes to the inefficient firm B (no shelving), then efficient firm A (shelving) and then again to B (no shelving).

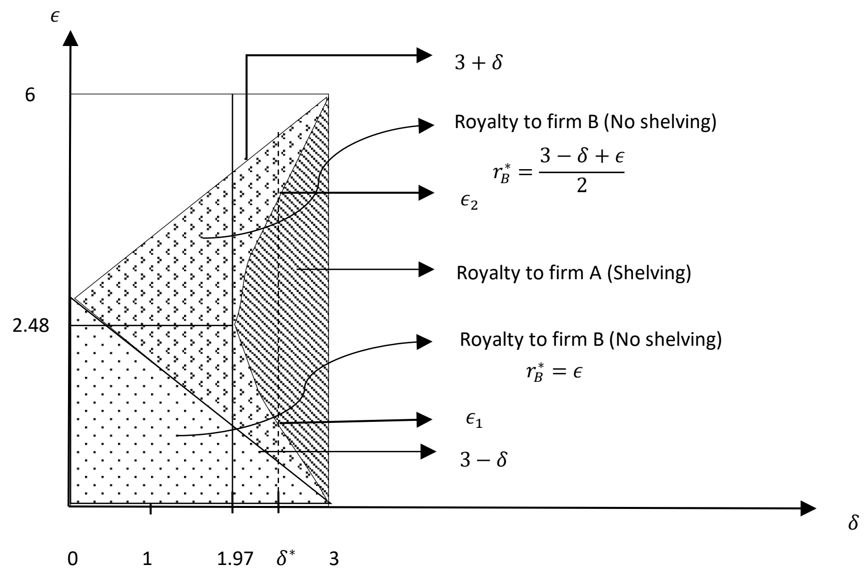


Figure 3: Optimal Royalty (Shelving and No Shelving, Non-monotonicity is visible at δ^*)

(Set $t = 1$)

The overall non-monotonicity on shelving as the size of the innovation gets bigger is also evident from figure 3. More precisely, when $(12\sqrt{2} - 15)t \approx 1.971t < \delta < 3t$ as the innovation size gets larger (within relevant range $0 < \epsilon < 3t + \delta$), we start with no shelving, then shelving and finally again no shelving. This is clear from the point δ^* on the horizontal axis of the diagram.

The broad intuitive explanation is as follows. When the license is offered to firm B with an equilibrium royalty rate same as the size of the innovation ϵ (happens when $0 < \epsilon < 3t - \delta$, i.e., for relatively small innovations), the efficient firm A has no incentive to acquire the technology as the relative cost positions of the firms do not change after licensing, and they remain same as pre-licensing. However, when the equilibrium royalty rate is strictly less than the size of the innovation (happens when $3t - \delta < \epsilon < 3t + \delta$, i.e., for relatively large innovations), firm B becomes more efficient and comes closer to firm A, and firm A starts losing some competitive advantage. As a result, firm A may now be incentivized to purchase the license at an appropriate royalty rate. But at the same time, if firm A gets the license, it may not secure enough additional revenue from the market to offer a good price to the innovator for the license. In turn, the innovator may not find it profitable to offer the license to firm A at this point. On the other hand, in this environment, firm B continues to get the license as the size of the innovation grows, i.e., up to ϵ_1 $(3t - \delta < \epsilon < \epsilon_1)$ albeit with a different royalty rate. Now when the innovation gets even bigger, i.e., when $(\epsilon > \epsilon_1)$, firm A gets the license as the innovator starts getting higher revenue from licensing to firm A with an appropriate royalty rate. Now firm A generates enough revenue from market competition to support this. The intuition is, at this size of innovation, the gain from maintaining the strategic advantage by firm A (we call it *strategic effect*) now outweighs the gain from cost reduction for production for firm B. Therefore, higher revenue is collected by the innovator by offering the license to firm A. However, when the size of the innovation gets even bigger (i.e., $\epsilon > \epsilon_2$), it becomes more profitable for the innovator to license it back to firm B as the gain from cost reduction generates higher surplus from firm B and dominates the gain from strategic effect due to shelving. This generates the overall non-monotonicity of royalty licensing.

Summing up, we show licensing to firm B is optimal to the innovator when the size of the innovation is small or large whereas licensing to firm A is optimal when the size of the innovation is medium. This also generates a non-monotonicity with respect to shelving and no-shelving as the size of the innovation gets bigger.

As for catch-up, as the innovation size gets bigger, within $0 < \epsilon \leq 3t - \delta$, there is no catch-up; but there will be catch-up when $3t - \delta < \epsilon < 3t + \delta$, and the difference between the two firms' effective marginal costs becomes $\frac{3t+\delta-\epsilon}{2}$ (see proof of Proposition 2 in Appendix A). However, there will be no leapfrogging for any level of innovation.

Finally, under exclusive royalty licensing the innovator's payoff is (see proof of Proposition 2) is as follows.

$Rev = \frac{\epsilon}{6t}(3t - \delta) \quad if \;\; 0 < \epsilon \leq 3t - \delta$

$$= \frac{(3t-\delta+\epsilon)^2}{24t} \quad if\ \delta \le (12\sqrt{2}-15)t \ \ and\ \ 3t-\delta < \epsilon < 3t+\delta$$

$$or\ (12\sqrt{2}-15)t < \delta < 3t\ and\ 3t-\delta < \epsilon < \epsilon_1 \ \ or\ \ \epsilon_2 < \epsilon < 3t+\delta$$

$$= \frac{1}{24t}(\epsilon+\delta-3t)(9t+\delta-\epsilon) \ \ if\ (12\sqrt{2}-15)t < \delta < 3t\ and\ \ \epsilon_1 < \epsilon < \epsilon_2$$

4.3 Optimal Licensing Contract - Comparing Fixed Fee and Royalty

The following result summarizes the overall profit maximizing contract for the innovator and the possibility of shelving (or no-shelving).

Proposition 3

Under the profit maximizing licensing contract of the innovator,

(i) *Only when* $max\{5\delta - 3t, 0\} < \epsilon < 3t - \delta$ *and* $\delta < t$, *royalty licensing is better than fixed fee licensing to the innovator. Under royalty licensing the technology goes to the inefficient firm, and it is not shelved. The licensing contract is given by* $r_B^* = \epsilon$.
Here we get no catch-up as the equilibrium royalty is equal to the size of the innovation. In all other situations, fixed fee licensing is better than royalty licensing.

(ii) *Under fixed fee licensing the technology is transferred to the efficient firm and it is shelved if* $\epsilon < 2\delta$; *no catch-up. The licensing contract is given by* $g_A = \frac{\epsilon(6t+2\delta-\epsilon)}{18t}$.

(iii) *Under fixed fee licensing the technology goes to the less efficient firm and there is no shelving if* $2\delta < \epsilon < 3t + \delta$. *Here we have catch-up and leapfrogging. The licensing contract is given by* $g_B = \frac{\epsilon(6t-2\delta+\epsilon)}{18t}$.

Proof: See Appendix A.

The following diagram shows clearly the regions where equilibrium outcome leads to fixed fee and royalty licensing as well as shelving and no-shelving of the technology.

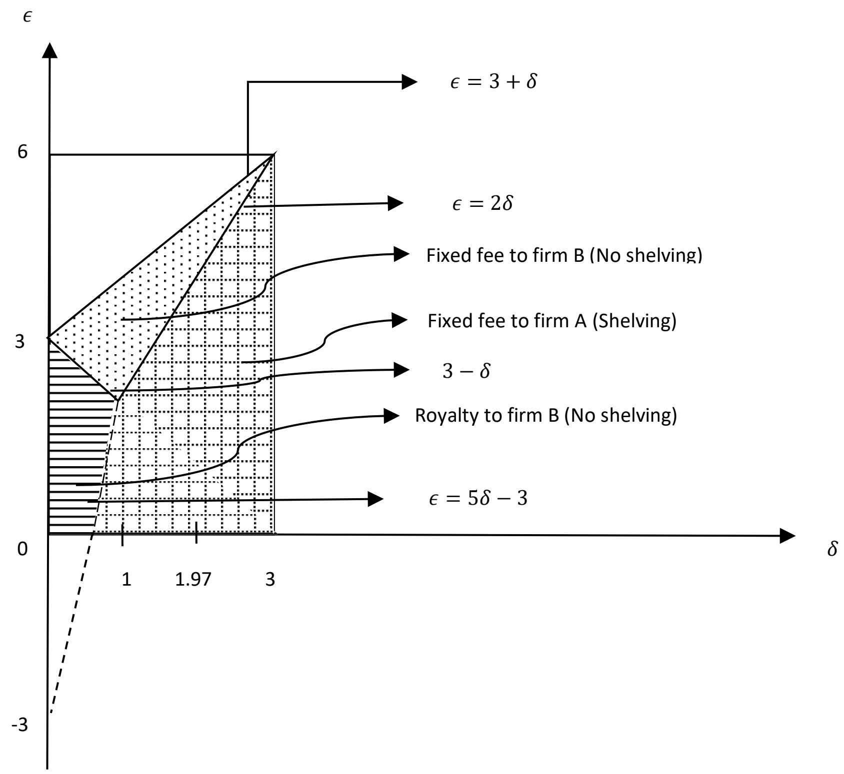


Figure 4: Optimal Licensing, Shelving and No Shelving (Set t =1)

When the size of the innovation is relatively small and the initial cost difference between the firms is low, a royalty contract is more profitable to the innovator than a fixed fee contract. This is where the optimal royalty rate is equal to the size of the innovation. However, when the innovation size gets larger and /or initial cost difference between the firms is high, fixed fee licensing provides higher revenue to the innovator. Now under fixed fee licensing whether the license goes to firm A or firm B depends jointly on relative sizes of the innovation and initial cost difference. For larger innovations compared to cost difference, it goes to the inefficient firm B, the large gain in production efficiency due to cost reduction generates a bigger profit, which is extracted by the innovator by fixed fee. For not so large innovations compared to cost difference, it is better for the innovator to allow firm A to maintain the strategic advantage (strategic effect) by transferring the license to firm A which is shelved. The surplus accrued due to strategic effect is then extracted with fixed fee by the innovator from firm A. The above intuition is also evident from the revenue expressions of the innovator from fixed fee licensing to firm A and firm B (refer to section 4.1).

