Firm Growth and Promotion Opportunities*

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Abstract

We develop a model in which a firm makes a sequence of production decisions and has to motivate each of its employees to exert effort. The firm motivates its employees through incentive pay and promotion opportunities, which may differ across different cohorts of workers. We show that the firm benefits from reallocating promotion opportunities across cohorts, resulting in an optimal personnel policy that is seniority-based. We also highlight a novel time-inconsistent motive for firm growth: when the firm adopts an optimal personnel policy, it may pursue future growth precisely to create promotion opportunities for existing employees.

Keywords: firm growth, promotions, internal labor markets, dynamic incentives

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1 Introduction

To meet the increased demand for explosives brought about by World War I, DuPont expanded its work force from 5,000 in 1914 to 85,000 in 1918. But its executives were keenly aware that the war would not last forever, and they formed plans for post-war diversification in part to ensure their employees would continue to have jobs. In addition to entering into related chemical-based industries, they made investments in other industries to have "a place to locate some managerial personnel who might not be absorbed by the expansion into chemical-based industries." (Chandler, 1962, p. 90)

This example illustrates that management requires planning ahead: it requires planning future production both to adapt to future business conditions and to set up future career opportunities for current employees. Such opportunities are abundant in fast-growing firms and can be used to great effect in motivating employees. In declining firms, they may be scarce or nonexistent (Bianchi et al., 2018). And when a firm's growth rate fluctuates, they may vary across different cohorts of employees who were hired at different times. Since firms need to keep *all* their employees motivated, production plans affect the kinds of personnel policies they should adopt.

At the same time, the firm's personnel policies influence its future production plans, as scholars of management and strategy have long argued. Barnard (1938), for example, points out that there is an "innate propensity for all organizations to expand... to grow seems to offer opportunity for the realization of all kinds of active incentives." (p. 159). Similarly, Jensen (1986) claims that using promotions to motivate employees "creates a strong organizational bias toward growth to supply the new positions that such promotion-based reward systems require." (p. 2)

Such growth is often criticized as "growth for growth's sake" or "empire building."[1] Yet, this type of "growth for opportunity's sake" may serve an important purpose. Bennett and Levinthal (2017), for example, argues that firm growth "can have implications for the firm's competitive advantage as a result of the impact of firm growth on the firm's ability to motivate and incentivize its employees." (p. 2006) Production plans influence and are influenced by personnel policies, and they should therefore be designed together.

This paper provides an attempt to understand how a firm's production plans interact with its promotion-based personnel policies. Existing economic theories are not well suited to explore this interaction, since they either focus on the forward-looking determinants of firm growth without

[1] Porter (1998), for example, notes that, "the challenge for general management is to find new ways to motivate and reward personnel," when firm growth begins to slow but cautions that it can be a "serious error" for firms "to diversify to provide the growth and advancement possibilities." (p. 251)

accounting for long-term employee incentives,[2] or they focus on long-term incentives for individual employees without exploring their implications for the size of the firm's workforce.[3]

Our contributions are twofold. First, we highlight how fluctuations in growth opportunities can lead firms to optimally adopt seniority-based promotion policies. Second, we demonstrate how the use of promotion-based incentives leads to a time-inconsistent opportunity-creation motive for firm growth. In doing so, we are able to assess when and how firms should pursue seemingly unprofitable growth strategies.

Model In order to study the interplay between firms' personnel policies and production plans, we develop a model with two key features. First, employees are motivated by their career prospects in the firm, and these career prospects depend on the firm's choice of pay and promotion policies. Second, the firm's ability to promote employees depends on how many employees it needs, which in turn depends on the demand for the products or services it produces.

In our model, a single principal interacts repeatedly with a pool of employees. The interaction between the principal and each employee is a dynamic moral hazard problem with a limited-liability constraint. The principal assigns each employee, in each period, to one of two jobs: a bottom job and a top job. In each job, the employee faces a moral hazard problem and must be provided incentives to exert effort. In the top job, the firm motivates the employee by paying a bonus for good performance. In the bottom job, the firm motivates the employee through a combination of bonuses for good performance and the prospect of being promoted to the top job.

Employees' promotion prospects depend both on how many new top positions there will be in the next period—which is determined by the firm's growth prospects—and on how many people are in line for these positions today—which is determined both by firm's production decisions today and by the personnel policies it has in place. The firm's problem is therefore to make its production plans and design its personnel policies jointly to maximize its profits.

Results and Implications Our first set of results shows how the firm optimally designs its personnel policies given its production plan. The firm's production plan determines how many promotion opportunities will arise and when, and the firm's problem is to figure out how to allocate these opportunities across different cohorts of employees who began employment at different times. Under an optimal promotion policy, promotion opportunities are not wasted, in the sense that they

[2] Lucas (1978), Jovanovic (1982), Hopenhayn (1992), Ericson and Pakes (1995).

[3] See Rogerson (1985) and Spear and Srivastava (1987) for early contributions and Biais, Mariotti, and Rochet (2013) for a recent survey with a focus on financial contracts.

are allocated in a way that minimizes the firm's wage bill.

The first main result is that firms optimally allocate promotion opportunities across cohorts according to a modified first-in-first-out rule that favors employees with more seniority. Under such a promotion policy, employees' promotion probabilities are weakly increasing in their seniority, and in each period t, employees in the bottom job can be divided into three categories depending on the period in which they were hired. Senior employees hired prior to some $\tau_1(t)$ are all promoted with the highest probability; junior employees hired after some $\tau_2(t)$ will be promoted with probability 0; employees hired in between $\tau_1(t)$ and $\tau_2(t)$ will be promoted with positive probability that is increasing in their seniority.

Our second set of results shows that optimal personnel policies lead firms to make time-inconsistent production plans: higher demand for the firm's product today can cause the firm to become "inefficiently large" in the future. Specifically, if we compare two firms that face the same future demand conditions, a firm facing better demand conditions today will have more employees both today and in the future. This path-dependence reflects an intertemporal trade-off: while the firm may be inefficiently large going forward, the firm can pay today's new hires with the opportunities this additional growth entails, therefore reducing the firm's overall wage bill.

Extensions The tools we develop to analyze optimal personnel policies also allow us to explore how firms should manage employees' careers when business conditions require the firm to downsize. An optimal personnel policy for a firm that has to make permanent cuts involves a first-in-last-out layoff policy and seniority-based severance payments: all laid-off employees are paid a severance payment upon dismissal, and less-senior employees are dismissed first and receive a smaller severance payment. If the cuts the firm has to make are only temporary, then an optimal personnel policy entails seniority-based temporary layoffs: less-senior employees are laid off, but once the firm begins hiring again, it rehires them before it hires new employees. The analysis also shows that optimal personnel policies might occasionally encourage employees to take time off, even when the firm is not downsizing.

Literature Review This paper contributes to the literature on internal labor markets.[4] An important set of features our model highlights is that optimal personnel policies involve seniority-based promotions. The existing literature argues that seniority-based promotion policies may be beneficial, since they can motivate efficient turnover (Carmichael, 1983), curtail rent seeking (Mil-

[4] See Gibbons (1997), Gibbons and Waldman (1999b), Lazear (1999), Lazear and Oyer (2013), and Waldman (2013) for reviews of the theory on and evidence regarding internal labor markets.

grom and Roberts, 1988; Prendergast and Topel, 1996), and allow firms to better capture employees' information rents (Waldman, 1990). While these papers establish benefits associated with seniority-based promotion policies, they do not establish that such policies are *optimal* ways to allocate scarce opportunities.

Our paper also contributes to the literature on the determinants of firm growth.[5] The idea that firms might grow in order to create promotion opportunities has been only informally articulated in the management, strategy, and finance literatures.[6] A paper that shares a similar motivation to ours is Bennett and Levinthal (2017). In their model, however, growth is not driven by the promotion opportunities it creates, which is central in our model.

Finally, our paper contributes to the literature on dynamic moral hazard problems. In dynamic moral hazard settings, nontrivial dynamics can arise in many different settings.[7] The closest papers are Board (2011) and Ke, Li, and Powell (2018). In Board's (2011) model, firms hire one supplier in each period, and its focus is on which supplier to utilize. Our model focuses on the number of employees to hire in each period, which can change from period to period. Ke, Li, and Powell (2018) examines how organizational constraints affect firms' personnel policies in a stationary environment in which the size of the firm is constant. In this case, there are no gains to reallocating promotion opportunities across cohorts, and optimal personnel policies are seniority blind. In our model, uneven growth leads to seniority-based personnel policies, and the need to provide long-term incentives leads the firm to adopt time-inconsistent production plans.

2 The Model

A firm interacts with a pool of risk-neutral workers in periods $t = 1, \ldots, T$, where T may be infinite, and all players share a common discount factor $\delta \in (0, 1)$. The firm's labor pool consists of a large mass of identical workers, and the firm chooses a personnel policy, which we will describe below, to maximize its discounted profits.

Production requires two types of activities to be performed, and each worker can perform a

[5] Jovanovic (1982), Hopenhayn (1992), Ericson and Pakes (1995), Albuquerque and Hopenhayn (2004), Clementi and Hopenhayn (2006).

[6] Barnard (1938), Chandler (1962), Drucker (1977), Jensen (1986), Baker, Jensen, and Murphy (1988), Porter (1998).

[7] Such dynamics are highlighted by DeMarzo and Fishman (2007) and Clementi and Hopenhayn (2006) in a financial contracting setting (see Biais, Mariotti, and Rochet (2013) for a survey), by Manove (1997), Halac (2012), Zhu (2012), Li and Matouschek (2013), Fong and Li (2017), Moroni (2017), Barron and Powell (2018), and Fudenberg and Rayo (2018) in an employment setting, by Boleslavsky and Kelly (2014) in a regulatory setting, by Padro i Miquel and Yared (2012) in a political economy setting, by Board (2011), Andrews and Barron (2016), and Urgun (2017) in a supplier allocation setting, and by Bird and Frug (2017), Li, Matouschek, and Powell (2017), Lipnowski and Ramos (2017), Fershtman (2018), Forand and Zápal (2018), and Guo and Hörner (2018) in a delegation setting.

single activity in each period. A worker performing activity $i \in \{1,2\}$ in period t chooses an effort level $e_t \in \{0,1\}$ at cost $c_i e_t$. A worker who chooses $e_t = 0$ is said to *shirk*, and a worker who chooses $e_t = 1$ is said to *exert effort*. We refer to a worker who exerts effort as *productive*. A worker's effort is his private information, but it generates a publicly observable signal $y_{i,t} \in \{0,1\}$ with $\Pr[y_{i,t} = 1 | e_t] = e_t + (1-q_i)(1-e_t)$, that is, shirking in activity i is contemporaneously detected with probability q_i. If the firm employs masses $N_{1,t}$ and $N_{2,t}$ of productive workers in the two activities, revenues are $\theta_t f(N_{1,t}, N_{2,t})$, where θ_t is the firm's period-t *demand parameter*, and f is continuously differentiable, concave, and satisfies $\lim_{N_{i,t}\to 0} \partial f(N_{1,t}, N_{2,t}) / \partial N_{i,t} = \infty$ and $\lim_{N_{i,t}\to\infty} \partial f(N_{1,t}, N_{2,t}) / \partial N_{i,t} = 0$ for $i = 1,2$. We will refer to $\theta = (\theta_1, \ldots, \theta_T)$ as a *demand path*.

In each period, the firm assigns each worker to an activity $A_t \in \mathcal{A} \equiv \{0,1,2\}$, where activity 0 is a non-productive activity. The worker either accepts the assignment or rejects the assignment and exits the firm's labor pool, receiving an outside option that yields utility 0. If $A_t \neq 0$, and the worker accepts the assignment, he then exerts effort e_t, his signal $y_{A_t,t}$ is realized, and then the firm pays the worker an amount $W_t \geq 0$. That is, the worker is protected by a limited-liability constraint. At the end of each period, each worker exogenously exits the firm's labor pool with probability d and receives 0 in all future periods, and a group of new workers enters the firm's labor pool.

Define a worker's *employment history* to be a sequence $h^t = (0, \ldots, 0, A_\tau, \ldots, A_t) \in \mathcal{H}^t$, where $A_s \in \mathcal{A}$ specifies the activity he was assigned to in period s, and τ is the time at which he first enters the firm's labor pool. By convention, we say that a worker is assigned to activity 0 in each period before he is in the firm's labor pool. We will say that a worker who is assigned to activity 1 or 2 for the first time in period t is a *new hire in period t* and that he is a *cohort-t worker*. Define $L(h^t)$ to be the mass of workers with employment history h^t.

Before we define a contract between the firm and a worker, we pause to make two observations that will simplify notation. First, if a worker is assigned to activity 1 or 2 and is not asked to exert effort this period, we can instead assign him to activity 0 this period. Second, if a worker is assigned to activity 1 or 2 and is asked to exert effort, it is without loss of generality to pay him 0 in this period and in all future periods if his signal is equal to 0. This follows because when a worker's signal is 0, the worker must have shirked, and this is the harshest punishment possible.

Given these two observations, we can now define a contract between the firm and a worker. A *contract* is a sequence of *assignment policies* $P_{i,t} : \mathcal{H}^t \to [0,1]$ specifying the probability the worker is assigned to activity i in period $t+1$ given employment history h^t and a sequence of *wage policies* $W_t : \mathcal{H}^t \to [0,\infty)$ specifying the wage the worker receives at the end of period t given his history.

A *personnel policy* is a set of contracts the firm has with each worker in its labor pool.

