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Model with production

Social welfare and optimal taxes

Growth, inequality, and taxation with risk loving and specialization

A. Araujo^{1,2}, J. Gama¹ and T. Kehoe³

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July 2, 2019

A. Araujo, J. Gama and T. Kehoe (IMPA, FGV-EPGE, U. of Minnesota.) Growth, inequality, and taxation with risk loving July 2, 2019 1 / 38

Model with production

Social welfare and optimal taxes

Motivation

- Different attitudes toward risk and ambiguity can be observed in different environments (for example: undergrad/MBA/PhD students).
- Araujo, Chateauneuf, Gama and Novinski (2018) found conditions for existence for risk lovers in AD economies.
- The analysis of inter-temporal economies becomes a very important task that it is still open.
- Atkinson (2002), Pikketty (2003), Piketty e Saez (2003), Saez and Veall (2003) documented the distribution of income in the last century for the UK, France, US and Canada respectively.

Introduction

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Related literature

- Income or wealth distribution with taxation: Benhabib, Bisin Zhu (2011, 2015, 2016), Jones (2015), and Acemoglu and Robinson (2015).
- Dynamics of income distribution with taxation: Gabaix, Lasry, Lions and Moll (2016) (analitically), and Aoki and Nirei (2017) (numerically).
- Risk aversion, taxation, and growth: García-Peñalosa and Wen (2008).

Introduction

Model with production

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Results

- In a simple overlapping generation model with a continuum of risk lovers and risk averters, no production and bequest:
 - The invariant distribution of the risk averters converge to a Dirac distribution that does not depend on the initial one.
 - The wealth distribution along time converges to the single one invariant distribution of wealth for any initial distribution for the risk lovers.
 - Inequality depends on the bequest rate, and it could be extremely high.

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Model with production

Social welfare and optimal taxes

Results

- With production:
 - Absence of taxes implies that a small group of risk lovers concentrate all the wealth in the long run.
 - Any positive marginal tax rate ensures that the concentration of wealth among the risk lovers converges. Also, the risk lovers concentrate most part of it.
 - We define a social welfare function. We show how it is affected by the tax rate, growth, inequality and the discount factors of the agents and the social planner.
 - Under some conditions, there exists an invariant distribution of wealth, and the optimal growth rate, taxation and inequality will depend on how the social planner discounts the future.

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Model without production

Model with production

Social welfare and optimal taxes

Model without production: Assumptions

Consider a sequential stochastic economy with:

- A countable number of dates t = 0, 1, ...
- In t = 0, there is a single initial state.
- For each state $s := (s_1, ..., s_t)$, there are two possible events that could occur in t + 1, each of them with probability 1/2.
- At each *s*, there is a continuum of risk lovers with measure 1 and a continuum of risk averters with measure 1.

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Model with production 00 00000 Social welfare and optimal taxes

Assumptions

- All agents live two periods.
- There is no consumption in t = 0. Every agent has an initial amount of the assets. There is a consumption good at every state s with date $t \ge 1$.
- At every state s, there are two assets with returns $(\overline{R}, \underline{R})$ and $(\underline{R}, \overline{R})$ with $\overline{R} > \underline{R} > 0$. In this economy there is **no** production.
- In the first period of life, an agent receives a bequest from his predecessor and an endowment $\omega_s^i \ge 0$, and he decides on purchases of assets.

• In the second one, an agent decides consumption and bequest.

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Assumptions

imal taxes

- At each state s, there is a wealth tax τ⁺_s(·) that will be imposed on any agent above some threshold W
 _s, and there is also a wealth subsidy τ⁻_s(·) that will be given for any agent below some threshold W
 _s. Therefore, the tax and the subsidy can be summarized by τ_s(·) = τ⁺_s(·) τ⁻_s(·).
- From now on,
 - τ_s^+ is a tax with an exogenous constant marginal rate $(\tau_s^+ \in [0,1))$ over the wealth above the threshold \overline{W}_s ,
 - τ_s^- is a subsidy with a constant marginal rate (τ_s^-) over the wealth below the benchmark \underline{W}_s . τ_s^- is *endogenously* determined in equilibrium to ensure a balanced government budget.