5. Welfare Analysis

In this section, we will first find out whether the optimal licensing contract chosen by the innovator improves social welfare, and then compare welfare between fixed fee licensing and royalty licensing.

5.1 Welfare Improving or Hurting?

To find whether the optimal licensing contract improves or hurts social welfare, we need to look at four distinct relevant cases ((i)- (iv)). Case (i) is about pre-licensing situation, the benchmark case; and cases ((ii) – (iv)) are all possible equilibrium situations after licensing. The social welfare (W) is equal to the society's benefit from the product (v) minus total costs, and the society's total costs (TC) is the sum of firms' total production costs (TPC), and consumers' total transportation cost (TTC). So, $W = v - TC = v - TPC - TTC$.

Let us use the notation PL as pre-licensing, FNSB as fixed fee licensing to B with no-shelving, RNSB as royalty licensing to B with no-shelving and FSA as fixed fee licensing to A with shelving.

Case (i) Welfare under pre-licensing

Under pre-licensing, $Q_A^{PL} = \frac{1}{6t}(3t + \delta)$, $Q_B^{PL} = \frac{1}{6t}(3t - \delta)$. It follows that

$$TPC^{PL} = c_A Q_A^{PL} + c_B Q_B^{PL} = \frac{1}{6t}\left(c_A(3t + \delta) + c_B(3t - \delta)\right) = c_B - \frac{1}{6t}\delta(3t + \delta),$$

$$TTC^{PL} = \frac{t}{2}\left(\left(Q_A^{PL}\right)^2 + \left(Q_B^{PL}\right)^2\right) = \frac{1}{36t}\left(9t^2 + \delta^2\right)^{8},$$

$$W^{PL} = v - c_B + \frac{1}{36t}\left(-9t^2 + 18t\delta + 5\delta^2\right).$$

[8] Note that since consumers are uniformly distributed over the linear city, the average transportation cost should be $\frac{tQ_i}{2}$ for consumers purchasing from firm i (for the nearest consumer the transportation cost is 0; for the farthest consumer it is tQ_i).

Welfare under licensing equilibria

Case (ii) Royalty licensing to the inefficient firm B (No Shelving)

In this case, the equilibrium royalty rate $r_B^* = \epsilon$ which is the same as the size of the innovation. Hence the effective marginal cost of firm B does not change, and the market competition after licensing remains unaltered. Firm B's production costs are reduced by $\epsilon Q_B^{PL} = \frac{\epsilon}{6t}(3t - \delta)$. Therefore, the social welfare under royalty equilibrium with no shelving is $W^{RNSB} = W^{PL} + \frac{\epsilon}{6t}(3t - \delta) > W^{PL}$.

Case (iii) Fixed fee licensing to the inefficient firm B (No Shelving)

In this case, firm B's marginal cost becomes $c_B - \epsilon$. We thus have $Q_A^{FNSB} = \frac{1}{6t}(3t + \delta - \epsilon)$, $Q_B^{FNSB} = \frac{1}{6t}(3t - \delta + \epsilon)$. Noting that this case arises when $2\delta < \epsilon < 3t + \delta$, it follows that

$$TPC^{FNSB} = c_A Q_A^{FNSB} + (c_B - \epsilon)Q_B^{FNSB} = c_B - \epsilon - \frac{1}{6t}(\delta - \epsilon)\big(3t + (\delta - \epsilon)\big) < TPC^{PL},$$

$$TTC^{FNSB} = \frac{t}{2}\left(\left(Q_A^{FNSB}\right)^2 + \left(Q_B^{FNSB}\right)^2\right) = \frac{1}{36t}\left(9t^2 + (\delta - \epsilon)^2\right) > TTC^{PL},$$

$$W^{FNSB} = v - (c_B - \epsilon) + \frac{1}{36t}\left(-9t^2 + 18t(\delta - \epsilon) + 5(\delta - \epsilon)^2\right).$$

We can see that, compared to under pre-licensing, firms' total production costs decrease while consumers' total transportation costs increase. Overall, $W^{FNSB} - W^{PL} = \frac{\epsilon}{36t}\big(18t + 5(\epsilon - 2\delta)\big) > 0$.

Case (iv) Fixed fee licensing to the efficient firm A (Shelving)

When firm A gets the license by paying the fixed fee, it is shelved; there is no change in costs of the firms and hence no change in competition after licensing and in transportation costs. Hence, $W^{FSA} = W^{PL}$.

The following proposition summarizes welfare implications due to shelving.

Proposition 4

Licensing with shelving is not welfare improving compared to the pre-licensing, while licensing with no shelving is always welfare improving.

The above result is not surprising. Since shelving prevents dissemination of technology (in this case a cost reducing technology), it impedes cost reduction and increase in competition, and therefore the society cannot benefit if a technology is shelved. In addition, the optimal licensing contract in our model is never royalty licensing to firm A, so shelving does not hurt social welfare either. On the contrary, when the technology is not shelved, in case (ii), it helps reduce the licensee's production costs and does not cause any other change in the other firm's production costs and consumers' transportation costs; and in case (iii), it helps reduce the licensee's production costs by so much that the licensee leapfrogs, thus the saving in production costs dominates the increase in consumers' transportation costs.

5.2 Welfare Comparison: Fixed Fee vs Royalty Licensing

If we examine Figures 1 and 3, we can see that the parameter space where royalty licensing goes to firm A (with shelving) is a proper subset of the parameter space where fixed fee licensing goes to firm A (with shelving). Does that mean fixed fee licensing is worse than royalty licensing in terms of social welfare? We investigate this issue in this subsection. The answer is no!

Let us use the notation FNSB as fixed fee licensing to B with no-shelving, RNSB as royalty licensing to B with no-shelving, FSA as fixed fee licensing to A with shelving and RSA as royalty licensing to A with shelving. Since there are two possibilities for royalty licensing to B with no-shelving (refer to Proposition 2 (ii) a and b), we use the notations RNSB1 and RNSB2 respectively.

In subsection 5.1, we have obtained $W^{RNSB1} = W^{PL} + \frac{\epsilon}{6t}(3t - \delta)$, $W^{FNSB} = W^{PL} + \frac{\epsilon}{36t}\big(18t + 5(\epsilon - 2\delta)\big)$, and $W^{FSA} = W^{PL}$, where $W^{PL} = v - c_B + \frac{1}{36t}\big(-9t^2 +$

$18t\delta + 5\delta^2\big)$. Now we derive W^{RNSB2} and W^{RSA}.

Case (v) Royalty licensing to the inefficient firm B (No Shelving, the second possibility)

In this case, $r_B^* = \frac{3t-\delta+\epsilon}{2}$. Hence the effective marginal cost of firm B is $c_B - \epsilon + r_B^*$. We thus have $Q_A^{RNSB2} = \frac{1}{6t}\left(3t + (\delta + r_B^* - \epsilon)\right)$, $Q_B^{RNSB2} = \frac{1}{6t}\left(3t - (\delta + r_B^* - \epsilon)\right)$. It follows that

$$TPC^{RNSB2} = c_A Q_A^{RNSB2} + (c_B - \epsilon) Q_B^{RNSB2} = c_B - \epsilon - \frac{(\delta-\epsilon)(9t+\delta-\epsilon)}{12t} < TPC^{PL},$$

$$TTC^{RNSB2} = \frac{t}{2}\left(\left(Q_A^{RNSB2}\right)^2 + \left(Q_B^{RNSB2}\right)^2\right) = \frac{1}{36t}\left(9t^2 + \left(\frac{3t+\delta-\epsilon}{2}\right)^2\right) < TTC^{PL},$$

$$W^{RNSB2} = v - (c_B - \epsilon) + \frac{1}{144t}\left(-45t^2 + 102t(\delta-\epsilon) + 11(\delta-\epsilon)^2\right) > W^{PL}.$$

We can see that, compared to under pre-licensing, firms' total production costs and consumers' total transportation costs decrease, then the social welfare must increase. However, it is worthwhile pointing out that the condition under which the optimal royalty rate to firm B is $r_B^* = \frac{3t-\delta+\epsilon}{2}$ is used when we derive $TPC^{RNSB2} < TPC^{PL}$.

