A Model of the Optimal Allocation of Government Expenditures

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Table 1. Government Expenditure by Function (% of Total Expenditure): 2012–2016 Average

	Belgium	Denmark	France	Germany	Greece	Italy	Netherlands	Spain	UK
Social protection	36.3	43.3	43.0	42.8	37.6	41.7	36.5	38.9	37.9
Education	11.6	12.6	9.6	9.6	8.2	7.9	11.8	9.1	11.6
Health	14.2	15.5	14.2	15.9	9.3	14.1	17.8	13.7	17.3
General public services	15.4	13.7	11.5	14.1	18.6	17.3	10.9	14.9	11.4
Economic affairs	12.7	6.4	9.4	7.2	13.7	8.0	8.9	11.3	7.2
Public order & safety	3.3	1.8	2.9	3.5	3.8	3.8	4.2	4.5	4.6
Defence	1.6	2.2	3.1	2.4	4.4	2.4	2.5	2.1	5.0
Environment protection	1.8	0.8	1.8	1.4	2.7	1.8	3.3	1.9	1.8
Recreation, culture & religion	2.3	3.2	2.4	2.2	1.3	1.5	3.2	2.6	1.7
Community amenities	0.7	0.5	2.1	0.9	0.5	1.4	0.9	1.1	1.6

Data source: *Eurostat* (accessed on July 25, 2018). Government expenditures are decomposed into ten broad functions based on the classification of the functions of government (COFOG) used in the European regime of National Accounts (ESA 2010). Figures are calculated as a simple average based on the available data for the most recent five years.

Introduction

- Government spending has a variety of socio-economic objectives. Table 1 lists the government expenditure breakdown by ten functions, and reports their percentage shares of total expenditures in several European countries between 2012 and 2016.

- In each country, the largest share is devoted to social protection, mainly in the form of income-redistribution programs that transfer wealth to the poor.

- Schooling is also an important target of public funds: most of these governments appropriate around 10% of their expenditures to establish and maintain the state education systems.

Introduction

- The other eight functions, including health care, public services, economic affairs (e.g., infrastructure construction and supply of water and electricity), environmental protection, etc., fit generally into the consumption of public goods and services. Public consumption accounts for half of government spending in almost every country.

- To the extent that various categories compete for resources under the fiscal budget constraint, an intriguing question arises as to how government spending can be optimally allocated between them?

The Model Setup

- Consider an overlapping generations economy where individuals live for three periods: childhood, young adulthood, and old adulthood.
- In each country, the largest share is devoted to social protection, mainly in the form of income-redistribution programs that transfer wealth to the poor.
- In childhood, an individual receives a publicly funded education but does not make decisions.
- In young adulthood, each individual earns a wage, chooses her private consumption and saving levels, and rears n children.
- In old adulthood, all of the individuals retire, spend their savings on private consumption without leaving bequests, and consume public goods provided by the government.

The Model Setup

- To highlight the essence of the model, we assume that individuals consume public goods in old adulthood only.
- Individuals who spend the young adulthood in period t are labeled as Generation t.
- A fraction $\theta_t \in (0, 1) of$ generation t become skilled workers (s), while the others serve as unskilled workers (u).
- Preferences of each i-type member where i $\in$(s,u) are given by

$$v_t^i = \ln c_{yt}^i + \alpha \ln c_0^i t + \beta q_t^i + \phi \ln G_t \tag{1}$$

  - where c_{yt}^i and c_{0t}^i denote the private consumption of an i-type member of generation t in her young and old adulthood, q_t^i denotes the quality of her children, G_t represents her public consumption in old adulthood.

The Model Setup

- The social welfare is defined as the following weighted sum:

$$V_t = \theta_t v_t^s + (1 - \theta_t) v_t^u \tag{2}$$

The Model Setup

- The quality of an i-type members child is measured by the probability that she will grow up to be a skilled worker, which depends on her family background (indicated by i) and the governments educational expenditure per child, E_t:

$$q_t^i = \begin{cases} \ln(1+\sigma E_t) & \text{if } i = u \\ \rho \ln(1+\sigma E_t) & \text{if i=s} \end{cases},$$

- where $q_t^i \in (0, 1)$. *The parameter* $\rho > 1$ measures the degree of elitism of public education. Holding the educational expenditure constant, a child whose parent is skilled is more likely to grow up to be skilled (i.e., $q^s > q^u$).

