NON-LINEAR PRICING WITH RENEGING*

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(Preliminary version, please do not circulate.)

Abstract

This paper studies a dynamic non-linear pricing problem, adding the possibility that the seller can costly renege on the initial contracts, which is common in reality in the forms of false advertising, add-on pricing and bait-and-switch. While reneging allows the seller to earn more surplus by offering a new full-extraction contract after learning the buyer's preference, a forward looking buyer will hide information. We fully characterize the equilibrium direct mechanism with the presence of this strategic interaction and show that the quality distortion may be mitigated and participation can be higher when the market moves from full-commitment to one with modest reneging cost. We establish the precise condition under which the welfare improvement happens and further relate it to whether the market is niche or mass. In addition, we show that in the sequential equilibrium, there always exists an implementable contract throughout the game even if the seller has already incurred the reneging cost and is free to modify it. By explicitly modeling seller's information extraction problem without full commitment, our results have policy implications on protecting consumers from deceptive business tactics.

Keywords: Non-linear pricing, renege, commitment, word-of-mouth.

JEL Classification: C73, D82, D86, L12, L14, L15

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1 Introduction

Canonical works such as Mussa and Rosen (1978) and Maskin and Riley (1984) have shown that by providing a spectrum of quality-differentiated products, a seller can segment consumers of different preferences. A crucial assumption of theirs is that the seller can commit to the contract terms offered. In practice, however, it is oftentimes observed that sellers fail to make such commitment. Instead, sellers may renege and offer new contracts upon consumers' choice, which is the common feature of business schemes such as add-on pricing, bait-and-switch and deceptive advertisements. Such deviation normally is not costless. It has consequences to sellers in the form of penalties from the regulator or loss of future profit due to damaged reputation. In this paper, we formalize the scenario by studying a dynamic mechanism design problem that allows the seller to costly renege on his initial offers and switch to a brand new menu.

In a typical direct mechanism with full-commitment power, information asymmetry causes seller to lose information rents and incur efficiency distortion. In our framework, the seller's main incentive to renege is to restore such losses if the buyer reveals type through her initial choice. On the other hand, a rational buyer will foresee the seller's strategy and hide her true preference. Therefore, if the seller would like to elicit information, he has to adjust the contract terms in order to convince the buyer that the cost of reneging surpasses the benefit. The seller then trades-off the value of the information against the loss from the contract adjustment. When will the seller decide to elicit information and avoid reneging? How will the seller adjust the contract terms accordingly? When and how will the seller renege on the initial offers? Do consumers always get hurt from the reneging possibility? The cost of reneging oftentimes depends on the strength of regulation. Are the harshest possible penalties always preferred?

The above questions is also motivated by the observed phenomenons in reality. In practice, it is commonly seen that some sellers fulfilling their initial offers while others unilaterally alternate the contract terms through business schemes such as add-on pricing, bait-and-switch and deceptive advertisements. Here we list two such examples where sellers renege on a subset of their offers. In this paper, we argue that such partial reneging could be optimal for sellers when they tend to separate the market but the commitment power is limited.

Staples. Stores like Staples offer in-store only discounts which are usually quite generous. However, an article in 2012[1] suggests that the low price may just be a bait and the store will switch terms when consumers try to buy. The article describes how Staples required their sales to sell on average $200 worth add-on products in addition to its advertised discounted items or the store refused to sell and guided customers to buy online instead.

Airlines. Airline companies usually advertise about cheap economy-class tickets. What the advertisements don't say is that the service passengers will receive may not be exactly the same as they expect and they may need to pay extra for checking luggages, beverage and snacks, on-board entertainment and so on[2]. For business-class tickets, however, they seldom conduct such tactics.

In effort to offer an explanation to these phenomenons from the perspective of information extraction, we construct a dynamic non-linear pricing model with costly reneging. Specifically, the game involves two stages. At the initial stage, a monopolistic seller (principal, he) proposes a menu of price-quality contracts and a buyer (agent, she) with private type chooses one of them. Given the buyer's choice, the seller may clinch the deal and end the game. Otherwise, the seller can pay a fixed cost k, renege on his initial offer and provide a new menu to the buyer. The seller can fully commit to the new contracts in the second stage. In our model, full commitment scenario corresponds to the case that k is sufficiently large, because the reneging cost is always larger than information rents and efficiency loss. When the cost approaches zero, the stage 1 contracts have no commitment power at all and a buyer will reveal no information. As a result, the game will move on to stage 2 for sure, and the equilibrium payoffs are still identical to the full commitment benchmark.

The challenge is to characterize the optimal dynamic direct mechanism at the intermediate level of the reneging cost k. We first consider "reneging-proof" equilibrium. Namely, we attempt to find the range of k under which the seller implements a direct mechanism to separate buyers and can perfectly commit to it. Under binary type space, the information rent and efficiency loss in the second-best solution are denoted as "Δ_H" and "Δ_L", respectively. Accordingly,

[1] See: https://www.nytimes.com/2012/09/09/your-money/sales-incentives-at-staples-draw\-complaints-the-haggler.html.

[2] See https://www.theglobeandmail.com/life/travel/how-does-a-224-flight-end-up\-costing-826/article1214981/

the seller cannot commit to the mechanism when k is lower than either "Δ_H" or "Δ_L", which implies that if the seller would like to elicit full information, he needs to adjust the mechanism so that a set of self-commitment constraints (SC) hold.

Given a direct mechanism, as k decreases, some of the SC constraints start to break down. The seller has to adjust the contracts to maintain it. To do so, the seller has to claim higher profit from the types (i.e., decreasing Δ). Specifically, if the contract for high type violates its SC, the seller will increase the price for the contract as high type buyers already enjoy quality at efficient level, while if the contract for low type violates its SC, the adjustment is to enhance its quality, which implies a decrease of distortion.

In the adjustment process, two observations worth noticing. First, compared with the second best mechanism, the optimal reneging-proof mechanism improves consumer surplus and social welfare with these modest levels of k if and only if the efficiency loss (Δ_L) is larger than the information rent (Δ_H). Second, under optimal adjustment, decrease in efficiency loss always implies an increase in information rent and vice versa. The above observation implies that as k becomes further smaller, reneging-proof mechanism is no longer feasible. At best, the seller can only extract partial information at the initial stage. We show that the lower bound of k for the equilibrium partial revelation is exactly the minimum of all Δ's, below which the first stage is simply "babbling".

In a partial revelation equilibrium, the mechanism at stage 1 consists of some implementable contracts, while others are "baits" that will never be executed. In equilibrium, rational buyers are completely aware of the seller's strategy. We show that the equilibrium with partial revelation is unique and can be of only two possible profiles. In a Partial-$L(H)$ profile, $L(H)$ type hide information while $H(L)$ type reveals preference. Meanwhile, the contract designed for $L(H)$ is implementable and the other is a "bait" in the Partial-$L(H)$ profile. Partial-L emerges if and only if information rent is higher than the efficiency loss.

Moreover, for contract terms, we show that the optimal implementable contract at stage 1 is exactly identical to the corresponding contract at stage 2, which belongs to the second best mechanism based on updated distribution. The result is surprising a priori, because after all, the seller has already paid the reneging cost and is free to offer any contract. Equivalently, it means that when a seller is offering a menu of contracts in the two-stage negotiation, there exists some options he will always honor. This result matches with our observation in the *Staples* example that the customers who chose the in-store price but refused to buy the add-on products were guided by the sales to make the order online, at where the contract is clear. In the *Airline* example, after finding out the additional charges, passengers can always buy business class instead if she find the discount is actually inferior. As for the "bait" contract, either the price or the quality or both can be different from the second period contract. This result can accommodate business schemes such as bait-and-switch, deceptive advertisement, add-on price and so on. In such practices, while the existing research focuses on that the sellers may renege on some of the listed deals, we argue that in the same maximization problem, they decide to commit to some contracts that are always available throughout. Another unexpected result is that despite the occasional reneging behavior, consumers are still better off compared to the full commitment scenario when Partial-H profile is an equilibrium.

Our results in both reneging-proof and partial revelation equilibria shows that the relative size between the information rent and efficiency loss in second best mechanism plays a crucial role in the equilibrium profiles as well as welfare implication. In short, consumer surplus and social welfare increase if and only if the efficiency loss is larger than the information rent. The two scenarios largely depend on market conditions. We show that efficiency loss is smaller than the information rent when the market is a niche in the sense that the proportion of low preference buyers is large enough, or when buyers' preferences are less diversified. On the other hand, the efficiency loss is larger than the information rent in a mass market or when buyers' preferences are more diversified.

We also consider the implications of reneging on the extensive margin when the second best mechanism does not have full participation. It is actually corresponding to an case of mass market such that the market has more high type agents or the buyers have large preference gaps. In this case, the information rent Δ_H is zero for high type and Δ_L is equal to first best surplus for low type. Therefore, the seller has the channel to include low type buyers into the market. We show that the possibility of reneging indeed increases participation of low demand agents and has a positive impact on consumer surplus and social welfare.

