ASPIRING PARENTS AND SCHOOL CONTEST[*†]

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ABSTRACT

Recently *The New York Times* ran a headline, "Harvard Rated Asian-American Applicants Lower on Personality Traits, Suit Says". The lawsuit against the esteemed university is a manifestation of wider disenchantment that elite universities in the United States engage in biased admissions hurting Asian-Americans. This paper builds a two-round contest model meant to reflect that high parental investments geared towards academic achievements can be detrimental to students in developing non-tangible attributes (positive personality). We find that excessive parental investments is an outcome of the competitive race. However, they do not necessarily crowd out children's efforts to compromise on their personality development. This result is predicated on the assumption of perfect substitutability between parental investments and children's effort in school performance. If the two types of investments are complements, it might even be plausible to think that parental investments would have a positive effect on personality development.

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Running Head: Aspiring parents

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1 Introduction

On June 15, 2018 *The New York Times* (NYT) published an article titled, "Harvard Rated Asian-American Applicants Lower on Personality Traits, Suit Says". Controversially, the representative body of Asian-American students - *Students for Fair Admissions* has filed a lawsuit accusing Harvard for its discrimination against Asian-American applicants in its admission procedures. The accusation was backed by the findings of an analysis involving more than 160,000 student records, which revealed that Asian-Americans were perceived to be less personable than individuals from other races. This discrimination was evident in the subjective lower ratings which were consistently conferred on Asian-Americans with regard to positive personality traits. The allegation of the biased admissions became more widespread following an internal review, which led university officers to concede that Asian-Americans were indeed somewhat disadvantaged in the admission process. Though the current proportion of Asian-Americans in Harvard stands at 19%, there would be a stark increase to 43% if academic achievement was the sole factor of consideration for admissions. Additionally, the proportion of Asian-Americans would drop to 31% and 26% if preferences for recruited athletes and legacy applicants as well as extracurricular and personal ratings were taken into account respectively. These changes in proportions contrast greatly with that of the Whites, whose share of the class would increase instead with the additional admission considerations. It was further noted that Harvard and other elite colleges have shared notes pertaining to the race of students who were eventually admitted, exacerbating the issue of unfair university admissions (The New York Times, 2018a).

The above excerpt encapsulates a growing resentment among Asian-Americans in the United States on how university admissions are biased and discriminatory. Despite Asian-Americans being more academically qualified than applicants of other races, they were perceived to be lacking outstanding qualities which would grant them admission as they were described to be "standard strong".

This disenchantment was given a voice in a parallel article in NYT, titled "College Admission Is Not a Personality Contest. Or Is It?" Admission officers in elite colleges have revealed that applicants were often evaluated on intangible aspects such as their personality traits, apart from their performance in standardized tests during the admission process (The New York Times, 2018b).

Even earlier in a 2011 *Wall Street Journal* article, "Why Chinese Mothers Are Superior," Yale Law School Professor Amy Chua extolled the virtue of being tiger mums. These parents tend to keep their children under close supervision, with a strong emphasis on their achievements even if attainment comes at the expense of their development of independence or personality (Chua, 2011).

As a counterpoint, Natalia Nedzhvetskaya wrote, "Why 'Chinese Mothers' Are Not Superior". It was argued that close parental supervision may raise high-achieving children in the short term. However, such authoritarian parenting methods and great emphasis on book-based learning might stunt their children's success in the long-term. This could be attributed to the neglect of the children's all-rounded development in other dimensions of life, where they might not be able to learn independently through trial-and-errors of their own initiatives or cultivate traits crucial for future success such as self-motivation (Nedzhvetskaya, 2011).

With the above ongoing debates in the United States, the following specific questions will be considered in the general context of a school contest:

(1) Do high parental investments in children's education help the children to achieve better long-term outcomes? This question should be seen acknowledging that exemplary performance in examinations does not necessarily develop *personality*.

(2) Under what situations can parents be expected to make high parental investments? Are such investments reactive in responding to children's low efforts or proactive in spurring the children to exert more efforts?

(3) Could it be that the ultra-competitive approach of Asian (or Asian-American) parents and the more hands-off/conservative approach of American parents (or parents from other ethnicities and nationalities) towards their children's education are the contributing factors which shape the respective group's social norms and bench-marking? An overly-competitive environment may prompt parents to adopt a more myopic approach towards their children's education. That is, parents become entirely focused on their children's short-term success measured by the attainment of exemplary academic grades. This could possibly be the underlying rationale behind the university admission authorities' so-called "biased" admission procedure where applicants are assessed not just on standard academic achievements, but also on other aspects such as positive personality.

In this paper, a model of school contest among parent-child pairs is formulated to address the above issues.

It is to be expected that in the face of strong societal pressure (i.e. norms, guilt or the sheer intensity of competition), parents are likely to become heavily involved at least in the early stages of their children's education. This is especially so during the pre-college school phase with parents providing various forms of support to ensure that their child excels academically to be able to gain entry into prestigious colleges. Top universities are often seen as platforms where one could be exposed to better networking opportunities, besides acquiring new social and knowledge-based skills which are beneficial in boosting one's future job prospects. The name brand and peer effects of top international universities are additional reasons, besides the quality of education, that cause parents from all over the world to vie for these selective limited slots for their children.

Excessive parental involvement, however, might be counter-productive as children's over-reliance on parental support could potentially crowd out the children's own efforts in independent learning and personality development. The personality necessary for one to exercise good judgement and being capable of thinking critically in the face of unfamiliar and challenging problems is often developed through years of schooling. Too much coaching in the form of private tutoring or parental guidance could in fact hinder the development of a child's own personality. Arguably, it is good judgement, strong personality and courage (The New York Times, 2018a) that could drive individuals towards eventual success in the later phase of life in colleges and the workplace – be it as academic researchers, scientists, innovators or entrepreneurs.

By parental support, we will consider the monetary support which parents could provide for their child's education in this paper. This encompasses support in terms of the provision of additional private tuition, the purchase of supplementary educational materials, or simply maintaining a conducive environment for the child's learning to boost their school exam grades.[1]

We build a contest model spanning two rounds. In the first round, students (or children) compete in schools to come out within a pre-specified top $\varkappa$-percentile. After crossing this threshold, students progress to colleges and the larger world where they participate in a more mature, second round

[1] The role of emotional support which parents could potentially provide for their children such as ensuring that their child's emotional health is optimal or creating a stable family and home environment, is an alternative form of investment that could have a positive long-term impact on the child's academic performance. Parents could also devote their time and other resources as a means of support for their child's education. We abstract away from these types of investments in this paper.

race. Those who succeed in the mature phase are the ultimate beneficiaries.

The first round race comprises of investments and efforts by parents and their children over two stages. We consider two orders of play in this paper. In the first protocol, parents simultaneously invest first (Stage 1) in their respective child's education and these parental investments will be observed by all children, followed by their simultaneous choices of effort in Stage 2. The combined profile of investments and efforts determine the successes (i.e., reaching top x-percentile) and failures (remaining below the top x-percentile) of the Round 1 race. In the second round race, the top x-percentile will compete for a single winner's slot. The seeds of success in the second round are already sown by the students' efforts in the first round: a winner is drawn randomly, where a relatively higher effort translates into a higher probability of success.

In a second protocol, students simultaneously exert efforts in Stage 1 of Round 1. These efforts are publicly observed, after which the parents can choose to supplement their child's effort with their own investments in Stage 2 of Round 1. The second round race runs in the same manner as that in the first protocol.

Both protocols are analysed with the assumption that the technology determining the child's success at school is one of perfect substitution. The case of complementary technology will be discussed in the conclusion.

The two orders of play induce very different dynamics of interactions between parent-child pairs. In the first protocol, benevolent parents who care about their children's success will all make zero investments irrespective of whether they value their child's ultimate success or place a fraction or even full weight to the child's interim success in the school contest (Propositions 1 and 2). The economic reasoning is that raising investment from zero to a positive amount by any parent given other parents' zero investments, would be *more-than-completely crowded out* by their own child lowering his effort in response, while all of the other children increase their efforts (Proposition 3). As a result, the parent choosing positive investment will see their child's chances of success both at the interim (end of Round 1) and the final race (end of Round 2) decreasing. This equilibrium is symbolic of the hands-off approach by parents resulting from the fact that they get to move first. With the first-mover advantage, parents can "commit" to not bailing their children out hence, prompting their children to exert more effort. That is, parents avoid the dilemma posed by the *rotten kids* (see Becker, 1974; Bruce and Waldman, 1990; Bergstorm 2000).

In the second protocol, one should expect the children to put in low efforts knowing well that their parents would bail them out eventually by sinking *all* their wealth. This would be in line with the idea of the *rotten kid theorem*. A parallel idea on how the first mover can take advantage of late movers by strategically committing to low investments in sequential voluntary contribution public good games appears in Varian (2005). However surprisingly in our formulation, while parents do indeed make full investments, these investments have no effect on the children's efforts. That is, the children tend to exert the same amount of effort as in the first protocol: the rotten kid problem gets completely neutralized. The reason is that, full symmetric parental investments *cancel out in the school contest*, leaving the children to sustain their chances of success as if, collectively, the parents had made no investments. The end result is that while both the parents and the children make investments in the school contest, the success odds across all parent-child pairs remain *uniform* i.e. *running to keep in the same place*[2] (Propositions 4–6).

The rest of the paper is organized as follows. The contest success functions as well as the children's and parents' payoffs are specified in Section 2. In Sections 3 and 4, we analyse the contest according to the first protocol. A comparative statics analysis linking parental investments to the composite inputs by the parent-child pairs under the first protocol is carried out in Section 5. The alternative second protocol is analysed in Section 6. Subsequently, a comparison between the two protocols is contained in Section 7. Section 8 summarises the results of the analysis and discusses possible extensions of the current analysis to a much richer setting.

2 Theoretical Framework

We consider the following model of academic race involving n parent-child pairs and two rounds of contest. We will assume that each parent has only one child. The first round will be regarded as the pre-college phase, where the winners at the end of this round will be successfully admitted into prestigious colleges. At the end of the second round – the college/post-college phase, the eventual winner of the contest will be chosen with the key factor of success being dependent on the child's effort in the first round.

[2] A metaphor for competition, adapted from Lewis Carroll (1871), *Through the Looking-Glass*.

There are two rounds in this contest. In Round 1, $n = 3$ parent-child pairs compete over two stages in a school contest for $m = 2$ slots. In Round 2, two winners compete in a more mature race where an eventual winner is selected according to the skills acquired by the students during the school contest. We denote the children by $\{j, k, l\}$ and their respective parents by $\{j', k', l'\}$. We make the following assumption about parental wealth throughout the paper:

ASSUMPTION 1. *All parents have identical wealth* $w_s = w > 0$*, where* $s \in \{j, k, l\}$.[3]

(i) Parents have no other use for their wealth, besides investing in their children's education.

(ii) Their wealth cannot be transferred onto their children.

(iii) Parents cannot borrow to spend on their children's education beyond the wealth they possess.

More specifically, the contest will be further elaborated in the subsequent sections.