Case (vi) Royalty licensing to the efficient firm A (Shelving)

In this case, $r_A^* = \frac{\delta+\epsilon-3t}{2}$. Hence the effective marginal cost of firm A is $c_A + r_A^*$. We thus have $Q_A^{RSA} = \frac{1}{6t}\left(3t + (\delta - r_A^*)\right)$, $Q_B^F = \frac{1}{6t}\left(3t - (\delta - r_A^*)\right)$. It follows that

$$TPC^{RSA} = c_A Q_A^{RSA} + c_B Q_B^{RSA} = c_B - \frac{\delta}{6t}\left(3t + (\delta - r_A^*)\right) > TPC^{PL}$$

$$TTC^{RSA} = \frac{t}{2}\left(\left(Q_A^{RSA}\right)^2 + \left(Q_B^{RSA}\right)^2\right) = \frac{1}{36t}\left(9t^2 + (\delta - r_A^*)^2\right) < TTC^{PL}$$

$$W^{RSA} = v - c_B + \frac{1}{36t}\left(-9t^2 + 18t(\delta - r_A^*) + 5(\delta - r_A^*)^2\right)$$

We can see that, compared to under pre-licensing, firms' total production costs increase while consumers' total transportation costs decrease. Overall, $W^{RSA} - W^{PL} = -\frac{r_A^*}{36t}(4\delta + r_A^*) < 0$. Interestingly, we get *welfare reducing* licensing here.

Having obtained the expressions of social welfare in all cases, we can now compare welfare between fixed fee licensing and royalty licensing. Straightforward algebraic calculations yield the results summarized in the following proposition:

Proposition 5

In terms of social welfare,

(i) When the innovator chooses to license to the same firm (either firm A or firm B) under both fixed fee licensing and royalty licensing, fixed fee licensing is better.

(ii) When the innovator chooses to license to firm B under royalty licensing and to firm A under fixed fee licensing, royalty licensing is better.

We can now go back to the question raised at the beginning of this subsection: Is fixed fee licensing worse than royalty licensing in terms of social welfare? The answer is no! First, fixed fee licensing to firm A never hurts social welfare while royalty licensing to firm A does hurt. Second, fixed fee licensing to firm B reduces firm B's effective marginal cost by the size of the innovation, while royalty licensing to firm B reduces firm B's effective marginal cost by less or even by zero. However, fixed fee licensing leads to shelving for greater parameter space.

6. Licensing – In the Standard Product Differentiation Cournot Model

In this section, to check the robustness of our main findings on licensing outcomes, we do the complete analysis in the standard Cournot model of horizontal product differentiation *a la* Singh and Vives (1984).

Two firms A and B (the potential licensees) produce two exogenously differentiated goods and compete in quantities. Firm i's $(i = A, B)$ marginal cost is assumed to be constant, denoted by c_i, and fixed cost is assumed to be zero. Like before, we assume that firm A is more efficient than firm B, implying $c_A < c_B$.

Firm i's inverse demand function is given by $p_i = á - \beta\left(q_i + \gamma q_j\right)$, where $\gamma \in (0,1)$ is an *inverse* measure of product differentiation --- the closer it is to one, the less differentiation there is between the goods.

Like before, there is an outside innovator who offers exclusive license of the cost reducing innovation of size $\epsilon > 0$. The innovation if licensed helps reduce the per-unit marginal cost of the inefficient firm B only by ϵ. That is, the innovation only benefits the inefficient firm B and has no value to the efficient firm A as far as cost reduction is concerned.

As before, the timing of the licensing game is given as follows:

Stage 1: The outside innovator offers a licensing scheme to firm A. Firm A accepts or rejects the offer. If firm A rejects, then the innovator offers a possibly different licensing scheme to firm B. Firm B accepts or rejects the offer.

Stage 2: Knowing which firm accepts the licensing contract, both firms compete in quantities and products are sold to consumers. We also look into catch-up and leapfrog after licensing takes place.

6.1 The Pre-Licensing Game

First, we examine the case where the outside innovator is not there, and the two asymmetric firms A and B are competing in the market. To ensure the less efficient firm's equilibrium quantity is positive, we need to assume $2(\alpha - c_B) - \gamma(\alpha - c_A) > 0$. Define $\delta \equiv \frac{\alpha - c_B}{\alpha - c_A}$. Then this assumption, together with $c_A < c_B$, can be expressed as

Assumption 4: $\frac{\gamma}{2} < \delta < 1$.

Note that δ is an *inverse* measure of cost asymmetry --- the closer it is to one, the less cost asymmetry there is between the firms.

The pre-licensing equilibrium quantities, prices and profits can be given as:

$$q_A = \frac{1}{\beta(4-\gamma^2)}\big(2(\alpha - c_A) - \gamma(\alpha - c_B)\big) = \frac{\alpha - c_A}{\beta(4-\gamma^2)}(2 - \gamma\delta),$$

$$q_B = \frac{1}{\beta(4-\gamma^2)}\big(2(\alpha - c_B) - \gamma(\acute{a} - c_A)\big) = \frac{\alpha - c_A}{\beta(4-\gamma^2)}(2\delta - \gamma),$$

$$p_A = c_A + \frac{1}{(4-\gamma^2)}\big(2(\alpha - c_A) - \gamma(\alpha - c_B)\big) = c_A + \beta q_A,$$

$$p_B = c_B + \frac{1}{(4-\gamma^2)}\big(2(\alpha - c_B) - \gamma(\alpha - c_A)\big) = c_B + \beta q_B,$$

$$\pi_A = \beta q_A^2 = \frac{1}{\beta(4-\gamma^2)^2}\big(2(\alpha - c_A) - \gamma(\alpha - c_B)\big)^2 = \frac{1}{\beta}\left(\frac{\alpha - c_A}{4-\gamma^2}(2 - \gamma\delta)\right)^2,$$

$$\pi_B = \beta q_B^2 = \frac{1}{\beta(4-\gamma^2)^2}\big(2(\alpha - c_B) - \gamma(\alpha - c_A)\big)^2 = \frac{1}{\beta}\left(\frac{\alpha - c_A}{4-\gamma^2}(2\delta - \gamma)\right)^2.$$

6.2 The Licensing Game

Now we look into the licensing game with two specific licensing contracts: fixed fee and royalty.

6.2.1 Fixed Fee

Suppose that firm A rejects the innovator's offer. Then the innovator will offer an attractive licensing scheme so that firm B will accept it. If firm B gets the technology by paying the fixed fee F_B, then it will accept if $F_B \le \frac{1}{\beta(4-\gamma^2)^2}\big(2(\alpha - c_B + \epsilon) - \gamma(\alpha - c_A)\big)^2 - \frac{1}{\beta(4-\gamma^2)^2}\big(2(\alpha - c_B) - \gamma(\alpha - c_A)\big)^2 = \frac{4\epsilon}{\beta(4-\gamma^2)^2}(2(\alpha - c_B) - \gamma(\alpha - c_A) + \epsilon) = \frac{4\varepsilon(\alpha - c_A)^2}{\beta(4-\gamma^2)^2}(2\delta - \gamma + å) \equiv g_B$, where $\varepsilon \equiv \frac{\epsilon}{\alpha - c_A}$., and firm A's profit will be $\frac{1}{\beta(4-\gamma^2)^2}\big(2(\alpha - c_A) - \gamma(\alpha - c_B + \epsilon)\big)^2$. Clearly, the innovator will set $F_B = g_B$.

Suppose that the innovator offers an attractive licensing scheme to firm A so that firm A will accept it. If firm A gets the technology by paying the fixed fee F_A, then it will accept as long as $F_A \le \frac{1}{\beta(4-\gamma^2)^2}\big(2(\alpha - c_A) - \gamma(\alpha - c_B)\big)^2 - \frac{1}{\beta(4-\gamma^2)^2}\big(2(\alpha -$

$c_A) - \gamma(\alpha - c_B + \epsilon)\big)^2 = \frac{\gamma\epsilon}{\beta(4-\gamma^2)^2}(4(\alpha - c_A) - 2\gamma(\alpha - c_B) - \gamma\epsilon) = \frac{\gamma\varepsilon(\alpha-c_A)^2}{\beta(4-\gamma^2)^2}(4 - 2\gamma\delta - \gamma\varepsilon) \equiv g_A$. Clearly, the innovator will set $F_A = g_A$.

To ensure firm A's equilibrium quantity is positive, we need to assume $2(\alpha - c_A) - \gamma(\alpha - c_B + \epsilon) > 0$. This assumption can be expressed as

Assumption 5: $\delta + \varepsilon < \frac{2}{\gamma}$.