Finally, we discuss the sources of the reneging cost k. The first possible source is the explicit punishment policy designed by regulators. Indeed, the Federal Trade Commission (FTC) in the

US and similar agencies in other countries all impose certain penalties on deceptive business acts. Surprisingly, our model suggests that the optimal policy might not be extremely large. When there is a large proportion of high demand buyers, and when the high demand is high enough or the low demand is low enough, a modest level of punishment schemes not only prevents deceptive acts, but also improves efficiency. In other scenarios, a modest level of punishment is also enough to preclude deceptive acts and ensure the second best welfare.

We also endogenize k by considering a dynamic word-of-mouth model. In the setup, we assume that in each period there are $B \in \mathbb{N}$ short-lived agents entering the market and transacting with a long-lived monopolistic seller. They are born unaware of the seller's reneging possibility, but otherwise are perfectly capable of choosing the best contract from a menu. The new buyers can communicate with all buyers from the last period and become aware of the reneging possibility (sophisticated) if any of the old buyers encountered reneging behavior. Given the setup, the reneging cost k is determined as the opportunity cost of turning a group of unaware buyers into sophisticated ones, since the seller is able to extract the first best surplus from the former. In a Markovian equilibrium, the endogenous k reflects the impact of the word-of-mouth. We show that the seller always reneges on the contract offered to an unaware L (H) type buyer if there are less of them and implement the contract offered to an unaware H (L) type buyer. Then the seller faces a cohort of sophisticated buyers next period with some probability. We further show that the seller will not renege when facing sophisticated buyers only if B is large enough, i.e., only when the word-of-mouth effect is significant enough.

Related Literature

This paper builds on canonical setup of non-linear pricing in the spirit of Mussa and Rosen (1978). Other seminal works include Spence (1977), Myerson (1981) and Maskin and Riley (1984). By providing a menu of price-quality contracts, the principal is able to induce agents to reveal information voluntarily[3]. Recent works consider extending non-linear models along different directions. For instance, many works such as Rochet and Stole (2002), Yang and Ye (2008) and Armstrong and Vickers (2010), consider non-linear pricing under competition.

[3] The survey by Riley (2001) summarizes all the important works of the models.

Instead, this paper considers a dynamic framework, such that the early stage contracts may act as a learning device. More importantly, compared with the benchmark in Mussa and Rosen (1978), our work shows that the welfare distortion on the low demand types may decrease in such dynamic framework.

It is quite common in practice that a seller does not have full commitment power and there is much research considering business tactics that involves the sellers' commitment problem. For instance, Ellison (2005) considers add-on pricing among competitive sellers who conduct price discrimination. It shows that even if a rational consumer expects the strategy, she may still accept the deal given it is costly to visit another seller. Later works such as Ellison and Ellison (2009) and Gabaix and Laibson (2006) also build on the framework. Our work considers a similar scenario under which the seller can deviate from the advertised offer term, but assume that the the seller is subject to a cost of deviation. In Ellison (2005), sellers have to keep the price of basic product (vertically low quality), which is always available to buyers. In our model, we show that the availability of basic product could be endogenously determined even if the seller is free to renege on any contracts initial design. In addition, we show that quality of basic product is not necessarily low quality. We characterize the equilibrium profile under which the high quality product is available throughout the game while the low quality ones might subject to change.

More broadly speaking, our work is also related to the literature concerning consumer protection against schemes such as deceptive advertisement and bait-and-switch. For instance Rhodes and Wilson (2018), considers the optimal regulation toward a seller who has incentive to misreport product quality and establishes conditions that allow a certain level of false ads that benefits social welfare. Our work also considers how the financial penalty, modelling as the reneging cost, influences the seller's contract design and finds the precise conditions that the distortion can be decreased. But our work differs by assuming that it is the buyer who has private information. Other examples include Inderst and Ottaviani (2013) who considers contract cancellation and Armstrong and Chen (2017) who considers discounting price quotation.

Theoretically, our work is related to strategic communication built on cheap talk models (Kartik, 2009; Kartik et al., 2007). In fact, in our model, when the reneging cost k is zero, the first stage communication is simple a cheap-talk and reveals no information. Kartik et al.

(2007) also discusses the sources of lying cost, such as regulations and reputation, which also apply to our framework. Moreover, we discuss the imperfection information inference from a mechanism design perspective (Green and Laffont, 1986). Deneckere and Severinov (2007) shows that under optimal mechanism, there is no exclusion. Our work shows that the design with reneging can enlarge the extensive margin since the principal has a channel to induce more agents to participate.

Finally, the sequential structure of our work is related to the growing literature of mechanism design with limited commitment, such as Bester and Strausz (2001), Skreta (2006) and Doval and Skreta (2018). In contrast to the research which attempts to find optimal mechanism and develop revelation principal under dynamic mechanism design, we characterize players equilibrium strategic interactions.

The rest of the paper is organized as follows. Section 2 introduces the framework and characterizes basic strategy profile structure. Section 3 discusses equilibrium and its welfare implications. Specifically, depending on the reneging cost, the equilibrium could be of reneging-proof, partial-revealing or babbling. In addition, we consider the effect on extensive margin by alternating model primitives. Section 4 discusses the policy implication of reneging cost from a legal and reputational perspective. A word-of-mouth model is introduced. Section 5 concludes.

2 Model

We consider a two-stage non-linear pricing problem. At Stage 1, a seller designs a menu of contracts. A buyer with private information visits and make a choice. Given the buyer's choice, the seller can renege on the contract by paying a fixed cost and providing another menu at Stage 2. On one hand, reneging may allow the seller to learn about the buyer and extract more surplus. One the other hand, a rational and forward looking buyer may hide information to avoid being squeezed.

2.1 Environment

The game has two stages without discounting. There are two parties - a monopolistic seller (Principal, he) and a buyer with private information (Agent, she). The seller produces quality q with cost $C(q)$ and we assume $C'(q) > 0, C''(q) > 0$. The buyer's private information is her marginal utility $\theta \in \Theta \subset \mathbb{R}^+$ of the good. We consider a binary type space $\Theta = \{L, H\}$ with distribution $f^s = (f^s_L, f^s_H)$, where $s = 1, 2$ indexes stages. Notice that the distribution is also equivalent to the principal's belief, which may be updated according to agent's behavior. Thus, f^1 and f^2 could be different.

We focus on contracts of format with price and quality - $(p, q) \in \mathbb{R}^{+2}$. Then buyer's utility given (p, q) is

$$u_\theta(p, q) = \theta q - p$$

while the seller's profit is

$$v(p, q) = p - C(q)$$

Assume both parties have outside option normalized to zero. The first best quality q^{FB}_θ satisfies[4]

$$C'(q^{FB}_\theta) = \theta$$

The timing of the game is as follows. At Stage 1, the seller proposes a mechanism with a menu of contracts $\Psi^1 \subset \mathbb{R}^{+2}$. Then the buyer chooses at most one in Ψ^1. Given buyer's choice, the seller can either clinch the deal immediately and the game ends. Alternatively, he can pay a fixed *reneging cost* $k \geq 0$ and initiate Stage 2, at which the seller proposes a fresh new mechanism $\Psi^2 \subset \mathbb{R}^{+2}$, while Ψ^1 becomes no longer available. Given Ψ^2, the buyer, chooses at most one contract from it. We assume the seller cannot renege on the deal again such that the game ends anyway at Stage 2.

[4] Since the seller is a monopolistic seller, the first best solution is also the perfect discrimination solution in which agents earn zero utility.

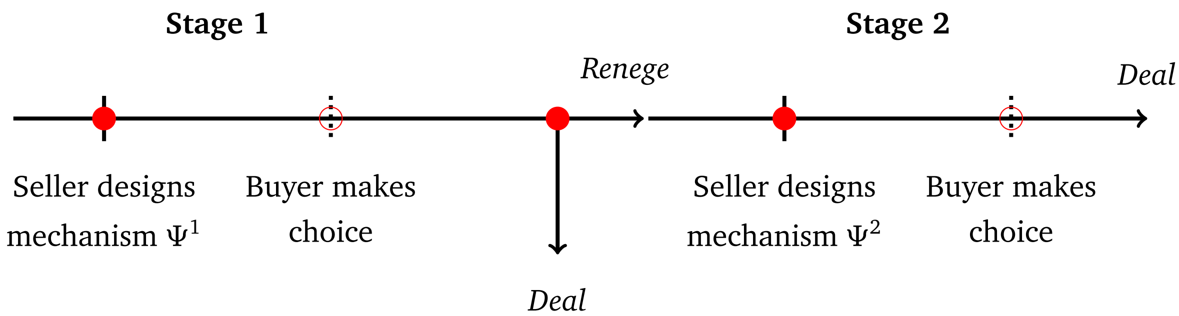

Figure 1: Timeline

One key feature of the model is that the seller is able to learn buyer's type through her choice at Stage 1. Hence, by reneging and initiating Stage 2, the seller can potentially extract more surplus. On the other hand, a rational and forward looking buyer will expect such strategy and hide information at Stage 1. The strategic interaction between the two parties makes the game nontrivial. Given the setup, we attempt to answer the following questions.