The firm's period-t profits are

$$\theta_t f(N_{1,t}, N_{2,t}) - \sum_{h^t \in H^t} W_t(h^t) L(h^t),$$

and each worker's period t utility is $W_t(h^t) - c_i e_t$. The firm's problem is to choose $(W_t)_{t=1}^T$ and $(P_{i,t})_{i,t}$ to maximize its expected discounted profits, and given the contract he faces, each worker chooses his acceptance decisions and effort decisions to maximize his expected discounted utility. Throughout most of the analysis, we will be focusing on contracts for which if a worker is ever assigned to activity 0 after he has been assigned to activity 1 or 2, he is assigned to activity 0 and receives a wage of 0 in all future periods. We will refer to such contracts as *full-effort contracts* because they motivate the worker to exert effort in every period they have been employed by the firm. In Section 7.2, we discuss situations where it may be optimal to permit workers to shirk in some periods.

Finally, we define a *production plan* to be a sequence $N = (N_{1,t}, N_{2,t})_{t=1}^T$ that specifies the mass of productive workers in each activity in each period. We will say that a production plan is *steady* if $N_{2,t+1} < (1-d)(N_{1,t} + N_{2,t})$ and $N_{i,t+1} \geq N_{i,t}(1-d)$ for $i = 1, 2$. The first condition says that the number of top positions in the firm does not grow too fast—it ensures that, in each period, there are enough incumbent workers to fill all the activity 2 positions. The second condition says that the firm does not shrink too fast.

To facilitate the exposition, we discuss the role of several of the model's assumptions after we state the main results. In particular, in section 6, we discuss the role of worker homogeneity, the monitoring structure, and deterministic demand paths.

3 Preliminaries

Our analysis solves the firm's problem in two steps. First, given any production plan N, we derive properties of *optimal personnel policies* that induce a mass $N_{i,t}$ of workers assigned to activity i to exert effort in period t. The second step of the firm's problem is to choose an optimal production plan N^* given a demand path θ. Section 5 analyzes the second step of the firm's problem.

3.1 Cost-Minimization Problem

Recall that a personnel policy is a set of contracts the firm has with each worker in its labor pool, where each contract describes the assignment policy and the wage policy the worker is subject to. Given a production plan N, we will say that a personnel policy *implements* N if, given the personnel policy, a mass $N_{1,t}$ and $N_{2,t}$ of workers exerts effort in activities 1 and 2 in period t. Denote a worker's *initial-hire history* by $n^t = (0, \ldots, 0, n_t)$, where $n_t \in \{1, 2\}$. The first lemma shows that the problem of characterizing cost-minimizing personnel policies can be simplified by focusing on a smaller class of personnel policies that do not depend on the identity of the individual worker. All the proofs are in the appendix.

Lemma 1. *Given N, if there is an optimal personnel policy, there is an optimal personnel policy in which workers with the same employment history face the same wage and assignment policies.*

In order to specify the firm's problem, define $c\left(h^t\right) = c_{A_t}$ if $A_t \in \{1, 2\}$ and 0 otherwise, and $q\left(h^t\right) = q_{A_t}$ if $A_t \in \{1, 2\}$. Denote by $w\left(h^t\right)$ the wage the worker receives if $y_{A_t,t} = 1$ and by $p_i\left(h^t\right)$ the probability the worker is assigned to activity i in the next period, conditional on remaining in the labor pool. Denote by $h^t A_{t+1} = (A_1, \ldots, A_{t+1})$ the concatenation of h^t with A_{t+1}. For all workers in the labor pool, we have

$$L\left(h^t i\right) = (1 - d)\, p_i\left(h^t\right) L\left(h^t\right).$$

Given a production plan N, the firm's problem is to minimize its *wage bill*

$$\min_{w(\cdot), p_i(\cdot)} \sum_{t=1}^{T} \sum_{h^t \in \mathcal{H}^t} \delta^{t-1} L\left(h^t\right) w\left(h^t\right)$$

subject to the following constraints.

Promise-Keeping Constraints. If we denote by $v\left(h^t\right)$ the worker's expected discounted payoffs at time t given employment history h^t, then workers' payoffs have to be equal to the sum of their current payoffs and their continuation payoffs:

$$v\left(h^t\right) = w\left(h^t\right) - c\left(h^t\right) + \delta\,(1 - d) \sum_{i \in \{1,2\}} p_i\left(h^t\right) v\left(h^t i\right). \tag{1}$$

Incentive-Compatibility Constraints. Workers prefer to exert effort in activity i if they cannot gain by shirking:

$$v\left(h^t\right) \geq (1 - q\,(h_t)) \left(w\left(h^t\right) + \delta\,(1 - d) \sum_{i \in \{1,2\}} p_i\left(h^t\right) v\left(h^t i\right) \right).$$

If we substitute (1) into this inequality, it becomes:

$$v\left(h^t\right) \geq \frac{1 - q_{A_t}}{q_{A_t}} c_{A_t} \equiv R_{A_t}, \tag{2}$$

where we refer to the quantity R_i as the *incentive rent for activity i*. Note that these incentive-compatibility constraints imply that workers receive positive surplus in equilibrium, and they imply that workers' participation constraints are also satisfied. We therefore do not explicitly include workers' participation constraints in the firm's problem.

Flow Constraints. In each period, the firm employs a mass $N_{i,t}$ workers in activity i:

$$\sum_{h^t|A_t=i} L\left(h^t\right) = N_{i,t}, \text{ for } i \in \{1,2\}. \tag{3}$$

Given these constraints, the firm maximizes its profits. For a given production plan, the firm's discounted profits are equal to the total discounted surplus net of the rents it pays to workers. Given a production plan, therefore, the firm's problem is to minimize these rents. Recall that a worker with employment history n^t is a worker who is first employed by the firm in period t.

Lemma 2. *Cost-minimizing personnel policies minimize the rents paid to new hires:*

$$\min \sum_{t=1}^{T} \sum_{h^t \in \mathcal{H}^t} \delta^{t-1} L\left(n^t\right) v\left(n^t\right)$$

subject to (1)*,* (2)*, and* (3)*.*

Lemma 2 shows that for a given production plan, the firm's cost-minimization problem is equivalent to minimizing the present discounted value of the rents that are paid to new hires. It will be conceptually convenient to decompose the rents paid to new hires into three components: the number of new hires, their necessary rents, and their excess rents. For the incentive-compatibility constraint to hold, the *necessary rents* that a new hire in period t must receive is R_{n_t}. We will refer to the quantity $v\left(n^t\right) - R_{n_t}$ as the *excess rents* paid to cohort-t workers. To reduce the rents that are paid to new hires, the firm therefore wants to reduce the number of new hires, as well as both the necessary and excess rents paid to new hires. We will now draw out the implications of this observation.

3.2 Internal Labor Markets

This section shows that internal labor markets are personnel policies that serve to minimize the number of new hires as well as the necessary rents for a given production plan. The firm's cost-minimization problem involves choosing an assignment probability and a wage payment for each worker at each history. For ease of exposition, we will first focus our analysis on steady production plans, returning to "unsteady" production plans in Section 7.1. Recall that a production plan N is steady if $N_{2,t+1} < (1-d)(N_{1,t} + N_{2,t})$ and $N_{i,t+1} \geq N_{i,t}(1-d)$ for $i = 1, 2$. In the lemma below, we show that we can focus on a narrower class of personnel policies without loss of generality.

Lemma 3. *Given a steady production plan N, if there is a cost-minimizing personnel policy, there is a cost-minimizing personnel policy with the following three properties:*

(i.) $v\left(h^t\right) \le R_2$ *if* $A_t = 1$ *and* $v\left(h^t\right) = R_2$ *if* $A_t = 2$.

(ii.) *All new workers are assigned to activity* 1, *except at* $t = 1$.

(iii.) $p_2\left(h^t\right) = 1$ *if* $A_t = 2$ *and* $p_1\left(h^t\right) + p_2\left(h^t\right) = 1$ *if* $A_t = 1$.

The first part of Lemma 3 shows that in a cost-minimizing personnel policy, the firm does not gain by rewarding workers with rents exceeding R_2. Part (ii.) of the lemma shows that new hires are always assigned to activity 1—our restriction to steady production plans rules out situations where the firm has grown so rapidly that it must necessarily hire new workers directly into activity 2. The exception to this is period 1, where the firm must hire $N_{2,1}$ workers into activity 2. The final part of the lemma shows that workers assigned to activity 2 will continue to be assigned to activity 2, and workers assigned to activity 1 will either continue to be assigned to activity 1 in the next period or will be "promoted" to activity 2. The restriction to steady production plans ensures that such an assignment policy is consistent with the firm's flow constraints.

Lemma 3 highlights several features that are consistent with Doeringer and Piore's (1971) description of internal labor markets: (1) there is a port of entry, (2) there is a well-defined career path, and (3) wages increase upon promotion. We will say that a personnel policy satisfying these three properties is an *internal labor market*. Lemma 3 immediately implies the following proposition.

Proposition 1. *Given a steady production plan N, if there is a cost-minimizing personnel policy, it can take the form of an internal labor market.*

Proposition 1 shows that in a relatively stable environment, a cost-minimizing personnel policy can be implemented as an internal labor market. This result generalizes Proposition 3 in Ke, Li, and Powell (2018). To develop some intuition for why an internal labor market is cost-minimizing, first notice that by setting $p_1\left(h^t\right) + p_2\left(h^t\right) = 1$ on the equilibrium path for all workers assigned to activity 1 or 2, the firm does not introduce unnecessary turnover, which in turn minimizes the total number of new hires. Next, to see why the firm optimally assigns new hires to activity 1, suppose the firm has an opening in activity 2. If it assigns a new hire to that opening, it would have to pay him at least R_2 in terms of first-period rents. If the firm instead assigns that new hire to activity 1, it can pay him weakly less in terms of first-period rents. Moreover, the firm can fill the opening with someone who is currently assigned to activity 1 by promoting him. This would make him value his future in the firm more, and the firm may be able to reduce his wages. Promotion opportunities therefore serve as a free incentive instrument the firm should allocate optimally.

4 Allocating Promotion Opportunities Across Cohorts

As we argued in the previous section, internal labor markets minimize the number of new hires and their necessary rents by using promotion opportunities as a free incentive instrument to motivate workers assigned to activity 1. The firm's objective therefore boils down to minimizing excess rents paid to new hires. To do so, the firm's problem is to allocate promotion opportunities, which arrive at different times, to different cohorts of workers.

Recall that in an internal labor market, workers assigned to activity 2 remain assigned to activity 2, and their wages are independent of when they began employment. An internal labor market can therefore be summarized by the pre-promotion wages, promotion probabilities, and values, $w_{1,t}^{\tau}$, p_t^{τ}, and $v_{1,t}^{\tau}$ for cohort-τ workers assigned to activity 1 in period t for each τ and t. We begin by defining a class of promotion policies that will turn out to include an optimal promotion policy.

Definition 1. A promotion policy is a *modified first-in-first-out (modified FIFO)* policy if, for every t, there exist two thresholds $\tau_1(t)$ and $\tau_2(t)$ with $0 \leq \tau_1(t) \leq \tau_2(t) \leq t$ such that:

($i.$) $p_t^{\tau} = \bar{p}_t$ for all $\tau \leq \tau_1(t)$ for some $\bar{p}_t \in (0, 1]$,

($ii.$) p_t^{τ} is weakly decreasing in τ for $\tau_1(t) < \tau \leq \tau_2(t)$, and

($iii.$) $p_t^{\tau} = 0$ for all τ for $\tau_2(t) \leq \tau \leq t$.

In a modified FIFO policy, sufficiently recent hires may not be promoted with positive probability, more senior workers are promoted with higher probability than less senior workers, and the promotion probability as a function of seniority is capped at some level $\bar{p}_t$. Two special cases of modified FIFO policies warrant special attention. The first are *FIFO* policies in which more-senior workers are always promoted first. The second special case are *seniority-blind* policies in which promotion opportunities are allocated evenly across cohorts. The following proposition describes cost-minimizing personnel policies.

Proposition 2. *Given a steady production plan N, if there is a cost-minimizing personnel policy, an internal labor market with the following properties is cost-minimizing:* ($i.$) $w_{1,t}^{\tau}$ *is weakly increasing in t,* ($ii.$) *if $\tau < \tau'$, $w_{1,t}^{\tau} \geq w_{1,t}^{\tau'}$ and $v_{1,t}^{\tau} \geq v_{1,t}^{\tau'}$, and* ($iii.$) *the promotion policy is a modified FIFO policy.*

The first part of Proposition 2 describes wage dynamics for a single cohort within the firm and shows that wages in activity 1 exhibit returns to tenure. This feature that wages are backloaded in a worker's career is familiar from models of optimal long-term contracts.[8] The second part of the proposition compares wage and value dynamics across cohorts: wages and values are higher for

[8] Becker and Stigler (1974), Lazear (1979), Ray (2002).

earlier cohorts than for later cohorts. The last part of the proposition describes promotion dynamics and shows that promotion opportunities can be optimally allocated according to a modified FIFO policy. This implies that workers' promotion prospects are optimally weakly seniority based.

To understand why promotion prospects may be strictly seniority based, suppose the firm grows slowly between periods 1 and 2 but quickly between periods 2 and 3. If the firm follows a seniority-blind promotion policy, it may allocate too many opportunities to cohort-2 workers, in the sense that their first-period incentive constraints are slack, and they are receiving excess rents, while the first-period incentive constraints for cohort-1 workers may be binding. In this case, the firm would gain by reducing the promotion rate of newer workers in order to increase the promotion rate for more-senior workers, allowing it to reduce their wages.

