Priority and Egalitarian Allocation in the Capability Approach

Inkee Jang* Biung-Ghi Ju†

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Abstract

Individuals have the different capabilities of transforming resources into basic human functionings. Our priority principles roughly say that the more disabled a person is, the greater the access to resources she should be provided. Extending Moreno-Ternero and Roemer (2006, "Impartiality, priority, and solidarity in the theory of justice," Econometrica 74, 1419-1427) and Chun, Jang, Ju (2014, "Priority, Solidarity, and Egalitarianism," Social Choice and Welfare, 43, 577-589) to a multidimensional setting, we provide a characterization of egalitarian allocation rules using our priority axioms and other standard axioms in the literature of fair allocation. Our egalitarian allocation rules choose allocations where all persons achieve the same level of capability index (the index function aggregate resources and basic human functionings into a real number representing the level of capability). Among these rules are resource-index and output-index egalitarian rules. We characterize resource-index egalitarian rules and output-index egalitarian rules. We also characterize HDI egalitarian rule, which is the central example in the family of output-index egalitarian rules. Keywords: priority, egalitarianism, capability approach, solidarity.

[Preliminary and Incomplete. Please do not circulate.]

*Department of Public Finance, School of Economics, Xiamen University, Xiamen, Fujian, China, 361005; inkeejang@xmu.edu.cn

†Department of Economics, Seoul National University, Gwanak-ro 1, Gwanak-gu, Seoul, South Korea, 151-746; bgju@snu.ac.kr

1 Introduction

TBC

2 Model

Consider a society with a finite number of sectors and a finite number of agents. The society allocates resources to the agents, and individual outputs are interpersonally comparable. Each individual, after they receive resources from the society, decides how much to assign into multiple sectors, and then produces multiple-dimensional outputs. Assume that the same kinds of outputs are interpersonally comparable, but outputs from different types are not comparable.

Let $N = \{1, 2, \cdots, n\}$ $(n \geq 3)$ be a set of agents, $M = \{1, 2, \cdots, m\}$ $(m \geq 2)$ be the set of sectors, and $W \in \mathbb{R}_+$ be the total available resource. Each agent $i \in N$ has an output function in each sector t, denoted by $y_{ti} \colon \mathbb{R}_+ \to \mathbb{R}_+$, which transforms the amount of resources used in sector t, x_{ti}, into the sectoral output $y_{ti}(x_{ti})$. Let $x_i \equiv (x_{ti})_{t \in M}$ and $y_i(x_i) \equiv (y_{ti}(x_{ti}))_{t \in M}$. We call y_i agent i's capability or profile of her output functions transforming resources into the level of achievement in each sector. Given any w_i of total resources, she divides w_i into sectoral resources according to her division rule $\gamma_i \colon \mathbb{R}_+ \to \mathbb{R}^m_+$. Let $\gamma_i \equiv (\gamma_{1i}, \cdots, \gamma_{mi})$. We assume that each sectoral output function $y_{ti} : \mathbb{R}_+ \to \mathbb{R}_+$ is continuous, strictly increasing, $y_{ti}(0) = 0$, and $\lim_{a_t \to \infty} y_{ti}(a_t) = \bar{y}$ for some $\bar{y} > 1$ for each $t \in M$. We also assume that her division rule $\gamma_i : \mathbb{R}_+ \to \mathbb{R}^m_+$ is continuous and satisfies

(i) (efficiency) $\gamma_i(x_i) > 0$[1] for any $x_i > 0$ and $\sum_{t \in M} \gamma_{ti}(x_i) = x_i$,

[1] For any vectors $x = (x_1, \cdots, x_n)$ and $x' = (x'_1, \cdots, x'_n)$, we denote $x' > x$ if $x'_k > x_k$

(ii) (monotonicity) $\gamma_i(x_i) > \gamma_i(x'_i)$ for any $x_i > x'_i$,[2]

(iii) (sector unboundedness) $\gamma_{ti}(x_i) \to \infty$ as $x_i \to \infty$ for each t.[3]

By *monotonicity*, for any $t, s \in M$, and $i \in N$, $y_{ti}(\gamma_{ti}(F_i(e))) = 0$ if and only if $y_{si}(\gamma_{si}(F_i(e))) = 0$ if and only if $W = 0$. Moreover, by *sector unboundedness*, for any $t, s \in M$, and $i \in N$, $y_{ti}(\gamma_{ti}(F_i(e))) \to \bar{y}$ if and only if $y_{si}(\gamma_{si}(F_i(e))) \to \bar{y}$ if and only if $W \to \infty$. That is, $y_i(\gamma_i(F_i(e))) \in (0, \bar{y}) \cup \{(0, \cdots, 0), (\bar{y}, \cdots, \bar{y})\}$. Denote $\bar{Y} = (0, \bar{y}) \cup \{(0, \cdots, 0), (\bar{y}, \cdots, \bar{y})\}$.

Let Γ be the set of all such division functions and $\mathcal{Y}^*$ be the set of all such output functions. An economy $e = (\gamma, y, W)$ consists of a profile of agents' division rules $\gamma \equiv (\gamma_i) \in \Gamma^n$, a profile of agents' capabilities $y \equiv (y_{1i}, \cdots, y_{mi})_{i \in N} \in (\mathcal{Y}^*)^{mn}$, and the available resource $W \in \mathbb{R}_+$. For notational simplicity, we denote $\gamma(F(e)) \equiv (\gamma_i(F_i(e)))_{i \in N}$. Let $\mathcal{E}^* \equiv \Gamma^n \times (\mathcal{Y}^*)^{mn} \times \mathbb{R}_+$ be the set of all economies, the *universal domain.* Domain $\mathcal{E}$ is a *covering domain* so that the graphs of output functions in $\mathcal{Y}$ cover the positive quadrant, that is, for all $a, b \in \mathbb{R}_{++}$, there is $y_{ti} \in \mathcal{Y}$ such that $y_{ti}(a) = b$. (We suppress * in $\mathcal{E}^*$ and $\mathcal{Y}^*$ when there is no confusion.)

An *allocation rule* $F\colon \mathcal{E} \to \mathbb{R}^n_+$ associates with each economy $e = (\gamma, y, W) \in \mathcal{E}$ an allocation of individual resources, $F(e) = (F_i(e))_{i \in N}$ satisfying the *resource constraint*, $\sum_{i \in N} F_i(e) = W$. Each sectoral output is produced by the same kind of resources. For example, $y_{1i}(\gamma_{1i}(F_i(e)))$ is agent i's first output with her share of the first resources. According to the resource constraint, we can consider that each individual, after receiving its individual resource, has its own division plan that the social rule cannot control so that the rule

for all $k = 1, \cdots, n$, and $x' \geq x$ if $x'_k \geq x_k$ for all $k = 1, \cdots, n$.

[2]If an individual gets more resource from the society, she assigns more resource into all sectors.

[3]Each individual's decision plan for both sectors are unbounded.

takes the division function profile into consideration when determining the distribution.

3 Axioms

3.1 Priority axioms

No-Domination. For all $e = (\gamma, y, W) \in \mathcal{E}$, there is no pair $i, j \in N$ such that $(\gamma_i(F_i(e)), y_i(\gamma_i(F_i(e))) \leq (\gamma_j(F_j(e)), y_j(\gamma_j(F_j(e))))$ and $(\gamma_{ti}(F_i(e)), y_{ti}(\gamma_{ti}(F_i(e)))) < (\gamma_{tj}(F_{tj}(e)), y_{tj}(\gamma_{tj}(F_j(e))))$ for some $t \in M$.

For all $i, j \in N$ and all $y_i, y_j \in \mathcal{Y}^n$, we say y_i is ***disabled relative to*** y_j if $y_i(x) \leq y_j(x)$ for all $x \in \mathbb{R}^m_+$ to denote as $y_i \leq y_j$, and y_i is ***strictly disabled relative to*** y_j if $y_i(x) < y_j(x)$ for all $x \in \mathbb{R}^m_+$ to denote as $y_i < y_j$.

No-Reversal (in Outputs). For all $e = (\gamma, y, W) \in \mathcal{E}$ and all $i, j \in N$, if $y_i \leq y_j$, then it cannot be that $y_i(\gamma_i(F_i(e))) \geq y_j(\gamma_j(F_j(e)))$ with $y_{ti}(\gamma_{ti}(F_i(e))) > y_{tj}(\gamma_{tj}(F_j(e)))$ for some $t \in M$.

Note that each of the two axioms implies *equal treatment of equals*; for all $e = (\gamma, y, W) \in \mathcal{E}$, if $(\gamma_i, y_i) = (\gamma_j, y_j)$, then $F_i(e) = F_j(e)$. Also notice that *no-domination* implies *no-reversal*.

Disability Monotonicity. For all $(\gamma, y, W) \in \mathcal{E}$, all $i \in N$, and all $y_i, y'_i \in \mathcal{Y}^m$, if $y'_i \leq y_i$, then $F_i(\gamma, (y'_i, y_{-i}), W) \geq F_i(\gamma, y, W)$.

3.2 Solidarity Axioms

Agreement. For all $e = (\gamma, y, W), e' = (\gamma', y', W') \in \mathcal{E}$, and all $N' \subseteq N$, if $(\gamma_{N'}, y_{N'}) = (\gamma'_{N'}, y'_{N'})$, then either $F_{N'}(e) = F_{N'}(e')$, $F_{N'}(e) > F_{N'}(e')$, or $F_{N'}(e) < F_{N'}(e')$.