2.1 Round 1

In the pre-college school phase, parents of child $s \in \{j, k, l\}$ with wealth $w_s > 0$ can invest $0 \leqslant I_s \leqslant I$ to support their child's education. Each child also chooses to exert effort, $0 \leqslant i_s \leqslant 1$, to perform well in their academic studies. Child's effort and parental investment, (i_s, I_s), determine a composite input

$$x_s = \alpha(i_s + I_s) \tag{1}$$

where $\alpha = \frac{1}{2(1+I)} > 0$. The profile of composite inputs $\{x_j, x_k, x_l\}$ determines the chance of success of child s in becoming one of the two winners at the end of Round 1 in the school contest. In defining the composite input x_s, we have set the specific value of α to ensure that a child's success

[3] This is a simplistic assumption made in this contest model. We abstract away from possible wealth inequality between parents. Additionally, we adopt the concept of mental accounting such that the analysis focuses on the parents setting aside some funds with the pure intention of supporting their children's education, instead of using the money for their own benefits. This further justifies why we are abstracting away from the potential cost that parents might incur from their investment. Bequest motives from parents to the children are also excluded from the analysis as these might disincentivise children from exerting effort in their studies. We further assume that parents cannot make any form of borrowing in addition to the wealth they hold, to abstract the model's analysis away from possible complications that might dilute the analysis.

probability in Round 1 of the contest remains within the natural bound, $[0, 1]$.[4] The contest success function will be specified below.

The variable i_s reflects the intensity of effort by child s. The intensity, $i_s = 1$, is indicative of the maximal number of hours per week, H, in studies that any child can devote to. Hence, we define the unit of effort such that $H = 1$. The variable I_s reflects the number of hours of private tutoring which the parents of child s could afford to supplement their child's effort. Wealth w_s allows a maximum number of I units of private tutoring to be bought by the parents at the prevailing wage rate. One unit of private tutoring is equivalent to full intensity (i.e. $i_s = 1$) of child's effort, leading to the parent-child competition intensity index x_s, as defined above.

Investment Choices: Within the first round, there are two stages. As mentioned previously, in the first protocol, parents decide on their investments simultaneously in the first stage while the children observe these parental investments and make their investments simultaneously in the second stage. The sequence of moves in Round 1 is reversed for the second protocol.

The children's cost of investment is increasing and convex in effort, as follows:

$$c(i_s) = \frac{1}{2} d_s i_s^2 \quad , \text{ where } d_s > 0$$

Payoffs from winning Round 1: Doing well academically not only boosts one's chances of getting into elite colleges, but one might also receive recognition and praise that further boost one's sense of achievement and self-esteem. These factors motivate the children to emerge as one of the top two winners from Round 1 where each winner will yield a positive payoff $\check{V}_s \in \{\check{V}_j, \check{V}_k, \check{V}_l\}$. Those who succeed could also confer high social prestige on their parents whom might thus gain satisfaction from witnessing their child's interim success at the end of Round 1 denoted by $\hat{V}_s \in \{\hat{V}_j, \hat{V}_k, \hat{V}_l\}$.

Round 1 can end in one of three possibilities, with the index pairs indicating the top two students irrespective of their rankings:

$$1)\ \{j, k\} \qquad 2)\ \{j, l\} \qquad 3)\ \{k, l\}$$

[4]Basically, a child can study up to a maximum number of hours per week, say $H = 40$ hours, which will be normalized to $\max i_s = 1$. Suppose parental wealth allows for $w_s = \mathsf{SGD}\ 150.00$ per week to be spent on private tuition. If private tuition costs SGD 30.00/hour, the maximum number of hours of private tuition that parents could afford would equal 5 hours/week. Suppose 1 hour of private tuition equals 4 hours of the child's studying time. The 5 hours of private tuition would hence be equivalent to 20 hours of the child's studying time. Therefore, the maximum parental investment can be set at $1/2$ unit. This way, given that $x_s = \alpha(i_s + I_s)$, we will have $\max x_s \equiv \alpha \cdot \max(i_s + I_s) = \alpha(1 + 1/2) = (3/2)\alpha$. For the success probability of child s to remain in the unit interval $[0, 1]$, we will require $0 \leqslant 2x_s \leqslant 1$ or $0 \leqslant \alpha \leqslant 1/3$.

Contest success function in Round 1. The probabilities for the aforementioned contingencies can be stated explicitly as follows:

$$
\begin{aligned}
&1) \quad \Pr(\{j,k\}) = \frac{x_j + x_k + 1 - 2x_l}{3} \\
&2) \quad \Pr(\{j,l\}) = \frac{x_j + x_l + 1 - 2x_k}{3} \\
&3) \quad \Pr(\{k,l\}) = \frac{x_k + x_l + 1 - 2x_j}{3}
\end{aligned}
$$

Given our definition of the composite input in (1) (see also footnote 4), it is easy to verify that the above probabilities will lie between zero and one.

For the three mutually-exclusive and exhaustive events of specific pairs emerging as the winners of Round 1, the following holds:

$$
\begin{aligned}
\Pr(\{j,k\} + \Pr(\{j,l\}) + \Pr(\{k,l\}) &= \frac{1}{3}\left(x_j + x_k + 1 - 2x_l + x_j + x_l + 1 - 2x_k + x_k + x_l + 1 - 2x_j\right) \\
&= \frac{1}{3}(3) \\
&= 1
\end{aligned}
$$

As the composite input of a parent-child pair x_s increases, holding constant that of other parent-child pairs, it becomes more probable that the child will emerge as one of the top two winners at the end of Round 1.

2.2 Round 2

Once a child succeeds in the first round and proceeds to the second round, the contest among $m = 2$ students for only a solitary position is purely driven by the accumulated experiences through the efforts exerted by the child in the first round.

The race now is a mechanical draw of an ultimate winner for the multi-pronged contest. We denote the two children who have progressed thus far by s_1 and s_2, irrespective of their rankings, such that $s_1, s_2 \in$ children $\{j,k,l\}$. The respective probabilities of either child s_1 or s_2 emerging as the eventual winner of the contest can be summarized by the following contest success function:

$$
\Pr(s_1 \text{ wins}) = \frac{i_{s_1} + 1 - i_{s_2}}{2} \quad , \quad s_1 \neq s_2
$$

$$
\Pr(s_2 \text{ wins}) = \frac{i_{s_2} + 1 - i_{s_1}}{2} \quad , \quad s_1 \neq s_2
$$

Since $0 \leqslant i_{s_1}, i_{s_2} \leqslant 1$, it follows that $0 \leqslant \Pr(s_1 \text{ wins}) \leqslant 1$ and $0 \leqslant \Pr(s_2 \text{ wins}) \leqslant 1$.

Evidently, these two probabilities sum up to 1 as well:

$$\begin{aligned}\Pr(s_1 \text{ wins}) + \Pr(s_2 \text{ wins}) &= \frac{i_{s_1} + 1 - i_{s_2}}{2} + \frac{i_{s_2} + 1 - i_{s_1}}{2} \\ &= \frac{1}{2}\left(i_{s_1} + 1 - i_{s_2} + i_{s_2} + 1 - i_{s_1}\right) \\ &= 1\end{aligned}$$

Investment Choices: As the child puts in more effort of his own in Round 1 – pre-college phase instead of solely relying on parental support, one learns to be independent and benefits from 'learning-by-doing'. Being capable of dealing with the complexities of one's academic work without the help of private tutors will equip individuals with essential skills such as critical thinking and problem-solving techniques. These are key contributing factors that aid an individual in succeeding in Round 2 where parental support tends to be scarce, be it attaining stellar results in the college phase or securing a job in the workforce upon graduation.

Payoffs: We assume that costs are sunk in Round 2 of the contest. The child who emerges as the eventual winner receives a fixed positive payoff of V in the form of monetary benefits or social recognition of one's achievements, while the loser in the second round receives zero payoff. Additionally, the parents of the ultimate winner will receive a positive payoff of $\widetilde{V}_s \in \{\widetilde{V}_j, \widetilde{V}_k, \widetilde{V}_l\}$ as they feel gratified witnessing their child's eventual success.

2.3 Protocols 1 and 2

The above contest game can take one of two forms, depending on which entity moves first and which entity moves second in Round 1. In the first game form (**Protocol 1**) to be analysed, parents would be making the first move. In the second game form (**Protocol 2**), the children would be moving first. These two game forms are depicted in Figs. 3.1 and 6.1 respectively.

The specific questions that we seek to answer are: (1) What are the optimal parental investments, (2) how intense should each child be studying, and (3) how does the order of moves influence parents' investment choices and the children's intensity of studying?

The higher the parental investment, the greater the chances of the child's success in the first round. However, greater parental investment might crowd out the child's own intensity of studying and result in lesser "learning", which would ultimately lower one's chances of succeeding in the second round. In the analysis to follow, we will study how this tradeoff plays out by solving two sequential move games using backward induction.

3 Stage 2 of Round 1: Children's Optimisation Problem (Protocol 1)

The game in Protocol 1 proceeds as illustrated by Fig. 3.1:

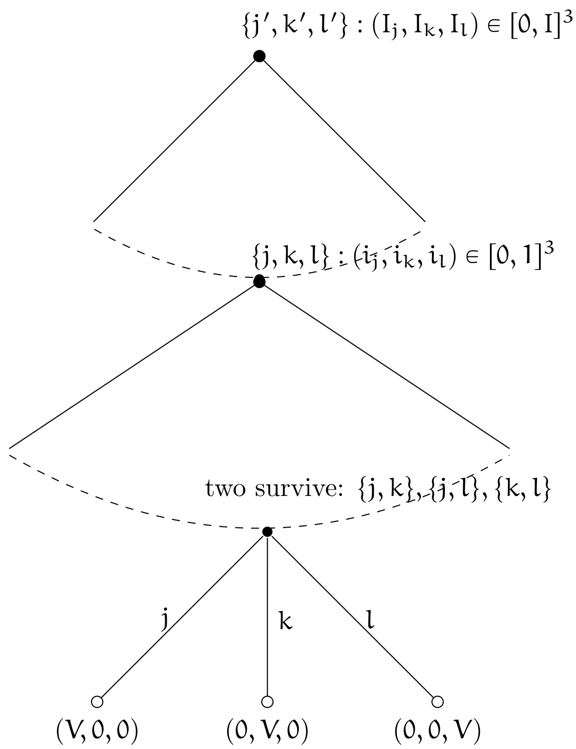

Figure 3.1: PROTOCOL 1

Using backward induction, we begin with the second stage of Round 1 by solving for the children's best response functions and determining the optimal effort levels they should be exerting. This is done while taking parental investments I_j, I_k, I_l as given since all parents would decide on their investments in the first stage of Round 1, before each child decides on their own effort levels i_j, i_k, i_l. Eventually, we solve for the Nash equilibrium of the children's investments in this continuation game.

3.1 Best Response Functions

We assume that each child has perfect information about the amount of parental support one is getting and that of their competitors. For generalisation purposes, we denote $s_1, s_2, s_3 \in$ children $\{j, k, l\}$ i.e. each of s_1, s_2, s_3 will be either one of the three children respectively and $s_1 \neq s_2 \neq s_3$.

From the point of view of child s_1, one's utility function U_{s_1} in consideration of the entire contest can be explicitly written as follows:

$$U_{s_1} = \Pr(\{s_1, s_2\}) \times \left(\check{V}_{s_1} + \frac{i_{s_1} + 1 - i_{s_2}}{2} \times V\right) + \Pr(\{s_1, s_3\}) \times \left(\check{V}_{s_1} + \frac{i_{s_1} + 1 - i_{s_3}}{2} \times V\right) - \frac{1}{2} d_{s_1} i_{s_1}^2$$

For illustration purposes, we adopt the perspective of parent-child pair l throughout this paper.