The Model Setup

- If N_t denotes the population of generation t and n denotes the fertility rate, the population of generation $t+1$ (i.e., the number of children in period t) amounts to

$$N_{t+1} = nN_t \tag{3}$$

The Model Setup

- Given that there are $\theta_t N_t$ skilled workers and $(1-\theta_t)N_t$ unskilled workers in generation t, they will bear $q_t^s(n\theta_t N_t)$ and $q_t^u[(n(1-\theta_t)N_t)]$ skilled children, respectively.

- With the total number of skilled workers in the next generation being $q_t^s n\theta_t N_t + q_t^u(n(1-\theta_t)N_t)$,the proportion of skilled workers in generation t + 1 can be written as

$$\theta_{t+1} = \frac{q_t^s n\theta_t N_t + q_t^u(n(1-\theta_t)N_t)}{N_{t+1}} = [1+(\rho-1)\theta_t]\ln(1+\sigma E_t) \tag{4}$$

which captures the evolution of θt.

The Model Setup

- We proceed to examine the production of the economy to determine the wage of workers. In any period, the economy consists of a skilled-labor and an unskilled-labor sector, both of which competitively produce the unique final good. Production in the unskilled labor sector requires unskilled labor only. Without loss of generality, one unit of unskilled labor can be converted into one unit of output:

$$Y^u = L^u \tag{5}$$

where Y^u and L^u are the output and labor input of the unskilled labor sector, respectively. With the price of a final good being normalized to unity, unskilled workers obtain an equilibrium wage of one in a perfectly competitive market.

The Model Setup

- In the skilled labor sector, the output Y^s is produced by capital K and skilled labor L^s according to a constant-return-to-scale production function F:

$$Y^s = F(K, L^s) = L^s f(K), \tag{6}$$

  where k K=Ls,

- where k denotes the ratio of capital to skilled labor. If both factor markets are competitive, we derive the interest rate r, and the wage rate of skilled labor w as

$$r = f'(k), w = f(k) - kf'(k) \tag{7}$$

The Model Setup

- In a small open economy with free capital mobility, the interest rate equals the worlds (exogenous) interest rate $r = \gamma$. Since k remains fixed at $f'^{-1}(r)$, w is constant and governed by r. In line with empirical findings (e.g., Broecke 2016), we assume that a workers (pre-tax) wage rate is positively related to her skill ($w > 1$). Note that w also represents the wage differential between skilled and unskilled workers, i.e., the ratio of a skilled workers wage to an unskilled workers wage.

The Model Setup

- The government can identify the sector in which each individual works, and impose a lumpsum income tax from workers. Denote T_t^i as the payment of each i-type member of generation t to the government, which is a tax if positive, and a social transfer if negative. Since T^u can be negative, the government can freely redistribute income from the skilled to the unskilled. Accordingly, the government obtains a total revenue of $N_t\theta_t T_t^s + N_t(1-\theta_t)T_t^u$ from generation t.

The Model Setup

- On the expenditure side, the government spends on childrens education amounting to $N_{t+1}E_t$ and the elderlys public consumption.
- We consider two channels of sponsoring public consumption in this study. Section 3 examines public consumption G_t as pre-funded by generation t themselves. In other words, the government appropriates a present value of $N_t G_t/(1+r)$ in period t for public consumption so that each member of generation t consumes G_t in period $t+1$ when she ages.
- Section 4 analyzes an inter-generational social contract, which requires generation $t+1$ to contribute a fixed fraction of their income to support the public consumption of generation t.

No Intergenerational Contract

- This section investigates the case in which every generation care about their own utility but not the previous generations utility. Although a parent regards her childrens educational attainments as a consumption good (equation (1)), a child has no incentive to enhance his parents welfare. In other words, altruism is assumed to be descending only.

