# Fiscal Capacity with Accumulating Defense Activities *†

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March 25, 2020

**Abstract:** To be able to impose taxes, states need to invest in their fiscal capacities—institutions enabling tax collection. Previous theoretical models cannot explain the time-series evolution of fiscal-capacity investment in many countries. Specifically, their results fail to hold around many states' external-war years. This paper develops a dynamic model to explain the evolution of fiscal-capacity investment over time. I analyze the effects of external-war probability, and defense activities' depreciation rate and price on a state's fiscal-capacity investment. I show that each of these variables have two-sided effects and provide some evidence supporting my model.

**Keywords:** Fiscal capacity, external war, durable public goods, defense activity.

**JEL Classification Numbers:** E62, H11, H20, H41

*This paper is based on my Ph.D. dissertation written under the supervision of Professors Ying Chen and M. Ali Khan at Johns Hopkins University. I am grateful to them for their guidance and support. I would also like to thank to Antonio Savoia, Vincenzo Galasso, İbrahim Ünalmış, Şevket Pamuk, Liuchun Deng, Semih Tümen, Tekin Köse, Zafer Yılmaz and seminar participants at the 2018 Public Economic Theory Conference, the 2018 Midwest Economic Theory Conference, the 3rd International Conference on the Political Economy of Democracy and Dictatorship, TOBB Economy and Technology University, and TED University for stimulating discussion and valuable comments. I also thank Hüsame Doğanay for excellent research assistance.

†This research is supported by TUBITAK Grant No: 1001.

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# Contents

# 1 Introduction

The economics literature has mostly assumed states have the ability to impose taxes at any rate they want. However, collecting taxes requires investments such as building tax administrations and monitoring processes, like the Internal Revenue Service in the US. Indeed, this presumption of the economics literature is valid only for the rich countries of the last century. Today high-income countries have much higher tax-to-GDP ratio than low-income countries. Additionally, for rich countries tax-to-GDP ratio is higher compared to what they had earlier with lower incomes. In rich countries, the increase in tax revenue was accomplished by investments in tax innovations and administrations, such as pay-as-you-earn method and progressive income tax (Besley et al., 2013). This raises the question of how a state's fiscal capacity—that is, ability to collect taxes—is determined.

Historical sociologists, such as Tilly (1990), have long worked on this question, emphasizing the role of external wars—wars with other states. There is also a growing literature in economics, including Besley and Persson (2009, 2011), that investigates states' fiscal capacity. This literature develops theoretical models to investigate the formation of a state's fiscal capacity. A main result of this literature is that a state's fiscal-capacity investment increases with an increase in the probability of an external war. This prediction is supported by cross-country fiscal-capacity differences.[1] However, it contradicts the time-series evolution of fiscal-capacity investment in many countries. To show this, in Figure 1, I present the change in the tax-to-GDP ratio, as a measure of fiscal-capacity investment, and war-mobilization years of some countries.[2] According to the previous literature, we expect to observe two trends in the figure: The change in a country's tax-to-GDP ratio increases as a war period approaches and decreases as a war period ends. But, as can be seen by the fitted values in the figure, the former prediction fails to hold in many situations, such as for Australia and Finland before any of their war periods, for Italy before its second and third war periods, for Canada before its first war period, and for the UK before its second war period. Additionally, the latter prediction fails to hold for almost all countries. Specifically, during the wars lasting more than a year, all countries', except the US's, tax-to-GDP-ratio change has an increasing trend towards the end of a war. Thus, the previous models are inconsistent with the time-series data.[3] In this paper, I develop a dynamic model of fiscal capacity to address this inconsistency.

In particular, I develop a three-period model departing from the fiscal-capacity model of Besley

[1] See, for example, Besley and Persson (2009, 2011).

[2] In the literature, including Besley and Persson (2009, 2011), one of the most common measures of fiscal capacity is the tax-to-GDP ratio. Additionally, according to the previous models, there is investment in fiscal capacity if and only if fiscal capacity is increasing, assuming that it does not depreciate. Thus, to measure fiscal-capacity investment, I use change in the tax-to-GDP ratio.

[3] In the previous models, besides the probability of an external war, another factor that affects a state's fiscal-capacity investment is its income. Thus, one can argue that the change-in-tax-to-GDP-ratio trends that I discussed can be explained by the income trends of the relevant countries. In Appendix A, I show that this argument is not valid.

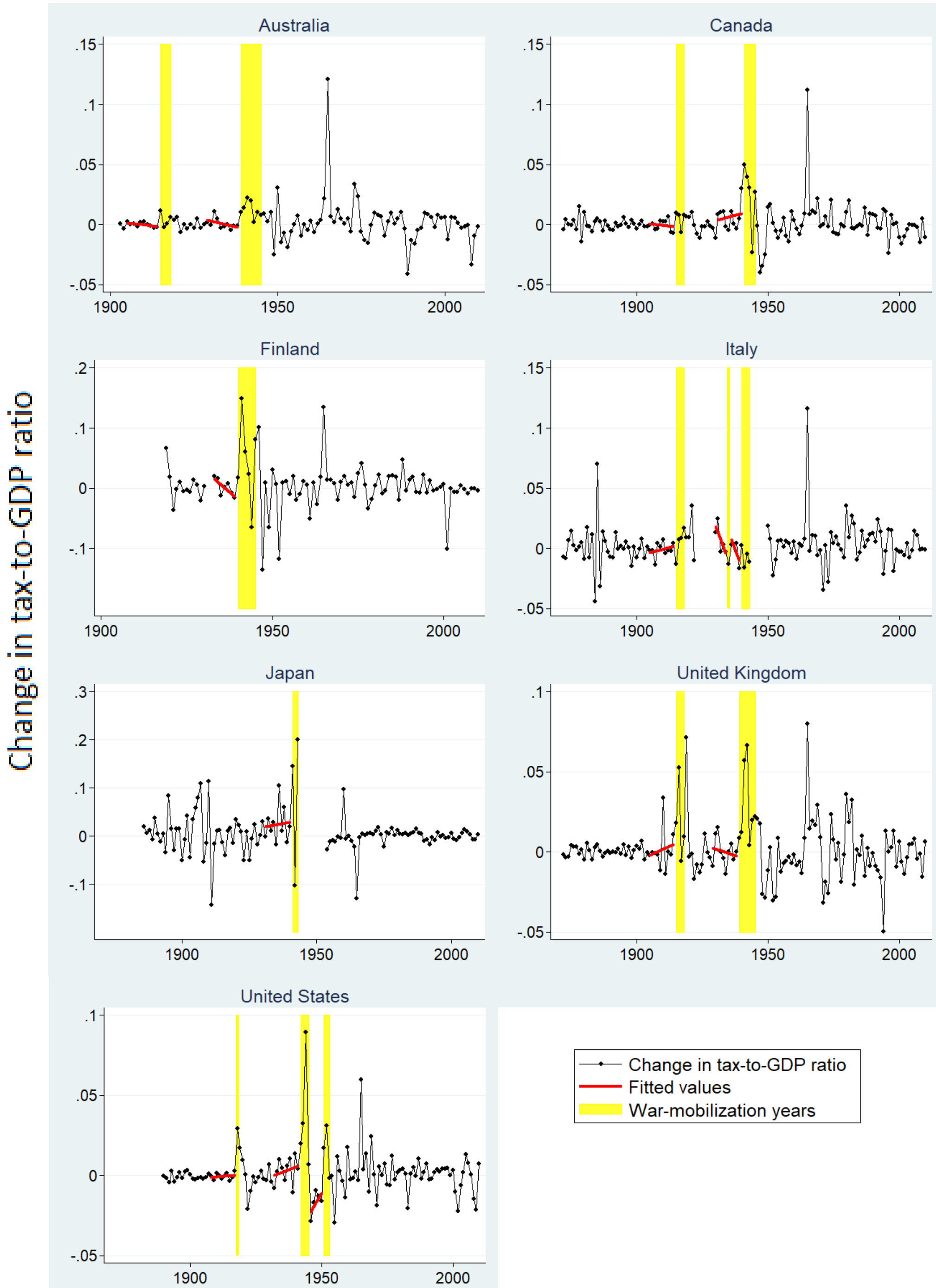


Figure 1: Change in tax-to-GDP ratio and war mobilization. Change in tax-to-GDP ratio in a year is calculated as the difference in that year's tax-to-GDP ratio from the previous year's, as measured by Rogers et al. (2018). Fitted values represent simple regression lines for the change in tax-to-GDP ratio covering ten years before a war-mobilization period, or the years between two war-mobilization periods. However, for Finland they cover seven years before the war-mobilization period, and for Italy they cover five years before its second war-mobilization period due to data availability. A country's war-mobilization period covers the time in which it mobilized a certain amount of its population for a war with other countries as measured by Rogers et al. (2018).