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Model with production

Social welfare and optimal taxes

Risk averter consumption problem

At a initial state t = 0, a risk averter $a_i \in [0, 1]$ solves the problem

 $\max U^{a_i}(c,b) = (1-\delta)\log c_1 + \delta\log b_1 + (1-\delta)\log c_2 + \delta\log b_2$ s.t.

$$\begin{array}{l} q_{0,1}\theta_1 + q_{0,2}\theta_2 \leq q_{0,1}\hat{\theta}_{0,1}^{a_i} + q_{0,2}\hat{\theta}_{0,2}^{a_i} \\ c_1 + b_1 \leq \sum_{j=1,2}(R_{1,j} + q_{(0,1),j})\theta_j - \tau(\sum_{j=1,2}(R_{1,j} + q_{(0,1),j})\theta_j), \\ c_2 + b_2 \leq \sum_{j=1,2}(R_{2,j} + q_{(0,2),j})\theta_j - \tau(\sum_{j=1,2}(R_{2,j} + q_{(0,2),j})\theta_j), \\ 0 \leq c_1, c_2, b_1, b_2. \end{array}$$

Here $\delta \geq 0$ will be the bequest rate, and $\hat{ heta}_0 \geq 0$ satisfies

$$\int_{i \in [0,1]} \hat{\theta}_{0,j}^{l_i} di + \int_{i \in [0,1]} \hat{\theta}_{0,j}^{a_j} di = 1 \ \forall j = 1, 2.$$

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At s with $t \ge 1$, $a_i \in [0, 1]$ faces the constraints:

$$\begin{array}{l} q_{s,1}\theta_1 + q_{s,2}\theta_2 \leq \omega_s^{a_i} + b_s^{a_i} \\ 0 \leq c_1 + b_1 \leq \sum_{j=1,2} (R_{1,j} + q_{(s,1),j})\theta_j - \tau(\sum_{j=1,2} (R_{1,j} + q_{(s,1),j})\theta_j) \\ 0 \leq c_2 + b_2 \leq \sum_{j=1,2} (R_{2,j} + q_{(s,2),j})\theta_j - \tau(\sum_{j=1,2} (R_{2,j} + q_{(s,2),j})\theta_j) \\ 0 \leq c_1, c_2, b_1, b_2 \end{array}$$

where $\omega_s^{a_i}$ is the endowment and $b_s^{a_i}$ is the bequest received from his predecessor.

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Model without production ○○○○ ○○○○○○○○ Model with production

Social welfare and optimal taxes

Risk lover consumption problem

A risk lover
$$l_i \in [0,1]$$
 at s with $t \ge 1$ solves

$$\max U^{l_i}(c,b) = \min \left\{ \frac{c_1}{1-\delta}, \frac{b_1}{\delta} \right\}^2 + \min \left\{ \frac{c_2}{1-\delta}, \frac{b_2}{\delta} \right\}^2$$
s.t.

$$\begin{array}{l} q_{s,1}\theta_1 + q_{s,2}\theta_2 \leq \omega_s^{l_i} + b_s^{l_i} \\ 0 \leq c_1 + b_1 \leq \sum_{j=1,2} (R_{1,j} + q_{(s,1),j})\theta_j - \tau(\sum_{j=1,2} (R_{1,j} + q_{(s,1),j})\theta_j) \\ 0 \leq c_2 + b_2 \leq \sum_{j=1,2} (R_{2,j} + q_{(s,2),j})\theta_j - \tau(\sum_{j=1,2} (R_{2,j} + q_{(s,2),j})\theta_j) \\ 0 \leq c_1, c_2, b_1, b_2 \end{array}$$

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Model without production

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Social welfare and optimal taxes

Equilibrium

For each state *s*, we require

$$\begin{split} \int_{i} \theta_{s,j}^{i} di &= 1 \; \forall j = 1, 2, \\ \int_{i} c_{s,k}^{i} di &= \overline{R} + \underline{R} + \overline{\omega}_{(s,k)} \; \forall k = 1, 2, \\ \int_{i} \tau_{s} \left(\sum_{j=1,2} \left(R_{1,j} + q_{(s,1),j} \right) \theta_{s,j}^{i} \right) &= 0, \end{split}$$

where

- $(c_s^i, b_s^i, \theta_s^i)$ is the optimal solution for the agent *i* and
- $\overline{\omega}_{(s,k)} = \int_i \omega_{(s,k)}^i di$ is the aggregate endowment in state (s,k).