- In period t, the interaction between the government and generation t proceeds in as a Stackelberg game. The government moves first to maximize the social welfare of generation t by choosing educational expenditure per child (E_t), public consumption per member of generation t in her old adulthood (G_t) and taxation and/or transfer (T_t^s, T_t^u). After observing government policies, each member of generation t optimally chooses her private consumption (c_{yt}^i,c_{0t}^i) where $i \in$(s,u).

No Intergenerational Contract

- In period t, the lifetime budget constraints of an i-type worker can be expressed by

$$c_{yt}^{i} + \frac{c_{0t}^{i}}{1+r} = I_{t}^{i}, \tag{8}$$

where I_t^i is an i-type members after-tax income net of child-rearing expenses in period t:

$$I_{t}^{i} = \begin{cases} 1 - T_{t}^{u} - \delta n & \text{if } i = u \\ w - T_{t}^{u} - \delta n & \text{if } i = s \end{cases},$$

where δ denotes the cost of rearing a child.

Proposition 1

- In period t, a higher proportion of skilled workers leads to
  (a) greater expenditure on public education per child
  ($\frac{\partial E_t}{\partial \theta_t} > 0$)
  (b) lower expenditure on public consumption ($\frac{\partial G_t}{\partial \theta_t} < 0$) if and only if

$$\frac{\rho - 1}{w - 1}[1 + (\sigma^{-1} - \delta)n] \geq \frac{1 + \alpha + \phi}{\beta} + 1 \qquad (9)$$

No Intergenerational Contract

- Proposition 1(a) suggests that all else being equal, a government tends to spend more on public education as the proportion of skilled workers (i.e., the rich) in the working-age population increases.
- Proposition 1(b) shows that G_t and θ_t have an ambiguous relationship. In particular, public consumption may decrease with the proportion of skilled workers if and only if condition (9) is satisfied.

No Intergenerational Contract

- Although the income effect and fiscal budget effect remain applicable in Proposition 1(b), the substitution (resources competition) between public education and public consumption potentially lowers the level of public consumption. This substitution effect become more prominent when people have a weak preference for public goods (small ϕ) and the children of skilled workers have improved learning efficiency (large ρ). Also, if people make significant gains from childrens quality (large β) but educational investments give low returns for children (small σ), then the government needs to increase its expenditure on public education, which in turn crowds out its expenditure on public consumption.

No Intergenerational Contract

- Since the expenditures on public goods are formulated as a component of the log-linear utility function, one may expect that it always increases with economic development, as measured by the proportion of skilled workers. However, Proposition 1(b) shows a somewhat surprising result that it is not always the case. The intriguing relationship between the expenditures on public goods and economic development depends critically on condition (9)
- In Proposition 1, we discuss a trait of (Et, Gt) in transitional dynamics. Following the earlier studies (Mookherjee and Napel 2007, Cremer, Gahvari, and Pestieau 2011), we define that labor force composition dynamics reach a steady state when the proportion of skilled workers remain unchanging over time:

Definition 1

The labor force composition dynamics is in the steady state if the proportion of skilled workers is constant, i.e., $\theta_t = \theta$ for all t.

Proposition 2

The steady-state proportion of skilled workers, θ, is determined by

$$\frac{\theta}{1+(\rho-1)} = \ln\left\{\frac{\beta[1+(\rho-1)\theta[1-\delta\sigma+\sigma/n+(w-1)\theta\sigma/n]}{1+\alpha+\beta+\phi+\beta(\rho-1)\theta}\right\} \tag{10}$$

The steady state always exists, which is nontrivial and stable.

Then, we develop the following proposition:

Proposition 3

In the steady state, the proportion of skilled workers and the educational spending per child

- (a) increase with the learning efficiency differential ($\frac{d\theta}{d\rho} > 0$ and $\frac{dE}{d\rho} > 0$);
- (b) decrease with the fertility rate ($\frac{d\theta}{dn} < 0$ and $\frac{dE}{dn} < 0$);
- (c) increase with the wage differential ($\frac{d\theta}{dw} > 0$ and $\frac{dE}{dw} > 0$).