and Persson (2011). In the model, there are two groups. Individuals in both groups take utility from consumption and a public good. As in Besley and Persson (2011), I interpret the public good as defense activities against external threats. The public good can take a high or a low value. The high value represents an external-war period and the low value a peace period. Different from Besley and Persson (2011), however, I let the public good accumulate over time following the durable-public-goods literature. This literature includes, for example, Battaglini and Coate (2007), who actually state that national-defense activities are the most prominent examples of accumulating public goods. In a period, one of the groups is appointed as the political power randomly. I call this group the incumbent, and the other group the opponent. The incumbent decides on the income-tax rate, transfers to each group, and investment in the public good. In her choice of the tax rate, the incumbent is constrained by the period's fiscal capacity, which is chosen by the previous period's incumbent. Specifically, an incumbent can increase the next period's fiscal capacity by making a costly investment.

I show that an incumbent's fiscal-capacity investment is higher than the socially optimal level if and only if political stability is high. Besides, her fiscal-capacity investment increases with income, political stability and lower investment cost. These results match the previous literature's results. However, contrary to the previous literature, I show that an increase in the probability of an external war can decrease the incumbent's fiscal-capacity investment. This result can explain the two trends, a decrease in fiscal-capacity investment as an external war approaches and an increase in it as an external war ends, that the previous models fail to explain. Additionally, I show that a decrease in the depreciation rate of the public good can increase fiscal-capacity investment. This result can also explain an increase in fiscal-capacity investment toward the end of an external war.

My paper mainly contributes to the literature on the theory of fiscal-capacity formation. Evolution of a state's fiscal capacity has long been investigated by political and economic historians (see, for example, Tilly, 1990; Levi, 1988; Brewer, 1989). In recent years, there is also a growing literature in economics that analyzes the determinants of a state's fiscal capacity relying on formal models. Besley and Persson (2009) develop a fiscal-capacity model focusing on the role of external wars and inclusiveness of political institutions. The same authors, in their book Besley and Persson (2011), extend their model in various directions, such as allowing for different preferences and income levels between groups in the society. While these papers develop two-period models, Besley et al. (2013) build a dynamic model of fiscal capacity with an infinite horizon to focus on the long-term determinants of fiscal capacity. However, all these papers conclude that an increase in the probability of an external war increases investment in fiscal capacity which contradicts with the time-series evolution of fiscal-capacity investment in many countries as I showed in Figure 1. I contribute to this literature by developing a dynamic model that can explain the time-series evolution of fiscal-capacity investment. Different from the previous models, in my model, I let the public good, which represents defense activities, accumulate over time.[4]

My paper also connects to the literature on durable public goods. Most of the papers in this literature—for example, Battaglini and Coate (2007) and Barseghyan and Coate (2014)—analyze the provision of durable public goods under various political institutions. Another emerging literature investigates the effects of durable public goods on institutions. For example, Karakas (2017) studies their effects on executive constraints. I contribute to this literature by investigating the effects of durable public goods on fiscal capacity.

The rest of the paper proceeds as follows. Section 2 presents the model. Section 3 analyzes the socially optimal fiscal-capacity investment. Section 4 focuses on the politically determined fiscal-capacity investment. Section 5 extends the model to investigate the effect of the public good's price on politically determined fiscal capacity. Section 6 provides some empirical evidence supporting the model. Finally, Section 7 concludes. The proofs of all propositions are given in Appendix F.

# 2 Model

The society has a unit mass of population divided into two groups with equal sizes. There are three time periods, denoted $s = 1, 2, 3$. In each period $s$, an incumbent group, $I_s$, holds political power and the other group, $O_s$, represents the opposition. The first period's incumbent is given exogenously and holds the political power with a probability $p_I$ in each period $s \in \{2, 3\}$. The period-$s$ incumbent decides on a government policy composed of a tax rate, $t_s \geq 0$, transfer to its own group, $r^{I_s} \geq 0$, transfer to the opposition group, $r^{O_s} \geq 0$, and investment in a public good, $g_s \geq 0$. At any period $s$, the level of the public good is given as

$$G_s = g_s + (1 - d)G_{s-1}$$

where $d \in [0, 1]$ is the deprecation rate and $G_0$ is given exogenously. The previous literature on fiscal capacity, including Besley and Persson (2009, 2011), assumes that the public good depreciates completely in a period—that is, $d = 1$. My model differs from the antecedents by letting the public good accumulate—that is, we can have $d < 1$.[5] The incumbent is constrained by the state's fiscal capacity, $\tau_s > 0$, in choosing the tax rate, so we have $t_s \leq \tau_s$. I explain how fiscal capacity is determined below. In each period $s$, each individual has an income $\omega > 0$.

[4]Another set of papers in the literature focuses on determinants of fiscal capacity different from Besley and Persson (2009, 2011). For example, Besley and Persson (2010) focus on the effects of internal conflicts on fiscal-capacity investment, Gennaioli and Voth (2015) analyze the role of war technology in state-capacity formation, Acemoglu et al. (2015) investigate local state capacity with a network approach, and Gillitzer (2017) studies the effect of macroeconomic income shocks on fiscal capacity. However, none of these papers can solve the inconsistencies between the theory and data I discussed in the introduction.

[5]By allowing accumulation of the public good, my model connects to the literature on durable public goods, which includes, for example, Battaglini and Coate (2007).

The period-$s$ utility of a group-$J_s$ member is given as

$$c^{J_s} + \alpha_s G_s = (1 - t_s)\omega + r^{J_s} + \alpha_s G_s$$

where $c^{J_s}$ is her consumption and $\alpha_s \in \{\alpha_L, \alpha_H\}$ with $\alpha_H \geq \alpha_L \geq 0$. I assume that $\alpha_s$ is equal to $\alpha_H$ with probability $\phi$ in each period $s$. Following Besley and Persson (2009, 2011), I interpret the public good as defense activities against external threats and $\phi$ as the risk of an external war.[6]

The first period's fiscal capacity is given exogenously as $\tau_1 > 0$. In any other period $s \in \{2, 3\}$, fiscal capacity, $\tau_s$, is chosen by the previous period's incumbent, $I_{s-1}$, with the restriction that $\tau_s \geq \tau_{s-1}$. To increase the next period's fiscal capacity, an incumbent must make an investment. The cost of this investment is given as $F(\tau_s - \tau_{s-1})$, where $F(.)$ is increasing, convex and twice continuously differentiable with $F(0) = F_\tau(0) = 0$, and $F_{\tau\tau}(\tau) \neq 0$ for any $\tau$. Fiscal capacity does not depreciate between the periods.