Next, to understand why workers in a later cohort may optimally be promoted before a worker in an earlier cohort, suppose the firm follows a FIFO policy. The logic is the flip side of the logic above. Such a policy may allocate too many opportunities to early cohorts, so that they are receiving excess rents, while later cohorts are not. The firm would gain by reducing the promotion rate for early cohorts in order to increase the promotion rate for later cohorts, allowing it to reduce their wages. A modified FIFO policy, which bases promotions on an "interior" degree of seniority minimizes the excess rents that are paid to new hires by allocating promotion opportunities to transfer slack across cohorts' first-period incentive constraints.

An important constraint on the firm's problem is that, while it can allocate period-t promotion opportunities to workers who started working at the firm prior to period t, it cannot allocate period-t promotion opportunities to workers who have yet to begin working at the firm. This "irreversibility of time" constraint (Grenadier, Malenko, and Malenko, 2016) ensures that excess rents for cohort-t workers are weakly decreasing in t: slack from later cohorts' first-period incentive constraints can be allocated to earlier cohorts but not vice versa.

We now discuss several implications of these results. First, the proposition shows that optimal promotion schemes are seniority based. Seniority-based promotion schemes have been historically common—according to Waldman (1990), "Doeringer and Piore (1971), Mincer (1974) and Edwards (1979) all suggest that seniority enters into the promotion process even after controlling for the effect which seniority may have on productivity." (p. 4) And they are still prevalent in many settings, including the U.S. manufacturing sector where Bloom et al. (2018) finds that a sizeable share of firms base promotions at least in part on seniority.

Next, the proposition shows that firms may make use of bonuses and promotions to motivate workers, even when workers do not differ in their abilities. Ekinci, Kauhanen, and Waldman's (forthcoming) model shows how firms' use of bonuses and promotions depends on worker-level characteristics, while our results show that it can also depend on firm-level characteristics such as its growth rate.

Finally, in the worker's incentive constraint, promotions and bonus payments serve as substitute instruments to motivate workers, yet in the personnel policies described in Proposition 2, promotions and bonus payments are positively related across workers: in a given period, workers hired earlier both receive higher wages and have greater promotion prospects than workers hired later. Like with many dynamic contracting problems,[9] optimal values are uniquely pinned down, although there are often many optimal solutions. We focus on optimal solutions with increasing wage profiles because they are consistent with evidence on tenure–wage profiles (Baker, Gibbs, and Holmström, 1994) but with the caveat that they are not uniquely optimal.

5 Personnel Policies and Production Plans

The previous section explored how differences in the firm's production plan shape the characteristics of the personnel policies it puts in place. We now turn to the firm's full problem in order to understand how dynamic incentive provision shapes the firm's optimal production plan. We will show that incentive considerations may lead the firm to adopt time-inconsistent production plans. We first describe the firm's full problem and we establish a lemma that shows that new workers never receive excess rents. This lemma allows us to reformulate the firm's problem. We take advantage of this formulation to completely characterize optimal production plans in a two-period model and establish some general properties of optimal production plans.

5.1 Preliminaries

We will first describe the firm's full problem of choosing an optimal production plan and a cost-minimizing personnel policy that implements that production plan. To do so, let H_t be the mass of new hires who are assigned to activity 1 in period t, and define z_t^s to be the fraction of cohort-s workers who are still assigned to activity 1 in period t. Using the result of Lemma 3, it is without loss of generality to set $v_{2,t}^s = R_2$ for every t and every cohort s, which pins down wages for activity 2 and ensures they are independent of when the worker was hired. Finally, recall that $w_{1,t}^s, v_{1,t}^s$, and p_t^s are cohort-s workers' activity-1 wages, activity-1 values, and promotion probabilities in period t.

[9] Lazear (1979), Board (2011).

The firm's full problem, which we will refer to as problem $(P1)$, is to

$$\max_{\left\{N_{i,t},H_t,w_{1,t}^s,z_t^s,v_{1,t}^s,p_t^s\right\}_{t=1}^T}\sum_{t=1}^T\delta^{t-1}\left[\theta_t f\left(N_{1,t},N_{2,t}\right)-\sum_{s=1}^t H_s w_{1,t}^s z_t^s-w_{2,t}N_{2,t}\right]$$

subject to a flow constraint for $N_{1,t}$ ensuring that the new hires in period t plus those new hires prior to t who remain in activity 1 in period t sum up to $N_{1,t}$:

$$H_t+\sum_{s=1}^{t-1}H_s z_t^s=N_{1,t}.$$

The firm must also satisfy a flow constraint for $N_{2,t}$,

$$N_{2,t-1}\left(1-d\right)+\sum_{s=1}^{t-1}H_s z_{t-1}^s p_{t-1}^s=N_{2,t},$$

ensuring that workers previously assigned to activity 2 and who remain, as well as other workers who were promoted prior to $t-1$, sum up to $N_{2,t}$. The fraction z_t^s must evolve according to

$$z_{t+1}^s=z_t^s\left(1-p_t^s\right)\left(1-d\right),$$

and the firm must also satisfy incentive compatibility for each worker in each period, that is, each worker assigned to activity i in period t must receive a value weakly exceeding R_i.

We now explore how dynamic incentives lead to time inconsistencies in the firm's optimal production plan. To do so, we first establish a lemma that imposes strong restrictions on optimal production plans. We will say that *workers receive no excess rents* if $v\left(n^t\right)=R_1$ for all new hires, except those who are hired in the first period and assigned to activity 2.

Lemma 4. *In an optimal production plan, workers receive no excess rents.*

Lemma 4 shows that if the firm optimally chooses its production plan, all cohorts receive the same first-period rents. To see why Lemma 4 is true, suppose to the contrary that there is some period t at which new hires receive excess rents. Such workers are necessarily motivated solely by future promotion prospects and receive zero wages in their first period of employment, or else the firm could reduce their first-period wages. In this case, the firm could just hire more workers in period t and pay them zero, increasing the firm's period-t profits. The proof of Lemma 4 shows how the firm can do so while satisfying the incentive constraints for these additional new hires as well as the existing new hires in period t. Lemma 4 also allows us to rewrite the firm's objective function and therefore reformulate the firm's problem of choosing an optimal production plan.

Define $\mathcal{N}$ to be the set of production plans for which there exists a personnel policy that satisfies incentive compatibility, the flow constraints described above, and gives new hires no excess rents. Define the function

$$\tilde{\pi}(N) = \sum_{t=1}^{T} \delta^{t-1} \left(\theta_t f(N_{1,t}, N_{2,t}) - c_1 N_{1,t} - c_2 N_{2,t} - H_t R_1 \right) - N_{2,1} R_2$$

where $H_t = N_{1,t} + N_{2,t} - (1-d)(N_{1,t-1} + N_{2,t-1})$ for $t > 1$ and $H_1 = N_1$. Note that, by Lemma 4, new hires receive no excess rents in an optimal production path N^*, so $\tilde{\pi}(N^*)$ coincides with the firm's objective function at N^* in problem $(P1)$.

Next, define problem $(P2)$ to be $\max_{N \in \mathcal{N}} \tilde{\pi}(N)$, let $\tilde{\pi}^*$ be the maximized profits under problem $(P2)$, and let π^* be the maximized profits under the firm's full problem $(P1)$. The following lemma shows that these two profit levels coincide.

Lemma 5. $\pi^* = \tilde{\pi}^*$.

Lemma 5 shows that the optimal-production problem amounts to maximizing the objective $\tilde{\pi}(N)$ subject to the constraint that new hires can be given no excess rents. While this constraint is still complicated in general, the firm's objective function is simple and depends only on N. This feature makes it easier to find the solution to the full problem and understand its properties.

5.2 Two-Period Example

We will now illustrate how past and future production decisions can become interlinked when firms motivate their workers with optimal personnel policies. Consider an example with $T = 2$ and with additively separable production within each period:

$$\theta_t f(N_{1,t}, N_{2,t}) = \theta_t \left[f_1(N_{1,t}) + f_2(N_{2,t}) \right]$$

for some strictly increasing and strictly concave functions f_1 and f_2. By Lemma 5, an optimal production plan solves the following problem:

$$\max_{\{N_{1,t}, N_{2,t}\}_{t=1}^{2}} \sum_{t=1}^{2} \delta^{t-1} \left(\theta_t \left[f_1(N_{1,t}) + f_2(N_{2,t}) \right] - c_1 N_{1,t} - c_2 N_{2,t} - H_t R_1 \right) - R_2 N_{2,1}$$

subject to the constraint that $\{N_{1,t}, N_{2,t}\}_{t=1}^{2}$ can be implemented with a personnel policy that gives new hires no excess rents. This condition is easy to check when $T = 2$. In particular, it requires only that cohort-1 workers assigned to activity 1 do not have "too many" promotion opportunities, so that it is feasible for the firm to motivate them without providing them excess rents.

The following proposition describes the main properties of the optimal production plan.

Proposition 3. *There is a continuous and increasing function $\bar{\theta}_2(\theta_1)$ such that the optimal production plan N^* satisfies the following:*

$(i.)$ **Intertemporal independence**: *If $\theta_2 < \bar{\theta}_2(\theta_1)$, then $\partial N^*_{i,1}/\partial\theta_2 = 0$ and $\partial N^*_{i,2}/\partial\theta_1 = 0$ for $i = 1, 2$.*

$(ii.)$ **Forward linkages:** *If $\theta_2 > \bar{\theta}_2(\theta_1)$, then $\partial N^*_{i,1}/\partial\theta_2 > 0$ for $i = 1, 2$.*

$(iii.)$ **Backward linkages**: *If $\theta_2 > \bar{\theta}_2(\theta_1)$, then $\partial N^*_{2,2}/\partial\theta_1 > 0$.*

Proposition 3 shows when and how optimal production decisions are linked across periods. Part $(i.)$ shows that when demand does not grow very quickly, optimal production decisions in a given period depend only on the demand parameter for that period. In this case, small changes in past or future demand conditions do not have intertemporal spillover effects. Parts $(ii.)$ and $(iii.)$ show that when demand grows quickly, optimal production features intertemporal linkages. Part $(ii.)$ shows that when future demand increases, the firm responds by optimally increasing production today. Part $(iii.)$ shows that stronger past demand conditions lead the firm to produce more today.

To see why intertemporal linkages can arise, first consider the forward linkages described in part $(ii.)$. Suppose tomorrow's demand parameter increases, leading the firm to increase the number of top positions tomorrow. All else equal, the rents of first-period new hires will increase because promotion opportunities are more abundant. The firm can increase its profits by reducing these rents and can do so either by reducing their bonuses in the first period or by reducing their promotion probability. When promotion opportunities are sufficiently abundant, first-period new hires already receive no bonus payments, so the firm can only reduce their promotion probability by increasing the number of workers in both positions in the first period, leading to what we refer to as *forward linkages* in production.

Part $(iii.)$ of the proposition shows that there are lingering effects of past demand conditions. When there are better demand conditions in the first period (i.e., higher values of θ_1), the firm increases its size in the first period. All else equal, the rents paid to new hires in the bottom job in the first period will decrease, since promotion opportunities will be more scarce when there are more workers vying for them. To see why the firm will also increase the number of top positions in the second period, we now argue that the marginal returns to increasing the number of such position will increase. By Lemma 2, the marginal returns to adding another top position in the second period is $\theta_2 f_2'\left(N^*_{2,2}\right) - c_2$ minus the rents that are paid to new hires in the *first* period. Notice that this is the rents paid to new hires in the first period rather than the second period because an optimal personnel policy takes the form of an internal labor market, and the incentive costs originate from the bottom job in the first period. Since these rents have decreased, the firm will expand the number of top positions in the second period, leading to what we refer to as *backward linkages* in production.

One implication of backward linkages in production is that two firms facing the same demand conditions in the second period may operate at different sizes because they had different demand conditions in the past. The firm that had better demand conditions in the past will be larger precisely in order to provide promotion opportunities for the workers it hired in the past. This implication formalizes the idea, present in the early works of Barnard (1938), Jensen (1986), and Porter (1998) that organizations may be biased towards growth in order to provide more opportunities for career advancement.

5.3 Forward and Backward Linkages

The example above provides precise conditions on the demand path $\theta = (\theta_1, \theta_2)$ under which optimal production features intertemporal linkages when $T = 2$. Deriving necessary and sufficient conditions for such linkages to arise when $T > 2$ is complicated in general. In this section, we will instead establish results for $T > 2$ that parallel those from the example and will continue to assume that $f(N_{1,t}, N_{2,t}) = f_1(N_{1,t}) + f_2(N_{2,t})$. Our objective is to demonstrate that both forward and backward linkages can, but need not necessarily, occur for general T.

In order to state the analogue of our first result on intertemporal independence, denote $N^*(\theta)$ to be an optimal production plan given demand path θ. We will say that $N^*(\theta)$ features *local intertemporal independence* if, for each t, small changes in θ_τ do not affect $N^*_{i,t}$ for $t \neq \tau$, that is, $\partial N^*_{i,t}/\partial\theta_\tau = 0$ for all $t \neq \tau$.

Recall from above that the firm's problem of choosing an optimal production plan is given by problem $(P2)$, which is to $\max_{N\in\mathcal{N}} \tilde{\pi}(N)$ where $\mathcal{N}$ is the set of production plans under which there is an incentive-compatible personnel policy that gives no excess rents to new hires. We will show that when optimal production does not yield "too many" promotion opportunities, the constraint that $N \in \mathcal{N}$ is slack at the optimum, and optimal production features local intertemporal independence.

