Loss aversion or preference imprecision? What drives the WTA-WTP disparity?*

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Abstract

We propose a framework that, while eliminating the endowment effect, accounts for the two leading explanations of the disparity between willingness to accept (WTA) and willingness to pay (WTP): loss aversion and preference imprecision. Our approach applies to incomplete preferences under both risk and ambiguity. We introduce axioms that characterize the disparity and each of its two components. We show that the WTA-WTP disparity is a monetary measure (i.e., a premium) of uncertainty aversion, with hedging, rather than risk, serving as the neutrality benchmark. We derive comparative statics results for this measure with respect to both individuals and prospects. Our framework is general and encompasses several models, including multiple-utility multiple-prior models, as special cases.

Keywords: willingness to accept, willingness to pay, uncertainty aversion, loss aversion, incomplete preferences, short-selling

JEL classification: D81, D91, C91

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1 Introduction

Large observed differences between willingness to accept (WTA) and willingness to pay (WTP) values (henceforth, the WTA–WTP gap or disparity) are among the most widely discussed phenomena in behavioral economics. In this paper, we study this disparity for uncertain prospects, which abound in finance, insurance, sports betting, and gambling (see, e.g., Horowitz, 2006; Eisenberger and Weber, 1995). The size of the gap in experiments varies with the study design, the elicitation method, and the precise definition employed. However, the gap is too large to be explained by standard utility theory, which attributes it solely to wealth effects arising from differences in initial positions in WTA and WTP elicitation tasks (Schmidt and Traub, 2009).

The predominant behavioral explanation of the gap is based on the asymmetric treatment of gains and losses: the joy of gaining a prospect is smaller than the pain of losing it (Kahneman et al., 1991; Marzilli Ericson and Fuster, 2014). This explanation was recently challenged by Chapman et al. (2023), who find that the disparity is at most weakly correlated with measures of loss aversion. This observation has sparked interest in explanations based on preference imprecision or caution (Dubourg et al., 1994; Cubitt et al., 2015; Cerreia-Vioglio et al., 2015, 2024; Bayrak and Hey, 2020). Despite differences in detail, these explanations share a common intuition: under uncertainty about relevant trade-offs, a cautious decision maker (DM) behaves conservatively, demanding more to sell and offering less to buy.[1]

When calibrated to the data, models that rationalize the gap - whether based on loss aversion or preference imprecision - typically ascribe the entire disparity to only one of these two effects. Hence, they are not useful for comparing the relative

[1] This idea is naturally captured by representing preferences with a set of utility functions rather than a single one. Although Cerreia-Vioglio et al. (2015, 2024) develop complete-preference models, the same underlying set-valued structure also appears in incomplete-preference frameworks (Dubra et al., 2004; Ok et al., 2012 for risk; Galaabaatar and Karni, 2013; Hara and Riella, 2023; Borie, 2023 for uncertainty). The main difference lies in how caution is interpreted. In Cerreia-Vioglio et al. (2015), caution is implemented as a form of pessimism in the evaluation of acts or certainty equivalents. In incomplete-preference models, caution manifests as inertia (Bewley, 2002): the DM adopts a new option only if it is better under all admissible utilities or beliefs.

magnitudes of these effects, either at the aggregate or the individual level. In this paper, we develop a model in which both effects are present and whose magnitudes can be measured and compared.

In studies of the WTA–WTP gap, WTP is typically elicited in tasks framed as buying a good, whereas WTA is usually elicited in tasks framed as selling a pre-owned good. Hence, the two tasks differ in terms of the DM's initial endowment. In classical utility theory, this difference generates an income effect, which is the only source of the disparity. In behavioral economics, this idea is further extended to the endowment effect: owning a good changes the way one values it. This has led many to view the WTA–WTP gap as equivalent to the endowment effect.[2]

To eliminate differences in initial endowments, we measure WTA using short-selling prices rather than selling prices. Taking a short-selling position in a prospect means taking a negative position in that prospect without owning it.[3] Because the payoffs of a buyer and a short seller are exact opposites and the status quo is identical in both cases, the WTA and WTP elicitation tasks isolate the agent's attitude toward gains and losses, with no endowment effect present.[4] To illustrate these concepts, we present a simple example.

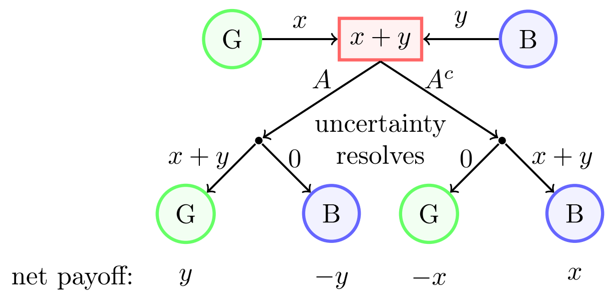
Motivating example Consider two positions in a gamble on an uncertain event A (e.g., whether a favorite team wins an upcoming soccer match), depicted in Figure 1. In position G, one puts x dollars in the pot; in position B, one puts y dollars. If A (resp. A^c) occurs, the person in position G (resp. B) wins the whole pot. Therefore,

[2]In models that distinguish these effects, evidence for the endowment effect is weaker than for loss aversion or the WTA–WTP disparity (see Plott and Zeiler, 2005 or Marzilli Ericson and Fuster, 2014 for surveys). For example, Brown (2005) find loss aversion not due to the loss of a good, but to the negative net outcome of buying or selling. Similarly, Shahrabani et al. (2008) find a positive correlation between short-selling prices and the WTA-WTP disparity. They test two explanations for the disparity - the status quo and endowment effects - and find evidence in favor of the former.

[3]This way of interpreting WTA - from the perspective of the organizer rather than the participant in a lottery, which is common in the literature on risk measures and insurance premiums (Bühlmann, 1970, p. 86) - is similar to the idea of taking a short position in finance.

[4]For a discussion of the WTA–WTP disparity under various definitions of buying and selling prices, see Eisenberger and Weber (1995). See also Lewandowski and Woźny (2022) for a discussion of selling versus short-selling prices.

Figure 1: There are 2 positions one can take in a gamble: B (blue) and G (green). In position G one is betting x, while in position B one is betting y. If event A (resp. A^c) occurs, position G (resp. B) receives the sum of the bets $x + y$.



the net profit of G is y if A occurs and $-x$ otherwise. The net profits in G and B are opposite. As a result, for a given probability of A, at most one side of the bet may have a positive expected value.

If the DM surely prefers taking either side of a bet to abstaining (i.e., both G and B are strictly preferred to not betting), we call such DM *uncertainty-loving*. Conversely, if the DM rejects at least one side of the bet, we call her *uncertainty-averse*. In our framework, a bet may be rejected for two distinct reasons. First, the DM may surely dislike it. Second, the DM may be uncertain about her trade-offs and, out of caution, decline to bet. We call the DM *surely uncertainty-averse* if she surely dislikes at least one side of every bet. The remaining case, when the DM is unable or unwilling to make a definitive choice, is interpreted as *preference imprecision*.

Sure uncertainty aversion (sure UA) is closely related to the idea that losses loom larger than gains. In a bet such as in Figure 1, the two positions produce exactly opposite net payoffs. Moreover, when $x = y$ and events A and A^c are symmetric (swapping positions leaves their attractiveness unchanged), so rejection of one implies rejection of the other for a surely uncertainty-averse DM. Hence our notion of UA extends the classical definition of loss aversion for risk (Kahneman and Tversky, 1979), in which individuals reject equal-chance bets involving the same gain and loss.

To quantify UA and sure UA, we use the short-selling price (WTA) and the buying price (WTP), along with their extensions proposed by Eisenberger and Weber

(1995); Cubitt et al. (2015).[5] Under complete preferences, WTP (resp. WTA) is the indifference price, i.e., the price at which the DM is indifferent between buying and not buying (or between issuing and not issuing) the ticket. Under incomplete preferences, such an indifference price need not exist. We therefore use boundary prices. The buying (resp. short-selling) price is the highest (lowest) price at which the DM prefers the prospect to the status quo. The no-buying (resp. no–short-selling) price is the lowest (highest) price at which the DM is confident that the status quo is preferable. The boundary prices partitions the price domain into three regions: (i) prices favoring trade, (ii) prices favoring the status quo, and (iii) prices for which the options are incomparable. These boundaries thus convey richer information than a simple buy/not-buy (or short-sell/no-short-sell) choices.

Contribution First, for potentially incomplete preferences over prospects (Savage (1954) acts), we axiomatically define UA. While weaker than risk aversion, UA extends some behavioral definitions of loss aversion. Our setting is rich and allows for objective probabilities, subjective probabilities, as well as partial or even full ambiguity regarding the underlying probabilities of events. Consequently, our definition differs from many standard definitions of ambiguity or UA in several respects. In particular, our definition uses hedging as the benchmark for neutrality rather than subjective expected utility or probabilistically sophisticated preferences (see, e.g., Ghirardato and Marinacci, 2002; Epstein, 1999; Schmeidler, 1989).[6]

Second, we distinguish the part of UA that the agent is certain or sure about, and the remaining part due to preference incompleteness. Third, we extend the standard definition of *loss aversion/not loss-loving* of Kahneman and Tversky (1979) from risk and complete preferences to ambiguity and incomplete preferences. Under mild

[5]Placing a bet can be viewed as a transaction involving the issuance and purchase of a lottery ticket. In the motivating example above, the DM choosing B offers the DM choosing G a ticket paying $x + y$ if A occurs and nothing otherwise, priced at x. The DM G accepts the bet if x does not exceed his WTP, while the DM B is willing to issue the ticket only if x is at least her WTA.

[6]Our notion treats uncertainty in the same way as it treats risk and compares both to certainty, whereas many standard definitions treat uncertainty as something layered on top of risk.

assumptions, we show the equivalence between not loss-loving and UA as well as loss aversion and the sure part of UA.

Fourth, we show how to measure UA, the sure part of UA, and the remaining part attributed to preference incompleteness using counterparts of indifference prices for incomplete preferences. Unlike standard measures of ambiguity aversion, which measure e.g. the size of the set of subjective beliefs and are unobservable in consequence, our measures are monetary and can be interpreted as *uncertainty premiums*, i.e., the amount DM is willing to pay to hedge a given prospect net of its buying price.

Fifth, we prove that UA is equivalent to WTA $>$ WTP. Thus, we provide an axiomatization of the gap. We also define its comparative version (*more uncertainty averse* agent and *more uncertain prospects*) to argue that the WTA-WTP disparity is a monetary measure of UA. We do the same for the sure part of UA. We illustrate some of our results within the Multi-Utility Multi-Prior (MUMP) model.

Sixth, we show how to decompose the WTA-WTP disparity using these measures: that is, one attributed to loss aversion (i.e. sure UA) and the other attributed to preferences incompleteness (here interpreted as preference imprecision). This decomposition allows one to disentangle the two channels that drive the WTA-WTP disparity.

2 The model and the main results

Let S represent a finite set of states, or, when the context is clear, their total count. Subsets of S are called events. The outcome set is $\mathbb{R}$, with elements designating income amounts. A prospect is a mapping from S to $\mathbb{R}$, identified with a vector in $\mathbb{R}^S$. $\mathcal{F}$ denotes the set of all prospects. We denote by λ ($\in \mathbb{R}$) a constant prospect whose values are λ for all states. Prospect 0 represents the status quo.

Prospects f, g are comonotonic if for all $s, t \in S$, $f(s) > f(t)$ implies $g(s) \geq g(t)$. We say that g is a perfect hedge of f if $f + g = \theta$ for some $\theta \in \mathbb{R}$. We write $f \geq g$ if

$f(s) \geq g(s)$ for all $s \in S$, $f > g$ if $f(s) > g(s)$ for all $s \in S$. For a prospect f, we also define $\underline{f} := \min_{s \in S} f(s)$ and $\bar{f} := \max_{s \in S} f(s)$. Given a nonempty event A and real numbers x, y, a prospect f such that $f(A) = x$, $f(A^c) = y$ is called a binary prospect and denoted by $(x, y; A)$. Our setup is that of uncertainty. Risk is a special case where $(S, \mathcal{S}, \Pi)$ is a probability space, and if the induced probability distributions of two prospects coincide, then the prospects are preferentially equivalent.

