Fiscal Policy as a Monetary Anchor*

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Abstract

If Ricardian equivalence fails, fiscal policy can provide an anchor for the price level even if the government commits to fully repay its debt in all contingencies. Therefore, it is possible to resolve the indeterminacy problem under an interest rate rule without resorting to either of the most popular (but theoretically controversial) selection devices: (i) ad hoc restrictions on explosive inflation paths, or (ii) the fiscal theory of the price level. I illustrate this point analytically in a Blanchard-Yaari-style overlapping generations economy and then prove it in a broader class of macroeconomic environments.

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1 Introduction

How does monetary policy determine the price level under an interest rate rule? What are the effects of interest rate shocks on inflation? These questions have familiar answers that are conventional wisdom among practitioners, but the theoretical foundation of these answers remains a subject of debate.

A simple example is instructive. Consider a flexible-price economy with a constant real interest rate r^*, an exogenous sequence of nominal interest rates $\{i_t\}$ and an endogenous log price level $\{p_t\} = \{\log P_t\}$. The Fisher equation, written in logs, is

$$r^* = i_t - (\mathbb{E}_t[p_{t+1}] - p_t). \tag{1}$$

A change in interest rates $\{di_t\}$ then changes the sequence of price levels according to

$$dp_t = -\sum_{s=0}^{\infty} di_{t+s} + \lim_{T\to\infty} \mathbb{E}_t[dp_T]. \tag{2}$$

With an exogenous interest rate, the effect of the shock on prices p_t is indeterminate. If agents expect that the future price level is *anchored* to some level, $p_\infty = \lim_{T\to\infty} \mathbb{E}_t[dp_T] = 0$, then the effect of the interest rate shock is conventional: an interest rate hike lowers prices for all t, whereas a cut raises prices. On the other hand, if expectations of future prices depend on the sequence of interest rates, the shock can have unconventional effects. For instance, if higher interest rates raise p_∞ via some mechanism, then one obtains the neo-Fisherian effect: higher interest rates can increase the price level for all t.

The standard approach is to resolve the indeterminacy via the *Taylor principle* plus a boundedness criterion: if the interest rate responds to past inflation, $i_t = \phi(p_t - p_{t-1})$ with $\phi > 1$, then for any sequence of interest rate shocks $\{di_t\}$, there exists a unique path $\{dp_t\}$ such that p_∞ remains bounded. Expectations are anchored by a belief that future prices will not explode in equilibrium. This equilibrium selection criterion has been the subject of extensive debate. Proponents argue that explosive prices could imply infeasible explosive real outcomes or that hyperinflationary paths can be ruled out with off-equilibrium changes in the monetary regime (Schmitt-Grohé and Uribe, 2000; Atkeson, Chari, and Kehoe, 2010). Opponents counter that the Taylor principle is simply a threat to blow up the economy, that off-equilibrium regime switches are unrealistic, and that a central bank would not be able to commit to an explosive price path (Cochrane, 2011; Neumeyer and Nicolini, 2025).

An alternative approach is the fiscal theory of the price level (the FTPL, see Leeper, 1991; Sims, 1994; Woodford, 1995). The price level satisfies the "government debt valuation equation": the real value of government debt must be equal to the present value of real government surpluses. If surpluses are not required to adjust so that the valuation equation holds for *any* realization of prices $\{p_t\}$ (*non-Ricardian* fiscal policy), then expectations of future prices are anchored by expectations of future surpluses. This view has been contested as well. Critics contend that the FTPL permits the government to run infeasible fiscal policies that violate its budget constraint (Buiter, 2002; Bassetto, 2002), that it is simply a different threat to blow up the economy (Kocherlakota and Phelan, 1999), that the FTPL equilibrium is not learnable (McCallum and Nelson, 2006), and that in heterogeneous-agent models, it is neither necessary nor sufficient to select an equilibrium (Hagedorn, 2024).

I demonstrate that whenever Ricardian equivalence fails, the indeterminacy problem can be resolved without accepting either perspective (and the associated theoretical conundrums). In this case, fiscal policy can anchor agents' expectations of p_∞, rendering outcomes determinate for all t. The basic intuition is that in non-Ricardian economies, changes in the *timing* of government surpluses can have wealth effects. Fiscal policy can target a price level $p_\infty = \overline{p}$ by committing to reduce surpluses when $p_\infty < \overline{p}$ and increase them when $p_\infty > \overline{p}$. (This strategy is exactly analogous to how fiscal policy would peg the price level in the FTPL.) Monetary policy governs agents' substitution of consumption across time, whereas fiscal policy determines aggregate nominal wealth. Nominal rate shocks can have unconventional effects only if they affect the nominal anchor $\overline{p}$. Hence, it is largely possible to resolve the controversies about the effects of monetary policy on inflation while circumventing the controversies that have plagued the earlier literature. These controversies are relevant only in the knife-edge case of Ricardian equivalence.

I illustrate this point in a standard Blanchard-Yaari overlapping generations model with an exogenous endowment. Households have log utility and die with a constant probability each period. Markets are incomplete because households cannot trade with yet-to-be-born generations, breaking Ricardian equivalence. A government sets the nominal rate according to a Taylor rule (with a time-varying coefficient), issues nominal bonds, and levies taxes. When the government balances its budget each period, the price level is indeterminate for all t, as in canonical monetary models with complete markets.

I study whether there exists a Ricardian fiscal policy that can select among the various equilibrium outcomes. In this economy, in fact, fiscal policy can select any such equilibrium, including one resembling that typically studied in the New Keynesian literature ("monetary dominance" with $\phi > 1$) and one resembling that studied in the FTPL literature ("fiscal dominance"). The intuition is as follows. Agents have log utility, so they consume a constant fraction of their total wealth, consisting of bond holdings plus the present value of future income minus taxes. The key observation is that when Ricardian equivalence fails, bonds represent net wealth for households: some of the taxes required to repay bond holders will fall on future generations. Therefore, fiscal policy can raise agents' wealth and boost demand by running deficits in the present and promising to repay in some future period (or vice-versa, if prices are too low). The government runs deficits or surpluses as required to target a given level of nominal demand and thus a given price level $\overline{p}$. There is an analogy with money supply rules. In the quantity theory, $MV = PY$ with constant V implies the government can target nominal income with a money supply rule. Here, by contrast, nominal income PY is a constant fraction of aggregate nominal wealth, so policy targets nominal wealth instead. (The main result would generalize, however, even if the marginal propensity to consume were to depend on the path of interest rates.)

After illustrating this result analytically, I turn to prove it in a more general environment. I consider a broad class of macroeconomic models characterized by an aggregate demand function (Farhi and Werning, 2019; Auclert, Rognlie, and Straub, 2024), an aggregate supply function, and a generalized Taylor rule for the nominal rate. This class of environments nests several types of models often studied in the literature, e.g., those with behavioral distortions, assets in the utility function, or HANK models. I define *strong Ricardian non-equivalence* (SRNE) as a property of the demand function: for any impulse $\{dc_t\}$ to consumption, there exists a sequence of changes in taxes $\{d\tau_t\}$ resulting in that impulse. By contrast, under Ricardian equivalence, only the present value of $\{d\tau_t\}$ – not its timing – matters.

I use sequence-space techniques to formally prove that when SRNE holds, there is an equivalence between the class of equilibria implemented by Ricardian and non-Ricardian fiscal rules. If a non-Ricardian fiscal rule can implement a given equilibrium, then so can a Ricardian rule. In particular, when attention is restricted to sequences of output and prices such that the present value of income is finite, then a Ricardian fiscal rule is sufficient to implement *any* equilibrium price vector.

This equivalence result sheds light on debates about the interactions between fiscal and monetary policy. For example, White (2025) argues that the "price puzzle" may arise from FTPL-style fiscal policy: if governments do not raise surpluses in response to nominal rate hikes, higher interest rates can result in higher inflation going forward. Skeptics contend that FTPL equilibria are pathological or knife-edge in some way – McCallum and Nelson (2006) argue that FTPL equilibria are not learnable even in the long run, whereas Angeletos and Lian (2023) point out that the FTPL hinges on delicate coordination across agents and strong assumptions about common knowledge. The equivalence between Ricardian and non-Ricardian fiscal rules demonstrates that the *equilibrium dynamics* induced by FTPL-style rules cannot be dismissed as unreasonable. There exist Ricardian rules that induce precisely the same empirical dynamics.

Related literature. It would be impossible to do justice to the vast literature on determinacy in monetary economies under interest rate targets, starting with Sargent and Wallace (1975). Instead, I list the most relevant areas of the literature as well as surveys and books that serve as a good starting point for the interested reader. The citations most relevant to my results are included above and in the main body of the paper.

My paper is most related to the extensive body of work on the interactions between monetary and fiscal policy. Sargent and Wallace (1981) provide one of the earliest results on these interactions, demonstrating that even under a money supply rule, a central bank cannot control inflation if it is eventually forced to monetize government debt. Cochrane (2023) surveys results on the FTPL, and Canzoneri, Cumby, and Diba (2001) document some empirical evidence regarding the FTPL's plausibility. Bassetto and Sargent (2020) provides a broader perspective on the coordination of monetary and fiscal policies that relates economies with commodity standards to modern fiat-money economies. Motivated by this literature, Bianchi, Faccini, and Melosi (2023) develop a structural model of "fiscal inflations" and fit it to U.S. data.

There is also a large literature on the Blanchard and Kahn (1980) approach to determinacy in New Keynesian economies under the boundedness criterion. Taylor (1993) originally proposed the principle $\phi > 1$. Goodfriend and King (1997), Clarida, Galí, and Gertler (1999), and Woodford (2001) provide classical textbook-style treatments of determinacy under interest rate rules. Clarida, Galí, and Gertler (2000) and Lubik and Schorfheide (2004) offer empirical evidence that the Taylor principle did not hold during the Great Inflation in the U.S. (Benhabib, Schmitt-Grohé, and Uribe, 2001) point out that even under the Taylor principle, some indeterminacy may remain due to the zero lower bound on nominal rates. Recently models with behavioral or information frictions have called the necessity of the Taylor principle into question (Gabaix, 2020; Angeletos and Lian, 2023).

Finally, my paper builds on the literature on monetary and fiscal policy in non-Ricardian economies. Samuelson (1958) and Diamond (1965) are early canonical models of money and debt in overlapping generations economies. Barro (1974) developed the concept of Ricardian equivalence, and (Seater, 1993) surveys the literature on its consequences. A failure of Ricardian equivalence is an important feature of heterogeneous-agent monetary economies as well, as in Bewley (1986). Nisticò (2012), Kaplan, Moll, and Violante (2018), and Farhi and Werning (2019) study the effects of monetary policy in non-Ricardian economies. Recently, Rachel and Ravn (2025) highlight how breaking Ricardian equivalence introduces new interactions between monetary and fiscal policy, which in turn alters the conditions required for determinacy (under the boundedness criterion).

Organization. Section 2 presents the main result in the stylized overlapping generations economy. Section 3 generalizes the results to a broader class of models. Section **??** concludes. Some minor proofs and supplementary results are postponed to the Appendix.

2 An Overlapping Generations Economy

I illustrate the main result in a flexible-price endowment economy with perpetual youth-style overlapping generations, as in Yaari (1965) or Blanchard (1985). I study log-linearized equilibria in this environment. As I show in the Appendix, it is actually easier for fiscal policy to select an equilibrium if I instead consider global equilibrium conditions, since they impose additional constraints on the paths followed by the nominal price level. Furthermore, a log-linear analysis keeps continuity with the general results in Section 3.

2.1 Environment

Time is discrete and infinite, $t \in \{0, 1, 2, ...\}$. There is no aggregate uncertainty, and agents hold rational expectations. The economy is populated by a unit measure of households in each period: each household dies with probability $1 - \delta \in [0, 1)$ per period, and a measure $1 - \delta$ of newborns enter. The aggregate endowment follows an exogenous path $Y_t \to Y^*$ as $t \to \infty$ and is distributed equally across households. All output is used for consumption. A government levies taxes, issues nominal bonds, and sets monetary policy.