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Model without production

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Social welfare and optimal taxes

Equilibrium allocation for risk lovers

Each risk lover specializes in one event. Therefore, for t = 0, if $q_{(0,1)} < q_{(0,2)}$, $c_0^{l_i}(1) > 0$ and $c_0^{l_i}(2) = 0, \forall i$,

and analogously for $t \ge 1$ and for $q_{(s,2)} < q_{(s,1)}$.

Taxes do not prevent the agent from having an extreme consumption.

Model without production

Model with production

Social welfare and optimal taxes

Equilibrium allocation for risk lovers

For the case in which $q_{s,1} = q_{s,2}$:

- Each risk lover is indifferent between consuming in event 1 or in event 2.
- There could be a set of agents with positive measure that consumes on the first event and also another set with positive measure that consumes on the second event.
- The existence of such sets with positive measure implies that there is **no uniqueness** of equilibrium.

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Model with production 0 00000 Social welfare and optimal taxes

Invariant distribution

Now, let us suppose that $\omega^{l_i} = \omega^{a_i} = \omega > 0$.

In absence of aggregate risk, $q_{s,1} = q_{s,2}$ which implies that all risk lovers are indifferent between investing in either events.

From now on, we will analyze equilibria in which the risk lovers decisions are independent in case of multiple optimal choices.

Therefore, half of the agents will specialize in each event .

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Model with production 0 00000 Social welfare and optimal taxes

Invariant distribution with no aggregate risk for risk averters without taxes

At t=0, the wealth of the risk averter is $w_0^{a_i}=q_{0,1}\hat{\theta}_{0,1}^{a_i}+q_{0,2}\hat{\theta}_{0,2}^{a_i}$.

Then,
$$w_{(s,1)}^{a_i} = w_{(s,2)}^{a_i} = \sum_{k=0}^t \left(\frac{\delta}{2\pi}\right)^k \omega + \left(\frac{\delta}{2\pi}\right)^t w_0^{a_i}$$
 where
 $\pi = \frac{1}{2} \left(\frac{\left(1 + \frac{\delta}{1-\delta}\right)(2\omega + \overline{R} + \underline{R}) - \delta(\overline{R} + \underline{R})}{\left(1 + \frac{\delta}{1-\delta}\right)(2\omega + \overline{R} + \underline{R})}\right).$

As a consequence, his wealth in the long run converges to $\lim_{t\to\infty} w_{(s,1)}^{a_i} = \frac{1}{1-\frac{\delta}{2\pi}} \omega = \left(1 + \frac{\delta}{1-\delta}\right) \left(\omega + 1/2\left(\overline{R} + \underline{R}\right)\right).$

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Invariant distribution of wealth for risk lovers

For any state s with $t\geq$ 1, the wealth distribution of the risk lovers, $w_s^{l_i},$ is:

$$\sum_{k=0}^{m} \left(rac{\delta}{\pi}
ight)^k \omega$$
 with measure $1/2^{m+1}$ for $m=1,\ldots,t-1$

for an agent who has been successful in the last m periods and failed m+1 periods ago.

$$\sum_{k=0}^{t-2} \left(\frac{\delta}{\pi}\right)^k \omega + \left(\frac{\delta}{\pi}\right)^{t-1} w_0^{l_i} \text{ with measure } 1/2^t$$

for an agent who has been successful in all periods.

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Figure: Invariant distribution of wealth for the risk lovers without taxes and a high bequest rate $(\delta \geq \frac{2\omega}{2\omega + \overline{R} + R})$.

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Figure: Invariant distribution of wealth for the risk lovers without taxes and a low bequest rate $(\delta < \frac{2\omega}{2\omega + \overline{R} + \underline{R}})$.