Letting $U_l =$ expected utility of child l in the entire contest,

$$U_l = \Pr(\{l, j\}) \times \left(\check{V}_l + \frac{i_l + 1 - i_j}{2} \times V\right) + \Pr(\{l, k\}) \times \left(\check{V}_l + \frac{i_l + 1 - i_k}{2} \times V\right) - \frac{1}{2} d_l i_l^2$$

We then proceed to express this function explicitly in terms of the composite inputs of parent-child pairs x_j, x_k, x_l, parental investment I_l and children's investments i_j, i_k, i_l as follows:

$$\begin{aligned}
U_l &= \frac{x_l + x_j + 1 - 2x_k}{3} \times \left(\check{V}_l + \frac{i_l + 1 - i_j}{2} \times V\right) + \frac{x_l + x_k + 1 - 2x_j}{3} \times \left(\check{V}_l + \frac{i_l + 1 - i_k}{2} \times V\right) - \frac{1}{2} d_l i_l^2 \\
&= \frac{\alpha(i_l + I_l) + x_j + 1 - 2x_k}{3} \times \check{V}_l + \frac{\alpha(i_l + I_l) + x_j + 1 - 2x_k}{3} \left(\frac{i_l + 1 - i_j}{2}\right) \times V \\
&\quad + \frac{\alpha(i_l + I_l) + x_k + 1 - 2x_j}{3} \times \check{V}_l + \frac{\alpha(i_l + I_l) + x_k + 1 - 2x_j}{3} \left(\frac{i_l + 1 - i_k}{2}\right) \times V - \frac{1}{2} d_l i_l^2
\end{aligned}$$

To maximize child l's utility, we obtain the first-order condition:

$$\begin{aligned}
\frac{\partial U_l}{\partial i_l} &= \frac{1}{3}\alpha\check{V}_l + \frac{\alpha(i_l + I_l) + x_j + 1 - 2x_k}{3}(V)\left(\frac{1}{2}\right) + \frac{i_l + 1 - i_j}{2}(V)\left(\frac{1}{3}\right)(\alpha) \\
&\quad + \frac{1}{3}\alpha\check{V}_l + \frac{\alpha(i_l + I_l) + x_k + 1 - 2x_j}{3}(V)\left(\frac{1}{2}\right) + \frac{i_l + 1 - i_k}{2}(V)\left(\frac{1}{3}\right)(\alpha) - \frac{1}{2} d_l (2 i_l) \\
&= \frac{2}{3}\alpha\check{V}_l + \frac{V}{6}\Big[\alpha(i_l + I_l) + x_j + 1 - 2x_k + \alpha(i_l + 1 - i_j) + \alpha(i_l + I_l) + x_k + 1 - 2x_j + \alpha(i_l + 1 - i_k)\Big] - d_l i_l \\
&= \frac{2}{3}\alpha\check{V}_l + \frac{V}{6}\Big[-x_j + 2 - x_k + \alpha(i_l + I_l + i_l + 1 - i_j + i_l + I_l + i_l + 1 - i_k)\Big] - d_l i_l \\
&= \frac{2}{3}\alpha\check{V}_l + \frac{V}{6}\Big[-x_j + 2 - x_k + \alpha(4i_l + 2I_l + 2 - i_j - i_k)\Big] - d_l i_l \\
&= 0
\end{aligned}$$

The second-order condition requires:

$$\frac{\partial^2 U_l}{\partial {i_l}^2} = \frac{2}{3}\alpha V - d_l < 0$$

$$\text{i.e.} \qquad \frac{2}{3}\alpha V < d_l \tag{2}$$

With reference to the first-order condition obtained, we proceed to derive the best response function of child l, which can be written as a function of i_j and i_k, with $\{I_j, I_k, I_l\}$ as exogenous terms:

$$i_l\left[d_l - \frac{2}{3}\alpha V\right] = \frac{2}{3}\alpha \check{V}_l + \frac{V}{6}\Big[-x_j + 2 - x_k + \alpha\,(2I_l + 2 - i_j - i_k)\Big]$$

$$\begin{aligned} \text{i.e.} \qquad i_l &= \frac{1}{d_l - \frac{2}{3}\alpha V}\left\{\frac{2}{3}\alpha \check{V}_l + \frac{V}{6}\Big[-\alpha(i_j + I_j) + 2 - \alpha(i_k + I_k) + \alpha(2I_l + 2 - i_j - i_k)\Big]\right\} \\ &= \frac{1}{d_l - \frac{2}{3}\alpha V}\left\{\frac{2}{3}\alpha \check{V}_l + \frac{V}{3} + \alpha\left(\frac{V}{6}\right)\Big[2I_l - (I_j + I_k) + 2 - 2(i_j + i_k)\Big]\right\} \end{aligned} \tag{3}$$

Given that the children have symmetric utility functions, their best response functions will also be symmetric. We can thus obtain child j's best response function and second-order condition:

$$i_j = \frac{1}{d_j - \frac{2}{3}\alpha V}\left\{\frac{2}{3}\alpha \check{V}_j + \frac{V}{3} + \alpha\left(\frac{V}{6}\right)\Big[2I_j - (I_l + I_k) + 2 - 2(i_l + i_k)\Big]\right\} \tag{4}$$

and $\frac{2}{3}\alpha V < d_j$ must be satisfied.

Similarly for child k, one's best response function and second-order condition are as such:

$$i_k = \frac{1}{d_k - \frac{2}{3}\alpha V}\left\{\frac{2}{3}\alpha \check{V}_k + \frac{V}{3} + \alpha\left(\frac{V}{6}\right)\Big[2I_k - (I_l + I_j) + 2 - 2(i_l + i_j)\Big]\right\} \tag{5}$$

and $\frac{2}{3}\alpha V < d_k$ must be satisfied.

3.2 Nash Equilibrium in Stage 2 continuation game

We then solve for the Nash Equilibrium in the continuation game, i.e. Stage 2 of Round 1 of the contest, for any given parental investment profile $\{I_j, I_k, I_l\}$. For simplicity, we adopt the following symmetric assumptions throughout the paper:

Assumption 2. $d_j = d_k = d_l = d$.

This implies that all children have the same specific cost of investment.

Assumption 3. $\check{V}_j = \check{V}_k = \check{V}_l = \check{V}$.

This implies that the children would receive a common positive payoff $\check{V}$, in the event that they emerge as one of the top two winners at the end of Round 1.

With reference to child l's best response function in (3), we express it as follows:

$$i_l + \frac{1}{d - \frac{2}{3}\alpha V}\left(\frac{V}{6}\right)(\alpha)(2)(i_j + i_k) = \frac{1}{d - \frac{2}{3}\alpha V}\left\{\frac{2}{3}\alpha\check{V} + \frac{V}{3} + \alpha\left(\frac{V}{6}\right)\left[2I_l - (I_j + I_k) + 2\right]\right\}$$

$$\text{i.e.}\quad i_l + \frac{\alpha V}{3\left(d - \frac{2}{3}\alpha V\right)}(i_j + i_k) = \frac{1}{d - \frac{2}{3}\alpha V}\left\{\frac{2}{3}\alpha\check{V} + \frac{V}{3} + \alpha\left(\frac{V}{6}\right)\left[2I_l - (I_j + I_k) + 2\right]\right\}$$

Given that $d > \frac{2}{3}\alpha V$ from (2), we define

$$\tau_1 = \frac{\alpha V}{3(d - \frac{2}{3}\alpha V)} \quad \text{where } \tau_1 > 0$$

$$\tau_2 = \frac{1}{d - \frac{2}{3}\alpha V} \quad \text{where } \tau_2 > 0$$

$$\text{such that}\quad \tau_1 = \frac{\alpha V}{3}(\tau_2).$$

We re-express child l's best response function in terms of τ_1 and τ_2 while adopting the same approach for children j and k with reference to (4) and (5). The following equations denote each child's best response function respectively:

$$\text{Child } l\text{'s: } i_l + \tau_1 i_j + \tau_1 i_k = \tau_2\left[\frac{2}{3}\alpha\check{V} + \frac{V}{3} + \alpha\left(\frac{V}{6}\right)(2I_l - I_{-l} + 2)\right] \quad \text{where } I_{-l} = I_j + I_k$$

$$\text{Child } j\text{'s: } \tau_1 i_l + i_j + \tau_1 i_k = \tau_2\left[\frac{2}{3}\alpha\check{V} + \frac{V}{3} + \alpha\left(\frac{V}{6}\right)(2I_j - I_{-j} + 2)\right] \quad \text{where } I_{-j} = I_l + I_k$$

$$\text{Child } k\text{'s: } \tau_1 i_l + \tau_1 i_j + i_k = \tau_2\left[\frac{2}{3}\alpha\check{V} + \frac{V}{3} + \alpha\left(\frac{V}{6}\right)(2I_k - I_{-k} + 2)\right] \quad \text{where } I_{-k} = I_l + I_j$$

We also denote the following:

$$\tau_2\left[\frac{2}{3}\alpha\check{V} + \frac{V}{3} + \alpha\left(\frac{V}{6}\right)(2I_l - I_{-l} + 2)\right] = \theta_l$$

$$\tau_2\left[\frac{2}{3}\alpha\check{V} + \frac{V}{3} + \alpha\left(\frac{V}{6}\right)(2I_j - I_{-j} + 2)\right] = \theta_j$$

$$\tau_2\left[\frac{2}{3}\alpha\check{V} + \frac{V}{3} + \alpha\left(\frac{V}{6}\right)(2I_k - I_{-k} + 2)\right] = \theta_k$$

Expressing the children's best-response functions in matrix notation,

$$\begin{pmatrix} 1 & \tau_1 & \tau_1 \\ \tau_1 & 1 & \tau_1 \\ \tau_1 & \tau_1 & 1 \end{pmatrix} \begin{pmatrix} i_l \\ i_j \\ i_k \end{pmatrix} = \begin{pmatrix} \theta_l \\ \theta_j \\ \theta_k \end{pmatrix}$$

$$\begin{pmatrix} i_l{}^* \\ i_j{}^* \\ i_k{}^* \end{pmatrix} = \begin{pmatrix} 1 & \tau_1 & \tau_1 \\ \tau_1 & 1 & \tau_1 \\ \tau_1 & \tau_1 & 1 \end{pmatrix}^{-1} \begin{pmatrix} \theta_l \\ \theta_j \\ \theta_k \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{\tau_1+1}{(\tau_1-1)(2\tau_1+1)} & \frac{\tau_1}{(\tau_1-1)(2\tau_1+1)} & \frac{\tau_1}{(\tau_1-1)(2\tau_1+1)} \\ \frac{\tau_1}{(\tau_1-1)(2\tau_1+1)} & -\frac{\tau_1+1}{(\tau_1-1)(2\tau_1+1)} & \frac{\tau_1}{(\tau_1-1)(2\tau_1+1)} \\ \frac{\tau_1}{(\tau_1-1)(2\tau_1+1)} & \frac{\tau_1}{(\tau_1-1)(2\tau_1+1)} & -\frac{\tau_1+1}{(\tau_1-1)(2\tau_1+1)} \end{pmatrix} \begin{pmatrix} \theta_l \\ \theta_j \\ \theta_k \end{pmatrix}$$