To make this argument precise, we introduce the relaxed production problem $(P3)$, which is to $\max_N \tilde{\pi}(N)$, ignoring the constraint that $N \in \mathcal{N}$. Let $\tilde{N}^*(\theta)$ denote a production plan that solves the firm's relaxed problem $(P3)$ given demand path θ. Next, define the average period-t promotion rate under a production plan $\tilde{N}$ by $\tilde{p}_t = \left(\tilde{N}_{2,t+1} - (1-d)\tilde{N}_{2,t}\right) / \left((1-d)\tilde{N}_{1,t}\right)$, and define $\hat{p}$ to be the solution to the following equation

$$R_1 = -c_1 + \delta(1-d)(\hat{p}R_2 + (1-\hat{p})R_1).$$

This equation defines a critical promotion rate $\hat{p}$ such that if the worker is paid a wage of 0 in each period he is assigned to activity 1, and he is promoted with probability $\hat{p}$, then his value for being assigned to activity 1 is exactly R_1. The following proposition provides sufficient conditions in terms of average promotion rates for the optimal production plan to feature local intertemporal independence.

Proposition 4. *Let $\tilde{N}^*$ be the solution to the relaxed problem $(P3)$ and $\tilde{p}_t^*$ be the corresponding average period-t promotion rate. If $\tilde{p}_t^* < \hat{p}$ for all t, then the optimal solution to problem $(P2)$ satisfies $N^* = \tilde{N}^*$ and features local intertemporal independence.*

Proposition 4 shows that when firms are limited in their promotion opportunities, their optimal production plan is locally intertemporally independent. If we compare the optimal production plan under two demand paths that differ only in period t, then the optimal production plans also only differ in period t if they both satisfy the conditions of Proposition 4. To see why this is the case, note that a small increase in θ_t will lead the firm to increase the number of positions it has in period t. In turn, the firm can promote more workers who were hired prior to t and reduce their pre-promotion wages. The firm can therefore ensure new hires receive no excess rents simply by adjusting its personnel policy rather than by altering its production levels in any other period. If the demand parameter θ_t increases significantly, the firm cannot reduce its pre-promotion wages further and therefore has to adjust the number of positions in other periods in order to maintain the no excess rents condition. This can lead to both forward linkages and backward linkages, which the following proposition explores.

Proposition 5. *The following are true:*

(i.) **Forward linkages:** *Fix a demand path θ* and a $t < T$. *There exists a demand path θ' with $\theta_\tau = \theta'_\tau$ for all $\tau \neq t$ and $\theta'_t > \theta_t$ under which $N^*_{i,t-1}(\theta) > N^*_{i,t-1}(\theta')$ for some i.*

(ii.) **Backward linkages:** *Fix θ_t for some $t > 1$. There exist demand paths θ' and θ'' with $\theta'_t = \theta''_t = \theta_t$ and $\theta'_\tau < \theta''_\tau$ for all $1 \leq \tau < t$ under which $N^*_{2,t}(\theta') < N^*_{2,t}(\theta'')$.*

Proposition 5 shows that optimal production plans can feature forward and backward linkages when $T > 2$. The first part of the proposition shows that as demand conditions in period t increase, eventually, the number of positions in period $t-1$ will increase.[10] This result does not rule out the possibility that as θ_t increases further, the number of positions in periods prior to $t-1$ will also increase.

[10] This argument for forward linkages in optimal production can also lead to the result that the firm is socially inefficiently large in some periods, that is, $\theta_t f'_i(N^*_{i,t}) < c_i$ is possible. This result is related to Fudenberg and Rayo's (2018) result that firms might optimally extract rents from workers by asking them to exert inefficiently high effort levels.

The second part of the proposition shows that past demand conditions can affect the number of positions in period t: two firms facing identical demand conditions in period t may therefore choose different production levels. In particular, the firm with better demand conditions in the past will have more top positions in period t. This result reflects an opportunity-creation motive for firm growth—firms might optimally expand the number of top positions in a period as a way of providing its workers with career incentives.

6 Discussion of Model Assumptions

Our discussion so far makes use of several simplifying assumptions to streamline the exposition. Specifically, we analyze a simple moral-hazard environment in which workers must receive rents in order to be motivated to exert effort, and we assume the firm puts in place a deterministic production plan. Below, we discuss how the paper's results are affected by each of several assumptions.

Non-zero minimum wage: The model assumes that the minimum wage payment is $\underline{w} = 0$. Our main results continue to hold for an interval of minimum wage levels. If the minimum wage is sufficiently negative, then workers' participation constraints can be made binding in their first period of employment, implying that workers need not receive rents (Carmichael, 1985). In this case, there would be no need to back-load worker compensation across activities and therefore no need to make use of promotion-based incentives. Moreover, optimal production would be intertemporally independent. As long as the minimum wage is not too low, so that workers must be given rents, seniority-based promotions and intertemporal linkages may still arise.

In terms of optimal production plans, as long as the minimum wage is not too high, our results remain unchanged as long as $\theta_t \partial f(N_{1,t}, N_{2,t}) / \partial N_{1,t} > \underline{w}$ for all $(N_{1,t}, N_{2,t})$. If this inequality fails to hold, then the optimal production plan will satisfy $\theta_t \partial f(N_{1,t}, N_{2,t}) / \partial N_{1,t} = \underline{w}$. In general, either new hires into activity 1 will receive rents R_1 or their marginal productivity will be equal to the minimum wage.

Worker heterogeneity: The model assumes that all workers are identical. One interpretation of our model is that the analysis applies to workers who are qualified to be promoted: even for qualified workers, promotion opportunities may be constrained by a firm's production plan, and they may be allocated according to seniority. The logic of our analysis can be extended to allow for heterogeneity in the degree to which workers are qualified to perform activity 2, and it suggests that even if an older worker is less talented than a recently hired worker, he may nevertheless get promoted before the more talented worker.

Specifically, consider a three-period model in which the optimal personnel policy features seniority-based promotions when workers are homogeneous. In other words, cohort-1 workers are promoted at a higher rate than cohort-2 workers at the end of the second period. Now suppose that each cohort-2 worker is as productive as $\kappa > 1$ cohort-1 workers when assigned to activity 2. If promotion decisions depend entirely on worker ability, then all cohort-2 workers will need to be promoted before cohort-1 workers. Under such a promotion policy, cohort-1 workers promotion opportunities are significantly reduced, and the firm must pay them more in terms of wages to keep them motivated. These extra wage payments may exceed the productivity gains associated with promoting cohort-2 workers when κ is close to 1. Promotion policies that depend only on worker ability may therefore be dominated by policies that take seniority into account.

Monitoring technology: The main model considers a monitoring technology in which a signal of $y_{i,t} = 0$ is perfectly indicative that the worker shirked. Another commonly studied monitoring technology is one in which a signal of $y_{i,t} = 1$ is perfectly indicative that the worker worked. For example, suppose $\Pr[y_{i,t} = 1 | e_t] = q_i e_t$. Under such a monitoring technology, there is no need to pay workers incentive rents: the firm can pay the worker 0 if $y_{i,t} = 0$ and a bonus of c/q_i if $y_{i,t} = 1$.[11] Since no incentive rents are required to motivate workers, optimal personnel policies are static, and optimal production plans would be intertemporally independent.

Stochastic production plans: Our main results for cost-minimizing personnel policies can be modified to allow $(N_{1,t}, N_{2,t})$ to be a stochastic process. In particular, suppose that $(N_{1,t}, N_{2,t})$ is a Markov process that takes on a countable number of values for each t and is steady along each path realization, in the sense that the firm never grows so fast that it needs to hire directly into activity 2, and the firm never shrinks so fast that it cannot assign all its incumbent workers to a productive activity. Because each worker is risk neutral, his continuation payoffs in period t depend on his expected promotion probability, taking expectations over the continuation process for the production plan beginning in period $t + 1$. In this case, optimal personnel policies again resemble an internal labor market, and it can still be strictly optimal to base promotions on seniority.

Other factors: Finally, we assume away many other important factors. For instance, we assume employees are risk neutral and make binary effort choices, and the firm has full commitment power. In addition, workers do not acquire human capital, and there is no uncertainty about their productivity.[12] Incorporating these factors can potentially improve our understanding of the

[11] Of course, if the minimum wage is sufficiently high, then this monitoring technology will also require workers to be given rents. In this case, as long as activity 2 requires more rents than activity 1, then many of the features of internal labor markets will arise. Additional features such as up-or-out promotions or firing on the equilibrium path might also be part of an optimal personnel policy even in a single-worker setting (Fong and Li, 2017).

[12] Many papers examine how these different features affect personnel and supplier dynamics and hence firm-level productivity dynamics but do not speak directly to the dynamics of firm size. For papers emphasizing the role of supplier and employee heterogeneity, see Board (2011), DeVaro and Waldman (2012), DeVaro and Morita (2013),

personnel policies firms adopt in richer environments and how they interact with firm growth, but doing so would obscure the main economic mechanisms we aim to highlight and is beyond the scope of this paper.

7 Unsteady Production and Partial-Effort Contracts

Our main analysis restricted attention to steady production plans and full-effort contracts to streamline the exposition. In this section, we relax these restrictions and show how doing so introduces additional nuances to optimal personnel policies. We first characterize properties of cost-minimizing personnel policies in environments in which the production plan is unsteady. We next explore whether and why a firm might want to adopt a partial-effort contract: a contract in which some workers are not expected to exert effort in some periods.

7.1 Unsteady Environments

We now explore the properties of cost-minimizing personnel policies for a wider class of production plans. Our analysis above shows that if the firm can choose both the production plan and the personnel policy at the same time, some of the cases we discuss below will not occur. Nevertheless, in some settings, the production plan is either partially or completely inflexible (for example, in a bureaucracy).

The analysis in Section 4 presumed that the firm's production plan N was a steady production plan. That is, we assumed that for each t, $N_{2,t+1} \leq (1-d)(N_{1,t}+N_{2,t})$ and, for each i, $N_{i,t+1} \geq (1-d)N_{i,t}$. In this section, we explore characteristics of optimal personnel policies when N is not a steady production plan. We will say that N involves *breakneck growth at* $t+1$ if $N_{2,t+1} > (1-d)(N_{1,t}+N_{2,t})$, and we will say that N involves *deep downsizing in activity* i *at* $t+1$ if $N_{i,t+1} < (1-d)N_{i,t}$, and *deep downsizing* at $t+1$ if $N_{1,t+1}+N_{2,t+1} < (1-d)(N_{1,t}+N_{2,t})$.

Breakneck Growth Suppose N involves breakneck growth for the first time at $t+1$, that is, even if the firm promotes all workers assigned to activity 1 in period t, it must assign some new hires at $t+1$ to activity 2. This implies that all workers hired prior to $t+1$ must earn a payoff of R_2 at the beginning of period $t+1$. We can then break the optimal personnel policy problem up into two problems.

Andrews and Barron (2016), Board, Meyer-ter-Vehn, and Sadzik (2017), and Foarta and Sugaya (2018); for papers emphasizing human capital acquisition, see Gibbons and Waldman (1999a, 2006); for papers emphasizing risk aversion and continuous effort, see Harris and Holmström (1982) and Holmström and Ricart i Costa (1986), Chiappori, Salanie, and Valentin (1999); for papers emphasizing lack of commitment, see Malcomson (1984), MacLeod and Malcomson (1988)

We first solve for optimal personnel policies for periods $1, \ldots, t$, treating t as effectively the last period of production but with the requirement that all incumbent workers at period t receive R_2 in continuation payoffs. For the second problem, we solve for optimal personnel policies for periods after $t+1$, and we take as given that all workers in cohorts prior to $t+1$ will initially be assigned to activity 2 and will therefore receive rents equal to R_2. In other words, the analysis can be carried out chunk-by-chunk, where each chunk starts with a period in which breakneck growth occurs and ends with the next period in which breakneck growth occurs. Within each chunk, the optimal personnel policy minimizes the rents that are paid to new hires assigned to activity 1, and the same type of analysis as in Section 4 can be applied, so the main results continue to hold.

Deep Downsizing In this section, we explore some features of personnel policies that might arise when firms go through periods of deep downsizing. Managing workers' careers is more complicated in this case because the firm will have to lay workers off, that is, it will have to assign some incumbent workers to activity 0 in the next period even if they have performed well in the past. Denote by $p_{0,t}^{\tau}$ the probability that a cohort-τ worker will be assigned to activity 0 in period $t+1$.

Proposition 6 describes optimal personnel policies when deep downsizing is permanent, in the sense that once there is deep downsizing in one period, there is deep downsizing in all future periods, the firm will never hire new workers, and it will shrink faster than by attrition alone. When this is the case, in order to motivate workers in their last period of employment, the firm has to pay severance pay to workers that it will not employ in the future.

Proposition 6. *Suppose N satisfies $N_{1,t+1} < (1-d)\,N_{1,t}$, $N_{2,t+1} > (1-d)\,N_{2,t+1}$, and $N_{1,t+1} + N_{2,t+1} < (1-d)\,(N_{1,t} + N_{2,t})$ for all t. There is an optimal personnel policy that implements N in which:*

(*i.*) *laid-off workers receive severance pay,*

(*ii.*) *if $\tau < \tau'$, then $p_{0,t}^{\tau} \leq p_{0,t}^{\tau'}$, and*

(*iii.*) *conditional on being laid off, workers with more seniority receive greater severance pay.*

This proposition describes an optimal personnel policy for a firm that experiences deep downsizing in every future period. The first part shows that when workers are laid off, they are given severance pay in their last period of employment. Severance pay is necessary to maintain workers' incentives to exert effort in their last period of employment. The second part of the proposition shows that an optimal personnel policy exhibits a last-in-first-out pattern for layoffs: workers with more seniority are less likely to be laid off in each period. The final part shows that if workers of different cohorts are laid off in the same period, their severance payments are higher the longer they have been employed by the firm.

