Technological changes, campaign spending and polarization

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Abstract

We focus on recent technological changes and search for possible explanations behind the well documented simultaneous increase in campaign spending and polarization. In our model, some voters are ideological and vote based on policy proposals (*à la Downs*), while others are impressionable and vote based on costly campaign spending (*à la Tullock*). If the distribution of voters between types is endogenous and depends on parties' platform choices, our results show that a) an increase in the effectiveness of electoral advertising or the share of impressionable voters, surely increases polarization and may also increase electoral spending, while b) a decrease in the cost of electoral advertising does not affect neither polarization nor electoral spending.

Keywords: *electoral competition, campaign spending, impressionable voters, semiorder lexicographic preferences*
JEL codes: *D72.*

1 Introduction

A well documented fact in US politics is the simultaneous increase of campaign spending and polarization. Updated measures indicate that polarization in 2015 was at the highest level since the era of Reconstruction, with this trend being well documented since earlier work by Poole and Rosenthal (1984).[1] Also campaign spending, with the exception of the 2016 presidential election, has been steadily increasing since 1960 with the increase in the 2008 and 2012 elections been noteworthy.[2] In this paper, we provide a new theoretical explanation for the occurrence of these phenomena by linking campaign spending

[1] See http://pooleandrosenthal.com/political_polarization_2015.htm.
[2] See https://www.fec.gov/data/elections/president/2016/.

and polarization with recent technological advances and changes in the management of electoral campaigns.

Focusing on technology is relevant because while it's still gaining importance (think of Obama's campaigns, the use of big data, and social media as recent examples (Nickerson and Rogers, 2014)), research so far has not drawn conclusive evidence on its effects on electoral competition (Herrera et al., 2008). Of course, what we today observe is probably just another snapshot in the history of political communication, since changes in campaign management have been gradually occurring during the last decades. Starting with the introduction of nationwide TV in the 1960s party centred non-professional campaigns were abandoned. Instead parties had to rely on professional campaign management due to the emergence of multiple channels and 24/7 news coverage and the internet revolution in the mid 1990s (Norris, 2000; Sabato, 1981).

In line with changes in technology and campaign management, we present a theory where technology affects a) the effectiveness of electoral campaigns –e.g., due to the professionalization of campaign management and advanced targeting technologies, b) the costs of electoral campaigns –e.g., due to the possibility of reaching larger masses at a low marginal cost, and c) the electorate's political awareness –e.g., due to the presence of various sources of information. As our results show, an increase in the effectiveness of electoral campaigns or a drop in the electorate's awareness increase polarization and may also increase campaign spending. On the contrary, changes in the marginal cost of campaigns can not explain changes neither in polarization nor campaign spending.

We analyze the effects of technological changes on campaign spending and polarization by combining the two seminal models of Downs (1957) and Tullock (1980). In particular, we assume that two office motivated parties first choose their electoral platforms and then decide upon the optimal level of costly campaign spending. Voters can be either impressionable or ideological as in the seminal papers by Baron (1994); Grossman and Helpman (1996). As common in this literature, non-impressionable voters, vote on the basis of their ideology and support the party that proposes the platform closest to their bliss point. Hence, parties compete for a share of non-impressionable voters as if they were competing in a Downsian model of electoral competition. On the contrary, impressionable voters are swayed towards one party or the other through costly electoral spending. Given each party's campaign spending, the effectiveness of the latter determines the fraction of impressionable voters that supports each of them.

Differing from previous literature, we endogenize the division of voters across impressionable and ideological. Such division depends on the differentiation between the proposed platforms, with the fraction of ideological voters assumed increasing in polarization. This assumption captures the idea that the more diverse platforms are, more voters vote based on their ideological preferences since platforms become "salient". On the contrary, when parties' platforms are similar, voters may have a "hard" time or little interest in distinguishing them, and turn to electoral spending that determines (probabilistically) their voting behavior. Alas intuitive, the way to formalize this behavior is that voters' preferences are described by a lexicographic semiorder (see Luce 1956; Tversky 1969; Manzini and Mariotti 2012; Rubinstein 1988; Leland 1994), consistent with the notion of the "just noticeable difference".[3] With such preferences, each individual chooses which party to support on the basis of platforms, but only if those are different "enough" (i.e., above a certain threshold). If the platforms are not sufficiently different, then the voter is influenced exclusively by parties' campaign spending.

The above sketched model encompasses the effect of technological changes on electoral competition through three distinct and non-mutually exclusive channels. The two first reflect the way campaigns for impressionable -*Tullock*- voters are conducted and how technology and changes in campaign management affect a) the *effectiveness*, and b) the *cost* of electoral campaigns. One could with reasonable confidence argue that recent technological advances have increased the *electoral effectiveness* since campaigns can be better targeted, and have decreased the *marginal cost of campaigns* given the possibility of reaching large masses.

The third channel captures how technology affects electoral competition through the endogenous division of voters to impressionable and ideological and can have evolved either way. One could for instance argue that for a given level of polarization, more voters have nowadays access to information about parties' platforms than before and therefore vote on an ideological basis. But also technology may affect the endogenous division of voters reversely due to "media malaise". If the rise of media is associated with a larger mistrust of politicians and disenchantment with politics, technology may lead to less interest in casting a vote on the basis of ideological preferences (Norris, 2000; Newton, 1999). Without taking any stance on how this division has evolved, we show how electoral competition is affected by the *conversion rate* at which impressionable voters become ideological as polarization increases.

Our setup proves relatively tractable. In contrast to previous contributions with endogenous platforms and electoral spending –that we later discuss, our model: i) has a unique Nash equilibrium in pure strategies when parties are symmetric, ii) can be solved allowing heterogeneity in parties' marginal cost of campaigning, and iii) can be solved for asymmetric distributions of voters' ideology. But probably more importantly, in equilibrium, several comparative statics arise regarding the effect of technological advances on

[3]In experimental psychology, the Weber-Fechner law remarks that the "just noticeable difference" is not necessarily influenced by the physiological but rather by psychological factors. The law states that the just-noticeable difference is increasing in the absolute level of the subtracts (Falmagne, 2002).

electoral competition.

Consider first an increase in the *effectiveness* of electoral spending. Since every dollar spent on campaigns leads to higher returns, parties have incentives to symmetrically increase their advertising spending. As a consequence, parties get involved in a rat race, increasing their electoral spending to attract impressionable voters without effectively improving their prospects of winning the election (spending effect). To mitigate such increase in electoral spending, parties have incentives to polarize their platforms and reduce the number of impressionable voters and hence their spending (polarization effect). If the spending effect dominates, electoral spending and polarization increase simultaneously and can explain the observed trends in the US politics. If the polarization effect dominates, an increase in electoral effectiveness is overcompensated by an increase in polarization and electoral spending decreases. Actually, the effectiveness of electoral spending proves crucial if one wants to explain the simultaneous increase in campaign spending and polarization in terms of campaign management. A decrease in the marginal *costs* of running a campaign does not affect neither polarization nor total spending. While a decrease in the marginal cost of campaigns leads to more advertising, the lower marginal cost of the latter leaves total campaign spending and hence polarization unaffected.

Finally, the *conversion rate* at which impressionable voters become ideological when polarization increases affects both campaign spending and polarization in a similar manner as a change in the effectiveness. An increase in this rate, for example because of easier access to information and hence less need to vote on the basis of campaigns, has the same effect as a decrease in the effectiveness of campaign spending. Since competition for impressionable voters becomes less fierce, parties propose less polarized platforms. Hence, if one wants to explain the recent simultaneous increase in polarization and spending through this channel, then either a) this rate has been decreasing, meaning that voters become more disengaged and care less about distinguishing parties, or b) while this rate has been increasing given for example more sources of information, the effect of the latter on electoral competition has been completely washed out by the increase in the effectiveness of electoral spending.

1.1 Related Literature

In terms of results, our work complements existing models of platform choice and advertising strategies. The closest paper in terms of research question and methodology is by Herrera et al. (2008) who explicitly model changes in the targeting effectiveness and its effect on polarization and spending. Contrary to us, they show that an improved campaign technology increases campaign spending while it reduces polarization. In our model, an improved campaign technology increases polarization and this is because polarization provides exactly opposite dynamics in the two models. While in our model polarization softens competition in the campaign spending stage (as the valence accumulation literature), in their model it has the exact opposite effect by strengthening such competition. Prummer (2018) focuses on changes in targeting technology and fragmentation of media networks as determinants of polarization. Moving away from a targeting story, Rivas (2017) provides an alternative justification for the simultaneous increase in polarization and campaign spending in a model where the latter is financed through lobbies. This occurs when elections become less salient, and therefore voters put less weight on policies and focus more on campaigns.

The structure of our model, where parties first choose platforms and then spending, resembles existing models of endogenous valence (e.g., Ashworth and Bueno de Mesquita (2009); Zakharov (2009); Carrillo and Castanheira (2008); Iaryczower and Mattozzi (2013) among others). In that literature, voters typically have additive separable preferences over platforms and valence (i.e., campaign spending). In the closest work to ours (Ashworth and Bueno de Mesquita, 2009; Zakharov, 2009), platform diversification softens the competition in the valence accumulation stage.[4] These dynamics are exactly the ones presented in our model through the endogenous division of voters to ideological and impressionable. Our model, however, proves more tractable and permits the analysis of the effects of technological changes on electoral competition. In contrast to Ashworth and Bueno de Mesquita (2009), our model under symmetry admits a pure strategy equilibrium in the campaign stage setting and does not require that voters' ideologies are uniformly distributed. A non-uniform distribution is also permitted in Zakharov (2009) but only when focusing on local Nash equilibria. In contrast, we are able to characterize Nash equilibria in pure strategies for a general distribution of voters' ideology (symmetric and log-concave) and perform relevant comparative statics. Also in contrast to the previous models, we are also able to solve the model and obtain results when parties have heterogeneous campaign costs, for example due to an incumbency advantage (Meirowitz, 2008; Pastine and Pastine, 2012).

Summing up, one could think of our model as one of endogenous valence where rather than additive separable preferences over platforms and campaigns, individuals have semiorder lexicogrpahic preferences (see Luce 1956; Tversky 1969; Manzini and Mariotti 2012; Rubinstein 1988; Leland 1994). Notice that similar considerations have been gaining relevance in economics through the idea of context-dependent choice. A prominent and close example are the salience models in which decision makers overweight attributes that exhibit greater differences in the available choice set (Bordalo et al. 2012, 2013a,b,

[4]The opposite effect may occur when parties have ideological motives (Epstein and Nitzan, 2004; Cardona et al., 2018) or there is uncertainty regarding the valence investment (Carrillo and Castanheira, 2008).

2015; Bushong et al. 2015; Kőszegi and Szeidl 2012). Semiorder lexicographic preferences as the ones assumed here can be seen as a particular case of salience in which: a) only the difference in one attribute (policy platforms) is relevant to assign the weight of each dimension, and b) individual weights on each attribute are discrete and take value 0 or 1. In aggregate terms, the effect of our assumption is similar to the one of salience at the individual level: the smaller the distance between platforms, the smaller is their weight on the final outcome of the electoral competition. Callander and Wilson (2006) and Nunnari and Zápal (2017) introduce such context-dependent preferences in political economy models. In the former, the utility of voting depends not only on the direct benefits of turnout but also on the context, i.e., the candidates' polarization. In the latter, the authors provide a model of "focusing", where voters attention is captured more on the issues that candidates' proposals differ more. Similarly, Amorós and Puy (2013); Aragonés et al. (2015); Denter (2017) focus on electoral competition models when parties have the ability to affect the relative salience of different issues or dimensions via their strategic actions (e.g., allocation of time or effort).

Finally, our model contributes in the contest theory literature (see Corchón (2007); Konrad (2009) for surveys) since parties compete for a share of impressionable voters as if they were competing in a Tullock contest (with the "noise" of the latter capturing the effectiveness of electoral spending). Notice that the value of the "prize" (of the contest) allocated based on campaign spending is endogenous and depends on platform selection. Consequently, platform selection turns crucial since by fixing closer platforms parties not only attract more voters from their competitor as in any Downsian model, but also increase the share of Tullock voters for which rent-dissipation arises and hence intensify the competition in the campaign stage (Nitzan, 1994; Tullock, 1980). In terms of results, the endogenous value of the prize provides a result in contrast to most of the contest literature since campaign spending need not be monotonically increasing in the noise of the contest success function.

