

Investment in Children, Social Security, and Intragenerational Risk Sharing

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People rear and educate children in exchange for old-age support

- Cheung (1972, p. 641): “[j]ust as dogs were raised to hunt for their masters before they were pets, so in early traditional China children were raised as a source of income and a store of wealth.
- Intrafamily contract in developing countries (Ehrlich and Lui 1991, 1998; Becker 1993)

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- Intrafamily contract in developing countries (Ehrlich and Lui 1991, 1998; Becker 1993)
- Pay-As-You-Go social security in developed countries
- A core function of PAYGO is to manage the risks of events leaving the old impoverished (Diamond 1977, Shiller 1999)

Literature regarding the risk-sharing role of social security ([Krueger and Kubler 2002](#), [Shiller 2003](#))

- Homogeneous individuals in each generation
- Pooling **aggregate** risks between different generations
- The old suffering a setback receive financial assistance from the young
- The old experiencing a favorable shock need less the young's contribution to the PAYGO pension plan

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Our paper analyzes **intra-generational risk sharing** in the presence of intergenerational links

- Individuals of each generation are either rich or poor

In each generation

- Members receive the same education but have different post-school earning capacities
- Wage differences are shaped by non-educational random factors ([Brown 1980](#), [Bowles, Gintis, and Osborne 2001](#))
- Such random factors matter for each generation, leading to **dynamic earnings heterogeneity**

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To finance their old-age consumption, young adults face two investment options:

- **Personal savings**, which earn a fixed but modest interest
- **Funding public education**, which raises their children's human capital/future earnings/transfer contributions and exhibits diminishing returns

Intra-Family (Ascending) Wealth Transfers

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PAYGO Social Security Scheme

- The old obtain the same pension funded by the young
- Insure parents against the risk tied to their children's future earnings
- Educational investments become as safe as personal savings
- People tend to invest more in children's education

Two welfare gains from PAYGO social security

- Insure against fluctuations in intergenerational transfer, which improves people's ability to **smooth consumption**
- Enhance **human capital development** and increase labor earnings

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PAYGO social security worsens income inequality

- **Gini coefficient** with respect to a generation's lifetime disposable incomes (i.e., wages minus transfer contributions plus retirement benefits)
- Movement along lifetime income ladder is blocked

Model Setup

- An overlapping-generations (OLG) small open economy
- A unit mass of *ex ante* identical individuals
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- A unit mass of *ex ante* identical individuals
- 3 periods: childhood, young adulthood, and old adulthood
- **Children** receive education without making decisions
- **Young adults** bear a child, earn a wage, choose consumption and saving levels, and contribute part of their wages to parents
- **Old adults** retire and live on private saving and the transfer
- Individuals who spend young adulthood are labelled as “generation t ”

Model Setup

- In period $t - 1$, the government provides every child with public education e_{t-1} financed by all parents.
- As children grow up in period t , their human capital accrues to $H(e_{t-1})$, which satisfies $H' > 0$, $H'' < 0$, $\lim_{e \rightarrow 0} H'(e) = \infty$

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- After getting on the job market, they earn a wage:

$$w_t = (1 + \epsilon)H(e_{t-1}),$$

where ϵ is the job market fortune:

$$\epsilon = \begin{cases} \epsilon_h > 0 & \text{with a probability of } p \\ \epsilon_l \in (-1, 0) & \text{with a probability of } 1 - p \end{cases},$$

- Assume that job perspective is uncertainty neutral

$$E(\epsilon) = p\epsilon_h + (1 - p)\epsilon_l = 0$$

- Each generation consists of a fraction p of h -type (lucky) members and a fraction $1 - p$ of l -type (unlucky) members
- **Social welfare** is

$$U_t = pU_{ht} + (1 - p)U_{lt}$$

- The lifetime utility of an i -type worker of generation t is

$$U_{it} = u(c_{it}) + v(d_{it}),$$

where c_{it} and d_{it} are young-age and old-age consumption levels for $i \in \{h, l\}$

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$$c_{it} = (1 - \delta)(1 + \epsilon_i)H(e_{t-1}) - s_{it} - e_t,$$

$$d_{it} = (1 + r)s_{it} + f_t$$

δ is the fraction of earnings transferred to the last generation

r is the (exogenous) interest rate

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- Intergenerational transfer f_t depends on the institution

$$f_t = \begin{cases} \delta(1 + \epsilon)H(e_t) & \text{No Social Security} \\ \delta E[(1 + \epsilon)H(e_t)] = \delta H(e_t) & \text{With Social Security} \end{cases}$$

Consider an individual receives old-age support from her own child

(Temporarily) assume the fraction of earnings transferred δ is exogenous

Comparing two investment options:

- Personal savings, with a risk-free return rate r
- Educational investments, with a risky return rate $\delta(1 + \epsilon)H'(\cdot)$

Proposition

Without the social security scheme, the educational expenditure per child changes over time, and satisfies

$$\frac{1+r}{\delta} < H'(e_t^n) < \frac{1+r}{\delta(1+\epsilon_l)}$$

- Government's educational spending is time-dependent (e_t^n is related to e_{t-1}^n)
- e_t^n will converge toward a constant level in the long run.