Separability. For all $e = (\gamma, y, W) \in \mathcal{E}, e' = (\gamma', y', W') \in \mathcal{E}$, and all $N' \subseteq N$ such that $(\gamma_{N'}, y_{N'}) = (\gamma'_{N'}, y'_{N'})$, if $\sum_{i \in N'}(F_i(e)) = \sum_{i \in N'}(F_i(e'))$, then $F_{N'}(e) = F_{N'}(e')$.

Separability is implied by *agreement*. It says that when the sum of resources remains unchanged for the unaffected agents, their allocation should be the same as before.

Resource Monotonicity. Let $e = (\gamma, y, W), e' = (\gamma, y, W') \in \mathcal{E}$ be such that $W' > W$. Then $F(e') > F(e)$.

Another implication of *agreement* is this axiom. When nothing but the amount of resources changes by a positive or negative shock, all agents should share its effect, that is, the amount of resources in each skill given to all agents should move in the same direction. The following axiom is also induced by *agreement*.

Resource Continuity. For all $(\gamma, y) \in \Gamma^n \times \mathcal{Y}^{mn}$, if a sequence of resources $\{W^n\}_{n \in \mathbb{N}}$ converges to W, then $\{F(\gamma, y, W^n)\}_{n \in \mathbb{N}}$ converges to $F(\gamma, y, W)$.

Evidently, an implication of *resource monotonicity* is *resource continuity.*

4 Main Results

We first show that *agreement* is equivalent to the combination of *separability* and *resource monotonicity.*

Proposition 1. *A rule satisfies agreement if and only if it satisfies separability and resource monotonicity.*

Proof. We skip the evident proof that *agreement* implies *separability* and *resource monotonicity.* To prove the converse, let F be a rule satisfying *separability* and *resource monotonicity.* Let $e = (\gamma, y, W) \in \mathcal{E}$ and $e' = (\gamma', y', W') \in \mathcal{E}$, and $N' \subseteq N$ be such that $y_{N'} = y'_{N'}$. We show that $F_{N'}(e) = F_{N'}(e')$ or $F_{N'}(e) > F_{N'}(e')$ or $F_{N'}(e) < F_{N'}(e')$. Without loss of generality, assume that $\sum_{i \in N'} F_i(e) \geq \sum_{i \in N'} F_i(e')$.

If $\sum_{i \in N'} F_i(e) = \sum_{i \in N'} F_i(e')$, then $F_{N'}(e) = F_{N'}(e')$ by *separability.*

Now consider the case $\sum_{i \in N'} F_i(e) > \sum_{i \in N'} F_i(e')$. By *resource continuity,* there is W^* such that $\sum_{i \in N'} F_i(\gamma, y, W^*) = \sum_{i \in N'} F_i(e')$. Then $W^* > W$ or $W^* < W$. By *resource monotonicity,* $W^* < W$. By *separability,* $F_{N'}(\gamma, y, W^*) = F_{N'}(e')$. By *resource monotonicity,* $F_{N'}(\gamma, y, W^*) < F_{N'}(e)$. Therefore, $F_{N'}(e') < F_{N'}(e)$. □

Proposition 2. *If a rule satisfies no-reversal, disability monotonicity, and agreement, then it satisfies no-domination.*

Proof. Let F be a rule satisfying *agreement*, *no-reversal*, and *disability monotonicity.*

Step 1. For all $e = (\gamma, y, W) \in \mathcal{E}$, $i \in N$, and $y'_i \leq y_i$, $F_i(\gamma, (y'_i, y_{-i}), W) \geq F_i(e)$ and $F_{N\setminus\{i\}}(\gamma, (y'_i, y_{-i}), W) \leq F_{N\setminus\{i\}}(e)$.

Let any $e = (\gamma, y, W), e' = (\gamma, (y'_i, y_{-i}), W) \in \mathcal{E}$ with $y'_i \leq y_i$. By *disability monotonicity* $F_i(\gamma, (y'_i, y_{-i}), W) \geq F_i(e)$. Therefore $\sum_{j\in N\setminus\{i\}} F_j(\gamma, (y'_i, y_{-i}), W) \leq \sum_{j\in N\setminus\{i\}} F_j(e)$. Finally, by *agreement*, $F_{N\setminus\{i\}}(e') \leq F_{N\setminus\{i\}}(e)$.

Step 2. F satisfies *no-domination*.

Suppose conversely that there exists $e = (\gamma, y, W)$ and $i, j \in N$ such that $(\gamma_i(F_i(e)), y_i(\gamma_i(F_i(e)))) \leq (\gamma_j(F_j(e)), y_j(\gamma_j(F_j(e))))$ and $(\gamma_{ti}(F_i(e)), y_{ti}(\gamma_{ti}(F_i(e)))) < (\gamma_{tj}(F_j(e)), y_{tj}(\gamma_{tj}(F_j(e))))$ for some $t \in M$. Let $y'_i \in \mathcal{Y}^n$ such that $y'_{si} \geq \max\{y_{si}, y_{sj}\}$ and $y'_{si}(\gamma_{si}(F_i(e))) \leq y_{sj}(\gamma_{sj}(F_j(e)))$ for each $s \in M$, and $y'_{ti}(\gamma_{ti}(F_i(e))) < y_{tj}(\gamma_{tj}(F_j(e)))$. Notice that both y_i and y_j are disabled relative to y'_i. Let $e' = (\gamma, (y'_i; y_{-i}), W)$. By Step 1, $F_i(e') \leq F_i(e)$ and $F_j(e') \geq F_j(e)$. Then $y'_{si}(\gamma_{si}(F_i(e'))) \leq y'_{si}(\gamma_{si}(F_i(e))) \leq y_{sj}(\gamma_{sj}(F_j(e))) \leq y_{sj}(\gamma_{sj}(F_j(e')))$ for all $s \in M$ and $y'_{ti}(\gamma'_{ti}(F_i(e'))) \leq y'_{ti}(\gamma_{ti}(F_i(e))) < y_{tj}(\gamma_{tj}(F_j(e))) \leq y_{tj}(\gamma'_{tj}(F_j(e')))$. That is, $y'_{si}(\gamma_{si}(F_i(e'))) \leq y_{sj}(\gamma_{sj}(F_j(e')))$ for all $s \in M$ with $y'_{ti}(\gamma'_i(F_i(e'))) < y_{tj}(\gamma'_{tj}(F_j(e')))$, which contradicts *no-reversal* at e'. □

4.1 Index-egalitarianism

We first define a family of rules that satisfy *no-domination* and *agreement*. Let Φ be the class of all continuous functions $\varphi : (\mathbb{R}^m_{++} \times \bar{Y}) \cup \{(0, \cdots, 0)\} \to \mathbb{R}_+$ satisfying $\varphi(0, 0, \cdots, 0) = 0$, and the following monotonicity property: for all $a, a' \in \mathbb{R}_{++} \cup \{(0, \cdots, 0)\}$ and $b, b' \in \bar{Y}$,

- $\varphi(a', b') \geq \varphi(a, b)$ if $(a', b') \geq (a, b)$,
- $\varphi(a', b') > \varphi(a, b)$ if $(a', b') \geq (a, b)$ with $(a'_t, b'_t) > (a_t, b_t)$ for some t.

Given an economy $e = (\gamma, y, W)$, for each $i \in N$, let $\psi_i : \mathbb{R}^m_+ \to \mathbb{R}_+$ be such that for all $a = (a_t)_{t \in M} \in \mathbb{R}_{++} \cup \{(0, \cdots, 0)\}$, $\psi_i(a) = \varphi(a, (y_{ti}(a_t))_{t \in M})$. Since agents have different output functions, each of them faces a different index function. Notice that ψ_i is non-decreasing in each resources because output functions are strictly increasing, and that $\psi_i(\gamma_{1i}(W), \cdots, \gamma_{mi}(W))$ is strictly increasing in W. Also notice that the properties of φ imply continuity and the following monotonicity for ψ_i for all $i \in N$: for $a = (a_t)_{t \in M}, a' = (a'_t)_{t \in M} \in \mathbb{R}_{++} \cup \{(0, \cdots, 0)\}$,

1. $\psi_i(a') \geq \psi_i(a)$ if $a' \geq a$, and
2. $\psi_i(a') > \psi_i(a)$ if $a' \geq a$ with $a'_t > a_t$ for some $t \in M$.

Notice that for each $\gamma_i \in \Gamma$, $(\psi_i \circ \gamma_i)(\cdot) \equiv \psi_i(\gamma_i(\cdot)) : \mathbb{R}_+ \to \mathbb{R}_+$ is a strictly increasing function. We now define the rule that equalizes the φ-value for all the agents.

Index-Egalitarian Rule E^φ(Moreno-Ternero and Roemer (2006)): For all $e \in \mathcal{E}$ and all $i \in N$, $E^\varphi(e) = w$, where w is chosen to equalize φ, that is,

$$\varphi(\gamma_1(w_1), y_1(\gamma_1(w_1))) = \cdots = \varphi(\gamma_n(w_n), y_n(\gamma_n(w_n)))$$

and

$$\sum_{i \in N} w_i = W.$$

Note that for each given $\varphi \in \Phi$, E^φ is well-defined by monotonicity and continuity properties of φ and strictly monotonicity of γ_i for each $i \in N$, and $(*)$.[4]

[4]Suppose conversely that for some $e = (\gamma, y, W) \in \mathcal{E}$ there exist $a = (a_i)_{\{i \in N\}}, a' = (a'_i)_{\{i \in N\}} \in \mathbb{R}^n_+$ such that $a \neq a'$ and $a, a' \in E^\varphi(e)$. Then, there exists $i \in N$ such that

Theorem 1. *A rule satisfies no-domination and agreement if and only if it is index-egalitarian.*

The proof is provided in the appendix.