Households: An individual household chooses its consumption $C_{i,t}$ and real bond holdings $A_{i,t}$. As is typical in perpetual youth models, I assume households have access to annuities markets that pass the assets of households who die on to those who survive until the next period. Hence, surviving households receive an ex-post return on savings $(1 + i_t)/\delta(1 + \pi_{t+1})$, where $1 + i_t$ is the nominal interest rate on government bonds and $\pi_t = P_t/P_{t-1}$ is inflation. Households have log preferences over consumption $C_{i,t}$, so the problem of a household born at time t is

$$\begin{aligned} \max_{C_{i,t+s}, A_{i,t+s}} \quad & \sum_{s=0}^{\infty} (\beta\delta)^s \log C_{i,t+s} \\ \text{s.t. } & A_{i,t+s+1} = \frac{1 + i_{t+s}}{\delta(1 + \pi_{t+s+1})} \big(A_{i,t,+s} + Y_{t+s} - \tau_{i,t+s} - C_{i,t+s}\big), \\ & \lim_{T\to\infty} \Big(\prod_{s=0}^{T-1} \frac{\delta(1 + \pi_{t+s+1})}{1 + i_{t+s}} \Big) A_{i,t+T} \geq 0, \quad A_{i,t} \text{ given.} \end{aligned} \tag{3}$$

where $\beta \in (0, 1)$ is the time rate of preference and $\tau_{i,t}$ denotes real taxes paid by household i (or transfers, if $\tau_{i,t} < 0$) at time t.

I assume, as in Angeletos, Lian, and Wolf (2024), that taxes levied on a household i are equal to an aggregate lump-sum tax τ_t that the government imposes on all households plus a transfer into (or out of) a "social fund." Specifically, all newborn households receive a transfer τ_{new} from the fund, which is financed by offsetting taxes $\tau_{\text{old}} = \frac{1-\delta}{\delta}\tau_{\text{new}}$ on households born in previous periods. The level of τ_{new} is set so that, in steady state, wealth is constant across generations, which guarantees that the real interest rate is equal to $1 + r^* = \beta^{-1}$. The social fund is simply a modeling device to streamline the analysis – all of the substantive results would go through in its absence.

Government policy: The government in this model sets the economy's nominal interest rate and debt issuance policy. For ease of exposition, I assume that the nominal interest rate follows an exogenously given path $\{i_t\}_{t=0}^{\infty}$ that approaches a limit i^* as $T \to \infty$.[1] In the more general setting, I allow nominal rates to follow a rule that depends on output and nominal prices, which nests Taylor rules as a special case.

I study how fiscal policy can be designed to anchor price level expectations given this exogenous monetary policy. The government begins at $t = 0$ with an outstanding portfolio of debt consisting of B_0 nominal bonds that mature at $t = 0$. From $t = 0$ forward, the value of the government debt portfolio evolves according to

$$B_{t+1} = (1 + i_t)\big(B_t - P_t\tau_t\big), \tag{4}$$

where τ_t denotes aggregate taxes.

I define a *fiscal rule* as a specification of taxes τ_t at each date, including past output, inflation, and nominal interest rates. Given the path of nominal rates and a fiscal rule, aggregate nominal debt must adjust to satisfy the flow budget constraint (4). The fiscal rule is permitted to depend on the observed history of events, $h^t = \{Y_{t-s}, P_{t-s}, i_{t-s}, B_{t-s}\}$ including past output, prices, nominal interest rates, and debt levels. A fiscal rule is Ricardian if it guarantees that debt equals the present value of taxes along *any* history (rather than only as an equilibrium condition), i.e., $\lim_{T\to\infty} I_{0,T}^{-1} B_T \leq 0$ for all h^t.

Market clearing and equilibrium: To close the model, I impose market-clearing conditions for goods and assets,

$$Y_t = (1-\delta)\sum_{s=0}^{\infty}\delta^s C_t^{t-s}, \tag{5}$$

and

$$\frac{B_t}{P_t} = (1-\delta)\sum_{s=0}^{\infty}\delta^s A_t^{t-s}, \tag{6}$$

[1] The determination of the steady-state nominal rate i^* is discussed further in Section **??**.

where C_t^{t-s} (resp. A_t^{t-s}) denotes time-t consumption (resp. asset holdings) of the generation born at $t-s$. An *equilibrium* consists of sequences $(P_t, B_t, \tau_t, \{C_{t+s}^t, A_{t+s}^t\}_{s=0}^\infty)$ such that consumption $\{C_{t+s}^t, A_{t+s}^t\}$ solves the household's problem for the generation born at t, taxes $\tau_t = \tau_t(h^t)$ satisfy the fiscal rule specification, and (4)-(6) hold at all dates.

2.2 Log-linear equilibrium conditions

I now present the linearized equilibrium conditions around a steady state with zero inflation. Henceforth, stars denote steady state values (e.g., B^* is the steady-state level of nominal debt), lowercase letters with hats denote log deviations (e.g., $\hat{p}_t \equiv (P_t - P^*)/P^*$), and the superscript N denotes a nominal quantity (e.g., $\hat{y}_t^N \equiv \hat{p}_t + \hat{y}_t$ is the log-deviation of nominal output). Boldface letters denote sequences, e.g., $\hat{\boldsymbol{y}}^N \equiv (\hat{y}_0^N, \hat{y}_1^N, \dots)$.

To define a linearized equilibrium, it is necessary to specify a sequence space in which equilibrium quantities may live. The key endogenous variable in this setting is the path of nominal income $\hat{\boldsymbol{y}}^N$. Since the path of real income is fixed, nominal income moves one-for-one with the price level. I consider sequences for nominal output with finite present value,

$$\hat{\boldsymbol{y}}^N : \quad \sum_{t=0}^{\infty} \beta^t \hat{y}_t^N < \infty. \tag{7}$$

This is weaker than the usual non-explosiveness criterion, which restricts attention to bounded sequences.

In the representative agent model ($\delta = 1$), this restriction follows directly from household optimization: if nominal income grows faster than β^{-1}, the present value of income diverges, and a solution to the optimization problem does not exist (see (9) below). As I show in the Appendix, in this non-Ricardian model, one can justify this restriction by interpreting $1-\delta$ as an *average* death rate in a model with heterogeneous death rates across individual households, where there exist near-Ricardian households with $1-\delta \to 0$. Regardless, as I demonstrate later, once one drops restriction (7), then *neither* Ricardian nor non-Ricardian fiscal policy can select an equilibrium.

With this notation, the equilibrium can be summarized by just two equations:

$$\hat{b}_{t+1}^N = \hat{i}_t + \beta^{-1}\hat{b}_t^N - (\beta^{-1} - 1)\hat{\tau}_t^N, \tag{8}$$

$$\hat{y}_t^N = \underbrace{(1-\beta\delta)}_{\text{MPC}} \times \underbrace{\left(\hat{v}_t^y + \kappa(\hat{a}_t^N - \hat{v}_t^\tau)\right)}_{\text{Wealth}}, \tag{9}$$

where $\hat{v}_t^y$ (resp. $\hat{v}_t^\tau$) denotes the present value from households' perspective of future income (resp. taxes),

$$\hat{v}_t^y \equiv \sum_{s=0}^{\infty}(\beta\delta)^s\big(\hat{y}_{t+s}^N - \frac{\beta\delta}{1-\beta\delta}\hat{i}_{t+s}\big),$$

$$\hat{v}_t^\tau \equiv \sum_{s=0}^{\infty}(\beta\delta)^s((1-\beta)\hat{\tau}_{t+s}^N - \beta\hat{i}_{t+s}),$$

and $\kappa \equiv B^*/P^*Y^*$ is the steady-state debt/GDP ratio. The first equation describes the evolution of government debt, where the initial debt level $\hat{b}_0$ is given. The second equilibrium condition equates nominal income to nominal demand, which is households' MPC $1-\beta\delta$ times aggregate wealth, which consists of private sector bond holdings (financial wealth) plus the present value of future income net of taxes (often referred to as "human wealth"). Note that I do not include a transversality condition (TVC) on private sector assets. The TVC for an invididual household is used to derive (9), so it does not provide an additional equilibrium restriction.

The aggregate demand function (9) clarifies the role of Ricardian non-equivalence in this model. When Ricardian equivalence holds ($\delta = 1$), the discount factor applied to future taxes by the private sector in $\hat{v}_t^\tau$ is the nominal interest rate $\beta^{-1} = 1 + i^*$, i.e., the discount rate on government debt. Under a Ricardian policy, forward iteration of (4) reveals that the value of government debt $\hat{b}_t^N$ is exactly equal to $\hat{v}_t^\tau$, so fiscal policy is decoupled from price level determination. When Ricardian equivalence fails ($\delta < 1$), households discount future taxes at a rate greater than $1 + i^*$. Intuitively, current generations do not bear the entire burden of taxation required to finance the public debt, since taxes are also paid by future generations. Government bonds represent net wealth for the private sector, so fiscal policy has wealth effects that influence aggregate demand. This is the key property in the derivation of my main results.

It remains to describe fiscal policy in the linearized model. Since the nominal rate and real output are exogenous, any dependence of a fiscal rule on prior history can be captured as dependence on prior realizations of the price level (or, equivalently, prior realizations of nominal income).

Definition 1. *A **fiscal rule** is a linear mapping from sequences of nominal income $\hat{\boldsymbol{y}}^N$ satisfying (7) to sequences of nominal taxes $\hat{\boldsymbol{\tau}}^N$ with finite present value,*

$$\hat{\tau}_t^N = \mathcal{T}_t(\{\hat{y}_{t-s}^N\}) \quad s.t. \ \sum_{t=0}^{\infty}\beta^t\tau_t^N < \infty \ \ \forall\ \hat{\boldsymbol{y}}^N s.t.\ (7)\ holds.. \tag{10}$$

. *It is* ***Ricardian*** *if*

$$\lim_{T\to\infty} \beta^T(\hat{b}_T^N - \sum_{s=0}^{T-1} \hat{i}_s) = 0 \ \ \forall\, \hat{\boldsymbol{y}}^N \ \textit{s.t. (7) holds.} \tag{11}$$

I conclude with the definition of a linearized equilibrium.

Definition 2. *A* ***linearized equilibrium*** *consists of sequences* $(\hat{\boldsymbol{y}}^N, \hat{\boldsymbol{b}}^N, \hat{\boldsymbol{\tau}}^N)$ *such that (7)-(10) hold.*

2.3 Constructing a fiscal policy that pins down the price level

I prove the main result by constructing a Ricardian fiscal policy that is sufficient to pin down the price level whenever Ricardian equivalence fails ($\delta < 1$). The first step of the argument can be understood intuitively through (9). I derive a fiscal policy such that at each t, for *any* path of anticipated future nominal income $\{\hat{y}_{t+s}^N\}$, fiscal policy adjusts so that wealth from government bond holdings net of taxes, $\kappa(\hat{b}_t^N - \hat{v}_t^\tau)$, exactly offsets the present value of income $\hat{v}_t^y$. Hence, this fiscal policy effectively stabilizes nominal demand and therefore the price level.

Proposition 1. *Suppose that Ricardian equivalence does not hold ($\delta < 1$). Then there exists a Ricardian fiscal policy such that*

$$\hat{v}_t^y + \kappa\big(\hat{b}_t^N - \hat{v}_t^\tau\big) = 0 \ \ \forall \ \ \hat{\boldsymbol{y}}^N \ \textit{s.t. (7)}. \tag{12}$$

Proof. I proceed in three steps. First, given nominal output $\hat{\boldsymbol{y}}^N$, I construct a candidate sequence of nominal taxes $\boldsymbol{\tau}^N$. Second, I verify that it satisfies (12). Finally, I demonstrate that the implied fiscal rule is Ricardian.