Letting $a = \frac{1}{(\tau_1-1)(2\tau_1+1)}$,

$$\begin{pmatrix} i_l{}^* \\ i_j{}^* \\ i_k{}^* \end{pmatrix} = \begin{pmatrix} -a(\tau_1+1) & a\tau_1 & a\tau_1 \\ a\tau_1 & -a(\tau_1+1) & a\tau_1 \\ a\tau_1 & a\tau_1 & -a(\tau_1+1) \end{pmatrix} \begin{pmatrix} \tau_2\left[\frac{2}{3}\alpha\check{V} + \frac{V}{3} + \alpha(\frac{V}{6})(2I_l - I_{-l} + 2)\right] \\ \tau_2\left[\frac{2}{3}\alpha\check{V} + \frac{V}{3} + \alpha(\frac{V}{6})(2I_j - I_{-j} + 2)\right] \\ \tau_2\left[\frac{2}{3}\alpha\check{V} + \frac{V}{3} + \alpha(\frac{V}{6})(2I_k - I_{-k} + 2)\right] \end{pmatrix}$$

Thus, the Nash Equilibrium of the continuation game for any parental investment profile is:

$$\begin{pmatrix} i_l{}^* \\ i_j{}^* \\ i_k{}^* \end{pmatrix} = \begin{pmatrix} \left(\frac{2}{3}\alpha\check{V} + \frac{V}{3}\right)(a\tau_2)(\tau_1 - 1) + a\tau_1(\tau_1 - 1) + a\tau_1\left(\tau_1 + \frac{1}{2}\right)(I_j + I_k - 2I_l) \\ \left(\frac{2}{3}\alpha\check{V} + \frac{V}{3}\right)(a\tau_2)(\tau_1 - 1) + a\tau_1(\tau_1 - 1) + a\tau_1\left(\tau_1 + \frac{1}{2}\right)(I_l + I_k - 2I_j) \\ \left(\frac{2}{3}\alpha\check{V} + \frac{V}{3}\right)(a\tau_2)(\tau_1 - 1) + a\tau_1(\tau_1 - 1) + a\tau_1\left(\tau_1 + \frac{1}{2}\right)(I_l + I_j - 2I_k) \end{pmatrix} \tag{6}$$

4 Stage 1 of Round 1: Parents' Optimisation Problem (Protocol 1)

Having derived the children's Nash equilibrium in the continuation game, we proceed to solve the first stage of the first round of the contest by considering the parents' optimization problem. Here, each parent chooses their optimal investment level, taking the investments of other parents as given and assuming that the children will play their Nash Equilibrium choices in the continuation game.

4.1 Utility Functions

Letting U'_l = expected utility of parent l' in the contest,

$$\begin{aligned}
U'_l &= \Pr(\{l,j\}) \times \left(\hat{V}_l + \frac{{i_l}^* + 1 - {i_j}^*}{2} \times \tilde{V}_l\right) + \Pr(\{l,k\}) \times \left(\hat{V}_l + \frac{{i_l}^* + 1 - {i_k}^*}{2} \times \tilde{V}_l\right) \\
&= \frac{x_l + x_j + 1 - 2x_k}{3} \times \left(\hat{V}_l + \frac{{i_l}^* + 1 - {i_j}^*}{2} \times \tilde{V}_l\right) + \frac{x_l + x_k + 1 - 2x_j}{3} \times \left(\hat{V}_l + \frac{{i_l}^* + 1 - {i_k}^*}{2} \times \tilde{V}_l\right) \\
&= \frac{\alpha({i_l}^* + I_l) + \alpha({i_j}^* + I_j) + 1 - 2\alpha({i_k}^* + I_k)}{3} \times \left(\hat{V}_l + \frac{{i_l}^* + 1 - {i_j}^*}{2} \times \tilde{V}_l\right) \\
&\quad + \frac{\alpha({i_l}^* + I_l) + \alpha({i_k}^* + I_k) + 1 - 2\alpha({i_j}^* + I_j)}{3} \times \left(\hat{V}_l + \frac{{i_l}^* + 1 - {i_k}^*}{2} \times \tilde{V}_l\right)
\end{aligned}$$

Substituting the Nash Equilibrium investment choices of the children ${i_j}^*, {i_k}^*, {i_l}^*$ from (6),

$$\begin{aligned}
U'_l &= \frac{1}{3}\Big[\alpha({i_l}^* + {i_j}^* - 2{i_k}^*) + \alpha(I_l + I_j - 2I_k) + 1\Big]\left\{\hat{V}_l + \frac{\tilde{V}_l}{2}\left[-3a\tau_1\left(\tau_1 + \frac{1}{2}\right)(I_l - I_j) + 1\right]\right\} \\
&\quad + \frac{1}{3}\Big[\alpha({i_l}^* + {i_k}^* - 2{i_j}^*) + \alpha(I_l + I_k - 2I_j) + 1\Big]\left\{\hat{V}_l + \frac{\tilde{V}_l}{2}\left[-3a\tau_1\left(\tau_1 + \frac{1}{2}\right)(I_l - I_k) + 1\right]\right\} \\
&= \left[-\alpha a\tau_1\left(\tau_1 + \frac{1}{2}\right)(I_l + I_j - 2I_k) + \frac{1}{3}\alpha(I_l + I_j - 2I_k) + \frac{1}{3}\right]\left\{\hat{V}_l + \frac{\tilde{V}_l}{2}\left[-3a\tau_1\left(\tau_1 + \frac{1}{2}\right)(I_l - I_j) + 1\right]\right\} \\
&\quad + \left[-\alpha a\tau_1\left(\tau_1 + \frac{1}{2}\right)(I_l + I_k - 2I_j) + \frac{1}{3}\alpha(I_l + I_k - 2I_j) + \frac{1}{3}\right]\left\{\hat{V}_l + \frac{\tilde{V}_l}{2}\left[-3a\tau_1\left(\tau_1 + \frac{1}{2}\right)(I_l - I_k) + 1\right]\right\} \\
&= \left\{\alpha\Big[I_l + I_j - 2I_k\Big]\left[\frac{1}{3} - a\tau_1\left(\tau_1 + \frac{1}{2}\right)\right] + \frac{1}{3}\right\}\left\{\hat{V}_l + \frac{\tilde{V}_l}{2}\left[-3a\tau_1\left(\tau_1 + \frac{1}{2}\right)(I_l - I_j) + 1\right]\right\} \\
&\quad + \left\{\alpha\Big[I_l + I_k - 2I_j\Big]\left[\frac{1}{3} - a\tau_1\left(\tau_1 + \frac{1}{2}\right)\right] + \frac{1}{3}\right\}\left\{\hat{V}_l + \frac{\tilde{V}_l}{2}\left[-3a\tau_1\left(\tau_1 + \frac{1}{2}\right)(I_l - I_k) + 1\right]\right\}
\end{aligned}$$

We then derive the marginal utility of parent l's investment as follows:

$$\begin{aligned}
\frac{\partial U'_l}{\partial I_l} &= \left\{\alpha\Big[I_l + I_j - 2I_k\Big]\left[\frac{1}{3} - a\tau_1\left(\tau_1 + \frac{1}{2}\right)\right] + \frac{1}{3}\right\}\left\{\left(\frac{\tilde{V}_l}{2}\right)\left[-3a\tau_1\left(\tau_1 + \frac{1}{2}\right)\right]\right\} \\
&\quad + \left\{\hat{V}_l + \frac{\tilde{V}_l}{2}\left[-3a\tau_1\left(\tau_1 + \frac{1}{2}\right)(I_l - I_j) + 1\right]\right\}\left\{\alpha\left[\frac{1}{3} - a\tau_1\left(\tau_1 + \frac{1}{2}\right)\right]\right\} \\
&\quad + \left\{\alpha\Big[I_l + I_k - 2I_j\Big]\left[\frac{1}{3} - a\tau_1\left(\tau_1 + \frac{1}{2}\right)\right] + \frac{1}{3}\right\}\left\{\left(\frac{\tilde{V}_l}{2}\right)\left[-3a\tau_1\left(\tau_1 + \frac{1}{2}\right)\right]\right\} \\
&\quad + \left\{\hat{V}_l + \frac{\tilde{V}_l}{2}\left[-3a\tau_1\left(\tau_1 + \frac{1}{2}\right)(I_l - I_k) + 1\right]\right\}\left\{\alpha\left[\frac{1}{3} - a\tau_1\left(\tau_1 + \frac{1}{2}\right)\right]\right\}
\end{aligned}$$

$$= \frac{\widetilde{V}_l}{2}\left[-3\alpha\tau_1\left(\tau_1+\frac{1}{2}\right)\right]\left\{\frac{2}{3}+\alpha\left[\frac{1}{3}-\alpha\tau_1\left(\tau_1+\frac{1}{2}\right)\right][I_l+I_j-2I_k+I_l+I_k-2I_j]\right\}$$
$$+\alpha\left[\frac{1}{3}-\alpha\tau_1\left(\tau_1+\frac{1}{2}\right)\right]\left\{2\widehat{V}_l+\widetilde{V}_l+\frac{\widetilde{V}_l}{2}\left[-3\alpha\tau_1\left(\tau_1+\frac{1}{2}\right)\right][I_l-I_j+I_l-I_k]\right\}$$
$$= \frac{\widetilde{V}_l}{2}\left[-3\alpha\tau_1\left(\tau_1+\frac{1}{2}\right)\right]\left\{\frac{2}{3}+\alpha\left[\frac{1}{3}-\alpha\tau_1\left(\tau_1+\frac{1}{2}\right)\right][2I_l-I_j-I_k]\right\}$$
$$+\alpha\left[\frac{1}{3}-\alpha\tau_1\left(\tau_1+\frac{1}{2}\right)\right]\left\{2\widehat{V}_l+\widetilde{V}_l+\frac{\widetilde{V}_l}{2}\left[-3\alpha\tau_1\left(\tau_1+\frac{1}{2}\right)\right][2I_l-I_j-I_k]\right\}$$
$$= \alpha\,[2I_l-I_j-I_k]\left[-3\alpha\tau_1\left(\tau_1+\frac{1}{2}\right)\right]\left[\frac{\widetilde{V}_l}{2}\right]\left[\frac{1}{3}-\alpha\tau_1\left(\tau_1+\frac{1}{2}\right)\right]\times 2$$
$$+\frac{\widetilde{V}_l}{2}\left[-3\alpha\tau_1\left(\tau_1+\frac{1}{2}\right)\right]\left(\frac{2}{3}\right)+\alpha\left[\frac{1}{3}-\alpha\tau_1\left(\tau_1+\frac{1}{2}\right)\right]\left[2\widehat{V}_l+\widetilde{V}_l\right]$$

$$\frac{\partial U_l'}{\partial I_l} = \alpha\widetilde{V}_l\Big[2I_l-I_j-I_k\Big]\left[-3\alpha\tau_1\left(\tau_1+\frac{1}{2}\right)\right]\left[\frac{1}{3}-\alpha\tau_1\left(\tau_1+\frac{1}{2}\right)\right]+\widetilde{V}_l\left[-\alpha\tau_1\left(\tau_1+\frac{1}{2}\right)\right]$$
$$+\alpha\left[\frac{1}{3}-\alpha\tau_1\left(\tau_1+\frac{1}{2}\right)\right]\left[2\widehat{V}_l+\widetilde{V}_l\right]$$
$$= \alpha\widetilde{V}_l\Big[2I_l-I_j-I_k\Big]\left\{-\alpha\tau_1\left(\tau_1+\frac{1}{2}\right)+3\left[\alpha\tau_1\left(\tau_1+\frac{1}{2}\right)\right]^2\right\}$$
$$+\widetilde{V}_l\left\{-\alpha\tau_1\left(\tau_1+\frac{1}{2}\right)+\alpha\left[\frac{1}{3}-\alpha\tau_1\left(\tau_1+\frac{1}{2}\right)\right]\right\}+2\alpha\widehat{V}_l\left[\frac{1}{3}-\alpha\tau_1\left(\tau_1+\frac{1}{2}\right)\right]$$