Note that we have implicitly assumed that the innovation can be fully used by firm B, i.e., $\epsilon < c_B$. To guarantee this, we make the following assumption:

Assumption 6: $c_B > \left(\frac{2}{\tilde{a}} - \delta\right)(\alpha - c_A) \Leftrightarrow c_A > \left(1 - \frac{\gamma}{2}\right)\alpha$.

Since $g_B - g_A = \frac{\varepsilon(\alpha-c_A)^2}{\beta(4-\gamma^2)^2}\left((4+\gamma^2)\varepsilon + 2(4+\gamma^2)\delta - 8\gamma\right)$, we have the results summarized in the following proposition.

Proposition 6

Under exclusive fixed fee licensing,

(i) *If and only if* $\frac{\gamma}{2} < \delta < \frac{4\gamma}{4+\gamma^2}$ *and* $\varepsilon < \frac{8\gamma}{4+\gamma^2} - 2\delta$, *i.e., if and only if firm B is much more inefficient than firm A and the size of the innovation is sufficiently small, the efficient firm gets the license and shelves; no catch-up. The licensing contract is given by* $g_A = \frac{\gamma\varepsilon(\alpha-c_A)^2}{\beta(4-\gamma^2)^2}(4 - 2\gamma\delta - \gamma\varepsilon)$.

(ii) *In all the other situations, the less efficient firm gets the license and there is no shelving. Here catch-up happens. Leapfrogging may or may not happen. The licensing contract is given by* $g_B = \frac{4\varepsilon(\alpha-c_A)^2}{\beta(4-\gamma^2)^2}(2\delta - \gamma + \varepsilon)$.

As in Proposition 1, when the size of the cost-reducing innovation is large, the less efficient firm will win the license. However, contrary to the result in Proposition 1, when the size of the cost-reducing innovation is sufficiently small, the less efficient firm will also win the license if it is not too inefficient. In both cases, the less efficient firm can benefit from the innovation sufficiently enough such that the efficient firm finds it too costly to prevent its rival to get the license.

What causes the difference between the result in Proposition 1 and that in Proposition 6? It is due to the fact that the market expands in the standard exogenous differentiation model while it is fixed in Hotelling model when the marginal cost of firm B decreases (if it wins the license). As a result, the increase of firm B's output exceeds the decrease of firm A's output. So firm B's willingness to pay may exceed firm A's even when the size of the innovation is sufficiently small.

In this situation we see that leapfrogging does not necessarily happen when firm B wins the license, unlike the case with exclusive fixed fee licensing in Hotelling model of product differentiation. The reason is that now firm B may win the license even if the size of the innovation is smaller than the size of the initial cost asymmetry while this is impossible in Hotelling model.

6.2.2 Royalty

We can use the same method as the one in subsection 6.2.1 to find the optimal exclusive royalty licensing contract, which is stated in the following proposition.

Proposition 7

(i) *If* $\frac{\gamma}{2} < \delta < \frac{2}{\gamma^3}\left((8-\gamma^2) - (4-\gamma^2)\sqrt{4+\gamma^2}\right)$ *and* $\varepsilon_1 < \varepsilon < \varepsilon_2$, *where* $\varepsilon_1 = \frac{\gamma(8-\gamma^2)-8\delta-2\sqrt{\gamma^4\delta^2-4\gamma(8-\gamma^2)\delta+4\gamma^2(5-\gamma^2)}}{2(4+\gamma^2)}$ and $\varepsilon_2 = \frac{\gamma(8-\gamma^2)-8\delta+2\sqrt{\gamma^4\delta^2-4\gamma(8-\gamma^2)\delta+4\gamma^2(5-\gamma^2)}}{2(4+\gamma^2)}$, *an exclusive royalty licensing to firm A is better than to firm B for the innovator. The licensing contract is given by* $r_A^* = \frac{\gamma}{8}(\alpha - c_A)\big(\gamma - 2(\delta-\varepsilon)\big)$. *Shelving happens and there is catch-up since firm A's effective marginal cost increases but no leapfrogging.*

(ii) *In all the other situations, exclusive royalty licensing to firm B is optimal for the innovator. License is not shelved.*

(a) When $0 < \varepsilon \leq \delta - \frac{\gamma}{2}$, *i.e., when the size of the innovation is small, the licensing contract is given by* $r_B^* = \epsilon = (\alpha - c_A)\varepsilon$. *No catch-up here.*

(b) When $\delta - \frac{\gamma}{2} < \varepsilon < \frac{2}{\gamma} - \delta$, *i.e., when the size of the innovation is large,*

except for the situation described in (i), the licensing contract is given by $r_B^* = \frac{\alpha - c_A}{4}(2(\delta+\varepsilon)-\gamma)$. *We have catch-up here, even leapfrogging if* $2 - \frac{\gamma}{2} - \delta < \varepsilon < \frac{2}{\gamma} - \delta$.

Proof: See Appendix B.

Here the result is qualitatively the same as in Hotelling model and the intuitive explanation provided below Proposition 2 applies.

Interestingly, here under royalty licensing we see that leapfrogging can happen whereas there is no leapfrogging (only catch-up) under royalty licensing in the Hotelling model. The reason why there is such a difference is hard to tell. One possible reason is that the condition determining the upper limit of the size of the innovation is weaker in the standard product differentiation model since the decrease of firm A's output is smaller than the increase of firm B's output when firm B wins the license (i.e., the market expansion effect explained below Proposition 6).

Finally, under exclusive royalty licensing the innovator's payoff is (see proof of Proposition 7) is as follows.

$$Rev^r = \begin{cases} \frac{(\alpha - c_A)^2\varepsilon}{\beta(4-\gamma^2)}(2\delta-\gamma) & \text{if } 0 < \varepsilon \le \delta - \frac{\gamma}{2} \\ \frac{(\alpha - c_A)^2}{8\beta(4-\gamma^2)}(2(\delta+\varepsilon)-\gamma)^2 & \text{if } \frac{2}{\gamma^3}\left((8-\gamma^2)-(4-\gamma^2)\sqrt{4+\gamma^2}\right) \le \delta < 1 \text{ and } \delta - \frac{\gamma}{2} < \varepsilon < \frac{2}{\gamma} - \delta \\ & \text{or } \frac{\gamma}{2} < \delta < \frac{2}{\gamma^3}\left((8-\gamma^2)-(4-\gamma^2)\sqrt{4+\gamma^2}\right) \text{ and } \delta - \frac{\gamma}{2} < \varepsilon < \varepsilon_1 \text{ or } \varepsilon_2 < \varepsilon < \frac{2}{\gamma} - \delta \\ \frac{\gamma(\alpha - c_A)^2}{32\beta(4-\gamma^2)}(\gamma - 2(\delta-\varepsilon))\left((8-\gamma^2)-2\gamma(\delta+\varepsilon)\right) & \text{if } \frac{\gamma}{2} < \delta < \frac{2}{\gamma^3}\left((8-\gamma^2)-(4-\tilde{a}^2)\sqrt{4+\gamma^2}\right) \text{ and } \varepsilon_1 < \varepsilon < \varepsilon_2 \end{cases}$$

6.3 Optimal Licensing Contract - Comparing Fixed Fee and Royalty

The following result summarizes the overall profit maximizing licensing contract for the innovator and the possibility of shelving (or no-shelving).

Proposition 8

The fixed fee exclusive licensing yields more revenue to the innovator than the royalty licensing. Therefore, the profit maximizing licensing contract of the innovator is as stated in Proposition 6.

Proof: See Appendix B.

Here we find a difference in the result with regard to the optimal licensing for the innovator. Under Hotelling model of product differentiation, the optimal licensing policy contains fixed fee for one set of parameters and per-unit royalty for another set, whereas in standard model of product differentiation the optimal licensing policy consists of fixed fee only. The intuition of the above result can be provided as follows: In the above exogenous product differentiation Cournot model a la Singh and Vives (1984), the total quantity demanded increases when either firm's effective marginal cost falls, while it remains fixed in Hotelling model. This makes fixed fee licensing more attractive to the innovator since firm B's effective marginal cost is reduced by a larger amount under fixed fee licensing than under royalty licensing. This market expansion possibility is a crucial difference although we assume that the firms compete in prices in Hotelling model and in quantities in the above standard product differentiation model.

7. Conclusion

The basic purpose of innovation gets defeated if the knowledge generated from the innovation is not used. When the innovation is a cost reducing technology, the value of innovation is not realized if that is not used in the production process of a firm. In this paper, we show that this may happen through patent licensing contracts. The phenomenon of acquiring a technology and not using it is called shelving. By shelving the technology, the technology-acquiring firm can strategically prevent its competitor from using it, and thus maintain its strategic advantage in the market, and hence the market dominance.

To understand this phenomenon in greater depth, we look into technology transfer arrangements through licensing in a strategic environment with an outside innovator and two potential licensees. The licensees have asymmetric cost structures leading to asymmetric absorptive capacities of the cost reducing technology. In particular, the innovation reduces the marginal cost of the inefficient firm only; the efficient firm does not benefit from it at all.