Let $\succcurlyeq$ be a binary relation on $\mathcal{F}$. For $f, g \in \mathcal{F}$, we say that f and g are *comparable* if $f \succcurlyeq g$ or $g \succcurlyeq f$, and *incomparable* if neither holds, denoted $f \bowtie g$. The relation $\succcurlyeq$ is *complete* if all pairs are comparable. The symmetric and asymmetric parts of $\succcurlyeq$ are denoted by $\sim$ and $\succ$, respectively. If $f \succcurlyeq 0$, we say that the DM prefers f over the status quo, and in a choice between f and 0, the DM accepts f. If $f \not\succcurlyeq 0$, the DM does not prefer f. If $0 \succ f$, the DM strictly dislikes f. If preferences are complete, $f \not\succcurlyeq g$ is equivalent to $g \succ f$. Under incomplete preferences, $f \not\succcurlyeq g$ can imply either $g \succ f$ or $g \bowtie f$, reflecting two possible reasons for rejecting f in a choice between f and g: either g is strictly preferred, or f and g are incomparable. We impose the following axioms on $\succcurlyeq$.

B0 (Preorder): $\succcurlyeq$ is reflexive and transitive.

B1 (Monotonicity): If $f \geq g$ then $f \succcurlyeq g$. If, in addition, $f \neq g$, then $f \succ g$.

B2 (Continuity): For any $f \in \mathcal{F}$, the sets nW:= $\{f \in \mathcal{F} : f \succcurlyeq 0\}$ and nB:= $\{f \in \mathcal{F} : 0 \succcurlyeq f\}$ are closed (with respect to the Euclidean topology on $\mathbb{R}^S$).

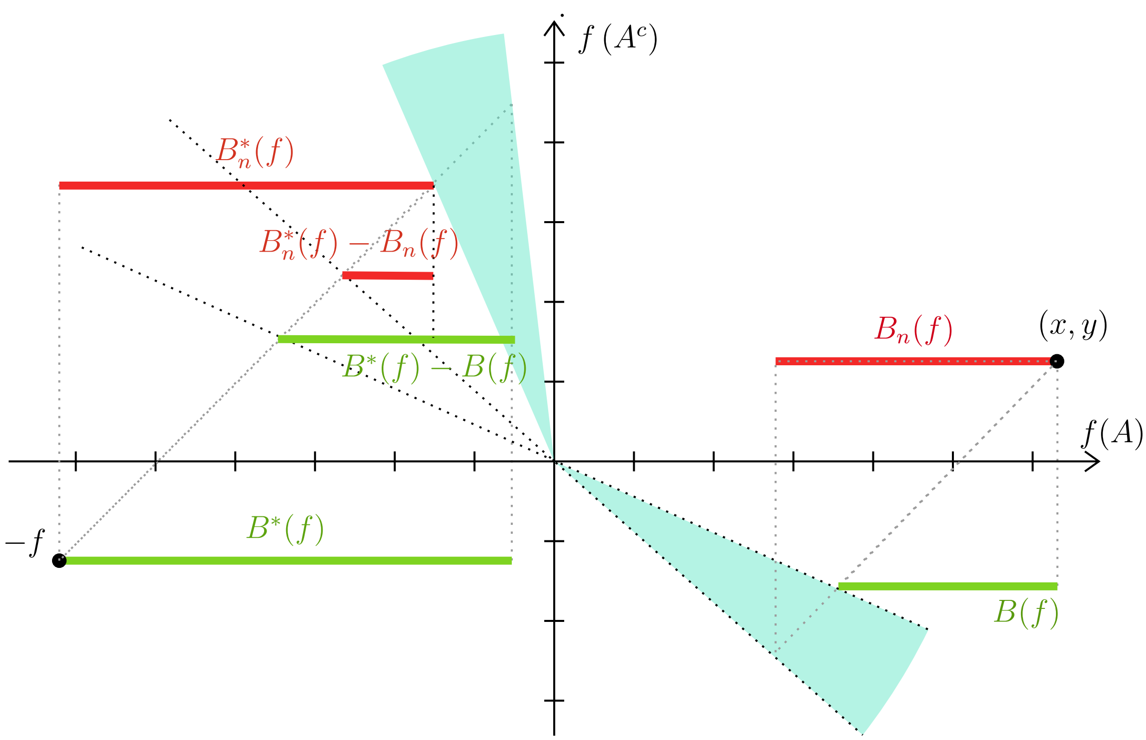
B0 and **B1** are standard; **B2** requires closedness, but only for the upper and lower contour sets at 0; notably, the corresponding strict contour sets need not be open.

2.1 Boundary prices and their basic properties

For prospect $f \in \mathcal{F}$, we define the following four price functionals:

$$\begin{aligned}
&\text{buying price} && B : \mathcal{F} \to \mathbb{R} && B(f) = \max\{\theta \in \mathbb{R} : f - \theta \succcurlyeq 0\}, && (1)\\
&\text{no buying price} && B_n : \mathcal{F} \to \mathbb{R} && B_n(f) = \min\{\theta \in \mathbb{R} : 0 \succcurlyeq f - \theta\}, && (2)\\
&\text{short-selling price} && B^* : \mathcal{F} \to \mathbb{R} && B^*(f) = \min\{\theta \in \mathbb{R} : \theta - f \succcurlyeq 0\}, && (3)\\
&\text{no short-selling price} && B^*_n : \mathcal{F} \to \mathbb{R} && B^*_n(f) = \max\{\theta \in \mathbb{R} : 0 \succcurlyeq \theta - f\}. && (4)
\end{aligned}$$

Figure 2: The boundary prices for a binary prospect $(x, y; A)$. The shaded area depicts prospects f for which neither $f \succcurlyeq 0$ nor $0 \succcurlyeq f$. We also illustrate construction of the WTA-WTP gap: $B^*(f) - B^*(f)$ as well as its sure counterpart: $B^*_n(f) - B^*_n(f)$.



The above prices have the following interpretation. The buying price $B(f)$ is the highest price θ at which the DM prefers $f - \theta$ to the status quo. Similarly, the no-buying price $B_n(f)$ is the smallest θ at which the DM prefers the status quo to $f - \theta$. $B^*(f)$ and $B^*_n(f)$ are defined analogously, as short-selling and no short-selling prices.

Observe that, in what follows, we use a short-selling price (not a selling price), when defining the WTA-WTP gap. This allows us to omit the endowment effects resulting from the differences in the initial positions between the buying and selling tasks. Figure 2 depicts the four prices defined for a binary prospect $(x, y; A)$. We state some basic properties of the prices. All proofs are in Section 7.

Lemma 1. *For $X \in \{B, B_n, B^*, B_n^*\}$ and every prospect f, a unique $X(f)$ exists and satisfies the mean property, i.e. $\underline{f} \leq X(f) \leq \bar{f}$. The prices satisfy*

$$B_n(f) \geq B(f), \quad B^*(f) \geq B_n^*(f). \tag{5}$$

Moreover, if there is prospect f such that at least one of the inequalities in (5) *is strict, then preferences are incomplete.*

Lemma 2. *For any prospect f and any scalar θ, the following holds:*

$$B^*(f) + B(\theta - f) = \theta \qquad \textit{and} \qquad B_n^*(f) + B_n(\theta - f) = \theta. \tag{6}$$

Equality (6) is known in the literature as a complementary symmetry between buying and short-selling prices. It has been proven to hold for complete preferences, see Lewandowski and Woźny (2022) for some recent results and the literature discussion. Here, we show that the complementary symmetry holds in settings allowing for incomplete preferences and provide a counterpart of the complementary symmetry between no buying and no short-selling prices. We refer to this result frequently later in the paper (in particular for $\theta = 0$).

2.2 Uncertainty aversion and preference imprecision

We now define UA and show it axiomatizes the positive WTA-WTP gap.

Definition 1 (UA)**.** *$\succcurlyeq$ is* ***uncertainty averse*** *if $f \succcurlyeq 0$ implies $-f \not\succcurlyeq 0$ for all $f \in \mathcal{F} \setminus \{0\}$.*

Interpreting the definition: an uncertainty-averse DM will never prefer either side of a bet, i.e., either f or $-f$, to not betting at all. The opposite behavior, where the DM prefers to bet regardless of which side, will be called uncertainty-loving. Intuitively, UA reflects a dislike of situations in which certainty is absent. We now proceed to our first main result.

Theorem 1. $\succcurlyeq$ *is uncertainty averse if and only if* $B^*(f) - B(f) > 0$ *holds for every* $f \in \mathcal{F} \setminus \{0\}$.

The theorem says that the strictly positive gap between short-selling and buying prices, the WTA-WTP gap, is equivalent to UA. We also establish the neutrality benchmark for UA. We say that the DM is *uncertainty neutral* if, for every prospect f, there exists a unique scalar θ such that $f - \theta \succcurlyeq 0$ and $\theta - f \succcurlyeq 0$.

Theorem 2. *A DM is uncertainty neutral if and only if* $B^*(f) - B(f) = 0 \;\; \forall f \in \mathcal{F}$.

Remark 1 (Uncertainty aversion versus risk aversion)**.** *In the risk setting, UA is weaker than risk aversion at* 0*. Indeed, for a prospect* f *with expected value* 0*, risk aversion implies* $0 \succ f$ *and* $0 \succ -f$*. While such a preference profile is consistent with UA, it is not necessarily implied by it. Specifically, UA requires that* at least *one side of the bet,* f *or* $-f$*, is not preferred to the status quo. Thus, it is possible for an uncertainty-averse DM to accept prospect* f *while still requiring compensation to accept the opposite prospect* $-f$*.*[7] *However, UA rules out risk neutrality at* 0*. For a prospect* f *with expected value* 0*, risk neutrality implies* $f \sim 0$ *and* $-f \sim 0$*, meaning the DM is indifferent between* f*,* $-f$*, and the status quo. This preference profile is not consistent with UA.*

Definition 2 (Imprecise preferences)**.** *The preferences of a DM are* imprecise with respect to prospect f *if there exists* $\theta \in \mathbb{R}$ *such that* $f + \theta \bowtie 0$*. Otherwise, the DM's preferences are* precise with respect to prospect f.

[7]Unlike risk aversion, UA allows for the coexistence of gambling and insurance, a behavioral phenomenon discussed since Friedman and Savage, 1948 and Markowitz, 1952. To illustrate, let (x, p) denote a prospect offering a large prize x with small probability p, and nothing otherwise. Many people are willing to pay more than its expected value, i.e., $B(x, p) > xp$, exhibiting risk-loving behavior. At the same time, they may require compensation exceeding the expected value to accept the opposite gamble $(-x, p)$, i.e., $B^*(x, p) > xp$, exhibiting risk-averse behavior. This pattern can coexist under UA whenever $B^*(x, p) > B(x, p) > xp$.

Preference imprecision (PI) is a local notion capturing incompleteness of preferences: if there is a prospect with respect to which the DM is imprecise, we say that her preferences are incomplete. Otherwise, they are complete.

Theorem 3. *$\succcurlyeq$ is imprecise with respect to prospect f if and only if $B_n(f) > B(f)$ and precise if and only if $B_n(f) = B(f)$. Similarly, $\succcurlyeq$ is imprecise with respect to prospect $-f$ if and only if $B^*(f) > B_n^*(f)$ and precise if and only if $B^*(f) = B_n^*(f)$.*

The above theorem shows that the preference imprecision is measured as the gap between no-buying and buying prices. In fact, for a given prospect, we have two such measures: $B_n(f) - B(f)$ as well as $B^*(f) - B_n^*(f)$. Generally, the two gaps can differ (for the same prospect), but later we identify cases for which they are the same.

Our objective in the next two parts is to decompose uncertainty aversion (UA) into two components: preference imprecision and a residual component capturing the portion of UA about which the decision maker is confident. The decomposition is based on boundary prices, which are experimentally elicitable (see the companion paper Lewandowski et al., 2026) and therefore observable. Our framework imposes no restrictions beyond those required for the existence of the boundary prices (1)–(4). Consequently, identification of preference imprecision and the confident component of UA is only partial, although full identification is possible in more structured models (see Example 2). Reflecting this partial identification, Subsections 2.3 and 2.4 present two complementary decompositions: one delivering an upper and the other a lower bound on preference imprecision. These decompositions are based on the notions of *sure* and *strong* UA, which correspond to alternative definitions that part of UA about which the decision maker is confident.

2.3 Sure uncertainty aversion and the decomposition of the WTA-WTP gap

Definition 3 (Sure UA)**.** $\succcurlyeq$ *is* ***surely uncertainty averse*** *if* $0 \not\succ f$ *then* $0 \succ -f$ *for all* $f \in \mathcal{F} \setminus \{0\}$.