I.e.,

$$\frac{\partial U_l'}{\partial I_l} = \alpha\widetilde{V}_l\Big[2I_l-I_j-I_k\Big]\left\{-\alpha\tau_1\left(\tau_1+\frac{1}{2}\right)+3\left[\alpha\tau_1\left(\tau_1+\frac{1}{2}\right)\right]^2\right\}$$
$$+\widetilde{V}_l\left\{-\alpha\tau_1\left(\tau_1+\frac{1}{2}\right)[1+\alpha]+\frac{1}{3}\alpha\right\}+2\alpha\widehat{V}_l\left[\frac{1}{3}-\alpha\tau_1\left(\tau_1+\frac{1}{2}\right)\right] \tag{7}$$

Utilising the symmetric properties of the parents' utility functions, we can obtain the following expressions for parents j' and k' using the same approach:

$$U_j' = \left\{\alpha\Big[I_j+I_l-2I_k\Big]\left[\frac{1}{3}-\alpha\tau_1\left(\tau_1+\frac{1}{2}\right)\right]+\frac{1}{3}\right\}\left\{\widehat{V}_j+\frac{\widetilde{V}_j}{2}\left[-3\alpha\tau_1\left(\tau_1+\frac{1}{2}\right)(I_j-I_l)+1\right]\right\}$$
$$+\left\{\alpha\Big[I_j+I_k-2I_l\Big]\left[\frac{1}{3}-\alpha\tau_1\left(\tau_1+\frac{1}{2}\right)\right]+\frac{1}{3}\right\}\left\{\widehat{V}_j+\frac{\widetilde{V}_j}{2}\left[-3\alpha\tau_1\left(\tau_1+\frac{1}{2}\right)(I_j-I_k)+1\right]\right\}$$

$$U_k' = \left\{\alpha\Big[I_k + I_l - 2I_j\Big]\left[\frac{1}{3} - a\tau_1\left(\tau_1 + \frac{1}{2}\right)\right] + \frac{1}{3}\right\}\left\{\widehat{V}_k + \frac{\widetilde{V}_k}{2}\left[-3a\tau_1\left(\tau_1 + \frac{1}{2}\right)(I_k - I_l) + 1\right]\right\}$$
$$+ \left\{\alpha\Big[I_k + I_j - 2I_l\Big]\left[\frac{1}{3} - a\tau_1\left(\tau_1 + \frac{1}{2}\right)\right] + \frac{1}{3}\right\}\left\{\widehat{V}_k + \frac{\widetilde{V}_k}{2}\left[-3a\tau_1\left(\tau_1 + \frac{1}{2}\right)(I_k - I_j) + 1\right]\right\}$$

Similarly, the marginal utility of parents' investments can be obtained as follows:

$$\frac{\partial U_j'}{\partial I_j} = \alpha\widetilde{V}_j\Big[2I_j - I_l - I_k\Big]\left\{-a\tau_1\left(\tau_1 + \frac{1}{2}\right) + 3\left[a\tau_1\left(\tau_1 + \frac{1}{2}\right)\right]^2\right\}$$
$$+ \widetilde{V}_j\left\{-a\tau_1\left(\tau_1 + \frac{1}{2}\right)[1 + \alpha] + \frac{1}{3}\alpha\right\} + 2\alpha\widehat{V}_j\left[\frac{1}{3} - a\tau_1\left(\tau_1 + \frac{1}{2}\right)\right]$$

$$\frac{\partial U_k'}{\partial I_k} = \alpha\widetilde{V}_k\Big[2I_k - I_l - I_j\Big]\left\{-a\tau_1\left(\tau_1 + \frac{1}{2}\right) + 3\left[a\tau_1\left(\tau_1 + \frac{1}{2}\right)\right]^2\right\}$$
$$+ \widetilde{V}_k\left\{-a\tau_1\left(\tau_1 + \frac{1}{2}\right)[1 + \alpha] + \frac{1}{3}\alpha\right\} + 2\alpha\widehat{V}_k\left[\frac{1}{3} - a\tau_1\left(\tau_1 + \frac{1}{2}\right)\right]$$

4.2 Contest equilibrium: Far-sighted parents

Consider now the first stage of Round 1 where parents make their investment decisions. We will derive a Nash Equilibrium of the parental contest, assuming that the children will respond in the continuation game by playing a Nash Equilibrium for *any* profile of parental investments.

Assumption 4. $I_j = I_k = I_l$.

This implies that all parents make symmetric investments, without imposing any restriction on their wealth levels i.e. $w_j = w_k = w_l = w_s > 0$ as in Assumption 1.

Assumption 5. *All parents are far-sighted. They are only concerned with their child's eventual success in Round 2 and would yield* $\widetilde{V}_s > 0$ *if their child emerges as the final winner. Parents do not derive any satisfaction from their child's interim success in Round 1 such that* $\widehat{V}_s = 0$.

Proposition 1 (**Contest with far-sighted parents**). *Consider Protocol 1 where parents invest first followed by their children deciding on their investments. Under Assumptions 4 and 5, a Subgame Perfect Nash Equilibrium (SPE) exists. The investment decisions are as follows:*

1. *Parents do not invest at all in their children's education and their investment choices are symmetric, i.e.* $I_j{}^* = I_k{}^* = I_l{}^* = 0$, *in Stage 1 of Round 1.*

2. *The children will make symmetric investments given by* $i_j{}^* = i_k{}^* = i_l{}^* > 0$ *in Stage 2 of Round 1, as given in* (6).

Proof. With reference to $\frac{\partial U_l'}{\partial I_l}$ from (7), it can thus be simplified to the following:

$$\frac{\partial U_l'}{\partial I_l} = \tilde{V}_l \left\{ -a\tau_1 \left(\tau_1 + \frac{1}{2}\right)[1+\alpha] + \frac{1}{3}\alpha \right\}$$

We verify that parents actually get disutility from their investments, i.e. $\frac{\partial U_l'}{\partial I_l} < 0$ and for this to be true, the following must be satisfied:

$$\tilde{V}_l \left\{ -a\tau_1 \left(\tau_1 + \frac{1}{2}\right)[1+\alpha] + \frac{1}{3}\alpha \right\} < 0 \tag{8}$$

Given that $\tilde{V}_l > 0$ as stated in Assumption 5,

$$-a\tau_1 \left(\tau_1 + \frac{1}{2}\right)[1+\alpha] + \frac{1}{3}\alpha < 0$$

$$a > \frac{\alpha}{3\tau_1(\tau_1 + \frac{1}{2})(1+\alpha)}$$

With $a = \frac{1}{(\tau_1 - 1)(2\tau_1 + 1)}$ and $\tau_1 = \frac{\alpha V}{3(d - \frac{2}{3}\alpha V)}$ where $d > \frac{2}{3}\alpha V$,

$$\frac{1}{(\tau_1 - 1)(2\tau_1 + 1)} > \frac{\alpha}{3\tau_1(\tau_1 + \frac{1}{2})(1+\alpha)}$$

$$\frac{1}{2(\tau_1 - 1)(\tau_1 + \frac{1}{2})} > \frac{\alpha}{3\tau_1(\tau_1 + \frac{1}{2})(1+\alpha)}$$

$$\frac{1}{2\alpha(\tau_1 - 1)} > \frac{1}{3\tau_1(1+\alpha)}$$

$$3\tau_1 + 3\tau_1\alpha > 2\tau_1\alpha - 2\alpha$$

$$\text{i.e.} \quad \tau_1\alpha + 3\tau_1 + 2\alpha > 0$$

This condition is satisfied since $\tau_1 > 0$ and $\alpha > 0$. Therefore, we are able to conclude that parents get disutility from investing in their child's education. Parents would thus be better off not investing any of their wealth despite not having any alternative use for it. The Nash Equilibrium of parental investments in Stage 1 of Round 1 is hence $I_j{}^* = I_k{}^* = I_l{}^* = 0$.

Since (8) has been proven to be true, $a > 0$ i.e. $\tau_1 > 1$ must hold given that $\tilde{V}_l > 0$, $\tau_1 > 0$ and

$\alpha > 0$. Following which, the children's investment choices at equilibrium are found to be symmetric as well according to (6), such that:

$$i_s{}^* = \left(\frac{2}{3}\alpha\check{V} + \frac{V}{3}\right)(a\tau_2)(\tau_1 - 1) + a\tau_1(\tau_1 - 1) \quad \text{where } i_j{}^* = i_k{}^* = i_l{}^* > 0. \qquad \textbf{Q.E.D.}$$

Intuition: With reference to (6), we seek to analyse the relationship between the Nash Equilibrium investment choices for the children in the continuation game and parental investments. The following observations can hence be generalised across all parent-child pairs accordingly.

First, there is an inverse relationship between the child's level of effort in the Nash Equilibrium of the continuation game and the amount of parental support one receives. When each parent invests more in his child's education, the child will in turn exert lesser effort i.e. an increase in I_l leads to a fall in $i_l{}^*$. This is economically intuitive as the child might choose to slacken, given that one's parent has already contributed a significant amount of investment which might potentially put him in a good stead to be one of the top two winners at the end of Round 1.

Second, it can be observed that there is a positive relationship between each child's effort and the parental investments received by one's competitors. When child l receives greater support from parent l i.e. there is an increase in I_l, children j and k will increase their stakes such that $i_j{}^*$ and $i_k{}^*$ turn out to be greater. In this scenario, child l's competitors might panic and choose to exert more effort upon realising that child l is receiving more parental support which could boost his chances of succeeding in the first round. There is thus a pressing need for children j and k to work harder as a result, if they care about competing on a fair ground with child l.

These observations will complement the elaborate explanation that follows in Section 5, to rationalise the contest equilibrium derived above – parents not investing at all due to disutility resulting from any positive parental investment, despite caring for their child's eventual success.

4.3 Contest equilibrium: Parents caring about interim and eventual success

We extend the analysis by relaxing one of the assumptions made previously. Assumption 4 continues to hold in this context, where parental investments are similarly assumed to be symmetric in equilibrium. However, Assumption 5 will be replaced by Assumption 6.

Assumption 6. *Parents are concerned about their child's interim as well as eventual success in* *both rounds of the contest, implying that* $\widehat{V}_s > 0$ *and* $\widetilde{V}_s > 0$ *respectively.*

Proposition 2 (**Contest where interim and eventual success matter**). *Consider Protocol 1 where parents invest first followed by their children deciding on their investments. Under Assumptions 4 and 6, an SPE exists. The investment decisions by both the parents and children are identical to that in Proposition 1, as follows:*

1. *Parents do not invest at all in their children's education and their investment choices are symmetric, i.e.* ${I_j}^* = {I_k}^* = {I_l}^* = 0$*, in Stage 1 of Round 1.*

2. *The children will make symmetric investments given by* ${i_j}^* = {i_k}^* = {i_l}^* > 0$ *in Stage 2 of Round 1, as given in* (6).