We find that under fixed fee licensing, shelving will happen when the size of the innovation is not too big compared to the initial cost difference between the potential licensees. This is where the technology goes to the efficient firm, and it is shelved. Obviously, there is no catch-up here for the inefficient firm. On the other hand, shelving will not happen for relatively large innovations, and this is where the technology goes to the inefficient firm. Moreover, here we observe not only catch-up but also leapfrogging, i.e., the inefficient firm becomes the efficient one after licensing (but leapfrogging does not necessarily happen in the standard product differentiation Cournot model). With per-unit royalty licensing, the analysis is much more nuanced, and we find an interesting non-monotonicity with respect to shelving and no shelving as the size of the innovation becomes bigger. Finally, in this context, we also find the optimal licensing contract and precisely identify regions of shelving and no shelving in the whole relevant parameter space, and derive corresponding welfare implications.

To check the robustness of our main findings we follow up with the same analysis but in the framework of a standard product differentiation model and mostly regenerate the same results with few exceptions. This points to the fact that our main findings are mostly invariant to the underlying basic model of horizontal production differentiation.

References

Badia, D. B., (2019), 'Patent Licensing and Technological Catch-Up in a Heterogeneous Duopoly', *Review of Industrial Organization*, 55, 287-300.

Cunningham, C., and Ederer, F., and Ma, S., (2021), 'Killer Acquisitions', *Journal of Political Economy*,129, 649-702.

Fumagalli, C., Motta, M., and Tarantino, E., (2020). 'Shelving or Developing? The Acquisition of Potential Competitors under Financial Constraints,' Economics Working Papers 1735, Department of Economics and Business, Universitat Pompeu Fabra.

Gilbert, R. J. and Newbery, D. (1982) 'Preemptive Patenting and the Persistence of Monopoly,' *American Economic Review*, 72, 514-526.

Hagiu, A. and Yoffie, D. (2013), 'The New Patent Intermediaries: Platforms, Defensive Aggregators, and Super-Aggregators' *Journal of Economic Perspectives*, 27, 45-66.

Hotelling, H. (1929), 'Stability in Competition', *The Economic Journal*, 93, 41-57.

Letina, I., Schmutzler, A., and Seibel, R., (2020), 'Killer Acquisitions and Beyond: Policy Effects on Innovation Strategies', Working paper 358, Department of Economics, University of Zurich.

Lu, Y. and Poddar, S. (2023), 'Exclusive and non-exclusive licensing with shelving', *Mathematical Social Sciences*, 13-29.

Norbäck, P., Olofsson, C., & Persson, L. (2020). 'Acquisitions for Sleep', *The B.E. Journal of Economic Analysis & Policy*, 20(2), https://doi.org/10.1515/bejeap-2019-0021.

Reinganum (1983). 'Uncertain Innovation and the Persistence of Monopoly', *American Economic Review*, 73, 741-748.

Singh, N., & Vives, X. (1984). 'Price and quantity competition in a differentiated duopoly', *The Rand Journal of Economics*, 546-554.

Torrisi, S., A. Gambardella, P. Giuri, D. Harhoff, K. Hoisl, and M. Mariani. 2016. "Used, Blocking and Sleeping Patents: Empirical Evidence from a Large-Scale Inventor Survey." *Research Policy*, 45 (7): 1374–85.

Appendix A

Proof of Proposition 2

Suppose that firm A rejects the innovator's offer. Then the innovator will offer an attractive licensing scheme so that firm B will accept it. Firm B's profit will be

$\frac{1}{18t}(3t-\delta+\epsilon-r_B)^2$, and its equilibrium quantity will be $\frac{1}{6t}(3t-\delta+\epsilon-r_B)$, where r_B denotes the per-unit royalty set in the exclusive royalty licensing to firm B. The innovator will set r_B to maximize $\frac{r_B}{6t}(3t-\delta+\epsilon-r_B)$. The optimum royalty rate is $r_B^* = \begin{cases} \frac{3t-\delta+\epsilon}{2} & if\ \epsilon > 3t-\delta \\ \epsilon\ \ if\ \ \epsilon \leq 3t-\delta \end{cases}$. It follows that the innovator's revenue is $Rev_B^r = \begin{cases} \frac{(3t-\delta+\epsilon)^2}{24t} & if\ \epsilon > 3t-\delta \\ \frac{\epsilon}{6t}(3t-\delta) & if\ \ \epsilon \leq 3t-\delta \end{cases}$ and firm A's profit is $\pi_A^{Rej} = \begin{cases} \frac{1}{18t}\left(\frac{9t+\delta-\epsilon}{2}\right)^2 & if\ \epsilon > 3t-\delta \\ \frac{1}{18t}(3t+\delta)^2 & if\ \ \epsilon \leq 3t-\delta \end{cases}$.

Suppose that the innovator offers an attractive licensing scheme to firm A so that firm A will accept it. Firm A's profit will be $\pi_A^R = \frac{1}{18t}(3t+\delta-r_A)^2$, where r_A denotes the per-unit royalty set in the exclusive royalty licensing to firm A. To determine the optimal royalty rate r_A, we distinguish the following two cases depending on the size of the innovation.

Case 1: $0 < \epsilon \leq 3t-\delta$

In this case, clearly, firm A does not accept any royalty licensing scheme with positive r_A. So the innovator offers $r_A > 0$ so that firm A rejects. After firm A's rejection, the innovator offers $r_B^* = \epsilon$, firm B accepts the technology and pre-licensing market outcome prevails. In this situation, the technology is used, therefore no shelving. There is no catch-up as the equilibrium royalty rate is equal to the size of the innovation.

Case 2: $3t-\delta < \epsilon < 3t+\delta$

In this case, if firm A rejects the offer, the optimal royalty rate set by the innovator to firm B will be $r_B^* = \frac{3t-\delta+\epsilon}{2} < \epsilon$. Firm B's effective marginal cost falls, adversely affecting firm A. Therefore, firm A has an incentive to preempt firm B from getting the technology and shelve it. Firm A will accept the license if and only if $\frac{1}{18t}(3t+\delta-r_A)^2 \geq \frac{1}{18t}\left(\frac{9t+\delta-\epsilon}{2}\right)^2$ implying $r_A \leq \frac{\delta+\epsilon-3t}{2}$. The innovator will maximize $\frac{r_A}{6t}(3t+\delta-r_A)$ subject to $r_A \leq \frac{\delta+\epsilon-3t}{2}$. The optimum r_A will be $r_A^* = \frac{\delta+\epsilon-3t}{2}$. It follows that the innovator's revenue is $Rev_A^r = \frac{1}{24t}(\epsilon + \delta - 3t)(9t + \delta - \epsilon)$. To determine whether the innovator licenses to firm B or firm A, we compare $\frac{(3t-\delta+\epsilon)^2}{24t}$ and $\frac{1}{24t}(\epsilon + \delta - 3t)(9t + \delta - \epsilon)$. We have $\frac{(3t-\delta+\epsilon)^2}{24t} - \frac{1}{24t}(\epsilon + \delta - 3t)(9t + \delta - \epsilon) = \frac{1}{12t}(\epsilon^2 - (3t + \delta)\epsilon + 18t^2 - 6t\delta)$.

Define $f(\epsilon) = \epsilon^2 - (3t + \delta)\epsilon + 18t^2 - 6t\delta$. Algebra tells us that (a) if $\delta < (12\sqrt{2} - 15)t \approx 1.971t$, then $f(\epsilon) > 0$ for all ϵ; (b) if $\delta = (12\sqrt{2} - 15)t$, then $f(\epsilon) > 0$ for all ϵ except $\epsilon = \frac{3t+\delta}{2}$ and $f\left(\frac{3t+\delta}{2}\right) = 0$; and (c) if $(12\sqrt{2} - 15)t < \delta < 3t$, then $f(\epsilon) > 0$ for $3t - \delta < \epsilon < \epsilon_1$ and $\epsilon_2 < \epsilon < 3t + \delta$, $f(\epsilon) < 0$ for $\epsilon_1 < \epsilon < \epsilon_2$ and $f(\epsilon) = 0$ for $\epsilon = \epsilon_1$ or ϵ_2, where ϵ_1 and ϵ_2 are as defined as in the text.

Hence, we obtain the results stated in Proposition 2(i) and (ii).

For the result on catch-up in 2(i), since firm A has to pay per-unit royalty rate, there will be catch-up. However, $c_B - (c_A + r_A^*) = \frac{3t+\delta-\epsilon}{2} > 0$ (as $\epsilon < 3t + \delta$), hence no leapfrogging.

For the result on catch-up in 2(ii)(b), since $c_B - \epsilon + r_B^* < c_B$ (as $r_B^* < \epsilon$), there will be catch-up. However, $c_B - \epsilon + r_B^* - c_A = \frac{3t+\delta-\epsilon}{2} > 0$, hence no leapfrogging.