Sure UA implies that the status quo must be strictly preferred to at least one side of any bet. It strengthens the notion of UA.

Theorem 4. $\succcurlyeq$ *is surely uncertainty averse if and only if* $B^*(f) - B(f) > 0$ *and* $B_n^*(f) - B_n(f) \geq 0$ *for every* $f \in \mathcal{F} \setminus \{0\}$.

Sure UA thus implies the non-negative gap between the no-short selling and no-buying prices. As the sure UA implies UA, it also means that a short-selling price is strictly larger than a buying price.

We now propose our first decomposition of the WTA-WTP gap. Consider an uncertainty averse DM and some prospect f. By Theorem 4, we have $B^*(f) > B(f)$. By definition of B^* and B, we know that for all θ in between $B(f)$ and $B^*(f)$, the agent will neither accept $f - \theta$ nor $\theta - f$. We partition this set to capture two motives (due to indecision or confidence) for why the DM rejects either one of the two betting positions: $\mathrm{PI}_f := \{\theta \in (B(f), B^*(f)) : \ 0 \bowtie f - \theta\}$, $\mathrm{PI}_{-f} := \{\theta \in (B(f), B^*(f)) : \ 0 \bowtie \theta - f\}$, sure UA $:= \{\theta \in (B(f), B^*(f)) : \ 0 \succcurlyeq f - \theta \ \wedge \ 0 \succcurlyeq \theta - f\}$. By definitions (1)–(4), the size of the above sets can be measured by the respective boundary prices leading to:

$$\text{decomp 1:} \quad \underbrace{B^*(f) - B(f)}_{\text{UA}} = \underbrace{B^*(f) - B_n^*(f)}_{\text{PI}_{-f}} + \underbrace{B_n^*(f) - B_n(f)}_{\text{sure UA}} + \underbrace{B_n(f) - B(f)}_{\text{PI}_f} \quad (7)$$

This decomposition splits the WTA–WTP gap into three components: one capturing the *sure* portion of the UA and two capturing preference imprecision with respect to f and $-f$. The *sure UA* refers to the minimal part of the UA that cannot be attributed to preference imprecision. Figure 2 provides a graphical representation of

Table 1: Preference patterns consistent with the three notions of uncertainty aversion for any nonzero prospect f. + indicates allowed patterns; − indicates ruled-out ones.

f vs. 0	$-f$ vs. 0	UA	strong UA	sure UA
$\succcurlyeq$	$\succcurlyeq$	$-$	$-$	$-$
$\succcurlyeq$	$\bowtie$	$+$	$-$	$-$
$\bowtie$	$\succcurlyeq$	$+$	$-$	$-$
$\bowtie$	$\bowtie$	$+$	$+$	$-$
$\prec$	$\not\succ$	$+$	$+$	$+$
$\not\succ$	$\prec$	$+$	$+$	$+$
$\prec$	$\prec$	$+$	$+$	$+$

this decomposition. We now propose an alternative decomposition that replaces the notion of *sure UA* with that of *strong UA*. Observe that the above decomposition measures preference imprecision twice: for f and $-f$. In some models (see Section 5) it is sufficient to measure it only once. This leads to our second decomposition.

2.4 Strong uncertainty aversion and the second decomposition of the WTA-WTP gap

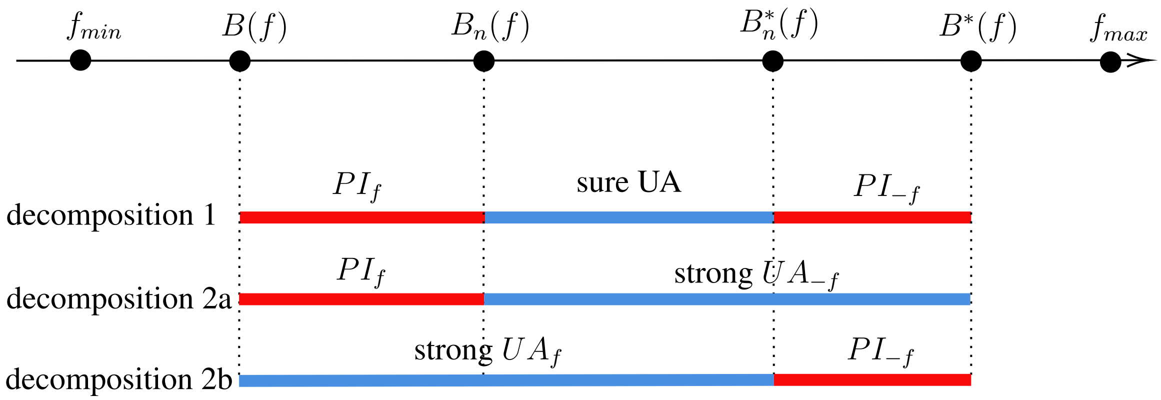
Strong UA captures the intuition that if the DM prefers bet f then he must strictly prefer the status quo to the opposite bet $-f$. This new notion lies in between UA and sure UA.

Definition 4 (Strong UA)**.** *$\succcurlyeq$ is **strongly uncertainty averse** if $f \succcurlyeq 0$ implies $0 \succ -f$ for all $f \in \mathcal{F} \setminus \{0\}$.*

Theorem 5. *$\succcurlyeq$ is strongly uncertainty averse if and only if $B^*(f) - B(f) > 0$, $B^*(f) - B_n(f) \geq 0$ and $B_n^*(f) - B(f) \geq 0$ for every $f \in \mathcal{F} \setminus \{0\}$.*

Note that sure UA implies strong UA and strong UA implies UA – this can be inferred directly, or through the above theorems that also characterize these three notions in terms of boundary prices. Table 1 shows possible patterns of preferences under the three notions of UA.

Figure 3: Uncertainty aversion, measured by the difference between the short-selling price and the buying price of a prospect, is decomposed into preference imprecision (red) and sure or strong uncertainty aversion (blue).



Strong UA leads to the second way we may partition the interval $(B(f), B^*(f))$ for an uncertainty averse individual. Since we have two betting positions, we define two partitions, one for each betting position: strong $\text{UA}_f := \{\theta \in (B(f), B^*(f)) : 0 \succcurlyeq f - \theta\}$, strong $\text{UA}_{-f} := \{\theta \in (B(f), B^*(f)) : \; 0 \succcurlyeq \theta - f\}$. This leads to the following two decompositions:

$$\text{decomp 2a:} \qquad \underbrace{B^*(f) - B(f)}_{\text{UA}} = \underbrace{B^*(f) - B_n(f)}_{\text{strong UA}_f} + \underbrace{B_n(f) - B(f)}_{\text{PI}_f}. \tag{8}$$

$$\text{decomp 2b:} \qquad = \underbrace{B^*(f) - B_n^*(f)}_{\text{PI}_{-f}} + \underbrace{B_n^*(f) - B(f)}_{\text{strong UA}_{-f}}. \tag{9}$$

Figure 3 depicts the three possible decompositions for the case where $B_n^*(f) \geq B_n(f)$. Intuitively, decomposition 1 attributes the smallest part of the WTA-WTP to the (sure) UA, while decompositions $2a$ and $2b$ attribute the smallest part of the WTA-WTP to the preference imprecision. See section 5 for examples and illustration.

2.5 Binary symmetric prospects

We say that events A and A^c are symmetric for $\succcurlyeq$ if, for all $x, y \in \mathbb{R}$, $(x, y; A) \succcurlyeq 0 \iff (x, y; A^c) \succcurlyeq 0$, and the same implication holds when $\succcurlyeq$ is replaced by $\preccurlyeq$.

We say that a binary prospect $(x, y; A)$ is symmetric if the events A and A^c are symmetric.[8] For such bets, the consequence of Lemma 2 is the following result.

Proposition 1. *For a binary symmetric prospect $f = (x, y; A)$, the following holds*

$$B_n(f) - B(f) = B^*(f) - B_n^*(f).$$

As a result, for a symmetric bet f, the gaps in PI_f and PI_{-f} are identical. This also implies that the strong UA_f and the strong UA_{-f} gaps are the same. These characteristics make binary symmetric prospects particularly useful in applications. We use them to compare UA with loss aversion for risk in Section 3. The result behind Proposition 1 is illustrated graphically in Figure 4, where for a binary symmetric bet $f = (x, y; A)$, its perfect hedge is $f^* = (y, x; A)$ (with $\theta = x + y$).

3 Uncertainty aversion versus loss aversion

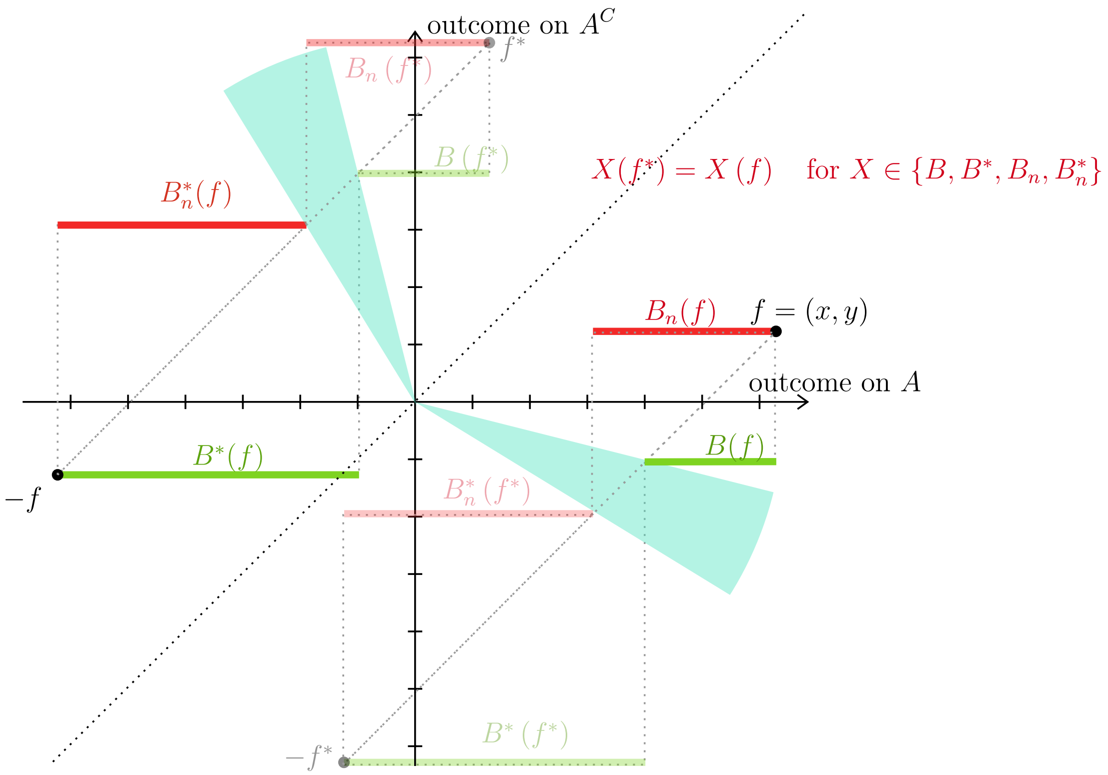
Our definition of UA measures the difference between the buying and short selling prices of f, that is, between the price of buying f and the price of buying $-f$. It is hence naturally related to the treatment of gains and losses. We will now establish a formal relationship between UA and loss aversion.

The standard definition of loss aversion for risk (Kahneman and Tversky, 1979) states that a DM dislikes equal-chance gambles of winning or losing the same nonzero amount. In this section, we extend this definition beyond (subjective) probability and beyond complete preferences. We replace equal-chance gambles with binary symmetric prospects. Since $\succ$ generally differs from $\not\preccurlyeq$ for incomplete preferences, we obtain two definitions instead of one.

Definition 5. $\succcurlyeq$ *is*

[8] The notion of binary symmetric events generalizes Ramsey's notion of $\frac{1}{2}$*-probability event* (see Parmigiani and Inoue, 2009, p.78 or Gul, 1992, Assumption 3).

Figure 4: Due to the symmetry of preferences with respect to the 45° line, all four prices for a binary symmetric prospect f are equal to those for f^*.