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Definition

The steady state is characterized by the time-invariant educational expenditure per child (i.e., $e_t = e$ for all t)

Each old adult receives the average transfer contributed by the young adults

Proposition

Under the social security scheme, the optimal educational spending per child is determined by

$$\forall t, \quad H'(e_t^k) = \frac{1+r}{\delta}$$

- e_t^k is independent of the job market uncertainty (ϵ).
- e_t^k does not depend on consumption taste (u' and v')
- e_t^k keeps constant over time (i.e., $e_t^k = e^k$ for all t)

Given the following specific functional forms

$$u(c_t) = \ln c_t$$

$$v(d_t) = \alpha \ln d_t$$

$$H(e_t) = \beta \ln e_t$$

Proposition

Under the social security scheme, an i -type individual's optimal saving amounts to

$$\forall t, s_{it}^k = \frac{\beta}{1 + \alpha} \left[\alpha(1 - \delta)(1 + \epsilon_i) - \frac{\delta}{1 + r} \right] \ln \frac{\beta\delta}{1 + r} - \frac{\alpha\beta\delta}{(1 + \alpha)(1 + r)}$$

which strictly increases with α , β , and ϵ_i .

A Comparative Analysis: With and Without Social Security

To measure the risk of generation t 's returns from investing in their children, we rely on **the coefficient of variation** of the transfer f_t

Proposition

Educational investment incurs a risk of $\sqrt{-\epsilon_l \epsilon_h}$ without social security but incurs no risk under the social security scheme.

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The introduction of social security tends to reward people for building children's human capital:

Proposition

Social security raises the educational spending per child ($e^k > e^n$).

Empirical support ([Bellettini and Ceroni 2000](#), [Lee and Chang 2006](#))

A Comparative Analysis: With and Without Social Security

According to Baland and Robinson (2000), an efficient educational investment satisfies

$$H'(e_t^*) = 1 + r$$

Proposition

Both ways of intergenerational transfer leads to an inefficient educational investment, but the implementation of social security improves efficiency, (i.e., $e_t^n < e_t^k < e_t^$).*

- When old-age support is provided by the next generation, educational investments remain inefficient no matter whether or not social security is present. This result holds even if the rate of return to saving is negative.

Impact of the PAYGO Social Security on Social Welfare

Proposition

(i) Given the educational spending, social security improves social welfare, i.e., $U^k(e_t) > U^n(e_t)$

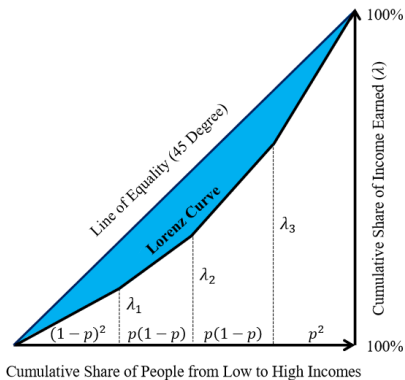
(ii) Social security improves social welfare in equilibrium, i.e., $\max\{U_t^k\} > \max\{U_t^n\}$

- Social security eliminates the uncertainty in an individual's old-age consumption level
- An extra benefit is to stimulate educational investment and hence enhance human capital development in equilibrium

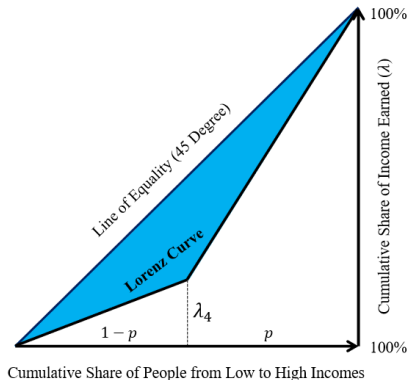
A Comparative Analysis: With and Without Social Security

- To gauge the degree of inequality in the income distribution, we rely on the Gini coefficient (G) based on the distribution of members' lifetime disposable incomes
- Without social security, there are **four types** of workers
 - ① A fraction p^2 of rich parent with a rich child
 - ② A fraction $p(1 - p)$ of rich parent with a poor child
 - ③ A fraction $p(1 - p)$ of poor parent with a rich child
 - ④ A fraction $(1 - p)^2$ of poor parent with a poor child
- With social security, there are **two types** of workers
 - ① A fraction of p of the rich
 - ② A fraction of $1 - p$ of the poor

(a) Without Social Security Scheme



(b) With Social Security Scheme



$$G^n = 1 - p^2 - (1-p)\lambda_1 - 2p(1-p)\lambda_2 - p\lambda_3$$
$$G^k = 1 - p - \lambda_4$$

Proposition

In the steady state, lifetime incomes are less equal under social security scheme than under intrafamily transfer scheme, i.e., $G^n < G^k$.