Step 1: Constructing a candidate $\hat{\boldsymbol{\tau}}^N$. If there exists such a sequence of taxes $\hat{\boldsymbol{\tau}}^N$, observe that

$$\begin{aligned}
-\hat{v}_t^y &= \kappa(\hat{b}_t^N - \hat{v}_t^\tau) \\
&= \kappa((1-\beta)\hat{\tau}_t^N - \beta\hat{i}_t + \beta\hat{b}_{t+1}^N - ((1-\beta)\hat{\tau}_t^N - \beta\hat{i}_t) - \beta\delta\hat{v}_{t+1}^\tau) \\
&= \kappa\beta(1-\delta)\hat{b}_{t+1}^N - \beta\delta\hat{v}_{t+1}^y
\end{aligned}$$

Then, a candidate policy $\hat{\boldsymbol{\tau}}^N$ should result in the sequence of nominal debt levels

$$\hat{b}_{t+1}^N = \frac{\beta\delta\hat{v}_{t+1}^y - \hat{v}_t^y}{\kappa\beta(1-\delta)} \ \ \forall\, t.$$

Here the role of Ricardian non-equivalence ($\delta < 1$) becomes clear: if $\delta = 1$, this path of debt would not be well-defined. It is supported by a sequence of taxes

$$\hat{\tau}_t^N = \frac{1}{1-\beta}\big(\hat{b}_t^N + \beta\hat{i}_t - \frac{\beta\delta\hat{v}_{t+1}^y - \hat{v}_t^y}{\kappa\beta(1-\delta)}\big) \;\; \forall\, t.$$

Given that $\hat{\boldsymbol{i}}$ is bounded and $\hat{\boldsymbol{y}}^N$ has finite present value, it must be that the candidate $\boldsymbol{\tau}^N$ has finite present value as well.

Step 2: Verifying that $\boldsymbol{\tau}^N$ satisfies (12). It suffices to show that (12) holds at $t = 0$. This amounts to showing that $\hat{v}_0^\tau = \hat{b}_0^N + \frac{1}{\kappa}\hat{y}_0^N$. Notice that

$$\begin{aligned}
\hat{v}_0^\tau &= \sum_{t=0}^{\infty}(\beta\delta)^t\big((1-\beta)\hat{\tau}_t^N - \beta\hat{i}_t\big)\\
&= -\sum_{t=0}^{\infty}(\beta\delta)^t\beta\hat{i}_t + \hat{b}_0^N + \hat{i}_0 - \frac{\beta\delta\hat{v}_1^y - \hat{v}_0^y}{\kappa(1-\delta)} + \sum_{t=1}^{\infty}(\beta\delta)^t\left(\frac{1}{\beta}\frac{\beta\delta\hat{v}_t^y - \hat{v}_{t-1}^y}{\kappa(1-\delta)} + \beta\hat{i}_t - \frac{\beta\delta\hat{v}_{t+1}^y - \hat{v}_t^y}{\kappa(1-\delta)}\right)\\
&= \hat{b}_0^N + \frac{\hat{v}_0^y}{\kappa} - \sum_{t=1}^{\infty}\frac{(\beta\delta)^{t-1}}{\kappa(1-\delta)}\big(\beta\delta\hat{v}_t^y - \beta\delta^2\hat{v}_t^y - \beta\delta\hat{v}_t^y + \beta\delta^2\hat{v}_t^y\big)\\
&= \hat{b}_0^N + \frac{\hat{v}_0^y}{\kappa},
\end{aligned}$$

as desired.

Step 3: Verifying that the policy is Ricardian. For any sequence of nominal income $\hat{\boldsymbol{y}}^N$ such that (7) holds,

$$\begin{aligned}
\lim_{T\to\infty}\beta^T\big(\tilde{\hat{b}}_T^N - \sum_{t=0}^{T-1}\hat{i}_t\big) &= \lim_{T\to\infty}\beta^T\hat{b}_T^N\\
&= \frac{\delta}{\kappa(1-\delta)(1-\beta\delta)}\lim_{T\to\infty}\beta^T\hat{i}_{T-1} - \frac{1}{\kappa\beta(1-\delta)}\lim_{T\to\infty}\beta^T\hat{y}_{T-1}^N\\
&= 0,
\end{aligned}$$

as desired. The first line follows from the boundedness of $\hat{i}_t$. The second follows from the definition of $\hat{b}_t^N$ and the identity $\hat{v}_t^y = \hat{y}_t^N - \beta\delta\hat{i}_t/(1-\beta\delta) + \beta\delta\hat{v}_{t+1}^y$. The third follows from the fact that $\hat{\boldsymbol{y}}^N$ has finite present value and the boundedness of $\hat{i}_t$, so both limits must be equal to zero. □

Note that the logic used in this proposition depends only on the aggregate demand function and not on aggregate supply, so this result is independent of whether prices are flexible or sticky, whether the endowment is exogenous or endogenous, and so on. Proposition 1 is just a specific instance of a more general principle derived in Section 3: when Ricardian equivalence fails, public debt represents net wealth for households. Hence, it is possible for the fiscal authority to affect the level of aggregate demand even if it commits to fully back its debt with tax revenues in all contingencies.

In this stylized model, a fiscal policy that anchors the price level takes a particularly simple form. With log preferences, households consume a constant fraction of their wealth in each period, so the government uses fiscal policy to target a given level of nominal wealth. If nominal wealth is too low to sustain the desired level of nominal demand, the government runs a runs a deficit and issues additional debt. The transfers that agents receive in the present outweigh the taxes that they expect to pay later, since those are borne in part by future generations. By contrast, if nominal wealth is too high, the government runs a surplus in order to repay debt. Households' expectation of future prices are anchored by the perception that the government will use fiscal policy to control the price level in the long run.

The fiscal policy derived in the proposition is quite simple: the prescribed level of nominal debt in the next period depends only on current nominal output and the short-term nominal rate,

$$\hat{b}_{t+1}^N = \frac{\delta}{1-\delta}\frac{\hat{i}_t}{\kappa(1-\beta\delta)} - \frac{\hat{y}_t^N}{\kappa\beta(1-\delta)}. \tag{13}$$

The government's *commitment* to this rule plays a key role: households understand that under this rule, no matter what level of nominal income is realized in the future, fiscal policy will offset it. If the government cannot commit, then households may not believe, for example, that it will pursue the required reduction in deficits if prices rise in the future. Running the rule (13) in a single period will then not be effective in stabilizing prices.

From (13), it is also clear why this fiscal rule is Ricardian. Nominal output has finite present value by assumption, so its growth must be strictly bounded by β^{-1}. However, this assumption is actually stronger than what is required from the proof: in the Appendix, I demonstrate that a modified version of (13) can achieve the same outcome as long as $\lim_{T\to\infty} \beta^T \hat{y}_T^N$ is merely assumed to be finite.[2] On the other hand, when $\beta^T \hat{y}_T^N$ is permitted to diverge, then even non-Ricardian policies (and, in particular, the canonical formulation of the FTPL) cannot pin down the price level, as I demonstrate in the next section. Therefore, the restriction to nominal income paths $\hat{\boldsymbol{y}}^N$ with finite present value does not change the conclusion that a Ricardian fiscal policy can select an equilibrium if and only if a non-Ricardian policy can do so.

The policy of targeting nominal wealth is conceptually similar, in certain respects, to monetarist prescriptions based on the quantity theory of money (Friedman, 1970). In the

[2] In this case, the rule is more complex and involves conditioning on long-run real interest rates.

quantity theory, the relationship $MV = PY$, plus the stability of velocity V, imply that the government can target nominal income by controlling the money supply. Here, there is a similar stable relationship between nominal income and nominal wealth given by (9). Proposition (1) implies that if (1) the government knows the exact marginal propensity to consume out of wealth, and (2) it can observe private wealth (here, just the present value of nominal output), then it can *exactly* implement a desired level of nominal income.

Although these may seem like extreme assumptions, the same logic would hold even if they were relaxed: if the government knew only an approximate relationship between wealth and demand, or if asset prices were to provide a noisy proxy for demand, then the government would still be able to *approximately* target a level of nominal income, with the quality of the approximation deteriorating in the level of noise. In this sense, the rule derived in Proposition 1 shares more in common with the canonical FTPL than with the Taylor principle. Under the Taylor principle, the policymaker does not need detailed knowledge of the underlying economy to stabilize prices: any sufficiently aggressive policy (coupled with the non-explosiveness criterion) is sufficient to eliminate indeterminacy. By contrast, in standard formulations of the FTPL, the price level is pinned down by the present value of surpluses. Hence, the better the government can estimate the private sector's discount factor, the more precisely it can target a given price level.

Proposition 1 immediately implies an equivalence between the classes of equilibria that can be implemented by non-Ricardian and Ricardian fiscal policies when Ricardian equivalence fails. Specifically, I prove the following corollary.

Corollary 1. *Suppose $\delta < 1$, and let $(\hat{\boldsymbol{y}}^N, \hat{\boldsymbol{b}}^N, \hat{\boldsymbol{\tau}}^N)$ is a linearized equilibrium with $\lim_{T\to\infty} \beta^T \hat{b}_T^N = 0$. Then there exists a Ricardian fiscal policy that uniquely implements this equilibrium.*

Proof. Let $\hat{\tau}^N_{\text{orig},t}$ be the realized sequence of taxes in the given equilibrium, and let $\hat{\boldsymbol{\tau}}^N_\Delta$ be the Ricardian fiscal rule that selects a unique equilibrium $\hat{\boldsymbol{y}}^N = 0$ given by Proposition 1. Then consider the fiscal rule

$$\hat{\tau}^N_t = \hat{\tau}^N_{\text{orig},t} + \hat{\tau}^N_{\Delta,t}.$$

The assumption $\beta^T \hat{b}^N_T \to 0$ for the original equilibrium implies that $\hat{\boldsymbol{\tau}}^N$ is Ricardian, since it is the sum of two Ricardian policies. If $\hat{\boldsymbol{y}}^N, \hat{\boldsymbol{y}}^{N'}$ are two equilibria under fiscal rule $\hat{\boldsymbol{\tau}}^N$, let

$$\hat{\boldsymbol{y}}^N_\Delta \equiv \hat{\boldsymbol{y}}^{N'} - \hat{\boldsymbol{y}}^N.$$

Then (9) implies

$$\hat{y}^N_{\Delta,t} = \hat{v}^y_{\Delta,t} + \kappa(\hat{b}^N_{\Delta,t} - \hat{v}^\tau_{\Delta,t}).$$

with the obvious notation. By construction $\hat{\boldsymbol{y}}^N_\Delta = \mathbf{0}$ is the only solution to this equation, so

$\hat{\boldsymbol{y}}^N = \hat{\boldsymbol{y}}^{N'}$, i.e., equilibrium is unique. $\square$

Whenever some equilibrium can be uniquely implemented by a non-Ricardian fiscal policy, there also exists a Ricardian policy that uniquely implements that equilibrium. Therefore, to rationalize an observed empirical regularity, it is never necessary to posit that the government is following a non-Ricardian fiscal policy off-path.

In representative-agent models satisfying Ricardian equivalence, it is well known that non-Ricardian fiscal policies can select equilibria that feature non-standard dynamics. For instance, the FTPL can support the "neo-Fisherian" conclusion that higher nominal interest rates *raise* inflation.[3] These conclusions have been challenged theoretically on the grounds that FTPL equilibria are in some way pathological, knife-edge, or unstable. For instance, McCallum and Nelson (2006) argue that FTPL equilibria are not "learnable" in the sense of Evans and Honkapohja (2001). Angeletos and Lian (2023) demonstrate that when Ricardian equivalence holds, FTPL equilibria depend delicately on common knowledge and unravel when common knowledge is broken.

The equivalence result in Corollary 1 casts doubt, however, on the notion that FTPL-style equilibria are merely theoretical curiosities. When Ricardian equivalence fails (even slightly), any equilibrium that could have been supported by a non-Ricardian fiscal rule can also be uniquely implemented by a Ricardian rule, which does not suffer from any of the associated theoretical controversies. Moreover, the associated Ricardian rule will often be qualitatively quite similar to the original non-Ricardian policy, e.g., (13) illustrates how expectations of deficits or surpluses in the future pin down the price level in the present. The equilibrium dynamics associated with non-Ricardian policies then have more robust theoretical support than the prior literature would suggest.