Proof. Simplifying the first-order condition previously obtained in (7) together with the underlying Assumption 4 where $I_j = I_k = I_l$, we arrive at the following:

$$\frac{\partial U_l'}{\partial I_l} = \widetilde{V}_l \left[-a\tau_1 \left(\tau_1 + \frac{1}{2} \right) (1 + \alpha) + \frac{1}{3}\alpha \right] + 2\alpha \widehat{V}_l \left[\frac{1}{3} - a\tau_1 \left(\tau_1 + \frac{1}{2} \right) \right]$$

It can be verified that parents actually gain disutility from their investment i.e. $\frac{\partial U_l'}{\partial I_l} < 0$, where the following has to be satisfied:

$$\widetilde{V}_l \left[-a\tau_1 \left(\tau_1 + \frac{1}{2} \right) (1 + \alpha) + \frac{1}{3}\alpha \right] + 2\alpha \widehat{V}_l \left[\frac{1}{3} - a\tau_1 \left(\tau_1 + \frac{1}{2} \right) \right] < 0 \tag{9}$$

We have previously validated that (8) is indeed true. With reference to (9), we can thus say that the first term $\widetilde{V}_l \left[-a\tau_1 \left(\tau_1 + \frac{1}{2} \right) (1 + \alpha) + \frac{1}{3}\alpha \right] < 0$. We proceed to prove that the second term $2\alpha \widehat{V}_l \left[\frac{1}{3} - a\tau_1 \left(\tau_1 + \frac{1}{2} \right) \right]$ in (9) is negative.

Given that $\alpha > 0$ and $\widehat{V}_l > 0$, the following has to be satisfied for $2\alpha \widehat{V}_l \left[\frac{1}{3} - a\tau_1 \left(\tau_1 + \frac{1}{2} \right) \right] < 0$:

$$\frac{1}{3} - a\tau_1 \left(\tau_1 + \frac{1}{2} \right) < 0$$

$$\frac{1}{3} < a\tau_1 \left(\tau_1 + \frac{1}{2} \right)$$

$$a > \frac{1}{3\tau_1(\tau_1 + \frac{1}{2})}$$

Given that $a = \frac{1}{(\tau_1-1)(2\tau_1+1)}$,

$$\frac{1}{(\tau_1-1)(2\tau_1+1)} > \frac{1}{3\tau_1(\tau_1+\frac{1}{2})}$$

$$\frac{1}{2(\tau_1-1)(\tau_1+\frac{1}{2})} > \frac{1}{3\tau_1(\tau_1+\frac{1}{2})}$$

$$\frac{1}{2(\tau_1-1)} > \frac{1}{3\tau_1}$$

$$3\tau_1 > 2\tau_1 - 2$$

$$\tau_1 > -2$$

Since the underlying condition $\tau_1 > -2$ is satisfied given that $\tau_1 > 0$, we can conclude that $\frac{\partial U_l'}{\partial I_l} < 0$ as it comprises of negative terms. Parents hence do not invest at all such that $I_j{}^* = I_k{}^* = I_l{}^* = 0$. Additionally, the fact that both terms in (9) are negative implies that $a > 0$ since $\alpha > 0$, $\widetilde{V}_l > 0$, $\widehat{V}_l > 0$ and $\tau_1 > 0$.

Similar to that in Proposition 1, the children's investment choices at equilibrium are found to be symmetric according to equation (6), such that:

$$i_s{}^* = \left(\frac{2}{3}\alpha\check{V} + \frac{V}{3}\right)(a\tau_2)(\tau_1-1) + a\tau_1(\tau_1-1) \quad \text{where } i_j{}^* = i_k{}^* = i_l{}^* > 0. \qquad \textbf{Q.E.D.}$$

Even though parents care about their children's interim and eventual success in the contest, they continue to make zero investments and this result is identical to that observed in Proposition 1. The explanation to these results lies in the more-than-complete crowding out effects arising from any positive parental investment, which will be elaborated in the next Section.

5 Effect of Parental Investment on Composite Input (Protocol 1)

It would be interesting to study the effect of a change in parental investment on the composite input x_s of a parent-child pair. This is due to the opposing effects on the latter resulting from the change in parental investment, which subsequently leads to a change in the children's investment choices in the Nash Equilibrium of the continuation game.

Proposition 3 (**More-than-complete crowding out**). *When $a > 0$, an increase in I_l results in:*

1. *a fall in* $i_l{}^*$
2. *a fall in* $x_l{}^*$
3. *a rise in* $x_j{}^*$
4. *a rise in* $x_k{}^*$

Proof. We consider the resulting effects from an increase in I_l. Parent l' will take I_j and I_k as given while the increase in I_l will prompt children j, k, l to observe this change and modify their investment choices accordingly. As discussed previously in Section 4.2, we have concluded that $i_l{}^*$ will fall with reference to (6). Given that the parent-child pair l's composite input $x_l = \alpha[i_l{}^*(\bar{I}_j, \bar{I}_k, I_l) + I_l]$, there are two opposing effects – an increase in I_l and a decrease in $i_l{}^*$. The cumulative effect on x_l is hence not as prominent, prompting the need to take the derivative of x_l with respect to I_l:

$$\begin{aligned}\frac{\partial x_l}{\partial I_l} &= \alpha\left[1 + \frac{\partial i_l{}^*}{\partial I_l}\right] \\ &= \alpha\left[1 - 2a\tau_1\left(\tau_1 + \frac{1}{2}\right)\right]\end{aligned}$$

We proceed to verify that $\frac{\partial x_l}{\partial I_l} < 0$, which implies that as parent l' invests more, the composite input of parent-child pair l will eventually fall.

For this to be true, the following has to be true given that $\alpha > 0$:

$$1 - 2a\tau_1\left(\tau_1 + \frac{1}{2}\right) < 0$$

$$1 < 2a\tau_1\left(\tau_1 + \frac{1}{2}\right)$$

$$a > \frac{1}{2\tau_1(\tau_1 + \frac{1}{2})}$$

Given that $a = \frac{1}{(\tau_1 - 1)(2\tau_1 + 1)}$,

$$\begin{aligned}\frac{1}{(\tau_1 - 1)(2\tau_1 + 1)} &> \frac{1}{2\tau_1(\tau_1 + \frac{1}{2})} \\ \frac{1}{2(\tau_1 - 1)(\tau_1 + \frac{1}{2})} &> \frac{1}{2\tau_1(\tau_1 + \frac{1}{2})}\end{aligned}$$

$$\frac{1}{\tau_1 - 1} > \frac{1}{\tau_1}$$

$$\tau_1 > \tau_1 - 1$$

Since $\tau_1 > 0$, this condition is satisfied and we can conclude that $\frac{\partial x_l}{\partial I_l} < 0$ when $a > 0$. Therefore, the observations as stated in Proposition 3 resulting from an increase in I_l can be made as follows.

First, child l will reduce his effort $i_l{}^*$ in the continuation game Nash Equilibrium as seen in (6).

Second, we have established that $\frac{\partial x_l}{\partial I_l} < 0$. The larger drop in $i_l{}^*$ as compared to the smaller rise in I_l causes $x_l{}^*$ to fall, implying that an increase in parental investment by parent l' is counter-productive. It results in more-than-complete crowding out of child l's efforts, to the extent of not just reducing his chances of succeeding in Round 1 due to the fall in $x_l{}^*$, but also in Round 2 of the contest due to the drop in $i_l{}^*$.

Third, the composite inputs by parent-child pairs j and k will increase in response. This can be accredited to the inverse relationship between I_l and children j's and k's investments as mentioned in Section 4.2. In short, the increase in I_l leads to an increase in $i_j{}^*$ and $i_k{}^*$ which ultimately results in an increase in $x_j{}^*$ and $x_k{}^*$ respectively while holding I_j and I_k fixed. The increase in $i_j{}^*$, $i_k{}^*$, $x_j{}^*$ and $x_k{}^*$ collectively reinforce the reduction in child $l's$ chances of succeeding in Rounds 1 and 2 of the contest. **Q.E.D.**

More-than-complete crowding out of parental investment by the child is a surprising result. In voluntary contribution public good and fundraising models, it has been observed that donors would collectively lower their contributions in response to an increase in government grants. That is, crowding out is *less than* a dollar-for-dollar, for what is known as *incomplete crowding out* (see Bergstrom, Blume and Varian, 1986; Andreoni and Payne, 2003).

Since an increase in parental investment will backfire and instead reduce the child's chances of success in both Rounds 1 and 2, making positive parental investment will render disutility to the parents. This will prompt parents not to invest in their children's education at all if parents wish to maximise their own utility. Hence, the above rationale accounts for the results obtained in Sections 4.2 and 4.3, where parents eventually decide on zero investment though they care about their child's success.

6 Alternative Game Form: Protocol 2

To extend our analysis of the game further, we reverse the sequence of moves in Round 1 of the contest by looking at a second protocol, as depicted in Fig. 6.1. There are still two rounds in the overall contest. However in Round 1, the children will be deciding on their investments simultaneously in the first stage while the parents will observe these decisions and decide on their investments simultaneously in the second stage. The second round race runs in the same way as that in the first protocol.

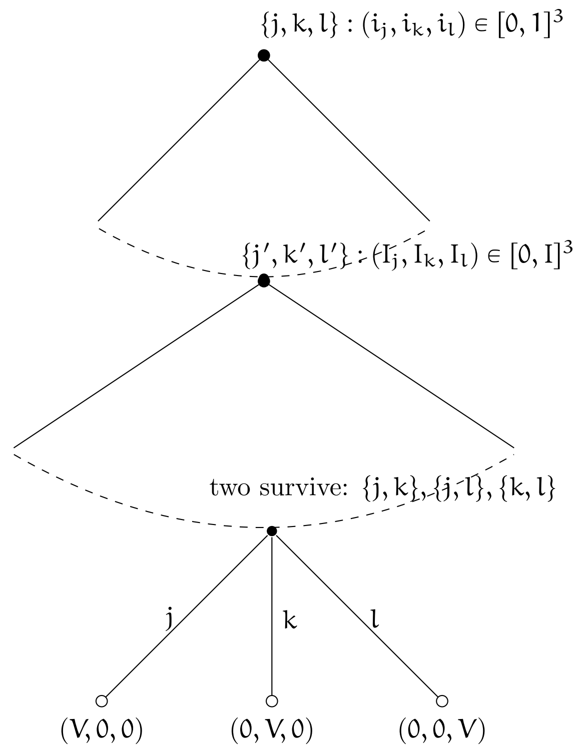

Figure 6.1: PROTOCOL 2

Proposition 4 (**Contest equilibrium under Protocol 2**). *Consider the school contest game under Protocol 2. Suppose children move first exerting efforts, followed by the parents making their investments. Under Assumption 6 where parents care about their child's interim and eventual success, the following is an SPE:*

1. *All parents have a dominant strategy in the continuation game – to invest all of their wealth into their children's education. They will make symmetric investments such that* $I_j{}^* = I_k{}^* =$

$I_l{}^* = I > 0$ *in Stage 2 of Round 1.*

2. *The children will make symmetric investments given by* $i_l{}^* = i_j{}^* = i_k{}^* > 0$ *in Stage 1 of Round 1, as illustrated by* (6).