No Intergenerational Contract

- Proposition 3(a) is intuitive: holding other things unchanged, if the children of skilled workers have improved learning efficiency (larger ρ), then the government tends to spend more on education (larger E), and children are less likely to become unskilled workers (larger θ).

No Intergenerational Contract

- Proposition 3(b) demonstrates that all else being equal, as people rear fewer children (smaller n), each child receives more educational spending (larger E), which leads to the improved quality of children. This result is in line with Becker and Lewiss (1973) theorem that parents face a trade off between childrens quantity and quality if children are treated as pure consumption goods.
  While Becker and Lewis (1973) and the subsequent research focus on the individual households decision on the number of children, our analysis differs in two aspects. The quality of children depends on the educational expenditure chosen by the government, which aims to maximize the utilitarian welfare on behalf of all parents. Also, we assume that the fertility rate is exogenous for simplicity.
  In other words, Proposition 3(b) presents a normative analysis of the optimal choice of the quality of children in response to different levels of fertility from the perspective of social

Proposition 4

The steady-state expenditure on public consumption per person
(a) decreases with the learning efficiency differential ($\frac{\partial G}{\partial \rho} < 0$) if condition (9) holds;
(b) increases with fertility rate ($\frac{\partial G}{\partial n} > 0$) if condition (9) holds and

$$\delta\sigma < 1 \tag{11}$$

(c) increases with the wage differential ($\frac{\partial G}{\partial w} > 0$) if condition (9) is not satisfied.

No Intergenerational Contract

- Proposition 4 reveals that the public consumption level critically depends on condition (9), which concerns the substitution effect between E and G.
- Proposition 4(a) indicates that the expenditure on public consumption may be inversely related to the learning efficiency of skilled workers children. Specifically, when public consumption and public education are sufficiently substitutable, increased educational spending induced by the improved learning efficiency of skilled workers children crowds out the expenditure on public consumption. Next, we find a positive effect of fertility rate on public consumption under some configurations.

No Intergenerational Contract

- Proposition 4(b) implies that given a lower population growth rate, people tend to sacrifice their old-age consumption of public goods if they have a weak preference for public goods (small ϕ) and a strong preference for childrens quality (large β), but the returns on educational investment are generally low (small σ).
- Proposition 4(c) demonstrates that a higher wage rate of skilled workers may result in a higher level of public consumption per person. This positive relationship can be explained by the income effect and fiscal budget effect, and it is more likely to be true when the substitution effect between E and G is dominated.

We next turn to features of income tax and transfer.

Proposition 5

The net income after taxation and subsidy is equal for skilled and unskilled workers.

- This result follows from our assumption that income taxation does not entail administrative cost or have any distortionary effect on labor supply. Nonetheless, it sheds light on the facts shown in Table 1 that social protection accounts for about 40 percent of all government expenditures in many European countries.
- The following proposition proceeds to investigate the effects of ρ, n, and w on (T^s, T^u):

No Intergenerational Contract

Proposition 6 In the steady state,

- (a) T^s and T^u increase with the learning efficiency differential ρ if condition (9) holds;
- (b) T^s and T^u increase with the fertility rate n if condition (9) and (11) hold;
- (c) T^s increases with the wage differential w if condition (9) holds, and T^u decreases with the wage differential w if condition (9) is not satisfied.

Intergenerational Social Contract

- In this section, we consider the case in which young adults support their parents public consumption through direct wealth transfers.
- Consider an intergenerational contract, under which young adults devote a fraction $\tau \in (0, 1)$ of their income to provide their parents with public consumption. Such a contract can be rationalized even when each generation is selfish, ignoring the welfare of other generations (Rangel 2003).
- If members of one generation do not fulfill their contractual obligation, they will be unable to enjoy public consumption provided by their children when they themselves become old, thereby suffering an infinitely negative utility (equation (1)).