So, in any period $s \in \{1, 2\}$, the government's budget constraint is given as

$$t_s\omega = g_s + \frac{r^{I_s} + r^{O_s}}{2} + F(\tau_s - \tau_{s-1}). \tag{1}$$

In the third period, it is given as

$$t_3\omega = g_3 + \frac{r^{I_3} + r^{O_3}}{2}. \tag{2}$$

In each period $s$, the exact timing of the events is

Stage 1. The initial conditions $\tau_s$ and $G_{s-1}$ are given.

Stage 2. The value of the public good, $\alpha_s$, is realized.

Stage 3. If it is the second or the third period, the incumbent, $I_s$, is realized.

Stage 4. The incumbent, $I_s$, chooses the government policy: tax rate, transfers and investment in the public good, $\{t_s, r^{I_s}, r^{O_s}, g_s\}$. In any period $s \in \{1, 2\}$, the incumbent also determines the fiscal capacity of the next period, $\tau_{s+1}$.

Stage 5. The payoffs for period $s$ are realized.

# 3 Social Planner's Solution

As a benchmark, I focus on the fiscal-capacity investment of a social planner that aims to maximize the total welfare in the society.[7] I assume that the social planner is at equal distance to both groups. Thus, she does not transfer income between the groups. Additionally, to simplify my analysis and focus on the interesting case, I follow Assumption 1 in this section.

[6] Battaglini and Coate (2007) state that national-defense activities are the most famous examples of durable public goods. Thus, my modeling of defense activities as accumulating over time is also supported by the literature on durable public goods.

[7] This is a common approach in the political-economy literature; see, for example, Persson and Tabellini (2000), Besley and Persson (2011), and Karakas (2017).

**Assumption 1.** *For the social planner, the public good's low value is small, and its high value is large enough. Specifically,* $\alpha_L + E(\alpha)(1-d) + E(\alpha)(1-d)^2 < 1 < \alpha_H$.

Assumption 1 ensures that the social planner invests in the public good if and only if there is an external war. Similar assumptions are also followed by Besley and Persson (2010, 2011).

The social planner's fiscal-capacity investment in the second period is given in Appendix B. Here, I focus on her fiscal-capacity investment in the first period, which is the period that gives us the main results of the paper. Since the social planner does not transfer income between the groups, her first-period problem is

$$\max_{t_1, g_1, \tau_2} \omega(1-t_1) + \alpha_1 G_1 + EV_2(\tau_2, G_1; \alpha_2)$$

subject to the constraints $0 \leq t_1 \leq \tau_1$, $0 \leq g_1$, $\tau_1 \leq \tau_2$,

$$t_1 \omega = g_1 + F(\tau_2 - \tau_1),$$

and

$$G_1 = (1-d)G_0 + g_1$$

where $EV_2(\tau_2, G_1; \alpha_2)$ is her expected utility in the second period when the fiscal capacity is $\tau_2$ and the stock of the public good is $G_1$. Expected utility of the social planner is obtained by her optimal choices in future periods. Its formula and derivation are given in Appendix B.

In her problem, the social planner imposes tax to finance public-good and fiscal-capacity investments. Because taxing decreases consumption, we can interpret her problem as decreasing consumption to invest in the public good and fiscal capacity. Additionally, since the benefits of consumption and the public good are linear, we can consider her consumption versus public-good-investment decision separate from her decision on fiscal-capacity investment.

For the social planner, consumption's marginal benefit is one, and that of the public-good investment is

$$\alpha_1 + E(\alpha) \sum_{t=1}^{2} (1-d)^t.$$

Thus, by Assumption 1, if the public good's value is low, consumption's marginal benefit is higher than that of the public-good investment. In this case, the social planner does not allocate any tax revenue for public-good investment. She collects tax only to finance fiscal-capacity investment. So, we have $t = \frac{F(\tau_2 - \tau_1)}{\omega}$. On the other hand, if the public good's value is high, public-good investment has a higher marginal benefit than consumption. Therefore, the social planner imposes the highest tax rate—that is, $t = \tau_1$—and spends the whole tax revenue on the public-good and fiscal-capacity investments. We can collect these results in the following proposition.

**Proposition 1.** *Given Assumption 1, if the public good's value is low, the social planner imposes a tax rate that is just enough to cover the cost of fiscal-capacity investment; otherwise, she imposes the highest tax rate and spends the whole tax revenue that is not invested in fiscal capacity on the public good. Specifically, if* $\alpha_1 = \alpha_L$*, then* $t = \frac{F(\tau_2 - \tau_1)}{\omega}$ *and* $g_1 = 0$*; otherwise,* $t = \tau_1$ *and* $g_1 = \tau_1 \omega - F_\tau(\tau_2 - \tau_1)$*.*

Given the social planner's policy decisions, we can show that her fiscal-capacity investment in the first period, $\tau_2^s - \tau_1$, satisfies the first-order conditions

$$\omega \left[ E\lambda_2 + E\lambda_3 - 2 \right] + \mu = \lambda_1 F_\tau(\tau_2^s - \tau_1), \tag{3}$$

$$\mu(\tau_2^s - \tau_1) = 0,$$

and

$$\mu \geq 0,$$

where

$$E\lambda_2 = \phi \left[ \alpha_H + E(\alpha)(1-d) \right] + (1 - \phi)$$

and

$$E\lambda_3 = \phi \alpha_H + (1 - \phi)$$

are the expected values of the public funds for the social planner in the second and third periods, respectively, and

$$\lambda_1 = \max \left\{ 1, \alpha_1 + (1-d)E(\alpha) + (1-d)^2 E(\alpha) \right\}$$

is the value of public funds for the social planner in the first period, and $\mu$ is the Lagrange multiplier.[8] If $\mu = 0$, Equation 3's left-hand side denotes fiscal-capacity investment's marginal benefit and its right-hand side represents fiscal-capacity investment's marginal cost for the social planner. Indeed, it is easy to show that the social planner invests in the second period's fiscal capacity and so that $\mu = 0$ as long as the probability of an external war is positive.

**Proposition 2.** *Given Assumption 1, the social planner invests in the second period's fiscal capacity if and only if the external-war probability is positive. Specifically,* $\tau_2^s - \tau_1 > 0$ *if and only if* $\phi > 0$*.*

In the following section, I use Equation 3 to compare the socially optimal and politically determined fiscal capacities.

# 4 Political Equilibrium

To analyze the politically determined fiscal capacity, I focus on the subgame-perfect equilibrium of the game described by the political process. To simplify my analysis and focus on the interesting

[8] Derivation of these conditions is given in Appendix C.

cases, I follow Assumption 2 in this section.

**Assumption 2.** *For the first period's incumbent, the public good's low value is small, and its high value is large enough. Specifically,* $\alpha_L + E(\alpha)(1-d) < 2 < \alpha_H$.

Assumption 2 gives two incentives to the first period's incumbent. First, in the last two periods, if the public good's value is low, the incumbent does not allocate any tax revenue for public-good investment. Second, in any period, if the public good's value is high, the incumbent does not allocate any tax revenue for transfers. A similar assumption is followed by Besley and Persson (2010).

I analyze the incumbent's second-period fiscal-capacity investment in Appendix D. Here, I focus on her first-period decisions, which give us the main results of the paper. In the first period, the problem of the incumbent is given as

$$\max_{t_1, r^{I_1}, r^{O_1}, g_1, \tau_2} \omega(1-t_1) + r^{I_1} + \alpha_1 G_1 + p_I EV^I_{I_2}(\tau_2, G_1; \alpha_2) + (1-p_I) EV^I_{O_2}(\tau_2, G_1; \alpha_2)$$

subject to the constraints $0 \leq t \leq \tau_1$, $0 \leq r^{I_1}$, $0 \leq r^{O_1}$, $0 \leq g_1$, $\tau_1 \leq \tau_2$,

$$t_1 \omega = g_1 + \frac{r^{I_1} + r^{O_1}}{2} + F(\tau_2 - \tau_1),$$

and

$$G_1 = (1-d)G_0 + g_1,$$

where $EV^I_{I_2}(\tau_2, G_1; \alpha_2)$ is the expected utility of the incumbent if she gets the political power in the second period when fiscal capacity is $\tau_2$ and the public-good stock is $G_1$. A similar interpretation also applies to $EV^I_{O_2}(\tau_2, G_1; \alpha_2)$ if the incumbent is the opponent in the second period. The incumbent's expected utilities are derived from the equilibrium choices in future periods as explained in Appendix D.