(i) ***loss averse*** *(LA) if* $0 \succ (x, -x; A)$ *holds for every* $x \in \mathbb{R} \setminus \{0\}$ *and any event* A *such that* A, A^c *are symmetric.*

(ii) ***not loss-loving*** *(not-LL) if* $(x, -x; A) \not\succ 0$ *for every* $x \in \mathbb{R} \setminus \{0\}$ *and any event* A *such that* A, A^c *are symmetric.*

Remark 2 (Alternative notions of loss aversion)**.** *Kahneman and Tversky (1979) defined loss aversion for risk within prospect theory using the following condition:*

$$x > y \geq 0 \implies (y, -y; 0.5) \succ (x, -x; 0.5), \tag{10}$$

where $(x, -x; 0.5)$ *denotes a monetary prospect yielding* x *or* $-x$ *with equal probability. Under the original version of prospect theory, condition* (10) *is reflected in the value function being steeper for losses than for gains. Many authors take these properties of the value function, rather than the behavioral condition* (10) *itself, as the defining feature of loss aversion, thereby anchoring the concept more firmly within specific parametric formulations of prospect theory.*[9] *Our measure builds directly on the original behavioral condition* (10)*, specifically the case where* $y = 0$*, and replaces the equal-probability lotteries with symmetric events to suit our ambiguity framework. A stronger version of the condition, allowing* $y \neq 0$*, is discussed in Remark 3.*

We say that the preferences $\succcurlyeq$ have *subjective expected utility* (SEU) representation if there exists unique beliefs $\mu \in \Delta(S)$ and a strictly increasing ratio-scale utility $u : \mathbb{R} \to \mathbb{R}$ with $u(0) = 0$ such that $f \succcurlyeq g$ iff $\int_S u(f(s))\mu(ds) \geq \int_S u(g(s))\mu(ds)$.

Proposition 2. *Assume* $\succcurlyeq$ *have SEU representation with beliefs* μ *and utility* u*. Then all of the following are equivalent: UA, sure UA, strong UA, LA, not-LL. Moreover*

[9]For example Wakker and Tversky (1993) offers a behavioral foundation that leads to the value function being steeper for losses than for gains under cumulative version of prospect theory. Schmidt and Zank (2005) propose an alternative behavioral measure of loss aversion for the original prospect theory. Köbberling and Wakker (2005) define an index of loss aversion as $\lambda = \frac{\lim_{x \to 0^-} v'(x)}{\lim_{x \to 0^+} v'(x)}$, based on the local curvature of the value function near the reference point. Abdellaoui et al. (2007) propose a parameter-free method for measuring loss aversion under prospect theory, and Abdellaoui et al. (2016) extend this approach to settings involving ambiguity. More recently, Alaoui and Penta (2025) decompose the utility function under expected utility into two components: one capturing the marginal rate of substitution, and the other reflecting attitudes toward risk and losses.

$\succcurlyeq$ is uncertainty neutral if and only if u is odd and uncertainty averse if and only if $-u(x) > u(-x)$ holds for all $x \neq 0$.

Clearly, under complete preferences different definitions of UA coincide. The same is true for loss aversion and not loss-loving. Interestingly, the proposition establishes that in the class of SEU preferences, UA is equivalent to loss aversion. In particular, a DM with an odd utility function is uncertainty neutral though not necessarily risk neutral. For preferences outside SEU, UA is more restrictive than loss aversion.

Theorem 6. *The following hold:*

(i) If $\succcurlyeq$ is uncertainty averse, then it is not loss-loving.

(ii) If $\succcurlyeq$ is surely uncertainty averse, then it is loss averse.

The reverse implications may not hold in general. Clearly, LA provides restrictions on preferences for binary symmetric prospects only. This, in general, is too weak to allow for extensions over arbitrary prospects. However, for preferences defined over Anscombe–Aumann acts, there exists an additional assumption allowing one to obtain such an extension and hence imply UA from loss aversion. This assumption is an incomplete-preferences version of the classical notion of UA due to Schmeidler (1989). To state it, we extend the set of prospects (only in this section) to $\mathcal{F} = \Delta(X)^S$, where X is a real interval. A Savage act in this set is represented by an act f such that for each state s there exists $x \in \mathbb{R}$ such that $f(s) = \delta_x$. We call such acts purely subjective. An act f is constant if $f(s) = p$ for any $s \in S$ and some $p \in \Delta(X)$. Given preferences $\succcurlyeq$ over $\Delta(X)^S$, we define preferences over $\Delta(X)$, denoted by $\overline{\succcurlyeq}$, as follows: $p \mathrel{\overline{\succcurlyeq}} q \iff f \succcurlyeq g$, where $f(s) = p$ and $g(s) = q$ for each $s \in S$. We now state an axiom similar to Schmeidler (1989) UA, but modified to incomplete preferences.

Definition 6 (Schmeidler uncertainty aversion: SUA)**.** *For any two purely subjective acts f, g, if $f \not\prec g$ then $\frac{1}{2}f + \frac{1}{2}g \succcurlyeq g$.*

Under SUA the DM prefers mixing. This allows us to extend loss aversion (not loss-loving) from binary symmetric prospects to the domain of purely subjective acts.

Theorem 7. *Let $\succcurlyeq$ be a preorder on $\Delta(X)^S$.*

(i) If $f(s) \overline{\not\succcurlyeq} g(s)$ for all $s \in S$ implies $f \not\succcurlyeq g$, then SUA and not-LL imply UA.

(ii) If $f(s) \overline{\succ} g(s)$ for all $s \in S$ implies $f \succ g$, then SUA and LA imply sure UA.

Theorem 7 shows that, under the additional monotonicity condition and Schmeidler UA, not-LL implies UA, and LA implies sure UA. Combined with Theorem 6, this yields the equivalence between not-LL and UA, and between LA and sure UA, confirming that our notion of UA extends behavioral measures of loss aversion within this class of preferences. The following example shows how UA, i.e. the gap, is driven by LA and SUA under Choquet Expected Utility (CEU) (Schmeidler, 1989).

Example 1. *Consider a CEU model with piecewise-linear utility u (equal to x for gains and λx for losses, $\lambda > 0$) and capacity v. Here, SUA reflects the subadditivity of v, and LA is captured by $\lambda > 1$. For a binary prospect $f = (1, 0; A)$ with $\emptyset \neq A \subset S$, one obtains*

$$v(A)(1 - B(f)) + \lambda(1 - v(A))(-B(f)) = 0$$

$$v(A^c)B^*(f) + \lambda(1 - v(A^c))(B^*(f) - 1) = 0$$

$$\text{and so} \quad B^*(f) - B(f) = 1 - \frac{v(A)}{v(A) + \lambda(1 - v(A))} - \frac{v(A^c)}{v(A^c) + \lambda(1 - v(A^c))}.$$

If $\lambda = 1$ (loss neutrality), the gap reduces to $1 - v(A) - v(A^c)$, the uncertainty-aversion index of Dow and da Costa Werlang (1992) based on SUA. If v is self-conjugate (Schmeidler-uncertainty neutrality), the gap depends only on λ; if f is additionally a symmetric prospect, the gap equals $(\lambda - 1)/(\lambda + 1)$.

4 WTA-WTP as an uncertainty premium and its comparative statics

In the literature, the WTA–WTP gap is often considered a behavioral phenomenon that should be rationalized by the asymmetric treatment of gains and losses, preference imprecision, caution, or the endowment effect. We now formalize two new interpretations of the WTA–WTP disparity as defined in our paper. Recall that f^* is a perfect hedge of f if $f^* = \theta - f$ for some $\theta \in \mathbb{R}$.

First, consider f and its buying price $B(f)$. By definition, $f - B(f) \succcurlyeq 0$, meaning that after purchase the DM faces the prospect $f - B(f)$. Now consider its perfect hedge with $\theta = 0$, that is, $B(f) - f$. By UA, $B(f) - f \not\succcurlyeq 0$. This relation implies that some monetary amount must be added to $B(f) - f$ to make it acceptable. Let the smallest such amount be ϵ, so that $\epsilon + B(f) - f \succcurlyeq 0$. By definition, $B^*(f) = \epsilon + B(f)$. Hence, the WTA–WTP gap is exactly ϵ, the smallest net amount required to compensate for the uncertainty faced after purchasing f (i.e. f net of its buying price). In other words, the WTA–WTP gap can be interpreted as an *uncertainty premium* for f, i.e. the minimal price to short-sell (or hedge) $f - B(f)$. Formally, this intuition yields formula (11), a simple consequence of Lemma 2, in the following proposition:

Proposition 3. *For any number θ we have*

$$B^*(f) - B(f) = B^*(f - B(f)), \tag{11}$$

$$= \theta - B(\theta - f) - B(f). \tag{12}$$

Second, one can also interpret the WTA–WTP gap in terms of perfect hedges. For some sure amount θ, consider f and its perfect hedge $f^* = \theta - f$. Taking each of these prospects individually entails facing uncertainty, but together they remove uncertainty and guarantee θ. The gap measures the difference between (sure amount) θ and (separate) buying prices of f and f^*. This leads to expression (12), which highlights the WTA–WTP gap as a *premium for the lack of certainty*, now seen from the perspective of both f and its perfect hedge f^*.

It the remaining subsections, we show the comparative statics results for the WTA-WTP gap: between individuals, between prospects and between sources of uncertainty. These results further justify WTA-WTP as a monetary measure of UA with the intuitive interpretation as an uncertainty premium.

4.1 More uncertainty averse individual

We start by defining across-individual comparison of UA and of sure UA, as captured by the respective price disparities. Formally, let $\succcurlyeq_i$ be a preference relation of agent i. Similarly, we denote by $B_i, B_i^*, B_{ni}, B_{ni}^*$ the buying, short-selling, no-buying and no-short-selling price of an individual i, respectively.

Definition 7. *$\succcurlyeq_1$ is* ***more UA*** *than $\succcurlyeq_2$ if for every $f \in \mathcal{F} \setminus \{0\}$ and some $\epsilon \in \mathbb{R}$:*

$$\big(f \succcurlyeq_1 0 \;\; \text{and} \;\; \epsilon - f \succcurlyeq_1 0\big) \;\Rightarrow\; \exists \delta \in \mathbb{R} : \big(f - \delta \succcurlyeq_2 0 \;\; \text{and} \;\; \delta + \epsilon - f \succcurlyeq_2 0\big).$$

$\succcurlyeq_1$ is ***more surely UA*** *than $\succcurlyeq_2$ if for every $f \in \mathcal{F} \setminus \{0\}$ and some $\epsilon \in \mathbb{R}$:*

$$\big(0 \succcurlyeq_2 f \;\; \text{and} \;\; 0 \succcurlyeq_2 \epsilon - f\big) \;\Rightarrow\; \exists \delta \in \mathbb{R} : \big(0 \succcurlyeq_1 f - \delta \;\; \text{and} \;\; 0 \succcurlyeq_1 \delta + \epsilon - f\big).$$

Theorem 8. *For any $f \in \mathcal{F} \setminus \{0\}$:*

(i) $\succcurlyeq_1$ is ***more UA*** *than $\succcurlyeq_2$ iff $B_1^*(f) - B_1(f) \geq B_2^*(f) - B_2(f)$.*

(ii) $\succcurlyeq_1$ is ***more surely UA*** *than $\succcurlyeq_2$ iff $B_{n1}^*(f) - B_{n1}(f) \geq B_{n2}^*(f) - B_{n2}(f)$.*

Observe that $B_1^*(f)$ is not necessarily higher than $B_2^*(f)$, nor is $B_2(f)$ necessarily higher than $B_1(f)$. This follows directly from the definition, noting that δ need not be positive. A more uncertainty-averse individual will exhibit a larger WTA–WTP gap than a less uncertainty-averse one. The above result together with Theorem 2 suggests a natural way to define UA: a DM is uncertainty-averse if her preferences exhibit more UA than those of an uncertainty-neutral DM. This reinforces the WTA–WTP gap as a measure of UA and highlights that its magnitude reflects the degree of

UA across individuals. The counterpart to this theorem concerns the measurement of preference imprecision.