Intergenerational Social Contract

- Ehrlich and Lui (1991) hold that an implicit social contract, in which a parent may invest in her childrens education in exchange for their old-age support, can be sustained by intergenerational reputation.
- In period t, the government and generation t engage in a Stackelberg game similar to that in Section 3. Specifically, the government moves first to maximize generation ts social welfare while providing public education and public consumption by collecting (net) taxes. After observing fiscal policies, each member of generation t, decides her private consumption during young and old adulthood $(ci_{yt},)ci_{ot}$ where $i \in \{s, u\}$.

Intergenerational Social Contract

- The governments balanced budget requires that the government spending on childrens education and the elderlys public consumption are all born by young adults:

$$N_{t+1}E_t+N_{t-1}G_{t-1} = N_t\theta_t T_t^s+N_t(1-\theta_t)T_t^u+\tau[w\theta_t N_t+(1-\theta_t)N_t] \tag{12}$$

- where the intergenerational contract requires that

$$N_{t-1}G_{t-1} = \tau[w\theta_t N_t + (1-\theta_t)N_t] \tag{13}$$

- Combining (12) and (13) shows that the government collects $(T_t^s,\ T_t^u)$ from skilled and unskilled workers to finance the public education of their children, namely,

$$nE_t = \theta_t T_t^s + (1-\theta_t)T_t^u \tag{14}$$

Proposition 7

The optimal educational expenditure per child in period t

- (a) increases with the proportion of skilled workers in generation t ($\frac{\partial E_t}{\partial \theta_t} > 0$);
- (b) increases with the learning efficiency differential ($\frac{\partial E_t}{\partial \rho} > 0$);
- (c) increases with the wage differential ($\frac{\partial E_t}{\partial w} > 0$);
- (d) decreases with the fertility rate ($\frac{\partial E_t}{\partial n} < 0$).

Proposition 7 derives some comparative statics that are qualitatively similar to those in Propositions 1(a) and 3.

Proposition 8

The optimal expenditure on public consumption per member of generation t

- (a) increases with the proportion of skilled workers in generation t ($\frac{\partial G_t}{\partial \theta_t} > 0$);
- (b) increases with the learning efficiency differential ($\frac{\partial G_t}{\partial \rho} > 0$);
- (c) increases with the wage differential ($\frac{\partial G_t}{\partial w} > 0$);
- (d) increases with the fertility rate ($\frac{\partial G_t}{\partial n} > 0$).

Intergenerational Social Contract

- Unlike Proposition 1(b), Proposition 8(a) presents a positive nexus between θ_t and G_t.
- Note that in section 3 one generation financially support their own public consumption, so they face a trade off between public education and public consumption. However, if public goods are supplied by the next generation as analyzed in this section such a trade off tends to fade. With economic development (larger θ_t), future generation is expected to consist of a larger proportion of skilled workers and a smaller proportion of unskilled workers. It is conceivable that as generation $t+1$ earn a higher average income, generation t will receive a larger intergenerational transfer payment from their children, which they can spend on the consumption of public goods.
- We next turn to the steady state, as defined in Definition 1.

Proposition 9

The steady-state proportion of skilled workers is characterized by

$$\frac{\theta}{e^{1+(\rho-1)\theta}}[1+\frac{(1+\alpha)[1+\theta(w-1)]}{[1+(\rho-1)\theta][\beta+(\beta\theta+\phi)(w-1)]}] \\ -\frac{\sigma[1+(w-1)\theta](1-\tau)}{n}=1-\sigma\delta \tag{15}$$

- We proceed to discuss the optimal intergenerational social contract, which maximizes the steady-state social welfare.
- By denoting $\tau^* = \text{argmaxf}\{V\}$, we establish the next proposition to show the existence of τ^*:

Proposition 10

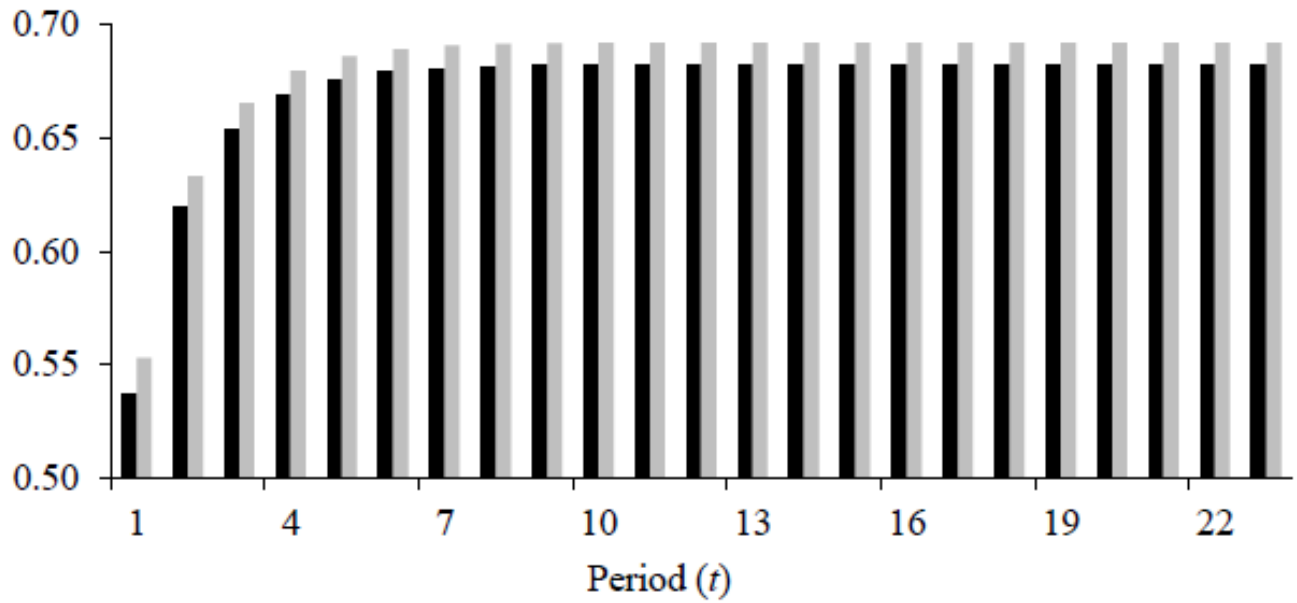
The optimal intergenerational contract τ^* exists.

- The intergenerational social contract essentially reflects a trade between different generations, which can be sustained by a self-fulfilling equilibrium. If people contribute more to their parents for their public consumption, they expect to enjoy more public consumption contributed by their children when they become old; but they also have to cut their private consumption and expenses on educating children.
- The optimal intergenerational contract,τ^*, which falls in the range [0, 1] and maximizes the exante utility of a representative individual in the long run, is characterized by equations (15).