Because transfers to the opponent group do not benefit the incumbent, she does not allocate any tax revenue for this purpose. So, we have $r^{O_1} = 0$. Additionally, while deciding on the tax-revenue allocation between transfers to her group and public-good investment, the incumbent compares their marginal benefits. The marginal benefit of transfers, $\lambda^I_r$, is 2, and the marginal benefit of the public-good investment, $\lambda^I_g$, is

$$\alpha_1 + E(\alpha)(1-d) + E(\alpha)(1-d)^2$$

for the incumbent. Thus, by Assumption 2, if the public good's value is low, the incumbent does not allocate any tax revenue for public-good investment; if its value is high, she does not allocate any tax revenue for transfers. Besides, since the marginal benefits of both types of spending are constant, the incumbent spends the whole tax revenue that is not invested in the second period's fiscal capacity either on transfers or on public-good investment.[9] Proposition 3 summarizes these results.

**Proposition 3.** *Given Assumption 2, the first-period incumbent's policies are given as follows.*

*(i) If the public good's marginal benefit is low, she chooses the highest tax rate and transfers the whole tax revenue that is not invested in fiscal capacity to her group. Specifically, if* $\lambda_g^I < \lambda_r^I$*, then* $t = \tau_1$*,* $r^{I_1} = 2\left[\tau_1 \omega - F(\tau_2 - \tau_1)\right]$*,* $r^{O_1} = 0$*, and* $g_1 = 0$*.*

*(ii) If the public good's marginal benefit is high, she chooses the highest tax rate and spends the whole tax revenue not invested in fiscal capacity on public-good investment. Specifically, if* $\lambda_g^I > \lambda_r^I$*, then* $t = \tau_1$*,* $r^{I_1} = r^{O_1} = 0$*, and* $g_1 = \tau_1 \omega - F(\tau_2 - \tau_1)$*.*

Then, we can show that the incumbent's fiscal-capacity investment, $\tau_2^* - \tau_1$, satisfies the first-order conditions

$$\omega \left\{E_I(\lambda_2) - 1 + p\left[E_I(\lambda_3) - 1\right]\right\} + \mu = \lambda_1^I F_\tau(\tau_2^* - \tau_1), \tag{4}$$

$$\mu(\tau_2^* - \tau_1) = 0,$$

and

$$\mu \geq 0,$$

where

$$E_I(\lambda_2) = \phi\left[\alpha_H + E(\alpha)(1-d)\right] + (1-\phi)p_I 2$$

and

$$E_I(\lambda_3) = \phi\alpha_H + (1-\phi)p_I 2$$

are the expected value of public funds for the incumbent in the second and third periods, respectively,

$$\lambda_1^I = \max\left\{\lambda_r^I, \lambda_g^I\right\}$$

is the value of public funds for the incumbent in the first period,

$$p = \begin{cases} 1 - p_I & \text{if only } O_1 \text{ invests in } \tau_3 \text{ in the second period,} \\ 1 & \text{if both } I_1 \text{ and } O_1 \text{ invests in } \tau_3 \text{ in the second period,} \\ p_I & \text{otherwise,} \end{cases}$$

and $\mu$ is the Lagrange multiplier.[10] Similar to the social planner's first-order conditions, if $\mu = 0$,

[9] To avoid the nongeneric cases, I ignore the case when the transfers' and the public-good investment's marginal benefits are equal.

[10] I give the value of $p$ depending on the model parameters and derive the incumbent's first-order conditions in Appendix E.

then Equation 4's left-hand side denotes the marginal benefit of fiscal-capacity investment, and its right-hand side is the incumbent's marginal cost of fiscal-capacity investment.

It is easy to show that the incumbent invests in the second period's fiscal capacity if and only if the expected value of public funds is positive for her.

**Proposition 4.** *Given Assumption 2, the first period's incumbent invests in fiscal capacity if and only if the expected value of public funds is positive for her. Specifically,* $\tau_2^* - \tau_1 > 0$ *if and only if* $E_I(\lambda_2) - 1 + p\,[E_I(\lambda_3) - 1] > 0$.

In the following proposition, I focus on the optimality of the incumbent's fiscal-capacity investment, comparing it with the social planner's.

**Proposition 5.** *Let Assumptions 1 and 2 hold. Further assume that both the social planner and the incumbent invest in the second period's fiscal capacity. If political stability is high (low), the incumbent's fiscal-capacity investment is more (less) than the social planner's. Specifically, there exists* $\bar{p} \in [0, 1]$ *such that if* $p_I > \bar{p}$ $(p_I < \bar{p})$, *then* $\tau_2^* > \tau_2^s$ $(\tau_2^* < \tau_2^s)$.

As I state in Proposition 5, if political stability is high, the first period's incumbent invests in the second period's fiscal capacity more than the social planner. This is because the only motive of the social planner to invest in fiscal capacity is financing public-good investment in case of a war in the future. In contrast, the incumbent invests in fiscal capacity both to finance public-good investment in case of a war and, if she gets the political power, to transfer the other group's income to her own group in the future. A high political stability means that the incumbent has a high probability of getting the political power in the future. This increases her fiscal-capacity investment due to the income-transfer motive. Thus, her fiscal-capacity investment exceeds the social planner's. If political stability is low, we have the opposite result. These results are independent of the depreciation rate of the public good. So, they are compatible with the previous literature, such as Besley and Persson (2011).

In the following, I focus on the effects of model parameters on the first-period incumbent's fiscal-capacity investment when she invests in fiscal capacity—that is, $\tau_2^* - \tau_1 > 0$.

## 4.1 Public Good with High Benefit

First, I focus on the effects of model parameters on the incumbent's fiscal-capacity investment when the marginal benefit of the public good is high for her—that is, $\lambda_g^I > \lambda_r^I$. In this situation, by Proposition 3, the incumbent allocates the whole tax revenue between the public-good and fiscal-capacity investments. A parameter affects the incumbent's fiscal-capacity investment by changing the marginal benefits of these two types of investments.

**Proposition 6.** *Let Assumption 2 hold. Further assume that the public good's marginal benefit is high for the first period's incumbent and she invests in fiscal capacity. The first-period incumbent's fiscal-capacity investment increases with each of the following.*

*(i) An increase in income,* $\omega$

*(ii) An increase in political stability,* $p_I$

*(iii) A decrease in the cost of fiscal-capacity investment,* $F_\tau$

An increase in income increases fiscal-capacity investment because it increases the tax revenue that can be collected in the future and thereby increases the marginal benefit of fiscal-capacity investment. An increase in political stability increases fiscal-capacity investment by increasing the probability that the incumbent benefits from the future fiscal capacity. Clearly, a decrease in the cost of fiscal-capacity investment increases investment in it. These results are aligned with the results in the previous literature, including Besley and Persson (2009, 2011).

Next, I focus on the external-war probability's effect on the incumbent's fiscal-capacity investment.