Definition 8. *$\succcurlyeq_1$ is **more imprecise wrt** f than $\succcurlyeq_2$ if for every $f \in \mathcal{F} \setminus \{0\}$ and some $\theta \in \mathbb{R}$: $\big(f \succcurlyeq_1 0$ and $0 \succcurlyeq_1 f + \theta\big) \Rightarrow \exists \delta \in \mathbb{R} : \big(f + \delta \succcurlyeq_2 0$ and $0 \succcurlyeq_2 f + \delta + \theta\big)$.*

Theorem 9. *For any $f \in \mathcal{F} \setminus \{0\}$, $\succcurlyeq_1$ is more imprecise wrt f than $\succcurlyeq_2$ iff*

$$B_{n1}(f) - B_1(f) \;\geq\; B_{n2}(f) - B_2(f).$$

Lemma 2 implies $B^*(f) = -B(-f)$ and $B^*_n(f) = -B_n(-f)$. Hence an immediate Corollary to Theorem 9 is that $\succcurlyeq_1$ is more imprecise wrt prospect $-f$ than $\succcurlyeq_2$ iff

$$B^*_1(f) - B^*_{n1}(f) \;\geq\; B^*_2(f) - B^*_{n2}(f), \qquad \forall f \in \mathcal{F} \setminus \{0\}.$$

4.2 More uncertain prospects

We now propose a notion of "more uncertain prospects" using only information encoded in preferences. Given two prospects f and g, we define g to be *more uncertain* than f if $g - f$ is a nonconstant prospect comonotonic to f. We say that f *(strongly) uncertainty-dominates* g if g is more uncertain than f and $g - f \not\succ 0$ (respectively, $g - f \not\succcurlyeq 0$). Finally, we say that $\succcurlyeq$ is *monotonic with respect to (strong) uncertainty-dominance* if $f \succcurlyeq g$ whenever f (strongly) uncertainty-dominates g.

Theorem 10. *If g is more uncertain than f, then*

$$B^*(f) - B(f) \leq B^*(g) - B(g), \tag{13}$$

$$\text{and} \quad B^*_n(f) - B_n(f) \leq B^*_n(g) - B_n(g), \tag{14}$$

and this statement is implied by each of the following two sets of conditions:

(i) $\succcurlyeq$ satisfies sure UA and is monotonic with respect to uncertainty-dominance,

(ii) $\succcurlyeq$ satisfies UA and is monotonic with respect to strong uncertainty-dominance.

In words, if an agent dislikes prospects that are uncertainty-dominated, the WTA–WTP gap for such a prospect becomes larger, indicating that the agent demands a higher uncertainty premium as a compensation. Note that uncertainty dominance implies neither $B(f) \geq B(g)$ nor $B^*(f) \leq B^*(g)$. Although such inequalities may hold in particular cases, in general the entire WTA–WTP gap captures the UA.

Remark 3 (A stronger version of loss aversion)**.** *Motivated by the original condition* (10) *in Kahneman and Tversky (1979), we define a stronger version of loss aversion as follows: for all $x > y \geq 0$ and all events A such that A and A^c are symmetric, $(y, -y; A) \succ (x, -x; A)$. This condition is implied by LA together with the strict version*[10] *of monotonicity with respect to strong uncertainty-dominance. Indeed, fix any event A such that A, A^c are symmetric, any $x > y \geq 0$, and set $\epsilon := x - y > 0$. Then $(y, -y; A)$ is comonotonic with $(\epsilon, -\epsilon; A)$ (with the constant act when $y = 0$ being comonotonic with any act). By LA, $(\epsilon, -\epsilon; A) \prec 0$, hence $(\epsilon, -\epsilon; A) \not\succcurlyeq 0$. Therefore, $(y, -y; A)$ strongly uncertainty-dominates $(x, -x; A) = (y, -y; A) + (\epsilon, -\epsilon; A)$. By the strict version of monotonicity with respect to strong uncertainty-dominance, we conclude that $(y, -y; A) \succ (x, -x; A)$.*

4.3 The Ellsberg preferences and more uncertain source

Uncertainty dominance captures both hedging behavior and greater variability in outcomes. However, we have not yet addressed source dependence (see, e.g., Baillon et al., 2025), one of the crucial aspects of ambiguity. To compare gambles that depend on different sources, we introduce the following property. Formally, a source is an algebra of events. For simplicity, we focus on binary partitions of the state space (E, E^c), where E is a nonempty proper subset of S and $E^c = S \setminus E$. We say that (E, E^c) *dominates* (F, F^c) if the following condition holds for all payoffs $x > y$:

$$(x, y; E) \succcurlyeq (x, y; F), \quad \text{and} \quad (x, y; E^c) \succcurlyeq (x, y; F^c). \tag{15}$$

[10] That is, replacing weak with strict preference in the definition.

To illustrate this concept, consider the classic single-urn Ellsberg paradox. An urn contains 30 black balls and 60 red and white balls in unknown proportions. A bet on an event A pays \$1 if A occurs and \$0 otherwise. Let event E denote drawing a black ball, and event F denote drawing a red ball. The standard pattern observed in the Ellsberg experiment consists of a preference for betting on E over F, and on E^c over F^c. Hence, (E, E^c) dominates (F, F^c).

Theorem 11. *If (E, E^c) dominates (F, F^c), then the following holds for all $x > y$*

$$B^*(x, y; E) - B(x, y; E) \leq B^*(x, y; F) - B(x, y; F), \tag{16}$$

$$\text{and} \quad B^*_n(x, y; E) - B_n(x, y; E) \leq B^*_n(x, y; F) - B_n(x, y; F). \tag{17}$$

This again highlights that the WTA–WTP gap is an appropriate measure of UA induced by source preferences.

5 WTA-WTP disparity in the MUMP model

We illustrate our results using the multi-utility multi-prior (MUMP) model (see Galaabaatar and Karni, 2013; Hara and Riella, 2023; Borie, 2023). MUMP is more specific than our setting, yet general enough to capture preference imprecision and UA at the same time. We follow Hara and Riella (2023) and assume in this section that the outcome set is $X = [a, b]$ for some $a, b \in \mathbb{R}$ with $a < 0 < b$ and that all the discussed properties hold on X rather than on $\mathbb{R}$.[11]

Definition 9 (MUMP). *$\succcurlyeq$ on $\mathcal{F}$ has a MUMP representation if there exist a compact set $\mathcal{U}$ of continuous strictly increasing real-maps on X and a compact convex set Π^u for every $u \in \mathcal{U}$, of probability measures on S such that for each $f, g \in \mathcal{F}$,*

$$f \succcurlyeq g \iff \int_S u(f) d\mu \geq \int_S u(g) d\mu \quad \textit{for every } (\mu, u) \in \Phi. \tag{18}$$

[11]MUMP was first formulated in the framework of Anscombe and Aumann (1963), where acts are defined as $\Delta(X)^S$, with $\Delta(X)$ representing the set of probability measures on X and S a finite set of states. In this paper, we restate MUMP within the Savage (1954) framework, using only AA-acts with degenerate lotteries, i.e., Dirac delta measures from $\Delta(X)$.

where $\Phi = \{(\mu, u) : \; u \in \mathcal{U}, \; \mu \in \Pi^u\}$.

MUMP contains several important special cases. Single-utility multi-prior (SUMP model of Bewley uncertainty), arises if there is only one utility in the set $\mathcal{U}$.[12]. Multi-utility single-prior (MUSP) is when the set of priors Π contains only one element and $\Pi^u = \Pi$ for all $u \in \mathcal{U}$. Finally, the case with a single utility and a single prior, corresponds to the Subjective Expected Utility model.

We illustrate our results in the MUMP class. The buying and short-selling prices of f for a "model" $(\mu, u) \in \Phi$, denoted by $B_{\mu,u}(f)$ and $B^*_{\mu,u}(f)$, are implicitly defined by

$$\sum_{s \in S} \mu(s) u(f(s) - B_{\mu,u}(f)) = 0, \tag{19}$$

$$\sum_{s \in S} \mu(s) u(B^*_{\mu,u}(f) - f(s)) = 0. \tag{20}$$

Proposition 4. *Suppose* $\succcurlyeq$ *has a MUMP representation with the set of priors and utilities* Φ. *Then for any* $f \in \mathcal{F}$, *we have*

$$B(f) = \min\nolimits_{(\mu,u)\in\Phi} B_{\mu,u}(f), \qquad B_n(f) = \max\nolimits_{(\mu,u)\in\Phi} B_{\mu,u}(f),$$
$$B^*_n(f) = \min\nolimits_{(\mu,u)\in\Phi} B^*_{\mu,u}(f), \qquad B^*(f) = \max\nolimits_{(\mu,u)\in\Phi} B^*_{\mu,u}(f).$$

Proposition 4 shows that under MUMP, the boundary prices correspond to the most optimistic and most pessimistic values across all "models" in Φ. Note that by definition, $B(f)$ represents the maximum price the DM is willing to pay for f. Since MUMP requires that $f \succcurlyeq 0$ if and only if the subjective expected utility of f exceeds that of 0 for each model in Φ, it follows that $B(f)$ must be the minimum buying price across all models in Φ. A similar interpretation holds for the other three prices. Finally, the gaps between the respective prices (e.g., WTA-WTP) can be interpreted as the monetary measure of the size of the set Φ when sampled at prospect f. We now present two numerical examples. The first illustrates two ways of rationalizing a

[12]In the context of buying and short-selling prices, where one alternative is always a deterministic status quo, the SUMP model is equivalent to the two-fold multiplier concordant preferences model of Echenique et al. (2022).

given price data set, as well as the distinction between sure and strong UA.

Example 2. *Let $f = (10, 0; A)$ be a symmetric prospect, and consider an individual reporting the following indifference prices: $B(f) = 2.39, B_n(f) = 4.05, B^*_n(f) = 5.95, B^*(f) = 7.61$. The two UA decompositions are given by:*

$$5.22 \text{ (UA)} = 1.90 \text{ (sure UA)} + 3.32 \ (PI_f + PI_{-f}) \tag{21}$$

$$5.22 \text{ (UA)} = 3.56 \ (\text{strong } UA_f = \text{strong } UA_{-f}) + 1.66 \ (PI_f = PI_{-f}) \tag{22}$$

Let $a < 0 < b$. We assume that the preference relation $\succcurlyeq$ has a MUMP representation with the set of utilities $\mathcal{U}$ and sets of priors Π^u for each $u \in \mathcal{U}$. For given $\alpha, \lambda \in \mathbb{R}_{++}$, the utilities $u_{\alpha,\lambda} : [a, b] \to \mathbb{R}$ in $\mathcal{U}$ are given by:

$$u_{\alpha,\lambda}(x) = \begin{cases} x^\alpha & \text{for } x \geq 0, \\ -\lambda(-x)^\alpha & \text{for } x < 0. \end{cases}$$

We denote by Π^u_A the set of probabilities $\mu(A)$ for $\mu \in \Pi^u$. For a binary gamble $(x, y; A)$, straightforward calculations yield the indifference prices for each $u \in \mathcal{U}$ and prior $\theta \in \Pi^u_A$:

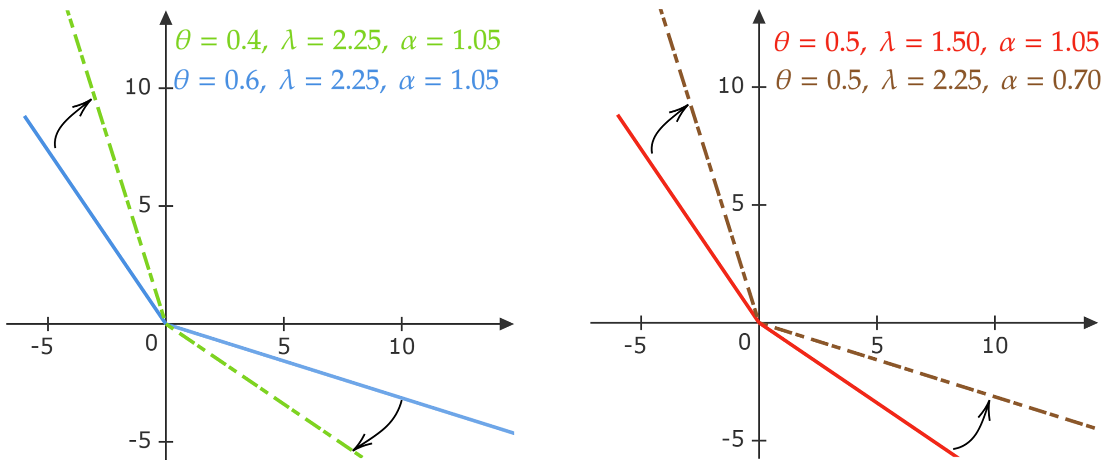
$$B_{\theta,\alpha,\lambda}(f) = p_{\theta,\alpha,\lambda}x + (1 - p_{\theta,\alpha,\lambda})y, \quad \text{where} \quad p_{\theta,\alpha,\lambda} = \frac{\theta^{1/\alpha}}{\theta^{1/\alpha} + ((1-\theta)\lambda)^{1/\alpha}},$$

$$B^*_{\theta,\alpha,\lambda}(f) = q_{\theta,\alpha,\lambda}x + (1 - q_{\theta,\alpha,\lambda})y, \quad \text{where} \quad q_{\theta,\alpha,\lambda} = \frac{(\theta\lambda)^{1/\alpha}}{(\theta\lambda)^{1/\alpha} + (1-\theta)^{1/\alpha}}.$$

Note that different θ's capture preference imprecision in belief, while different α's and λ's capture imprecision in taste. A MUMP model is specified by the set of triples $(\theta, \lambda, \alpha)$, which defines the utilities and priors in the set Φ. Consider two such models: M1 with $(\theta, \lambda, \alpha) \in \{(0.4, 2.25, 1.05), (0.6, 2.25, 1.05)\}$, and M2 with $(\theta, \lambda, \alpha) \in \{(0.5, 1.50, 1.05), (0.5, 2.25, 0.70)\}$. Note that M1 is a SUMP model, while M2 is a MUSP model. The indifference curves of the utilities at 0 in each of the models are graphically presented in Figure 5.