2.4 A comparison with the FTPL

Before moving on to the general model, I compare the equilibrium selection method derived in Proposition 1 to that of the FTPL. To understand equilibrium selection (both here and in the general model), it will be useful to formulate equilibrium in terms of an *excess demand* function.

In the analysis of the previous section, the market-clearing conditions for goods and assets, $\hat{c}_t = \hat{y}_t$ and $\hat{a}_t = \hat{b}_t$, were combined with household optimality conditions to obtain just two equilibrium equations, (8) and (9). To characterize the excess demand function, however, it

[3] The Taylor principle can select such equilibria as well, in fact. King (2000) demonstrates that for *any* desired equilibrium path of inflation $\overline{\pi}_t$, there exists a Taylor-like rule that selects that equilibrium by reacting aggressively to deviations $\pi_t - \overline{\pi}_t$.

is necessary to keep consumption distinct from output and private sector assets distinct from government debt.

Given a vector of nominal price levels $\hat{\boldsymbol{p}}$, nominal supply is simply $\hat{\boldsymbol{y}}^N = \hat{\boldsymbol{p}} + \hat{\boldsymbol{y}}$. Nominal demand is derived from households' optimality conditions:

$$\hat{c}_t^N = (1-\beta\delta)\big(\hat{v}_t^y + \kappa(\hat{a}_t^N - \hat{v}_t^\tau)\big), \tag{14}$$

where household nominal savings $\hat{a}_t^N$ evolve according to

$$\hat{a}_{t+1}^N = \hat{i}_t + \beta^{-1}\hat{a}_t^N - (\beta^{-1}-1)\hat{\tau}_t^N - (\beta\kappa)^{-1}(\hat{c}_t^N - \hat{y}_t^N), \tag{15}$$

with initial condition $\hat{a}_0^N = \hat{b}_0^N$.

These conditions can be combined with the evolution of government debt to derive the excess nominal demand function. Define *forward-looking wealth*

$$\hat{w}_t \equiv \hat{v}_t^y + \kappa(\tilde{v}_t^\tau - \hat{v}_t^\tau),$$

where

$$\tilde{v}_t^\tau \equiv \sum_{s=0}^{\infty} \beta^s\big((1-\beta)\hat{\tau}_{t+s}^N - \beta\hat{i}_{t+s}\big)$$

denotes the present value of taxes discounted by the nominal rate $1+i_t$ (rather than the household's discount factor $(1+i_t)/\delta$). Forward looking wealth $\hat{w}_t$ is what household wealth would be if total household assets were equal to the present value of taxes. Off-equilibrium, however, these quantities can differ. Then, the excess demand function can be written in terms of forward-looking household wealth, output, and the gap between assets and the present value of taxes.

Proposition 2. *In the OLG economy, excess nominal demand can be written as*

$$\hat{c}_t^N - \hat{y}_t^N = \underbrace{(1-\beta\delta)\hat{w}_t - \hat{y}_t^N - \frac{1-\beta\delta}{\beta\delta}\sum_{s=0}^{t-1}\delta^{t-s}\big((1-\beta\delta)\hat{w}_s - \hat{y}_s^N\big)}_{substitution} + \underbrace{\delta^t(\hat{b}_0^N - \tilde{v}_0^\tau)}_{wealth}. \tag{16}$$

Notice that all terms on the right-hand side depend only on the price vector $\hat{\boldsymbol{p}}$, exogenous output $\hat{\boldsymbol{y}}$, and government policy summarized by $(\hat{\boldsymbol{i}}, \hat{\boldsymbol{\tau}}^N, \hat{b}_0^N)$.

The first two terms in (16) reflect *substitution effects.* There is excess demand at t is high when current forward-looking wealth is greater than nominal income. Excess demand in previous periods s, $(1-\beta\delta)\hat{w}_s > \hat{y}_s^N$, depresses current consumption because it implies that households have run down savings (i.e., $\hat{a}_t^N$ has decreased).

The third term reflects a *wealth effect* that materializes only when fiscal policy is non-Ricardian. Demand is higher (lower) when initial assets $\hat{b}_0^N$ are greater (less) than the present value of taxes $\tilde{v}_0^\tau$ at $t = 0$. When policy is Ricardian, the present value of taxes must be exactly equal to initial debt for *all* price vectors, so this term is identically zero for all $\hat{\boldsymbol{p}}$.

In what follows, I describe how, exactly, the fiscal policies used in "Ricardian" equilibrium selection and the FTPL guarantee that there exists a unique price vector such that excess demand is equal to zero. In short, a Ricardian fiscal rule operates through the substitution effect in aggregate demand, whereas the FTPL operates through the wealth effect. I also consider how equilibrium determinacy depends on the class of admissible price vectors – in particular, whether attention is restricted to nominal income paths satisfying (7), the larger class of price vectors such that

$$\hat{\boldsymbol{y}}^N : \lim_{T\to\infty} \beta^T |\hat{y}_t^N| < \infty, \tag{17}$$

or, finally, an unrestricted class of price vectors.

Ricardian equilibrium selection: Consider first the class of nominal income paths satisfying the finite-present value restriction (7). Under the Ricardian fiscal rule (13), the excess demand function (16) is

$$\hat{c}_t^N - \hat{y}_t^N = \frac{1-\beta\delta}{\beta\delta} \sum_{s=0}^{t-1} \delta^{t-s} \hat{y}_s^N - \hat{y}_t^N. \tag{18}$$

This simplification follows from the fact that this rule is Ricardian, so the present value of taxes is equal to government debt for all price vectors, $\hat{b}_t^N = \tilde{v}_t^\tau$ identically. The original rule satisfied $\hat{v}_t^y + \kappa(\hat{b}_t^N - \hat{v}_t^\tau) = 0$ for all price vectors, so forward-looking wealth $\hat{w}_t = 0$. Excess demand is then equal to excess savings accrued in prior periods (the term in the summation) minus current nominal income.

The excess demand mapping (18) has two key properties that are important for the general formulation of Ricardian equilibrium selection policies in Section 3.

1. The excess demand function is *injective* as a function of nominal income $\hat{y}^N$, meaning the right-hand side of (18) is equal to zero for all t if and only if $\hat{\boldsymbol{y}}^N = \mathbf{0}$. If $\hat{\boldsymbol{y}}^N \neq \mathbf{0}$, then let t^* be the first t such that $\hat{y}_t^N \neq 0$. Then $\hat{c}_{t^*}^N - \hat{y}_{t^*}^N = -\hat{y}_{t^*}^N \neq 0$.

2. The excess demand function maps into the space of sequences with zero present value.

That is, for any $\hat{\boldsymbol{y}}^N$,

$$\sum_{t=0}^{\infty}\beta^t(\hat{c}_t^N-\hat{y}_t^N)=\sum_{t=0}^{\infty}\beta^t\Big(\frac{1-\beta\delta}{\beta\delta}\sum_{s=0}^{t-1}\delta^{t-s}\hat{y}_s^N-\hat{y}_t^N\Big)$$
$$=\sum_{t=0}^{\infty}\delta^{-t}\frac{1-\beta\delta}{\beta\delta}\hat{y}_t^N\sum_{s=t+1}^{\infty}(\beta\delta)^s-\sum_{t=0}^{\infty}\beta^t\hat{y}_t^N=0.$$

This property follows from the fact that the fiscal rule is Ricardian. The present value of consumption must be equal to initial assets plus the present value of net income $\hat{y}_t^N-\hat{\tau}_t^N$. When taxes are set so that their present value is equal to initial debt $\hat{b}_0^N$ for any price vector, it follows that $\sum_{t=0}^{\infty}\hat{c}_t^N=\sum_{t=0}^{\infty}\beta^t\hat{y}_t^N$ identically.

The first property guarantees that the fiscal rule can pin down an equilibrium price vector, and the second property implies that the rule accomplishes this goal through the substitution channel in (16).

What if the set of admissible nominal income paths is expanded? There may be nothing in the full non-linear model that rules out nominal income paths whose present value diverges. In the larger class of nominal income paths that do not grow faster than β^{-1}, i.e., the class (17), the fiscal rule must be modified slightly. Specifically, the original rule (13) is no longer Ricardian because

$$L(\hat{\boldsymbol{y}}^N)\equiv\lim_{T\to\infty}\beta^T\hat{y}_T^N$$

may be nonzero. A simple change guarantees that the rule is Ricardian while still pinning down a unique equilibrium: let

$$\hat{b}_{t+1}^N=\frac{\delta}{1-\delta}\frac{\hat{i}_t}{\kappa(1-\beta\delta)}-\frac{\hat{y}_t^N-\beta^{-t}L(\hat{\boldsymbol{y}}^N)}{\kappa\beta(1-\delta)}. \tag{19}$$

This modification is accomplished with an additional one-time tax of $\Delta\tau_0^N=-L(\hat{\boldsymbol{y}}^N)/(\kappa\beta(1-\delta))$ at $t=0$, which keeps $\hat{b}_t^N-\hat{v}_t^\tau$ unchanged for all t. For example, the original plan (13) would have required the government to roll over debt indefinitely in a deflation where nominal income diverges downwards. This modification increases taxes at $t=0$ by just enough to offset the long-run debt. It is feasible to implement this plan by observing asset prices at $t=0$: since real output $\hat{y}_t$ and the nominal rate $\hat{i}_t$ are bounded, $\lim_{T\to\infty}\beta^T(\hat{y}_T^N-\hat{y}_0^N)=\lim_{T\to\infty}\beta^T(\sum_{t=0}^{T-1}\hat{r}_t)$, where $\hat{r}_t\equiv\hat{i}_t-(\hat{p}_{t+1}-\hat{p}_t)$ is the nominal rate. Hence, $L(\hat{\boldsymbol{y}}^N)$ can be read off of long-run real interest rates.

With this modified Ricardian rule, the excess demand function (18) becomes

$$\hat{c}_t^N - \hat{y}_t^N = \frac{1-\beta\delta}{1-\delta}L(\hat{\boldsymbol{y}}^N) - \hat{y}_t^N - \frac{1-\beta\delta}{\beta\delta}\sum_{s=0}^{t-1}\delta^{t-s}\big(\frac{1-\beta\delta}{1-\delta}L(\hat{\boldsymbol{y}}^N) - \hat{y}_s^N\big).$$

From this equation, it follows that the modified policy selects a unique nominal income path $\hat{\boldsymbol{y}}^N = \mathbf{0}$ as well. Note that if $\hat{\boldsymbol{y}}^N$ is an equilibrium, then it must be that $\frac{1-\beta\delta}{1-\delta}L(\hat{\boldsymbol{y}}^N) = \hat{y}_t^N$ for all t, that is, any equilibrium must have constant nominal output. Otherwise, if t^* were the first period t such that $\frac{1-\beta\delta}{1-\delta}L(\hat{\boldsymbol{y}}^N) - \hat{y}_t^N$, the summation term in (18) would be nonzero, and there would be excess demand. If nominal output is constant, however, then $L(\hat{\boldsymbol{y}}^N) = 0$. This implies $\hat{y}_t^N = 0$ for all t in any equilibrium.

When expanding the class of admissible nominal income paths even further to those that grow faster than β^{-1}, it is no longer possible to design a Ricardian fiscal rule that selects an equilibrium. In this case, however, even a non-Ricardian rule (as in the FTPL) cannot select an equilibrium either. To demonstrate this point, I consider FTPL-style policies next.