The proof will be developed in the next two sections.

6.1 Stage 2 of Round 1: Parents' Optimisation Problem

Here, each parent simultaneously chooses their optimal investment level while taking the investments of the children $\{i_j, i_k, i_l\}$ as given.

Proof of Proposition 4.1. As derived in Section 4.1, the utility function of parent l' is as follows:

$$\begin{aligned} U_l' = &\frac{\alpha(i_l + I_l) + \alpha(i_j + I_j) + 1 - 2\alpha(i_k + I_k)}{3} \times \left(\widehat{V}_l + \frac{i_l + 1 - i_j}{2} \times \widetilde{V}_l\right) \\ &+ \frac{\alpha(i_l + I_l) + \alpha(i_k + I_k) + 1 - 2\alpha(i_j + I_j)}{3} \times \left(\widehat{V}_l + \frac{i_l + 1 - i_k}{2} \times \widetilde{V}_l\right) \end{aligned}$$

Obtaining parent l's marginal utility from investing,

$$\begin{aligned} \frac{\partial U_l'}{\partial I_l} &= \frac{1}{3}\alpha\left(\widehat{V}_l + \frac{i_l + 1 - i_j}{2} \times \widetilde{V}_l\right) + \frac{1}{3}\alpha\left(\widehat{V}_l + \frac{i_l + 1 - i_k}{2} \times \widetilde{V}_l\right) \\ &= \frac{2}{3}\alpha\widehat{V}_l + \frac{1}{3}\alpha\widetilde{V}_l\left(\frac{i_l + 1 - i_j + i_l + 1 - i_k}{2}\right) \\ &= \frac{2}{3}\alpha\widehat{V}_l + \frac{1}{6}\alpha\widetilde{V}_l\left(2i_l + 2 - i_j - i_k\right) \end{aligned} \tag{10}$$

In Section 2.1, we have established that $0 \leqslant i_j, i_k, i_l \leqslant 1$ must be satisfied. Referring back to (10), this condition implies that $\frac{\partial U_l'}{\partial I_l} > 0$ will be satisfied, given that $\alpha > 0$, $\widehat{V}_l > 0$ and $\widetilde{V}_l > 0$. Since parent l' gains utility from investing, one will invest all his wealth w_l into his child's education to maximise his own utility in consideration that one has no other use for his wealth. This result is also applicable to parents j' and k' as the utility functions of the parents are symmetric. Since all parents are assumed to have positive and identical wealth as stated in Assumption 1, the dominant strategy is such that all parents will make positive and symmetric investments in the second stage of Round 1 of the contest: $I_j{}^* = I_k{}^* = I_l{}^* = I > 0$. **Q.E.D.**

6.2 Stage 1 of Round 1: Children's Optimisation Problem

Proof of Proposition 4.2. Using backward induction, we proceed to solve the children's optimisation problem. As derived previously in Section 3.1, child l's utility function can be written as follows:

$$\begin{aligned} U_l = & \frac{\alpha(i_l + I_l{}^*) + \alpha(i_j + I_j{}^*) + 1 - 2\alpha(i_k + I_k{}^*)}{3} \times \check{V}_l \\ & + \frac{\alpha(i_l + I_l{}^*) + \alpha(i_j + I_j{}^*) + 1 - 2\alpha(i_k + I_k{}^*)}{3} \left(\frac{i_l + 1 - i_j}{2}\right) \times V \\ & + \frac{\alpha(i_l + I_l{}^*) + \alpha(i_k + I_k{}^*) + 1 - 2\alpha(i_j + I_j{}^*)}{3} \times \check{V}_l \\ & + \frac{\alpha(i_l + I_l{}^*) + \alpha(i_k + I_k{}^*) + 1 - 2\alpha(i_j + I_j{}^*)}{3} \left(\frac{i_l + 1 - i_k}{2}\right) \times V - \frac{1}{2} d_l i_l^2 \end{aligned}$$

Given that $I_j{}^* = I_k{}^* = I_l{}^*$ in the continuation game Nash Equilibrium, it can be simplified to:

$$\begin{aligned} U_l = & \frac{\alpha i_l + \alpha i_j + 1 - 2\alpha i_k}{3} \times \check{V}_l + \frac{\alpha i_l + \alpha i_j + 1 - 2\alpha i_k}{3} \left(\frac{i_l + 1 - i_j}{2}\right) \times V + \frac{\alpha i_l + \alpha i_k + 1 - 2\alpha i_j}{3} \times \check{V}_l \\ & + \frac{\alpha i_l + \alpha i_k + 1 - 2\alpha i_j}{3} \left(\frac{i_l + 1 - i_k}{2}\right) \times V - \frac{1}{2} d_l i_l^2 \end{aligned}$$

To maximise the child's utility, we obtain the first order condition:

$$\begin{aligned} \frac{\partial U_l}{\partial i_l} & = \frac{1}{3}\alpha \check{V}_l + \frac{\alpha i_l + \alpha i_j + 1 - 2\alpha i_k}{3}(V)\left(\frac{1}{2}\right) + \frac{i_l + 1 - i_j}{2}(V)\left(\frac{1}{3}\alpha\right) \\ & \quad + \frac{1}{3}\alpha \check{V}_l + \frac{\alpha i_l + \alpha i_k + 1 - 2\alpha i_j}{3}(V)\left(\frac{1}{2}\right) + \frac{i_l + 1 - i_k}{2}(V)\left(\frac{1}{3}\alpha\right) - \frac{1}{2} d_l (2 i_l) \\ & = \frac{2}{3}\alpha \check{V}_l + \frac{V}{6}(4\alpha i_l - 2\alpha i_j + 2 - 2\alpha i_k + 2\alpha) - d_l i_l \\ & = 0 \end{aligned}$$

Making i_l the subject, we derive the best response function of child l as a function of i_j and i_k:

$$i_l\left(d_l - \frac{2}{3}\alpha V\right) = \frac{2}{3}\alpha \check{V}_l + \frac{V}{6}\Big[2 + 2\alpha - 2\alpha(i_j + i_k)\Big]$$

$$\text{i.e.} \qquad i_l = \frac{1}{d_l - \frac{2}{3}\alpha V}\left\{\frac{2}{3}\alpha \check{V}_l + \frac{V}{3}\Big[1 + \alpha - \alpha(i_j + i_k)\Big]\right\}$$

Adopting the same approach for children j and k, the following equations denote each child's best response function respectively under Assumptions 2 and 3:

$$\text{Child } l\text{'s: } i_l + \tau_1 i_j + \tau_1 i_k = \tau_2 \left[\frac{2}{3}\alpha\check{V} + \frac{V}{3}(1+\alpha)\right]$$
$$\text{Child } j\text{'s: } \tau_1 i_l + i_j + \tau_1 i_k = \tau_2 \left[\frac{2}{3}\alpha\check{V} + \frac{V}{3}(1+\alpha)\right]$$
$$\text{Child } k\text{'s: } \tau_1 i_l + \tau_1 i_j + i_k = \tau_2 \left[\frac{2}{3}\alpha\check{V} + \frac{V}{3}(1+\alpha)\right]$$

We also denote the following:

$$\tau_2 \left[\frac{2}{3}\alpha\check{V} + \frac{V}{3}(1+\alpha)\right] = \theta_s.$$

Expressing these best response functions in matrix notation,

$$\begin{pmatrix} 1 & \tau_1 & \tau_1 \\ \tau_1 & 1 & \tau_1 \\ \tau_1 & \tau_1 & 1 \end{pmatrix} \begin{pmatrix} i_l \\ i_j \\ i_k \end{pmatrix} = \begin{pmatrix} \theta_s \\ \theta_s \\ \theta_s \end{pmatrix}$$

$$\begin{pmatrix} i_l{}^* \\ i_j{}^* \\ i_k{}^* \end{pmatrix} = \begin{pmatrix} 1 & \tau_1 & \tau_1 \\ \tau_1 & 1 & \tau_1 \\ \tau_1 & \tau_1 & 1 \end{pmatrix}^{-1} \begin{pmatrix} \theta_s \\ \theta_s \\ \theta_s \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{\tau_1+1}{(\tau_1-1)(2\tau_1+1)} & \frac{\tau_1}{(\tau_1-1)(2\tau_1+1)} & \frac{\tau_1}{(\tau_1-1)(2\tau_1+1)} \\ \frac{\tau_1}{(\tau_1-1)(2\tau_1+1)} & -\frac{\tau_1+1}{(\tau_1-1)(2\tau_1+1)} & \frac{\tau_1}{(\tau_1-1)(2\tau_1+1)} \\ \frac{\tau_1}{(\tau_1-1)(2\tau_1+1)} & \frac{\tau_1}{(\tau_1-1)(2\tau_1+1)} & -\frac{\tau_1+1}{(\tau_1-1)(2\tau_1+1)} \end{pmatrix} \begin{pmatrix} \theta_s \\ \theta_s \\ \theta_s \end{pmatrix}$$

Given that $a = \frac{1}{(\tau_1-1)(2\tau_1+1)}$ as previously,

$$\begin{pmatrix} i_l{}^* \\ i_j{}^* \\ i_k{}^* \end{pmatrix} = \begin{pmatrix} -a(\tau_1+1) & a\tau_1 & a\tau_1 \\ a\tau_1 & -a(\tau_1+1) & a\tau_1 \\ a\tau_1 & a\tau_1 & -a(\tau_1+1) \end{pmatrix} \begin{pmatrix} \theta_s \\ \theta_s \\ \theta_s \end{pmatrix}$$

$$= \begin{pmatrix} -a(\tau_1+1)\theta_s + a\tau_1\theta_s + a\tau_1\theta_s \\ a\tau_1\theta_s - a(\tau_1+1)\theta_s + a\tau_1\theta_s \\ a\tau_1\theta_s + a\tau_1\theta_s - a(\tau_1+1)\theta_s \end{pmatrix}$$

$$= \begin{pmatrix} -a\tau_1\theta_s - a\theta_s + a\tau_1\theta_s + a\tau_1\theta_s \\ a\tau_1\theta_s - a\tau_1\theta_s - a\theta_s + a\tau_1\theta_s \\ a\tau_1\theta_s + a\tau_1\theta_s - a\tau_1\theta_s - a\theta_s \end{pmatrix}$$

$$= \begin{pmatrix} a\theta_s(\tau_1 - 1) \\ a\theta_s(\tau_1 - 1) \\ a\theta_s(\tau_1 - 1) \end{pmatrix}$$

The Nash Equilibrium for the children is found to be:

$$\begin{pmatrix} i_l^* \\ i_j^* \\ i_k^* \end{pmatrix} = \begin{pmatrix} a\tau_2\left[\frac{2}{3}\alpha\check{V} + \frac{V}{3}(1+\alpha)\right](\tau_1 - 1) \\ a\tau_2\left[\frac{2}{3}\alpha\check{V} + \frac{V}{3}(1+\alpha)\right](\tau_1 - 1) \\ a\tau_2\left[\frac{2}{3}\alpha\check{V} + \frac{V}{3}(1+\alpha)\right](\tau_1 - 1) \end{pmatrix} \tag{11}$$

We have previously established that $\alpha > 0$, $\tau_2 > 0$, $\check{V} > 0$ and $V > 0$. In the context of this alternative game form, we conjecture that the children will still be making positive investments (i.e. putting in efforts) in the equilibrium if they are concerned about winning the overall contest.