- How large will the reneging cost have to be to refrain the seller from reneging?
- Which type of buyers will be more likely to trade at Stage 2?
- How does the limited commitment affect welfare, buyer's and seller's payoff compared with non-linear pricing environment discussed by Mussa and Rosen (1978)?

We restrict the game within two periods. Hence, as long as the game moves to Stage 2, subgame perfection requires that the seller will implement second best mechanism to maximize expected profit based on updated belief $f^2 = (f_L^2, f_H^2)$. We briefly review the important results of the one-stage mechanism as follows.

Stage 2 Optimum

In the spirit of Mussa and Rosen (1978), the "second best" mechanism, $\Psi^{SB}(f)$ is the solution to the following problem,

$$v^{SB}(f) = \max_{p_\theta, q_\theta} f_H(p_H - C(q_H)) + f_L(p_L - C(q_L)) \qquad (\mathscr{P}^{SB})$$

subject to

$$Hq_H - p_H \geq Hq_L - p_L \tag{IC$_H$}$$

$$Lq_L - p_L \geq Lq_H - p_H \tag{IC$_L$}$$

$$Hq_H - p_H \geq 0 \tag{IR$_H$}$$

$$Lq_L - p_L \geq 0 \tag{IR$_L$}$$

In the second best mechanism, IC_H and IR_L are binding,

$$Hq_H^{SB} - p_H^{SB} = Hq_L^{SB} - p_L^{SB}; Lq_L^{SB} - p_L^{SB} = 0$$

And hence, the mechanism satisfies the following FOCs,

$$C'(q_H^{SB}) = H; C'(q_L^{SB}) = \frac{L}{f_L} - \frac{f_H H}{f_L} \tag{1}$$

We are interested in separating equilibrium in this section[5]. Thus, we restrict our attention to the parameters that satisfy the following assumption[6],

Assumption 1. *The type space and the respective distribution satisfies*

$$L \geq f_H H$$

Accordingly, we summarize some important properties of the mechanism as follows,

Proposition 1. *(Mussa and Rosen '78) The second best mechanism $\Psi^{SB}(f)$ is separating and satisfies the following properties*

P1. High type always participate. Low type participates if and only if $L \geq f_H H$;

P2. High type buyer consumes at efficient level $q_H^{SB} = q_H^{FB}$ and earns information rent $u_H^{SB} > 0$;

[5] The other case with $L < f_H H$ will be discussed in Section 3.

[6] Notice that when Assumption 1 is violated, then the optimal mechanism is to only serve high type, i.e.

$$p_H = p_H^{FB}, q_H = q_H^{FB}$$

P3. Low type buyer consumes below efficient level $q_L^{SB} < q_L^{FB}$ and earns zero surplus $u_L^{SB} = 0$;

P4. High type's utility u_H^{SB} is decreasing in f_H;

P5. The profits earned from each type $v_H^{SB} \equiv p_H^{SB} - C(q_H^{SB})$ and $v_L^{SB} \equiv p_L^{SB} - C(q_L^{SB})$ are increasing and decreasing in f_H, respectively.

P6. The seller's expected profit v^{SB} is increasing in f_H with

$$\frac{\partial v^{SB}}{\partial f_H} = v_H^{SB} - v_L^{SB}$$

The second best mechanism will act as a benchmark in the two-stage game. Also, for subgame perfection, we assume that when the game move to Stage 2, the seller will implement $\Psi^{SB}(f^2)$.

2.2 Strategy Profiles

Now we consider the problem at Stage 1. The strategy profile consists of a set of contracts designed by the seller, the buyer's initial choices, and the seller's response to the seller's report. Of course, both parties understand that as long as the game moves to Stage 2, they will follow the second best solution with full-commitment.

We restrict of our attention to direct mechanisms. That is, the seller designs two contracts at Stage 1. The buyer chooses the contract by reporting $\hat{\theta} \in \{\hat{L}, \hat{H}\}$. Define the contracts at Stage 1

$$\Psi^1 = \left\{p^1(\hat{\theta}), q^1(\hat{\theta})\right\}$$

On the other hand, we treat the buyer's choice as reporting her type. Denote the probability that a type θ buyer reports $\hat{\theta}$ as[7]

$$\mathbf{g} = \{g(\hat{\theta}|\theta) \in [0,1] : g(\hat{H}|H) \geq g(\hat{H}|L), g(\hat{H}|\theta) + g(\hat{L}|\theta) = 1\}$$

Also, define $g(\hat{\theta}) \equiv \sum_\theta f_\theta g(\hat{\theta}|\theta)$ as the overall probability that the seller receives a report $\hat{\theta}$.

[7] Because $\hat{\theta}$ is simply a signal, we only focus on the profile with $g(\hat{H}|H) \geq g(\hat{H}|L)$. Obviously, any profile with $g(\hat{H}|H) < g(\hat{H}|L)$ is equivalent to having $\tilde{H} = \hat{L}, \tilde{L} = \hat{H}$, which gives $g(\tilde{H}|H) \geq g(\tilde{H}|L)$.

We refer the scenario in which $g(\hat{H}|H) = g(\hat{L}|L) = 1$ as "perfect revelation", while $g(\hat{H}|H) + g(\hat{L}|L) = 1$ as "no information". And any case in between will be "partial revelation".

Based on $\mathbf{g}$ and $\hat{\theta}$, the seller decides whether to set the deal and earn

$$v^1(\hat{\theta}) \equiv p^1(\hat{\theta}) - C(q^1(\hat{\theta}))$$

or to renege, pay k and implement $\Psi^{SB}(f^2(\hat{\theta}))$. Denote the seller's probability of reneging as

$$\mathbf{r} = \{r(\hat{\theta}) \in [0,1]\}$$

The equilibrium profile is a triple $\{\Psi^1, \mathbf{g}, \mathbf{r}\}$ that satisfies both sides' incentive compatibility and also maximizes the seller's expected profit, which is formally defined as follows.

Definition 1. *The equilibrium consists of the triple $\{\Psi^1, \boldsymbol{g}, \boldsymbol{r}\}$ which maximizes the seller's expected profit*

$$V(k) = \max \sum_{\hat{\theta} \in \{\hat{L}, \hat{H}\}} g(\hat{\theta}) \cdot \Big((1 - r(\hat{\theta})) \cdot v^1(\hat{\theta}) + r(\hat{\theta}) \cdot (v^{SB}(f^2(\hat{\theta})) - k) \Big) \qquad (\mathscr{P}^{RE})$$

subject to the following constraints,

1. *Given* $\mathbf{g}$*, Bayesian update gives*

$$f_H^2(\hat{\theta}) = \frac{f_H g(\hat{\theta}|H)}{f_H g(\hat{\theta}|H) + f_L g(\hat{\theta}|L)} \qquad (2)$$

2. *The seller is rational in reneging decision* $\boldsymbol{r}$*,*

$$r(\hat{\theta}) \begin{cases} = 1 & \text{if} \quad v^1(\hat{\theta}) > v^{SB}(f^2(\hat{\theta})) - k \\ \in [0,1] & \text{if} \quad v^1(\hat{\theta}) = v^{SB}(f^2(\hat{\theta})) - k \\ = 0 & \text{if} \quad v^1(\hat{\theta}) > v^{SB}(f^2(\hat{\theta})) - k \end{cases} \qquad (3)$$

3. *Both types of buyer is incentive compatible in choosing* **g***,*

$$g(\hat{H}|H)\begin{cases} =1 & \textit{if} \quad u_H(\hat{H}) > u_H(\hat{L}) \\ \in[0,1] & \textit{if} \quad u_H(\hat{H}) = u_H(\hat{L}) \\ =0 & \textit{if} \quad u_H(\hat{H}) < u_H(\hat{L}) \end{cases} \tag{4}$$

and

$$g(\hat{L}|L)\begin{cases} =1 & \textit{if} \quad u_L(\hat{L}) > u_L(\hat{H}) \\ \in[0,1] & \textit{if} \quad u_L(\hat{L}) = u_L(\hat{H}) \\ =0 & \textit{if} \quad u_L(\hat{L}) < u_L(\hat{H}) \end{cases} \tag{5}$$

where $u_\theta(\hat{\theta}) \equiv (1-r(\hat{\theta}))\cdot(\theta q^1(\hat{\theta}) - p^1(\hat{\theta})) + r(\hat{\theta})\cdot u_\theta^{SB}(f^2(\hat{\theta}))$ *represents buyer* θ*'s expected payoff if she reports* $\hat{\theta}$*.*

Since all three parties may randomize their actions, which will change the objective and constraints significantly, we have to discuss the equilibrium by cases. In general, our analysis strategy consists two steps. First, we rule out the impossible strategy profiles. That is, we will exclude the profiles, that either violates incentive compatibility or fails to maximize profit, regardless of k and type distribution. Second, for those possible profiles, we will pin down the range of reneging cost k that supports them. For instance, as we will discuss later, there exists equilibrium without renege if and only if k is sufficiently large.