FTPL-style equilibrium selection: In canonical formulations of the FTPL, the price level is pinned down by a government debt valuation equation: the government sets the present value of surpluses exogenously, and then the price level adjusts so that the value of nominal debt is equal to that present value. Real interest rates may vary over time in the OLG model, so here I capture a non-Ricardian FTPL-style fiscal rule by assuming that the government sets real taxes

$$\hat{\tau}_t = \frac{\beta}{1-\beta}\hat{r}_t, \tag{20}$$

where $\hat{r}_t = \hat{i}_t - (\hat{p}_{t+1} - \hat{p}_t)$ is the real interest rate from t to $t+1$. Then, for any sequence of prices, the present value of surpluses is unchanged:

$$\hat{\tau}_0 + \sum_{t=1}^{\infty}\beta^t\big(\hat{\tau}_t - \sum_{s=0}^{t-1}\hat{r}_s\big) = 0.$$

The excess demand function under this fiscal rule is

$$\hat{c}_t^N - \hat{y}_t^N = \delta^t(\hat{b}_0^N - \hat{p}_0) + \frac{1-\beta\delta}{\beta\delta}\sum_{s=0}^{t-1}\delta^{t-s}u_s - u_t, \tag{21}$$

where

$$u_t \equiv (1-\beta\delta)\hat{v}_t^y - \hat{y}_t^N - \frac{1-\delta}{\delta}\kappa\sum_{s=0}^{\infty}(\beta\delta)^s\hat{p}_{t+s}.$$

The main difference from the excess demand function under a Ricardian rule is the first term,

$\delta^t(\hat{b}_0^N - \hat{p}_0)$. This term captures a wealth effect: the value of households' initial asset holdings is not offset by future taxes, so the larger initial asset holdings, the higher demand will be at all future dates.

When attention is restricted to nominal income paths with finite present value (i.e., those satisfying (7)), it is simple to see how a non-Ricardian policy selects an equilibrium. Taking the present value of both sides of the excess demand function,

$$\sum_{s=0}^{\infty} \beta^t(\hat{c}_t^N - \hat{y}_t^N) = \frac{\hat{b}_0^N - \hat{p}_0}{1 - \beta\delta}.$$

If the right-hand side is nonzero, then it must be that excess demand $\hat{c}_t^N - \hat{y}_t^N \neq 0$ for some t. Therefore, since the present value of surpluses is fixed, the equilibrium price level must adjust so that the real value of initial bond holdings is unchanged, $\hat{p}_0 = \hat{b}_0^N$. When initial bond holdings is greater than the present value of taxes, aggregate household wealth exceeds the present value of output, so there must be excess demand for some t.

Going beyond the space of output sequences with finite present value, consider paths $\hat{\boldsymbol{y}}^N$ that do not grow faster than β^{-1} (i.e., those satisfying (17)). Even under this mild relaxation of the set of admissible paths, the FTPL does not typically yield a unique equilibrium (unlike the Ricardian fiscal rule outlined previously). One can show that if the excess demand is equal to zero for all t, the sequence u_t in (21) must satisfy

$$u_t = \beta^{-t}(\hat{b}_0^N - \hat{p}_0).$$

The terms in u_t involve nominal income, so if there exists such an equilibrium, it must be that nominal income grows at rate β^{-1}. In fact, the government budget constraint (8) implies that debt $\hat{b}_t^N$ grows at rate β^{-1} as well as long as $\hat{b}_0^N \neq \hat{p}_0$. That is, in any such equilibrium, the government budget constraint does not hold even on-path, so government debt is a bubble.

When does a bubble equilibrium exist? Under Ricardian equivalence ($\delta = 1$), this type of equilibrium is ruled out by the aggregate demand condition (9), which incorporates households' transversality condition. The present value of income to households, $\hat{v}_t^y$, diverges, so there does not exist a solution to households' optimization problem. Hence, in representative-agent models, there is no loss of generality in restricting to nominal income sequences satisfying (7).

On the other hand, when Ricardian equivalence fails, this type of equilibrium may or may not be ruled out by individual optimization conditions in the full non-linear model. As Diamond (1965) illustrates, overlapping generations economies can feature bubbles in equilibrium. The Appendix shows that even if two overlapping generations economies have the same linearized equilibrium demand function (9), the full non-linear models might put different restrictions on the existence of bubbles in equilibrium. Thus, the Ricardian fiscal rule (19) is more robust in pinning down a unique equilibrium than the basic FTPL in this setting. (Of course, there exist other non-Ricardian rules that can rule out equilibria here, since there exists a Ricardian rule that does so.)

3 The General Model

The overlapping generations model serves as a proof of concept and helps to illustrate the intuition underlying the main results. However, one might wonder to what extent the results in the previous section are specific to that model. For instance, the OLG model assumed flexible prices, an exogenous sequence of nominal rates, and a demand block with a constant marginal propensity to consume out of wealth. To address this question, I generalize to a much broader class of macroeconomic models and characterize the conditions under which Ricardian and non-Ricardian fiscal rules implement identical sets of equilibria. The general setting demonstrates that the results carry over to environments in which many of the OLG model's strong assumptions are relaxed.

3.1 The general non-linear environment

Time is again discrete and infinite. There is a single consumption good. The model has three "blocks": a demand block, a supply block, and government policy. I maintain the assumption that there is no aggregate uncertainty, but unlike in the OLG model, this general setting does not impose rational expectations.

Demand block: I consider the demand block to be comprised of a continuum of households who consume and save, as in the OLG model. As before, The aggregate real asset holdings of the household sector, A_t, evolve according to

$$A_{t+1} = \frac{1+i_t}{1+\pi_{t+1}}\big(A_t + Y_t - \tau_t - C_t\big), \tag{22}$$

where, as before, $1+\pi_t = P_t/P_{t-1}$ is the rate of inflation, Y_t is total output (labor income plus dividends), and τ_t denotes aggregate taxes levied by the government. As in Farhi and Werning (2019), Auclert, Rognlie, and Straub (2024), or Wolf (2025), I characterize optimal household behavior via an *aggregate consumption function*

$$C_t = \mathcal{C}\big(A_0, \{Y_s\}, \{\tau_s\}, \{1+i_s\}, \{1+\pi_s\}\big), \tag{23}$$

where $\{x_s\}$ denotes the full time path of a variable x. That is, the path of consumption depends on all of the terms in the private-sector budget constraint (22): (1) the private sector's initial asset holdings, (2) the paths of income and taxes, and (3) the path of nominal interest rates and inflation. Such a consumption function can be written down in almost *any* standard macroeconomic model – the inclusion of future realizations of income, taxes, and so on does not imply that agents hold rational expectations about those outcomes. Indeed, Farhi and Werning (2019) show how the consumption function (23) can capture behavioral distortions.

In some (but not all) of the results, I impose the following assumption on the demand function.

Assumption 1. *The aggregate consumption function $\mathcal{C}(\cdot)$ satisfies*

$$\sum_{t=0}^{\infty} R_t^{-1} C_t = A_0 + \sum_{t=0}^{\infty} R_t^{-1}(Y_t - \tau_t), \tag{24}$$

where $R_t \equiv \prod_{s=1}^{t} \frac{1+i_{s-1}}{1+\pi_s}$ is the interest earned on an investment from 0 to t.

Condition (24) is effectively a transversality condition on aggregate demand. Again, this condition is satisfied by most standard models (including those with heterogeneous agents or behavioral distortions). The combination of Assumption 1 with a Ricardian constraint on government policy will imply that, by Walras' Law, one equilibrium condition will be redundant given all of the others. Later, I present some canonical examples of models satisfying these conditions along with my main assumption regarding Ricardian non-equivalence.

Supply block: I assume that the supply side is comprised of firms whose aggregate supply schedule satisfies

$$Y_t = \mathcal{Y}(\{P_s\}). \tag{25}$$

That is, output depends on the sequence of nominal prices. This assumption is consistent with both flexible- and sticky-price specifications. In a flexible-price setting, output is typically exogenous: output may come from an exogenous endowment, as in the benchmark model, or from the combination of static labor-leisure optimization and firm first-order conditions, e.g., $W_t u'(C_t) = v'(L_t)$ with $W_t = 1$ and $Y_t = L_t$. On the other hand, with sticky prices, one often obtains expressions relating output to the path of nominal prices directly. For example, the (linearized) New Keynesian Phillips curve can be written as $\hat{y}_t = \frac{\hat{\pi}_t - \beta\hat{\pi}_{t+1}}{\kappa}$, where $\hat{\pi}_t \equiv \hat{p}_t - \hat{p}_{t-1}$.

Government: The government, as before, sets the nominal interest rate and issues debt financed with taxes. The nominal rate is set according to a generalized Taylor-type rule

$$1 + i_t = \Phi(\{P_s\}, \{Y_s\}, \{\varepsilon_s\}), \tag{26}$$

i.e., the nominal rate in any period can depend on the entire sequence of prices and output as well as a sequence of *monetary policy shocks* $\{\varepsilon_s\}$ whose value is realized at $t = 0$. This allows for sophisticated rules, e.g., that take into account arbitrary lags of inflation or even expectations of future macroeconomic outcomes. The shocks correspond to an exogenous deviation from this rule, e.g., the deterministic path of interest rates in the OLG model.

The government budget constraint is (4), as before, and initial government debt is given by (**??**). Now, I define a fiscal rule as a path of taxes that, in principle, can depend on the entire path of prices, nominal rates, output, and the monetary policy shock,

$$\tau_t = \mathcal{T}(\{P_s\}, \{1 + i_s\}, \{Y_s\}, \{\varepsilon_s\}) \tag{27}$$

like the path of nominal rates. Even though nominal rates are determined by the other variables via the generalized Taylor rule (32), it is important for the policy condition on the path of nominal rates independently so that the government budget constraint can hold after any history (rather than only for equilibrium sequences $\{1 + i_s\}$). A Ricardian fiscal rule satisfies $B_0 = \sum_{s=0}^{\infty} R_t^{-1}\tau_t$ for any path $\{P_s, 1 + i_s, Y_s, \varepsilon_s\}$.

Market clearing and equilibrium: The market-clearing conditions for goods and assets are

$$C_t = Y_t \;\; \forall\, t, \tag{28}$$

$$A_t = B_t/P_t \;\; \forall\, t. \tag{29}$$

An *equilibrium* consists of sequences $\{C_t, Y_t, A_t, B_t, \tau_t, P_t, 1 + i_t, 1 + r_t, \varepsilon_t\}$ such that (**??**), (**??**), (4), 22-23, and 25-29 hold.

3.2 Linearization

I assume that the model has a zero-inflation steady state $\{Y^*, B^*, 1+r^*, P^*\}$ and linearize around it. Some functional-analytic preliminaries are used throughout the analysis. I denote sequences with boldface letters (e.g., $\boldsymbol{x} = (z_0, z_1, \ldots)$) and linear operators on sequences by bold capital letters, e.g., $\boldsymbol{z} = \boldsymbol{M}\boldsymbol{x}$. Hats again denote log deviations from steady state.

It is necessary to specify a sequence space in which the endogenous variables, such as nominal prices and output, will lie. Define the *weighted-ℓ^p space*

$$\ell^p_\alpha \equiv \big\{\boldsymbol{x} \in \mathbb{R}^{\mathbb{N}} : \sum_{t=0}^{\infty} \alpha^t |x_t| < \infty\big\}.$$

where $\alpha > 0$ is a *weighting constant.* I equip this space with the usual norm

$$||\boldsymbol{x}||^p_\alpha = \big(\sum_{t=0}^{\infty} \alpha^t |x_t|^p\big)^{1/p}.$$

The non-explosiveness criterion typically used in DSGE models requires $\boldsymbol{x} \in \ell^\infty_1$ for all endogenous variables $\boldsymbol{x}$: this is simply the restriction that x_t remain bounded, i.e., $\sum_t |x_t| < \infty$. (Note that here the weighting constant is one, and the p-norm approaches the max operator as $p \to \infty$.)

I will instead typically work on the space ℓ^1_β, where the weighting constant $\beta = 1/(1+r^*)$ is the inverse of the steady-state real interest rate. All sequences $\boldsymbol{x}$ in this space have finite present value when discounted by β,

$$\boldsymbol{V}(\boldsymbol{x}) = \sum_{t=0}^{\infty} \beta^t x_t < \infty,$$

where I refer to $\boldsymbol{V} : \ell^1_\beta \to \mathbb{R}$ as the *present value functional.* This restriction is necessary to make sense of the Ricardian constraint on fiscal policy and Assumption 1, which relates the present value of consumption demand to the present value of net income. Moreover, I demonstrate that once the ℓ^1_β restriction is dropped, then *neither* Ricardian nor non-Ricardian fiscal policy can select an equilibrium. That is, canonical formulations of the FTPL impose the ℓ^1_β restriction as well.