Proof of Proposition 3

The innovator's revenue from fixed fee licensing is

$$Rev = \frac{\epsilon}{18t}\{6t + 2\delta - \epsilon\} \quad if\ \epsilon < 2\delta$$

$$= \frac{\epsilon}{18t}\{6t - 2\delta + \epsilon\} \quad if\ 2\delta < \epsilon < 3t + \delta.$$

And royalty revenues are summarized in the text and replicated below.

$$Rev = \frac{\epsilon}{6t}(3t - \delta) \quad if\ \ 0 < \epsilon \le 3t - \delta$$

$$= \frac{(3t-\delta+\epsilon)^2}{24t} \quad if\ \delta \le (12\sqrt{2} - 15)t\ \ and\ \ 3t - \delta < \epsilon < 3t + \delta$$

$$or\ (12\sqrt{2} - 15)t < \delta < 3t\ and\ 3t - \delta < \epsilon < \epsilon_1\ \ or\ \ \epsilon_2 < \epsilon < 3t + \delta$$

$$= \frac{1}{24t}(\epsilon + \delta - 3t)(9t + \delta - \epsilon) \ \ if \ \left(12\sqrt{2} - 15\right)t < \delta < 3t \ and \ \ \epsilon_1 < \epsilon < \epsilon_2$$

Consider the following cases:

(1) Case 1: $0 < \epsilon \leq 3t - \delta$

Case 1.1: $\delta < t \ (thus \ 2\delta < 3t - \delta)$

Case 1.1.1: When $\epsilon < 2\delta$

Since

$$\frac{\epsilon}{18t}\{6t + 2\delta - \epsilon\} - \frac{\epsilon}{6t}(3t - \delta) = \frac{\epsilon}{18t}\{5\delta - 3t - \epsilon\}\begin{cases} > 0 \ if \ 0 < \epsilon < 5\delta - 3t \\ < 0 \ if \ 5\delta - 3t < \epsilon < 2\delta \end{cases},$$

fixed fee licensing is better than royalty licensing if $0 < \epsilon < 5\delta - 3t$ (this requires $\delta > \frac{3t}{5}$), while royalty licensing is better than fixed fee licensing if $5\delta - 3t < \epsilon < 2\delta$.

Case 1.1.2: When $2\delta < \epsilon \leq 3t - \delta$

Since

$$\frac{\epsilon}{18t}\{6t - 2\delta + \epsilon\} - \frac{\epsilon}{6t}(3t - \delta) = \frac{\epsilon}{18t}\{-3t + \delta + \epsilon\} < 0,$$

royalty licensing is better than fixed fee licensing.

In sum, in case 1.1, fixed fee licensing is better than royalty licensing if $0 < \epsilon < 5\delta - 3t$ (this requires $\delta > \frac{3t}{5}$), royalty licensing is better than fixed fee licensing if $5\delta - 3t < \epsilon \leq 3t - \delta$.

Case 1.2: $\delta \geq t \ (thus \ 2\delta \geq 3t - \delta)$

Since

$$\frac{\epsilon}{18t}\{6t + 2\delta - \epsilon\} - \frac{\epsilon}{6t}(3t - \delta) = \frac{\epsilon}{18t}\{5\delta - 3t - \epsilon\} \geq 0,$$

fixed fee licensing is better than royalty licensing.

(2) Case 2: $\delta \leq \left(12\sqrt{2} - 15\right)t \ \ and \ \ 3t - \delta < \epsilon < 3t + \delta$

Case 2.1: $\delta < t \ (thus \ 2\delta < 3t - \delta)$

It is straightforward to calculate $\frac{\epsilon}{18t}\{6t - 2\delta + \epsilon\} - \frac{(3t-\delta+\epsilon)^2}{24t} = \frac{1}{72t}\{\epsilon^2 + 2\epsilon(3t - \delta) - 3(3t - \delta)^2\} = \frac{1}{72t}[\epsilon - (3t - \delta)][\epsilon + 3(3t - \delta)] > 0$. Therefore, fixed fee licensing is better than royalty licensing.

Case 2.2: $t \leq \delta \leq (12\sqrt{2} - 15)t\ (thus\ 2\delta \geq 3t - \delta)$

Case 2.2.1: When $3t - \delta < \epsilon < 2\delta$

Since

$\frac{\epsilon}{18t}\{6t + 2\delta - \epsilon\} - \frac{(3t-\delta+\epsilon)^2}{24t} = \frac{\epsilon}{9t}\{2\delta - \epsilon\} + \frac{\epsilon}{18t}\{6t - 2\delta + \epsilon\} - \frac{(3t-\delta+\epsilon)^2}{24t} =$

$\frac{\epsilon}{9t}\{2\delta - \epsilon\} + \frac{1}{72t}[\epsilon - (3t - \delta)][\epsilon + 3(3t - \delta)] > 0,$

fixed fee licensing is better than royalty licensing.

Case 2.2.2: When $2\delta < \epsilon < 3t + \delta$

For the same reason as in case 2.1, fixed fee licensing is better than royalty licensing.

In sum, in case 2, fixed fee licensing is better than royalty licensing.

Before moving to next case, recall we defined $\epsilon_1 = \frac{3t+\delta-\sqrt{-63t^2+\delta^2+30t\delta}}{2}$ and $\epsilon_2 = \frac{3t+\delta+\sqrt{-63t^2+\delta^2+30t\delta}}{2}$

(3) Case 3: $(12\sqrt{2} - 15)t < \delta < 3t\ and\ 3t - \delta < \epsilon < \epsilon_1\ \ or\ \ \epsilon_2 < \epsilon < 3t + \delta$

Note that $2\delta > \epsilon_2$.

Case 3.1: When $3t - \delta < \epsilon < \epsilon_1$ or $\epsilon_2 < \epsilon < 2\delta$

For the same reason as in case 2.2.1, fixed fee licensing is better than royalty licensing.

Case 3.2: When $2\delta < \epsilon < 3t + \delta$

For the same reason as in case 2.1, fixed fee licensing is better than royalty licensing.

In sum, in case 3, fixed fee licensing is better than royalty licensing.

(4) Case 4: $(12\sqrt{2} - 15)t < \delta < 3t\ and\ \epsilon_1 < \epsilon < \epsilon_2$

For the same reason as in case 2.2.1, fixed fee licensing is better than royalty licensing.

After analyzing the above four cases, we obtain the results stated in Proposition 3.

Appendix B

Proof of Proposition 7

Suppose that firm A rejects the innovator's offer. Then the innovator will offer an attractive licensing scheme so that firm B will accept it. Firm B's profit will be $\frac{1}{\beta(4-\gamma^2)^2}\left(2(\alpha-c_B+\epsilon-r_B)-\gamma(\alpha-c_A)\right)^2$, and its equilibrium quantity will be $\frac{1}{\beta(4-\gamma^2)}\left(2(\alpha-c_B+\epsilon-r_B)-\gamma(\alpha-c_A)\right)$, where r_B denotes the per-unit royalty set in the exclusive royalty licensing to firm B. The innovator will maximize $\frac{r_B}{\beta(4-\gamma^2)}\left(2(\alpha-c_B+\epsilon-r_B)-\gamma(\alpha-c_A)\right)$ subject to $\frac{1}{\beta(4-\gamma^2)^2}\left(2(\alpha-c_B+\epsilon-r_B)-\gamma(\alpha-c_A)\right)^2 \geq \frac{1}{\beta(4-\gamma^2)^2}\left(2(\alpha-c_B)-\gamma(\alpha-c_A)\right)^2$ which is equivalent to $r_B \leq \epsilon$. It is straightforward to obtain the optimal royalty rate

$$r_B^* = \begin{cases} \frac{1}{4}\left(2(\alpha-c_B+\epsilon)-\gamma(\alpha-c_A)\right) = \frac{\alpha-c_A}{4}(2(\delta+\varepsilon)-\gamma) & if\ \varepsilon \geq \delta-\frac{\gamma}{2} \\ \epsilon = (\alpha-c_A)\varepsilon & otherwise \end{cases}.$$

It follows that the innovator's revenue is

$$Rev_B^r = \begin{cases} \frac{(\alpha-c_A)^2}{8\beta(4-\gamma^2)}(2(\delta+\varepsilon)-\gamma)^2 & if\ \varepsilon \geq \delta-\frac{\gamma}{2} \\ \frac{(\alpha-c_A)^2\varepsilon}{\beta(4-\gamma^2)}\ (2\delta-\gamma) & otherwise \end{cases},$$

and firm A's profit is $\pi_A^{Rej} =$

$$\begin{cases} \frac{1}{16\beta(4-\gamma^2)^2}\left((8-\gamma^2)(\alpha-c_A)-2\gamma(\alpha-c_B+\epsilon)\right)^2 & if\ \varepsilon \geq \delta-\frac{\gamma}{2} \\ \frac{1}{\beta(4-\gamma^2)^2}\left(2(\alpha-c_A)-\gamma(\alpha-c_B)\right)^2 & otherwise \end{cases}.$$

Suppose that the innovator offers an attractive licensing scheme to firm A so that firm A will accept it. Firm A's profit will be $\pi_A^R = \frac{1}{\beta(4-\gamma^2)^2}\left(2(\alpha-c_A-r_A)-\gamma(\alpha-c_B)\right)^2$, where r_A denotes the per-unit royalty set in the exclusive royalty licensing to firm A. To determine the optimal royalty rate r_A, we distinguish the following two cases depending on the size of the innovation.