2 Model

Let two political parties $i \in \{L, R\}$ first propose (and commit to) platforms x_i in the policy space $X = [0, 1]$ and then choose the level of campaign advertising $e_i \geq 0$. Without loss of generality, we assume $x_L \leq x_R$. Let $S_i(x_L, x_R, e_L, e_R)$ be the vote share for party i and $c_i(e_i) = \mu_i e_i$ the cost of advertising, with $\mu_i > 0$ denoting the constant marginal cost of advertising. Without loss of generality, we assume $\mu_L \leq \mu_R$. Parties' are office motivated with payoffs $\Pi_i = S_i(x_L, x_R, e_L, e_R) - c_i(e_i)$.[5]

Voters have a preferred policy x drawn from distribution $G(x)$ with corresponding density $g(x)$ symmetric and log-concave (i.e., $(\ln g(x))'' \leq 0$), with full support in X.[6] Independent of their ideal policy some voters are ideological and some are impressionable. The ideological citizens vote sincerely for the party whose proposed platform is closer to them and split their vote in case of identical proposed platforms (*à la Downs*). The utility of a voter with ideology x that votes for party i is $u_x(i) = -|x - x_i|$.[7]

The impressionable citizens' vote depends only on campaign spending. In particular, we assume that given parties' advertising, the probability that an impressionable citizen votes for party i is determined *à la Tullock* and hence equal to $e_i^\eta/(e_L^\eta + e_R^\eta)$, where parameter $\eta \geq 0$ captures the *effectiveness of electoral campaigns*.[8]. In the extreme case where $\eta = 0$ the impressionable voters split equally across the two parties. However, as η increases the allocation of impressionable voters across parties becomes more responsive to campaign spending. Impressionable voters voting on the basis of persuasive campaign spending is a standard assumption in this literature (see for example the seminal papers by Baron (1994); Grossman and Helpman (1996) and a large literature thereafter). The specific proposed function is the seminal contest success function introduced by Tullock (1980). This function is extensively used in the literature and apart from tractability also satisfies relevant axiomatic properties (Skaperdas, 1996) and can be micro-founded in a reasonable manner (also in our setting, see the Appendix).

Let $y = x_R - x_L \in X$ be the platforms' polarization, $F(y)$ a continuous cumulative distribution function, log-concave (i.e., $(\ln F(y))'' \leq 0$), with corresponding density $f(y)$ with full support in X. The share of ideological voters is $F(y)$, and therefore the share of impressionable voters is $1 - F(y)$.

The timing of the game is as follows: At $t = 1$, the political parties simultaneously choose the political platforms that maximizes their payoff. At $t = 2$, having observed the platforms choices and the share of impressionable voters determined by the polarization, parties choose the advertising levels. Finally, at $t = 3$, voters vote. Given the nature of our game, we focus on subgame perfect Nash equilibria in pure strategies.

[5] Here each party's objective is to maximize its vote share net of the campaign costs. Alternatively, $S_i(x_L, x_R, e_L, e_R)$ can also be interpreted as the probability of winning by assuming parties' uncertainty on voters' ideology as in Aragonès and Xefteris (2017).

[6] Under symmetric campaign costs, our results can be extended to asymmetric distributions of ideology (available upon request).

[7] The assumption of a particular distance function is made without loss of generality.

[8] Here the campaign contest for impressionable voters is resolved via Tullock's ratio-form CSF that facilitates the comparative statics of our model. In the symmetric case, our results would be identical if we considered the difference-form CSF proposed by Alcalde and Dahm (2007), the tractable noise CSF proposed by Amegashie (2006) or the relative-difference CSF by Beviá and Corchón (2015) under the parameter restrictions proposed by Balart et al. (2017).

3 Results

Given the described game, let $\bar{x} = \frac{x_L + x_R}{2}$ be the indifferent ideological voter for $x_L \neq x_R$. Ideological voters with $x_i \leq \bar{x}$ vote for L, while the remaining ones vote for R. Thus, party L obtains a share $S^L_{Idl} = G(\bar{x})$ of the ideological voters and party R, $S^R_{Idl} = 1 - G(\bar{x})$. If $x_L = x_R$, then $S^R_{Idl} = S^L_{Idl} = \frac{1}{2}$. Given that the individual probability that an impressionable citizen votes for party i is $\frac{e_i^\eta}{e_L^\eta + e_R^\eta}$, the expected share of impressionable votes to party i is $S^i_{Imp} = \frac{e_i^\eta}{e_L^\eta + e_R^\eta}$ for party i. Hence, the expected vote share obtained by the parties can be then written as a weighted average of the previous two:

$$S_i(x_L, x_R, e_L, e_R) = F(y) S^i_{Idl}(x_L, x_R) + (1 - F(y)) S^i_{Imp}(e_L, e_R). \tag{1}$$

This expression clearly highlights the effect that platform choices have in our game. First, there is a direct effect on the vote share of the ideological voters (via $S^i_{Idl}(x_L, x_R)$). Second, there is also an indirect effect in terms of the ideological-impressionable composition of the electorate (via $F(y)$). As common in Downsian type models, convergence by one party towards the proposal of the other party increases the share of ideological voters via the relocation of the indifferent voter. However, the indirect effect leads to an increase in the amount of impressionable voters and hence a tougher competition in the advertising stage. Hence, platforms' choice is a non-trivial task and the equilibrium levels of polarization will depend on the parameters of our model.

3.1 Symmetric parties

For illustrative purposes, and to highlight our main results in the simplest framework, we first pay attention to the case where parties have identical marginal costs, i.e., $\mu_A = \mu_B = \mu$. Recall that voters' behavior is essentially parametric and hence the last stage in our backward induction reasoning is the choice of advertising. Equilibrium advertising can be solved as effort in a Tullock contest with symmetric players, in which the share of impressionable citizens is the prize of winning. Hence, in equilibrium each party is expected to obtain half of the impressionable votes. In particular, following Corchón (2007), Konrad (2009) or Nti (1999), the equilibrium in this stage is described in the lemma below

Lemma 1. *For all $\eta \leq 2$ there exists a unique Nash equilibrium in the campaign stage and advertising is given by $e_i^*(x_L, x_R) = (1 - F(x_R - x_L)) \frac{\eta}{4\mu}$, for all i.*

All proofs appear in the Appendix.

Our first Lemma draws from previous results in the contest theory literature. It characterizes the equilibrium advertising levels while stating a condition on the campaigns' effectiveness η such that an equilibrium in pure strategies exists. If the campaigns are not effective "enough" (i.e., $\eta \leq 2$), an equilibrium in pure strategies exists and is unique with advertising being: a) increasing in the campaign effectiveness η, b) decreasing in the marginal cost μ, and c) decreasing in the platforms' polarization y (recall that $y = x_R - x_L$).[9] Note that the symmetric spending in equilibrium implies that in equilibrium impressionable voters split between the two parties (i.e., $S^L_{Imp} = S^R_{Imp} = 0.5$).

Anticipating the advertising levels in the second stage, the political parties' maximization problem in the first stage is to choose the level of platform differentiation that maximizes their payoff. For instance, for party L, the payoff at $t = 1$ is $\Pi_L\left(x_L, x_R, e^*_L(x_L, x_R), e^*_R(x_L, x_R)\right)$, which can be written as:

$$S_L(x_L, x_R) - c_L(e^*_L(x_L, x_R)) = F(x_R - x_L)S^L_{Idl}(x_L, x_R) + (1 - F(x_R - x_L))S^L_{Imp} - \mu e^*_L(x_L, x_R)$$

where $S^L_{Imp} = \frac{1}{2}$ and $S^L_{Idl} = G(\bar{x})$ if $x_L \neq x_R$ or $S^L_{Idl} = \frac{1}{2}$ if $x_L = x_R$.

The platform's choice is subject to a trade-off, that is clear in the party's payoff. Consider that for a given set of platforms (x_L, x_R), the leftist party chooses to propose a less extreme platform. On the one hand, the indifferent voter is more to the right, which has a positive effect on S^L_{Idl} as in a standard Downsian model. On the other hand, it converts some ideological voters to impressionable ones, which by Lemma 1 increases the spending on advertising in the second stage of game. Similar to Tirole (1988) and Ashworth and Bueno de Mesquita (2009) divergence is a tool of softening competition in the vertical dimension (the advertising stage in our case).[10]

The importance of each of the two forces present in the trade-off is determined by the rate at which impressionable voters become ideological as polarization increases (i.e., $\frac{f(y)}{F(y)}$) and the mass of voters around the indifferent voter $g(\bar{x})$. The relative importance of these two forces is moderated by the campaign effectiveness η. When $\eta = 0$, the trade-off disappears and the parties always converge to the median. We characterize the equilibrium of the platform stage in the following proposition (the proof can be found in the appendix).

[9] If campaigns are effective "enough" (i.e., $\eta > 2$), there is no equilibrium in pure strategies in that stage and all mixed-strategy equilibria are payoff equivalent (Alcalde and Dahm, 2010) with parties' expected payoffs in that stage zero and $E(e^*_i(x_L, x_R)) = (1 - F(x_R - x_L))\frac{1}{2\mu}$). For recent advances on the properties of such mixed equilibria and relevant literature refer to Ewerhart (2015).

[10] An alternative assumption could shed some light into the problem of turnout using this same trade-off. Suppose that voters who cannot distinguish between platforms do not turn out, and that the effect of advertising is mobilizing them – as "get out the vote" efforts. Then, advertising would increase when there is a large share of non-voters, who behave as if they were indifferent between platforms. This interpretation can be empirically supported with Greco (2018), and references therein.

Proposition 1. *Let* $0 < \eta \leq 2$ *and* $\bar{y}$ *be implicitly defined by* $\frac{f(\bar{y})}{F(\bar{y})} = \frac{2}{\eta} g(\frac{1}{2})$. *For any* $\mu > 0$ *and* $\in (0,1]$ *there exists a unique subgame perfect Nash equilibrium. For* $F(0) > 0$, *the equilibrium platforms depend on* $G(x)$ *and* $F(y)$ *in the following way:*

- *(Convergent equilibrium)* $x_L^* = x_R^* = \frac{1}{2}$, *if* $\frac{f(0)}{F(0)} \leq \frac{2}{\eta} g(\frac{1}{2})$,
- *(Interior equilibrium)* $x_L^* = \frac{1}{2} - \frac{\bar{y}}{2}$, *and* $x_R^* = \frac{1}{2} + \frac{\bar{y}}{2}$, *if* $\frac{f(1)}{F(1)} < \frac{2}{\eta} g(\frac{1}{2}) < \frac{f(0)}{F(0)}$
- *(Extremism equilibrium)* $x_L^* = 0$ *and* $x_R^* = 1$, *if* $\frac{2}{\eta} g(\frac{1}{2}) \leq \frac{f(1)}{F(1)}$.