**Proposition 7.** *Let Assumption 2 hold. Further assume that the public good's marginal benefit is high for the first period's incumbent and she invests in fiscal capacity. An incremental increase in the probability of an external war increases the first-period incumbent's fiscal-capacity investment if and only if this probability is high. Specifically, there exists* $\bar{\phi} \in [0,1]$ *such that* $\frac{\partial(\tau_2^* - \tau_1)}{\partial \phi} \geq 0$ *if and only if* $\bar{\phi} \leq \phi$ *(with* $\frac{\partial(\tau_2^* - \tau_1)}{\partial \phi} < 0$ *if* $\bar{\phi} < \phi$*).*

As stated in Proposition 7, an incremental increase in the probability of an external war increases fiscal-capacity investment if and only if this probability is high. This is because, when the public good's marginal benefit is high, the incumbent faces a trade-off between spending the tax revenue on fiscal-capacity investment and spending it on public-good investment. An increase in the probability of an external war increases both investments' marginal benefits. However, as I illustrate in Figure 2, while the probability of an external war increases the fiscal-capacity investment's marginal benefit at an increasing rate, it increases the public-good investment's at a constant rate. Thus, if the probability of an external war is low, an incremental increase in this probability increases the marginal benefit of public-good investment more than it increases the fiscal-capacity investment's. As a result, the incumbent increases her public-good investment and decreases her fiscal-capacity investment. If the probability of an external war is high, an incremental increase in this probability increases fiscal-capacity investment's marginal benefit more than the public-good investment's. Thus, the incumbent increases her fiscal-capacity investment and decreases her public-good investment.

An increase in the external-war probability increases fiscal-capacity and public-good investments' marginal benefits at different rates because it affects them at different layers. An increase in the external-war probability raises fiscal-capacity investment's marginal benefit at an increasing rate

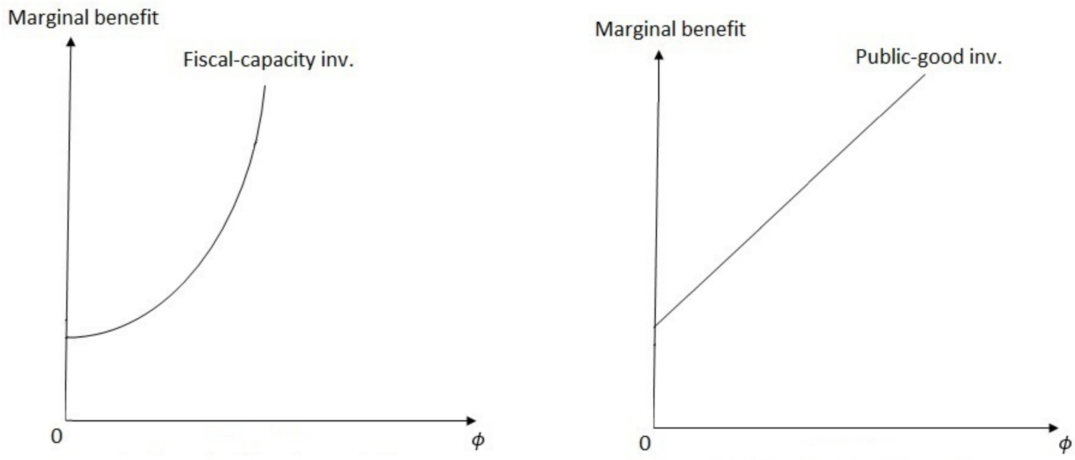


Figure 2: Effects of the external-war probability on the marginal benefits of fiscal-capacity and public-good investments.

because of its two-layer effect. In particular, first, it increases the probability that the second period's fiscal capacity will be used for public-good investment. Second, it increases the probability that these public-good investments will have a high value in the third period. An increase in the probability of an external war increases the public-good investment's marginal benefit at a constant rate. Because it affects the public-good investment's marginal benefit, only by increasing the probability will this investment have a high value in future periods.

Proposition 7 explains the time evolution of fiscal-capacity investment in many countries. In particular, it offers an explanation for the two trends the previous models fail to explain in Figure 1. The first of these trends is that as a war period approaches, the tax-to-GDP-ratio change of many countries decreases. According to Proposition 7, in these countries, if the probability of war is below the threshold level, $\bar{\phi}$, as the war approaches and so its probability increases, fiscal-capacity investment decreases. Thus, the change in their tax-to-GDP-ratio exhibits a decreasing trend. Second, in Figure 1, towards the end of a war period, tax-to-GDP-ratio change of almost all countries increases. According to Proposition 7, in these countries, if the probability of war is below the threshold level, as the war winds down and so its probability decreases, fiscal-capacity investment increases. Therefore, the change in their tax-to-GDP-ratio exhibits an increasing trend.

Proposition 7 also shows that if the public good accumulates, the antecedent literature's main result, an increase in the external-war probability always increases fiscal-capacity investment, as stated in Besley and Persson (2009, 2011), no longer holds. The difference between my result and the antecedent literature's stems from the fact that when the public good does not accumulate, as in the antecedent literature, an increase in the external-war probability only increases the marginal benefit of fiscal-capacity investment. However, when the public good accumulates, as in my model,

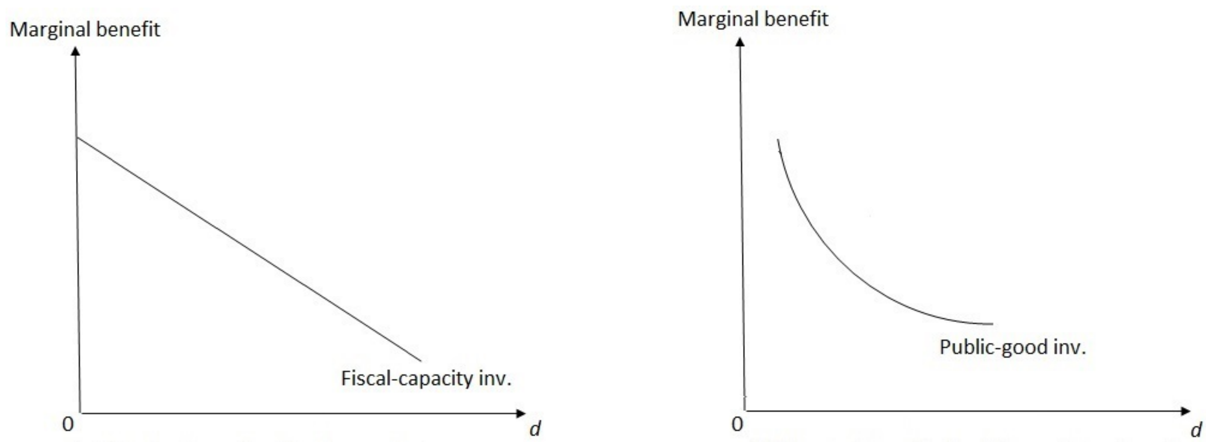


Figure 3: Effects of the public good's depreciation rate on the marginal benefits of fiscal-capacity and public-good investments.

an increase in the external-war probability increases the marginal benefits of both fiscal-capacity and public-good investments.

Proposition 8 focuses on the effect of the public good's depreciation rate on fiscal-capacity investment. Similar to the effect of the external-war probability, the depreciation rate's effect on fiscal-capacity investment depends on a critical value of the depreciation rate.

**Proposition 8.** *Let Assumptions 2 hold. Further assume that the public good's marginal benefit is high for the first period's incumbent and she invests in fiscal capacity. An incremental increase in the public good's depreciation rate increases the first-period incumbent's fiscal-capacity investment if and only if this rate is low. Specifically, there exists* $\bar{d} \in [0, 1]$ *such that* $\frac{\partial(\tau_2^* - \tau_1)}{\partial d} \geq 0$ *if and only if* $d \leq \bar{d}$ *(with* $\frac{\partial(\tau_2^* - \tau_1)}{\partial d} > 0$ *if* $d < \bar{d}$*).*

As given in Proposition 8, an increase in the public good's depreciation rate increases fiscal-capacity investment if and only if this rate is below a threshold level. This is because an increase in the public good's depreciation rate decreases the marginal benefits of both fiscal-capacity and the public-good investments. However, as I present in Figure 3, while this effect is constant for fiscal-capacity investment, it is decreasing for the public-good investment. Thus, if the depreciation rate is low, an incremental increase in this rate decreases the public-good investment's marginal benefit more than the fiscal-capacity's. So, the incumbent increases her fiscal-capacity investment. If the depreciation rate is high, we have the opposite effect.