Our first result shows that there exists no equilibrium in which the seller randomizes between reneging and not. Formally, we have

Lemma 1. *There exists no equilibrium, in which* $r(\hat{\theta}) \in (0,1)$ *for any* $\hat{\theta} \in \{\hat{L}, \hat{H}\}$*.*

We prove Lemma 1 by ruling out all the profiles with $r(\hat{\theta}) \in (0,1)$. Intuitively, the seller prefers to separate the buyers to the maximum extent, so that he can extract more surplus. Thus, given any partial revelation profile, a profitable adjustment is to increase either $g(\hat{H}|H) \in (0,1)$ or $g(\hat{L}|L) \in (0,1)$. If $r(\hat{\theta}) \in (0,1)$, the adjustment is feasible since any violation of buyer's indifference conditions can be resolved by varying $r(\hat{\theta})$. Second, when there is perfect revelation, we further show that the seller always prefers either reneging and earning first best surplus net of the cost or settling the deal at the first stage depending on k.

Now we restrict our attention on the profiles with $r(\hat{\theta}) \in \{0,1\}$. The following lemma further shows that the two types of buyers will not implement different mixed strategy at the same time.

Lemma 2. *There exists no equilibrium, in which $g(\hat{L}|L)+g(\hat{H}|H) \neq 1$ and both $g(\hat{L}|L), g(\hat{H}|H) \in (0,1)$.*

In the expression, $g(\hat{L}|L)+g(\hat{H}|H) \neq 1$ implies partial revelation, such that $f \neq f^2$. By the same logic, the seller wishes to separate the two types until some $g(\hat{\theta}|\theta) = 1$.

Proposition 2. *For any k and type distribution that satisfies Assumption 1, only profiles including the following four reneging behaviors are possible*

- *Reneging-proof: Perfect revelation at stage 1 without reneging,*

$$r(\hat{H}) = r(\hat{L}) = 0; g(\hat{H}|H) = g(\hat{L}|L) = 1;$$

- *Partial-H: Some type-H lies, the seller reneges to $\hat{L}$,*

$$r(\hat{H}) = 0; r(\hat{L}) = 1; g(\hat{H}|H) \in (0,1); g(\hat{L}|L) = 1;$$

- *Partial-L: Some type-L lies, the seller reneges to $\hat{H}$,*

$$r(\hat{H}) = 1; r(\hat{L}) = 0; g(\hat{H}|H) = 1; g(\hat{L}|L) \in (0,1);$$

- *Babbling (no information at stage 1),*

$$r(\hat{H}) = r(\hat{L}) = 1; g(\hat{H}|H) + g(\hat{L}|L) = 1;$$

3 Equilibrium

In Section 2, we introduce the model ingredients and discuss possible strategy profiles in e-quilibrium. By extending the classical non-linear pricing model to a dynamic game, we significantly enrich players' action space as well as strategies. As indicated in Proposition 2, we show that only four strategy profiles are possible in equilibrium. In this section, we attempt to investigate the profiles. Namely, we will fully characterize contract terms under different equilibrium profiles and find model primitives that support them.

3.1 Reneging-proof mechanisms

We start with discussing equilibrium under which the seller commits to the mechanism at stage 1, i.e.

$$r(\hat{H}) = r(\hat{L}) = 0; g(\hat{H}|H) = g(\hat{L}|L) = 1.$$

Then the optimal *reneging-proof mechanisms* $\Psi(k) = \{(p_H, q_H), (p_L, q_L)\}$ is the solution to

$$V(k) = \max_{\Psi} \sum_{\theta} f_\theta v_\theta = \sum_{\theta} p_\theta - C(q_\theta)$$

subject to IC_θ, IR_θ and SC_θ for both $\theta \in \{H, L\}$. Since the game ends at stage 1, SC_θ can be written as

$$k \geq w_\theta^{FB} - v_\theta$$

Namely, the seller's most profitable reneging strategy is to use the first best contract so that the net gain is $w_\theta^{FB} - v_\theta$. The mechanism is reneging-proof if the cost k is larger than the gain. For notation neatness, we use symbol "Δ" to represent the net gain from reneging.

Definition 2. *Given a separating mechanism $\Psi = \{(p_H, q_H), (p_L, q_L)\}$, define $\Delta_\theta(\Psi)$ as*

$$\Delta_\theta(\Psi) \equiv w_\theta^{FB} - (p_\theta - C(q_\theta))$$

Specifically, under Ψ^{SB}, denote

$$\Delta_\theta^{SB} \equiv w_\theta^{FB} - v_\theta^{SB}$$

When $k \geq \bar{k} \equiv \max\{\Delta_L^{SB}, \Delta_H^{SB}\}$, both SC constraints are satisfied under the second best mechanism Ψ^{SB}. Hence, Ψ^{SB} is indeed the seller's optimal choice.

When $k = \bar{k} - \varepsilon$, then second best is no longer reneging proof. Actually, when ε is not too large, reneging-proof mechanism still exists. In the following analysis, we will analysis the range of k that supports reneging-proof and characterize the mechanisms. To solve the problem, first we have the following result which significantly simplifies our solution concept

Lemma 3. *Under the optimal reneging-proof mechanism, if exists, is separating with IC_H and IR_L binding and $q_H = q_H^{FB}$.*

We call the mechanisms that bind IC_H and IR_L with $q_H = q_H^{FB}$ *binding mechanism*. And a binding mechanism can be only represented by q_L, i.e.,

$$V(k) = \max_{q_L} f_H(Hq_H^{FB} - (H-L)q_L - C(q_H^{FB})) + f_L(Lq_L - C(q_L))$$

subject to

$$k \geq \Delta_H = (H-L)q_L \tag{SC_H}$$

$$k \geq \Delta_L = w_L^{FB} - (Lq_L - C(q_L)) \tag{SC_L}$$

Given Ψ is a binding mechanism, Δ_L measures the welfare distortion from low type's first best output, while Δ_H represents the information rent paid to the high type. As an illustration, Figure 2 demonstrate Δ's using a simple example. Consistent with the figure, Δ_θ and q_L has monotone relationships.

Lemma 4. *For binding mechanisms Ψ represented by q_L, Δ_H is increasing in q_L and Δ_L is decreasing in q_L. There exists a unique binding mechanism $\underline{\Psi}$ such that*

$$\Delta_H(\underline{\Psi}) = \Delta_L(\underline{\Psi})$$

Lemma 4 implies that any adjustment of q_L will induce Δ_H and Δ_L move to the different directions. Intuitively, when q_L increases, it will decrease low type's welfare distortion, but it will increase high type's incentive to mimic low type, such that the seller has to pay higher information rent. Therefore, we may expect that (i) how to adjust q_L depends on whether $\Delta_L^{SB} > \Delta_H^{SB}$ or not, and (ii) when k is sufficiently small, there exist no binding mechanism that can support both SC constraints. Accordingly, we divide our analysis into two cases. In particular, we are interested in how q_L differs from q_L^{SB}.

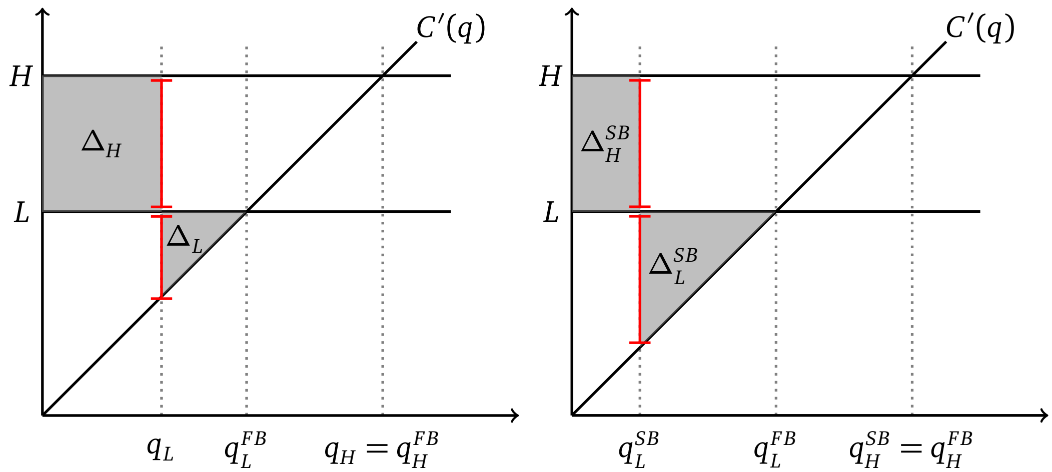

Figure 2: Δ and binding mechanism

Note: (1) The graph demonstrates an environment in which seller's cost is quadratic in q (linear marginal cost) and $f_H = f_L = 0.5$; (2) The left panel demonstrates one example of binding mechanism. It is not optimal since the marginal benefit of decreasing q_L is $0.5(H - L)$, the longer red segment. The marginal loss is the shorter red segment. Thus, decreasing q_L will increase the seller's expected profit. (3) The right panel shows that as the marginal loss becomes increasingly larger, the process will stop at the second best scenario, in which the two segments are of the same length.