The consumption function linearizes to

$$\hat{\boldsymbol{c}} = \boldsymbol{\mathcal{C}}_{A_0} \hat{a}_0 + \boldsymbol{\mathcal{C}}_Y \hat{\boldsymbol{y}} + \boldsymbol{\mathcal{C}}_\tau \hat{\boldsymbol{\tau}} + \boldsymbol{\mathcal{C}}_i \hat{\boldsymbol{i}} + \boldsymbol{\mathcal{C}}_\pi \hat{\boldsymbol{\pi}}, \tag{30}$$

where the notation $\boldsymbol{\mathcal{C}}_{\boldsymbol{x}}$ denotes the Fréchet derivative of the consumption function $\mathcal{C}$ with respect to a variable $\boldsymbol{x}$ evaluated at the steady state,[4]

$$\boldsymbol{\mathcal{C}}_{\boldsymbol{x}} \equiv \frac{d\mathcal{C}}{d\boldsymbol{x}}\big|_{S.S.}.$$

[4]Here, I assume that the Fréchet derivatives exist and are bounded mappings from the space ℓ^p_α of endogenous variables to itself.

The aggregate supply function linearizes to

$$\hat{\boldsymbol{y}} = \boldsymbol{\mathcal{Y}}\hat{\boldsymbol{p}}. \tag{31}$$

The government block of the model can be written as

$$\hat{\boldsymbol{i}} = \boldsymbol{\Phi}_P\hat{\boldsymbol{p}} + \boldsymbol{\Phi}_Y\hat{\boldsymbol{y}} + \Phi_\varepsilon\boldsymbol{\varepsilon}, \tag{32}$$

$$\hat{\boldsymbol{\tau}} = \boldsymbol{\mathcal{T}}_P\hat{\boldsymbol{p}} + \boldsymbol{\mathcal{T}}_i\hat{\boldsymbol{i}} + \boldsymbol{\mathcal{T}}_Y\hat{\boldsymbol{y}} + \boldsymbol{\mathcal{T}}_\varepsilon\boldsymbol{\varepsilon}, \tag{33}$$

$$\hat{b}_0 = -\sum_{t=1}^{\infty}\beta^t\frac{\overline{B}_{0,t}}{B^*}\sum_{s=0}^{t-1}\hat{i}_s, \tag{34}$$

In this linearized environment, I restrict to fiscal rules that always yield a path of taxes with finite present value.

Definition 3. *A **fiscal rule on** ℓ^p_α consists of bounded linear mappings $\boldsymbol{\mathcal{T}}_P, \mathcal{T}_i, \boldsymbol{\mathcal{T}}_Y, \boldsymbol{\mathcal{T}}_\varepsilon : \ell^p_\alpha \to \ell^1_\beta$. It is **Ricardian** if*

$$\hat{b}_0 = \boldsymbol{V}\big((1-\beta)(\hat{\boldsymbol{p}} + \hat{\boldsymbol{\tau}}) - \beta\hat{\boldsymbol{i}}\big) \ \ \forall\ (\hat{\boldsymbol{p}}, \hat{\boldsymbol{i}}, \hat{\boldsymbol{y}}, \boldsymbol{\varepsilon}) \in (\ell^p_\alpha)^4. \tag{35}$$

I conclude with the market clearing conditions,

$$\hat{\boldsymbol{c}} = \hat{\boldsymbol{y}}, \tag{36}$$

$$\hat{a}_0 = \hat{b}_0 - \hat{p}_0. \tag{37}$$

and the definition of a linearized equilibrium.

Definition 4. *An ℓ^p_α **linearized equilibrium** consists of sequences $(\hat{\boldsymbol{c}}, \hat{\boldsymbol{y}}, \hat{\boldsymbol{p}}, \hat{\boldsymbol{b}}, \hat{\boldsymbol{i}}, \hat{\boldsymbol{\tau}}, \hat{\varepsilon}) \in \ell^p_\alpha$ and $\hat{a}_0 \in \mathbb{R}$ such that (30)-(37) hold.*

I say that a fiscal rule *uniquely implements* a given equilibrium in ℓ^p_α if it is the only ℓ^p_α equilibrium consistent with that rule.

Notice that all equilibrium variables can be written as (bounded) linear functions of prices and the interest rate shock, $(\hat{\boldsymbol{p}}, \boldsymbol{\varepsilon})$. This is because the equilibrium equations are block-recursive: output $\hat{\boldsymbol{y}}$ depends only on prices; policies $(\hat{\boldsymbol{i}}, \hat{\boldsymbol{\tau}})$, depend only on prices, monetary shocks, and output; and all other variables depend on some combination of $(\boldsymbol{\varepsilon}, \hat{\boldsymbol{y}}, (\hat{\boldsymbol{i}}, \hat{\boldsymbol{\tau}})$. Henceforth, the notation $\hat{\boldsymbol{x}}(\hat{\boldsymbol{p}}, \boldsymbol{\varepsilon})$ will be used to denote the mapping from $(\hat{\boldsymbol{p}}, \boldsymbol{\varepsilon})$ to a variable $\hat{\boldsymbol{x}}$, e.g.,

$$\hat{\boldsymbol{i}}(\hat{\boldsymbol{p}}, \boldsymbol{\varepsilon}) = \boldsymbol{\Phi}_p\hat{\boldsymbol{p}} + \boldsymbol{\Phi}_Y\hat{\boldsymbol{y}}(\hat{\boldsymbol{p}}) + \boldsymbol{\Phi}_\varepsilon\boldsymbol{\varepsilon},$$

where $\hat{\boldsymbol{y}}(\hat{\boldsymbol{p}}) = \boldsymbol{\mathcal{Y}}\hat{\boldsymbol{p}}$.

3.3 Stabilizing prices when Ricardian equivalence fails

This section formalizes the key intermediate result in proving the equivalence between Ricardian and non-Ricardian fiscal rules as equilibrium selection mechanisms. I demonstrate that when Ricardian equivalence fails in a particular sense, then there exists a Ricardian fiscal rule that uniquely implements an equilibrium with stable prices, $\hat{\boldsymbol{p}} = \mathbf{0}$.

Ricardian (non)-equivalence is a property of the consumption function, as defined below.

Definition 5. *The consumption function $\mathcal{C}$ satisfies **Ricardian equivalence** if*

$$\exists\ \boldsymbol{\mathcal{C}}_w : \mathbb{R} \to \ell^p_\alpha \quad s.t. \quad \boldsymbol{\mathcal{C}}_{A_0}\hat{a}_0 + \boldsymbol{\mathcal{C}}_\tau \hat{\boldsymbol{\tau}} = \boldsymbol{\mathcal{C}}_w(\hat{a}_0 - (1-\beta)\boldsymbol{V}(\hat{\boldsymbol{\tau}})).$$

*It satisfies **strong Ricardian non-equivalence (SRNE)** if $\mathcal{C}_\tau$ has a bounded right inverse,*

$$\exists\ \boldsymbol{\mathcal{C}}_\tau^{-1} : \ell^p_\alpha \to \ell^p_\alpha \quad s.t. \quad \boldsymbol{\mathcal{C}}_\tau \boldsymbol{\mathcal{C}}_\tau^{-1} = \boldsymbol{I} \quad and \quad ||\boldsymbol{\mathcal{C}}_\tau^{-1}||^p_\alpha < \infty,$$

where $\boldsymbol{I}$ is the identity operator and $||\cdot||^p_\alpha$ denotes the operator norm on ℓ^p_α.

Intuitively, the consumption function satisfies Ricardian equivalence if demand depends only on initial assets minus the present value of taxes. Then, the timing of taxes to finance a given quantity of debt is irrelevant. The consequence of Ricardian equivalence is that the image of $\boldsymbol{\mathcal{C}}_\tau$ is one-dimensional.

On the other hand, the consumption function specifies SRNE if, for any impulse $\Delta\hat{\boldsymbol{c}}$ to the path of consumption, there exists *some* path of taxes that induces that impulse in demand. Under Assumption 1, this property is more intuitive: for any impulse with present value zero, there exists a change in the *timing* of taxes inducing that impulse. Wolf (2025) demonstrates that in addition to the OLG model presented in the previous section, there are many other macroeconomic models whose demand block satisfies SRNE, such as bond-in-utility models.

A concise set of equations characterizing equilibrium illustrates why the properties of $\boldsymbol{\mathcal{C}}_\tau$ are crucial. Define *excess supply absent fiscal policy* at a vector of prices and shocks $(\hat{\boldsymbol{p}}, \boldsymbol{\varepsilon})$ by

$$\boldsymbol{\Delta}(\hat{\boldsymbol{p}}, \boldsymbol{\varepsilon}) \equiv \hat{\boldsymbol{y}}(\hat{\boldsymbol{p}}) - \hat{\boldsymbol{c}}_0(\hat{\boldsymbol{p}}, \boldsymbol{\varepsilon}),$$

where the first term is supply given the price vector $\hat{\boldsymbol{p}}$ and the second term is demand absent fiscal policy,

$$\hat{\boldsymbol{c}}_0(\hat{\boldsymbol{p}}, \boldsymbol{\varepsilon}) \equiv \boldsymbol{\mathcal{C}}_{A_0}(\hat{b}_0(\hat{\boldsymbol{p}}, \boldsymbol{\varepsilon}) - \hat{p}_0) + \boldsymbol{\mathcal{C}}_Y \hat{\boldsymbol{y}}(\hat{\boldsymbol{p}}) + \boldsymbol{\mathcal{C}}_i \hat{\boldsymbol{i}}(\hat{\boldsymbol{p}}, \boldsymbol{\varepsilon}) + \boldsymbol{\mathcal{C}}_\pi \hat{\boldsymbol{\pi}}(\hat{\boldsymbol{p}}),$$

This captures demand coming from initial assets, changes in income, nominal interest rates, and inflation (after plugging in the supply curve (25) to obtain income at each price vector and the policy rule (32) to obtain interest rates).

The other component of demand comes from fiscal policy. Define

$$\boldsymbol{\mathcal{C}}_\tau(\hat{\boldsymbol{p}},\boldsymbol{\varepsilon})\hat{\boldsymbol{\tau}}(\hat{\boldsymbol{p}},\boldsymbol{\varepsilon}) \equiv \boldsymbol{\mathcal{C}}_\tau\big(\boldsymbol{\mathcal{T}}_p\hat{\boldsymbol{p}} + \boldsymbol{\mathcal{T}}_i\hat{\boldsymbol{i}}(\hat{\boldsymbol{p}},\boldsymbol{\varepsilon}) + \boldsymbol{\mathcal{T}}_Y\hat{\boldsymbol{y}}(\hat{\boldsymbol{p}}) + \boldsymbol{\mathcal{T}}_\varepsilon\boldsymbol{\varepsilon}\big).$$

This component of demand captures the effects of the fiscal rule on demand.

With this notation, an equilibrium can be expressed simply as a solution to the equation

$$\boldsymbol{\Delta}(\hat{\boldsymbol{p}},\boldsymbol{\varepsilon}) = \boldsymbol{\mathcal{C}}_\tau\hat{\boldsymbol{\tau}}(\hat{\boldsymbol{p}},\boldsymbol{\varepsilon}). \tag{38}$$

Excess supply absent fiscal policy (the left-hand side) must equal excess demand from fiscal policy. The key intermediate result is that under SRNE, there exists a *Ricardian* fiscal rule that guarantees a unique solution $\hat{\boldsymbol{p}} = 0$ to this equation whenever $\boldsymbol{\varepsilon} = \mathbf{0}$. Heuristically, a Ricardian fiscal rule can eliminate all multiplicity due to self-fulfilling fluctuations in prices.

I begin with a lemma.