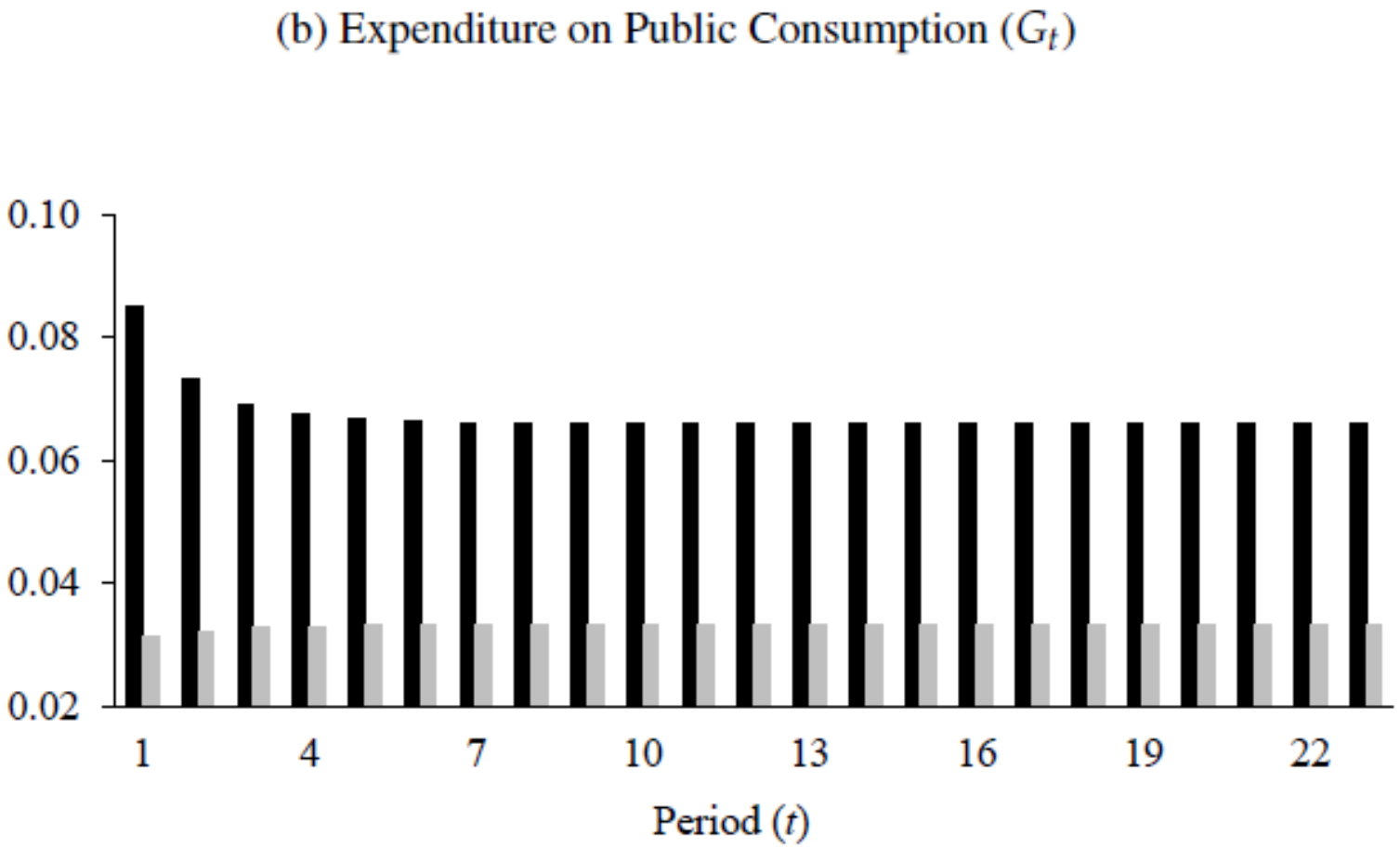
Simulation: A Comparative Analysis

Figure 1: Dynamics of Public Expenditure and Social Welfare ($n = 1.4$ and $w = 3$)