Case 1: $\Delta_L^{SB} > \Delta_H^{SB}$ Suppose $k = \bar{k} - \varepsilon$, then SC_L is violated under Ψ^{SB}, while SC_H is still slack if ε is sufficiently small. So the optimal mechanism $\Psi^*(k)$ is the solution to the following problem

$$V(k) = \max_{q_L} f_H(Hq_H^{FB} - (H - L)q_L - C(q_H^{FB})) + f_L(p_L - C(q_L)) \tag{$\mathscr{P}_L^*$}$$

subject to

$$Lq_L - C(q_L) = Lq_L^{FB} - C(q_L^{FB}) - k \tag{SC_L}$$

The binding SC_L implies in the optimal reneging-proof mechanism, $p_L - C(q_L) > v_L^{SB}$ and $q_L > q_L^{SB}$. That is, to convince the a low type buyer that the there will be no reneging, the seller needs to retain more profit at the first stage. Since the optimal mechanism is binding, higher profit from low type is equivalent to having larger q_L in a binding mechanism. Compared with the second best mechanism, the optimal reneging mechanism under $k = \bar{k} - \varepsilon$ reduces low type's quality distortion, which implies welfare improving, as shown in Figure 3.

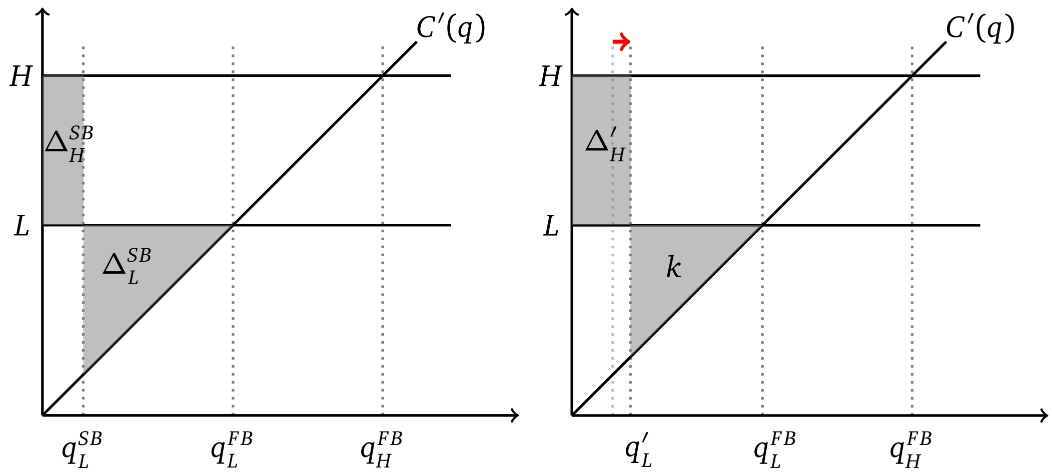

Figure 3: Adjustment of q_L when $\Delta_L^{SB} > \Delta_H^{SB}$

Note: (1) The left panel shows the second best solution when $\Delta_L^{SB} > \Delta_H^{SB}$. (2) To satisfy SC_L, q_L will be increased until the triangle's area is equal to k as shown in right panel. (3) Meanwhile, to maintain SC_H, we also need to make sure that $\Delta_H' \leq k$.

Case 2: $\Delta_L^{SB} < \Delta_H^{SB}$ If $k \equiv \bar{k} - \varepsilon$, then SC_H is violated under Ψ^{SB}, while SC_L is still slack. Then the seller's problem becomes

$$V(k) = \max_{q_L} f_H(Hq_H^{FB} - (H-L)q_L - C(q_H^{FB})) + f_L(p_L - C(q_L)) \qquad (\mathscr{P}_H^*)$$

subject to

$$p_H - C(q_H) = Hq_H^{FB} - C(q_H^{FB}) - k \qquad (\text{SC}_H)$$

The binding SC_H implies that $p_H - C(q_H) > v_H^{SB}$. In this case, the seller has to convince high type buyer that he will not renege. As shown in Figure 4, the optimal reneging-proof mechanism induces lower q_L compared with Ψ^{SB}.

The above analysis shows that for some $k < \max\{\Delta_L^{SB}, \Delta_H^{SB}\}$, by adjusting q_L, there exists reneging-proof mechanism exists. Compared with Ψ^{SB}, the mechanism generates higher profit from type θ if SC_θ is binding. Given different model primitives, the optimal reneging-proof mechanism has different welfare implications. Further, we realize that reneging-proof mechanism does not exist if k is sufficiently small. By Lemma 4, the lowest reneging cost that

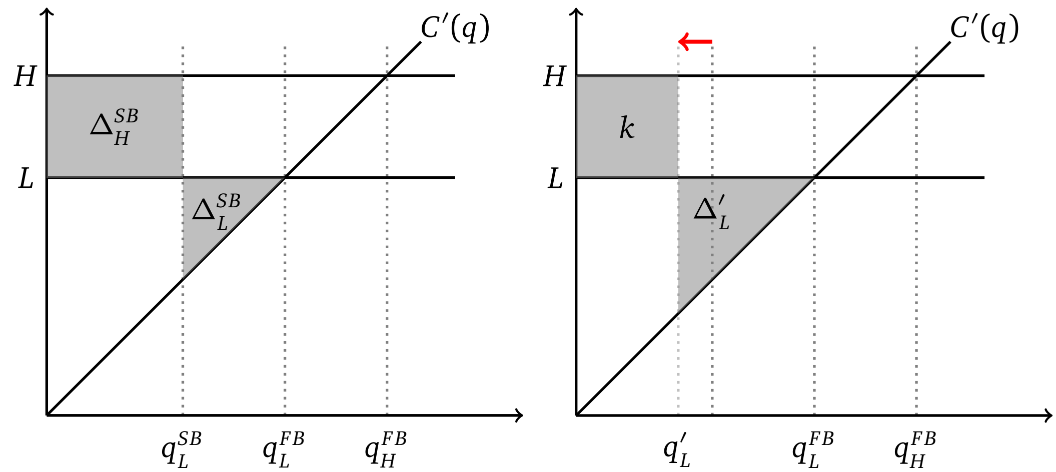

Figure 4: Adjustment of q_L when $\Delta_L^{SB} < \Delta_H^{SB}$

Note: (1) The left panel shows the second best solution when $\Delta_L^{SB} < \Delta_H^{SB}$. (2) To satisfy SC_H, q_L will be decreased until the rectangle's area is equal to k as shown in right panel. (3) Meanwhile, to maintain SC_L, we also need $\Delta_L' \leq k$.

maintains reneging-proof mechanism is

$$\tilde{k} \equiv \Delta_L(\tilde{\Psi}) = \Delta_H(\tilde{\Psi}) \leq \bar{k}$$

Based on the analysis above, we summarize our results as the following proposition.

Proposition 3. *Reneging-proof mechanism exists if $k \geq \tilde{k}$ and the optimal reneging-proof mechanism Ψ^* satisfies the following properties,*

P1. Ψ^* is a binding mechanism;

P2. When $k \geq \bar{k}$, $\Psi^* = \Psi^{SB}$;

P3. When $k \in [\tilde{k}, \bar{k})$, if $\Delta_L^{SB} > \Delta_H^{SB}$, then $q_L^* > q_L^{SB}$;

P4. When $k \in [\tilde{k}, \bar{k})$, if $\Delta_L^{SB} < \Delta_H^{SB}$, then $q_L^* < q_L^{SB}$.

The third and forth properties indicates that the committing mechanism can improve social welfare if and only if $\Delta_L^{SB} > \Delta_H^{SB}$, since low type buyers suffer less distortion. In terms of expected payoff, in both cases, the seller is less profitable than the second best scenario. On the other side, high type buyers enjoy more information rent when $\Delta_L^{SB} > \Delta_H^{SB}$.

3.2 Equilibrium with partial revelation

In this section, we consider the two profiles with partial revelation.