Table 2 shows that in M1, the WTA-WTP gap for each individual utility function is the same, equal to 3.56. In this case, the imprecision is entirely due to uncertainty

Figure 5: Indifference curves of the utilities in Model 1 (left panel) and Model 2 (right panel). The curves show the same UA, strong UA, and sure UA generated either by imprecision in belief or imprecision in taste.



about the prior, and equals $5.22 - 3.56 = 1.66$, *as captured by decomposition* (22). *In contrast, in M2, the maximal WTA-WTP gap for individual utility functions equals 5.22, capturing the whole UA, while the minimal gap is 1.91, reflecting sure UA. Here, the imprecision is solely due to uncertainty about taste, and equals* $5.22 - 1.91 = 3.31$, *as captured by decomposition* (21). *In summary, in this example strong UA is more appropriate for measuring the "sure" part of UA in the SUMP model, while sure UA is more relevant for the MUSP model. Without knowing the true model family, decomposition* (21) *provides an upper bound on preference imprecision, while decomposition* (22) *provides a lower bound.*

Table 2: The same UA, sure UA, and strong UA generated in two different models.

model	$(\theta, \lambda, \alpha)$'s	$B_{\theta,\lambda,\alpha}$	$B^*_{\theta,\lambda,\alpha}$	$B^*_{\theta,\lambda,\alpha} - B_{\theta,\lambda,\alpha}$	UA	sure UA	strong UA
M1	$(0.4, 2.25, 1.05)$	2.39	5.95	**3.56**	5.22	1.91	**3.56**
	$(0.6, 2.25, 1.05)$	4.05	7.61	**3.56**			
M2	$(0.5, 1.50, 1.05)$	4.05	5.95	**1.91**	**5.22**	**1.91**	3.56
	$(0.5, 2.25, 0.70)$	2.39	7.61	**5.22**			

Our second example is a MUSP model with utility functions based on Kőszegi and Rabin (2006) preferences as specified in O'Donoghue and Sprenger (2018). The only source of imprecision is the location of a reference point.

Example 3. *Given a reference point $r \in \mathbb{R}$ and two parameters $\eta, \lambda > 0$ let $u(\cdot|r) : \mathbb{R} \to \mathbb{R}$ be*

$$u_r(x) = \begin{cases} x + \eta(x - r) & \text{if } x \geq r, \\ x + \eta\lambda(x - r) & \text{if } x < r. \end{cases}$$

Let $f = (x, y; A)$ be a binary symmetric bet where $x > y$. Let $\eta, \lambda > 0$ be given and $\mu(A) = 0.5$. For $a, b \in \mathbb{R}$ such that $a < b$ we assume that $\mathcal{U} = \{u_r : \ r \in [a, b]\}$. Buying and short-selling prices of f for an individual utility u_r are given by:

$$B_r(f) = \frac{x + y + \eta(x - r) + \eta\lambda(y - r)}{2 + \eta + \eta\lambda} + \begin{cases} \frac{2\eta r}{2+\eta+\eta\lambda} & \text{if } r < 0, \\ \frac{2\eta\lambda r}{2+\eta+\eta\lambda} & \text{if } r \geq 0. \end{cases}$$

$$B_r^*(f) = \frac{x + y + \eta(y + r) + \eta\lambda(x + r)}{2 + \eta + \eta\lambda} - \begin{cases} \frac{2\eta r}{2+\eta+\eta\lambda} & \text{if } r < 0, \\ \frac{2\eta\lambda r}{2+\eta+\eta\lambda} & \text{if } r \geq 0. \end{cases}$$

We consider WTA and WTP as functions of r for $x = 200$, $y = -50$, $\eta = 2$, $\lambda = 2$ and $r \in [-50, 100]$. We thus have $B_r(f) = 43.75 + 0.25|r|$, $B_r^*(f) = 106.25 - 0.25|r|$, and $B^*(f) - B(f) = \max_{r \in [-50,100]}[B_r^*(f) - B_r(f)] = B_0^*(f) - B_0(f) = 62.5$, $B_n^*(f) - B_n(f) = \min_{r \in [-50,100]}[B_r^*(f) - B_r(f)] = B_{100}^*(f) - B_{100}(f) = 12.5$. Hence, the entire gap of 62.5 is divided into sure UA (12.5) and preference imprecision (50).

6 Discussion and concluding remarks

Cautious expected utility Cerreia-Vioglio et al. (2024) propose an explanation of the WTA–WTP gap based on caution. Their approach differs from ours in several respects. First, they discuss the WTA–WTP disparity in the context of the endowment effect and therefore treat WTA as the selling price of an initially owned object. However, under their assumption that the (stochastic) status quo serves as the reference point, there is no difference between the selling and short-selling prices. Second, our domain consists of prospects (Savage acts mapping states to payoffs), whereas their domain consists of lotteries over bundles. We can therefore model ambiguity, while their model captures trade-offs between goods in a risk setting. Third, our approach is model-independent,[13] whereas Cerreia-Vioglio et al. (2024) derive the existence of the WTA–WTP gap and loss aversion for risk from their (symmetric) cautious utility representation. By contrast, we *characterize* the WTA–WTP gap and loss aversion axiomatically. Finally, while Cerreia-Vioglio et al. (2024) show that loss aversion and the WTA–WTP gap are not necessarily related (neither implies the other, even under cautious expected utility), in our setting the WTA–WTP gap implies loss aversion, but the reverse implication requires additional structure and assumptions.

WTA-WTP disparity in the cautious completion of the MUMP model Assume $\succcurlyeq$ has a MUMP representation with the set of priors and utilities is Φ. We may consider a cautious completion of $\succcurlyeq$ denoted by $\succcurlyeq^*$ on $\mathcal{F}$ defined as follows: for any $f, g \in \mathcal{F}$, $f \succcurlyeq^* g \iff \min_{(\mu,u)\in\Phi} u^{-1}\left(\int_S u(f)d\mu\right) \geq \min_{(\mu,u)\in\Phi} u^{-1}\left(\int_S u(g)d\mu\right)$. Hara and Riella (2023)[14] suggests the following interpretation: $\succcurlyeq$ represents choices that can be made with certainty, while $\succcurlyeq^*$ represents forced choices that are made even if the DM is not confident. Under $\succcurlyeq^*$ we have the following observation for any $f \in \mathcal{F}$: $B(f) = B_n(f) = \min_{(\mu,u)\in\Phi} B_{\mu,u}(f)$ and $B^*(f) = B^*_n(f) = \max_{(\mu,u)\in\Phi} B^*_{\mu,u}(f)$. The above is a simple counterpart of Proposition 4 for $\succcurlyeq^*$.

Correlation between WTA and WTP and between the WTA–WTP gap and loss aversion. Recently, Chapman et al. (2023) show that WTA and WTP are uncorrelated and that the disparity between them is, at best, only weakly related to loss aversion. These findings challenge the view that loss aversion is the primary explanation for the WTA–WTP disparity. Our framework provides tools to re-examine

[13]For example, the definitions of short-selling and buying prices are robust to changes in reference-point determination rules (Lewandowski and Woźny, 2022).

[14]See also Gilboa et al. (2010).

these results using alternative measures of WTA and loss aversion. Such empirical analysis is conducted in a companion paper (Lewandowski et al., 2026). That paper replicates the finding of no correlation between WTA and WTP and, in addition, documents a positive correlation between the WTA–WTP gap, interpreted as uncertainty aversion (UA), and our measure of loss aversion, namely sure uncertainty aversion.

Decomposition and drivers of the WTA–WTP gap. The companion paper (Lewandowski et al., 2026) also investigates the decomposition of the WTA–WTP gap and its underlying drivers. Buying and no-buying prices (and analogously, short-selling and no-short-selling prices) are elicited using a modified multiple price list (MPL) procedure proposed by Cubitt et al. (2015) (see also Agranov and Ortoleva, 2025). Prices are listed in ascending order across rows, and for each price subjects choose among three options: *I certainly would buy (short-sell)*, *I am not sure*, and *I certainly would not buy (short-sell)*. The switching point away from the first option defines the buying (or short-selling) price, while the switching point away from the third option defines the no-buying (or no-short-selling) price.

Using these elicited boundary prices for symmetric prospects under risk, partial uncertainty, and full ignorance, the companion paper reports three main findings. First, the WTA–WTP gap is positive for the vast majority of subjects. Second, preference imprecision accounts for a substantial share of the gap, ranging from 34% to 59%, corresponding to the lower and upper bounds implied by the decompositions in (8), (9), and (7). Third, subjects cluster into three broad groups according to the primary driver of the WTA–WTP gap: (i) no preference imprecision, with the entire gap driven by sure or strong UA; (ii) a combination of sure or strong UA and preference imprecision; and (iii) predominantly preference imprecision. Many subjects in the third group exhibit negative sure UA (capturing sure uncertainty loving), yet display a positive overall UA due to substantial preference imprecision. We refer the reader to Lewandowski et al. (2026) for further results and discussion.

7 Proofs

Proof of Lemma 1 Take any $f \in \mathcal{F}$. We prove all statements for $B(f)$. The remaining cases are proved similar. We first show existence. If $f = \theta^*$ for some $\theta^* \in \mathbb{R}$ then by **B0**–**B1** $B(f) = \theta^* = \underline{f} = \bar{f}$. Assume that f is nonconstant and define: $\mathcal{B}(f) := \{\theta \in \mathbb{R} : f - \theta \succcurlyeq 0\}$, $A := \{g \in \mathcal{F} : g = f - \theta,\ \theta \in \mathbb{R}\}$, $A' := \{g \in \mathcal{F} : g = f - \theta,\ \theta \in \mathcal{B}(f)\}$. We first show that $\mathcal{B}(f)$ is nonempty. Indeed it contains $\underline{f}$: $f - \underline{f} \geq 0$ and $f \neq \underline{f}$, which, in view of **B1**, implies $f - \underline{f} \succ 0$. Hence, $\underline{f} \in \mathcal{B}(f)$. We now show that $\mathcal{B}(f)$ is bounded from above. Indeed since $f - \theta \leq 0$, $f \neq \theta$, for $\theta \geq \bar{f}$, so by **B1** $0 \succ f - \theta$ which implies that $f - \theta \not\succcurlyeq 0$. So $\mathcal{B}(f)$ does not contain any $\theta \geq \bar{f}$. We next show that $\mathcal{B}(f)$ is closed. A' is the intersection of A, which is closed, and nW, which is also closed by **B2**. So A' is also closed. Define a function $\gamma : \mathbb{R} \to \mathcal{F}$ by $\gamma(\theta) = f - \theta$. Note that γ is a continuous function. Hence a preimagine of any closed set is closed. Note that a preimage of A' is $\mathcal{B}(f)$, and since the former is closed, the latter must also be. We have shown that $\mathcal{B}(f)$ is a nonempty and closed set bounded from above. So $\mathcal{B}(f)$ contains its maximum, which proves that $B(f)$ exists. It is also unique by monotonicity.