Theorem 2. *A rule satisfies no-reversal, disability monotonicity, and agreement if and only if it is index-egalitarian.*

Proof. The "only-if" part follows from Theorem 1 and Proposition 2. It suffices to prove that all index-egalitarian rules satisfy *disability monotonicity.* Let $F = E^{\varphi}$ for some $\varphi \in \Phi$, $e = (\gamma, y, W) \in \mathcal{E}$, $i \in N$, and $e' = (\gamma, y', W)$ with $y' \equiv (\hat{y}_i, y_{-i})$ for some $\hat{y}_i \leq y_i$. Then there exist $\lambda, \lambda' \geq 0$ such that for all $j \in N$, $\varphi(\gamma_j(F_j(e)), y_j(\gamma_j(F_j(e)))) = \lambda$ and $\varphi(\gamma_j(F_j(e')), y'_j(\gamma_j(F_j(e')))) = \lambda'$. First, suppose that $\lambda' > \lambda$. Then $\varphi(\gamma_j(F_j(e)), y_j(\gamma_j(F_j(e)))) < \varphi(\gamma_j(F_j(e')), y'_j(\gamma_j(F_j(e'))))$ for all $j \in N \backslash \{i\}$, which implies that $F_j(e) < F_j(e')$. In the case of i, $\varphi(\gamma_i(F_i(e)), y_i(\gamma_i(F_i(e)))) < \varphi(\gamma_i(F_i(e')), \hat{y}_i(\gamma_i(F_i(e')))) \leq \varphi(\gamma_i(F_i(e')), y_i(\gamma_i(F_i(e'))))$, which implies that $F_i(e) < F_i(e')$. Altogether, $W = \sum_{j \in N} F_j(e) < \sum_{j \in N} F_j(e') = W$, which is a contradiction. Therefore, $\lambda' \leq \lambda$. Then $\varphi(\gamma_j(F_j(e)), y_j(\gamma_j(F_j(e)))) \geq \varphi(\gamma_j(F_j(e')), y'_j(\gamma_j(F_j(e'))))$ for all $j \in N \backslash \{i\}$, which implies that $F_j(e) \geq F_j(e')$. And by *the resource constraint,* $F_i(e) \leq F_i(e')$, as required by *disability monotonicity.* □

$a_i \neq a'_i$. Without loss of generality, assume $a'_i > a_i$, which indicates $\gamma_i(a'_i) > \gamma_i(a_i)$ and $y_i(\gamma_i(a'_i)) > y_i(\gamma_i(a_i))$, and therefore $\varphi(\gamma_i(a'_i), y_i(\gamma_i(a'_i))) > \varphi(\gamma_i(a_i), y_i(\gamma_i(a_i)))$. Since both a' and a are φ-value equalizers, $\varphi(\gamma_j(a'_j), y_j(\gamma_j(a'_j))) > \varphi(\gamma_j(a_j), y_j(\gamma_j(a_j)))$ for all $j \in N$, and therefore $\sum_{j \in N}(a'_j) > \sum_{j \in N}(a_j)$, which contradicts the fact that $\sum_{j \in N}(a'_j) = \sum_{j \in N}(a_j) = W$.

4.2 Resource-index egalitarianism and output-index egalitarianism

We define two refinements of the family of index. Let $\mathcal{G}$ be the class of all functions $g : \mathbb{R}^m_{++} \cup \{(0, 0, \cdots , 0)\} \to \mathbb{R}_+$, continuous on its domain and nondecreasing, satisfying $g(0, \cdots , 0) = 0$, and the monotonicity property[5]. For any $a, b \in \mathbb{R}^m_{++} \cup \{(0, 0, \cdots , 0)\}$, we call that an index $\varphi \in \Phi$ is a *resource-index* if $\varphi(a, b) = g(a)$ for some $g \in \mathcal{G}$, and an *output-index* if $\varphi(a, b) = g(b)$ for some $g \in \mathcal{G}$. Let Φ^R be the class of all resource-index and Φ^O be the class of all output-index. A rule F is ***resource-index-egalitarian*** if it is a φ-index egalitarian rule for any $\varphi \in \Phi^R$, and a rule F is ***output-index-egalitarian*** if it is a φ-index egalitarian rule for any $\varphi \in \Phi^O$.

We introduce additional axioms for the sake of the axiomatization of these refinements of index-egalitarianism.

We call a transformation by a continuous and strictly increasing mapping $z_t : \mathbb{R}_+ \to \mathbb{R}_+$ with $z_t(0) = 0$. $z = (z_t)_{t\in M} : \mathbb{R}^m \to \mathbb{R}^m$ is a transformation profile if each z_t is a transformation and $z(a) = (z_t(a_t))_{t\in M}$ for each $a \in \mathbb{R}_+$. For any $(x_{ti})_{t\in M,i\in N} \in \mathbb{R}^{mn}_+$ and $z \in \mathcal{Z}^m$, we let $\iota_z(x) = (z(x_i))_{i\in N}$ for simplicity. Let $\mathcal{Z}$ be the set of transformations and $\mathcal{Z}^m$ be the set of transformation profiles. For any transformation profile $z \in \mathcal{Z}^m$, any division function profile $\gamma = (\gamma_{ti})_{t\in M,i\in N} \in \Gamma^n$, and $(x_{ti})_{t\in M,i\in N} \in \mathbb{R}^{mn}_+$, we denote $(z \circ \gamma_i)(x_i) = (z_t(\gamma_{ti}(x_{ti})))_{t\in M}$ a transformed division function for each agent i and $\iota_z \circ \gamma = ((z \circ \gamma_i)(x_i))_{i\in N}$. Similarly, for any transformation profile $z = (z_t)_{t\in M} \in \mathcal{Z}$ and any productivity function profile $y = (y_{ti})_{t\in M,i\in N} \in \mathcal{Y}^{mn}$, we denote $y_i \circ z^{-1} = (y_{ti} \circ z_t^{-1})_{t\in M}$ a transformed productivity function for each agent i and $y \circ \iota_z^{-1} = (y_i \circ z^{-1})_{i\in N}$. Notice that $z \circ \gamma_i \in \Gamma$ and $y \circ \iota_z^{-1} \in \mathcal{Y}^{mn}$.

[5] For any $x, x' \in \mathbb{R}^m_+$, $g(x') \geq g(x)$ if $x' \geq x$ and $g(x') > g(x)$ if $x' > x$.

Resource Scale Invariance requires solutions to be invariant under any continuous and non-decreasing transformation of problems. It is a strengthening of scale invariance (formally defined in Moulin (2000), or Thomson (2003), for instance,), which requires invariance under any linear transformation. *Resource Scale Invariance* requires that if the total amount of resource is also transformed so that the original allocation (in terms of capacity) can be covered exactly, then we should stick to the original allocation. Therefore, *resource scale invariance* is an invariance requirement which aims at canceling the re-distributive effect of common 'unit' shocks unrelated to the underlying demands.

Resource Scale Invariance. For all $(\gamma, y, W) \in \mathcal{E}$ and $(a_t)_{t\in M} \in \mathbb{R}^M_{++}$,

$$F(\gamma, y, W) = F((a_t \cdot \gamma_{ti})_{t\in M, i\in N}, (y_{ti} \circ (\iota/a_t))_{t\in M, i\in N}, W).$$

($\iota : \mathbb{R}_+ \to \mathbb{R}_+$ is the identity function.)

Output Scale Invariance requires that if everyone's productivity profile is transformed consistently, and then the allocation should be the same as before.

Output Scale Invariance. For all $(\gamma, y, W) \in \mathcal{E}$ and $(a_t)_{t\in M} \in \mathbb{R}^M_{++}$,

$$F(\gamma, y, W) = F(\gamma, (a_t \cdot y_{ti})_{t\in M, i\in N}, W).$$

Theorem 3. *A rule satisfies no-domination, agreement, and resource scale invariance if and only if it is output-index-egalitarian.*

Proof. Let any *output-index-egalitarian* rule $F = E^\varphi$ for some $\varphi \in \Phi^O$. Then there exists $g \in \mathcal{G}$ such that $\varphi(a, b) = g(b)$ for each $a, b \in \mathbb{R}^m_+$. We first show that F satisfies all the three axioms. Since it is *index-egalitarian*, it satisfies *no-domination* and *agreement* by Theorem 1. Let any $(a_t)_{t\in M} \in \mathbb{R}^M_{++}$, $e = (\gamma, y, W) \in \mathcal{E}$, and $e^a = ((a_t \cdot \gamma_{ti})_{t\in M, i\in N}, (y_{ti} \circ (\iota/a_t))_{t\in M, i\in N}, W) \in \mathcal{E}$. We need to show that $F(e^a) = F(e)$. Let $x = (x_{ti})_{t\in M, i\in N} = \gamma(F(e)) \in \mathbb{R}^{mn}_+$. Since $F = E^\varphi$, for each $i, j \in N$, $\varphi(x_i, y_i(x_i)) = \varphi(x_j, y_j(x_j))$, that is, $g(y_i(x_i)) = g(y_j(x_j))$ for some $g \in \mathcal{G}$. Notice that $\varphi((a_t x_{ti})_{t\in M}, (y_{ti}(\iota/a_t(a_t x_{ti})))_{t\in M}) = \varphi((a_t x_{ti})_{t\in M}, y_i(x_i)) = g(y_i(x_i))$ for each $i \in N$. That is, an allocation $w = (w_i)_{i\in N}$ satisfying $(a_t \gamma_{ti}(w_i))_{t\in M, i\in N} = (a_t x_{ti})_{t\in M, i\in N}$ satisfying $\sum_{i\in N} w_i = W$ equalizes the φ-value in e^a. Since the rule E^φ is uniquely defined, $F(e^a) = w$. Finally, from $a_t \gamma_{ti}(w_i) = a_t x_{ti} = a_t \gamma_{ti}(F_i(e))$ for each $t \in M, i \in N$, $w = F(e)$, which implies $F(e^a) = F(e)$.