Case 1: $0 < \varepsilon \leq \delta - \frac{\gamma}{2}$

In this case, clearly, firm A does not accept any royalty licensing scheme with positive r_A. So the innovator offers $r_A > 0$ so that firm A rejects. After firm A's rejection, the innovator offers $r_B^* = \epsilon$, firm B accepts the technology and pre-licensing market outcome prevails. In this situation, the technology is used, therefore no shelving.

There is no catch-up as the equilibrium royalty rate is equal to the size of the innovation.

Case 2: $\delta - \frac{\gamma}{2} < \varepsilon < \frac{2}{\gamma} - \delta$

In this case, if firm A rejects the offer, the optimal royalty rate set by the innovator to firm B will be $r_B^* = \frac{\alpha - c_A}{4}(2(\delta+\varepsilon)-\gamma) < \epsilon$. Firm B's effective marginal cost falls, adversely affecting firm A. Therefore, firm A has an incentive to preempt firm B from getting the technology and shelve it. Firm A will accept the license if and only if $\frac{1}{\beta(4-\gamma^2)^2}\left(2(\alpha-c_A-r_A)-\gamma(\alpha-c_B)\right)^2 \geq \frac{1}{16\beta(4-\gamma^2)^2}\left((8-\gamma^2)(\alpha-c_A)-2\gamma(\alpha-c_B+\epsilon)\right)^2$ implying $r_A \leq \frac{\gamma}{8}\left(\gamma(\alpha-c_A)-2(\alpha-c_B-\epsilon)\right)$. The innovator will maximize $\frac{r_A}{\beta(4-\gamma^2)}\left(2(\alpha-c_A-r_A)-\gamma(\alpha-c_B)\right)$ subject to $r_A \leq \frac{\gamma}{8}\left(\gamma(\alpha-c_A)-2(\alpha-c_B-\epsilon)\right)$. The optimum r_A will be $r_A^* = \frac{\gamma}{8}\left(\gamma(\alpha-c_A)-2(\alpha-c_B-\epsilon)\right) = \frac{\gamma}{8}(\alpha-c_A)\left(\gamma-2(\delta-\varepsilon)\right)$. It follows that the innovator's revenue is $Rev_A^r = \frac{\gamma(\alpha-c_A)^2}{32\beta(4-\gamma^2)}\left(\gamma-2(\delta-\varepsilon)\right)\left((8-\gamma^2)-2\gamma(\delta+\varepsilon)\right)$. To determine whether the innovator licenses to firm B or firm A, we compare $\frac{(\alpha-c_A)^2}{8\beta(4-\gamma^2)}(2(\delta+\varepsilon)-\gamma)^2$ and $\frac{\gamma(\alpha-c_A)^2}{32\beta(4-\gamma^2)}\left(\gamma-2(\delta-\varepsilon)\right)\left((8-\gamma^2)-2\gamma(\delta+\varepsilon)\right)$ We have

$$\frac{(\alpha-c_A)^2}{8\beta(4-\gamma^2)}(2(\delta+\varepsilon)-\gamma)^2 - \frac{\gamma(\alpha-c_A)^2}{32\beta(4-\gamma^2)}\left(\gamma-2(\delta-\varepsilon)\right)\left((8-\gamma^2)-2\gamma(\delta+\varepsilon)\right) = \frac{\gamma(\alpha-c_A)^2}{32\beta(4-\gamma^2)}\left(4(4+\gamma^2)\varepsilon^2 + 4\left(8\delta-\gamma(8-\gamma^2)\right)\varepsilon + (4-\gamma^2)\left(4\delta^2-\gamma^2\right)\right).$$

Define $g(\varepsilon) = 4(4+\gamma^2)\varepsilon^2 + 4\left(8\delta-\gamma(8-\gamma^2)\right)\varepsilon + (4-\gamma^2)(4\delta^2-\gamma^2)$.

Clearly, if $\delta \geq \frac{\gamma(8-\gamma^2)}{8}$, then $g(\varepsilon) > 0$ and thus $Rev_B^r > Rev_A^r$.

What if $\frac{\gamma}{2} < \delta < \frac{\gamma(8-\gamma^2)}{8}$?

Since

$$\left(4\left(8\delta-\gamma(8-\gamma^2)\right)\right)^2 - 4\times 4(4+\gamma^2)\times(4-\gamma^2)\left(4\delta^2-\gamma^2\right) = 64\gamma\left(\gamma^3\delta^2 - 4(8-\gamma^2)\delta + 4\gamma(5-\gamma^2)\right)\begin{cases} < 0 & if\ \delta > \frac{2}{\gamma^3}\left((8-\gamma^2)-(4-\gamma^2)\sqrt{4+\gamma^2}\right) \\ > 0 & if\ \delta < \frac{2}{\gamma^3}\left((8-\gamma^2)-(4-\gamma^2)\sqrt{4+\gamma^2}\right) \end{cases},$$

we have $g(\varepsilon) > 0$ for all ε if $\delta > \frac{2}{\gamma^3}\left((8-\gamma^2)-(4-\gamma^2)\sqrt{4+\gamma^2}\right)$.

If $\frac{\gamma}{2} < \delta < \frac{2}{\gamma^3}\left((8-\gamma^2)-(4-\gamma^2)\sqrt{4+\gamma^2}\right)$, define $\varepsilon_1 = \frac{\gamma(8-\gamma^2)-8\delta-2\sqrt{\gamma^4\delta^2-4\gamma(8-\gamma^2)\delta+4\gamma^2(5-\gamma^2)}}{2(4+\gamma^2)}$ and $\varepsilon_2 = \frac{\gamma(8-\gamma^2)-8\delta+2\sqrt{\gamma^4\delta^2-4\gamma(8-\gamma^2)\delta+4\gamma^2(5-\gamma^2)}}{2(4+\gamma^2)}$, we have $g(\varepsilon) > 0$ for $\delta - \frac{\gamma}{2} < \varepsilon < \varepsilon_1$ or $\varepsilon_2 < \varepsilon < \frac{2}{\gamma} - \delta$, and $g(\varepsilon) < 0$ for $\varepsilon_1 < \varepsilon < \varepsilon_2$.

Hence, we obtain the results stated in Proposition 7(i) and (ii).

For the result on catch-up in 7(i), since firm A has to pay per-unit royalty rate, there will be catch-up. However, $c_B - (c_A + r_A^*) = (\alpha - c_A)\left(1 - \delta - \frac{\gamma}{8}(\gamma - 2(\delta - \varepsilon))\right) = \frac{1}{8}(\alpha - c_A)(8 - \gamma^2 - 2(4-\gamma)\delta - 2\gamma\varepsilon) > \frac{1}{8}(\alpha - c_A)\left(8 - \gamma^2 - 2(4-\gamma)\delta - 2\gamma\left(\frac{2}{\gamma} - \delta\right)\right) = \frac{1}{8}(\alpha - c_A)(4 - \gamma^2 - 4(2-\gamma)\delta) > \frac{1}{8}(\alpha - c_A)\left(4 - \gamma^2 - 4(2-\gamma)\frac{2}{\gamma^3}\left((8-\gamma^2)-(4-\gamma^2)\sqrt{4+\gamma^2}\right)\right) > 0$, where the first inequality is due to Assumption 5, the second is due to $\delta < \frac{2}{\gamma^3}\left((8-\gamma^2)-(4-\gamma^2)\sqrt{4+\gamma^2}\right)$ and the third one holds for $\gamma \in (0,1)$, hence no leapfrogging.

For the result on catch-up in 7(ii)(b), since $c_B - \epsilon + r_B^* < c_B$ (as $r_B^* < \epsilon$), there will be catch-up. However, $c_B - \epsilon + r_B^* = \frac{1}{2}(c_B + \alpha - \epsilon) - \frac{\tilde{a}(\alpha - c_A)}{4} = c_A + \frac{1}{2}\left(2 - \frac{\gamma}{2} - \delta - \varepsilon\right)(\alpha - c_A)$. Hence there is leapfrogging if $2 - \frac{\gamma}{2} - \delta < \varepsilon < \frac{2}{\gamma} - \delta$.

Proof of Proposition 8

The innovator's revenue from fixed fee licensing is

$$Rev^f = \begin{cases} \frac{\gamma\varepsilon(\alpha-c_A)^2}{\beta(4-\gamma^2)^2}(4 - 2\gamma\delta - \gamma\varepsilon) & if\ \frac{\gamma}{2} < \delta < \frac{4\gamma}{4+\gamma^2}\ and\ \varepsilon < \frac{8\gamma}{4+\gamma^2} - 2\delta \\ \frac{4\varepsilon(\alpha-c_A)^2}{\beta(4-\gamma^2)^2}(2\delta - \gamma + \varepsilon) & otherwise \end{cases}$$

And royalty revenues are summarized in the text and replicated below.