We now discuss some features of optimal personnel policies for a firm that experiences temporary deep downsizing, that is, the firm must downsize in one period, and there is a future period at which it will need to hire again. To do so, we enable the firm to assign workers to any activity $A_t \in \{0,1,2\}$, including the null activity $A_t = 0$, in any period.[13] We will say that a worker is *permanently laid off* in period t if he is assigned to activity 0 in all future periods with probability 1. We say that a worker is *temporarily laid off* in period t if he is assigned to activity 0 in period $t+1$ and is assigned to activity 1 or 2 in a future period with positive probability. The next proposition partially characterizes optimal personnel policies when a firm experiences temporary deep downsizing.

Proposition 7. *Suppose there is a t_1 at which $N_{1,t_1+1} < (1-d)\,N_{1,t_1}$ and $N_{1,t_1+1} + N_{2,t+1} < (1-d)\,(N_{1,t_1} + N_{2,t_1})$, and there is a $t_2 > t_1$ at which $N_{1,t_2+1} + N_{2,t_2+1} > (1-d)\,(N_{1,t_2} + N_{2,t_2})$. Then:*

(i.) no workers are permanently laid off in period t_1, and

(ii.) $v^{\tau}_{1,t_2+k} \geq v^{t_2+k}_{1,t_2+k}$ for all $\tau < t_2$ and for all $k \geq 1$.

The conditions for Proposition 7 imply that the firm must downsize at t_1, and at time t_2+1, it recovers and must hire workers into one of the two positions. This proposition shows that whenever this is the case, the firm favors rehiring laid-off workers. If, instead, the firm hired new workers, it would have to pay them rents in their first period of employment. By rehiring laid-off workers, the firm can allocate these rents to these workers and reduce the overall rents it has to pay. The second part of the proposition shows that temporarily laid off workers will be rehired before the firm hires a worker who has never worked for the firm in the past, and moreover, these workers receive higher continuation payoffs than new hires. The rationale for this result is similar to the logic underlying why seniority-based promotions can be optimal.

7.2 Partial-Effort Contracts

Throughout our analysis, we have focused on full-effort contracts. In this subsection, we will show how and why it may be optimal to put in place a partial-effort contract—that is, to ask workers to exert no effort in a given period—even when the firm does not go through a period of deep downsizing. When promotion prospects in the future look more promising than they do today, one way to relax a worker's incentive constraint today is to allow them to shirk. Doing so, of course,

[13] Recall that in Section 2, we assumed that if a worker is ever assigned to activity 0 after he has been assigned to activity 1 or 2, he is assigned to activity 0 in all future periods.

comes at the cost that the firm will need to hire other workers in the next period who will exert effort. If future promotion prospects are sufficient to guarantee those workers can be motivated at zero cost, then such a policy will reduce the firm's overall wage bill.

Proposition 8. *Suppose there is an optimal full-effort contract in which $p_2^1 = 1$, $v_{1,1}^1 = R_1$, $p_2^2 < 1$, and $v_{1,2}^2 > R_1$. Then there is a partial-effort contract that has a lower wage bill.*

Proposition 8 shows that partial-effort contracts may be optimal when there are a lot of promotion opportunities in a period, and the firm has already exhausted its ability to transfer slack to earlier cohorts' first-period incentive constraints by increasing their promotion probabilities. At that point, the only further instrument the firm has to transfer slack is to reduce the earlier cohort's effort costs and increase the effort costs of the later cohort that is receiving excess rents. Doing so is feasible as long as some of the earlier cohort was exerting effort in this period, and as long as the firm can reduce hiring in the next period.

In contrast to the existing work on hiring and sourcing decisions,[14] which highlights the benefit of biasing such decisions towards insiders, the optimality of partial-effort contracts suggests that the firm's personnel policies can also exhibit a temporary "outsider bias." These results, therefore, show that future production plans can impact current hiring and sourcing decisions.

8 Conclusion and Discussion

This paper develops a model in which firms jointly optimize their personnel policies and production plans. We first show that optimal personnel policies resemble internal labor markets in which seniority plays an important role in promotion and wage decisions. We also show that optimal production plans may feature forward and backward linkages in the sense that today's optimal production level might depend on demand conditions both in the future and in the past.

One implication of our model is that firms may pursue inefficient growth in order to provide incentives at a lower cost, consistent with Baker, Jensen, and Murphy's (1988) observation that an "important problem with promotion-based reward systems is that they require organizational growth to feed the reward system." (p. 600) Of course, another important reason for inefficient firm growth is the empire-building motives on the part of the executive team. Finance scholars suggest that firms should leverage up in order to commit their executives not to pursue inefficient growth strategies (Jensen, 1986). Our results caution that organizational solutions aimed at curtailing empire building by forcing management to make sequentially optimal production decisions might

[14] Board (2011), Ke, Li, and Powell (2018).

throw out the baby with the bathwater, since they undermine the firm's ability to make efficient use of long-term incentives for its workforce.

The opportunity-creation motive for inefficient growth we highlight abstracts from, but has implications for, the important strategic choices firms have to make when they decide to expand. Anecdotal evidence suggests such motives underlie firm diversification plans. Interwar DuPont, as we mentioned in the introduction, pursued growth through diversification, expanding into other lines of business rather than expanding its existing business. Drucker (1977), in discussing why Callahan Associates diversified beyond their core capabilities, wrote:

> "Callahan deeply believed that the company had to expand to give people promotion opportunities. And since he also believed that no one chain should grow beyond the point where one person could easily manage it and know every nook and cranny of it, this meant going purposefully into new businesses every six or seven years." (p. 13)

When a firm decides to diversify, an important issue it faces is whether to do so organically or through acquisition. Our model suggests that organic growth may create additional career opportunities for existing employees that growth through acquisition might not. Future work examining the personnel implications of different ways of expanding can help improve our understanding of the dynamics of corporate strategy.

Appendix A: Cost-Minimizing Personnel Policies

Lemma 1. *Given N if there is an optimal personnel policy, there is an optimal personnel policy in which workers with the same employment history face the same wage and assignment policies.*

Proof of Lemma 1. If there is an optimal personnel policy in which two workers with the same employment history receive different wage and/or assignment policies, then we can consider an alternative assignment and wage policy that is a public randomization between these policies, and if both players are subject to this same alternative policy, their incentive constraints and the firm's flow constraints remain satisfied.■

Lemma 2. *Cost-minimizing personnel policies minimize the rents paid to new hires:*

$$\min \sum_{t=1}^{T} \sum_{h^t \in \mathcal{H}^t} \delta^{t-1} L\left(n^t\right) v\left(n^t\right)$$

subject to (1), (2), *and* (3).

Proof of Lemma 2. The PDV of the firm's wage bill, times δ is

$$\sum_{t=1}^{T} \sum_{h^t \in \mathcal{H}^t} \delta^{t-1} L\left(h^t\right) w\left(h^t\right).$$

For all workers who currently work in the firm in period t, that is for those for which $A_t \neq 0$, the flow constraint gives us

$$\begin{aligned} L\left(h^t 1\right) &= (1-d)\, p_1\left(h^t\right) L\left(h^t\right) \\ L\left(h^t 2\right) &= (1-d)\, p_2\left(h^t\right) L\left(h^t\right). \end{aligned}$$

In addition, for $i \in \{1,2\}$, we can write $N_{i,t} = \sum_{h^t | A_t = i} L\left(h^t\right)$. We can write the period-t wages paid to workers with employment history h^t as $L\left(h^t\right) w\left(h^t\right)$, which equals

$$\begin{aligned} & L\left(h^t\right) v\left(h^t\right) + L\left(h^t\right) c\left(h^t\right) - \delta(1-d) L\left(h^t\right)\left(p_1\left(h^t\right) v\left(h^t 1\right) + p_2\left(h^t\right) v\left(h^t 2\right)\right) \\ = \; & L\left(h^t\right) v\left(h^t\right) + L\left(h^t\right) c\left(h^t\right) - \delta L\left(h^t 1\right) v\left(h^t 1\right) - \delta L\left(h^t 2\right) v\left(h^t, 2\right), \end{aligned}$$

where the first equality plugs in the promise-keeping constraint for workers with employment history h^t, and the second equality plugs in the flow constraint.

The total wage bill is the sum of these expressions over time and over employment histories and is therefore

$$
\begin{aligned}
& \sum_{t=1,h^t}^{T} \delta^{t-1} L\left(h^t\right) w\left(h^t\right) \\
= & \sum_{t=1}^{T} \sum_{h^t \in \mathcal{H}^t} \delta^{t-1}\left(L\left(h^t\right) v\left(h^t\right)+L\left(h^t\right) c\left(h^t\right)-\delta L\left(h^t 1\right) v\left(h^t 1\right)-\delta L\left(h^t 2\right) v\left(h^t 2\right)\right) \\
= & \sum_{t=1}^{T} \sum_{h^t \in \mathcal{H}^t} \delta^{t-1} L\left(h^t\right) c\left(h^t\right)+\sum_{t=1}^{T} \sum_{h^t \in \mathcal{H}^t} \delta^{t-1}\left(L\left(h^t\right) v\left(h^t\right)-\delta L\left(h^t 1\right) v\left(h^t 1\right)-\delta L\left(h^t 2\right) v\left(h^t 2\right)\right) \\
= & \sum_{t=1}^{T} \delta^{t-1}\left(N_{1,t} c_1+N_{2,t} c_2\right)+\sum_{t=1}^{T} \delta^{t-1} L\left(n^t\right) v\left(n^t\right),
\end{aligned}
$$

where recall that $L\left(n^t\right)$ are the new workers hired into the firm in period t. It follows that the firm's objective is simply to minimize

$$
\sum_{t=1}^{T} \delta^{t-1} L\left(n^t\right) v\left(n^t\right),
$$

which establishes the lemma.■

Lemma 3. *Given a steady production plan N, if there is an optimal personnel policy, there is an optimal personnel policy with the following three properties:*
(i.) $v\left(h^t\right) \leq R_2$ if $A_t = 1$ and $v\left(h^t\right) = R_2$ if $A_t = 2$.
(ii.) All new workers are assigned to activity 1, except at $t = 1$.
(iii.) $p_2\left(h^t\right) = 1$ if $A_t = 2$ and $p_1\left(h^t\right) + p_2\left(h^t\right) = 1$ if $A_t = 1$.

Proof of Lemma 3. To establish part (i.), we will first show that for all h^t, we do not need to have both $w\left(h^t\right) > 0$ and $v\left(h^t\right) > R\left(A_t\right)$. To establish this intermediate result, there are two cases to consider. First, suppose the worker is a new hire in period t. In this case, if both $w\left(h^t\right) > 0$ and $v\left(h^t\right) > R\left(A_t\right)$, the firm can reduce the wage bill by reducing $w\left(h^t\right)$ without violating the incentive constraint. Second, if the worker was a new hire prior to period t, the firm can reduce $w\left(h^t\right)$ and increase $w\left(h^{t-1}\right)$ to maintain $v\left(h^{t-1}\right)$. This establishes the intermediate result and shows that it is without loss of generality to focus on personnel policies in which in each period, either the minimum wage constraint is binding or the IC constraint is binding. We will use this result to establish part (i.), but we do not make use of it in our other results.

For part (i.), there are two cases to consider. First, suppose $w\left(h^t\right) > 0$. Then by the previous result, we have $v\left(h^t\right) = R\left(h^t\right) \leq R_2$. Next, suppose $w\left(h^t\right) = 0$. We can then consider all histories that follow h^t. With probability 1, the workers must eventually receive a strictly positive wage, or else there must be some employment history following h^t at which his incentive constraint is violated. If $w\left(h^t\right) = 0$, we can write $v\left(h^t\right)$ as

$$
v\left(h^t\right)=\sum_{h^\tau} \operatorname{Pr}\left[h^\tau \mid h^t\right]\left(\sum_{s=0}^{\tau-t-1} \delta^s\left(-c_1\right)+\delta^{\tau-t} v\left(h^\tau\right)\right),
$$

where h^τ is the first history following h^t such that $w(h^\tau) > 0$. We can again use the previous result to get

$$\begin{aligned} v\left(h^t\right) &= \sum_{h^\tau} \Pr\left[h^\tau \middle| h^t\right] \left(\sum_{s=0}^{\tau-t-1} \delta^s (-c_1) + \delta^{\tau-t} R(h^\tau)\right) \\ &< \sum_{h^\tau} \Pr\left[h^\tau \middle| h^t\right] R(h^\tau) \leq R_2, \end{aligned}$$

which establishes part (*i.*).

For part (*ii.*), note that part (*i.*) implies that $v\left(h^t\right) \leq R_2$ if $A_t = 1$. As a result, if a new worker is assigned to activity 2, it is better instead to assign them to activity 1 and promote an existing worker assigned to activity 1 to instead be assigned to activity 2. This would relax the existing workers' incentive constraints and reduce the wage bill.