The rationale being, the child's chances of success in the second round is completely contingent on one's effort exerted in the first round through 'learning-by-doing'. Not exerting any effort in the contest such that $i_s = 0$ will put the child in a great disadvantage, due to the lower chances of succeeding not just in Round 1, but also in Round 2 of the contest. Therefore, we rule out the possibility of $i_s = 0$ in the contest.

Given that $a = \frac{1}{(\tau_1-1)(2\tau_1+1)}$, there are two possibilities which correspond to the conjecture that i_l^*, i_j^*, i_k^* will be positive, stated as follows:

1. If $\tau_1 > 1$, then $\tau_1 - 1 > 0$. This implies that $a > 0$ as well. Therefore, $i_l^* = i_j^* = i_k^* > 0$ according to (11).

2. If $\tau_1 < 1$, then $\tau_1 - 1 < 0$. This implies that $a < 0$ as well. Therefore, $i_l^* = i_j^* = i_k^* > 0$

according to (11). **Q.E.D.**

The equilibrium of the contest with this alternative game form would hence be that the children will be exerting symmetric and positive effort in their academics, while their parents will invest all of their wealth into their respective child's education as well.

6.3 Economic Intuition

With a change in the order of moves in the game, the stark differences in outcomes obtained in the second protocol as compared to the first protocol are evident.

In the former, parents gain satisfaction from investing and it becomes a dominant strategy for them to invest all of their wealth into their child's education. The rationale is as such: given that children are the ones making the first move followed by the parents, children's sunk investments would have determined their chances of success in the more mature race. The chances of the children's success in the second round cannot be influenced by the parents' investment decisions as the outcome is purely contingent on the children's own efforts. However, parents could nonetheless boost their children's odds of proceeding from Round 1 to the more mature race (Round 2) by investing, which increases the parent-child pair's composite input x_s. Parents would hence choose to do so to maximise their utility given the positive marginal utility from their investment.

7 Comparison Between Protocols 1 and 2

7.1 Investment choices by children

Since the differences in parental investment choices at equilibrium are evident across both game forms, we are interested in observing if there are any changes to the children's investment choices in the equilibrium as well.

Proposition 5 (**Identical investment by children**). *Despite the change in sequence of moves for the parents and children, as well as differences in parental investment choices across the two game forms, the children's investment choices remain unchanged for both the first and second protocols.*

Proof. For Protocol 1, we have considered two scenarios where firstly, parents only care about the child's eventual success (Proposition 1) and secondly, child's interim and eventual success both matter to the parents (Proposition 2). With reference to (6), we have derived the children's investment choices in equilibrium to be the following:

$$i_s{}^* = \left(\frac{2}{3}\alpha\check{V} + \frac{V}{3}\right)(a\tau_2)(\tau_1 - 1) + a\tau_1(\tau_1 - 1)$$

In Protocol 2, we have considered the case where parents are concerned about the child's interim and eventual success as stated in Proposition 4. With reference to (11), the Nash Equilibrium of the children's investments is such that:

$$\begin{aligned} i_s{}^* &= a\tau_2\left[\frac{2}{3}\alpha\check{V} + \frac{V}{3}(1+\alpha)\right](\tau_1 - 1) \\ &= \left(\frac{2}{3}\alpha\check{V} + \frac{V}{3}\right)(a\tau_2)(\tau_1 - 1) + a\tau_2\left(\frac{V}{3}\right)\alpha(\tau_1 - 1) \\ &= \left(\frac{2}{3}\alpha\check{V} + \frac{V}{3}\right)(a\tau_2)(\tau_1 - 1) + a\tau_1(\tau_1 - 1) \end{aligned}$$

It is evident that the children's investment choices in equilibrium are the same across both Protocols 1 and 2, despite the reverse in sequence of moves in Round 1 and the disparity between parental investment choices. The intuition for this surprising result is simple. In the second protocol, the children who are moving first would have been expected to under-invest (relative to the first protocol) since their parents are expected to sink in all their wealth in their education. However, the neutrality derives from the very fact that symmetric (i.e. identical) parental investments cancel out in the children's payoff functions. Hence, when the children are making their investment decisions in Protocol 2, it is as if, collectively, the parents did not invest at all (which is the same as that in Protocol 1, where zero parental investment choices in equilibrium are observed). As a result, the eventual outcome can be described as: *running to keep in the same place.*

7.2 Probability of success

With differing composite inputs by the parent-child pairs across both game forms, we are also interested in the potential disparity between the odds of success for each parent-child pair in the first and second protocol of the contest model.

PROPOSITION 6 (**Identical odds of success**). *The chances of success for each parent-child pair are the same for the following events across both game forms:*

1. *Emerging as one of the top two winners at the end of Round 1 and proceeding to Round 2*

2. *Emerging as the ultimate winner of the overall contest at the end of Round 2*

Proof. In the first game form in Propositions 1 and 2, we have established the *SPE* such that $I_j^* = I_k^* = I_l^* = 0$ and $i_j^* = i_k^* = i_l^* > 0$. The composite inputs $x_j^* = x_k^* = x_l^*$ according to (1) are thus identical across all parent-child pairs. Therefore, the probability for which each of the three possible events that could occur according to the contest success function in Round 1 would be identical such that:

$$\Pr(\{j,k\}) = \Pr(\{j,l\}) = \Pr(\{k,l\}) = \frac{1}{3}$$

Similarly, each of the top two winners will have an equal chance of being the ultimate winner according to the contest success function in Round 2 where

$$\Pr(s_1 \text{ wins}) = \Pr(s_2 \text{ wins}) = \frac{1}{2}$$

. In the second game form in Proposition 4, we have concluded that the *SPE* is such that $I_j^* = I_k^* = I_l^* = I$ and $i_j^* = i_k^* = i_l^* > 0$. The aforementioned outcomes for x_s and the contest success functions in both Rounds 1 and 2 for the first game form will hence be applicable for the second protocol as well.

Hence, we conclude that the probabilities of success across both Protocols 1 and 2 for the respective rounds of contest are identical as follows:

$$\text{Round 1:} \quad \Pr(\{j,k\}) = \Pr(\{j,l\}) = \Pr(\{k,l\}) = \tfrac{1}{3}$$
$$\text{Round 2:} \quad \Pr(s_1 \text{ wins}) = \Pr(s_2 \text{ wins}) = \tfrac{1}{2}$$

Q.E.D.

It would be better for parents not to invest at all in their children's education, since their investments would not have any long-term value in enhancing their children's *personality* given that parental investments are simply dissipative. On the contrary, these investments would not have gone to waste if positive parental investments could instead motivate the children to work harder. This would have benefited the society overall as children would develop on their *personality* in the process of 'learning-by-doing'.

7.3 Generality of Propositions 5 and 6

We have assumed that parental investments I_s up to the maximum amount of I have zero opportunity cost throughout this paper. We rationalise that the results obtained in Propositions 5 and 6 are not dependent on this assumption.

Suppose parents incur some opportunity cost that is concave in I_s denoted by $\nu(I_s)$, with $\nu'(I_s) > 0$ and $\nu''(I_s) < 0$. This might yield a different outcome where parents do not sink all of their wealth into their children's education as what was observed in Propositions 5 and 6, simply because the interior solution could possibly be $0 \leqslant {I_s}^* \leqslant I$. If the cost of investment is large, parents would have to decide on their optimal investments by striking a balance between the incremental gain reflected by the increase in their children's odds of success in both Rounds 1 and 2, as well as the marginal utility loss incurred as reflected by $\nu'(I_s)$.

We argue that as long as parents face symmetric opportunity cost in their investments, their investment choices in the equilibrium will be symmetric. In turn, these parental investments would *cancel out* in the school contest and prompt the children to exert the same amount of efforts as that observed in both Protocols 1 and 2, regardless of whether parents incur an opportunity cost from their investments. Propositions 5 and 6 would hence remain valid as well.

8 Conclusion

8.1 Summary of Analysis

This paper examines a multi-pronged contest model comprising of two rounds (two stages within the first round) between parent-child pairs. The model seeks to shed some light on the issue of how parental investments and the efforts exerted by their children towards their education would affect the children's odds of success not just in the short-term, but also in the long-term.

Two orders of play are analysed in the paper, with the distinguishing factor being the sequence of moves within the first round of the contest. In the first protocol, parents would be making the first move in Stage 1 of Round 1 followed by the children, who would then decide on their effort levels in Stage 2 of Round 1. The reversed order of moves is true for the second protocol. Subsequently, the second round of the contest runs identically across both protocols.

In the first protocol, we conclude that parents would not invest at all in their children, regardless of whether they care about their child's eventual and/or interim success in the school contest. The underlying rationale is simply because any positive parental investment would prompt the children to exert lower efforts such that more-than-complete crowding out arises hence, decreasing one's chances of success in the short-term and long-term. Zero parental investment thus induces the children to exert their own effort in order to have a go at succeeding in the contest, whose choices of effort turn out to be positive and symmetric in equilibrium.

In the second protocol, parents are observed to have sunk all of their wealth into their children's education. This is a surprising result contrary to natural economic intuition. Despite the children (first-mover) anticipating that their parents (second-mover) would fully invest in their education, they chose to exert the same amount of effort as that observed in the first protocol. This neutralises the rotten kid problem, which would instead predict that the children would exert lower efforts than those in Protocol 1, with the expectation that their parents will bail them out eventually. The underlying rationale is that full symmetric parental investments would *cancel out* in the contest, such that it appears as if collectively, the parents have made zero investments. As a result, the children would put in their own efforts in order to sustain their odds of success in the contest.

8.2 Possible Extensions

The paper has made several simplifying assumptions: (i) perfect substitution between parental investment and children's investment, (ii) identical parental wealth, (iii) no long term impact of parental investment on the development of child's intrinsic skills. Extensions of the current analysis relaxing each of these assumptions would enrich the current contest model.

For instance, are Asian and Asian-American parents right in playing the role of 'tiger mums'? If so, does this imply that American parents are wrong in their relative hands-off approach towards their children's development? Answers to these questions may well vary if complementary technology was assumed in the model, instead of perfect substitution technology. With the former, complementarity suggests that positive parental investments will motivate the children to work harder due to an increase in the marginal effectiveness of children's efforts. This in turn suggests that the aggressive parenting role adopted by Asian and Asian-American parents could be justified. While parents push their children to work harder in their studies, this would facilitate their learning process not just academically, but also in other aspects such as boosting of their self-confidence that could potentially translate into greater chances of future success. This possibility definitely calls for the need for further research to further explore the potential economic insights that could be gained.

Additionally, heterogeneity in parental wealth could possibly provide interesting insights pertaining to issues such as how the prevalence of inequality arises, not because of genetic reasons e.g. one's innate talent but rather, due to the differing economic backgrounds that individuals have. This is attributed to the fact that some individuals are born economically rich while others are born poor. Therefore, this implication might complicate the inference problem of the university admission authorities: are exemplary academic grades the result of children receiving greater parental support or the child's own effort? This also poses a question as to whether granting admission for valuable scarce university slots to applicants with good grades is necessarily better as grades become less informative for admissions. Practising affirmative actions for university admissions based on wealth backgrounds, nationality and ethnicity may well have some economic justifications as such information might provide authorities with a better understanding of the contributing factors towards the student's success.

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