Let any F that satisfies *no-domination*, *agreement*, and *resource scale invariance*. We show that F is *output-index-egalitarian*. By Theorem 1, $F = E^\varphi$ for some $\varphi \in \Phi$. Let any $(a_t)_{t\in M} \in \mathbb{R}^M_{++}$, $e = (\gamma, y, W) \in \mathcal{E}$, and $e^a = ((a_t \gamma_{ti})_{t\in M, i\in N}, (y_{ti} \circ (\iota/a_t))_{t\in M, i\in N}, W) \in \mathcal{E}$. By *resource scale invariance*, $F(e) = F(e^a)$, that is, in economy e^a, allocation $F(e)$ equalizes the φ-value. To be specific, for each $i, j \in N$ in e^a, $\varphi((a_t \gamma_{ti}(F_i(e)))_{t\in M}, (y_{ti}(\iota/a_t(a_t \gamma_{ti}(F_i(e)))))_{t\in M}) = \varphi((a_t \gamma_{tj}(F_j(e)))_{t\in M}, (y_{tj}(\iota/a_t(a_t \gamma_{tj}(F_j(e)))))_{t\in M})$, that is, $\varphi((a_t \gamma_{ti}(F_i(e)))_{t\in M}, y_i(\gamma_i(F_i(e)))) = \varphi((a_t \gamma_{tj}(F_j(e)))_{t\in M}, y_j(\gamma_j(F_j(e))))$. Since $(a_t)_{t\in M} \in \mathbb{R}^M_{++}$ and $e = (\gamma, y, W) \in \mathcal{E}$ are arbitrary, for each γ and y, $\varphi((\gamma_{ti}(F_i(e)))_{t\in M}, y_i(\gamma_i(F_i(e)))) = \varphi((\gamma_{tj}(F_j(e)))_{t\in M}, y_j(\gamma_j(F_j(e))))$ implies $\varphi((a_t \gamma_{ti}(F_i(e)))_{t\in M}, y_i(\gamma_i(F_i(e)))) = \varphi((a_t \gamma_{tj}(F_j(e)))_{t\in M}, y_j(\gamma_j(F_j(e))))$ for any $(a_t)_{t\in M} \in \mathbb{R}^M_{++}$, which in turn implies $\varphi \in \Phi^O$, that is, F is *output-index-egalitarian*. □

Theorem 4. *A rule satisfies no-domination, agreement, and output scale invariance if and only if it is resource-index-egalitarian.*

Proof. Let any *resource-index-egalitarian* rule $F = E^{\varphi}$ for some $\varphi \in \Phi^R$. Then there exists $g \in \mathcal{G}$ such that $\varphi(a,b) = g(a)$ for each $a, b \in \mathbb{R}^m_+$. We first show that F satisfies all the three axioms. Since it is *index-egalitarian*, it satisfies *no-domination* and *agreement* by Theorem 1. Let any $(a_t)_{t\in M} \in \mathbb{R}^M_{++}$, $e = (\gamma, y, W) \in \mathcal{E}$, and $e^a = (\gamma, (a_t y_{ti})_{t\in M, i\in N}, W) \in \mathcal{E}$. Since $F = E^{\varphi}$, for each $i, j \in N$, $\varphi(\gamma_i(F_i(e)), y_i(\gamma_i(F_i(e)))) = \varphi(\gamma_j(F_j(e)), y_j(\gamma_j(F_j(e))))$, that is, $g(\gamma_i(F_i(e))) = g(\gamma_j(F_j(e)))$ for some $g \in \mathcal{G}$. From the fact that $\varphi(\gamma_i(F_i(e^a)), (a_t y_{ti}(\gamma_{ti}(F_i(e^a))))_{t\in M}) = g(\gamma_i(F_i(e^a)))$ for each $i \in N$, $F(e)$ equalizes the φ-value in e^a. Since the rule E^{φ} is uniquely defined, $F(e^a) = F(e)$, which indicates that F satisfies *output scale invariance.*

Let any F that satisfies *no-domination*, *agreement*, and *output scale invariance.* We show that F is *resource-index-egalitarian.* By Theorem 1, $F = E^{\varphi}$ for some $\varphi \in \Phi$. Let any $(a_t)_{t\in M} \in \mathbb{R}^M_{++}$, $e = (\gamma, y, W) \in \mathcal{E}$, and $e^a = (\gamma, (a_t y_{ti})_{t\in M, i\in N}, W) \in \mathcal{E}$. By *output scale invariance*, $F(e^a) = F(e)$, that is, in economy e^a, allocation $F(e)$ equalizes the φ-value. To be specific, for each $i, j \in N$ in e^a, $\varphi(\gamma_i(F_i(e)), (a_t y_{ti}(\gamma_{ti}(F_i(e))))_{t\in M}) = \varphi(\gamma_j(F_j(e)), (a_t y_{tj}(\gamma_{tj}(F_j(e))))_{t\in M})$. Since $(a_t)_{t\in M} \in \mathbb{R}^M_{++}$ is arbitrary, for each γ and y, $\varphi(\gamma_i(F_i(e)), y_i(\gamma_i(F_i(e)))) = \varphi(\gamma_j(F_j(e)), y_j(\gamma_j(F_j(e))))$ implies $\varphi(\gamma_i(F_i(e)), (a_t y_{ti}(\gamma_{ti}(F_i(e))))_{t\in M}) = \varphi(\gamma_j(F_j(e)), (a_t y_{tj}(\gamma_{tj}(F_j(e))))_{t\in M})$, which in turn implies $\varphi \in \Phi^R$, that is, F is *resource-index-egalitarian.* □

4.3 Human development index

We define a refinement of the family of output-index. We call an output-index is *comprehensive-output-index* if it strictly increases in each sector output. That is, $\Phi^{CO} = \{\varphi \in \Phi^O : \exists g \in \mathcal{G} \ s.t. \ \forall a, b \in \mathbb{R}^m \cup \{(0, 0, \cdots, 0)\} \ \varphi(a,b) = g(b)$ and $g(b_t + \epsilon, b_{-t}) > g(b_t)$ for each $t \in M$ and any $\epsilon > 0$ whenever $b >$

$0\}$. A rule F is ***comprehensive-output-index-egalitarian*** if it is a φ-index egalitarian rule for any $\varphi \in \Phi^{CO}$.

Sector Disability Monotonicity. For any $e = (\gamma, y, W) \in \mathcal{E}$ with $W > 0$, $t \in M, i \in N$, and $y'_{ti} \in \mathcal{Y}$, if $y'_{ti} < y_{ti}$ then $F_i(\gamma, ((y'_{ti}, y_{-ti}), y_{-i}), W) > F_i(\gamma, y, W)$.

Corollary 1. *A rule satisfies no-domination, agreement, ordinality, and sector disability monotonicity if and only if it is comprehensive-output-index-egalitarian.*

Proof. Let any *comprehensive-output-index-egalitarian* rule $F = E^{\varphi}$ for some $\varphi \in \Phi^{CO}$. Then there exists $g \in \mathcal{G}$ such that $\varphi(a, b) = g(a)$ for each $a, b \in \mathbb{R}^m_+$ and g strictly increasing in each sector element. We first show that F satisfies all the four axioms. Since it is *output-index-egalitarian*, it satisfies *no-domination*, *agreement,* and *ordinality* by Theorem 3. Let any $e = (\gamma, y, W) \in \mathcal{E}$ with $W > 0$, and let $x = (x_i)_{i \in N} = \gamma(F(e))$. Notice that $x \in \mathbb{R}^{mn}_{++}$ since $W > 0$. Since F is *output-index-egalitarian*, $g(y_i(x_i)) = g(y_j(x_j))$ for each $i, j \in N$. Let any $y'_{ti} < y_{ti}$, $y'_i = ((y'_{ti}, y_{-ti})$, and $x' = \gamma(F(\gamma, (y'_i, y_{-i}), W)$.

Let any F that satisfies *no-domination*, *agreement*, *ordinality*, and *sector disability monotonicity*. We show that F is *comprehensive-output-index-egalitarian*. By Theorem 3, $F = E^{\varphi}$ for some $\varphi \in \Phi^{CO}$. (Omit this part. Trivial.) □

One example of a noteworthy comprehensive-output-index is *Human Development Index (HDI)*: $\varphi^{HDI}(a, b) = \Pi_{t \in M} b_t$. A rule is *HDI-egalitarian* if it is a φ-index egalitarian rule with $\varphi = \varphi^{HDI}$, and denote $E^{\varphi^{HDI}}$. We introduce additional axioms for the sake of the axiomitization of this social rule.