Finally, for part (*iii.*), suppose $p_1\left(h^t\right) + p_2\left(h^t\right) < 1$. Because $N_{i,t+1} \geq (1-d) N_{i,t}$ for $i = 1, 2$, we must have that $L\left(n^{t+1}\right) > 0$, so there must be positive hiring into either position 1 or position 2. We will construct a perturbation to the personnel policy in which any rents that would be paid out to new hires are paid out, instead, to currently employed workers. This perturbation will introduce slack into some current workers' incentive constraints, and it will not increase the total wage bill. If a positive mass of new workers is hired and assigned to activity 1, $L\left(\vec{0}1\right)$, let $\tilde{p}_1\left(h^t 1\right) = p_1\left(h^t 1\right) + \varepsilon$, and let $\tilde{L}\left(\vec{0}1\right) = L\left(\vec{0}1\right) - \varepsilon(1-d) L\left(h^t\right)$. This perturbation preserves the flow constraint, and it relaxes workers' incentive constraints in periods $s \leq t$ for those workers with history h^t. This perturbation therefore weakly decreases the firm's overall wage bill. A similar perturbation can be constructed if $L\left(\vec{0}2\right) > 0$. Result (i) of this lemma implies that $v\left(h^t\right) = R_2$ if $A_t = 2$, which implies that $p_2\left(h^t\right) = 1$ if $A_t = 2$.■

Proposition 1. *If N is a steady production plan, an internal labor market is an optimal personnel policy.*

Proof of Proposition 1. Follows directly from Lemma 3 and the definition of an internal labor market.■

Proposition 2. *Given a steady production plan N, if there is a cost-minimizing personnel policy, an internal labor market with the following properties is cost-minimizing:* (*i.*) *$w_{1,t}^\tau$ is weakly increasing in t,* (*ii.*) *if $\tau < \tau'$, $w_{1,t}^\tau \geq w_{1,t}^{\tau'}$ and $v_{1,t}^\tau \geq v_{1,t}^{\tau'}$, and* (*iii.*) *the promotion policy is a modified FIFO policy.*

Proof of Proposition 2. We will establish properties (*i.*), (*ii.*), and what we will refer to as property (*iii′.*) which is that if $p_t^\tau, p_t^{\tau'} \in (0,1)$, then $v_{1,t+1}^\tau = v_{1,t+1}^{\tau'}$.

First, note that for any t, in any optimal personnel policy, it must be the case that $v\left(h^t\right) \geq v\left(n^t\right)$, that is, new hires receive lower rents than incumbent workers. Suppose to the contrary that $v\left(h^t\right) < v\left(n^t\right)$. We can then "switch" the future history of a worker with employment history h^t with a new worker. This switch preserves the total wage bill and relaxes the incentive constraints of workers whose employment histories are consistent with h^t.

Now, suppose $\tau_1 < \tau_2$. We can write the rents workers that each cohort receives in period t if they are assigned to activity 1 as follows:

$$\begin{aligned} v_{1,t}^{\tau_1} &= w_{1,t}^{\tau_1} - c_1 + \delta(1-d)\left(p_t^{\tau_1} R_2 + (1-p_t^{\tau_1})\, v_{1,t+1}^{\tau_1}\right) \\ v_{1,t}^{\tau_2} &= w_{1,t}^{\tau_2} - c_1 + \delta(1-d)\left(p_t^{\tau_2} R_2 + (1-p_t^{\tau_2})\, v_{1,t+1}^{\tau_2}\right). \end{aligned}$$

Take a rent $v_{1,t}^{\tau_1}$. We can always reduce $w_{1,t}^{\tau_1}$ by ε and increase $v_{1,t+1}^{\tau_1}$ by $\varepsilon / \left[\delta(1-d)(1-p_t^{\tau_1})\right]$, maintaining rents $v_{1,t}^{\tau_1}$, unless either $w_{1,t}^{\tau_1} = 0$ or $v_{1,t+1}^{\tau_1} = R_2$. We can do this similar for the τ_2 cohort. Let $\bar{w}_{1,t}^1$ and $\bar{w}_{1,t}^2$ denote the resulting activity-1 wages at which this procedure terminates and $\bar{v}_{1,t+1}^{\tau_1}$ and $\bar{v}_{1,t+1}^{\tau_2}$ the resulting continuation payoffs. There are four cases to consider: (a) $\bar{w}_{1,t}^1 = 0$, $\bar{w}_{2,t}^1 = 0$; (b) $\bar{w}_{1,t}^1 > 0$, $\bar{w}_{1,t}^2 > 0$; (c) $\bar{w}_{1,t}^1 = 0$, $\bar{w}_{1,t}^2 > 0$; and (d) $\bar{w}_{1,t}^1 > 0$, $\bar{w}_{1,t}^2 = 0$.

The first observation is that case (d) is impossible, because it would imply that $v_{1,t}^{\tau_1} > v_{1,t}^{\tau_2}$. If $\bar{w}_{1,t}^1 > 0$, this implies that $\bar{v}_{1,t+1}^{\tau_1} = R_2$, and as a result, it must be the case that cohort-τ_1's continuation payoff weakly exceeds cohort-τ_2's continuation payoff, and so do the wages.

Next, in case (b), $\bar{v}_{1,t+1}^{\tau_1} = \bar{v}_{1,t+1}^{\tau_2} = R_2$, so both cohorts have the same continuation payoffs. Moreover, if $v_{1,t}^{\tau_1} \leq v_{1,t}^{\tau_2}$, it must be the case that $\bar{w}_{1,t}^{\tau_1} \leq \bar{w}_{1,t}^{\tau_2}$. Define $\tilde{p}_t = \left(L_{1,t} p_t^{\tau_1} + L_{2,t} p_t^{\tau_2}\right) / \left(L_{1,t} + L_{2,t}\right)$, where $L_{i,t}$ is the mass of cohort-τ_i workers assigned to activity 1 in period t. Promoting both cohorts at rate $\tilde{p}_t$ maintains the flow constraints, and it does not affect $v_{1,t}^{\tau_1}$ or $v_{1,t}^{\tau_2}$, so such a personnel policy is optimal if the original personnel policy is optimal, and it satisfies property $(ii.)$ of the proposition. It also satisfies property $(i.)$, which means that after period t, both cohorts earn wages $\hat{w} = c_1 + (1 - \delta(1-d)) R_2$, which must weakly exceed $\bar{w}_{1,t}^{\tau_1}$ and $\bar{w}_{1,t}^{\tau_2}$, or else $v_{1,t}^{\tau_1}$ or $v_{1,t}^{\tau_2}$ would exceed R_2. Moreover, property $(iii'.)$ is satisfied because $\bar{v}_{1,t+1}^{\tau_1} = \bar{v}_{1,t+1}^{\tau_2} = R_2$.

In case (c), $\bar{v}_{1,t+1}^{\tau_2} = R_2$, which necessarily exceeds $\bar{v}_{1,t+1}^{\tau_1}$. If $p_t^{\tau_1} \leq p_t^{\tau_2}$, then properties $(i.)$ and $(ii.)$ are automatically satisfied. We can then decrease $p_t^{\tau_1}$ by ε, increase $p_t^{\tau_2}$ by $L_{1,t}\varepsilon / L_{2,t}$. This perturbation does not affect $v_{1,t}^{\tau_2}$, since $\bar{v}_{1,t+1}^{\tau_2} = R_2$, and in order to maintain $v_{1,t}^{\tau_1}$, we increase $\tilde{v}_{1,t+1}^{\tau_1}$. We can keep doing this until either $\tilde{p}_t^{\tau_1} = 0$ or $\tilde{p}_t^{\tau_2} = 0$. Now, suppose $p_t^{\tau_1} > p_t^{\tau_2}$. Then choose $\tilde{p}_t^{\tau_1} = \tilde{p}_t^{\tau_2} = \left(L_{1,t} p_t^{\tau_1} + L_{2,t} p_t^{\tau_2}\right) / \left(L_{1,t} + L_{2,t}\right)$. This construction maintains cohort-τ_2's continuation payoff. Increase $\bar{v}_{1,t+1}^{\tau_1}$ to $\tilde{v}_{1,t+1}^{\tau_1}$ which maintains the same continuation payoff for cohort-τ_1. This construction satisfies properties $(i.)$ and $(ii.)$. Further, we can alter this construction just as we did in the proof of case 2 in order to construct an optimal personnel policy that satisfies property $(iii'.)$.

Finally, consider case (a). Set $\tilde{p}_t^{\tau_1} = \tilde{p}_t^{\tau_2} = \left(L_{1,t} p_t^{\tau_1} + L_{2,t} p_t^{\tau_2}\right) / \left(L_{1,t} + L_{2,t}\right)$, and choose $\tilde{v}_{1,t+1}^{\tau_1}$ and $\tilde{v}_{1,t+1}^{\tau_2}$ to maintain the same continuation payoffs for both cohorts. Since $v_{1,t}^{\tau_1} \leq v_{1,t}^{\tau_2}$, it must be the case that $\tilde{v}_{1,t+1}^{\tau_1} \leq \tilde{v}_{1,t+1}^{\tau_2}$. This establishes properties $(i.)$ and $(ii.)$, and we can use a similar argument as above to construct an optimal personnel policy that satisfies property $(iii'.)$. Properties $(ii.)$ and $(iii.)$ imply that the promotion policy is a modified FIFO policy.$\blacksquare$

Appendix B: Optimal Production

This appendix characterizes the firm's problem of choosing an optimal production plan. The first lemma establishes the result that under an optimal production plan, new hires receive no excess rents.

Lemma 4. *In an optimal production plan, workers receive no excess rents.*

Proof of Lemma 4. In order to get a contradiction, suppose that $v\left(n^t\right) > R_1$ for some n^t with $n_t = 1$. Then it must be the case that $w^t_{1,t} = 0$ or else the firm could reduce $w^t_{1,t}$ while still satisfying the incentive constraint for new hires in period t. We will construct a perturbation that holds fixed the firm's profits for all periods $\tau > t$ and under which the firm produces more in period t without paying any additional wages.

Suppose the firm hires ε additional workers into the bottom job in period t. Let the firm promote these workers with probability 1 in period τ, where τ is the first period in which existing new hires in period t are promoted with positive probability, and let the firm pay these workers $\tilde{w}^t_{1,t'} = 0$ for all $t \le t' \le \tau$. Notice that these workers' incentive constraints are satisfied. To see this, note that by Lemma 3, existing new hires can be paid $w^t_{1,t'} = 0$ for all $t \le t' \le \tau$ and have their incentive constraints satisfied in periods t to τ. The additional new workers are promoted with probability 1 in period τ, so their continuation payoffs in each period prior to τ are weakly higher than it is for existing new hires.

For the existing new hires, in period τ, reduce their promotion probability in period τ by ε/H_t and fire these workers with the same probability. This perturbation preserves the flow constraints and continues to satisfy the incentive constraints for existing new hires. Notice that it increases production in period t by $\theta_t f\left(N_{1,t} + \varepsilon, N_{2,t}\right) - \theta_t f\left(N_{1,t}, N_{2,t}\right) > 0$, and it preserves the firm's wage bill, which contradicts the claim that $v\left(n^t\right) > R_1$.■

The next lemma shows that the solution to the firm's reformulated production problem is a solution to its full problem.

Lemma 5. $\pi^* = \tilde{\pi}^*$.

Proof of Lemma 5. For all N, we have that $\tilde{\pi}(N) \ge \pi(N)$, because the optimal personnel policy under an exogenously given N may pay excess rents to new hires. First, we will show that $\tilde{\pi}^* \ge \pi^*$. To see why this is the case, note that by Lemma 4, $\tilde{\pi}(N^*) = \pi(N^*)$ and that at N^*, there exists a feasible personnel policy that gives new hires no excess rents. This means that $\tilde{\pi}^* \ge \tilde{\pi}(N^*) = \pi(N^*) = \pi^*$.

Next, we will show that $\pi^* \ge \tilde{\pi}^*$. To see this, let $\tilde{N}^*$ maximize $\tilde{\pi}\left(\tilde{N}\right)$ subject to the constraint that $\tilde{N} \in \mathcal{N}$. At this $\tilde{N}^*$, there exists a feasible personnel policy that generates profits $\tilde{\pi}\left(\tilde{N}^*\right)$ and gives no excess rents to new hires. We therefore have $\tilde{\pi}^* = \tilde{\pi}\left(\tilde{N}^*\right) = \pi(N^*) \le \pi^*$, completing the proof.■

The following proposition characterizes the solution to the $T = 2$ example.

Proposition 3. *There is a continuous and increasing function $\bar{\theta}_2(\theta_1)$ such that the optimal production plan N^* satisfies the following:*

(i.) **Intertemporal independence**: *If $\theta_2 < \bar{\theta}_2(\theta_1)$, then $\partial N^*_{i,1}/\partial\theta_2 = 0$ and $\partial N^*_{i,2}/\partial\theta_1 = 0$ for $i = 1, 2$.*

(ii.) **Forward linkages:** *If $\theta_2 > \bar{\theta}_2(\theta_1)$, then $\partial N^*_{i,1}/\partial\theta_2 > 0$ for $i = 1, 2$.*

(iii.) **Backward linkages**: *If $\theta_2 > \bar{\theta}_2(\theta_1)$, then $\partial N^*_{2,2}/\partial\theta_1 > 0$.*

Proof of Proposition 3. First, note that because $T = 2$, there is no need to give excess rents to new hires in period 2 because there are no future promotion opportunities for them. Moreover, any worker hired at $t = 1$ directly into activity 2 will also not receive excess rents. The only additional constraint that needs to be checked, therefore, is that the firm can give cohort-1 workers assigned to activity 1 no excess rents. This constraint can be written as $N_{2,2} \le (1-d)\,\hat{p}N_{1,1} + (1-d)\,N_{2,1}$.

To see why this additional constraint is necessary, note that if it is not satisfied, cohort-1 workers must be promoted at a rate exceeding $\hat{p}$, which means that even if they receive $w_{1,1}^1 = 0$, their rents will exceed R_1. This constraint is also sufficient, since if it is satisfied, cohort-1 workers assigned to activity 1 will be promoted at rate $(N_{2,2} - (1-d) N_{2,1}) / ((1-d) N_{1,1}) < \hat{p}$, and their first-period incentive constraints can be satisfied with a positive wage $w_{1,1}^1 > 0$.