Rev^r

$$= \begin{cases} \frac{(\alpha-c_A)^2\varepsilon}{\beta(4-\gamma^2)}\ (2\delta-\gamma) & if\ 0<\varepsilon\le\delta-\frac{\gamma}{2} \\ \frac{(\alpha-c_A)^2}{8\beta(4-\gamma^2)}(2(\delta+\varepsilon)-\gamma)^2 & if\ \frac{2}{\gamma^3}\left((8-\gamma^2)-(4-\gamma^2)\sqrt{4+\gamma^2}\right)\le\delta<1\ and\ \delta-\frac{\gamma}{2}<\varepsilon<\frac{2}{\gamma}-\delta, \\ & or\ \frac{\gamma}{2}<\delta<\frac{2}{\gamma^3}\left((8-\gamma^2)-(4-\gamma^2)\sqrt{4+\gamma^2}\right)\ and\ \delta-\frac{\gamma}{2}<\varepsilon<\varepsilon_1\ or\ \varepsilon_2<\varepsilon<\frac{2}{\gamma}-\delta \\ \frac{\gamma(\alpha-c_A)^2}{32\beta(4-\gamma^2)}(\gamma-2(\delta-\varepsilon))((8-\gamma^2)-2\tilde{a}(\delta+\varepsilon)) & if\ \frac{\gamma}{2}<\delta<\frac{2}{\gamma^3}\left((8-\gamma^2)-(4-\gamma^2)\sqrt{4+\gamma^2}\right)\ and\ \varepsilon_1<\varepsilon<\varepsilon_2 \end{cases}$$

Consider the following cases:

(1) Case 1: $0<\varepsilon\le\delta-\frac{\gamma}{2}$

Case 1.1: $\frac{\gamma}{2}<\delta<\frac{4\gamma}{4+\gamma^2}$ and $\varepsilon\le min\left\{\frac{8\gamma}{4+\gamma^2}-2\delta,\delta-\frac{\gamma}{2}\right\}$

In this case, we compare the innovator's revenue from fixed fee licensing to firm A and revenue from royalty licensing to firm B ($r_B^*=\epsilon=(\alpha-c_A)\varepsilon$).

Since

$$\frac{\gamma\varepsilon(\alpha-c_A)^2}{\beta(4-\gamma^2)^2}(4-2\gamma\delta-\gamma\varepsilon)-\frac{(\alpha-c_A)^2\varepsilon}{\beta(4-\gamma^2)}\ (2\delta-\gamma)=\frac{\varepsilon(\alpha-c_A)^2}{\beta(4-\gamma^2)^2}(8\gamma-8\delta-\gamma^3-\gamma^2\varepsilon)>0$$

for $\frac{\gamma}{2}<\delta<\frac{4\gamma}{4+\gamma^2}$ and $\varepsilon\le min\left\{\frac{8\gamma}{4+\gamma^2}-2\delta,\delta-\frac{\gamma}{2}\right\}$,

fixed fee licensing (to firm A) is better than royalty licensing (to firm B).

Case 1.2: $\frac{\gamma(20+\gamma^2)}{6(4+\gamma^2)}<\delta\le 1$ and $max\left\{\frac{8\gamma}{4+\gamma^2}-2\delta,0\right\}\le\varepsilon\le\delta-\frac{\gamma}{2}$

In this case, we compare the innovator's revenue from fixed fee licensing to firm B and revenue from royalty licensing to firm B ($r_B^*=\epsilon=(\alpha-c_A)\varepsilon$).

Since

$$\frac{4\varepsilon(\alpha-c_A)^2}{\beta(4-\gamma^2)^2}(2\delta-\gamma+\varepsilon)-\frac{(\alpha-c_A)^2\varepsilon}{\beta(4-\gamma^2)}\ (2\delta-\gamma)=\frac{\varepsilon(\alpha-c_A)^2}{\beta(4-\gamma^2)^2}\left(4\varepsilon+\gamma^2(2\delta-\gamma)\right)>0,$$

fixed fee licensing (to firm B) is better than royalty licensing (to firm B).

(2) Case 2: $\frac{\gamma}{2}<\delta<\frac{2}{\gamma^3}\left((8-\gamma^2)-(4-\gamma^2)\sqrt{4+\gamma^2}\right)$ $and\ \varepsilon_1<\varepsilon<\varepsilon_2$

In this case, we compare the innovator's revenue from fixed fee licensing to firm A and revenue from royalty licensing to firm A.

Since

$$\frac{\gamma\varepsilon(\alpha-c_A)^2}{\beta(4-\gamma^2)^2}(4-2\gamma\delta-\gamma\varepsilon)-\frac{\gamma(\alpha-c_A)^2}{32\beta(4-\gamma^2)}(\gamma-2(\delta-\text{å}))\left((8-\gamma^2)-2\gamma(\delta+\varepsilon)\right)=$$

$$-\frac{\gamma(\alpha-c_A)^2}{32\beta(4-\gamma^2)^2}\Big(4\gamma(4+\gamma^2)\varepsilon^2-4(16+8\gamma^2-\gamma^4-16\gamma\delta)\varepsilon+(4-\gamma^2)(2\delta-\gamma)(\gamma^2+$$

$2\gamma\delta - 8)\Big)\Big) > 0$ for $\varepsilon_1 < \varepsilon < \varepsilon_2$ (this can be shown by evaluating the difference at the two extreme values of ε and verifying it positive), fixed fee licensing (to firm A) is better than royalty licensing (to firm A).

(3) Case 3: $\frac{\gamma}{2} < \delta < \frac{2}{\gamma^3}\left((8-\gamma^2) - (4-\gamma^2)\sqrt{4+\gamma^2}\right)$ $and\ \delta - \frac{\gamma}{2} < \varepsilon < \varepsilon_1\ or\ \varepsilon_2 < \varepsilon < \frac{8\gamma}{4+\gamma^2} - 2\delta$, or $\frac{2}{\gamma^3}\left((8-\gamma^2) - (4-\gamma^2)\sqrt{4+\gamma^2}\right) \le \delta < \frac{\gamma(20+\gamma^2)}{6(4+\gamma^2)}$ $and\ \delta - \frac{\gamma}{2} < \varepsilon < \frac{8\gamma}{4+\gamma^2} - 2\delta$

In this case, we compare the innovator's revenue from fixed fee licensing to firm A and revenue from royalty licensing to firm B ($r_B^* = \frac{\alpha - c_A}{4}(2(\delta+\varepsilon) - \gamma)$).

Since $\frac{\gamma\varepsilon(\alpha-c_A)^2}{\beta(4-\gamma^2)^2}(4 - 2\gamma\delta - \gamma\varepsilon) - \frac{(\alpha-c_A)^2}{8\beta(4-\gamma^2)}(2(\delta+\varepsilon)-\gamma)^2 = -\frac{(\alpha-c_A)^2}{8\beta(4-\gamma^2)^2}\Big(4(4+\gamma^2)\varepsilon^2 - 4(12\gamma - \gamma^3 - 2(4+\gamma^2)\delta)\varepsilon + (4-\gamma^2)(2\delta-\gamma)^2\Big) > 0$ for $\delta - \frac{\gamma}{2} < å < \frac{\gamma(20+\gamma^2)}{6(4+\gamma^2)}$ (this can be shown by evaluating the difference at the two extreme values of ε and verifying it positive), fixed fee licensing (to firm A) is better than royalty licensing (to firm B).

(4) Case 4: $\frac{\gamma}{2} < \delta < 1\ and\ max\left\{\frac{8\gamma}{4+\gamma^2} - 2\delta, \delta - \frac{\gamma}{2}\right\} < \varepsilon < \frac{2}{\gamma} - \delta$

In this case, we compare the innovator's revenue from fixed fee licensing to firm B and revenue from royalty licensing to firm B ($r_B^* = \frac{\alpha - c_A}{4}(2(\delta+\varepsilon) - \gamma)$).

Since $\frac{4\varepsilon(\alpha-c_A)^2}{\beta(4-\gamma^2)^2}(2\delta - \gamma + \varepsilon) - \frac{(\alpha-c_A)^2}{8\beta(4-\gamma^2)}(2(\delta+\varepsilon)-\gamma)^2 = \frac{(\alpha-c_A)^2}{8\beta(4-\gamma^2)^2}\Big(4(4+\gamma^2)\varepsilon^2 + 4(4+\gamma^2)(2\delta-\gamma)\varepsilon - (4-\gamma^2)(2\delta-\gamma)^2\Big) > 0$ for $\frac{\gamma}{2} < \delta < 1\ and\ max\left\{\frac{8\gamma}{4+\gamma^2} - 2\delta, \delta - \frac{\gamma}{2}\right\} < \varepsilon < \frac{2}{\gamma} - \delta$, fixed fee licensing (to firm B) is better than royalty licensing (to firm B).

After analyzing the above four cases, we obtain the results stated in Proposition 8.