If $F(0) = 0$, *then only interior and extremism equilibrium exists with interior equilibrium arising if and only if* $\frac{f(1)}{F(1)} < \frac{2}{\eta} g(\frac{1}{2})$ *and extremism otherwise. The advertising effort at any SPNE is uniquely defined in Lemma 1 by the level of equilibrium polarization.*

First, notice that in contrast to previous literature – where equilibrium platforms require mixed strategies– Proposition (1) shows that for any distribution of parameters there exists a unique pure strategy SPNE[11]. Such result follows from endogenizing the weight given to platforms and advertising. While the relationship between these two resembles that of horizontal and vertical differentiation, our model allows for an additional strategic effect: modifying the composition of the electorate between ideological or impressionable voters. In this unique equilibrium, the level of polarization (y^*) can be zero (convergent equilibrium), one (extremism equilibrium) or $\bar{y}$ (interior equilibrium). The emerging type of equilibrium depends on: a) the concentration of voters around the median $g(\frac{1}{2})$, b) the rate at which impressionable voters become ideological when polarization increases $\frac{f(y)}{F(y)}$, and c) the effectiveness of electoral campaigns η.[12]

Starting with the effectiveness of the electoral campaigns, a large value of η makes the competition for impressionable votes aggressive which exacerbates electoral costs in the second stage (Lemma 1). Therefore, a high value of η provides incentives to reduce the number of impressionable voters, making platform polarization in the first stage attractive. That is, as η increases, we may move across types of equilibria (with convergence occurring for a larger set of parameters), but also platforms become less polarized in the interior equilibrium (since $\bar{y}$ is decreasing in η). Similarly, when many voters are concentrated around the median (i.e., a high value of $g(\frac{1}{2})$), there are strong

[11] Note here that we have restricted attention to $\eta \leq 2$ due to the mixed strategies in the campaign stage for $\eta > 2$. However, the unique pure strategy equilibrium characterized in the platform substage –which is the main difference to Ashworth and Bueno de Mesquita 2009– is also an equilibrium for $\eta > 2$ with equilibrium platforms the same as the the ones characterized for $\eta = 2$. This is due to the payoff equivalence result described in the previous footnote.

[12] Our characterization would never involve convergent equilibria if we were to permit $F(0) = 0$ and this would be the only relevant difference with our results. However, assuming $F(0) \geq 0$ would involve some additional cost in notation that we prefer to avoid. Note also that our characterization is valid for any $F(0) = \epsilon > 0$ and hence ϵ can be arbitrarily small.

incentives to propose moderate platforms. Thus, a strong presence of moderate voters leads to equilibria of "low" polarization (again, either across equilibria types or within the interior equilibrium).

The *conversion rate* at which uniformed voters become ideological when polarization increases (i.e, the reverse hazard rate $\frac{f(y)}{F(y)}$) is also crucial in understanding our result. This rate captures the incentives of increasing polarization as a tool of reducing electoral campaign costs. By log-concavity of $F(y)$, the rate is monotonically decreasing and hence takes its maximum value at $y = 0$ and its minimum value at $y = 1$. Our results show that if its minimum value is large "enough" (i.e., $\frac{2}{\eta}g(\frac{1}{2}) \leq \frac{f(1)}{F(1)}$), polarization is very effective in restraining the electoral spending and extremism emerges because small changes toward polarization convert a large percentage of voters into ideological ones, diminishing the incentives to spend in the advertising. Analogously, if the maximum value of the conversion rate is small "enough" (i.e., $\frac{f(0)}{F(0)} \leq \frac{2}{\eta}g(\frac{1}{2})$), polarization has a very poor influence on electoral spending and the original Downsian result of platform convergence emerges. The conversion rate is so small that increasing polarization does not increase the percentage of ideological voter enough to diminish the number of advertisements in a profitable fashion. A distributional change in the function determining the distribution of voters across types gives interesting comparative statics.

Notation 1. *Let ρ parametrize the sensitivity of the conversion rate due to inputs other than polarization (e.g., education or interest in politics). We say that the conversion rate $F(y;\rho)/f(y;\rho)$ satisfies the monotone likelihood ratio property (MLRP) in y if for any $\rho_1 > \rho_0$ it holds that*

$$\frac{f(y;\rho_1)}{F(y;\rho_1)} > \frac{f(y;\rho_0)}{F(y;\rho_0)}$$

where $F(y;\rho)$ and $f(y;\rho)$ differentiable in ρ.

Any increase in ρ which makes the conversion rate more sensitive to changes may move platforms across types of equilibria (favoring more polarization). But also at any interior equilibrium, polarization is increasing in ρ (it follows from applying implicit differentiation to the interior equilibrium condition $\frac{f(\bar{y})}{F(\bar{y})} = \frac{2}{\eta}g(\frac{1}{2})$).

Finally, notice that our platforms' characterization does not depend on the costs of campaigning μ in any manner. Given that this cost is symmetric for the two parties, increasing or decreasing it would only rescale the equilibrium levels of advertising e_i (from Lemma 1) but will not modify the actual level of electoral spending μe_i and hence polarization.

Effects on Electoral Spending

A technological change that increases electoral effectiveness has an ambiguous effect on electoral spending. This is apparent when we look at the relevant expression:

$$\frac{\partial \mu e_i^*(x_L^*, x_R^*)}{\partial \eta} = \overbrace{\frac{1-F(y^*)}{4}}^{\text{Spending effect (+)}} \quad \overbrace{-f(y^*)\frac{\partial y^*}{\partial \eta}\frac{\eta}{4}}^{\text{Polarization effect (-)}} \tag{2}$$

On the one hand, ceteris paribus an increase in η increases advertising (Lemma 1). On the other hand, it also increases polarization (Proposition 1), which in turn, decreases the share of impressionable voters and so the levels of advertising (Lemma 1). We call the former the spending effect, while we label the latter as the polarization effect.

At the fully divergent and fully convergent equilibria there is no polarization effect, $\frac{\partial y^*}{\partial \eta} = 0$, and spending either increases monotonically with η due to the spending effect or is unaffected. At the interior equilibrium however, the polarization effect takes place and mitigates the spending effect. If polarization increases disproportionally with η, the polarization effect may even overturn the spending effect, and hence observe η and campaign spending move in opposite directions. In Lemma (2) below, we provide the conditions for a simultaneous increase in electoral spending and polarization, and we use an example to illustrate it.

Lemma 2. *In any interior equilibrium, a technological change that increases the campaign effectiveness η – and hence polarization – also increases electoral spending due to the spending effect dominating the polarization effect if and only if effectiveness is low "enough". Formally, in any interior equilibrium $\frac{\partial \mu e_i^*(x_L^*, x_R^*)}{\partial \eta} \geq 0$ if and only if $\eta \leq 2g(\frac{1}{2})\frac{1-F(\bar{y})}{f(\bar{y})}\left[\frac{F(\bar{y})}{f(\bar{y})}\right]'$.*

Example 1: $F(y)$ is uniformly distributed over $(0, 1]$ with a mass at zero $F(0) = \frac{1}{10}$. The "conversion rate" now – from impressionable to ideological voters – is proportional to polarization: $\frac{f(y)}{F(y)} = \frac{\frac{9}{10}}{\frac{1}{10}+\frac{9}{10}y}$. From Proposition (1), we are at a convergent equilibrium for $\eta < \frac{2}{9}g(\frac{1}{2})$, at an interior equilibrium for $\frac{2}{9}g(\frac{1}{2}) \leq \eta \leq \frac{20}{9}g(\frac{1}{2})$ and at an extremism equilibrium for $\eta > \frac{20}{9}g(\frac{1}{2})$. These conditions highlight how the concentration of voters around the median gives rise to different equilibrium types. If for example $g(\frac{1}{2})$ is large then extremism can be excluded as an outcome for any level of campaign effectiveness. In the interior equilibrium polarization given by $y^* = \bar{y} = \frac{\eta}{2g(\frac{1}{2})} - \frac{1}{9}$ and it is straightforward to see that polarization is increasing in η. Using the condition in Lemma (2) we can also get the non-monotone comparative statics on campaign spending in the interior equilibrium and show that campaign spending is increasing in the campaign

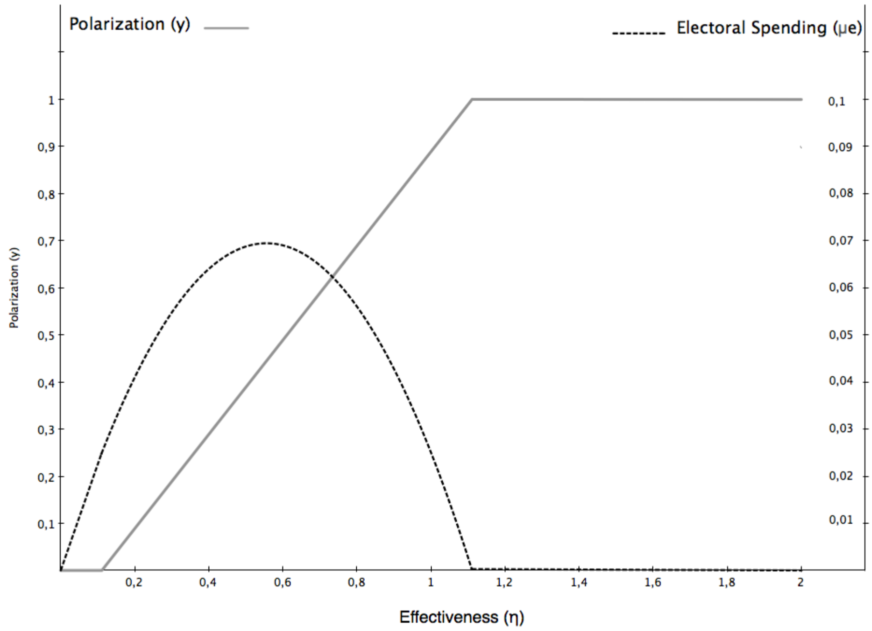


Figure 1: Comparative statics on η for electoral spending and polarization. Uniform distribution of y with $F(0) = \frac{1}{10}$ and $g(1/2) = 1/2$.

effectiveness for η low enough (i.e., $\eta \leq \frac{10}{9} g(\frac{1}{2})$) and decreasing otherwise.

In Figure 1 we graphically represent the comparative statics of changes of η on polarization and electoral spending also assuming that $g(\frac{1}{2}) = \frac{1}{2}$. Consider first the equilibrium levels of polarization (solid line). For η lower than $\frac{1}{9}$ or greater than $\frac{10}{9}$ the convergent and extremist equilibria respectively arise. For $\frac{1}{9} \leq \eta \leq \frac{10}{9}$ the interior equilibrium arises and polarization is strictly increasing in η. Let's now turn to electoral spending (dashed line). For $\eta < \frac{1}{9}$, polarization is constant and equal to zero and electoral spending is monotonically increasing in η (as in any standard Tullock contest). If $\eta > \frac{10}{9}$ the extremism equilibrium arises and due to all voters being ideological campaign spending is zero.[13] In the interior equilibrium interval (i.e., $\eta \in [\frac{1}{9}, \frac{10}{9}]$), the non-monotonicity arises. Electoral spending increases until reaching $\eta = \frac{5}{9}$ due to the spending effect being larger than the polarization one. On the contrary, spending decreases for values of η larger than $\frac{5}{9}$ due to the polarization effect overcoming the spending effect and till extremism arises.

Polarization and campaign spending are also affected by the rate at which impressionable voters become ideological as polarization increases as parametrized by ρ. A technological change that increases ρ has an ambiguous effect on electoral spending. As

[13] Zero spending in the case of extremism arises because of the simplification of no mass of impressionable voters under maximal platform separation. One could trivially extend our model by including a mass of impressionable voters $0 < \delta < 1$ even under extreme polarization. In that case, in the extreme equilibrium campaign spending is positive and strictly increasing in η. Our characterization in Proposition 1 would remain unaffected and campaign efforts in Lemma 1 will be simply rescaled but not qualitatively affected.

above, the condition comes from looking at the derivative of campaign spending with respect to ρ.

$$\frac{\partial \mu e_i^*(x_L^*, x_R^*)}{\partial \rho} = \overbrace{-\frac{\partial F(y^*)}{\partial \rho}\frac{\eta}{4}}^{\text{Spending effect (+)}} \overbrace{-\frac{\partial F(y^*)}{\partial y^*}\frac{\partial y^*}{\partial \rho}\frac{\eta}{4}}^{\text{Polarization effect (-)}}$$

On the one hand, an increase in ρ increases the "stock" of impressionable voters $1 - F(y)$. The spending effect then suggests that, for a given level of polarization y, an increase in ρ increases advertising (Lemma 1).[14] On the other hand, an increase in ρ affects the "conversion" of voters from impressionable to ideological due to polarization by making them more responsive, and therefore provides incentives to increase polarization (Proposition 1). But this increased polarization in turn, decreases the share of impressionable voters and so decreases the levels of advertising (Lemma 1). As before, we label this latter effect as the polarization effect.