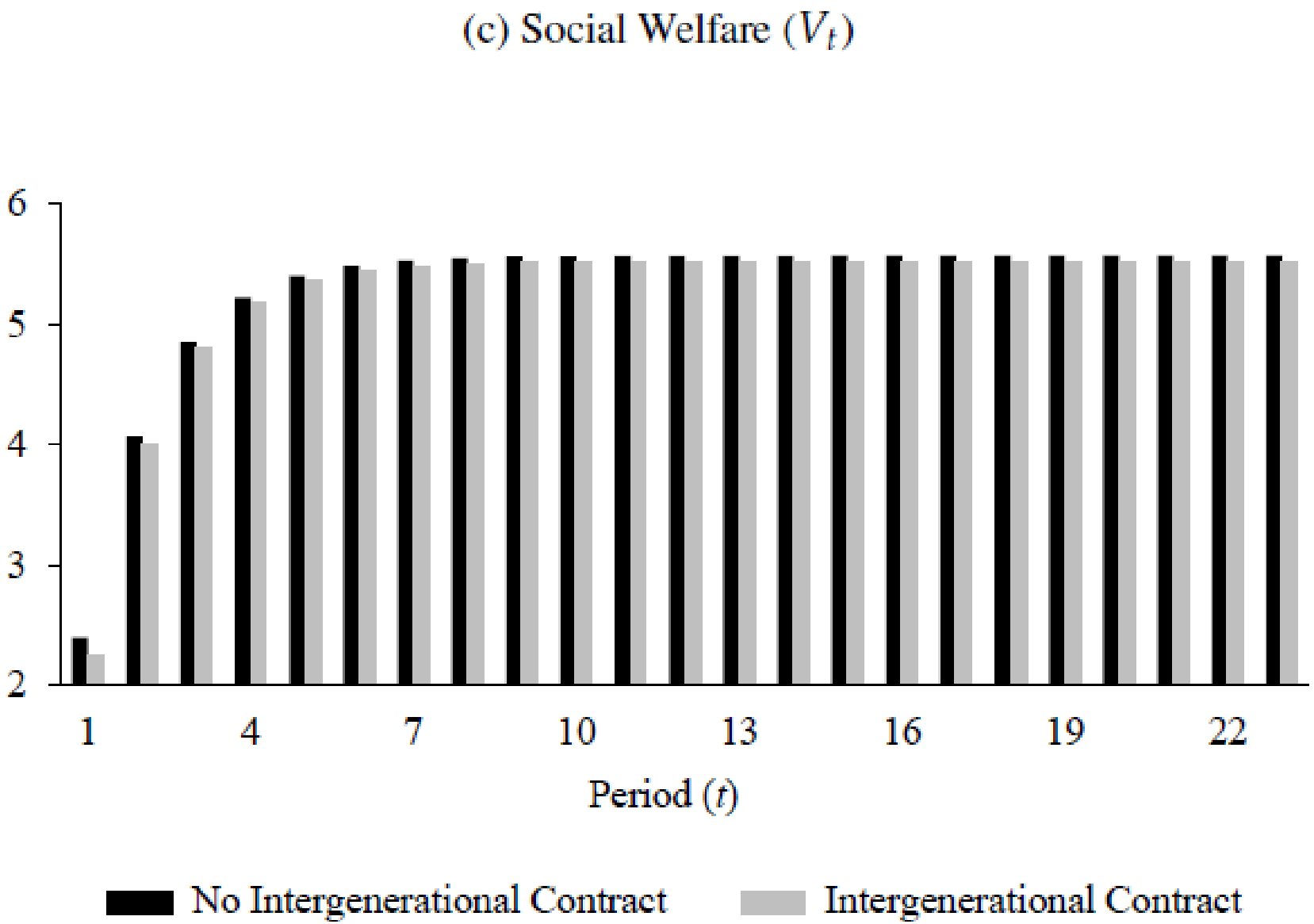
(a) Expenditure on Public Education (E_t)



Simulation: A Comparative Analysis



Simulation: A Comparative Analysis



Conclusion

- From the perspective of public finance, tax revenue and government spending are two sides of the same coin.
- This paper adds to the literature on the optimal allocation of government spending by incorporating individual heterogeneity and intergenerational mobility.
- In our model, a government plans its expenditures according to the objectives of social protection, public education, and consumption of public goods and services, each of which aims to address a different aspect of social welfare.
- The total public revenues, which come from the income taxation, imposes a constraint that necessitates trade-offs between these competing objectives, thereby affecting the composition of government spending.

Conclusion

- We first analyze the case that each generation maximizes their own expected utility without making direct wealth transfers to their parents.
- Our analysis shows that in the steady state the educational expenditure increases with the learning efficiency of skilled workers children and the wage differential, but decreases with the fertility rate.
- Also, under some configurations, expenditure on public consumption is inversely related to the learning efficiency of skilled workerss children but positively related to the wage differential and the fertility rate.

Conclusion

- We then investigate the optimal allocation of government expenditures with intergenerational wealth transfers. If there exists a PAYG program allowing that the young generation devote a certain proportion of their income to support the public consumption of the old generation, our model delivers some different implications. By examining the transitional dynamics of the model, we show that educational expenditure and public consumption tend to increase with economic development.
- We also characterize the optimal intergenerational contract, which maximizes the steady-state social welfare.
- Finally, our simulation exercise illustrates that the introduction of optimal intergenerational contract improves social welfare under some circumstances.