- Partial-H: Some type-H lies; the seller reneges to $\hat{L}$,

$$r(\hat{H}) = 0; r(\hat{L}) = 1; g(\hat{H}|H) \in (0,1); g(\hat{L}|L) = 1;$$

- Partial-L: Some type-L lies; the seller reneges to $\hat{H}$,

$$r(\hat{H}) = 1; r(\hat{L}) = 0; g(\hat{H}|H) = 1; g(\hat{L}|L) \in (0,1);$$

Partial-H: Denote $x \equiv g(\hat{H}|H) \in (0,1)$. When the game moves to stage 2, the type distribution f^2 becomes

$$f_H^2 = \frac{f_H(1-x)}{f_H(1-x)+f_L} < f_H$$

And the seller will implement $\Psi^{SB}(f^2)$, in which q_L satisfies

$$C'(q_L) = \frac{L - f_H^2 H}{f_L^2} = \frac{Lf_L - (H-L)(1-x)f_H}{f_L} \tag{6}$$

so that $q_L^{SB}(f^2) > q_L^{SB}$. $x \in (0,1)$ also indicates that type-H buyer is indifferent between reporting $\hat{H}$ and $\hat{L}$. Since $r(\hat{H}) = 0$ and $r(\hat{L}) = 1$, the condition becomes $Hq_H^1 - p_H^1 = Hq_H^{FB} - p_H^{SB}(f^2)$. Given that the stage 2 mechanism is $\Psi^{SB}(f^2)$, the seller's optimal design is to equalize (q_H^1, p_H^1) and $(q_H^{FB}, p_H^{SB}(f^2))$. Effectively, the equilibrium involves only 2 contracts that form a binding mechanism. The choice variables can be reduced to q_L and x,

$$V(k) = \max_{q_L, x} f_H(Hq_H^{FB} - (H-L)q_L - C(q_H^{FB})) + f_L(Lq_L - C(q_L)) - (f_L + f_H(1-x)) \cdot k$$

subject to Equation (6) and SC_H, which can be written as

$$k \geq (H-L)q_L$$

The above objective is no larger than $v^{SB}(f)$, therefore, when $k > \bar{k}$, $\Psi^{SB}(f)$ without reneging is still the optimal solution. When $k < \bar{k}$, the profile is not feasible if $\Delta_L^{SB} < \Delta_H^{SB}$. Since $\Delta_H^{SB} = \bar{k} > k \geq (H-L)q_L$ requires $q_L < q_L^{SB}$, which is impossible given $x \in (0,1)$. In addition, the profile cannot exist anyway when $k < \underline{k} \equiv \min\{\Delta_H^{SB}, \Delta_L^{SB}\}$.

Partial-L: Denote $y \equiv g(\hat{L}|L) \in (0,1)$. When the game moves to stage 2, the type distribution f^2 becomes

$$f_H^2 = \frac{f_H}{f_H + f_L(1-y)} > f_H$$

And the seller will implement $\Psi^{SB}(f^2)$, in which q_L satisfies

$$C'(q_L) = \frac{L - f_H^2 H}{f_L^2} = \frac{Lf_L(1-y) - (H-L)f_H}{f_L(1-y)} \tag{7}$$

so that $q_L^{SB}(f^2) < q_L^{SB}$. $y \in (0,1)$ also indicates that type-L buyer is indifferent between reporting $\hat{H}$ and $\hat{L}$. Since $r(\hat{H}) = 1$ and $r(\hat{L}) = 0$, the condition becomes $Lq_L^1 - p_L^1 = 0$. Meanwhile, we also need to guarantee that high type buyer has no incentive to mimic low type. Given that the stage 2 mechanism is $\Psi^{SB}(f^2)$, the seller's optimal design is to equalize (q_L^1, p_L^1) and $(q_L^{SB}(f^2), p_L^{SB}(f^2))$. Similar to the previous profile, the choice variables are only q_L and y,

$$V(k) = \max_{q_L, y} f_H(Hq_H^{FB} - (H-L)q_L - C(q_H^{FB})) + f_L(Lq_L - C(q_L)) - (f_L(1-y) + f_H) \cdot k$$

subject to Equation (7) and SC_L,

$$k \geq w_L^{FB} - (Lq_L - C(q_L))$$

Obviously, $q_L < q_L^{SB}$ so that profile is not feasible when $k < \underline{k}$ and when $k \in [\underline{k}, \bar{k}]$, the profile is feasible only if $\Delta_H^{SB} > \Delta_L^{SB}$.

Proposition 4. *The equilibrium $\{\Psi^*, \boldsymbol{g}, \boldsymbol{r}\}$ is partial revealing if $k \in (\underline{k}, \tilde{k})$ and satisfies the following properties,*

P1. The equilibrium is of Partial-$H(L)$ if and only if only if $\Delta_L^{SB} > (<) \Delta_H^{SB}$.

P2. Under Partial-H profile, $q_L^* > q_L^{SB}$;

P3. Under Partial-L profile, $q_L^* < q_L^{SB}$.

3.3 Discussion on Δ_θ

The results are largely depending on the relative sizes of Δ_L^{SB} and Δ_H^{SB}, which are determined by model primitives such as H, L and f_θ. It is important to discuss the key determinants that makes $\Delta_H^{SB} \geq \Delta_L^{SB}$ and vice versa.

We impose the following assumption for the comparative statics.

Assumption 2. $C'''(q) \geq 0$ *for any* $q \geq 0$.

The quadratic cost function which is commonly used in the literature satisfies the assumption.

Proposition 5. *For given H and L, there exist a unique* $\widehat{f}_L$*, such that* $\Delta_H^{SB} \geq \Delta_L^{SB}$ *if* $f_L \in [\widehat{f}_L, 1]$ *and* $\Delta_H^{SB} < \Delta_L^{SB}$ *if* $f_L \in [1 - \frac{L}{H}, \widehat{f}_L)$.

For given L and f_L*, there exist a unique* $\widehat{H}$*, such that* $\Delta_H^{SB} \geq \Delta_L^{SB}$ *if* $H \in [L, \widehat{H}]$ *and* $\Delta_H^{SB} < \Delta_L^{SB}$ *if* $H \in (\widehat{H}, \frac{L}{f_H}]$.

For given H and f_L*, there exist a unique* $\widehat{L}$*, such that* $\Delta_H^{SB} \geq \Delta_L^{SB}$ *if* $L \in [\widehat{L}, H]$ *and* $\Delta_H^{SB} < \Delta_L^{SB}$ *if* $L \in [f_H H, \widehat{L})$.

Proof. Define $D := \Delta_H^{SB} - \Delta_L^{SB}$. We can calculate

$$D = Hq_L^{SB} - Lq_L^{FB} + C(q_L^{FB}) - C(q_L^{SB})$$

where q_H^{SB} and q_L^{SB} are determined by

$$\begin{aligned} C'(q_H^{SB}) &= H \\ C'(q_L^{SB}) &= \frac{L - f_H H}{f_L} \end{aligned}$$

First, notice that

$$\frac{\partial q_L^{SB}}{\partial f_L} > 0$$
$$\frac{\partial D}{\partial f_L} = [H - C'(q_L^{SB})]\frac{\partial q_L^{SB}}{\partial f_L} > 0$$

The permissible f_L ranges from $1-\frac{L}{H}$ to 1. When $f_L = 1-\frac{L}{H}$, $q_L^{SB} = 0$ and $D = -Lq_L^{FB} + C(q_L^{FB}) < 0$. When $f_L = 1$, $q_L^{SB} = q_L^{FB}$ and $D = (H-L)q_L^{FB} > 0$. This proves the first part of the proposition.

Next, take derivative of D with respect to H, we have

$$\frac{\partial D}{\partial H} = q_L^{SB} - \frac{(H-L)f_H}{(f_L)^2}\frac{1}{C''(q_L^{SB})}$$
$$\frac{\partial^2 D}{\partial H^2} = -\frac{f_H}{(f_L)^2} + \frac{\partial q_L^{SB}}{\partial H}[1 + \frac{(H-L)f_H}{(f_L)^2}\frac{C'''(q_L^{SB})}{(C''(q_L^{SB}))^2}] < 0$$

The permissible range of H is from L to $\frac{L}{f_H}$. When $H = L$, $q_L^{SB} = q_L^{FB}$, $D = 0$ and $\partial D/\partial H > 0$. When $H = \frac{L}{f_H}$, $q_L^{SB} = 0$, $D = -Lq_L^{FB} + C(q_L^{FB}) < 0$ and $\partial D/\partial H < 0$. Therefore, as H increases, D first increases from 0, then starts to decrease and finally reaches a negative value. This proves the second part of the proposition.

Finally, we calculate

$$\frac{\partial D}{\partial L} = -q_L^{FB} + \frac{H-L}{(f_L)^2}\frac{1}{C''(q_L^{SB})}$$
$$\frac{\partial^2 D}{\partial L^2} = -\frac{1}{(f_L)^2} - \frac{\partial q_L^{SB}}{\partial L}[1 + \frac{H-L}{(f_L)^2}\frac{C'''(q_L^{SB})}{(C''(q_L^{SB}))^2}] < 0$$

From the first equation, we know that D decreases in L when L is close to H and D may be increasing in L when L is small.