At the convergent equilibria there is no polarization effect since $\frac{\partial y^*}{\partial \rho} = 0$ and spending increases monotonically with ρ. That is, an increase in ρ keeping polarization constant would increase the number of impressionable voters and their weight in the parties' maximization problem and parties would have higher incentives to increase advertising. In the interior equilibrium the polarization effect kicks in and the net effect on spending depends on the magnitude of these two effects. Recall that changes in ρ enter in the spending effect due to changes in "stock", while they enter in the polarization effect due to changes in the "conversion". Finally, polarization is constant in ρ once the extremism equilibrium is reached and spending is constant and equal to zero since there are no impressionable voters in that case. The following lemma summarizes the above and provides the formal conditions for the interior case.

Lemma 3. *In any interior equilibrium, a technological change that increases ρ – and hence polarization – also increases electoral spending due to the spending effect dominating the polarization effect if and only if the the effect of ρ on the "stock" of impressionable voters is "large" enough. Formally, in any interior equilibrium $\frac{\partial \mu e_i^*(x_L^*, x_R^*)}{\partial \rho} \geq 0$ if and only if $-\frac{\partial F(\bar{y})}{\partial \rho} \geq f(\bar{y})\frac{\partial \bar{y}}{\partial \rho}$.*

Finally, notice that in the symmetric case, the marginal cost of advertising plays no role in equilibrium: neither the platforms nor the campaign spending depend on μ. When we introduce a cost-asymmetry between the parties, the equilibrium changes, favoring the party with the lower marginal cost.

[14] This is due to MLRP implying first order stochastic dominance, hence $\frac{\partial F(y^*)}{\partial \rho} < 0$ making the spending effect positive.

3.2 Asymmetric parties

In contrast to previous models with endogenous valence, the tractability and intuition of the model remain in place when we incorporate cost asymmetries. In this section we assume, without loss of generality, that $\mu_L < \mu_R$. We first characterize campaign spending in the second stage and then show how equilibrium platforms are affected by the asymmetry. Following Baik (1994) and Nti (1999) the following Lemma arises.

Lemma 4. *Let $\bar{\eta}$ implicitly defined by $\mu_L^{\bar{\eta}} + \mu_R^{\bar{\eta}} = \bar{\eta}\mu_R^{\bar{\eta}}$. For all $\eta \leq \bar{\eta}$ there exists a unique Nash equilibrium in the campaign stage and advertising is given by $e_i^*(x_L, x_R) = (1 - F(y))\frac{\eta}{\mu_i}\frac{\mu_L^\eta \mu_R^\eta}{(\mu_L^\eta + \mu_R^\eta)^2}$ for all i.*

Lemma (4) shows that, in equilibrium, parties choose different levels of advertising ($e_i^*(x_L, x_R)$) although spend equal total amounts ($\mu_i e_i^*(x_L, x_R)$).[15] The share of impressionable voters is not any longer equally split across parties, giving an advantage to the party with the lower marginal cost. This generates an asymmetry in parties' incentives to use polarization as a device to reduce electoral spending and eliminates the interior equilibria where $x_L^* + x_R^* = 1$. In any interior equilibrium parties propose asymmetric platforms. The convergent and extremism equilibria described previously also arise. We present the conditions for the rise of each type of equilibrium in the following propositions.

Proposition 2. *(Convergent equilibrium) For any $0 < \eta \leq \bar{\eta}$, $\mu_L < \mu_R$ and $F(0) > 0$, there exists a unique SPNE with $x_L^* = x_R^* = \frac{1}{2}$ if and only if $\frac{f(0)}{F(0)} \leq g(\frac{1}{2}) \Big/ \Big(\frac{\mu_L^\eta \mu_R^\eta}{(\mu_L^\eta + \mu_R^\eta)^2} 2\eta + \frac{\mu_R^\eta - \mu_L^\eta}{\mu_L^\eta + \mu_R^\eta} \Big)$. For $\eta = 0$, there is always a convergent equilibrium. Campaign spending for the convergent equilibrium is uniquely characterized in Lemma 4.*

As in the symmetric case, the convergent equilibrium takes place when a lot of voters are concentrated around the median (i.e., high $g(\frac{1}{2})$) and the conversion rate if they were to start polarizing their platforms (i.e., $\frac{f(0)}{F(0)}$) is low. In the asymmetric case, the size of the asymmetry is also a determinant of platform convergence. If the asymmetry converges to zero, then the relevant inequality converges to the one of Proposition 1. As the asymmetry however increases, platform convergence becomes less likely (the denominator at the right hand side of the inequality is increasing in the asymmetry). This is because, as the asymmetry increases, the symmetric (convergent) equilibrium becomes less attractive for the disadvantaged party that loses the competition for impressionable voters and hence has incentives to diversify and propose distinct platforms. This potentially leads to the following interior equilibria types.

[15] The condition on η that guarantees equilibrium in pure strategies is more restrictive than in the symmetric case, since $\bar{\eta}$ is lower or equal than 2. For $\eta \in (\bar{\eta}, 2)$ the equilibrium is characterized by Wang (2010) and Ewerhart (2017), while payoff equivalence by Alcalde and Dahm (2010) can be used to solve the platform stage for $\eta > 2$.

Proposition 3. *(Interior Equilibrium) For any* $0 < \eta \leq \bar{\eta}$ *and* $\mu_L < \mu_R$*, let* $\bar{x}^* = G^{-1}(\frac{\mu_R^\eta}{\mu_L^\eta + \mu_R^\eta})$ *and* $\bar{y}$ *be implicitly defined by* $\frac{f(\bar{y})}{F(\bar{y})} = \frac{g(\bar{x}^*)}{2\eta}\frac{(\mu_L^\eta + \mu_R^\eta)^2}{\mu_L^\eta \mu_R^\eta}$*, there exists a unique interior SPNE if and only if one of the following conditions is satisfied*

$$\frac{F(2\bar{x}^* - 1)}{f(2\bar{x}^* - 1)}g(\bar{x}^*) < 2\eta\frac{\mu_L^\eta \mu_R^\eta}{(\mu_L^\eta + \mu_R^\eta)^2} < \frac{F(1)}{f(1)}g(0.5) + \frac{\mu_R^\eta - \mu_L^\eta}{\mu_R^\eta + \mu_L^\eta} \quad \text{if} \quad \frac{\mu_R^\eta}{\mu_L^\eta + \mu_R^\eta} < G(\frac{3}{4}) \tag{3}$$

$$\frac{F(\frac{1}{2})}{f(\frac{1}{2})}g(\frac{1}{2}) - 2G(\frac{3}{4}) + \frac{2\mu_R^\eta}{\mu_R^\eta + \mu_L^\eta} < 2\eta\frac{\mu_L^\eta \mu_R^\eta}{(\mu_L^\eta + \mu_R^\eta)^2} \quad \text{if} \quad \frac{\mu_R^\eta}{\mu_L^\eta + \mu_R^\eta} \geq G(\frac{3}{4}) \tag{4}$$

An equilibrium is interior if at least one of the parties chooses an interior platform. That is, a platform between the median and the corner (either 0 or 1, depending on the party). Taking this definition into account, Proposition 3 describes the necessary and sufficient conditions for the existence and uniqueness of an interior equilibrium, which depend on the extent of the parties' cost asymmetries. Then, within the scope of the proposition, the corollary below characterizes the interior equilibria.

Corollary 1. *Within Proposition 3, there are two types of equilibria:*

- *(Interior/Interior)* $x_L^* = \bar{x}^* - \frac{\bar{y}}{2}$ *and* $x_R^* = \bar{x}^* + \frac{\bar{y}}{2}$ *if and only if Condition 3 and* $2\eta\frac{\mu_L^\eta \mu_R^\eta}{(\mu_L^\eta + \mu_R^\eta)^2} < \frac{F(2-2\bar{x}^*)}{f(2-2\bar{x}^*)}g(\bar{x}^*)$ *are satisfied.*
- *(Interior/Corner)* $x_L^* < 0.5$ *and* $x_R^* = 1$*, otherwise.*

Campaign spending for each of the above SPNE is uniquely characterized in Lemma 4

All interior equilibria are asymmetric. It can be that both propose interior platforms *(Interior/Interior)* or that the cost advantaged party proposes an interior platform while the disadvantaged party proposes an extreme platform *(Interior/Corner)*. The cost asymmetry plays an important role in determining the type but also polarization levels of the interior equilibrium. The cost disadvantaged party R has more incentives than the advantaged party to reduce the share of (costly) impressionable voters. Consequently, R has more incentives to separate its platform from L. Thus, in any interior equilibrium platforms are shifted towards the cost-disadvantaged party R. That is, in the *(Interior/Interior)* equilibrium the point around which parties propose symmetric platforms is to the right (i.e., $\bar{x}^* > 0.5$), while in the *(Interior/Corner)* it is the disadvantaged party that proposes an extreme platform. Note that in general, if the asymmetry is low "enough" (i.e., (3) is satisfied) the disadvantaged party may also propose an interior equilibrium. If this asymmetry however is high "enough" (i.e., (4) is satisfied) only the *(Interior/Corner)* equilibrium arises, the disadvantaged party would like to separate more but can not. In that sense, "large" asymmetries never give rise to extremism equilibria either as the following proposition summarizes.

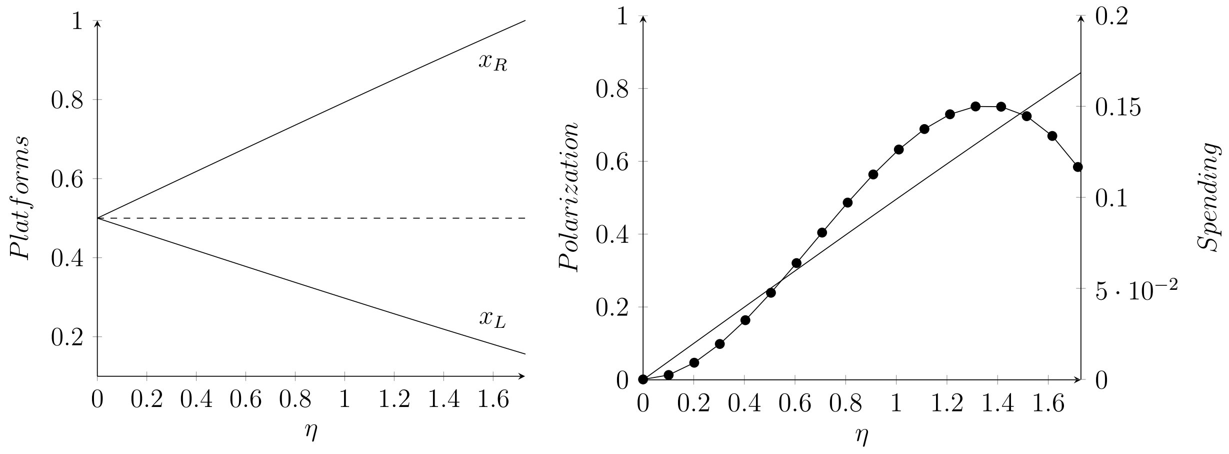
Proposition 4. *(Extremism Equilibrium) For any* $0 < \eta \leq \bar{\eta}$ *and* $\mu_L < \mu_R$*, there exists a unique SPNE with* $x_L^* = 0$ *and* $x_R^* = 1$ *if and only if* $g(0.5) \Big/ \left(2\eta \frac{\mu_L^\eta \mu_R^\eta}{(\mu_L^\eta + \mu_R^\eta)^2} - \frac{\mu_R^\eta - \mu_L^\eta}{\mu_L^\eta + \mu_R^\eta} \right) \leq \frac{f(1)}{F(1)}$ *and campaign spending as characterized in Lemma 4.*

If the asymmetry converges to zero, the relevant inequality converges to the one of Proposition 1. As the asymmetry however increases, extremism becomes less likely (the denominator on the left hand side of the inequality is decreasing in the asymmetry). This is because, the advantaged party has no incentives to polarize. Also, as in the symmetric case, extremism is more likely when the rate at which impressionable voters becomes ideological is great enough under maximal platform separation (i.e., $\frac{f(1)}{F(1)}$ is high).