We now prove that $B(f) \in [\underline{f}, \bar{f}]$. We have already shown that $\underline{f} \in \mathcal{B}(f)$ so by the definition of the latter $B(f) \geq \underline{f}$. Now observe that $f - \bar{f} \leq 0$, $f \neq \bar{f}$, so **B1** implies that $0 \succ f - \bar{f}$. On the other hand $f - B(f) \succcurlyeq 0$. By **B0**, $f - B(f) \succcurlyeq f - \bar{f}$. By **B1** we must have $\bar{f} \geq B(f)$ which finishes the proof of (i).

We now show $B_n(f) \geq B(f)$. By definition $f - B(f) \succcurlyeq 0$ and $0 \succcurlyeq f - B_n(f)$. So by **B0**, $f - B(f) \succcurlyeq f - B_n(f)$. By **B1** we have $B_n(f) \geq B(f)$.

We now prove the last statement. Suppose that for some prospect f one of the inequalities in (5) are strict, say $B_n(f) > B(f)$. To show that preferences are incomplete, it suffices to show that there is a pair of noncomparable prospects. Take $\theta \in \mathbb{R}$

such that $B_n(f) > \theta > B(f)$. By the definition of $B(f)$, $f - \theta \not\succcurlyeq 0$. By the definition of $B_n(f)$, $0 \not\succcurlyeq f - \theta$. So 0 and $f - \theta$ are not comparable and $\succcurlyeq$ is incomplete.

Proof of Lemma 2 We show that for $X \in \{B^*, B, B_n^*, B_n\}$ it holds: $X(f + \lambda) = X(f) + \lambda$ for any $\lambda \in \mathbb{R}$, $f \in \mathcal{F}$. We show it for $X = B$. The rest is analogous:

$$B(f + \lambda) = \max\{\theta \in \mathbb{R} : f + \lambda - \theta \succcurlyeq 0\} = \lambda + \max\{\theta \in \mathbb{R} : f - \theta \succcurlyeq 0\}.$$

Moreover, for all $f \in \mathcal{F}$, the following holds: $B(-f) = -B^*(f)$. Indeed:

$$\begin{aligned} -B(-f) &= -\max\{\theta \in \mathbb{R} : -f - \theta \succcurlyeq 0\} = \min\{-\theta \in \mathbb{R} : -\theta - f \succcurlyeq 0\} = \\ &= \min\{\theta' \in \mathbb{R} : \theta' - f \succcurlyeq 0\} = B^*(f). \end{aligned}$$

Hence $B^*(f) = -B(-f) = \theta - B(\theta - f)$ and thus the first equation of (6) holds. Similarly, the second equation holds because B_n is translation invariant and, for all $f \in \mathcal{F}$, $B_n(-f) = -B_n^*(f)$.

Proof of Theorem 1 Suppose that UA holds. By the definition of B, for any nonzero prospect f, $f - B(f) \succcurlyeq 0$. UA implies that $B(f) - f \not\succcurlyeq 0$, which in view of the definition of B^* implies that $B(f) < B^*(f)$. Now assume that $B^*(f) > B(f)$ for some nonzero prospect f such that $f \succcurlyeq 0$. We must prove that $-f \not\succcurlyeq 0$. By the definition of B and in view of the monotonicity of $\succcurlyeq$, we have $B(f) \geq 0$ and thus $B^*(f) > 0$. From the definition of B^*, we obtain that $-f \not\succcurlyeq 0$.

Proof of Theorem 3 We only prove the first part, as the second part follows similar reasoning. Suppose that the DM is imprecise with respect to f. Then there is a $\theta \in \mathbb{R}$ such that $f + \theta \not\succcurlyeq 0$ and $0 \not\succcurlyeq f + \theta$. By the definition of B, $-\theta > B(f)$. Similarly, by the definition of B_n, $B_n(f) > -\theta$. It follows that $B_n(f) > B(f)$. Similarly, if $B_n(f) > B(f)$ holds for some prospect f, take $\theta \in \mathbb{R}$ such that $B_n(f) > -\theta > B(f)$. By the definition of B and B_n, it holds: $f + \theta \not\succcurlyeq 0$ and $0 \not\succcurlyeq f + \theta$.

Proof of Theorem 4 We first prove the $\Rightarrow$ part. Assume that sure UA holds and suppose, by way of contradiction, that for some nonzero prospect f, $B^*(f) \leq B(f)$

or $B_n^*(f) < B_n(f)$. If $B^*(f) \leq B(f)$, then take $\theta \in \mathbb{R}$ such that $B^*(f) \leq \theta \leq B(f)$. By the definitions of B^* and B, this implies that $f - \theta \succcurlyeq 0$ and $\theta - f \succcurlyeq 0$, which implies that $0 \not\succ f - \theta$ and $0 \not\succ \theta - f$, a contradiction to sure UA. If $B_n^*(f) < B_n(f)$, then take $\theta \in \mathbb{R}$ such that $B_n^*(f) < \theta < B_n(f)$. By the definitions of B_n and B_n^*, we have $0 \not\succcurlyeq f - \theta$ and $0 \not\succcurlyeq \theta - f$. This implies $0 \not\succ f - \theta$ and $0 \not\succ \theta - f$, a contradiction to sure UA. This finishes this part of the proof.

We now prove the $\Leftarrow$ part. We assume that for any nonzero prospect f, $B^*(f) > B(f)$ and $B_n^*(f) \geq B_n(f)$. We take an arbitrary nonzero prospect f such that $0 \not\succ f$. This means that $0 \not\succcurlyeq f$ or $f \succcurlyeq 0$. If $0 \not\succcurlyeq f$, then, by the definition of B_n, $B_n(f) > 0$. Hence $B_n^*(f) > 0$ and $B^*(f) > 0$, by assumption. In view of the definitions of B_n^* and B^*, we obtain $0 \succcurlyeq -f$ and $-f \not\succcurlyeq 0$ which implies $0 \succ -f$. If $f \succcurlyeq 0$, then, by the definition of B, $B(f) \geq 0$, so, by assumption, $B^*(f) > 0$ and $B_n^*(f) \geq 0$. In view of the definitions of B^* and B_n^*, we obtain $-f \not\succcurlyeq 0$ and $0 \succcurlyeq -f$, which implies $0 \succ -f$.

Proof of Theorem 5 Suppose that strong UA holds and consider an arbitrary nonzero prospect f. By the definition of B, $f - B(f) \succcurlyeq 0$, which implies, by strong UA, $0 \succ B(f) - f$ which means $0 \succcurlyeq B(f) - f$ and $B(f) - f \not\succcurlyeq 0$. By the definition of B_n^* and B^*, these imply $B_n^*(f) \geq B(f)$ and $B^*(f) > B(f)$. As f is arbitrary, the same holds for $-f$ and hence, in view of Lemma 2 (applied for $\theta = 0$), we have $-B_n(f) \geq -B^*(f)$ and conclude that $B^*(f) \geq B_n(f)$. This finishes the proof of the first part of the proposition. To prove the converse, we take an arbitrary nonzero prospect f and assume that $B_n^*(f) \geq B(f)$ and $B^*(f) > B(f)$ holds. We also assume that $f \succcurlyeq 0$. This, by the definition of B implies that $B(f) \geq 0$. By our assumptions it implies that $B^*(f) > 0$ and $B_n^*(f) \geq 0$ and, by the definitions of B^* and B_n^*, implies that $0 \succcurlyeq -f$ and $-f \not\succcurlyeq 0$, which implies $0 \succ -f$. This completes the proof.

Proof of Proposition 1 Let A, A^c be symmetric events and let $x, y \in \mathbb{R}$. By the definition of B, $(x - B(x, y; A), y - B(x, y; A); A) \succcurlyeq 0$. Because A, A^c are symmetric, $(y - B(x, y; A), x - B(x, y; A); A) \succcurlyeq 0$. By the definition of B, $B(y, x; A) \geq B(x, y; A)$. Repeating the same argument with $B(y, x; A)$ instead of $B(x, y; A)$ shows that $B(x,y;A) \geq B(y,x;A)$, which together with the previous inequality yields $B(x,y;A) = B(y,x;A)$. Similarly, one can show that $B_n(x,y;A) = B_n(y,x;A)$. Applying Lemma 2 (with $\theta = x+y$) and the already proved part, we get $B^*(x,y;A) - B^*_n(x,y;A) = x+y-B(y,x;A)-x-y+B_n(y,x;A) = B_n(x,y;A)-B(x,y;A)$.

Proof of Proposition 2 Assume $\succcurlyeq$ has a SEU representation with utility u and probability μ. We first prove that if (A, A^c) are symmetric events then $\mu(A) = \frac{1}{2} = \mu(A^c)$. Indeed, by the definition of symmetric events, for any $x, y \in X$, $(x,y;A) \sim (y,x;A)$. By the definition of SEU, this is equivalent to $\mu(A)u(x) + (1-\mu(A))u(y) = \mu(A)u(y) + (1-\mu(A))u(x)$ or $(u(x)-u(y))(2\mu(A)-1) = 0$, and since u is strictly increasing, $\mu(A) = \frac{1}{2}$. We now prove that $\succcurlyeq$ is loss averse if and only if $u(x) < -u(-x)$ for all $x \in X \setminus \{0\}$. Take arbitrary symmetric events (A, A^c) and an arbitrary $x \in X \setminus \{0\}$. By the definition of LA, $0 \succ (x,-x;A)$, or equivalently $0 \succcurlyeq (x,-x;A)$ and $0 \not\preccurlyeq (x,-x;A)$. By SEU and the fact that $\mu(A) = \frac{1}{2}$, this is equivalent to $\frac{1}{2}u(x) + \frac{1}{2}u(-x) < u(0) = 0$ or $u(x) < -u(-x)$ for all $x \in X \setminus \{0\}$.

SEU preferences are complete. Hence, LA and not-LL are equivalent and so are different notions of uncertainty aversion: UA, strong UA and sure UA. In view of Theorem 6 (proved below), it suffices to show that LA implies UA. Assume $\succcurlyeq$ is loss averse and $f \succcurlyeq 0$ for some nonzero f. By SEU, $\int_S u(f)d\mu \geq 0$. By loss aversion, $u(x) < -u(-x)$ for all $x \in X \setminus \{0\}$ and hence $\int_S u(-f)d\mu < 0$. By SEU $-f \not\succcurlyeq 0$ and hence UA holds. The proof that $\succcurlyeq$ is uncertainty neutral if and only if u is odd follows similar logic and hence is omitted.

Proof of Theorem 6 Assume that $\succcurlyeq$ is surely uncertainty averse. It implies that for any nonzero prospect f, $0 \succ f$ or $0 \succ -f$. Take any $x \neq 0$ and a pair of symmetric events (A, A^c). Set $f = (x,-x;A)$. Then $-f = (-x,x;A)$ and by the definition of symmetric events $f \sim -f$. By transitivity (**B0**), $0 \succcurlyeq f \iff 0 \succcurlyeq -f$ and $f \not\succcurlyeq 0 \iff -f \not\succcurlyeq 0$. Hence $0 \succ f \iff 0 \succ -f$ and therefore $0 \succ f$ and $0 \succ -f$. Since $-f = (-x,x;A) = (x,-x;A^c)$, we have proved that $0 \succ (x,-x;A)$ and $0 \succ (x,-x;A^c)$. Because x and A were arbitrary, the proof of the first implication is completed. The proof of the second implication is similar. The only difference is that by transitivity, if $f \sim -f$, then $f \not\succcurlyeq 0 \iff -f \not\succcurlyeq 0$.

Proof of Theorem 7 We prove (i). First note that for any f, g we must have $f \not\succ g$ or $g \not\succ f$: otherwise $f \succ g$ and $g \succ f$, which by definition of $\succ$ would imply both $f \succcurlyeq g$ and $f \not\succcurlyeq g$, a contradiction. For any purely subjective act f, let $-f$ be the act assigning to each state the negative of the payoff assigned by f. Then either $f \not\prec -f$ or $f \not\succ -f$. Suppose $f \not\prec -f$. By SUA, $\frac{1}{2}f + \frac{1}{2}(-f) \succcurlyeq -f$. Recall that the expression $\frac{1}{2}f(s) + \frac{1}{2}(-f(s))$ denotes the constant act delivering the lottery $\frac{1}{2}f(s) + \frac{1}{2}(-f(s))$ in every state. Not-LL implies $\frac{1}{2}f(s) + \frac{1}{2}(-f(s)) \overline{\not\succcurlyeq} 0$ for all s, and by the additional monotonicity condition we obtain $\frac{1}{2}f + \frac{1}{2}(-f) \not\succcurlyeq 0$. We claim that $-f \not\succcurlyeq 0$: otherwise $-f \succcurlyeq 0$ would contradict the previous conclusion or transitivity. The symmetric case $f \not\succ -f$ yields $f \not\succcurlyeq 0$. Thus, for any f, either $f \not\succcurlyeq 0$ or $-f \not\succcurlyeq 0$, completing the proof of (i). The proof of (ii) is analogous and omitted.