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Model without production

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Invariant distribution for risk lovers without taxes

Proposition

There is an invariant distribution of wealth for the agents that, with any initial wealth distribution w_0 , the wealth distribution converges to the invariant distribution w_{∞} when t goes to infinity.

In any equilibrium, the aggregate wealth of each group (risk lovers and risk averters) converges to $\left(1 + \frac{\delta}{1-\delta}\right)\left(\omega + \frac{1}{2}(\overline{R} + \underline{R})\right)$.

Therefore, each group has half of the total wealth in the long run.

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Model with production

Social welfare and optimal taxes

Invariant distribution for risk lovers with taxes

Now, for taxes with
$$\overline{W}_s = \underline{W}_s = \left(1 + \frac{\delta}{1-\delta}\right) \left(\omega + \frac{1}{2}(\overline{R} + \underline{R})\right)$$
, we have:

Proposition

There is an invariant distribution for the agents that, with any initial wealth distribution w_0 , the wealth distribution converges to the invariant distribution $w_{\tau,\infty}$ when t goes to infinity.

In any equilibrium, the aggregate wealth of each group (risk lovers and risk averters) converges also to $\left(1 + \frac{\delta}{1-\delta}\right)\left(\omega + \frac{1}{2}(\overline{R} + \underline{R})\right)$.

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Model without production

Model with production

Social welfare and optimal taxes

Model with production: Assumptions

- There is no consumption in t = 0. Every agent has an initial amount of the assets. There is a consumption good at every state s with date $t \ge 1$.
- There are two different technologies, a risky one $(\overline{R}_R, \underline{R}_R)$, and a safe one, $(\overline{R}_S, \underline{R}_S)$ satisfying that

$$\overline{R}_R > \overline{R}_S \geq \underline{R}_S > \underline{R}_R.$$

• An agent who invests in the risky technology will receive \overline{R}_R if the good event occurs, and \underline{R}_R if the event bad occurs.

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Model with production

Social welfare and optimal taxes

Assumptions

- In the first period of life, an agent receives a bequest from his predecessor and an endowment $\omega_s^i \ge 0$, and he decides on purchases of assets.
- In the second period of life, an agent decides on consumption and the bequest that he leaves to his descendant.

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Model with production

Social welfare and optimal taxes

Taxation and growth rate

 au_s can be rewritten as $au_s(x) = au\left(rac{1}{2}\int_i w_s^i di - x
ight)$ where $au \in [0,1]$.

Any increment in taxes will decrease the growth rate, $g_{\tau,t} = \overline{w}_{\tau,t+1}/\overline{w}_{\tau,t} - 1$, of the economy.

Proposition

There exists $\tau^* \in (0,1)$ such that the growth rate of the economy, $g_{t,\tau}$, is maximum for all $t \ge 1$. Moreover, $g_{t,\cdot}$ is an increasing in $[0,\tau^*]$ and decreasing in $[\tau^*,1]$.

Model without production

Model with production

Social welfare and optimal taxes

Invariant aggregate concentration of wealth

Under general hypotheses, taxes ensure the convergence of the aggregate concentration of wealth between the type of agents to an invariant distribution.

Proposition

For a fixed marginal tax rate $\tau \in [0,1]$ with technology returns such that $\overline{R}_R > \overline{R}_S = R_S = \underline{R}_S > \underline{R}_R \ge 0$ and $\mathbb{E}[R_R] \ge \frac{1}{\delta} \ge \mathbb{E}[R_S]$, $\lim_{t\to\infty} \frac{\overline{w}_t}{\overline{w}_t^a} = \gamma_{\tau}$ where $\gamma_{\tau} \in [1,\infty]$.

Note that $\gamma_0 = \infty$ and $\gamma_1 = 1$. Moreover, we have that γ_t is a strictly decreasing C^1 function such that $\frac{d}{d\tau}\gamma_\tau < 0$ for all $\tau \in [0,1)$.

Model without production

Model with production ○ ○ ○ ○ Social welfare and optimal taxes

Invariant aggregate concentration of wealth

Corollary

For any fixed tax rate $\tau \in [0,1]$, the growth rate of the economy, $g_{\tau,t}$ converges to g_{τ} when t goes to infinity.