An increase in the public good's depreciation rate decreases the public-good investment's marginal benefit at a decreasing rate because of its two-layer effect. First, an increase in the depreciation rate decreases the public-good stock transferred from the first to the second period. Second, it decreases the public-good stock transferred from the second to the third period. An increase in the public good's depreciation rate decreases the marginal benefit of the fiscal-capacity investment at a constant rate because of its one-layer effect. Specifically, it decreases the fiscal-capacity investment's marginal benefit by decreasing the marginal benefit of the public-good investment in the second period, which is a way to use the first-period fiscal-capacity investment.

Proposition 8 provides another explanation for the trend in almost all countries' fiscal-capacity investment which cannot be explained by the previous models. Remember that almost all countries' tax-to-GDP-ratio change increases towards the end of a war period as given in Figure 1. According to Proposition 8, two different scenarios can be at work. First, the depreciation rate of the public good can be low but increasing towards the end of a war. Second, the depreciation rate of the public good can be high but decreasing. Under both scenarios, fiscal-capacity investment increases as the war ends. As the antecedent literature (e.g. Besley and Persson, 2009, 2011) assumes that the depreciation rate of the public good is always equal to one, we cannot observe these effects in the previous models.

## 4.2 Public Good with Low Benefit

So far, I have analyzed the effects of model parameters on the first-period incumbent's fiscal-capacity investment when the public good has a high marginal benefit for her. In the following, I continue analyze when the public good's marginal benefit is low for the incumbent—that is, $\lambda_g^I < \lambda_r^I$. In this case, by Proposition 3, the incumbent spends all tax revenues on fiscal-capacity investment and transfers to her own group. Since, the marginal benefit of transfers is constant, a parameter affects the incumbent's fiscal-capacity investment by changing the marginal benefit of that investment.

**Proposition 9.** *Let Assumption 2 hold. Further assume that the public good's marginal benefit is low for the first period's incumbent and she invests in fiscal capacity. The first-period incumbent's fiscal-capacity investment increases with one of the following.*

*(i) An increase in income,* $\omega$

*(ii) An increase in political stability,* $p_I$

*(iii) A decrease in the cost of fiscal-capacity investment,* $F_\tau$

*(iv) An increase in the probability of an external war,* $\phi$

*(v) A decrease in the depreciation rate of the public good,* $d$

As I state in Parts i–iii of Proposition 9, when the public good's marginal benefit is low for the first period's incumbent, an increase in income or political stability or a decrease in the cost of fiscal-capacity investment increases the incumbent's fiscal-capacity investment. These results and their reasoning are the same when the public good's marginal benefit is high for the incumbent. Additionally, as I state in Parts iv–v of the proposition, an increase in the probability of an external war or a decrease in the depreciation rate of the public good also increases the incumbent's fiscal-capacity investment. These results are different from when the public good's marginal benefit is high for the incumbent.

The effects of the external-war probability and the public good's depreciation rate on the incumbent's fiscal-capacity investment differ under the two cases, public good with a high marginal benefit and with a low marginal benefit, due to different use of tax revenues. In particular, when the public good's marginal benefit is high, the incumbent allocates the whole tax revenue between fiscal-capacity and public-good investments. The probability of an external war and the depreciation rate of the public good affect the marginal benefits of both investments. Thus, a change in them affects the incumbent's fiscal-capacity investment in both an increasing and decreasing way. However, when the public good's marginal benefit is low, the incumbent allocates the whole tax revenue between fiscal-capacity investment and transfers. Because the marginal benefit of transfers is constant, in this case, the two parameters only affect the fiscal-capacity investment's marginal benefit. Therefore, a change in these parameters affects the incumbent's fiscal-capacity investment only in one direction.

Proposition 9 gives the same results with the previous literature, including Besley and Persson (2009, 2011). Thus, when the public good's marginal benefit is low for the incumbent, accumulation of the public good does not change the results in the antecedent literature.

# 5 Public Good's Price

The previous literature on fiscal capacity, including Besley and Persson (2009, 2011), assumes that the public good's price is equal to the consumption good's—that is, it is equal to one. Until now, I have also followed this assumption. However, the durable-public-goods literature, which motivates this paper, lets the consumption and public goods have different prices (e.g. Battaglini and Coate, 2007). In this section, I extend my model, letting the two goods have different prices. The extended model does not significantly change the results I have presented so far.[11] Therefore, in the following, I focus on the effect of the public good's price on the first-period incumbent's fiscal-capacity investment.

To this end, I denote the public good's price with $y > 0$. In any period $s \in \{1, 2\}$, the government's budget constraint is given as

$$t_s\omega = yg_s + \frac{r^{I_s} + r^{O_s}}{2} + F(\tau_s - \tau_{s-1}).$$

In the third period, it is given as

$$t_3\omega = yg_3 + \frac{r^{I_3} + r^{O_3}}{2}.$$

[11] The formal statements and proofs of the propositions under the extended model is available from the author.

In this section, as an analog of Assumption 2, I follow Assumption 3.

**Assumption 3.** *For the first period's incumbent, the public good's low value is small, and its high value is large enough. Specifically,* $\frac{1}{y}[\alpha_L + E(\alpha)(1-d)] < 2 < \frac{1}{y}\alpha_H$.

Additionally, when the public good's price is $y$, the marginal benefit of the public-good investment for the first period's incumbent, $\lambda^I_{g,y}$, is given as

$$\frac{1}{y}\left[\alpha_1 + E(\alpha)(1-d) + E(\alpha)(1-d)^2\right].$$

I analyze the effect of the public good's price on the first-period incumbent's fiscal-capacity investment first, when the marginal benefit of the public good is high for the incumbent—that is, $\lambda^I_{g,y} > \lambda^I_r$.

**Proposition 10.** *Let Assumption 3 hold. Further assume that the public good's marginal benefit is high for the first period's incumbent and she invests in fiscal capacity. An increase in the public good's price,* $y$*, increases the first-period incumbent's fiscal-capacity investment if and only if the value of fiscal capacity is high for the incumbent to transfer income from the other group in the future—that is,* $2(1-\phi)p_I > 1$.

As I state in Proposition 10, when the marginal benefit of the public good is high for the incumbent, an increase in the public good's price increases fiscal-capacity investment if and only if the future fiscal capacity is valuable enough for the incumbent to transfer income from the other group. When the marginal benefit of the public good is high for the incumbent, she allocates the whole tax revenue between fiscal-capacity and the public-good investments. An increase in the public good's price decreases the marginal benefit of both investments. However, if the value of future fiscal capacity for transferring income form the other group is high for the incumbent, fiscal-capacity investment's marginal benefit decreases less than the public-good investment's. So, the incumbent increases her fiscal-capacity investment. Otherwise, fiscal-capacity investment's marginal benefit decreases more than the public-good investment's and the incumbent decreases her fiscal-capacity investment.

Next, I focus on the effect of the public good's price on the first-period incumbent's fiscal-capacity investment when the marginal benefit of the public good is low for the incumbent—that is, $\lambda^I_{g,y} < \lambda^I_r$.