Proof of Theorem 8 and 9 We prove only (i) of Theorem 8; proofs of part (ii) and Theorem 9 are analogous. For the "only if" direction, take any nonzero prospect f. By the definitions of B_1 and B_1^*, agent 1 prefers both $f - B_1(f)$ and $B_1^*(f) - f$ to the status quo. Let $\epsilon := B_1^*(f) - B_1(f)$. If agent 1 is more uncertainty averse than agent 2, then there exists $\delta \in \mathbb{R}$ such that $f - B_1(f) - \delta \succcurlyeq_2 0$ and $\delta + B_1^*(f) - f \succcurlyeq_2 0$. By the definitions of B_2 and B_2^*, this implies $B_1(f) + \delta \leq B_2(f)$ and $\delta + B_1^*(f) \geq B_2^*(f)$. Hence $B_1^*(f) - B_1(f) \geq B_2^*(f) - B_2(f)$. Since f was arbitrary, this completes the "only if" part. For the "if" part, assume the antecedent. We must show that agent 1 is more uncertainty averse than agent 2. Take any f and $\epsilon \in \mathbb{R}$ such that $f \succcurlyeq_1 0$ and $\epsilon - f \succcurlyeq_1 0$. By the definitions of B_1, B_1^*, we have $B_1(f) \geq 0$, $B_1^*(f) \leq \epsilon$, hence $B_1^*(f) - B_1(f) \leq \epsilon$. By assumption, $B_2^*(f) - B_2(f) \leq \epsilon$ $(*)$. By the definition of B_2, $f - B_2(f) \succcurlyeq_2 0$. Let $\delta := B_2(f)$, so $f - \delta \succcurlyeq_2 0$. From $(*)$, $B_2^*(f) = B_2(f) + (B_2^*(f) - B_2(f)) \leq \delta + \epsilon$, and by the definition of B_2^* this means $\delta + \epsilon - f \succcurlyeq_2 0$. Since f was arbitrary, the proof is complete.

Proof of Theorem 2 We first prove the "only if" part. Take an arbitrary prospect f. For an UA neutral DM there is a unique scalar θ^* such that $f - \theta^* \succcurlyeq 0$ and $\theta^* - f \succcurlyeq 0$. Note that for all $\theta \geq \theta^*$, $\theta - f \succcurlyeq 0$ by monotonicity (B1). By uniqueness of θ^*, it follows that $f - \theta \not\succcurlyeq 0$. Hence, by the definition of B, $\theta^* = B(f)$. Similarly, for all $\theta \leq \theta^*$, $f - \theta \succcurlyeq 0$ and $\theta - f \not\succcurlyeq 0$, and hence, in view of the definition of B^*, $\theta^* = B^*(f)$. So $B^*(f) - B(f) = 0$. We now prove the converse. Take an arbitrary prospect f. Define $\theta^* := B(f)$. By assumption, $\theta^* = B^*(f)$. By Lemma 1, such θ^* is unique. By the definition of B and B^*, it follows that $f - \theta^* \succcurlyeq 0$ and $\theta^* - f \succcurlyeq 0$. Furthermore, there is no other θ satisfying these conditions, because for all $\theta < \theta^*$, $\theta - f \not\succcurlyeq 0$ and for all $\theta > \theta^*$, $f - \theta \not\succcurlyeq 0$.

Proof of Theorem 10 We need to prove that each of the two, (i) and (ii), implies that (13)–(14) hold whenever g is more uncertain than f. As the proofs in the two cases, (i) and (ii), are very similar, we will proceed with one proof and highlight the differences in the two cases. Take two prospects f, g such that g is more uncertain than f, i.e., $h := g - f$ is a nonconstant prospect comonotonic with f. We observe that, for any $\theta \in \mathbb{R}$, the prospect $g - B_n(f) - \theta$ is more uncertain than $f - B_n(f)$, and their difference is given by $g - B_n(f) - \theta - (f - B_n(f)) = h - \theta$. Similarly, since $-g$ is more uncertain than $-f$ whenever g is more uncertain than f, we note that, for any $\theta \in \mathbb{R}$, the prospect $B^*_n(f) + \theta - g$ is more uncertain than $B^*_n(f) - f$, and their difference is $B^*_n(f) + \theta - g - (B^*_n(f) - f) = \theta - h$. Hence, for θ in the set

$$\{\theta \in \mathbb{R} : \; h - \theta \nsucc 0 \;\wedge\; \theta - h \nsucc 0\}, \tag{23}$$

prospect $f - B_n(f)$ uncertainty-dominates $g - B_n(f) - \theta$ and prospect $B^*_n(f) - f$ uncertainty-dominates $B^*_n(f) + \theta - g$. Similarly, for θ in the set

$$\{\theta \in \mathbb{R} : \; h - \theta \not\succcurlyeq 0 \;\wedge\; \theta - h \not\succcurlyeq 0\}, \tag{24}$$

prospect $f - B_n(f)$ strongly uncertainty-dominates $g - B_n(f) - \theta$ and prospect $B^*_n(f) - f$ strongly uncertainty-dominates $B^*_n(f) + \theta - g$. So, for θ in the corresponding set and monotonicity with respect to the corresponding dominance, uncertainty-dominance in the case of (i) and strong-uncertainty dominance in the case of (ii), would imply $f - B_n(f) \succcurlyeq g - B_n(f) - \theta$ and $B_n^*(f) - f \succcurlyeq B_n^*(f) + \theta - g$, which in view of the definitions of $B_n(f)$ and $B_n^*(f)$ as well as transitivity of $\succcurlyeq$, yields $0 \succcurlyeq g - B_n(f) - \theta$ and $0 \succcurlyeq B_n^*(f) + \theta - g$. By the definitions of $B_n(g)$ and $B_n^*(g)$, we get $B_n(g) \leq B_n(f) + \theta$ and $B_n^*(g) \geq B_n^*(f) + \theta$, or, after combining the two inequalities, $B_n^*(g) - B_n(g) \geq B_n^*(f) - B_n(f)$. So, in order to prove that (14) is implied by (i), respectively (ii), we need to show that the set defined by (23), respectively by (24), is nonempty.

Similarly, we observe that for any $\theta \in \mathbb{R}$, prospect $g - B(g)$ is more uncertain than $f + \theta - B(g)$ and their difference is given by $g - B(g) - (f + \theta - B(g)) = h - \theta$. Moreover, prospect $B^*(g) - g$ is more uncertain than $B^*(g) - \theta - f$ and their difference is $B^*(g) - g - (B^*(g) - \theta - f) = \theta - h$. So for θ in the set defined by (23), prospect $f + \theta - B(g)$ uncertainty-dominates $g - B(g)$ and prospect $B^*(g) - \theta - f$ uncertainty-dominates $B^*(g) - g$. Similarly, for θ in the set defined by (24), prospect $f + \theta - B(g)$ strongly uncertainty-dominates $g - B(g)$ and prospect $B^*(g) - \theta - f$ strongly uncertainty-dominates $B^*(g) - g$. So, for θ in the corresponding set and monotonicity with respect to the corresponding dominance, uncertainty-dominance in the case of (i) and strong-uncertainty dominance in the case of (ii), would imply $f + \theta - B(g) \succcurlyeq g - B(g)$ and $B^*(g) - \theta - f \succcurlyeq B^*(g) - g$, which in view of the definitions of $B(f)$ and $B^*(f)$ as well as transitivity of $\succcurlyeq$, yields $f + \theta - B(g) \succcurlyeq 0$ and $B^*(g) - \theta - f \succcurlyeq 0$. By the definitions of $B(g)$ and $B^*(g)$, we would thus get $B(f) \geq B(g) - \theta$ and $B^*(f) \leq B^*(g) - \theta$, or, after combining the two inequalities, $B^*(g) - B(g) \geq B^*(f) - B(f)$.

So, in order to prove that (13) is implied by (i), respectively (ii), we need to show that the set (23), respectively (24), is nonempty. In the case (i), $\succcurlyeq$ satisfies sure UA. Theorem 4 implies $B_n^*(h) \geq B_n(h)$ and so, there is $\theta \in \mathbb{R}$ such that $B_n(h) \leq \theta \leq B_n^*(h)$. By the definitions of B_n^* and B_n, $0 \succcurlyeq h - \theta$ and $0 \succcurlyeq \theta - h$, and hence we have $h - \theta \not\succcurlyeq 0$ and $\theta - h \not\succcurlyeq 0$. This proves that the set of defined by (23) is nonempty.

In the case (ii), $\succcurlyeq$ satisfies UA. Theorem 1 implies that $B^*(h) > B(h)$ and so, there is $\theta \in \mathbb{R}$ such that $B(h) < \theta < B^*(h)$. By the definitions of B^* and B, $h - \theta \not\succcurlyeq 0$ and $\theta - h \not\succcurlyeq 0$. This proves that the set defined by (24) is nonempty and finishes the proof.

Proof of Theorem 11 Take arbitrary two payoffs $x > y$ and events E, F such that (E, E^c) dominates (F, F^c). Let $B_1 := B(x, y; F)$. By the definition of B and in view of the dominance, $(x - B_1, y - B_1; E) \succcurlyeq (x - B_1, y - B_1; F) \succcurlyeq 0$. By transitivity, $(x - B_1, y - B_1; E) \succcurlyeq 0$. Hence, by the definition of B, we have $B(x, y; E) \geq B_1 = B(x, y; F)$ (*). Similar argument yields $B(x, y; E^c) \geq B_1 = B(x, y; F^c)$. By Lemma 2 where $\theta = x + y$, we have $\theta - B^*(y, x; E^c) \geq \theta - B^*(y, x; F^c)$ or $B^*(x, y; F) \geq B^*(x, y; E)$ (**). Combining (*) and (**) gives: $B^*(x, y; E) - B(x, y; E) \leq B^*(x, y; F) - B(x, y; F)$, which proves the first part of the Theorem.
Similarly, let $B_{n1} := B_n(x, y; E)$. By the definition of B_n and in view of the dominance, $0 \succcurlyeq (x - B_{n1}, y - B_{n1}; E) \succcurlyeq (x - B_{n1}, y - B_{n1}; F)$. By transitivity, $0 \succcurlyeq (x - B_{n1}, y - B_{n1}; F)$, which, taking into account the definition of B_n gives $B_n(x, y; F) \leq B_{n1} = B_n(x, y; E)$ (*). Similar argument yields $B_n(x, y; F^c) \leq B_{n1} = B_n(x, y; E^c)$. Making use of Lemma 2 with $\theta = x + y$, we get $B_n^*(x, y; E) \leq B_n^*(x, y; F)$ (**). Combining (*) and (**) gives: $B_n^*(x, y; E) - B_n(x, y; E) \leq B_n^*(x, y; F) - B_n(x, y; F)$, which completes the proof.

Proof of Proposition 4 We only prove it for B as the rest is similar. Take an arbitrary $f \in \mathcal{F}$. We can rewrite the definition of B as follows

$$B(f) = \max \left\{ \theta \in \mathbb{R} : \sum_{s \in S} \mu(s) u(f(s) - \theta) \geq 0 \quad \text{for all } (\mu, u) \in \Phi \right\}. \tag{25}$$

We will prove that $B(f) = \hat{\theta} := \min_{(\mu,u) \in \Phi} B_{\mu,u}(f)$. Note that

$$\sum_{s \in S} \mu(s) u(f(s) - \hat{\theta}) \geq 0 \quad \text{for all } (\mu, u) \in \Phi. \tag{26}$$

Hence, by (25), $B(f) \geq \hat{\theta}$. Suppose that $\theta' > \hat{\theta}$ and let

$$(\mu^*, u^*) := \arg \min_{(\mu,u)\in\Phi} B_{\mu,u}(f). \tag{27}$$

Then, by monotonicity $\sum_{s\in S} \mu^*(s)u^*(f(s) - \theta') < 0$, but this, in view of (25), implies that $\theta' \neq B(f)$. So it must be that $B(f) = \hat{\theta}$.

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