The firm's problem is

$$\begin{aligned} &\max_{\{N_{1,t}, N_{2,t}\}_{t=1}^2} \theta_1 f(N_{1,1}, N_{2,1}) - (c_1 + R_1) N_{1,1} - (c_2 + R_2) N_{2,1} \\ &+\delta \left[\theta_2 f(N_{1,2}, N_{2,2}) - c_1 N_{1,2} - c_2 N_{2,2} - H_2 R_1\right], \end{aligned}$$

subject to the constraint that $N_{2,2} \leq (1-d) N_{1,1}\hat{p} + (1-d) N_{2,1}$. Let μ be the Lagrange multiplier on this constraint. The Kuhn-Tucker conditions for the firm's problem are

$$\begin{aligned} \theta_1 f_1'\left(N_{1,1}^*\right) &= c_1 + R_1 - \delta(1-d) R_1 - \mu^*(1-d)\hat{p} \\ \theta_1 f_2'\left(N_{2,1}^*\right) &= c_2 + R_2 - \delta(1-d) R_1 - \mu^*(1-d) \\ \theta_2 f_1'\left(N_{1,2}^*\right) &= c_1 + R_1 \\ \theta_2 f_2'\left(N_{2,2}^*\right) &= c_2 + R_1 + \mu^*/\delta, \end{aligned}$$

as well as $\mu^* \geq 0$, $N_{2,2}^* \leq (1-d) N_{1,1}^*\hat{p} + (1-d) N_{2,1}^*$, and complementary slackness.

Suppose the constraint is slack. Then the associated solution, N^*, in fact solves the constrained maximization problem if $N_{2,2}^* < (1-d) N_{1,1}^*\hat{p} + (1-d) N_{2,1}^*$, or

$$f_2'^{-1}\left(\frac{c_2 + R_1}{\theta_2}\right) > (1-d)\hat{p} f_1'^{-1}\left(\frac{c_1 + (1-\delta(1-d)) R_1}{\theta_1}\right) + (1-d) f_1'^{-1}\left(\frac{c_2 + R_2 - \delta(1-d) R_1}{\theta_1}\right).$$

Since f_1 and f_2 are strictly concave, the left-hand side of this inequality is increasing in θ_2, and the right-hand side is increasing in θ_1. Given θ_1, define $\bar{\theta}_2(\theta_1)$ so that this inequality holds with equality. This function is increasing in θ_1, and it is continuous. Moreover, for all $\theta_2 < \bar{\theta}_2(\theta_1)$, the inequality is satisfied, and therefore N^* solves the firm's relaxed and full problems and is locally intertemporally independent.

Next, suppose $\theta_2 > \bar{\theta}_2(\theta_1)$. Then it must be the case that $N_{2,2}^* = (1-d) N_{1,1}^*\hat{p} + (1-d) N_{2,1}^*$. The firm's optimality conditions can be combined to give us

$$\theta_1 f_1'\left(N_{1,1}^*\right) + \delta(1-d)\hat{p}\theta_2 f_2'\left(N_{2,2}^*\right) = c_1 + (1-\delta(1-d)) R_1 + \delta(1-d)\hat{p}(c_2 + R_1) \tag{1}$$

and

$$\theta_1 f_2'\left(N_{2,1}^*\right) + \delta(1-d)\theta_2 f_2'\left(N_{2,2}^*\right) = c_2 + R_2 - \delta(1-d) R_1 + \delta(1-d)(c_2 + R_2). \tag{2}$$

Note that the right-hand sides of these equations do not depend on θ_1 or θ_2.

We will first show that $\partial N_{i,1}^*/\partial\theta_2 > 0$. Differentiating (1) with respect to θ_2, we get

$$\theta_1 f_1''\left(N_{1,1}^*\right)\frac{\partial N_{1,1}^*}{\partial\theta_2} = -\delta(1-d)\hat{p}\left[f_2'\left(N_{2,2}^*\right) + \theta_2 f_2''\left(N_{2,2}^*\right)\frac{\partial N_{2,2}^*}{\partial\theta_2}\right].$$

In order to get a contradiction, suppose $f_2'\left(N_{2,2}^*\right) + \theta_2 f_2''\left(N_{2,2}^*\right)\partial N_{2,2}^*/\partial\theta_2 < 0$. Since $f_1'' < 0$, this implies that $\partial N_{1,1}^*/\partial\theta_2 < 0$. A similar argument establishes that $\partial N_{2,1}^*/\partial\theta_2 < 0$. But since

$N^*_{2,2} = (1-d) N^*_{1,1}\hat{p} + (1-d) N^*_{2,1}$, it must be the case that $\partial N^*_{2,2}/\partial\theta_2 < 0$, but this contradicts the assumption that $f'_2 \left(N^*_{2,2}\right) + \theta_2 f''_2 \left(N^*_{2,2}\right) \partial N^*_{2,2}/\partial\theta_2 < 0$. It must therefore be the case that $\partial N^*_{i,1}/\partial\theta_2 > 0$.

Next, we will snow that $\partial N^*_{2,2}/\partial\theta_1 > 0$. Differentiating (1) with respect to θ_1, we get

$$f'_1 \left(N^*_{1,1}\right) + \theta_1 f''_1 \left(N^*_{1,1}\right) \frac{\partial N^*_{1,1}}{\partial\theta_1} = -\delta (1-d) \hat{p}\theta_2 f''_2 \left(N^*_{2,2}\right) \frac{\partial N^*_{2,2}}{\partial\theta_1},$$

and differentiating (2) with respect to θ_1, we get

$$f'_1 \left(N^*_{2,1}\right) + \theta_1 f''_2 \left(N^*_{2,1}\right) \frac{\partial N^*_{2,1}}{\partial\theta_1} = -\delta (1-d) \hat{p}\theta_2 f''_2 \left(N^*_{2,2}\right) \frac{\partial N^*_{2,2}}{\partial\theta_1}.$$

In order to get a contradiction, suppose $\partial N^*_{2,2}/\partial\theta_1 < 0$. Then it must be the case that $f'_i \left(N^*_{i,1}\right) + \theta_1 f''_i \left(N^*_{i,1}\right) \partial N^*_{i,1}/\partial\theta_1 < 0$ for $i = 1, 2$. But then we must have $\partial N^*_{i,1}/\partial\theta_1 > 0$ for $i = 1, 2$, and since $N^*_{2,2} = (1-d) N^*_{1,1}\hat{p} + (1-d) N^*_{2,1}$, $\partial N^*_{2,2}/\partial\theta_1 > 0$, which is a contradiction. It must therefore be the case that $\partial N^*_{2,2}/\partial\theta_1 > 0$.■

Propositions 4 and 5 partially characterize optimal production plans when $T > 2$.

Proposition 4. *Let $\tilde{N}^*$ be the solution to the relaxed problem and $\tilde{p}^*_t$ be the corresponding average period-t promotion rate. If $\tilde{p}^*_t < \hat{p}$ for all t, then the optimal solution $N^* = \tilde{N}^*$ and features local intertemporal independence.*

Proof of Proposition 4. Suppose $\tilde{N}^*$ is a solution to the relaxed problem and that $\tilde{p}^*_t < \hat{p}$ for all t. Consider the following personnel policy. In each period t, all workers who have not yet been promoted are promoted with probability $\tilde{p}^*_t$, and they receive a wage $\tilde{w}_{1,t}$ satisfying

$$R_1 = \tilde{w}_{1,t} - c_1 + \delta (1-d) \left(\tilde{p}^*_t R_2 + (1-\tilde{p}_t) R_1\right).$$

Since $\tilde{p}_t < \hat{p}$ for each t, the wage $\tilde{w}_{1,t} > 0$. This personnel policy satisfies incentive compatibility in each period and provides new hires with no excess rents, so $\tilde{N}^*$ solves the full problem.

Fix t and increase θ_τ to $\theta_\tau + \varepsilon$ for $\varepsilon > 0$ small. Denote the perturbed demand path by $\theta'(\varepsilon)$, and let $\tilde{N}^* \left(\theta'(\varepsilon)\right)$ be the solution to the relaxed problem under demand path $\theta'(\varepsilon)$. We can again construct an incentive-compatible personnel policy under $\tilde{N}^* \left(\theta'(\varepsilon)\right)$ in which new hires receive no excess rents. To do so, consider a seniority-blind promotion policy with promotion rates $\tilde{p}'^*_t(\varepsilon)$ in period t and with wages satisfying

$$R_1 = \tilde{w}'_{1,t} - c_1 + \delta (1-d) \left(\tilde{p}'^*_t(\varepsilon) R_2 + (1-\tilde{p}_t) R_1\right).$$

Since $\tilde{w}_{1,t} > 0$ for all t, it must be the case that for ε sufficiently small, $\tilde{w}'_{1,t} > 0$ for all t as well. Such a policy therefore satisfies incentive compatibility in each period and also provides new hires with no excess rents, establishing that $\tilde{N}^* \left(\theta'(\varepsilon)\right)$ is a solution to the full problem. Moreover, since the relaxed problem has no intertemporal linkages, it must be the case that $\tilde{N}^*_{i,t} \left(\theta'(\varepsilon)\right) = \tilde{N}^*_{i,t}(\theta)$ for all $t \neq \tau$, so the optimal production plan features local intertemporal independence.■

Proposition 5. *The following are true:*

(*i.*) **Forward linkages:** *Fix a demand path θ* and a $t < T$. *There exists a demand path θ' with $\theta_\tau = \theta'_\tau$ for all $\tau \neq t$ and $\theta'_t > \theta_t$ under which $N^*_{i,t-1}(\theta) > N^*_{i,t-1}\left(\theta'\right)$ for some i.*

($ii.$) **Backward linkages:** *Fix θ_t for some $t > 1$. There exist demand paths θ' and θ'' with $\theta'_t = \theta''_t = \theta_t$ and $\theta'_\tau < \theta''_\tau$ for all $1 \leq \tau < t$ under which $N^*_{2,t}(\theta') < N^*_{2,t}(\theta'')$.*

Proof of Proposition 5. For the first part of the proposition, suppose θ is a constant demand path. Then the solution to the relaxed problem is a solution to the full problem, and it will be the case that in each period, $H^*_{t-1} > 0$. Consider a family of demand paths $\theta'(\kappa)$ that satisfy $\theta'_\tau = \theta_\tau$ for all $\tau \neq t$, and $\theta'_t = \theta_t + \kappa$ for $\kappa > 0$. We want to show that either $N^*_{1,t}(\theta'(\kappa)) > N^*_{1,t}(\theta'(0))$ or $N^*_{2,t}(\theta'(\kappa)) > N^*_{2,t}(\theta'(0))$ for some $\kappa > 0$.

Suppose to the contrary that $N^*_{1,t}(\theta'(\kappa)) \leq N^*_{1,t}(\theta'(0))$ and $N^*_{2,t}(\theta'(\kappa)) \leq N^*_{2,t}(\theta'(0))$ for all $\kappa > 0$. Let $H^*_t(\theta'(\kappa))$ denote the mass of new hires in period t under the optimal production plan under demand path $\theta'(\kappa)$. Workers hired in period $t-1$ under $\theta'(\kappa)$ must be promoted with probability exceeding

$$\frac{N^*_{2,t}(\theta'(\kappa)) - (1-d)N^*_{2,t-1}(\theta'(\kappa)) - (1-d)\left(N^*_{1,t-1}(\theta'(\kappa)) - H^*_{t-1}(\theta'(\kappa))\right)}{H^*_{t-1}(\theta'(\kappa))},$$

where notice that $N^*_{2,t}(\theta'(\kappa)) - (1-d)N^*_{2,t-1}(\theta'(\kappa))$ is the mass of vacancies available in the top job at the end of period $t-1$, and $(1-d)\left(N^*_{1,t-1}(\theta'(\kappa)) - H^*_{t-1}(\theta'(\kappa))\right)$ is the mass of workers assigned to activity 1 in period $t-1$ who were hired prior to period $t-1$. The numerator therefore represents the smallest mass of these vacancies that must be allocated to period-$t-1$ new hires. It must be the case that this probability is less than $\hat{p}$, or else new hires in period $t-1$ would have to be paid excess rents. This observation implies that we must have

$$\begin{aligned} N^*_{2,t}(\theta'(\kappa)) &\leq \hat{p}H^*_{t-1}(\theta'(\kappa)) + (1-d)\left[N^*_{2,t-1}(\theta'(\kappa)) + N^*_{1,t-1}(\theta'(\kappa)) - H^*_{t-1}(\theta'(\kappa))\right] \\ &= (1-d)\left[N^*_{2,t-1}(\theta'(\kappa)) + N^*_{1,t-1}(\theta'(\kappa))\right] + (\hat{p}+d-1)H^*_{t-1}(\theta'(\kappa)) \\ &\leq (1-d)\left[N^*_{2,t-1}(\theta'(\kappa)) + N^*_{1,t-1}(\theta'(\kappa))\right] + \max\left\{0, (\hat{p}+d-1)N^*_{1,t-1}(\theta'(\kappa))\right\} \\ &\leq (1-d)\left[N^*_{2,t-1}(\theta'(0)) + N^*_{1,t-1}(\theta'(0))\right] + \max\left\{0, (\hat{p}+d-1)N^*_{1,t-1}(\theta'(0))\right\}, \end{aligned}$$

where the last inequality holds by our assumption that $N^*_{i,t-1}(\theta'(\kappa)) \leq N^*_{i,t-1}(\theta'(0))$ for all $\kappa > 0$.