**Proposition 11.** *Let Assumption 3 hold. Further assume that the public good's marginal benefit is low for the first period's incumbent and she invests in fiscal capacity. An increase in the public good's price,* $y$*, decreases the first-period incumbent's fiscal-capacity investment.*

If the marginal benefit of the public good is low for the incumbent, increasing the public good's price decreases her fiscal-capacity investment. This is because, when the marginal benefit of the public good is low for her, the incumbent allocates the whole tax revenue between fiscal-capacity investment and transfers to her own group. An increase in the public good's price decreases fiscal-capacity investment's marginal benefit by decreasing the amount of public good that can be financed by this investment in the future. However, it does not affect the marginal benefit of transfers, which is constant. As a result, the incumbent decreases her fiscal-capacity investment.

## 6 Some Evidence

In this section, I provide some empirical evidence supporting my model. In particular, according to my model, when the public good's marginal benefit is high for the first period's incumbent, an increase in the external-war probability increases her fiscal-capacity investment if and only if this probability is high. Additionally, fiscal-capacity and public-good investments move in opposition directions. I show that these results are supported by Australia and the UK data.

To this end, in Figure 4, I present the fitted values of change in tax-to-GDP and military-expenditure-to-GDP ratios for Australia and the UK around World War II. I use change in tax-to-GDP ratio to measure fiscal-capacity investment, as before, and military-expenditure-to-GDP ratio to measure the public-good investment. To present my evidence, in the figure, I divide the time into four periods for both countries using dashed lines.[12] In Period 1, both countries' change in tax-to-GDP ratios decrease and the military-expenditure-to-GDP ratios increase. Interpreting that during this period the probability of war is low but increasing, these trends support my result that if the probability of war is low, an increase in this probability decreases fiscal-capacity investment and increases public-good investment. In Period 2, both countries' change in tax-to-GDP and military-expenditure-to-GDP ratios increase. Interpreting that during this period the probability of war is high and increasing, the increases in change in tax-to-GDP ratios support my result that if the probability of war is high, an increase in this probability increases fiscal-capacity investment. However, in this period, the increases in military-expenditure-to-GDP ratios contradict my result. According to my model, the military-expenditure-to-GDP ratios move in the opposite direction from the change in tax-to-GDP ratios, so they should be decreasing. The reason that my result on the military-expenditure-to-GDP ratios does not hold can be because the countries used public debt, in addition to tax revenues, to finance their military expenditures during this period.[13] In Period 3, both countries' change in tax-to-GDP ratios increase and the military-expenditure-to-GDP ratios decrease. Interpreting that during this period the probability of war is high but decreasing,

[12] For Australia, Period 1 covers the years between 1929 and 1938, Period 2 the years between 1938 and 1941, Period 3 the years between 1941 and 1943, and Period 4 the years between 1943 and 1947. For the UK, Period 1 covers the years between 1929 and 1938, Period 2 the years between 1938 and 1942, Period 3 the years between 1942 and 1943, and Period 4 the years between 1943 and 1947.

[13] In Period 2, in the UK, while the tax-to-GDP ratio is 0.25, the military-expenditure-to-GDP ratio is 0.28. So, it is very likely that some of the military expenditures were financed by public debt. In Period 2, in Australia, the tax-to-GDP ratio is 0.07 and the military-expenditure-to-GDP ratio is 0.06. Although the tax-to-GDP ratio is higher than the military-expenditure-to-GDP ratio, considering that the government must also spent some of the tax revenues on non-military items, its very likely that some of the military expenditures were financed by public debt.

these trends support my result that if the probability of war is high, a decrease in this probability decreases fiscal-capacity investment and increases public-good investment. Lastly, in Period 4, both countries' change in tax-to-GDP ratios decrease and their military-expenditure-to-GDP ratios increase. Now, interpreting that during this period the probability of war is low and decreasing, these trends support my result that if the probability of war is low, a decrease in this probability decreases investment in fiscal capacity and increases investment in the public good.

Previous models, such as the models of Besley and Persson (2009, 2011), state that an increase in the probability of an external war always increases fiscal-capacity investment and decreases public-good investment. Thus, with the earlier models, we cannot explain the trends in the change-in-tax-to-GDP and the military-expenditure-to-GDP ratios either in Australia or in the UK. In particular, according to the previous models, we should be observing trends in both countries, for example, as in Figure 5.[14] The change-in-tax-to-GDP-ratio trends in this figure do not match Australia's and the UK's data in Periods 1 and 4. Additionally, the military-expenditure-to-GDP-ratio trends conflict with Australia's and the UK's data in all periods except in Period 3.

## 7 Conclusion

In this paper, I analyze a dynamic fiscal-capacity model with two stock variables: fiscal capacity and public good. In the model, one of the two groups in the society takes the political power in each period randomly. The group that holds the political power decides on the income-tax rate, transfers to each group, and public-good investment. Additionally, it decides on fiscal-capacity investment, which determines the maximum tax rate the next period's political power can choose. Following the antecedent literature, I interpret the public good as national-defense activities against external threats. However, different from the previous literature, I let the public good accumulate over time.

Similar to the previous literature, I show that if political stability is high, there is overinvestment in fiscal capacity. However, contrary to the antecedent literature, I show that if the external-war probability is low, an increase in this probability decreases fiscal-capacity investment. Additionally, I show that if the public good's depreciation rate is low, a decrease in this rate decreases fiscal-capacity investment. The latter two results can explain the time evolution of fiscal-capacity investment in many countries, which the previous models fail to explain.

My model can be extended in two interesting directions. First, my model comprises three periods. Extending it to an infinite horizon would improve our understanding of the long-run determinants of fiscal capacity. Second, my model does not allow for government borrowing. Extending it to include government borrowing would improve our understanding of the time-series evolution of fiscal capacity.

[14]Note that in Figure 5, only the variables' directions of changes over time reflect the previous models' prediction. The exact amounts of the changes are arbitrary.