Lemma 1. *Suppose that $\mathcal{C}(\cdot)$ satisfies SRNE. Then there exists a fiscal rule $\hat{\boldsymbol{\tau}}(\hat{\boldsymbol{p}},\boldsymbol{\varepsilon})$ such that $\boldsymbol{\Delta}(\hat{\boldsymbol{p}},\mathbf{0}) - \boldsymbol{\mathcal{C}}_\tau\hat{\boldsymbol{\tau}}(\hat{\boldsymbol{p}},\mathbf{0})$ is injective as an operator from $\ell^1_\beta \to \ker(\boldsymbol{V})$.*

Proof. First, let $\boldsymbol{K} \subset \ell^1_\beta$ denote the subspace of constant sequences, and note that $\ell^1_\beta = \ker(\boldsymbol{V}) \oplus \boldsymbol{K}$. For any $\boldsymbol{x} \in \ell^1_\beta$, it is possible to write

$$\boldsymbol{x} = \underbrace{\big(\boldsymbol{x} - \boldsymbol{V}(\boldsymbol{x})\mathbf{1}\big)}_{\in\ker(\boldsymbol{V})} + \underbrace{\boldsymbol{V}(\boldsymbol{x})\mathbf{1}}_{\in\boldsymbol{K}},$$

where $\mathbf{1} = (1,1,1,\dots)$ is a constant sequence. Hence, $\ker(\boldsymbol{V})$ is complemented by a one-dimensional set, and since $\ell^1_\beta \simeq \ell^1_\beta \oplus \mathbb{R}$ (Pelczynski's decomposition theorem), we immediately obtain $\ell^1_\beta \simeq \ker(\boldsymbol{V})$, so there exists an isomorphism $\boldsymbol{J} : \ell^1_\beta \to \ker(\boldsymbol{V})$.[5]

Given this isomorphism $\boldsymbol{J}$, I show that there exists a fiscal rule (not necessarily Ricardian) such that $\boldsymbol{\Delta}(\hat{\boldsymbol{p}},\mathbf{0}) - \boldsymbol{\mathcal{C}}_\tau\hat{\boldsymbol{\tau}}(\hat{\boldsymbol{p}},\mathbf{0}) = \boldsymbol{J}$. Let the fiscal rule be simply $\hat{\boldsymbol{\tau}} = \boldsymbol{\mathcal{T}}_p\hat{\boldsymbol{p}}$ where

$$\boldsymbol{\mathcal{T}}_p\hat{\boldsymbol{p}} = \boldsymbol{\mathcal{C}}_\tau^{-1}\big(\Delta(\hat{\boldsymbol{p}},\mathbf{0}) - \boldsymbol{J}\big),$$

which is a bounded operator from ℓ^1_β to itself. Then

$$\Delta(\hat{\boldsymbol{p}},\mathbf{0}) - \boldsymbol{\mathcal{C}}_\tau\hat{\boldsymbol{\tau}}(\hat{\boldsymbol{p}},\mathbf{0}) = \Delta(\hat{\boldsymbol{p}},\mathbf{0}) - \boldsymbol{\mathcal{C}}_\tau\boldsymbol{\mathcal{C}}_\tau^{-1}\big(\Delta(\hat{\boldsymbol{p}},\mathbf{0}) - \boldsymbol{J}\big) = \boldsymbol{J},$$

[5] One can also take a constructive approach to prove the existence of $\boldsymbol{J}$. Note that the sequences $\boldsymbol{u}_t = \boldsymbol{e}_t - \beta^{-1}\boldsymbol{e}_{t+1}$ form a basis for $\ker(\boldsymbol{V})$, where $\boldsymbol{e}_t$ is the unit sequence with t-th element equal to one. Simply construct $\boldsymbol{J}$ that maps $\boldsymbol{e}_t \to \boldsymbol{u}_t$.

as desired. $\square$

This result is analogous to Lemma **??** in the OLG model, which demonstrates the existence of a fiscal policy that exactly offsets the wealth effect induced by any path of anticipated future income. Specifically, the lemma constructs a fiscal policy such that for any path of nominal income $\hat{\boldsymbol{y}}^N$, one obtains a path of demand $\hat{\boldsymbol{c}}$ such that (1) $\hat{\boldsymbol{c}}^N \neq \hat{\boldsymbol{y}}^N$, and (2) $\boldsymbol{V}(\hat{\boldsymbol{y}}^N - \hat{\boldsymbol{c}}^N) = 0$. The first property corresponds to the injectivity property in Lemma 1: for any price vector $\hat{\boldsymbol{p}}$, one obtains a non-zero path of excess supply. The second property corresponds to the mapping into $\ker(\boldsymbol{V})$: the resulting path $\hat{\boldsymbol{y}}^N - \hat{\boldsymbol{c}}^N$ has present value zero.

Next, I prove the analogue of Proposition 1: there exists a *Ricardian* fiscal policy that uniquely implements $\hat{\boldsymbol{p}} = \boldsymbol{0}$ as an equilibrium.

Proposition 3. *There exists a Ricardian fiscal rule $\hat{\boldsymbol{\tau}}(\hat{\boldsymbol{p}}, \boldsymbol{\varepsilon})$ that uniquely implements $\hat{\boldsymbol{p}} = \boldsymbol{0}$ when $\boldsymbol{\varepsilon} = \boldsymbol{0}$.*

Proof. Take the fiscal rule constructed in the proof of Lemma 1, such that $\boldsymbol{\Delta}(\hat{\boldsymbol{p}}, \boldsymbol{0}) - \boldsymbol{\mathcal{C}}_\tau \hat{\boldsymbol{\tau}}(\hat{\boldsymbol{p}}, \boldsymbol{0})$ is an injection from ℓ^1_β to $\ker(\boldsymbol{V})$. Recall that this fiscal rule depended only on the price vector, and denote it by $\tilde{\hat{\boldsymbol{\tau}}} = \tilde{\boldsymbol{\mathcal{T}}}_P \hat{\boldsymbol{p}}$.

Define the modified fiscal rule

$$\hat{\boldsymbol{\tau}} = \boldsymbol{\mathcal{T}}_P \hat{\boldsymbol{p}} + \boldsymbol{\mathcal{T}}_i \hat{\boldsymbol{i}},$$

where

$$\boldsymbol{\mathcal{T}}_P \hat{\boldsymbol{p}} = \tilde{\boldsymbol{\mathcal{T}}}_P \hat{\boldsymbol{p}} - (1-\beta)\big(\boldsymbol{V}(\hat{\boldsymbol{p}} + \tilde{\boldsymbol{\mathcal{T}}}_P \hat{\boldsymbol{p}})\big)\boldsymbol{1},$$

$$\boldsymbol{\mathcal{T}}_i \hat{\boldsymbol{i}} = \big(\boldsymbol{V}(\beta \hat{\boldsymbol{i}}) + \hat{b}_0(\hat{\boldsymbol{i}})\big)\boldsymbol{1},$$

where $\hat{b}_0(\hat{\boldsymbol{i}}) = -\sum_{t=1}^{\infty} \beta^t \frac{\overline{B}_{0,t}}{B^*} \sum_{s=0}^{t-1} \hat{i}_s$ and $\boldsymbol{1} = (1, 1, \dots)$ is the unit sequence, with $\boldsymbol{V}(\boldsymbol{1}) = (1-\beta)^{-1}$. It is simple to check that the rule is Ricardian by construction.

Now, suppose that given this fiscal rule, for a given $\boldsymbol{\varepsilon}$, there exist two equilibria with price vectors $\hat{\boldsymbol{p}}$ and $\hat{\boldsymbol{p}}'$. Define $\boldsymbol{dp} = \hat{\boldsymbol{p}}' - \hat{\boldsymbol{p}}$. By (38),

$$\begin{aligned}
\boldsymbol{\Delta}(\boldsymbol{dp}, \boldsymbol{0}) &= \boldsymbol{\mathcal{C}}_\tau \hat{\boldsymbol{\tau}}(\boldsymbol{dp}, \boldsymbol{0}) \\
&= \boldsymbol{\mathcal{C}}_\tau \big(\tilde{\hat{\boldsymbol{\tau}}}(\boldsymbol{dp}, \boldsymbol{0}) - \boldsymbol{V}((1-\beta)(\boldsymbol{dp} + \tilde{\hat{\boldsymbol{\tau}}}(\boldsymbol{dp}, \boldsymbol{0})) - \beta \hat{\boldsymbol{i}}(\boldsymbol{dp}, \boldsymbol{0}) - \hat{b}_0(\boldsymbol{dp}, \boldsymbol{0}))\boldsymbol{1}\big)
\end{aligned}$$

Then we have

$$\boldsymbol{\Delta}(\boldsymbol{dp}, \boldsymbol{0}) - \boldsymbol{\mathcal{C}}_\tau \tilde{\hat{\boldsymbol{\tau}}}(\boldsymbol{dp}, \boldsymbol{0}) = -\boldsymbol{V}((1-\beta)(\boldsymbol{dp} + \tilde{\hat{\boldsymbol{\tau}}}(\boldsymbol{dp}, \boldsymbol{0})) - \beta \hat{\boldsymbol{i}}(\boldsymbol{dp}, \boldsymbol{0}) - \hat{b}_0(\boldsymbol{dp}, \boldsymbol{0}))\boldsymbol{1}.$$

The left-hand side is the operator constructed in Lemma 1, which is injective into $\ker \boldsymbol{V}$. Since the left-hand side is nonzero, we can restrict attention to $\hat{\boldsymbol{d}}p$ such that the right-hand side is nonzero as well. The right-hand side is nonzero only at price vectors $\boldsymbol{dp}$ where the original policy, $\tilde{\hat{\boldsymbol{\tau}}}$, does not satisfy the government budget constraint.

Taking $\boldsymbol{V}(\cdot)$ of both sides,

$$\boldsymbol{V}\big(\boldsymbol{\Delta}(\boldsymbol{dp},\boldsymbol{0}) - \boldsymbol{\mathcal{C}}_\tau \tilde{\hat{\boldsymbol{\tau}}}(\boldsymbol{dp},\boldsymbol{0})\big) = (1-\beta)^{-1} V((1-\beta)(\boldsymbol{dp} + \tilde{\hat{\boldsymbol{\tau}}}(\boldsymbol{dp},\boldsymbol{0})) - \beta \hat{\boldsymbol{i}}(\boldsymbol{dp},\boldsymbol{0}) - \hat{b}_0(\boldsymbol{dp},\boldsymbol{0})),$$

a contradiction, since the left-hand side maps into $\ker \boldsymbol{V}$. Therefore, in any equilibrium, $\hat{\boldsymbol{p}} = 0$. □

This proof follows exactly the logic of (1). The fiscal rule originally developed in Lemma 1 guarantees that at any non-zero price vector $\hat{\boldsymbol{p}}$, excess supply is non-zero for some t. If the original fiscal rule formulated in the lemma is non-Ricardian, I simply modify it to a Ricardian one, and the argument goes through regardless.

3.4 The equivalence result

These results lay the groundwork for the main equivalence result, which I state here but prove in the Appendix.

Theorem 1 (Equivalence)**.** *Suppose that Assumption 1 holds and that $\mathcal{C}(\cdot)$ satisfies SRNE. Then:*

1. *Let $(\hat{\boldsymbol{c}}, \hat{\boldsymbol{y}}, \hat{\boldsymbol{p}}, \hat{\boldsymbol{b}}, \hat{\boldsymbol{i}}, \hat{\boldsymbol{\tau}}, \hat{\boldsymbol{\varepsilon}})$ be an ℓ^1_α equilibrium that is uniquely implemented by some fiscal rule, where $0 < \alpha \leq \beta$. Then there exists a Ricardian rule that uniquely implements it.*

2. *Let $\alpha > \beta$. Then no fiscal rule can uniquely implement an equilibrium on ℓ^1_α.*

These results demonstrate that when Ricardian equivalence fails, the set of equilibria that can be implemented by *non-Ricardian* policy (i.e., the FTPL) are precisely the same as those implemented by *Ricardian policy.* The first part of the result states that either type of fiscal policy can select an equilibrium when one restricts attention to sequences of nominal income with finite present value. As shown in the previous section, the Ricardian policy that selects an equilibrium in this case can be viewed as a form of nominal wealth targeting.

Conversely, when moving beyond sequence spaces where nominal income must have finite present value (ℓ^1_α for $\alpha > \beta$), then it is no longer possible for *any* fiscal policy to uniquely select an equilibrium. That is, in this case, even the FTPL is not an equilibrium selection device. Linearized models using the FTPL for equilibrium selection *must* restrict attention to finite present-value equilibria. Sometimes this restriction comes directly from agents' optimization problems, but sometimes it does not – in these cases, equilibria with bubbles are effectively ruled out by assumption.