Next, notice that

$$(\theta_t + \kappa) f'_2\left(N^*_{2,t}(\theta'(\kappa))\right) \leq c_2 + R_2,$$

or else the firm would increase its profits by hiring an additional worker in period t, assign him to activity 2, and then assign him to activity 0 in all future periods. This implies that

$$N^*_{2,t}(\theta'(\kappa)) \geq f'^{-1}_2\left(\frac{c_2 + R_2}{\theta_t + \kappa}\right),$$

which in turn implies that $N^*_{2,t}(\theta'(\kappa)) \to \infty$ as $\kappa \to \infty$, since f_2 satisfies $f'_2(N_{2,t}) \to 0$ as $N_{2,t} \to \infty$. This is a contradiction, so we must have that $N^*_{i,t-1}(\theta'(\kappa)) > N^*_{i,t-1}(\theta'(0))$ for some $\kappa > 0$ and some i. This establishes the first part of the proposition.

We prove part ($ii.$) by construction. Let θ'' be the demand path that satisfies $\theta''_\tau = \theta_t$ for all τ. By the previous proposition, the solution to the relaxed problem, $\tilde{N}^*(\theta'')$, is a solution to the firm's full problem: $\tilde{N}^*(\theta'') = N^*(\theta'')$. However, notice that under the relaxed problem,

$N_{2,1}^{*}(\theta'') < N_{2,t}^{*}(\theta'')$, since they satisfy

$$\theta_1'' f_2'\left(N_{2,1}^{*}(\theta'')\right) = c_2 + R_2$$

and for $1 < t < T$,

$$\theta_t'' f_2'\left(N_{2,t}^{*}(\theta'')\right) = c_2 + (1 - \delta(1-d)) R_1 < c_2 + R_2,$$

and

$$\theta_T'' f_2'\left(N_{2,T}^{*}(\theta'')\right) = c_2 + R_1 < c_2 + R_2,$$

and f_2 is concave.

Now consider a demand path θ' under which $\theta_\tau' = \theta_t$ for all $\tau \geq t$ and $\theta_\tau' = \varepsilon$ for $\tau < t$ and $\varepsilon > 0$ small. Notice that as $\varepsilon \to 0$, the firm's optimal production plan converges to the optimal production plan for a problem in which period t is the first period. We therefore have that $N_{2,t}^{*}(\theta') \to N_{2,1}^{*}(\theta'')$, which is strictly smaller than $N_{2,t}^{*}(\theta'')$. For ε sufficiently small, it must be the case that $N_{2,t}^{*}(\theta') < N_{2,t}^{*}(\theta'')$, establishing the first part of the proposition.■

Appendix C: Unsteady Production and Partial-Effort Contracts

Proposition 6. *Suppose N satisfies $N_{1,t+1} < (1-d) N_{1,t}$, $N_{2,t+1} > (1-d) N_{2,t+1}$, and $N_{1,t+1} + N_{2,t+1} < (1-d)(N_{1,t} + N_{2,t})$ for all t. There is an optimal personnel policy that implements N in which:*

(*i.*) *laid-off workers receive severance pay,*

(*ii.*) *if $\tau < \tau'$, then $p_{0,t}^{\tau} \leq p_{0,t}^{\tau'}$, and*

(*iii.*) *conditional on being laid off, workers with more seniority receive greater severance pay.*

Proof of Proposition 6. Using a similar argument as in the proof of Proposition 2, we may assume that $v_{1,t}^{\tau}$ is decreasing in τ. That is, later-cohort workers value being assigned to activity 1 in period t more than newer-cohort workers. Given an optimal personnel policy, we now construct an optimal personnel policy with the desired properties by specifying $w_{1,t}^{\tau}, p_{0,t}^{\tau}, p_{2,t}^{\tau}, v_{1,t+1}^{\tau}$, and $v_{0,t+1}^{\tau}$. To do this, we proceed in three steps.

First, we will assign promotion opportunities in each period to workers so that workers with positive promotion probabilities all receive the same continuation payoff if they are not promoted, and the average rate of promotion for workers satisfies the flow constraint for activity 2, that is, $p_{2,t} = [N_{2,t+1} - (1-d) N_{2,t}] / [(1-d) N_{1,t}]$. In particular, it can be shown that there exists a k such that for all $\tau > k$, we have $p_{2,t}^{\tau} = 0$, and for all $\tau, \tau' \leq k$, promotion probabilities will satisfy the following two sets of equations. First, for all $\tau, \tau' \leq k$

$$\frac{1 - p_{2,t}^{\tau}}{1 - p_{2,t}^{\tau'}} = \frac{v_{1,t}^{\tau} + c_1 - \delta(1-d) R_2}{v_{1,t}^{\tau'} + c_1 - \delta(1-d) R_2},$$

which ensures that, if they receive a wage of 0 this period, workers promoted with positive probability receive the same continuation conditional on not being promoted. Second, the flow constraint for activity 2 is satisfied $\sum_{\tau=1}^{k} N_{1,t}^{\tau} p_{2,t}^{\tau} = p_{2,t} N_{1,t}$. These two sets of equations pin down k and $p_{2,t}^{\tau}$ for all τ. Given the associated promotion probabilities, we can write, for each τ,

$$v_{1,t}^{\tau} = -c_1 + \delta(1-d)\left(p_{2,t}^{\tau} R_2 + \left(1 - p_{2,t}^{\tau}\right) \tilde{v}_{t+1}^{\tau}\right).$$

Notice that our construction ensures that $\tilde{v}^{\tau}_{t+1} = \tilde{v}^{\tau'}_{t+1}$ for all $\tau, \tau' \leq k$, and $\tilde{v}^{\tau}_{t+1} \geq \tilde{v}^{\tau'}_{t+1}$ for all $k \leq \tau \leq \tau'$.

In the second step, we construct wages $w^{\tau}_{1,t}$ and continuation payoffs $\hat{v}^{\tau}_{t+1}$ for each cohort to guarantee that each cohort is promoted with the same probability as in the previous step, they receive the same payoffs $v^{\tau}_{1,t}$, and $\hat{v}^{\tau}_{t+1} \leq R_2$ for all τ. That is,

$$w^{\tau}_{1,t} = \max\left\{v^{\tau}_{1,t} + c_1 - (1 - \delta(1-d)) R_2, 0\right\}.$$

This implies that we can write

$$v^{\tau}_{1,t} = w^{\tau}_{1,t} - c_1 + \delta(1-d)\left(p^{\tau}_{2,t} R_2 + \left(1 - p^{\tau}_{2,t}\right)\hat{v}^{\tau}_{t+1}\right).$$

Notice that this construction implies there is some k' such that $w^{\tau}_{1,t} = 0$ for all $\tau \geq k'$.

Finally, we construct severance probabilities $p^{\tau}_{0,t}$ so that workers with the least seniority are laid off first, and we construct severance values $v^{\tau}_{0,t+1}$ so that the incentive constraints for laid-off workers remain satisfied. To this end, let $v^{\tau}_{0,t+1} = \hat{v}^{\tau}_{t+1} = v^{\tau}_{1,t+1}$, and write $p^{\tau}_{0,t} = \left(1 - p^{\tau}_{2,t}\right)\rho^{\tau}_t$, where ρ^{τ}_t is the probability of being laid off conditional on not being promoted. The flow constraint for activity 1 requires that the number of workers who are laid off is equal to the number of workers the firm has to get rid of, or

$$\sum_{\tau=1}^{t} p^{\tau}_{0,t} N^{\tau}_{1,t} = (1-d)(N_{1,t} + N_{2,t}) - (N_{1,t+1} + N_{2,t+1}).$$

This constraint implies there exists a k'' such that $\rho^{\tau}_t = 1$ for all $\tau > k''$, and $\rho^{\tau}_t = 0$ for all $\tau < k''$. This constructed policy satisfies all the conditions in the statement of the proposition.■

Proposition 7. *Suppose there is a t_1 at which $N_{1,t_1+1} < (1-d) N_{1,t_1}$ and $N_{1,t_1+1} + N_{2,t+1} < (1-d)(N_{1,t_1} + N_{2,t_1})$, and there is a $t_2 > t_1$ at which $N_{1,t_2+1} + N_{2,t_2+1} > (1-d)(N_{1,t_2} + N_{2,t_2})$. Then:*

(*i.*) *no workers are permanently laid off in period t_1, and*

(*ii.*) *$v^{\tau}_{1,t_2+k} \geq v^{t_2+k}_{1,t_2+k}$ for all $\tau < t_2$ and for all $k \geq 1$.*

Proof of Proposition 7. Suppose $L\left(n^{t_2+1}\right) > 0$ and there is a positive mass of workers who worked for the firm by t_2 but are assigned to activity 0 in period t_2 and receive payoff v^{τ}_{0,t_2+1} for some $\tau < t_2$. Suppose such workers are permanently laid off. There are then two cases: either $v^{\tau}_{0,t_2+1} \geq v^{t_2+1}_{i,t_2+1}$ or $v^{\tau}_{0,t_2+1} < v^{t_2+1}_{i,t_2+1}$.

In the first case, consider an alternative personnel policy in which the firm does not hire the new worker and instead rehires the old worker and treats him the way the firm would have treated the new worker but pays him an additional $v^{\tau}_{0,t_2+1} - v^{t_2+1}_{i,t_2+1}$ in period $t_2 + 1$. This new personnel policy still satisfies the flow constraint for activity i, and it satisfies the promise-keeping constraint and the incentive constraints for the re-hired worker, and it pays out less in rents to new hires, so it reduces the overall wage bill.

In the second case in which $v^{\tau}_{0,t_2+1} < v^{t_2+1}_{i,t_2+1}$, similarly consider an alternative personnel policy in which the firm does not hire the new worker and instead rehires the old worker and treats him exactly the same ways as the firm would have treated the new worker. This new personnel policy is again feasible and reduces the overall wage bill because it pays out less in rents to new hires. This establishes part (*i.*).

For part $(ii.)$, if it is ever the case that $v^{\tau}_{1,t_2+k} < v^{t_2+k}_{1,t_2+k}$, then we can instead give the new worker initial rents of v^{τ}_{1,t_2+k} and give the cohort-τ worker period t_2+k rents of $v^{t_2+k}_{1,t_2+k}$. The associated personnel policy relaxes the cohort-τ worker's incentive constraint for all periods $t \leq t_2+k$, and it reduces the initial rents of the cohort-t_2+k worker while maintaining their incentive constraint. It therefore reduces the firm's overall wage bill.■

Proposition 8. *Suppose there is an optimal full-effort contract in which $p^1_2 = 1$, $v^1_{1,1} = R_1$, $p^2_2 < 1$, and $v^2_{1,2} > R_1$. Then there is a partial-effort contract that has a lower wage bill.*

Proof of Proposition 8. Consider the following perturbation. Take a small mass ε of cohort-1 workers who have not yet been promoted by period 2, and ask them not to exert effort. Instead, hire an additional ε cohort-2 workers, and promote all cohort-2 workers with probability $\tilde{p}^2_2 = p^2_2 \frac{N_{1,2}}{N_{1,2}+\varepsilon}$ instead of with probability p^2_2. These workers will still receive excess rents for ε sufficiently small. Finally, hire $(1-d)\,\varepsilon$ fewer workers in period 3. Under this perturbation, the total rents for cohort-1 workers remains $N_{1,1}R_1$. The total rents for cohort-2 workers also remains the same, even though each cohort-2 worker receives slightly lower rents. And the total rents for cohort-3 workers falls because there are fewer of them. We argue that the total rents paid to new hires therefore falls. Each cohort-2 worker's value in period 2 under the original personnel policy is

$$v_2 = -c_1 + \delta\,(1-d)\left[p^2_2 R_2 + \left(1-p^2_2\right) R_1\right],$$

and their value in period 2 under the perturbed policy is

$$\tilde{v}_2 = -c_1 + \delta\,(1-d)\left[\tilde{p}^2_2 R_2 + \left(1-\tilde{p}^2_2\right) R_1\right],$$

where $\tilde{p}^2_2 = \frac{N_{1,2}}{N_{1,2}+\varepsilon} p^2_2$. The total rents paid to cohort 2 are therefore:

$$\begin{aligned}
(N_{1,2}+\varepsilon)\,\tilde{v}_2 &= -\,(N_{1,2}+\varepsilon)\,c_1 + \delta\,(1-d)\left[(N_{1,2}+\varepsilon)\,\tilde{p}^2_2 R_2 + (N_{1,2}+\varepsilon)\left(1-\tilde{p}^2_2\right) R_1\right] \\
&= -\,(N_{1,2}+\varepsilon)\,c_1 + \delta\,(1-d)\left[N_{1,2} p^2_2 R_2 + \left((N_{1,2}+\varepsilon) - N_{1,2} p^2_2\right) R_1\right] \\
&= -\varepsilon c_1 + \delta\,(1-d)\,\varepsilon R_1 - N_{1,2} c_1 + \delta\,(1-d)\left[N_{1,2} p^2_2 R_2 + \left(N_{1,2} - N_{1,2} p^2_2\right) R_1\right] \\
&= \varepsilon\left(-c_1 + \delta\,(1-d)\,R_1\right) + N_{1,2} v_2.
\end{aligned}$$

Thus, this perturbation increases cohort 2's total rents by $\varepsilon\left(-c_1 + \delta\,(1-d)\,R_1\right)$. From period 2's perspective, this perturbation also reduces cohort-3's rents by $\delta\varepsilon\,(1-d)\,R_1$ since there are $\varepsilon\,(1-d)$ fewer cohort-3 workers. It therefore reduces the total rents of cohorts 2 and 3 by εc_1, while leaving the total rents of cohort 1 the same.■

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