Full Compensation for Extreme Disability (FCED). For any $(\gamma, y, W) \in \mathcal{E}$ where $W < \infty$, and any $i \in N$, $t \in M$,

$$\lim_{\epsilon \to 0} F_i(\gamma, ((y_\epsilon, y_{-t,i}), y_{-i}), W) = W,$$

where $y_\epsilon(a_t) = y_{ti}(\epsilon \cdot a_t)$.

Full compensation for extreme disability says that if y_{ti} gets relatively smaller for any $t \in M$ and $i \in N$, then $F_i(e)$ approaches to W while all the other agents' resources get close to zero.

Capability Separability. For any $y_i, y'_i \in \mathcal{Y}^m$, any $t \in M$, if there are ecomonies $(\gamma, y, W), (\gamma, ((y'_{ti}, y_{-t,i}), y_{-i}), W') \in \mathcal{E}$ such that $F_i(\gamma, y, W) = F_i(\gamma, ((y'_{ti}, y_{-t,i}), y_{-i}), W')$, then

$$F_i(\gamma, ((y_{ti}, y'_{-t,i}), y_{-i}), W) \geq F_i(\gamma, y, W)$$

implies

$$F_i(\gamma, (y'_i, y_{-i}), W') \geq F_i(\gamma, ((y'_{ti}, y_{-t,i}), y_{-i}), W').$$

Capability separability says that if agent i needs to be compensated by a capability change from $y_{-t,i}$ to $y'_{-t,i}$ when he was receiving a same amount of resource, then it should be independent of y_{ti}.

Capability Symmetry. For any $(\gamma, y, W) \in \mathcal{E}$, $i \in N$, and any permutation π,

$$F_i(\gamma, y, W) = F_i((\pi(\gamma_i), \gamma_{-i}), ((\pi(y_i)), y_{-i}), W).$$

Capability symmetry says that all characteristics are equally important.

Theorem 5. *A rule satisfies no-domination, agreement, resource scale invariance, FCED, capability separability, and capability symmetry, and if and only if it is HDI-egalitarian.*

The proof is provided in the appendix.

5 Appendix

5.1 Proof of Theorem 1

Proof. Fix $\tilde{\gamma} = (\tilde{\gamma}_i)_{\{i \in N\}} \in \Gamma$ and $\tilde{y} = (\tilde{y}_i)_{\{i \in N\}} \in \mathcal{Y}^m$. Given a rule F and $\alpha \in \mathbb{R}_+$, let $\mathcal{E}(\alpha)$ be the set of economies in which there exists an agent with $(\tilde{\gamma}, \tilde{y})$ who receives individual resource α, that is,

$$\mathcal{E}(\alpha) = \{e \in \mathcal{E} : \text{ there exists } i \in N \text{ such that } (\gamma_i, y_i) = (\tilde{\gamma}, \tilde{y}) \text{ and } F_i(e) = \alpha\}.$$

Notice that for any $e = (\gamma, y, W) \in \mathcal{E}(\alpha)$, any $j \in N$ with $(\gamma_j, y_j) = (\tilde{\gamma}, \tilde{y})$ receives α by *equal treatment of equals*, implied by *no-domination.*

Lemma 1. *If F satisfies no-domination and resource continuity, then for all $y \in \mathcal{Y}^{mn}$, all $N' \subset N$, and all $\alpha \in \mathbb{R}_+$, there exists $W^* \in \mathbb{R}_+$ such that $\sum_{i \in N'} F_i(\gamma, y, W^*) = \alpha$.*

Proof. Let any $y \in \mathcal{Y}^{mn}$, $N' \subset N$, and $\alpha \geq 0$. If $\alpha = 0$, then W^* satisfies the statement, and we are done. Now assume $\alpha > 0$. Let $x_L \geq 0$ be such that $x_L < \alpha$. Since $\sum_{i \in N}(F_i(\gamma, y, x_L)) = x_L$ and for all $i \in N$ $F_i(\gamma, y, x_L) \geq 0$, $\sum_{i \in N'} F_i(\gamma, y, x_L) < \alpha$.

We next show that there is $x_H \geq 0$ such that $\sum_{i \in N'} F_i(\gamma, y, x_H) > \alpha$. Consider a sequence $(W^n \colon n \in \mathbb{N})$ such that $\lim_{n \to \infty} W^n = \infty$. Since

$\sum_{i\in N} F_i(\gamma, y, W^n) = W^n$ for all n, there exists $j \in N$ such that $(F_j(\gamma, y, W^n) : n \in \mathbb{N})$ is an unbounded sequence, which implies, by *sector unboundedness*, that $(\gamma_{tj}(F_j(\gamma, y, W^n)) : n \in \mathbb{N})$ is unbounded for each $t \in M$.

We show that there is $\bar{n}$ such that $\sum_{i\in N'} F_i(\gamma, y, W^{\bar{n}}) > \alpha$. Suppose conversely that $\sum_{i\in N'} F_i(\gamma, y, W^n) \leq \alpha$ for all $n \in \mathbb{N}$. Then $\gamma_{ti}(F_i(\gamma, y, W^n)) \leq \alpha$ and $y_{ti}(\gamma_{ti}(F_i(\gamma, y, W^n))) \leq y_{ti}(\alpha) < \bar{y}$ for each $i \in N'$ and $t \in M$. Since $(\gamma_{tj}(F_j(\gamma, y, W^n)) : n \in \mathbb{N})$ is unbounded for each t, there exists n such that $\alpha < \gamma_{tj}(F_j(\gamma, y, W^n))$ for all t, and $(y_{1i}(\alpha), \cdots, y_{mi}(\alpha)) < (y_{1j}(\gamma_{1j}(F_j(\gamma, y, W^n))), \cdots, y_{mj}(\gamma_{mj}(F_j(\gamma, y, W^n))))$ for all $i \in N'$. Hence for such n, for all $t \in M, i \in N'$, $\gamma_{ti}(F_i(\gamma, y, W^n)) \leq \alpha < \gamma_{tj}(F_j(\gamma, y, W^n))$ and $y_{ti}(\gamma_{ti}(F_i(\gamma, y, W^n))) \leq y_{ti}(\alpha) < y_{tj}(\gamma_{tj}(F_j(\gamma, y, W^n))) < \bar{y}$, which contradicts *no-domination*.

Now let $x_H \equiv W^{\bar{n}}$. Then $\sum_{i\in N'} F_i(\gamma, y, x_H) > \alpha$. Since $\sum_{i\in N'} F_i(\gamma, y, x_L) < \alpha < \sum_{i\in N'} F_i(\gamma, y, x_H)$, by *resource continuity,* there exists $W^* \in \mathbb{R}_+$ such that $\sum_{i\in N'} F_i(\gamma, y, W^*) = \alpha$. □

Lemma 2. *Assume that F satisfies no-domination and agreement. For all $e \equiv (\gamma, y, W)$ and all three distinct $i, j, k \in N$, there is $e' \equiv (\gamma', y', W')$ such that $(\gamma'_i, y'_i) = (\gamma_i, y_i)$, $(\gamma'_j, y'_j) = (\gamma'_k, y'_k) = (\gamma_j, y_j)$, $F_i(e') = F_i(e)$, and $F_j(e') = F_k(e') = F_j(e)$.*

Proof. Let $e = (\gamma, y, W)$ and i, j, k are distinct. Let $(\gamma', y') \in \Gamma^n \times \mathcal{Y}^{mn}$ be such that $(\gamma'_i, y'_i) = (\gamma_i, y_i)$, $(\gamma'_j, y'_j) = (\gamma'_k, y'_k) = (\gamma_j, y_j)$. By Lemma 1, there is W' such that $F_i(e') + F_j(e') = F_i(e) + F_j(e)$, where $e' \equiv (\gamma', y', W')$. By *separability* (implied by *agreement*), $F_i(e') = F_i(e)$ and $F_j(e') = F_j(e)$. Since $(\gamma'_j, y'_j) = (\gamma'_k, y'_k)$ and by *equal treatment of equals* (implied by *no-domination*), $F_k(e') = F_j(e') = F_j(e)$. □

Lemma 3. *If F satisfies no-domination and agreement, then for any $\alpha \in \mathbb{R}_+$ and any $e = (\gamma, y, W), e' = (\gamma', y', W') \in \mathcal{E}(\alpha)$, there is no pair $i, j \in N$ such that $(\gamma_i(F_i(e)), y_i(\gamma_i(F_i(e)))) \leq (\gamma'_j(F_j(e')), y'_j(\gamma'_j(F_j(e'))))$ and $(\gamma_{ti}(F_i(e)), y_{ti}(\gamma_{ti}(F_i(e)))) < (\gamma'_{tj}(F_j(e')), y'_{tj}(\gamma'_{tj}(F_j(e'))))$ for some $t \in M$.*

Proof. Suppose conversely that there exist $\alpha \geq 0, i, j \in N$, and $e = (\gamma, y, W), e' = (\gamma', y', W') \in \mathcal{E}(\alpha)$ such that $(\gamma_i(F_i(e)), y_i(\gamma_i(F_i(e)))) \leq (\gamma'_j(F_j(e')), y'_j(\gamma'_j(F_j(e'))))$ and $(\gamma_{ti}(F_i(e)), y_{ti}(\gamma_{1i}(F_i(e)))) < (\gamma'_{tj}(F_j(e')), y'_{tj}(\gamma'_{tj}(F_j(e'))))$ for some $t \in M$. By Lemma 2, we may let $(\gamma_1, y_1) = (\gamma'_1, y'_1) = (\tilde{\gamma}, \tilde{y})$ and assume that $1, i, j$ are three distinct agents. Note that $F_1(e) = F_1(e') = \alpha$. Let $\hat{\gamma}$ and $\hat{y}$ be such that $(\hat{\gamma}_{\{1,i,j\}}, \hat{y}_{\{1,i,j\}}) = (\gamma'_{\{1,i,j\}}, y'_{\{1,i,j\}})$ and $(\hat{\gamma}_{N\setminus\{1,i,j\}}, \hat{y}_{N\setminus\{1,i,j\}}) = (\gamma_{N\setminus\{1,i,j\}}, y_{N\setminus\{1,i,j\}})$. By Lemma 1, there is $\hat{W}$ such that $\hat{e} \equiv (\hat{\gamma}, \hat{y}, \hat{W})$ and $F_1(\hat{e}) + F_i(\hat{e}) + F_j(\hat{e}) = F_1(e') + F_i(e') + F_j(e')$. By *separability* (implied by *agreement*), $F_{\{1,i,j\}}(\hat{e}) = F_{\{1,i,j\}}(e')$.