As with the symmetric case, the inequalities in all equilibrium conditions also depend on the effectiveness of campaigns, the mass of ideological voters and conversion rates at different points. While we have covered the most relevant ones, especially what concerns the cost asymmetries, in general, the incentives to polarize (or not) and move across equilibria types remain similar to the symmetric case. Proceeding with examples, similar intuition to the results of the symmetric case arise. The simultaneous increase in polarization and electoral spending observed can be reconciled also in the presence of an asymmetry.

Example 2: $F(y)$ and G(y) uniformly distributed in X. In Figure 2 we show two cases, in Panel (a) we plot the case of "low" asymmetry ($\mu_R/\mu_L = 1.2$) while in Panel (b) we plot the case of "high" asymmetry ($\mu_R/\mu_L = 2$). For both levels of asymmetry, we are at a convergent equilibrium only when $\eta = 0$. That is, when impressionable voters are split equally across the two parties regardless of campaign spending parties do not have incentives to differentiate their platforms. As long as $\eta > 0$ we illustrate the divergent and asymmetric *(Interior/Interior)* equilibrium platforms as characterized in Corollary 1. Note that in both panels, and as described in our results, the disadvantaged party R is proposing a relatively more extreme platform compared to the advantaged party L. In the example of "low" asymmetry, platforms diverge monotonically as η increases (as in the symmetric case) and hence polarization is also increasing. In the example of "high" asymmetry, and despite polarization being again increasing in the effectiveness of campaign spending, the advantaged party does not respond to increases in η in a monotonic manner. Finally, when it comes to campaign spending, as the upper right panel shows, we may again encounter a situation of a non-monotonic relationship between campaign spending and η (as in Figure 1 and the symmetric case). But perhaps more importantly, our results can again sustain the simultaneous increase in polarization and campaign spending due to technological changes even in the presence of asymmetries for a wide range of parameters.

Panel **(a)**: "low" asymmetry: $\mu_R/\mu_L = 1.2$



Panel **(b)**: "high" asymmetry: $\mu_R/\mu_L = 2$

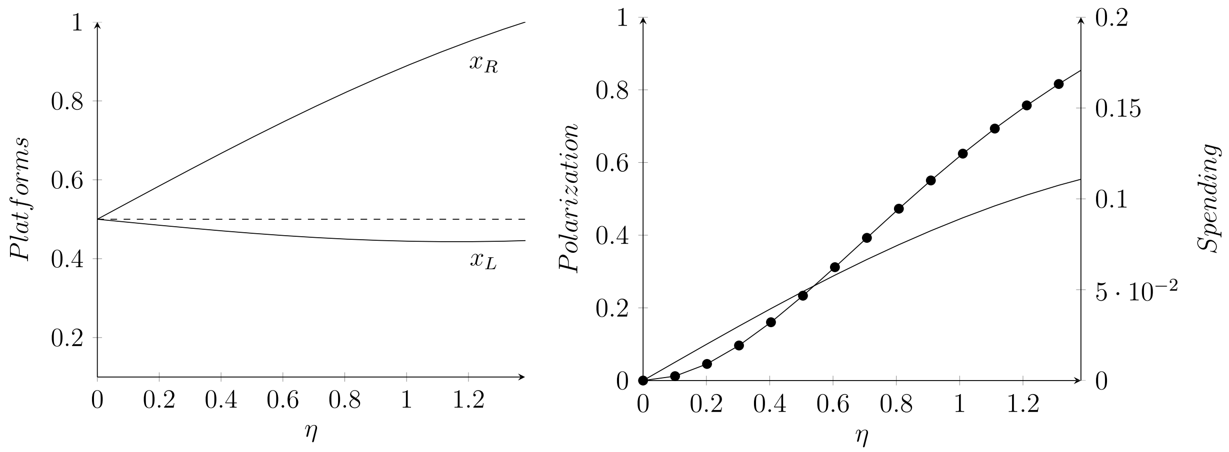


Figure 2: Solid lines depict equilibrium platforms (x_L^*, x_R^*) in the left panels and polarization $(x_R^* - x_L^*)$ in the right panels. Dotted lines depict campaign spending. Graphs are plotted on the interval of η that guarantees an equilibrium in pure strategies in the advertising stage.

4 Conclusion

Citing Herrera et al. (2008), *"commentators have suggested that the reason for both the increased polarization and campaign spending is that skilled political operatives using sophisticated statistical tools and purchasing advertising in local markets are better able to target particular voters"* (see for example, NBC 2017). However, existing results so far were linking such technological advances with a reduction in polarization, and therefore favored alternative channels that may drive polarization such as more volatile preferences.

Our results in contrast, are the first to justify the simultaneous increase in polarization and electoral spending due to recent technological changes and better targeting of the electoral campaigns.

Given that one would naturally expect further advances in campaign technology two natural questions arise: a) should we expect a further increase in polarization, and b) what about campaign spending? Our theory would say yes, further advances in targeting will lead to further polarization. Ways to go against this trend would require policies that improve the awareness of theawareness electorate and a shift of voters' attention from uninformative campaigns to political platforms. With regards to campaign spending, recall that our theory does not provide a monotone comparative static. As the targeting technology improves, parties have incentives to increase their campaign spending, but at the same time to polarize (which reduces campaign spending). Hence, our results are not incompatible with the observations of the 2016 presidential US election where campaign spending dropped. Furthermore, following this presidential campaign, the diffusion of plausible but false information received the name of "fake news". This notion, widely used during and after the campaigns, have direct implications on the electorate's awareness: for any given "real" platforms, the stock of ideological voters decreases with the amount of fake news. In terms of our model, the reversed hazard rate would increase, causing an increase in polarization as well.

While one of our contributing messages could be the importance of the electorate's awareness as a way to affect campaign spending and polarization, political pundits have paid special attention to caps on electoral spending. In the context of symmetric costs, our model provides very clear and intuitive implications regarding the effects of this policy. If a cap is below the equilibrium campaign spending, it will completely shut down the polarization effect in our model. That is, when a cap is binding, the parties will not induce an increase in electoral spending by moving their platform towards their competitor. This eliminates one of the elements of the parties trade-off in the choice of platform location. As a consequence, platform convergence is the only possible outcome. In other words, any cap on electoral spending smaller or equal to the equilibrium spending, will automatically induce convergence at the median voter. Thus our model predicts an important impact of electoral caps on party polarization. The same intuition can be sustained under asymmetric costs if the inverse hazard rate is great enough under full convergence.

5 Appendix - Proofs

This section is organized as follows. In Subsection 5.1 we show the pure strategy equilibrium in advertising for asymmetric costs. Additionally, we discuss the extension for $\eta > \bar{\eta}$ in the symmetric case. In Subsection 5.2 we write and solve the platform selection problem for the general case with asymmetric costs. Thus, taking into account the Kuhn-Tucker conditions, we solve this maximization problem and we show the first and second order conditions. Lemmi A.1, A.2 and Remark 1 in that subsection provide the conditions for existence and uniqueness of platforms that maximize the Lagrangian for the general case. Next, we show uniqueness and existence of a unique (pure) Subgame Perfect Nash Equilibrium, and we characterize the equilibrium platforms for the symmetric case (Proposition 1). Later on, we do the same for asymmetric costs. In this case, we show separatedly the convergent equilibrium (Proposition 2), the interior equilibrium (Proposition 3 and Corollary 1) and the extremism one (Proposition 4).

5.1 Advertising

We begin proving the more general Lemma 4, i.e., $\mu_L \neq \mu_R$, and after the proof we discuss the implications of symmetry, i.e., Lemma 1.

Proof. For any given pair (x_L, x_R), parties simultaneously choose e_i that maximizes $(1 - F(y))\frac{e_i^\eta}{e_L^\eta + e_R^\eta} - \mu_i e_i(x_L, x_R)$. Hence, from the FOC, we obtain the first necessary condition for an interior PSNE: $e_i = \frac{1-F(y)}{\mu_i}\eta\frac{\mu_L^\eta \mu_R^\eta}{(\mu_L^\eta + \mu_R^\eta)^2}$. The equilibrium payoffs can be written as

$$(1 - F(y))[\frac{\mu_i^\eta}{\mu_L^\eta + \mu_R^\eta} - \eta\frac{\mu_L^\eta \mu_R^\eta}{(\mu_L^\eta + \mu_R^\eta)^2}],$$

which are positive if and only if $(\mu_L^\eta + \mu_R^\eta)\mu_i^\eta - \eta\mu_L^\eta\mu_R^\eta \geq 0$. Therefore, the second necessary condition for PSNE is that $\eta \leq \bar{\eta} : \mu_R^{\bar{\eta}} + \mu_L^{\bar{\eta}} = \bar{\eta}\mu_R^{\bar{\eta}}$. Moreover, for all η lower or equal than $\bar{\eta}$, the SOC holds with strict inequality, assuring not only existence but also uniqueness of the equilibrium. □

The symmetric case. Notice that $\bar{\eta} \leq 2$ holds with strict equality if and only if $\mu_L = \mu_R$. In the symmetric case, for all $\eta \leq 2$, the conditions on the existence and uniqueness of PSNE are satisfied, and therefore $e_i = \frac{1-F(y)}{4\mu_i}\eta$. For $\eta > 2$, Alcalde and Dahm (2010) show that "the contest possesses an all-pay auction equilibrium" (Theorem 3.2 in their paper). In a symmetric contest, this theorem implies full dissipation, which means that the expected payoff would be 0 for parties L and R, and the corresponding expected bids would be $\frac{1}{2}\frac{1-F(y)}{\mu_i}\eta$.

5.2 Polarization and Spending

The objective function, existence and uniqueness

Throughout this section let $y = x_R - x_L$, $\bar{x} = \frac{x_R+x_L}{2}$, and $S^L_{Idl}(x_L, x_R) = G(\bar{x}) = 1 - S^R_{Idl}(x_L, x_R)$ for $x_L \neq x_R$ and $S^L_{Idl}(x_L, x_R) = S^R_{Idl}(x_L, x_R) = \frac{1}{2}$ otherwise. By backward induction and using the equilibrium expressions of the of advertising subgame, we can write the first-stage payoff for the political parties as follows

$$\Pi_i(x_L, x_R) = F(y)S^i_{Idl}(x_L, x_R) + (1 - F(y))\frac{\mu^\eta_{-i}}{\mu^\eta_R + \mu^\eta_L} - (1 - F(y))\eta\frac{\mu^\eta_R\mu^\eta_L}{(\mu^\eta_R + \mu^\eta_L)^2}.$$

Lemma A.1. *In any equilibrium $x_L \leq 1/2$ and $x_R \geq 1/2$.*

Proof. Remind that we have assumed without loss of generality that $x_L \leq x_R$. First consider the case of a divergent equilibrium, i.e., $x_L \neq x_R$. We proceed by contradiction. Assume an equilibrium such that $1/2 < \tilde{x}_L < \tilde{x}_R$. Then party R is strictly better of by deviating to $x_R = 2\tilde{x}_L - \tilde{x}_R$, which maintains unchanged the proportion of each type of voter and the share of impressionable votes and strictly increases the share of ideological votes of party R. Analogously we can show that party L always has a profitable deviation to $x_L = 2\tilde{x}_R - \tilde{x}_L$ when $\tilde{x}_L < \tilde{x}_R < 1/2$

Now consider the case of a convergent equilibrium $x_L = x_R$. Given the assumption of $x_L \leq x_R$ two necessary conditions for a convergent equilibrium at $x_L = x_R = x$ are:

$$\lim_{x_L \to x^-} \Pi_L(x_L, x) \leq \Pi_L(x, x)$$
$$\lim_{x_R \to x^+} \Pi_R(x, x_R) \leq \Pi_R(x, x)$$

For $x > \frac{1}{2}$, $lim_{x_L \to x^-} G(x_L, x) > \frac{1}{2} = G(x, x)$, which implies that $lim_{x_L \to x^-} \Pi_L(x_L, x) > \Pi_L(x, x)$ and violates the equilibrium condition for party L. A similar argument applies for party R if $x < \frac{1}{2}$. $\square$

Remind that when $x_L = x_R \neq 1/2$, a discontinuity can arise in the objective function, due to the discontinuity of $S^i_{Idl}(x)$. However, the previous lemma restricts convergence to the case $x_L = x_R = 0.5$, in which $S^i_{Idl}(x)$, and consequently $\Pi_i(x_L, x_R)$, are continuous. Thus, by including the equilibrium constraints ($x_L \leq 1/2$ and $x_R \geq 1/2$) in the maximization problem, the Lagrangians below are continuous, and provided that the second order conditions are met, will be solved by the equilibrium platforms (due to the presence of equilibrium constraints solving the Lagrangian is a necessary but not a sufficient condition for equilibrium, as we show in Remark 1 below).