Lemma In absence of taxes, g_t converges to $E[R_R]\delta - 1$.

However, taxes not only reduce the growth rate for any period t, they also reduce the growth rate in the long run.

For taxes with marginal rates bounded away from zero at any date $t, -1 < R_S \delta - 1 < g_{\tau,t} < E[R_R] \delta - 1$ for all $t \ge 1$.

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Model without production

Model with production

Social welfare and optimal taxes

Invariant distribution with taxes

In absence of taxes, there is no invariant distribution for the risky agents since a small proportion of the agents accumulate all the wealth and this proportion tends to zero in the long run.

However, with taxes, it is possible to find an invariant distribution if $\underline{R}_R = 0$.

Given a fixed marginal tax rate $\tau \in (0,1]$, for any initial distribution of endowment $(w_0^i)_i$, there is an invariant distribution of wealth among the agents. Moreover, the invariant distribution is unique.

The invariant distribution has the form showed in slides 18 and 19.

Note that invariant distribution changes if marginal tax rate changes too. A. Araujo, J. Gama and T. Kehoe (IMPA, FGV-EPGE, U. of Minnesota.) Growth, inequality, and taxation with risk loving July 2, 2019 27 / 38

Model without production

Model with production

Social welfare and optimal taxes

Define an economy:
$$\mathbb{E}[R_R] = 2.43$$
 with $\underline{R}_R = 0$,
 $\underline{R}_S = \overline{R}_S = \mathbb{E}[R_S] = 1.6$ and $\delta = 0.5$



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Introduction	Model without production	Model with production	Social welfare and optimal taxes
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Economy	90th-95th	95th-99th	99th-100th
United States	.113	.231	.347
au = 0.15	.038	.142	.519
au = 0.175	.044	.155	.458
au = 0.2	.048	.166	.403
$\tau = 0.225$.052	.175	.352

Table: Percentiles of the top tail

US data obtained from Rodriguez et al. (2002).

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Model without production

Model with production

Social welfare and optimal taxes •••• ••••••

Social welfare and optimal tax rate

To do so, let us consider a social welfare function

$$W\left(\left(U^{i}\right)_{i},\left(c_{\tau,t}^{i}\right)_{i},\left(b_{\tau,t}^{i}\right)_{i}\right) := \sum_{t=1}^{\infty} \hat{\delta}^{t} \int \log U^{i}\left(c_{\tau,t}^{i},b_{\tau,t}^{i}\right) di$$

with a discount rate given by $\hat{\delta} \in (0,1)$ and $((c_{\tau,t}^i)_i, (b_{\tau,t}^i)_i)$ is the equilibrium for a fixed marginal tax rate $\tau \in [0,1]$. The idea is to find a marginal tax rate that maximizes the social welfare function mentioned above.

We can interpret the optimal tax as the solution of a social planner taxation problem with a discount rate $\hat{\delta}$.

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From now on, we will consider that the initial allocation for an economy with a marginal tax rate $\tau \in [0,1]$ is $(x_{\tau}^i)_i$ where x_{τ} denotes the invariant concentration of wealth.

The equilibrium allocation is given by $\left(\left(\left(1-\delta\right)x_{\tau}^{i}(1+g_{\tau})^{t}\right)_{t}\right)_{i},\left(\left(\delta x_{\tau}^{i}(1+g_{\tau})^{t}\right)_{t}\right)_{i}\right)$ where the first component is the consumption of the agent *i* in each date of the tree, and the second component is the bequest of the agent *i* in each date of the tree.