Let γ'' and y'' such that $(\gamma'', y'') = ((\gamma_i, \gamma'_{-i}), (y_i, y'_{-i}))$. By Lemma 1, there is $W'' \geq 0$ such that $e'' \equiv (\gamma'', y'', W'')$ and

$$F_1(e'') + F_i(e'') + F_j(e'') = \alpha + F_i(e) + F_j(e'). \tag{1}$$

Suppose $F_1(e'') > \alpha$. By applying *agreement* to e and e'', we get $F_i(e'') > F_i(e)$. Likewise, by applying *agreement* to $\hat{e}$ and e'', we get $F_j(e'') > F_j(e')$. Altogether, $F_1(e'') + F_i(e'') + F_j(e'') > \alpha + F_i(e) + F_j(e')$, contradicting (1). Therefore $F_1(e'') \leq \alpha$. Similarly, we can show $F_1(e'') \geq \alpha$. Hence $F_1(e'') = \alpha$. Then, by applying *separability* to e'', $F_i(e'') = F_i(e)$ and $F_j(e'') = F_j(e')$. That is, $(\gamma''_i(F_i(e'')), y''_i(\gamma''_i(F_i(e'')))) = (\gamma_i(F_i(e)), y_i(\gamma_i(F_i(e))))$, $(\gamma''_j(F_j(e'')), y''_j(\gamma''_j(F_j(e'')))) = (\gamma'_j(F_j(e')), y'_j(\gamma'_j(F_j(e'))))$. That, however, implies $(\gamma''_i(F_i(e'')), y''_i(\gamma''_i(F_i(e'')))) \leq (\gamma''_j(F_j(e'')), y''_j(\gamma''_j(F_j(e''))))$ and $(\gamma''_{ti}(F_i(e'')), y''_{ti}(\gamma''_{ti}(F_i(e'')))) < (\gamma''_{tj}(F_j(e'')), y''_{tj}(\gamma''_{tj}(F_j(e''))))$, which violates *no-domination* at e''. □

Let $C(\alpha)$ be the set of all resource-output pairs for all sectors in all economies in $\mathcal{E}(\alpha)$, that is, $C(\alpha) = \{c \in \mathbb{R}^{2m}_{+} :$ there exists $e \in \mathcal{E}(\alpha)$ such that $c = (\gamma_i(F_i(e)), y_i(\gamma_i(F_i(e))))$ for some $i \in N\}$.

We show that $C(\cdot)$ is downward sloping for each sector.

Lemma 4. *If F satisfies no-domination and agreement, then for any $\alpha \geq 0$, $C(\alpha)$ is downward sloping for each sector, that is, for all $(a, b), (a', b') \in C(\alpha)$ with $a'_{M\setminus t} = a_{M\setminus t}, b'_{M\setminus t} = b_{M\setminus t}, a'_t > a_t$ for some t, we have $b'_t \leq b_t$.*

Proof. Assume that F satisfies *no-domination* and *agreement*. To prove that $C(\alpha)$ has the sector downward sloping property, suppose to the contrary that for some $(a, b), (a', b') \in C(\alpha)$ and $t \in M$, $a'_{M\setminus t} = a_{M\setminus t}, b'_{M\setminus t} = b_{M\setminus t}, (a'_t, b'_t) > (a_t, b_t)$. By definition of $C(\alpha)$, there exist $e = (\gamma, y, W), e' = (\gamma', y', W') \in \mathcal{E}$ such that for some $i, j \in N$, $(\gamma_i(F_i(e)), y_i(\gamma_i(F_i(e)))) = (a, b)$ and $(\gamma'_j(F_j(e')), y'_j(\gamma'_j(F_j(e')))) = (a', b')$. However, it indicates that $(\gamma_i(F_i(e)), y_{ti}(\gamma_{ti}(F_i(e)))) < (\gamma'_j(F_j(e')), y'_{tj}(F_{1j}(e')))$ and $(\gamma_{si}(F_i(e)), y_{si}(\gamma_{si}(F_i(e)))) = (\gamma_{sj}(F_j(e')), y_{sj}(\gamma_{sj}(F_j(e'))))$ for all $s \in M\setminus\{t\}$, which contradicts Lemma 3. $\square$

The next lemma says that $C(\alpha)$ and $C(\alpha')$ for any $\alpha \neq \alpha'$ do not intersect.

Lemma 5. *$\{C(\alpha) : \alpha \in \mathbb{R}_+\}$ is a collection of disjoint sets.*

Proof. Let $\alpha_1 > \alpha_2$ and suppose that $(a, b) \in C(\alpha_1) \cup C(\alpha_2)$. Then there exist $e_1 = (\gamma, y, W_1)$ and $i \in N$ such that $(\gamma_1, y_1) = (\tilde{\gamma}, \tilde{y})$, $F_1(e_1) = \alpha_1$, and $(\gamma_i(F_i(e_1)), y_i(\gamma_i(F_i(e_1)))) = (a, b)$. By Lemma 1, there is W_2 such that $e_2 = (\gamma, y, W_2)$ and $F_1(e_2) = \alpha_2$. By *resource monotonicity*, $F_i(e_2) > F_i(e_1)$, and therefore $(\gamma_i(F_i(e_2)), y_i(\gamma_i(F_i(e_2)))) > (a, b)$. Since $(\gamma_i(F_i(e_2)), y_i(\gamma_i(F_i(e_2)))), (a, b) \in$

$C(\alpha_2)$, $C(\alpha_2)$ is not downward sloping, which violates the result of Lemma 3. $\square$

The next lemma says that, by varying $\alpha \in \mathbb{R}_+$, $C(\alpha)$'s can cover $\mathbb{R}^m_{++} \times (0, \bar{y})^m$.

Lemma 6. *For all* $(a, b) \in \mathbb{R}^m_{++} \times (0, \bar{y})^m \cup \{(0, \cdots, 0)\}$, *there is a unique* $\alpha \geq 0$ *such that* $(a, b) \in C(\alpha)$.

Proof. Let any $(a, b) \in \mathbb{R}^m_{++} \times (0, \bar{y})^m \cup \{(0, \cdots, 0)\}$. Since $\mathcal{Y}$ covers $\mathbb{R}_{++} \times (0, \bar{y})$, there exists $y \in \mathcal{Y}^{mn}$ and $i \in N \backslash 1$ such that $y_1 = \tilde{y}$ and $y_i(a) = b$. Let any $\gamma \in \Gamma^n$ such that $\gamma_i(w_i) = a$ for some $w_i \in \mathbb{R}_+$. By Lemma 1, there exists $W \in \mathbb{R}_+$ such that $F_i(\gamma, y, W) = w_i$. Then we get $(a, b) \in C(F_1(\gamma, y, W)$. Finally, the uniqueness of α is implied by Lemma 5. $\square$

The next lemma says that for each $t \in M$, if $\alpha_1 > \alpha_2$, then $C_t(\alpha_1)$ lies above $C_t(\alpha_2)$, which can be shown using Lemma 1, 4, and 6 as in Chun, Jang, and Ju (2014).

Lemma 7. *If* $\alpha_1 > \alpha_2$, *then*

1. *for all* $(a, b) \in C(\alpha_2)$ *there exists* $(a', b') \in C(\alpha_1)$ *such that* $(a, b) < (a', b')$, *and*

2. *there is no* $(a'', b'') \in C(\alpha_2)$ *and* $(a, b) \in C(\alpha_1)$ *such that* $(a'', b'') > (a, b)$.

Now we construct an 'index' from the above discussion.

Lemma 8. *Define* $\varphi : \mathbb{R}^{2m}_{++} \cup (0, \cdots, 0) \to \mathbb{R}+$ *such that for all* $(a, b) \in \mathbb{R}^{2m}_{++}$*,* $\varphi(a, b) = \alpha$*, where* $\alpha \in \mathbb{R}_+$ *is such that* $(a, b) \in C(\alpha)$*. Then* φ *is well-defined and is continuous,* $\inf\{\varphi(a, b) : (a, b) \in \mathbb{R}^2_{++}\} = \varphi(0, 0) = 0$*, and satisfies the monotonicity property*[6]*.*

Proof. By Lemma 6, there exists a unique $\alpha \in \mathbb{R}_+$ such that $(a, b) \in C(\alpha)$. By *no domination*, $\varphi(0, \cdots, 0) \leq \varphi(c)$ for any $c \in \mathbb{R}^{2m}_{++}$ and $\varphi(0, \cdots, 0) = 0$.