For $i \in \{L, R\}$, let $\lambda_i \geq 0$ be the multipliers associated with the feasibility constraints and $\nu_i \geq 0$ with the equilibrium constraints. The Lagrangians are:

$$\mathcal{L}_L = \Pi_L(x_L, x_R) - \lambda_L(-x_L - 0) - \nu_L(x_L - \frac{1}{2}) \tag{5}$$

$$\mathcal{L}_R = \Pi_R(x_L, x_R) - \lambda_R(x_R - 1) - \nu_R(\frac{1}{2} - x_R). \tag{6}$$

The first order conditions (FOC from now on) are:

$$\frac{\partial \mathcal{L}_L}{\partial x_L} = \frac{\partial \Pi_L(x_L, x_R)}{\partial x_L} + \lambda_L - \nu_L = 0, \tag{7}$$

$$\frac{\partial \mathcal{L}_R}{\partial x_R} = \frac{\partial \Pi_R(x_L, x_R)}{\partial x_R} - \lambda_R + \nu_R = 0. \tag{8}$$

Where,

$$\Pi'_L \equiv \frac{\partial \Pi_L(x_L, x_R)}{\partial x_L} = F(y)\frac{g(\bar{x})}{2} - f(y)G(\bar{x}) + f(y)\frac{\mu_R^\eta}{\mu_R^\eta + \mu_L^\eta} - f(y)\eta\frac{\mu_R^\eta \mu_L^\eta}{(\mu_R^\eta + \mu_L^\eta)^2}, \tag{9}$$

$$\Pi'_R \equiv \frac{\partial \Pi_R(x_L, x_R)}{\partial x_R} = -F(y)\frac{g(\bar{x})}{2} + f(y)[1 - G(\bar{x})] - f(y)\frac{\mu_L^\eta}{\mu_R^\eta + \mu_L^\eta} + f(y)\eta\frac{\mu_R^\eta \mu_L^\eta}{(\mu_R^\eta + \mu_L^\eta)^2} \tag{10}$$

Lemma A.2. *Let $F(x)$ be log-concave and $g(x)$ be symmetric and log-concave. The objective functions are strictly quasiconcave in the policy space when $x_L \leq \frac{1}{2} \leq x_R$, hence the second order conditions (SOC) are satisfied.*

The constrained optimization problem includes two linear constraints for each party, thus focusing on the quasiconcavity of the payoff functions suffices.

Proof. Without loss of generality, consider party L. Under the conditions of Lemma (A.1), continuity of Π_L is assured. Then, let us modify Equation (9) by dividing it over the densities $f(y)$ and $g(\bar{x})$:

$$\tilde{\Pi}'_L \equiv \frac{\Pi'_L}{f(y)g(\bar{x})} = \frac{F(\bar{y})}{2f(\bar{y})} - \frac{1}{g(\bar{x})}\left[G(\bar{x}) - \frac{\mu_R^\eta}{\mu_R^\eta + \mu_L^\eta} + \eta\frac{\mu_R^\eta \mu_L^\eta}{(\mu_R^\eta + \mu_L^\eta)^2}\right] \tag{11}$$

Let $\tilde{\Pi}_L$ be the primitive of $\tilde{\Pi}'_L$. $\tilde{\Pi}_L$ is strictly quasiconcave if and only if $\tilde{\Pi}'_L(x)(x' - x) > 0$ whenever $\tilde{\Pi}_L(x') > \tilde{\Pi}_L(x)$. Since strict quasiconcavity is determined by the sign of $\tilde{\Pi}'_L(x)$, which is the same of the sign of $\Pi'_L(x)$ (because $f(y)g(\bar{x})$ is strictly positive), the strict quasiconcavity of $\tilde{\Pi}_L(x)$ guarantees the strict quasiconcavity of $\Pi_L(x)$.

Therefore, by showing the strict concavity of $\tilde{\Pi}_L$ (i.e., $\tilde{\Pi}''_L = \frac{\partial \frac{\Pi'_L}{f(y)g(\bar{x})}}{\partial x_L} < 0$), we will be proving that $\Pi_L(x_L, x_R)$ is strictly quasiconcave too. Hence $\tilde{\Pi}''_L$ is

$$\frac{\partial \frac{\Pi'_L}{f(y)g(\bar{x})}}{\partial x_L} = -\frac{1}{2}\left[\frac{F(\bar{y})}{f(\bar{y})}\right]' - \frac{1}{2g(\bar{x})^2}\left\{g(\bar{x})^2 - g'(\bar{x})\left[G(\bar{x}) - \frac{\mu_R^\eta}{\mu_R^\eta + \mu_L^\eta} + \eta\frac{\mu_R^\eta \mu_L^\eta}{(\mu_R^\eta + \mu_L^\eta)^2}\right]\right\}$$

By log-concavity of $F(y)$, the term $-[\frac{F(\bar{y})}{2f(\bar{y})}]'$ is negative, so we can focus on the negativity of $H = -\left\{g(\bar{x})^2 - g'(\bar{x})\left[G(\bar{x}) - \frac{\mu_R^\eta}{\mu_R^\eta+\mu_L^\eta} + \eta\frac{\mu_R^\eta\mu_L^\eta}{(\mu_R^\eta+\mu_L^\eta)^2}\right]\right\}$ in the expression above to guarantee strict concavity of $\tilde{\Pi}_L(x)$ (and hence strict quasi-concavity of $\Pi_L(x)$) . Let consider two cases.

- If $g'(\bar{x}) \geq 0$, then $-g'(\bar{x})[\frac{\mu_R^\eta}{\mu_R^\eta+\mu_L^\eta} - \eta\frac{\mu_R^\eta\mu_L^\eta}{(\mu_R^\eta+\mu_L^\eta)^2}])$ is negative (strictly negative for $g'(\bar{x}) > 0$) because the term in brackets is always positive given $\eta < \bar{\eta}$ (see proofs of Lemmi 1 and 4). Log-concavity of $g(x)$ implies log-concavity of $G(\bar{x})$, thus $-[g(\bar{x})^2 - g'(\bar{x})G(\bar{x})]$ is negative (strictly negative for $g'(\bar{x}) = 0$). Hence H is strictly negative and $\Pi_L(x)$ strictly quasi-concave.

- If $g'(\bar{x}) < 0$, suppose there exists $\hat{x}_L : G(\frac{\hat{x}_L + x_R}{2}) = \frac{\mu_R^\eta}{\mu_R^\eta+\mu_L^\eta} - \eta\frac{\mu_R^\eta\mu_L^\eta}{(\mu_R^\eta+\mu_L^\eta)^2}$. Since $G(\bar{x})$ is increasing in x, for $x_L > \hat{x}_L$, H would be strictly negative and $\Pi_L(x)$ strictly quasi-concave. For $x_L \leq \hat{x}_L$, $G(\bar{x}) \leq \frac{\mu_R^\eta}{\mu_R^\eta+\mu_L^\eta} - \eta\frac{\mu_R^\eta\mu_L^\eta}{(\mu_R^\eta+\mu_L^\eta)^2}$ implies that $\frac{\partial\Pi_L(x_L,x_R)}{\partial x_L}$ is strictly positive which directly implies that $\Pi_L(x_L, x_R)$ is strictly quasiconcave for $x_L \in [0, \hat{x}_L]$.

We can proceed similarly to show that $\Pi_R(x_L, x_R)$ is also strictly quasiconcave (in that case we use that $g(x)$ implies that $1 - G(x)$ is log-concave). □

Remark 1. *Let (x_L, x_R) be a solution to the Lagrangians. Then, in the platforms stage, there is a divergent equilibrium ($x_L \neq x_R$) only when $\nu_L = \nu_R = 0$ and there is a convergent one ($x_L = x_R$) only when $\nu_L \geq 0$ and $\nu_R \geq 0$.*

Proof. For the divergent equilibria, let $x_L \neq x_R$ be an equilibrium with $\nu_R = 0$. Suppose $\nu_L > 0$. Then $x_L = \frac{1}{2} < x_R$ and $\Pi'_L(\frac{1}{2}, x_R) > 0$. Thus, party L has incentives to deviate to a platform strictly larger than $\frac{1}{2}$ which violates Lemma (A.1). Then $x_L \neq x_R$ with $\nu_R = 0 < \nu_L$ cannot be an equilibrium. Hence, ν_L must be 0 for this type of equilibrium to exist. The same holds true for $\nu_L = 0 < \nu_R$.

For the convergent equilibrium, Lemma (A.1) implies $x_L = x_R = \frac{1}{2}$ is the only candidate to equilibrium. Hence, at the solution to the Lagrangian, $\nu_L \geq 0$ and $\nu_R \geq 0$ is true. Notice that even if $\Pi'_L(\frac{1}{2}, \frac{1}{2}) \geq 0$ and $\Pi'_R(\frac{1}{2}, \frac{1}{2}) \leq 0$, like in a standard Dawnsian game, they do not have incentives to deviate. Suppose L deviates to a platform strictly larger than $\frac{1}{2}$, then it would become the Right party, which implies that if would have incentives to converge to $\frac{1}{2}$. □

Proof of Proposition 1

Proof. Lemma A.1 shows that we can constraint the analysis to the case with $x_L \leq \frac{1}{2} \leq x_R$ without loss of generality. Lemma A.2 shows strict quasiconcavity of the maximization problem. Hence, the first order conditions are necessary and sufficient to characterize the equilibrium. With $\mu_L = \mu_R$ equations 9 and 10 become:

$$\Pi'_L \equiv \frac{\partial \Pi_L(x_L, x_R)}{\partial x_L} = F(y)\frac{g(\bar{x})}{2} - f(y)G(\bar{x}) + f(y)\frac{1}{2} - f(y)\eta\frac{1}{4}, \tag{12}$$

$$\Pi'_R \equiv \frac{\partial \Pi_R(x_L, x_R)}{\partial x_R} = -F(y)\frac{g(\bar{x})}{2} + f(y)[1 - G(\bar{x})] - f(y)\frac{1}{2} + f(y)\eta\frac{1}{4} \tag{13}$$

Remark 1 shows that convergence in the symmetric case implies $\Pi'_L \geq 0$ and $\Pi'_R \leq 0$. If these conditions are met, $x_L = \frac{1}{2} = x_R$ implies $y = 0$. Notice that if $F(0) = 0$, then there is no convergent equilibrium unless $\eta = 0$. For $\eta = 0$, there is only a convergent equilibrium. When $F(0) > 0$, both conditions can be re-written as $\frac{f(0)}{F(0)} \leq \frac{2}{\eta}g(\frac{1}{2})$. Similarly, from Remark 1 we have full polarization when $\Pi'_L \leq 0$ and $\Pi'_R \geq 0$, evaluated at $x_L = 0$, $x_R = 1$. Using that $x_L = 0$ and $x_R = 1$ imply $y = 1$, both conditions above can be re-written as $\frac{f(1)}{F(1)} \geq \frac{2}{\eta}g(\frac{1}{2})$. And when there is a divergent equilibrium (without full polarization), we obtain what we call the interior/interior equilibrium when $\Pi'_L = \Pi'_R = 0$. This equality implies that polarization is implicitly defined by $\frac{f(\bar{y})}{F(\bar{y})} = \frac{2}{\eta}g(\frac{1}{2})$, when there is no convergent or extremism equilibrium, i.e., when the two inequalities above do not hold:

$$\frac{f(1)}{F(1)} < \frac{2}{\eta}g(\frac{1}{2}) < \frac{f(0)}{F(0)}$$

. □

Proof of Proposition 2

Proof. Because of Lemma (A.2) and Remark (1), solving the Lagrangian in the case $\lambda_L = \lambda_R = 0$ and $\nu_L \geq 0$ $\nu_R \geq 0$ suffices for a convergent equilibrium. By Lemma A.1, a convergent equilibrium can only take place at $x_L = x_R = \frac{1}{2}$, which implies $\lambda_i = 0$ for $i = L, R$. Given that ν_L and ν_R are positive, the Kuhn-Tucker conditions for a convergent equilibrium imply $\frac{\partial \Pi_L(x,x)}{\partial x_L} \geq 0$ and $\frac{\partial \Pi_R(x,x)}{\partial x_R} \leq 0$ and using that $G(0.5) = \frac{1}{2}$ they can be written for $i = L, R$ and $-i \neq i$ as:

$$\frac{F(0)}{f(0)}g(0.5) \geq 2\eta\frac{\mu_L^\eta \mu_R^\eta}{(\mu_L^\eta + \mu_R^\eta)^2} + \frac{\mu_i^\eta - \mu_{-i}^\eta}{\mu_L^\eta + \mu_R^\eta}.$$

Given that $\mu_R \geq \mu_L$, if the equation for R is satisfied, it will also be so for L. Finally, note that by substituting $\mu_L = \mu_R$ we obtain the convergent condition for the proof of Proposition (1). For $F(0) = 0$, there is not convergent equilibrium unless $\eta = 0$.