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Social welfare and optimal tax rate Therefore, the welfare function can be rewritten as

$$W\left(\left(U^{i}\right)_{i},\left(c_{\tau,t}^{i}\right)_{i},\left(b_{\tau,t}^{i}\right)_{i}\right) = \frac{\hat{\delta}}{1-\hat{\delta}} \int \log\left(\mathbf{x}_{\tau}^{i}\right) d\mathbf{i} + \sum_{t=1}^{\infty} 2\hat{\delta}^{t} t \log\left(1+g_{\tau}\right) + \frac{2\hat{\delta}}{1-\hat{\delta}} \log\left(\delta^{\delta}(1-\delta)^{1-\delta}\right).$$

Proposition

The social welfare function, W, in the equilibrium allocation can be written as $W\left(\left(U^{i}\right)_{i},\left(c_{\tau,t}^{i}\right)_{i},\left(b_{\tau,t}^{i}\right)_{i}\right) = X\left(\hat{\delta},\tau\right) + G\left(\hat{\delta},\tau\right) + D\left(\hat{\delta},\delta\right)$ where

1.
$$X(\hat{\delta}, \cdot)$$
 depends on inequality,
2. $G(\hat{\delta}, \cdot)$ depends on growth, and
3. $D(\hat{\delta}, \cdot)$ depends on the bequest rate of the agents.

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Example

Define an economy with $\underline{R}_R = 0$, $\overline{R}_R = 4.9$, $\underline{R}_S = \overline{R}_S = 1.5$ and $\delta = 0.5$



Figure: Social welfare function with $\hat{\delta} = \delta$ and $\hat{\delta} = (1 + \delta)/2$ for different marginal tax rates τ .

The critical tax for $\hat{\delta} = \delta$ is 68%, and for $\hat{\delta} = \frac{1+\delta}{2}$ is 47.9%. If $\delta = \frac{1+\delta}{2}$ is 47.9% is $\frac{1+\delta}{2}$ is $\frac{1+\delta}{2}$ is $\frac{1+\delta}{2}$. If $\delta = \frac{1+\delta}{2}$ is $\frac{1+\delta}{2}$ is $\frac{1+\delta}{2}$. If $\delta = \frac{1+\delta}{2}$ is $\frac{1+\delta}{2}$ is $\frac{1+\delta}{2}$. If $\delta = \frac{1+\delta}{2}$ is $\frac{1+\delta}{2}$ is $\frac{1+\delta}{2}$. If $\delta = \frac{1+\delta}{2}$ is $\frac{1+\delta}{2}$ is $\frac{1+\delta}{2}$. If $\delta = \frac{1+\delta}{2}$ is $\frac{1+\delta}{2}$ is $\frac{1+\delta}{2}$. If $\delta = \frac{1+\delta}{2}$ is $\frac{1+\delta}{2}$ is $\frac{1+\delta}{2}$. If $\delta = \frac{1+\delta}{2}$ is $\frac{1+\delta}{2}$ is $\frac{1+\delta}{2}$. If $\delta = \frac{1+\delta}{2}$ is $\frac{1+\delta}{2}$ is $\frac{1+\delta}{2}$. If $\delta = \frac{1+\delta}{2}$ is $\frac{1+\delta}{2}$ is $\frac{1+\delta}{2}$. If $\delta = \frac{1+\delta}{2}$ is $\frac{1+\delta}{2}$ is $\frac{1+\delta}{2}$. If $\delta = \frac{1+\delta}{2}$ is $\frac{1+\delta}{2}$ is $\frac{1+\delta}{2}$. If $\delta = \frac{1+\delta}{2}$ is $\frac{1+\delta}{2}$ is $\frac{1+\delta}{2}$. If $\delta = \frac{1+\delta}{2}$ is $\frac{1+\delta}{2}$ is $\frac{1+\delta}{2}$.



Figure: Social welfare function with $\hat{\delta} = (2 + \delta)/3$ for different marginal tax rates τ .

The critical tax for $\hat{\delta} = \frac{2+\delta}{3}$ is around 34,6%. Conjecture For every $\hat{\delta} \in (0,1)$, there is only one marginal tax rate, $\tau_{\hat{\delta}}$, that maximizes the social welfare function. A. Araujo, J. Gama and T. Kehoe (IMPA, FGV-EPGE, U. of Minnesota.) Growth, inequality, and taxation with risk loving July 2, 2019 34 / 38

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Remark

The optimal tax rate increases when the social planner becomes more impatient.

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Remark The optimal tax rate decreases when the the spread of the risky technology increases.

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Thank you!