To prove the monotonicity property, let any $a, a', b, b' \in \mathbb{R}^m_+$ such that $(a', b') \geq (a, b)$. We show that $\varphi(a', b') \geq \varphi(a, b)$. If $\varphi(a, b) > \varphi(a', b')$, then $(a, b) \in C(\varphi(a, b))$ and $(a', b') \in C(\varphi(a', b'))$. By Lemma 7, there exists $a'', b'' \in \mathbb{R}^m_+$ such that $(a'', b'') > (a', b')$ and $(a'', b'') \in C(\varphi(a, b))$. However, from the fact that $(a, b), (a'', b'') \in C(\varphi(a, b))$ and $(a'', b'') > (a, b)$, $C(\varphi(a, b))$ is not downward sloping, which contradicts Lemma 4. Now assume that $(a', b') \geq (a, b)$ with $(a'_t, b'_t) > (a_t, b_t)$ for some t. We show that $\varphi(a', b') > \varphi(a, b)$. We know that $\varphi(a', b') \geq \varphi(a, b)$ so that we only need to show $\varphi(a', b') \neq \varphi(a, b)$. Suppose conversely that $\varphi(a', b') = \varphi(a, b) = \alpha$. Then $(a', b'), (a, b) \in C(\alpha)$ so that there exists $(\gamma, y, W) \in \mathcal{E}$ and $i, j \in N \backslash 1$ such that $(\gamma_1, y_1) = (\tilde{\gamma}, \tilde{y})$, $(\gamma_i(F_i(e)), y_i(\gamma_i(F_i(e)))) = (a, b)$, and $(\gamma_j(F_j(e)), y_j(\gamma_j(F_j(e)))) = (a', b')$. However, the fact that $(\gamma_j(F_j(e)), y_j(\gamma_j(F_j(e)))) \geq (\gamma_i(F_i(e)), y_i(\gamma_i(F_i(e))))$ and $(\gamma_{tj}(F_{tj}(e)), y_{tj}(\gamma_{tj}(F_j(e)))) \geq (\gamma_{ti}(F_i(e)), y_{ti}(\gamma_{ti}(F_i(e))))$ contradicts *no domination*.

Finally, the continuity of φ can be shown as in Moreno-Ternero and Roemer (2006). □

[6]For any $a, a', b, b' \in \mathbb{R}^m_+$,

- $\varphi(a', b') \geq \varphi(a, b)$ if $(a', b') \geq (a, b)$,
- $\varphi(a', b') > \varphi(a, b)$ if $(a', b') \geq (a, b)$ with $(a'_t, b'_t) > (a_t, b_t)$ for some t.

Now we are ready to prove Theorem 1.

For all $(a,b) \in \mathbb{R}^m_{++} \times (0,\bar{y})^m \cup (0,\cdots,0)$, let $\varphi(a,b) = \alpha$ where α is such that $(a,b) \in C(\alpha)$. By Lemma 8, $\varphi(\cdot)$ is well-defined and $\varphi \in \Phi$. We show that $F(e) = E^{\varphi}(e)$ for all $e = (\gamma, y, W) \in \mathcal{E}$. If $(\gamma_i, y_i) = (\tilde{\gamma}, \tilde{y})$ for some $i \in N$, then by letting $\lambda = F_i(e)$, for all $j \in N$, we have $(\gamma_j(F_j(e)), y_j(\gamma_j(F_j(e)))) \in C(\lambda)$, that is, $\varphi(\gamma_j(F_j(e)), y_j(\gamma_j(F_j(e)))) = \lambda$. Since $\sum_{j\in N} F_j(e) = W$, $F(e) = E^{\varphi}(e)$.

We now consider when there is no $i \in N$ with $(\gamma_i, y_i) = (\tilde{\gamma}, \tilde{y})$. Consider $\gamma' = (\tilde{\gamma}, \gamma_{N\setminus 1})$ and $y' = (\tilde{y}, y_{N\setminus 1})$. By Lemma 1, there exists W' such that $e' = (\gamma', y', W')$ and $\sum_{i\in N\setminus\{1\}} F_i(e') = \sum_{i\in N\setminus\{1\}} F_i(e)$. As we let $\lambda = F_1(e')$, $(\gamma'_i(F_i(e')), y'_i(\gamma'_i(F_i(e')))) \in C(\lambda)$ so that $\varphi(\gamma'_i(F_i(e')), y'_i(\gamma'_i(F_i(e')))) = \lambda$. Moreover, by *separability* (implied by *agreement*), for all $i \in N\setminus\{1\}$, $F_i(e') = F_i(e)$, and therefore $F_i(e') = F_i(e)$. Then, for all $i \in N\setminus\{1\}$, $\varphi(\gamma_i(F_i(e)), y_i(\gamma_i(F_i(e)))) = \varphi(\gamma'_i(F_i(e')), y'_i(\gamma'_i(F_i(e')))) = \lambda$. Similarly, we can show that $\varphi(\gamma_1(F_1(e)), y_1(\gamma_1(F_1(e)))) = \lambda$. Therefore, $F(e) = E^{\varphi}(e)$. □

5.2 Proof of Theorem 5

Proof. Since HDI egalitarian rule is output-index-egalitarian, it satisfies *no-domination*, *agreement*, and *resource scale invariance* by theorem 3. It is also obvious that the HDI egalitarian rule satisfies the other axioms.

Let an allocation rule F satisfying *no-domination*, *agreement*, *resource scale invariance*, *FCED*, *capability separability*, and *capability symmetry*. By theorem 3, $F = E^{\varphi}$ where $\varphi \in \Phi^O$. Then there exists $g \in \mathcal{G}$ such that $\varphi(a,b) = g(b)$, that is, F is a g-value equalizer.

The following lemma argues that we can consider that $g \in G$ has a normalization property, i.e., $g(\beta \cdot \mathbf{1}_m) = \beta$ for all $\beta \in [0,\bar{y}]$.

Lemma 9. *If F satisfies no-domination, agreement, and resource scale invariance, then there exists $g \in G$ such that $g(\beta \cdot \mathbf{1}_m) = \beta$ for each $\beta \in [0, \bar{y}]$ and for any $e = (\gamma, y, W) \in \mathcal{E}$ and $i, j \in N$,*

$$g(y_i(\gamma_i(F_i(e)))) = g(y_j(\gamma_j(F_j(e)))).$$

Proof. Since $\varphi \in \Phi^O$, there exists $\hat{g} \in \mathcal{G}$ such that $\varphi(a, b) = \hat{g}(b)$, that is, $\hat{g}(y_i(\gamma_i(F_i(e)))) = \hat{g}(y_j(\gamma_j(F_j(e))))$ for all $i, j \in N$. Let $f : [0, \hat{y}] \to [0, \hat{y}]$ be such that for each $\beta \in [0, \hat{y}]$ $f(\hat{g}(\beta \cdot \mathbf{1}_m)) = \beta$. Then f is continuous and strictly increasing, according to the properties of $\hat{g}$. Therefore as we let $g = f \circ \hat{g}$, $\hat{g}(y_i(\gamma_i(F_i(e)))) = \hat{g}(y_j(\gamma_j(F_j(e))))$ if and only if $g(y_i(\gamma_i(F_i(e)))) = g(y_j(\gamma_j(F_j(e))))$. Finally, $g(\beta \cdot \mathbf{1}_m) = f(\hat{g}(\beta \cdot \mathbf{1}_m)) = \beta$. □

Notice that g is cardinal: there is no $g' \in \mathcal{G} \backslash \{g\}$ that satisfies $g'(\beta) = \beta$ for all $\beta \in [0, \bar{y}]$ and

$$g'(y_i(\gamma_i(F_i(e)))) = g'(y_j(\gamma_j(F_j(e))))$$

for any $e = (\gamma, y, W) \in \mathcal{E}$. Also notice that since g is strictly increasing, $0 \leq g \leq \bar{y}$. We first show from the following claims that g satisfies several useful properties.