□

Proof of Proposition 3 and Corollary 1

Below we provide the conditions for the existence (Proposition 3) and the characterization (Corollary 1) of the divergent equilibrium in which both parties play interior platforms. Following the classification of Corollary 1 we prove the interior/interior equilibrium in Lemma A.3 and the interior/corner equilibrium in Lemma A.4.

Lemma A.3. *Let $\bar{x}^* = G^{-1}(\frac{\mu_R^\eta}{\mu_L^\eta+\mu_R^\eta})$ and $y^* = \bar{y}$ be implicitly defined by $\frac{f(\bar{y})}{F(\bar{y})} = \frac{g(\bar{x}^*)}{2\eta}\frac{(\mu_L^\eta+\mu_R^\eta)^2}{\mu_L^\eta\mu_R^\eta}$. Hence, there is a unique divergent interior equilibrium $x_L^* = \bar{x}^* - \frac{\bar{y}}{2}$, $x_R^* = \bar{x}^* + \frac{\bar{y}}{2}$ if and only if*

$$\frac{F(2\bar{x}^*-1)}{f(2\bar{x}^*-1)}g(\bar{x}^*) < 2\eta\frac{\mu_L^\eta\mu_R^\eta}{(\mu_L^\eta+\mu_R^\eta)^2} < \frac{F(2-2\bar{x}^*)}{f(2-2\bar{x}^*)}g(\bar{x}^*)$$

which only holds for $\frac{\mu_R^\eta}{\mu_R^\eta+\mu_L^\eta} < G(\frac{3}{4})$.

Proof. Because of Lemma (A.2) and Remark (1), solving the Lagrangian in the case $\lambda_i = \nu_i = 0$ for all i suffices for a divergent interior equilibrium. Using $\bar{x} = (x_L + x_R)/2$ and $y = x_R - x_L$, the pair (x_L^*, x_R^*) is uniquely defined by the pair $(\bar{x}^*, y^*)$ with $y^* = \bar{y}$. We begin by proving that there is a unique $(\bar{x}^*, \bar{y})$ that solves the FOCs when $\lambda_i = 0 = \nu_i$ for all i. From $\frac{\partial\mathcal{L}_L}{\partial x_L} + \frac{\partial\mathcal{L}_R}{\partial x_R} = 0$ and $G(x)$ strictly increasing in x, we obtain the unique $\bar{x}^*$ that solves the FOCs:

$$G(\bar{x}^*) = \frac{\mu_R^\eta}{\mu_L^\eta+\mu_R^\eta} \iff \bar{x}^* = G^{-1}(\frac{\mu_R^\eta}{\mu_L^\eta+\mu_R^\eta}).$$

Plugging $\bar{x}^*$ in 7 or 8, we obtain $\frac{f(\bar{y})}{F(\bar{y})} = \frac{g(\bar{x}^*)}{2\eta}\frac{(\mu_L^\eta+\mu_R^\eta)^2}{\mu_L^\eta\mu_R^\eta}$. Since the conversion rate is increasing and $g(\bar{x}^*)$ can be treated as a constant, there is a unique polarization level $\bar{y}$ that solves the FOCs. Hence, we obtain the unique

$$x_L^* = \bar{x}^* - \frac{\bar{y}}{2}$$

and

$$x_R^* = \bar{x}^* + \frac{\bar{y}}{2}$$

that solve the FOCs. Note that for the case $\mu_L = \mu_R$ we immediately obtain that $G(\bar{x}^*) = \frac{1}{2}$ and by symmetry o $G(x)$ that $\bar{x}^* = \frac{1}{2}$

Finally, we have to check that the above solutions, x_L^* and x_R^* lie within the corresponding policy sub-space $x_L^* \in (0, \frac{1}{2})$ and $x_R^* \in (\frac{1}{2}, 1)$. First, $G(x)$ is increasing, so its inverse is as well. Hence, from $x_L^* \in (0, \frac{1}{2})$, we obtain that $\bar{y} > 2G^{-1}(\frac{\mu_R^\eta}{\mu_R^\eta+\mu_L^\eta}) - 1$, and from $x_R^* \in (\frac{1}{2}, 1)$, we obtain that $\bar{y} < 2 - 2G^{-1}(\frac{\mu_R^\eta}{\mu_R^\eta+\mu_L^\eta})$. Hence, it must be the case that

$$\bar{y} \in \left(2G^{-1}(\frac{\mu_R^\eta}{\mu_R^\eta + \mu_L^\eta}) - 1, 2 - 2G^{-1}(\frac{\mu_R^\eta}{\mu_R^\eta + \mu_L^\eta})\right) \iff \bar{y} \in (2\bar{x}^* - 1, 2 - 2\bar{x}^*), \tag{14}$$

which only holds if $\frac{\mu_R^\eta}{\mu_R^\eta+\mu_L^\eta} < G(\frac{3}{4})$. Also, in Equation 14, we can use the definition of $\bar{y}$ and that the rate $\frac{F(x)}{f(x)}$ is increasing and invertible to obtain the conditions of the equilibrium in terms of the conversion rate:

$$\frac{F\left(2G^{-1}(\frac{\mu_R^\eta}{\mu_R^\eta+\mu_L^\eta}) - 1\right)}{f\left(2G^{-1}(\frac{\mu_R^\eta}{\mu_R^\eta+\mu_L^\eta}) - 1\right)} < \frac{F(\bar{y})}{f(\bar{y})} \frac{1}{g\left(G^{-1}(\frac{\mu_R^\eta}{\mu_R^\eta+\mu_L^\eta})\right)} < \frac{F\left(2 - 2G^{-1}(\frac{\mu_R^\eta}{\mu_R^\eta+\mu_L^\eta})\right)}{f\left(2 - 2G^{-1}(\frac{\mu_R^\eta}{\mu_R^\eta+\mu_L^\eta})\right)}$$

$$\iff$$

$$\frac{F(2\bar{x}^* - 1)}{f(2\bar{x}^* - 1)} < \frac{F(\bar{y})}{f(\bar{y})} \frac{1}{g(\bar{x}^*)} < \frac{F(2 - 2\bar{x}^*)}{f(2 - 2\bar{x}^*)}$$

Note that for the symmetric cost case, $\mu_L = \mu_R$, the above simplifies to $\frac{F(0)}{f(0)}g(\frac{1}{2}) < \frac{\eta}{2} < \frac{F(1)}{f(1)}g(\frac{1}{2})$

□

Lemma A.4. $(0 < x_L^* \leq \frac{1}{2}, x_R^* = 1)$ *Let* $\bar{x}^* = G^{-1}(\frac{\mu_R^\eta}{\mu_L^\eta+\mu_R^\eta})$ *and* $\bar{y}$ *be implicitly defined by* $\frac{f(\bar{y})}{F(\bar{y})} = \frac{g(\bar{x}^*)}{2\eta}\frac{(\mu_L^\eta+\mu_R^\eta)^2}{\mu_L^\eta\mu_R^\eta}$. *Let* x_L^* *be implicitly defined by* $2G(\frac{1+x_L^*}{2}) = \frac{F(1-x_L^*)}{f(1-x_L^*)}g(\frac{1+x_L^*}{2}) + 2\frac{\mu_R^\eta}{\mu_R^\eta+\mu_L^\eta} - 2\eta\frac{\mu_R^\eta\mu_L^\eta}{(\mu_R^\eta+\mu_L^\eta)^2}$.

For $\frac{\mu_R^\eta}{\mu_R^\eta+\mu_L^\eta} < G(\frac{3}{4})$, *there is a unique equilibrium* $(x_L^*, 1)$ *if and only if*

$$\frac{F(2-2\bar{x}^*)}{f(2-2\bar{x}^*)}g(\bar{x}^*) < 2\eta\frac{\mu_L^\eta\mu_R^\eta}{(\mu_L^\eta+\mu_R^\eta)^2} < \frac{F(1)}{f(1)}g(0.5) + \frac{\mu_R^\eta - \mu_L^\eta}{\mu_R^\eta + \mu_L^\eta}$$

For $\frac{\mu_R^\eta}{\mu_R^\eta+\mu_L^\eta} \geq G(\frac{3}{4})$, *there is a unique equilibrium* $(x_L^*, 1)$ *if and only if*

$$\frac{F(\frac{1}{2})}{f(\frac{1}{2})}g(\frac{3}{4}) - 2G(\frac{3}{4}) + \frac{2\mu_R^\eta}{\mu_R^\eta + \mu_L^\eta} < 2\eta\frac{\mu_L^\eta\mu_R^\eta}{(\mu_L^\eta+\mu_R^\eta)^2}$$

Proof. Because of Lemma (A.2) and Remark (1), solving the Lagrangian in the case where $\nu_L = \nu_R = \lambda_L = 0$ and $\lambda_R \geq 0$ is sufficient to find an equilibrium where $x_L^* \leq \frac{1}{2}$ and $x_R^* = 1$. Let $\bar{x} = \frac{1+x_L}{2}$ and $y = 1 - x_L$, then taking into account the conditions on the Lagrange-multipliers:

$$\begin{aligned}\frac{\partial \Pi_L(x_L, x_R=1)}{\partial x_L}+\frac{\partial \Pi_R(x_L, x_R=1)}{\partial x_R} &= \lambda_R \geq 0\\ \frac{\mu_R^\eta}{\mu_R^\eta+\mu_L^\eta} &\geq G(\frac{x_L+1}{2})\\ 2G^{-1}(\frac{\mu_R^\eta}{\mu_R^\eta+\mu_L^\eta}) &\geq x_L+1 \end{aligned} \tag{15}$$

Lemma A.1 and Equation (15) imply that $x_L \leq \min\{2G^{-1}(\frac{\mu_R^\eta}{\mu_R^\eta+\mu_L^\eta})-1, \frac{1}{2}\}$. Taking into account that $2G^{-1}(\frac{\mu_R^\eta}{\mu_R^\eta+\mu_L^\eta})-1<\frac{1}{2} \iff \frac{\mu_R^\eta}{\mu_R^\eta+\mu_L^\eta}<G(\frac{3}{4})$, we solve the following:

- If $\frac{\mu_R^\eta}{\mu_R^\eta+\mu_L^\eta}<G(\frac{3}{4})$, then $\frac{\partial \Pi_L(x_L,1)}{\partial x_L}|_{x_L=2\bar{x}^*-1} \leq 0$ must hold in equilibrium. Plugging in $x_L=2\bar{x}^*-1$ and $x_R=1$ and using $G(\bar{x}^*)=\frac{\mu_R^\eta}{\mu_R^\eta+\mu_L^\eta}$ in Equation (7) we obtain

$$\frac{F(2-2\bar{x}^*)}{f(2-2\bar{x}^*)}g(\bar{x}^*) \leq 2\eta\frac{\mu_L^\eta\mu_R^\eta}{(\mu_L^\eta+\mu_R^\eta)^2} \tag{16}$$