The permissible range of L is from $f_H H$ to H. When $L = f_H H$, $q_L^{SB} = 0$ and $D < 0$. When $L = H$, $q_L^{SB} = q_L^{FB}$ and $D = 0$. Therefore, as L increases, D first increases from a negative value to some positive value, then starts to decrease and finally reaches zero. This proves the third part of the proposition. □

This proposition shows that when the majority of buyers are low type, or when buyers have similar preferences, the information rent is higher than the efficiency loss. The majority of buyers are of low type in a niche market. Then our result shows that in a niche market or when buyers' preferences are less diversified, the seller has to further enlarges the quality distortion in order to extract the private information, and the "bait" is the high-quality product.

On the contrary, when the market is a mass market (in the sense that majority of buyers are high type), or when buyers have diversified preferences, the efficiency loss is larger than the information rent. Then the seller has to narrows the quality distortion in order to learn buyers' types, and the "bait" is the low-quality product.

To understand the result in the proposition, notice that when q_L^{SB} is higher, holding other things constant, the efficiency loss is smaller and the information rent is larger. Then the information rent tends to be larger than the efficiency loss. q_L^{SB} is larger if a buyer is more likely to be of low type, if the high demand is lower and if the low demand is higher. Notice that at the same time, when the high demand gets lower, the information rent decreases, and when the low demand gets higher, the efficiency loss increases and the information rent decreases. These two effects move the comparison of Δ's in the opposite direction as the impact of q_L^{SB}. Indeed, the proof of the proposition shows that as H gets closer to L, eventually the difference between Δ_H^{SB} and Δ_L^{SB} decreases. Similarly, as L gets closer to H, eventually the difference decreases. However, we show that these trends cannot overturn the impact of q_L^{SB}.

3.4 Extensive Margin

The analysis in Section 2 considers the equilibrium relying on Assumption 1, such that in the second best benchmark, both H and L buyers participate. In this section, we consider the alternative case.

Assumption 3. *The type space and the respective distribution satisfy*

$$L < f_H H$$

Accordingly, the benchmark mechanism is summarized as follows.

Lemma 5. *Under Assumption 3, the second best mechanism $\Psi^{SB}(f)$ excludes low type and the* *seller use the first best contract for H, i.e.*

$$q_L^{SB} = q_H^{FB}, p_H^{SB} = Hq_H^{SB}$$

And

$$\Delta_L^{SB} = w_L^{FB}, \Delta_H^{SB} = 0$$

Lemma 5 indicates that the lower type is excluded in from the market while all the surplus from the high type is earned by the seller. More importantly, the case indicates that

$$\Delta_L^{SB} > \Delta_H^{SB} = 0$$

for sure. Therefore, when $k < \Delta_L^{SB} = w_L^{FB}$, a reneging proof mechanism has SC_L binding. Consistent with the illustration by Figure 5, the seller can commit to a reneging-proof mech-

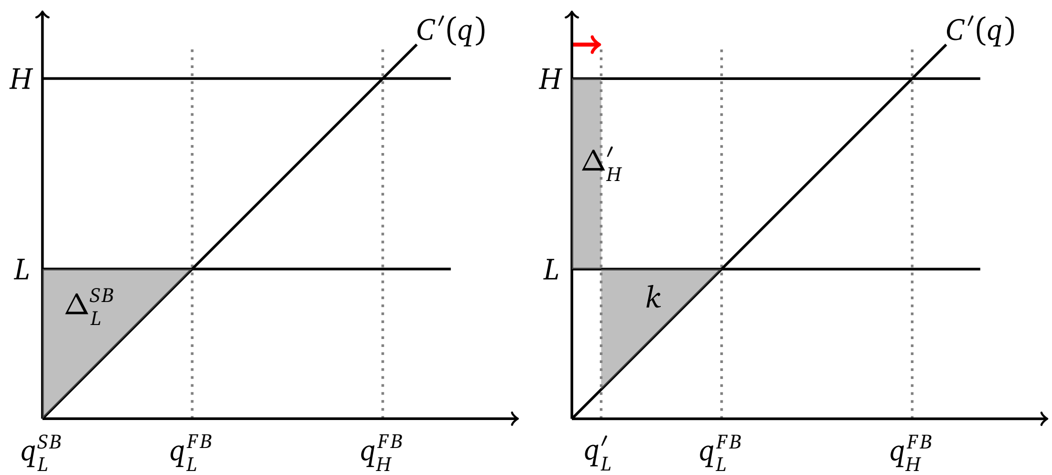

Figure 5: Adjustment at extensive margin

Note: (1) The left panel shows the second best solution under Assumption 3 and $\Delta_L^{SB} = w_L^{FB}$. (2) SC_L demands the seller to choose $q_L > 0$. (3) Meanwhile, to maintain SC_H, we also need to make sure that $\Delta_H' \leq k$.

anism when k is sufficiently large. By monotonicity of Δ in q_L, there exists a $\tilde{k}$ above which reneging-proof is guaranteed.

When $k < \tilde{k}$, the seller cannot commit to both types. Similar to the analysis in Section 2, we need to consider which self-commitment constraint to maintain. Following the similar logic, it is easy to see that the equilibrium is of partial-H profile. In addition, since $\Delta_H^{SB} = 0$, Proposition 6 implies that as long as k is positive, L buyers will participate in equilibrium. Proposition 6 summarizes the key result on extensive margin. In add

Proposition 6. *Under Assumption 3, reneging-proof mechanism exists if any only if $k \geq \tilde{k}$. When $k \in (0, \tilde{k})$, the equilibrium is of partial-H profile.*

Discussion

Indeed, Assumption 3 is a special case of $\Delta_L^{SB} > \Delta_H^{SB} = 0$. In this case, the buyers who are supposed to be excluded participates. One may ask, if there exists a set of model primitives that reduce participation given the seller can renege, which makes the extensive margin results less robust. Actually, it is easy to see that such scenario is impossible. Image that both types participate in benchmark (under Assumption 1. Suppose in equilibrium, Stage 2 distribution satisfies Assumption 3, then it has to be the case that the implementable stage 1 contract has $q_L = 0$. If so, SC_L implies that $k \geq w_L^{FB}$. However, such a large k can guarantees a reneging-proof mechanism for sure.

4 Sources of the Reneging Cost

In the previous analysis, we treat the reneging cost k as an exogenous parameter without discussing its sources. In this section, we provide two possible interpretations and discuss the corresponding policy implications.

4.1 Financial Punishment Imposed by Regulators

The reneging costs k can represent the financial punishment a seller needs to pay if the deceptive acts are caught by the regulator. Such measures are commonly adopted by government agencies in many countries to protect consumers. The regulators normally have fairly large freedom to design how harsh the punishments are. The main implication of our work is then, to stop the reneging behavior and achieve the (constraint) highest social welfare when the seller uses non-linear pricing, very large punishment is not necessary. Moreover, in a mess market or when buyers have diverse preferences, modest level of punishment is the best and the resulting social welfare is even higher than the welfare achieved with infinite amount of punishment.

To be more precise, when $\Delta_L^{SB} > \Delta_H^{SB}$, our earlier results show that the mechanism is reneging-proof as long as $k \geq \tilde{k}$. In addition, q_L at $\tilde{k}$ is larger than q_L^{SB} and is the largest for any possible k. Therefore, to maximize the social welfare as well as buyer's expected payoff, the regulator should optimally choose the modest punishment level $k = \tilde{k}$. With this amount of punishment, reneging is completely eliminated.

When Δ_H^{SB} is larger, the welfare is constant and reaches the highest level for any $k \geq \Delta_H^{SB}$. At the same time, the induced mechanism is reneging-proof. This implies that the modest punishment level $k = \Delta_H^{SB}$ is also enough to completely eliminate reneging and yield the (constraint) highest social welfare.

4.2 Word-of-Mouth and Buyer Unawareness

Now we endogenize the reneging cost k by embedding the main one-shot mechanism into a dynamic environment with "word-of-mouth" and "buyer unawareness". The dynamic model is a rather stylized setting. Here, we are not trying to capture all the realistic aspects of

Time is discrete and runs forever, indexed by t. There is a long-lived seller, who may encounter death shock with probability of $\delta \in (0,1)$ at the end of each period and $B \in \mathbb{N}$ one-period-lived buyers in each period. Each buyer has a two-dimensional type. The first dimension is the preference $\theta \in \{H, L\}$, which is randomly and independently drawn from the distribution with probability f_H, f_L, respectively. The second dimension is the awareness, which is determined by the following learning process.

