Rational Asset Pricing Bubbles with Collateral and Short-Sale Constraints

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Abstract: In this paper we develop a theory of rational asset pricing bubbles under financial constraints. The analysis is bottom-up. We begin with the frictionless benchmark: using Luenberger (1969)'s optimization approach, we derive a constructive proof of the Farkas Lemma and establish the classical Fundamental Theorem of Asset Pricing. We then extend this framework to economies with restricted portfolio trading by introducing a notion of no-unlimited-arbitrage under financial constraints. Our main pricing result shows that linear financial constraints preserve the portfolio independence of asset prices: security prices admit a decomposition into standard state-price terms and constraint-shadow-price adjustments that depend only on constraint parameters, not on individual portfolio choices. Building on this foundation, we characterize complete and incomplete financial market structures under financial constraints –in particular, we establish when the constrained pricing formula yields a unique set of state prices and when it does not, extending the classical completeness criterion to the constrained setting. Finally, we use these results to address two central concepts in the rational bubbles debate: the notions of the Fundamental Value of a security and Rational Bubble Pricing, both defined and characterized under financial constraints.