- If $\frac{\mu_R^\eta}{\mu_R^\eta+\mu_L^\eta} \geq G(\frac{3}{4})$, then $\frac{\partial \Pi_L(x_L,1)}{\partial x_L}|_{x_L=0.5} \leq 0$ must hold in equilibrium. Using $x_L=0.5$ in Equation (7) we obtain

$$\frac{F(0.5)}{f(0.5)}-\frac{2}{g(\frac{3}{4})}G(\frac{3}{4})+\frac{2}{g(\frac{3}{4})}\frac{\mu_R^\eta}{\mu_R^\eta+\mu_L^\eta}-\frac{2}{g(\frac{3}{4})}\eta\frac{\mu_R^\eta\mu_L^\eta}{(\mu_R^\eta+\mu_L^\eta)^2} \leq 0$$
$$\iff$$
$$\frac{F(0.5)}{f(0.5)}g(\frac{3}{4})-2G(\frac{3}{4})+\frac{2\mu_R^\eta}{\mu_R^\eta+\mu_L^\eta} \leq 2\eta\frac{\mu_L^\eta\mu_R^\eta}{(\mu_L^\eta+\mu_R^\eta)^2} \tag{17}$$

Finally, the fully divergent equilibrium is excluded when we assure $x_L>0$, i.e., when $2\eta\frac{\mu_L^\eta\mu_R^\eta}{(\mu_L^\eta+\mu_R^\eta)^2}<\frac{F(1)}{f(1)}g(0.5)+\frac{\mu_R^\eta-\mu_L^\eta}{\mu_R^\eta+\mu_L^\eta}$ (see Proposition 4 below). Notice that when $\frac{\mu_R^\eta}{\mu_R^\eta+\mu_L^\eta} \geq G(\frac{3}{4})$, the latter inequality always holds.[16] □

Proof of Proposition 4

Proof. Because of Lemma (A.2) and Remark (1), solving the Lagrangian in the case where $\nu_i=0$ and $\lambda_i \geq 0$ for all i is sufficient to find an equilibrium with $x_L=0$ and

[16]Recall that $\eta \leq \bar{\eta}$ implies that $2\eta\frac{\mu_L^\eta\mu_R^\eta}{(\mu_L^\eta+\mu_R^\eta)^2}<2\frac{\mu_L^\eta+\mu_R^\eta}{\mu_R^\eta}\frac{\mu_L^\eta\mu_R^\eta}{(\mu_L^\eta+\mu_R^\eta)^2}=2\frac{\mu_L^\eta}{\mu_L^\eta+\mu_R^\eta}$. Also $2\frac{\mu_L^\eta}{\mu_L^\eta+\mu_R^\eta} \leq \frac{\mu_R^\eta-\mu_L^\eta}{\mu_R^\eta+\mu_L^\eta} \iff \mu_R^\eta \geq 3\mu_L^\eta$. Then, single-peakedness and symmetry of $g(x)$ imply $G(\frac{3}{4})>\frac{3}{4}$. Finally $\mu_R^\eta \geq 3\mu_L^\eta$ follows directly from $\frac{\mu_R^\eta}{\mu_R^\eta+\mu_L^\eta} \geq G(\frac{3}{4})$

$x_R = 1$. Hence, from Equation 7 and 8,

$$\lambda_L \geq 0 \text{ iff } \frac{F(1)}{f(1)} g(0.5) + \frac{\mu_{-i}^{\eta} - \mu_i^{\eta}}{\mu_L^{\eta} + \mu_R^{\eta}} \leq 2\eta \frac{\mu_L^{\eta} \mu_R^{\eta}}{(\mu_L^{\eta} + \mu_R^{\eta})^2}.$$

As $\mu_R \geq \mu_L$, the condition for party L is the sufficient one. Last, $\frac{F(1)}{f(1)} g(0.5) > 0$, so a necessary condition for a fully divergent equilibrium is

$$2\eta \mu_L^{\eta} \mu_R^{\eta} \geq \mu_R^{2\eta} - \mu_L^{2\eta} = (\mu_R^{\eta} - \mu_L^{\eta})(\mu_R^{\eta} + \mu_L^{\eta}) \geq (\mu_R^{\eta} - \mu_L^{\eta}) \eta \mu_R^{\eta},$$

where the last inequality follows from $\eta \leq \bar{\eta}$. And the expression above simplifies to $3\mu_L^{\eta} \geq \mu_R^{\eta}$. □

5.3 Comparative statics

Proof of Lemma (2).

Proof. By Proposition (1), the interior equilibrium arises if $\eta \in [2g(\frac{1}{2})\frac{F(0)}{f(0)}, 2g(\frac{1}{2})\frac{F(1)}{f(1)}]$. Using the implicit function theorem we can write: $\frac{\partial \bar{y}}{\partial \eta} = \frac{1}{2g(0.5)\left[\frac{F(\bar{y})}{f(\bar{y})}\right]'}$. Then,

$$\frac{\partial \mu e_i^*(x_L^*, x_R^*)}{\partial \eta} = -f(\bar{y}) \frac{\partial \bar{y}}{\partial \eta} \frac{\eta}{4} + \frac{1 - F(\bar{y})}{4} = -\frac{f(\bar{y})}{2g(0.5)\left[\frac{F(\bar{y})}{f(\bar{y})}\right]'} \frac{\eta}{4} + \frac{1 - F(\bar{y})}{4}.$$

Hence,

$$\frac{\partial \mu e_i^*(x_L^*, x_R^*)}{\partial \eta} \geq 0 \iff 1 - F(\bar{y}) \geq \frac{f(\bar{y})}{2g(0.5)\left[\frac{F(\bar{y})}{f(\bar{y})}\right]'} \eta \iff \eta \leq \frac{1 - F(\bar{y})}{f(\bar{y})} 2g(0.5) \left[\frac{F(\bar{y})}{f(\bar{y})}\right]'$$

where $\left[\frac{F(\bar{y})}{f(\bar{y})}\right]'$ is a positive number by log-concavity of $F(y)$.

□

Proof of Lemma (3)

Proof. By Proposition (1), the interior equilibrium arises if $\eta \in [2g(\frac{1}{2})\frac{F(0)}{f(0)}, 2g(\frac{1}{2})\frac{F(1)}{f(1)}]$. Then,

$$\frac{\partial \mu e_i^*(x_L^*, x_R^*)}{\partial \rho} = -\frac{\partial F(y^*)}{\partial \rho} \frac{\eta}{4} - \frac{\partial F(y^*)}{\partial y^*} \frac{\partial y^*}{\partial \rho} \frac{\eta}{4}$$

and hence $\frac{\partial \mu e_i^*(x_L^*, x_R^*)}{\partial \rho} \geq 0$ if and only if $-\frac{\partial F(\bar{y})}{\partial \rho} \geq f(\bar{y}) \frac{\partial \bar{y}}{\partial \rho}$. □

5.4 Voters' behavior

In the main text we preferred to introduce as an assumption a) the endogenous division of voters across ideological and impressionable, and b) the Tullock contest in the campaign stage to avoid unnecessary additional notation. The following arguments can formally justify such behavior.

5.4.1 Endogenous division of voters

Semiorder (or weak) lexicographic preferences

Semiorder lexicographic preferences (Tversky, 1969) are described as lexicographic preferences "disregarding" small differences Fishburn (1974). Instead of requiring the available alternatives to be identical in the dominant attribute to consider the dominated one as standard lexicographic preferences, semiorder lexicographic preferences require the dominant attribute of the alternatives to be similar enough.

The introduction of this type of preference in our model can be done as follows. Assume a population of voters of measure 1. Voters are heterogeneous in two dimensions. First, they are heterogeneous regarding their favorite platform, which is represented by $x \in [0,1]$ and is distributed according to $G(x)$. Second, voters are heterogeneous in their sensitivity towards differences in the ideology space. We denote by ϕ the value of the minimal distance between the two platforms that a voter considers to be "relevant". In the terms of the experimental literature in human perception (or psychophysics), ϕ can be interpreted as the just-noticeable difference.

If the distance between the two platforms is less than ϕ, then the voter considers the two parties identical in terms of their policy proposal and moves to the dominated attribute which is electoral advertising, e_i. Note that voters with $\phi = 0$ have standard lexicographic preferences, while if $\phi > 0$ voters do not have a bliss point as usual but a bliss interval. Although x is irrelevant for impressionable voters, all individuals are identified by the pair (x, ϕ).

The above features can be represented in an analytical manner by adapting the semiorder lexicographic structure proposed by Luce (1978). Considering voter x, ϕ, we can write:

$$\vartheta_{x,\phi}(i) = - \mid x - x_i \mid \Upsilon(\phi \leq x_R - x_L) + e_i^{\eta} \theta_i^{x} [1 - \Upsilon(\phi \leq x_R - x_L)] \tag{18}$$

where $\Upsilon(\phi \leq x_R - x_L)$ is an indicator function taking value 1 when $\phi \leq x_R - x_L$. Given a pair of policy platforms x_L and x_R, voters with $\phi > x_R - x_L$ are indifferent in terms of the policy space and have been called impressionable in the main text. Individuals with $\phi \leq x_R - x_L$ have been called ideological.

Given the above indicators and denoting by $F(\phi)$ the distribution function of ϕ over the population, the proportion of ideological voters is $F(x_R - x_L)$, and consequently, $1 - F(x_R - x_L)$ for impressionable voters.

Platform preferences and ideology sensitivity are assumed to be independent, i.e., $G(x|\phi) = G(x)$ and $F(\phi|x) = F(\phi)$. Consequently, for a given pair of policy platforms x_L and x_R, the votes of ideological and impressionable voters can be independently aggregated. By taking into account the considerations presented in the main text, parties' vote shares can be immediately written as in 1.

Salience and attention

In salience models decision makers overweight attributes that exhibit greater heterogeneity in the available choice set (Bordalo et al. 2012, 2013a,b, 2015). The endogenous segmentation between the two type of voters can also be obtained under a particular example of salient thinking.

An important difference between our model and the ones above is that while in their context their consider two vertical attributes, our case combines a bounded horizontal attribute (policy space) and a vertical attribute (campaign spending). This makes difficult to define a salience function comparable in terms of the two attributes. As an alternative, one can fix a constant level of salience of campaign spending ϕ. For ideology, we can define a salience function for ideology $\sigma(x_L, x_R)$ as in Bordalo et al. 2012. In this context, ideology can be defined as salient if and only if $\sigma(x_L, x_R) \geq \phi$. Finally, one can consider the rank-based weighting salience function proposed by Bordalo et al. 2012, 2013b, 2015, where:

$$\vartheta_{x,\phi}(i) = \begin{cases} -(\mid x - x_i \mid) + \omega e_i^{\eta} \theta_i^{x} & \text{if } \sigma(x_L, x_R) \geq \phi \\ -\omega \mid x - x_i \mid + e_i^{\eta} \theta_i^{x} & \text{if } \sigma(x_L, x_R) < \phi \end{cases}$$

By assuming the salience distortion to be $\omega = 0$ and that individuals are heterogeneous in $\phi \sim F(\phi)$ (as well as a mass $1-$ of individuals for which electoral advertising is always salient) one obtains the vote shares in 1.

5.4.2 Campaign stage as a Tullock contest

The Tullock contest for the impressionable voters could be due to voters' behavior in a model where the utility of an impressionable voter with ideology x that votes for party i is $v_x(i) = e_i^{\eta} \theta_i$, impressionable citizens vote the party that gives a higher $v_x(i)$, where θ_L and θ_R either follow independent inverse exponential distributions (Jia, 2008) or are drawn from any *iid* process where parties spend $\mu_i e_i$ to have e_i^{η} independent attempts and the electoral competition takes a best-shot form (Baye and Hoppe, 2003; Fullerton and McAfee, 1999; Fu and Lu, 2012).[17]

[17] The assumption that party shocks are identical to all individuals is made without loss of generality. Alternatively, one can assume indivdual specific party shocks θ_i^x that are iid across individuals and parties.

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