Buyers' Learning Process: Word-of-Mouth

We assume that buyers are born to be unaware of the seller's reneging possibility. However, upon entry, they can observe the transactions in the last period. If the seller did not renege at all, all the buyers remain unaware. As long as the seller reneged to one old buyer, then all B new buyers know the reneging possibility immediately. We call unaware buyers naive buyers and aware buyers sophisticated buyers. This learning process captures the “word-of-mouth" phenomenon and the larger is B, the wider the news about reneging is spread.

In each period, buyers are either all naive or all sophisticated, which also correspond to two states of the economy, N and S.

Transaction Process

The trading process between the seller and each new buyer is of the same format as the two-stage game described in the main model. Briefly speaking, the seller can renege on the first stage contracts and propose a new one. The difference is that first, the new buyers could be naive whereas they are all sophisticated in the static game. Also, there is no explicit cost of reneging. Instead, the reneging cost is endogenized as an opportunity cost, because whether the seller will be facing a generation of naive or sophisticated buyer is determined by the seller's reneging behavior and the seller gets higher expected payoff when facing naive buyers. Finally, the seller is facing more than one buyer each period. We assume that the seller designs the contracts in each period before the types of that generation of buyers are realized.

We seek for conditions under which the contracts for sophisticated buyers can be reneging-proof.

Endogenous Reneging Cost

The reneging cost associated with buyer i is equal to the expected change in the continuation value of the seller

$$k = \text{Prob(the seller does not renege to other buyers)}(1-\delta)(V_N - V_S)$$

where V_N and V_S denote the seller's expected profit at state N and S, respectively. The probability in the expression is the probability that one single buyer is pivotal. It is included because we assume that once the seller reneged on one contract, whether he reneged on other contracts or not does not affect the continuation value.

Optimal Contracts When Facing Naive Buyers

When facing a naive buyer, if the seller provides a binding mechanism at the first stage, then he can perfectly learn the buyer's type. In principle, we can classify all possible binding mechanisms offered to naive buyers into the following four categories, according to his reneging decision conditional on buyer's report. We can show that only two of them can be optimal.

a. The seller reneges on the contract offered to H.

   Given that the seller will renege only on the contract offered to H, the optimal way is to extract the first best surplus from low type in the first stage and give high type a large information rent to maintain IC_H. Specifically, the mechanism has IC_H and IR_L binding and

$$q_L = q_L^{FB}, p_L = Lq_L^{FB}$$

   Then, the seller's expected payoff will be

$$V_N = Bw^{FB} + (f_L)^B(1-\delta)V_N + (1-(f_L)^B)(1-\delta)V_S.$$

b. The seller reneges on the contract offered to L.

   Given that the seller will renege only on the contract offered to L, the optimal way is to extract the first best surplus from high type and provide low type zero to maintain IC_H. Specifically, the mechanism has

$$q_H = q_H^{FB}, p_H = Hq_H^{FB}, q_L = 0, p_L = 0$$

   The mechanism is separating for sure and the seller's expected profit will be

$$V_N = Bw^{FB} + (f_H)^B(1-\delta)V_N + (1-(f_H)^B)(1-\delta)V_S.$$

c. The seller always reneges.

If the seller always renege, he will get

$$V_N = Bw^{FB} + (1-\delta)V_S,$$

which is always smaller than case a or b.

d. The seller never reneges.

Given that the seller has decided to never reneges, the seller should use the second best mechanism. Then $V_N = Bv^{SB}/\delta$. If this is indeed optimal, then the difference in V_N and V_S is large enough such that with sophisticated buyers, the optimal mechanism is also the second best mechanism and it is reneging-proof. This means that $V_S = Bv^{SB} + (1-\delta)V_N$. However, the resulting k is then 0. This is a contradiction.

The above discussion is summarized in the following lemma.

Lemma 6. *Facing naive buyers, the seller either reneges on the contract offered to H or renege on the contract offered to L. He will choose the former if and only if $f_H \leq f_L$. The value function of state N is*

$$V_N = Bw^{FB} + (1-\delta)\Big((1-f^B)V_S + f^B V_N\Big) \tag{8}$$

where $f \equiv \max\{f_H, f_L\} \geq 1/2$.

The lemma shows that the seller gets the first best surplus minus the cost associated with reneging when facing naive buyers. The seller chooses over (a) and (b) to minimize the chance of reneging. Notice that the optimal mechanism for naive buyers does not depend on what is happening in state S. It only depends on whether f_L or f_H is larger.

Optimal Contracts when Facing Sophisticated Buyers

Next, we study the seller's optimal strategy at state S. We are interested to see under what conditions a Committing Equilibrium defined below exists.

Definition 3. *A Committing Equilibrium holds when the mechanism offered to sophisticated buyers is reneging-proof.*

The reason why we focus on the Committing Equilibrium is that the social welfare is maximized in a Committing Equilibrium in any case.

In the dynamic model, a *Committing Equilibrium* exists when the endogenously determined reneging cost k is greater than $\tilde{k}$. If so, the first stage profit will be $Bv(k)$. Therefore, the seller's value function can be written as

$$V_S = Bv(k) + (1-\delta)V_N$$

Together with Equation (8), the reneging cost is equal to

$$k = \Xi(k) \equiv \frac{B(1-\delta)(w^{FB} - v(k))}{1+(1-\delta)(1-f^B)}$$

A *committing equilibrium* holds if there exists a $k > \tilde{k}$ such that $\Xi(k) = k$.

Proposition 7. *There exist a $\overline{B} > 1$, such that a Committing Equilibrium exists only if $B \geq \overline{B}$. When the Committing Equilibrium exists, it is unique.*

Proof. Because $v(k)$ weakly increases in k, the function $\Xi(\cdot)$ is weakly decreasing in k, This means that if the Committing Equilibrium exists, it must be unique. This also implies that a *Committing Equilibrium* exists only if

$$\Xi(\tilde{k}) \geq \tilde{k}.$$

In addition, we can show that $\Xi(k)$ is larger for any given k when B is larger. Taking derivative of $\Xi(k)$ with respect to B, the derivative is proportional to

$$2-\delta+(1-\delta)f^B(B\ln(f)-1)$$

The derivative of $f^B(B\ln(f)-1)$ is $Bf^B(\ln(f))^2$, which is strictly positive. This implies $f^B(B\ln(f)-1) \geq -1$, where the right-hand-side of the inequality is the value of the expression when $B = 0$. As a result, the derivative of $\Xi(k)$ with respect to B is strictly positive.

Then $\Xi(\tilde{k}) \geq \tilde{k}$ if and only if B is large enough. Finally, we prove that $\overline{B} > 1$. Suppose that there exists a *Committing Equilibrium* when $B = 1$, then $\Xi(\tilde{k})_{\{B=1\}} \geq \tilde{k}$. However, when $B = 1$,

$$\Xi(\tilde{k}) = \frac{(1-\delta)(w^{FB} - v(\tilde{k}))}{1 + (1-\delta)(1-f)} = \frac{(1-\delta)\tilde{k}}{1 + (1-\delta)(1-f)} < \tilde{k}$$

This is a contradiction. □

A direct corollary of the proposition is that there exist no Committing Equilibrium when there is only one buyer in each period.

Corollary 1. *There is no Committing Equilibrium when $B = 1$.*

In this extension, we use B to measure how wide the news on any reneging behavior is spread. The above proposition shows that to eliminate reneging, the news must be spread wide enough. Only then, the endogenous reneging cost is large enough.

5 Conclusion

This paper considers a dynamic non-linear pricing problem. By allowing reneging, the model provides the seller a channel to learn the buyer's preference and extract more surplus. On the other hand, a rational buyer realizes the scheme and will hide information given the seller's strategy. We fully characterize the equilibrium considering that the seller will use direct mechanism and provide precise conditions for different welfare implications.

Our main results concern both intensive margin and extensive margin. On the intensive margin, we predict that in mass market, a modest level of reneging cost can help reduce welfare distortion caused by information asymmetry. On the extensive margin, there is always a potential to increase the buyer's participation since the principal seeks to grasp all the surplus from the non-participating types.

In addition, our model depicts the seller's reneging behavior and the seller's reporting strategy. Although the model allows the seller to renege on all contracts at the initial stage, he will endogenously design an implementable contract. Such behavior provides a new angle to understand business tactics such as add-on pricing, deceptive advertisement and bait-and-switch. That is, the seller has the incentive to differentiate the market and learn buyers' types.

The results shed some lights on the regulation regarding consumer protection.

There are several possible directions for future work. In terms of model setup, this paper considers a binary type space as well as a two-stage game. One may consider extending it to a continuous type space or infinite time spectrum. Actually, our model is concise enough and able to embed to many other frameworks. For instance, based on our word-of-mouth model in Section 4, one may consider a firm dynamic model and investigate how a diffusion effect may affect firm growth. Also, besides seller's type, we can consider the possibility that reneging cost is the seller's private information.

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