According to this equivalence result, much of the debate surrounding the FTPL has been somewhat misguided. The question of whether the government can violate its budget constraint off-equilibrium matters only in the knife-edge case where Ricardian equivalence holds. Theorem 1 demonstrates that once Ricardian equivalence fails, one can assert that fiscal policy selects an equilibrium without adopting the FTPL. The mechanism, however, is heuristically similar to the FTPL: the government increases deficits when the long-run price level is expected to be low relative to target, and it decreases deficits when it is expected to be too high. Therefore, generically, one can study economies in which monetary policy pins down the path of prices in the short run while fiscal policy provides a long-term nominal anchor *without* wading into the extensive controversies surrounding non-Ricardian fiscal policies.

4 Conclusion

Discussions of price-level determinacy under an interest rate rule in monetary economies have long centered around either the Taylor principle or non-Ricardian fiscal policies. Both methods of determining equilibria have been subject to extensive theoretical debate and critique. I demonstrate that when Ricardian equivalence fails, it is possible to sidestep this debate entirely. A Ricardian fiscal policy is sufficient to provide an anchor for the price level in the long run, so that monetary policy can focus on smoothing inflation fluctuations in the short run. This result extends readily to a variety of non-Ricardian macroeconomic models, such as those with behavioral distortions or bonds in the utility function.

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A Household demand in the non-linear model

In this section, I derive aggregate demand in the non-linear model, which is in turn used to derive (9) in the linearized model.

Optimal individual consumption: A consumption-savings plan $(C_{i,t}, A_{i,t})$ maximizes (**??**) subject to (**??**) and the no-Ponzi condition if and only if the following hold:

$$1 = \beta(1 + r_{t+s})\frac{C_{i,t+s}}{C_{i,t+s+1}}, \tag{39}$$

$$\lim_{T\to\infty} R_{t,t+T}^{-1}\delta^T A_{i,t+T} \leq 0, \tag{40}$$

where $R_{t,t+s} \equiv \prod_{s=0}^{T-1}(1 + r_{t+s})$ denotes the real return on an investment from t to $t + s$.

The transversality condition (40), together with the no-Ponzi condition in the household's problem, imply that for an individual household i, consumption satisfies the intertemporal budget constraint

$$\sum_{s=0}^{\infty} R_{t,t+s}^{-1}\delta^s C_{i,t+s} = A_{i,t} + \sum_{s=0}^{\infty} R_{t,t+s}^{-1}\delta^s\big(Y_{t+s} - \tau_{t+s}\big), \tag{41}$$

using (**??**) to substitute out wages W_t. The individual Euler equation (39) yields $C_{i,t+s} = \beta^s R_{t,t+s}C_{i,t}$. Combining these two relationships, rearranging, and multiplying by P_t on both sides,

$$P_tC_{i,t} = (1 - \beta\delta)\Bigg(P_tA_{i,t} + \sum_{s=0}^{\infty} I_{t,t+s}^{-1}\delta^s P_{t+s}\big(Y_{t+s} - \tau_{i,t+s}\big)\Bigg). \tag{42}$$

Aggregation: In the benchmark model, where all households have identical (β, δ) and pay taxes into the social fund when old, (42) can be aggregated across households to obtain a formula for $P_tC_t = \sum_{s=0}^{\infty} \delta^s P_tC_t^{t-s}$:

$$P_tC_t = (1 - \beta\delta)\Bigg(P_tA_t + \sum_{s=0}^{\infty} I_{t,t+s}^{-1}\delta^s P_{t+s}\big(Y_{t+s} - \tau_{t+s} - \mathbf{1}\{s \geq 1\}\tau_{\text{old}}\big)\Bigg). \tag{43}$$

I also consider a modified environment in which households may have heterogeneous (β_i, δ_i). Let $(1 - \delta)m(\beta, \delta)$ denote the mass of newborns with $(\beta_i, \delta_i) = (\beta, \delta)$ born in each period, and suppose $\int_{\beta,\delta} m(\beta, \delta)d\beta d\delta = 1$. The mass of age-$s$ households with parameters (β, δ) is then $q(\beta, \delta, s) = (1 - \delta)\delta^s m(\beta, \delta)$, and the total measure of agents with parameters (β, δ) is $m(\beta, \delta)$.

All households supply one unit of labor in each period and thus earn identical incomes, whereas taxes may differ across households. Specifically, I assume the government levies taxes to keep wealth constant across the distribution of (β, δ). Let $A_t(\beta, \delta) = \int\limits_{i:(\beta_i,\delta_i)=(\beta,\delta)} A_{i,t} di$ denote total assets held by households with parameters (β, δ) at the beginning of period t. The total stock of assets in the economy is B_t/P_t. I suppose all agents pay taxes τ_t, and in addition, newborn households with parameters (β, δ) pay taxes

$$\Delta\tau_t(\beta, \delta) = \frac{A_t(\beta, \delta) - B_t/P_t}{(1 - \delta)m(\beta, \delta)}.$$

Then (42), integrated across such agents, implies

$$P_t C_t(\beta, \delta) = (1 - \beta\delta)\big(B_t + \sum_{s=0}^{\infty} I_{t,t+s}^{-1} \delta^s P_{t+s}(Y_{t+s} - \tau_{t+s})\big) \tag{44}$$

(Note that the tax adjustments $\Delta\tau$ drop out of the forward-looking term because these taxes are levied only on newborns.) Summing over all households, aggregate consumption obeys a formula analogous to (43),

$$P_t C_t = M\tilde{P}C\big(B_t + \sum_{s=0}^{\infty} I_{t,t+s}^{-1} \tilde{\delta}^s P_{t+s}(Y_{t+s} - \tau_{t+s})\big), \tag{45}$$

where

$$M\tilde{P}C = \int\limits_{\beta,\delta} (1 - \beta\delta)m(\beta, \delta) d\beta d\delta$$

is the average MPC of agents in the economy and

$$\tilde{\delta} = \int\limits_{\beta,\delta} \delta m(\beta, \delta) d\beta d\delta$$

is the average survival rate.

Note that (45) is identical to (43) under $\delta \leftrightarrow \tilde{\delta}$, $\beta \leftrightarrow (1 - M\tilde{P}C)/\tilde{\delta}$. However, (44) imposes additional optimality conditions. In particular, if $m(\beta, \delta)$ has positive mass for all δ sufficiently close to 1, then a solution to *all* households' optimization problems does not exist unless $\sum_{s=0}^{\infty} I_{t,t+s}^{-1} P_{t+s}(Y_{t+s} - \tau_{t+s})$ is finite.

B Log-linear equilibrium

This section derives the log-linear equilibrium conditions. I begin with the budget constraint (**??**) aggregated across households and recast in nominal terms. First, I derive steady-state conditions. From (4),

$$\tau^{N*} = \frac{i^*}{1+i^*}B^*.$$

Then, (43) yields

$$Y^{N*} = (1-\beta\delta)\big(B^* + \frac{(1+i^*)Y^{N*} - i^*B^*}{1+i^*-\delta} - \frac{\delta}{1+i^*-\delta}P^*\tau_{\text{old}}\big),$$

so rearranging yields an expression for the steady-state nominal rate required for zero inflation,

$$1+i^* = \frac{1}{\beta}\big(1 + \frac{1-\beta\delta}{\delta}\frac{(1-\delta)B^* - \delta P^*\tau_{\text{old}}}{Y^{N*}}\big). \tag{46}$$

To obtain a steady state with $1+i^* = \beta^{-1}$, set $P^*\tau_{\text{old}} = \frac{1-\delta}{\delta}B^*$.

Next, I linearize the aggregate private-sector budget constraint.

$$\begin{aligned}\hat{a}^N_{t+1} &= \hat{i}_t + \frac{B^*}{B^*-\tau^{N*}}\hat{a}^N_t - \frac{i^*}{1+i^*}\frac{B^*}{B^*-\tau^{N*}}\hat{\tau}^N_t + \frac{Y^{N*}}{B^*-\tau^{N*}}(\hat{y}^N_t - \hat{c}^N_t)\\ &= \hat{i}_t + \beta^{-1}\hat{a}^N_t - (\beta^{-1}-1)\hat{\tau}^N_t + \frac{Y^{N*}}{B^*}\beta^{-1}(\hat{y}^N_t - \hat{c}^N_t).\end{aligned}$$

The linearized government budget constraint is derived in a nearly identical manner.

It remains to linearize the aggregate demand equation (43).

$$\begin{aligned}\hat{c}^N_t &= \sum_{s=0}^{\infty}(\beta\delta)^s(1-\beta\delta)\big(\hat{y}^N_t - \sum_{u=0}^{s-1}\hat{i}_{t+u}\big) + (1-\beta\delta)\frac{B^*}{Y^{N*}}\left(\hat{a}^N_t - \frac{i^*}{1+i^*}\sum_{s=0}^{\infty}(\beta\delta)^s\big(\hat{\tau}^N_t - \sum_{u=0}^{s-1}\hat{i}_{t+u}\big)\right)\\ &\quad - \sum_{s=1}^{\infty}(\beta\delta)^s\frac{1-\delta}{\delta}\frac{(1-\beta\delta)B^*}{Y^{N*}}\sum_{u=0}^{s-1}\hat{i}_{t+u}\\ &= \sum_{s=0}^{\infty}(\beta\delta)^s\big((1-\beta\delta)\hat{y}^N_t - \beta\delta\hat{i}_{t+u}\big) + (1-\beta\delta)\frac{B^*}{Y^{N*}}\left(\hat{a}^N_t - \sum_{s=0}^{\infty}(\beta\delta)^s(1-\beta)\hat{\tau}^N_t - \frac{1-\beta\delta}{\delta}\sum_{s=1}^{\infty}(\beta\delta)^s\sum_{u=0}^{s-1}\hat{i}_{t+u}\right)\\ &= \sum_{s=0}^{\infty}(\beta\delta)^s\big((1-\beta\delta)\hat{y}^N_{t+s} - \beta\delta\hat{i}_{t+s}\big) + \kappa\left(\hat{a}^N_t - \sum_{s=0}^{\infty}(\beta\delta)^s\big((1-\beta)\hat{\tau}^N_{t+s} - \beta\hat{i}_{t+s}\big)\right),\end{aligned}$$

as in (9).

C Omitted proofs

Proof of Proposition 2. Observe that

$$\hat{c}_t^N - \hat{y}_t^N = (1-\beta\delta)\hat{w}_t - \hat{y}_t^N + (1-\beta\delta)\kappa(\hat{a}_t^N - \hat{b}_t^N + \hat{b}_t^N - \tilde{v}_t^\tau). \tag{47}$$

Then the difference of (8) and (15) yields

$$\begin{aligned}
\hat{a}_t^N - \hat{b}_t^N &= \beta^{-1}(\hat{a}_{t-1}^N - \hat{b}_{t-1}^N) - (\beta\kappa)^{-1}(\hat{c}_{t-1}^N - \hat{y}_{t-1}^N) \\
&= \delta(\hat{a}_{t-1}^N - \hat{b}_{t-1}^N) - (\beta\kappa)^{-1}\big((1-\beta\delta)(\hat{v}_{t-1}^y + \kappa(\hat{b}_{t-1}^N - \hat{v}_{t-1}^\tau)) - \hat{y}_{t-1}^N\big) \\
&= -\frac{1}{\beta\kappa}\sum_{s=0}^{t-1}\delta^{t-1-s}\big((1-\beta\delta)(\hat{v}_s^y + \kappa(\hat{b}_s^N - \hat{v}_s^\tau) - \hat{y}_s^N\big),
\end{aligned}$$

since $\hat{a}_0^N = \hat{b}_0^N$. From (8), it is immediate that $\hat{b}_t^N - \tilde{v}_t^\tau = \beta^{-t}(\hat{b}_0^N - \tilde{v}_0^\tau)$. Combining this relationship with (47), one obtains (16). □