Claim 1. *(Minimal lower boundedness) For any $b \in (0, \bar{y})^m$ and $t \in M$,*

$$\lim_{\beta \to 0} g(\beta, b_{-t}) \to 0.$$

Proof. Let any $b \in (0, \bar{y})^m$ and $e = (\gamma, y, W) \in \mathcal{E}$ such that $y_i(\gamma_i(F_i(e))) = b$. Since $b \neq (\bar{y}, \cdots, \bar{y})$, $W < \infty$. Denote $y_\epsilon(\cdot) = y_{ti}(\epsilon \cdot)$ and $e^\epsilon = (\gamma, ((y_\epsilon, y_{-t,i}), y_{-i}), W)$ for each $\epsilon \in (0, 1)$. By *FCED*, $\lim_{\epsilon \to 0} F_i(e^\epsilon) = W$, which also implies $\lim_{\epsilon \to 0} F_j(e^\epsilon) = 0$ for any $j \neq i$. Denote $e'^\epsilon = (\gamma, ((y_\epsilon, y_{-t,i}), y_{-i}), W^\epsilon)$ where

W^ϵ is chosen such that $F_i(e) = F_i(e'^\epsilon)$ for each $\epsilon \in (0,1)$. Since F is a g-value equalizer, $g(y_i(\gamma_i(F_i(e^\epsilon)))) = g(y_j(\gamma_j(F_i(e^\epsilon))))$ for each ϵ. Moreover, by *agreement*, since $\epsilon < 1$, $W^\epsilon < W$. Thus

$$\lim_{\beta\to 0} g(\beta, b_{-t}) = \lim_{\epsilon\to 0} g(y_i(\gamma_i(F_i(e'^\epsilon)))) \leq \lim_{\epsilon\to 0} g(y_i(\gamma_i(F_i(e^\epsilon)))) = \lim_{\epsilon\to 0} g(y_j(\gamma_j(F_i(e^\epsilon)))) \to 0,$$

which completes the proof. □

Claim 2. *(Sector Independence) For any $b, b' \in (0, \bar{y})^m$,*

$$g(b) \geq g(b_t, b'_{-t}) \text{ implies } g(b'_t, b_{-t}) \geq g(b').$$

Proof. Let any $b, b' \in (0, \bar{y})^m$. Assume $g(b) \geq g(b_t, b'_{-t})$. Also let any $e = (\gamma, y, W) \in \mathcal{E}$ and $y'_i \in \mathcal{Y}^m$ such that $y_i(\gamma_i(F_i(e))) = b$ and $y'_i(\gamma_i(F_i(e))) = b'$. Notice that, as we denote $\hat{y}_i = (y'_{ti}, y_{-t,i})$ and $\hat{y}'_i = (y_{ti}, y'_{-t,i})$, $\hat{y}_i(\gamma_i(F_i(e))) = (b'_t, b_{-t})$ and $\hat{y}'_i(\gamma_i(F_i(e))) = (b_t, b'_{-t})$.

We argue that $F_i(\gamma, (\hat{y}'_i, y_{-i}), W) \geq F_i(e)$. Suppose conversely that $F_i(\gamma, (\hat{y}'_i, y_{-i}), W) < F_i(e)$. Then there exists $W_2 > W$ such that $F_i(e^2) = F_i(e)$ where $e^2 = (\gamma, (\hat{y}'_i, y_{-i}), W_2)$. Moreover, by *agreement*, $F_j(\gamma, (\hat{y}'_i, y_{-i}), W) > F_j(e)$ for any $j \neq i$ so that there exists $W_1 < W$ such that $e^1 = (\gamma, (\hat{y}'_i, y_{-i}), W_1)$ and $F_j(e^1) = F_j(e)$. Since F is a g-value equalizer,

$$g(\hat{y}'_i(\gamma_i(F_i(e^1)))) = g(y_j(\gamma_j(F_j(e^1)))) = g(y_j(\gamma_j(F_j(e)))) = g(y_i(\gamma_i((F_i(e)))) = g(b).$$

Moreover, by *agreement* and since $W_2 > W_1$, $F_k(e^2) > F_k(e^1)$ for all $k \in N$. However, since $\hat{y}'_i(\gamma_i(F_i(e^2))) = \hat{y}'_i(\gamma_i(F_i(e))) = (b_t, b'_{-t})$,

$$g(\hat{y}'(\gamma_i(F_i(e^2)))) = g(b_t, b'_{-t}) \leq g(b) = g(y_i(\gamma_i(F_i(e^1)))),$$

which violates the fact that g is strictly increasing.

Now, by *capability separability*, by letting $e' = (\gamma, (y'_i, y_{-i}), W')$ and $\hat{e} = (\gamma, (\hat{y}_i, y_{-i}), W')$ with $F_i(\hat{e}) = F_i(e)$, we have $F_i(e') \geq F_i(\hat{e}) = F_i(e)$. Then, by *agreement*, $F_j(e') \leq F_j(\hat{e})$ for each $j \neq i$ and therefore $g(y'_i(\gamma_i(F_i(e')))) = g(y_j(\gamma_j(F_j(e')))) \leq g(y_j(\gamma_j(F_j(\hat{e})))) = g(\hat{y}_i(\gamma_i(F_i(\hat{e})))) = (b'_t, b_{-t})$, considering that F is a g-equalizer. Finally,

$$g(b') = g(y'_i(\gamma_i(F_i(e)))) \leq g(y'_i(\gamma_i(F_i(e')))) \leq g(\hat{y}_i(\gamma_i(F_i(\hat{e})))) = g(b'_t, b_{-t}).$$

□

Claim 3. *(Sector Symmetry) For any permutation π and any $b \in \bar{Y}$,*

$$g(b) = g(\pi(b)).$$

Proof. Let any permutation π and any $b \in \bar{Y}$. It is trivial if $b \in \{(0, \cdots, 0), (\bar{y}, \cdots, \bar{y})\}$. Suppose that $b \in (0, \bar{y})^m$. Let any $e = (\gamma, y, W) \in \mathcal{E}$ such that $y_i(\gamma_i(F_i(e))) = b$. Then $0 < W < \infty$ and $(\pi(y_i))(\pi(\gamma_i)(F_i(e_\pi))) = \pi(b)$ as we denote $e_\pi = ((\pi(\gamma_i), \gamma_{-i}), (\pi(y_i), y_{-i}), W)$. We also have $F_i(e) = F_i(e_\pi)$ by *capability symmetry*, which implies $F(e) = F(e_\pi)$ by *agreement*. Since F is g-value equalizer, $g(y_i(\gamma_i(F_i(e)))) = g((\pi(y_i))(\pi(\gamma_i)(F_i(e_\pi))))$, which completes the proof. □

For any $\gamma \in \Gamma, y \in \mathcal{Y}^{mn}$,

$$F(\gamma, y, W) = (0, \cdots, 0) \text{ if } W = 0 \text{ and } \lim_{W \to \infty} F_i(\gamma, y, W) \to \infty \text{ for all } i \in N,$$

which coincides with the HDI egalitarian rule. For the rest of the proof, we restrict our attention to positive and finite social resources. We prove the rest through an adaptation of the proof used by Herrero et al. (2010).

Let any $b \in (0, \bar{y})^m$ and let any $t \in M$. By *sector independence* and *cardinality*, since b_t is independent of b_{-t}, there exist real valued mappings u and v such that:[7]

$$g(b) = u(b_t) + v(b_t)g(1, b_{-t}). \tag{2}$$

Take any $s \in M \backslash \{t\}$. Since $u(b_t)$ is independent of b_{-t}, *minimal lower boundedness* implies that

$$u(b_t) = \lim_{b_s \to 0} (g(b) - v(b_t)g(1, b_{-t})) = 0$$

for all b_t. Now, plugging in $b = (b_t, \mathbf{1}_{m-1})$ into (2), we get $v(b_t) = g(b_t, \mathbf{1}_{m-1})$, and consequently,

$$g(b) = g(b_t, \mathbf{1}_{m-1}) \cdot g(1, b_{-t}).$$

Define now the function $\hat{g}(b_{-t}) = g(1, b_{-t})$, that satisfies *sector independence* and *minimal lower boundedness*, and apply to $\hat{g}$ the argument above for some sector $s \in M \backslash \{t\}$. That is, by *sector independence* and *cardinality*, there are two real valued mappings $\hat{u}$ and $\hat{v}$ such that

$$\hat{g}(b_{-t}) = \hat{u}(b_s) + \hat{v}(b_s)\hat{g}(1, b_{-ts}). \tag{3}$$

We can show $\hat{u}(\cdot) = 0$ by the same logic above, and by plugging in $b_{-t} = (b_s, \mathbf{1}_{m-2})$ into (3), we get $\hat{v}(b_s) = \hat{g}(b_s, \mathbf{1}_{m-2})$ so that

$$\hat{g}(b_{-t}) = \hat{g}(b_s, \mathbf{1}_{m-2}) \cdot \hat{g}(1, b_{-ts}).$$

Since, by definition, $\hat{g}(b_{-t}) = g(1, b_{-t})$, we have that

$$g(1, b_{-t}) = g(b_s, \mathbf{1}_{m-1}) \cdot g(1, 1, b_{-ts}).$$

[7] See Keeney and Raiffa (1976, ch. 5,6), and Herrero et al. (2010, Theorem 1).

Replacing $g(1, b_{-t})$ in $g(b)$, we have

$$g(b) = g(b_t, \mathbf{1}_{m-1}) \cdot g(b_s, \mathbf{1}_{m-1}) \cdot g(1, 1, b_{-ts}).$$

By repeating this procedure for all sectors, we arrive at:

$$g(b) = g(b_1, \mathbf{1}_{m-1}) \cdot g(b_2, \mathbf{1}_{m-1}) \cdot \ldots \cdot g(b_m, \mathbf{1}_{m-1}).$$

Therefore, for each $t \in M$ if we define $f_t(b_t) = g(b_t, \mathbf{1}_{m-1})$, we get

$$g(b) = \Pi_{t=1}^{m} f_t(b_t),$$

with $f_t(b_t) \in (0, \bar{y})$ for each $b_t \in (0, \bar{y})$. By *sector symmetry*, $f_t(\cdot) = f_s(\cdot) = f(\cdot)$ for all $t, s \in M$. Therefore, we can write:

$$g(b) = \Pi_{t=1}^{m} f(b_t).$$

For each $\beta \in (0, \bar{y})$, $\beta = g(\beta) = [f(\beta)]^m$, thus, $f(\beta) = \beta^{\frac{1}{m}}$. Therefore

$$g(b) = \Pi_{t=1}^{m} b_t^{\frac{1}{m}}.$$

□

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