- A result running contrary to popular belief: social security propagates income inequality.
- Intra-family transfers give rise to the possibility of social mobility over the life cycle.
- Intragenerational mobility is blocked by social security
- Our result is consistent with [Gokhale, Kotlikoff, Sefton, and Weale's \(2001\)](#) calibration finding that were it not for social security, U.S. income inequality would be lower.

The Optimal Intergenerational Contract

We now endogenize δ characterizing the optimal intergenerational contract that maximizes steady-state welfare, $\delta_g := \arg \max \{U\}$.

Proposition

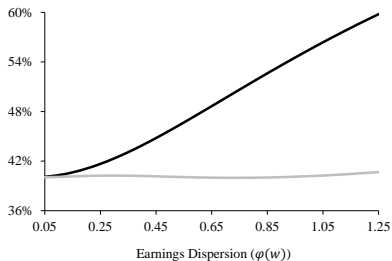
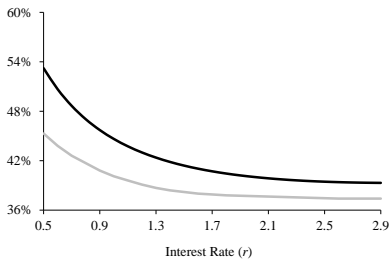
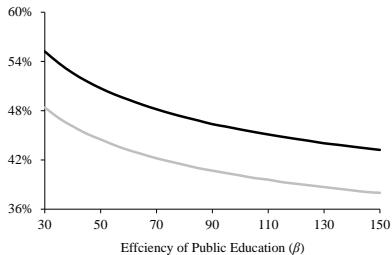
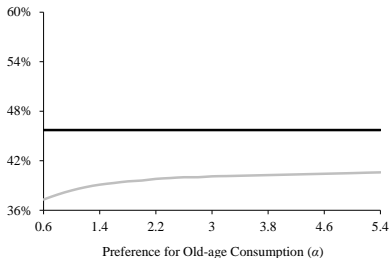
Under the social security scheme, the optimal intergenerational contract, δ_g^k , is characterized by

$$\begin{aligned} & \frac{(r + \epsilon_h + r\epsilon_h)(\delta - 1 + \delta \ln \frac{\beta\delta}{1+r}) + \delta - 1}{[1 + (r + \epsilon_h + r\epsilon_h)(1 - \delta)] \ln \frac{\beta\delta}{1+r} - \delta} \\ = & \frac{\epsilon_h (r + \epsilon_l + r\epsilon_l)(\delta - 1 + \delta \ln \frac{\beta\delta}{1+r}) + \delta - 1}{\epsilon_l [1 + (r + \epsilon_l + r\epsilon_l)(1 - \delta)] \ln \frac{\beta\delta}{1+r} - \delta} \end{aligned}$$

which is not a function of α .

Simulation: the Optimal Intergenerational Contract

- It is hard to find an analytical solution to the optimal intrafamily contract (δ_g^n).
- A numerical exercise is needed
- Parameter Benchmark Values: $\alpha = 3$, $\beta = 100$, $r = 1$, $\epsilon_h = 0.5$, $\epsilon_l = -0.5$
- We compare δ_g^n with δ_g^k in a simple simulation



Social Security (dark curve) vs. No Social Security (gray curve)

- The analysis of risk sharing is central to public economics in general and social protection in particular (Boadway and Keen 2000, Doepke and Tertilt 2016, Fleurbaey and Maniquet 2018).
- We build a model with heterogeneous workers analyzing the role of the social security in sharing **intragenerational** risk on labor market through **intergenerational** links.
- Social security system enables different families to hedge against the risk of children's earnings

Conclusion

- Replacing traditional kinship practices with social security transforms **investments in children** from a risky to safe asset
- Social security leads to an increase in **children's human capital** and their earnings, which creates welfare gains.
- Social security **worsens the inequality** of lifetime disposable income in steady state.
- Our simulation exercises suggest that the **optimal transfer payment** to the old is generally higher under social security scheme.