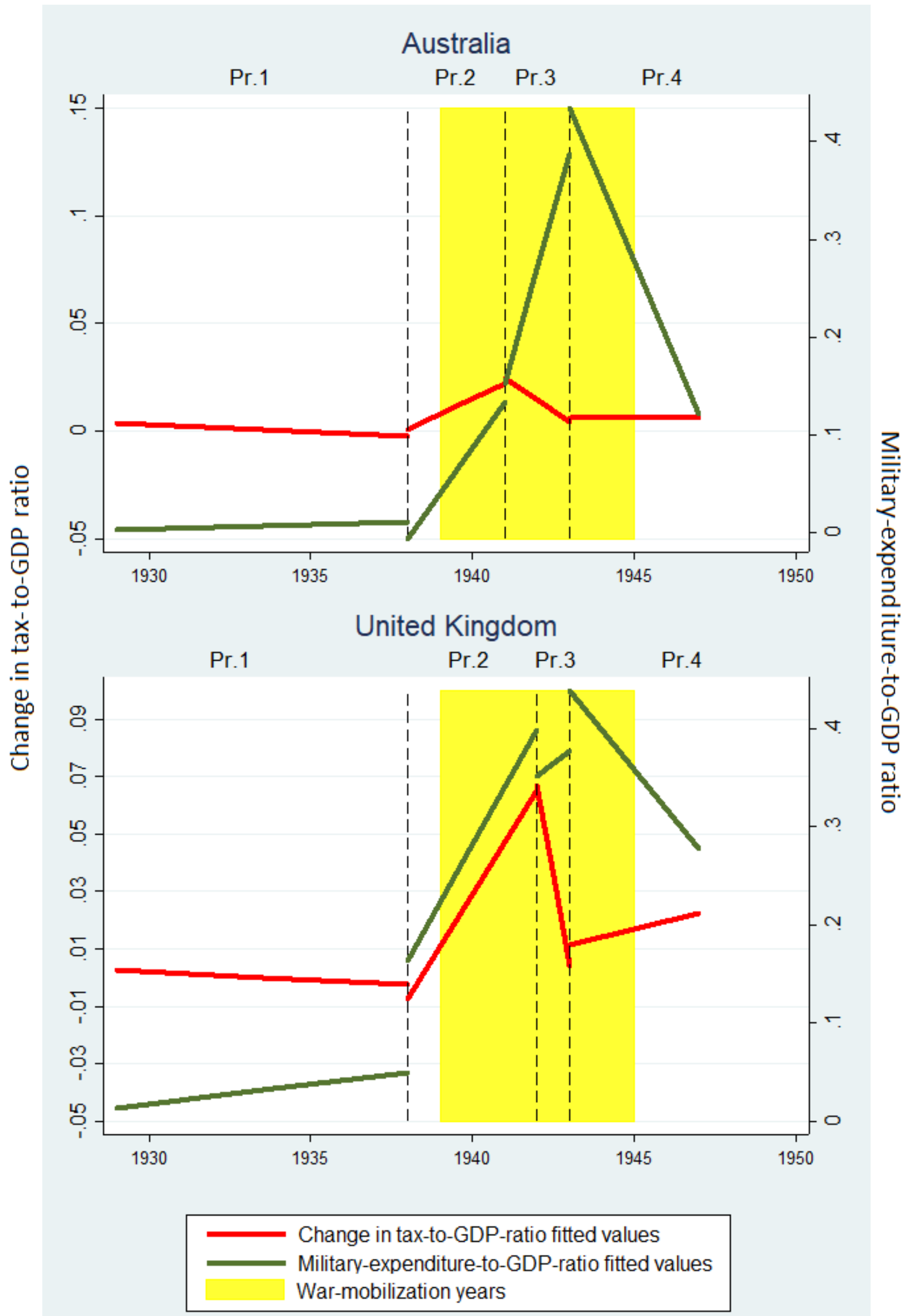


Figure 4: Australia's and the UK's fitted values for the change in tax-to-GDP and military-expenditure-to-GDP ratios, and war-mobilization years. "Pr." is an abbreviation for "Period." Change in tax-to-GDP ratio in a year is calculated as the difference of that year's tax-to-GDP ratio from the previous year's as measured by Rogers et al. (2018). Military-expenditure-to-GDP ratio is taken as measured by Roser and Nagdy (2020). Fitted values represent simple regression lines for the relevant variables and time periods. A country's war-mobilization years covers the time in which it mobilized a certain amount of its population for a war with other countries as measured by Rogers et al. (2018).

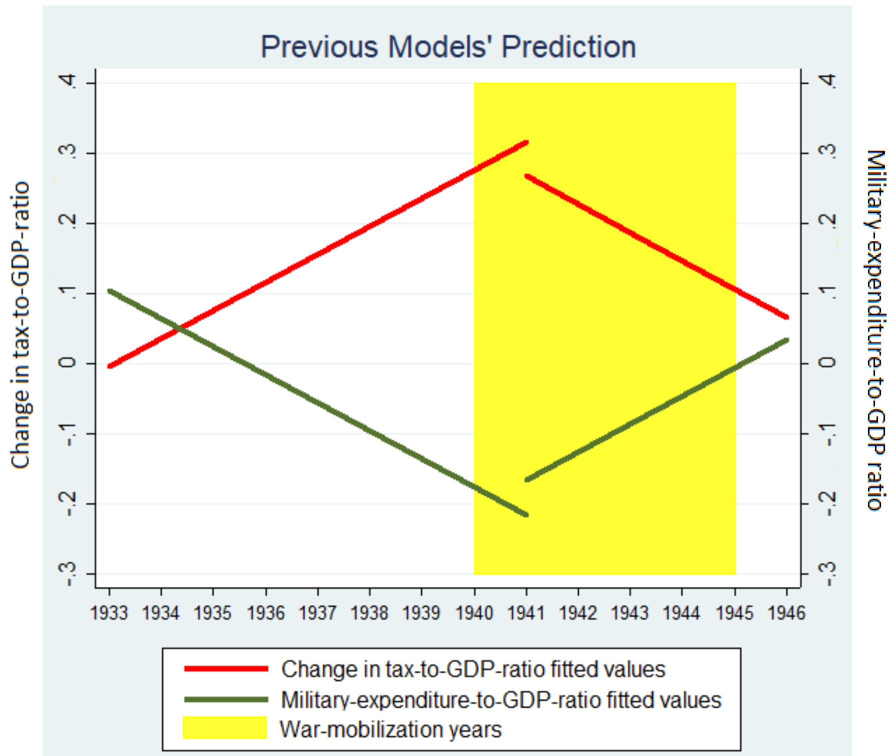


Figure 5: An illustration of the previous models' prediction for a country's change in tax-to-GDP and military-expenditure-to-GDP ratios around an external-war period. Only the signs of the lines' slope reflect the previous models' prediction. The absolute values of the slopes are arbitrary.

# Appendices

## A Income and Fiscal-Capacity Investment

In the introduction, I discussed that the previous models' main result, an increase in the external-war probability increases fiscal-capacity investment, is incompatible with the trends in fiscal-capacity investment of many countries. Thus, I claimed that the antecedent literature fails to explain the time-series evolution of fiscal-capacity investment in those countries. Yet, one can argue that, according to the previous models, an explanation of the trends that I discussed can be the changes in the those countries' incomes because the previous models, including Besley and Persson (2009, 2011), show that a country's fiscal-capacity investment moves in the same direction as its income. In this section, I present some evidence showing that the previous models cannot explain the trends that I discussed in the introduction even after we consider changes in the related countries' incomes.

To this end, in Figure 6, I present the real national income per capita and change in tax-to-GDP ratio of the same countries in Figure 1 around their war-mobilization years. In the introduction, the first trend I claimed cannot be explained with the previous models was that as a war-mobilization period approaches, many countries' change in tax-to-GDP ratio decreases. As given in Figure 6, we see such a trend in Australia and Finland before any of their war periods, in Italy before its second and third war periods, in Canada before its first war period, and in the UK before its second war period. According to the previous models, as a war period approaches, we should observe an increasing trend in the change in tax-to-GDP ratio of these countries since the war probability is increasing. So, there is an incompatibility between the data and the previous models. However, the previous models also conclude that as a country's income decreases its fiscal-capacity investment decreases. Thus, according to the previous models, an explanation of the trends in the change in tax-to-GDP ratio of these countries can be a decreasing trend in their incomes. Yet, as I show in Figure 6, all these countries' real national income per capita has an increasing trend before the war periods, except Italy's before its second war period. Thus, the previous models cannot explain the decreasing trends in the change in tax-to-GDP ratios as a war-mobilization period approaches even after we consider income trends.

The second trend which I claimed cannot be explained with the previous models was that as a war-mobilization period ends, almost all countries' change in tax-to-GDP ratio increases. As given in Figure 6, we see such a trend in all countries, except the US, during their any war-mobilization period that is longer than a year. According to the previous models, as a war period ends, we should be observing a decreasing trend in the change in tax-to-GDP ratio of a country since the war probability is decreasing. So, we have another incompatibility between the data and the previous models. However, the previous models also conclude that as a country's income increases its fiscal-capacity investment increases. Thus, according to the previous models, an explanation of the increasing trend in the change in tax-to-GDP ratios that we observe in the data can be a decreasing trend in the real national incomes. Yet, as I show in Figure 6, this explanation can hold only for Canada during its first war period. In all other countries, where we have data, real national income per capita has a decreasing trend towards the end of war periods. So, the previous models cannot explain the increasing trends in the change in tax-to-GDP ratios towards the ends of war periods even after we consider the income trends.

As a result, the previous models cannot explain the time-series evolution of fiscal-capacity investment in many countries even after we consider the income trends in these countries.

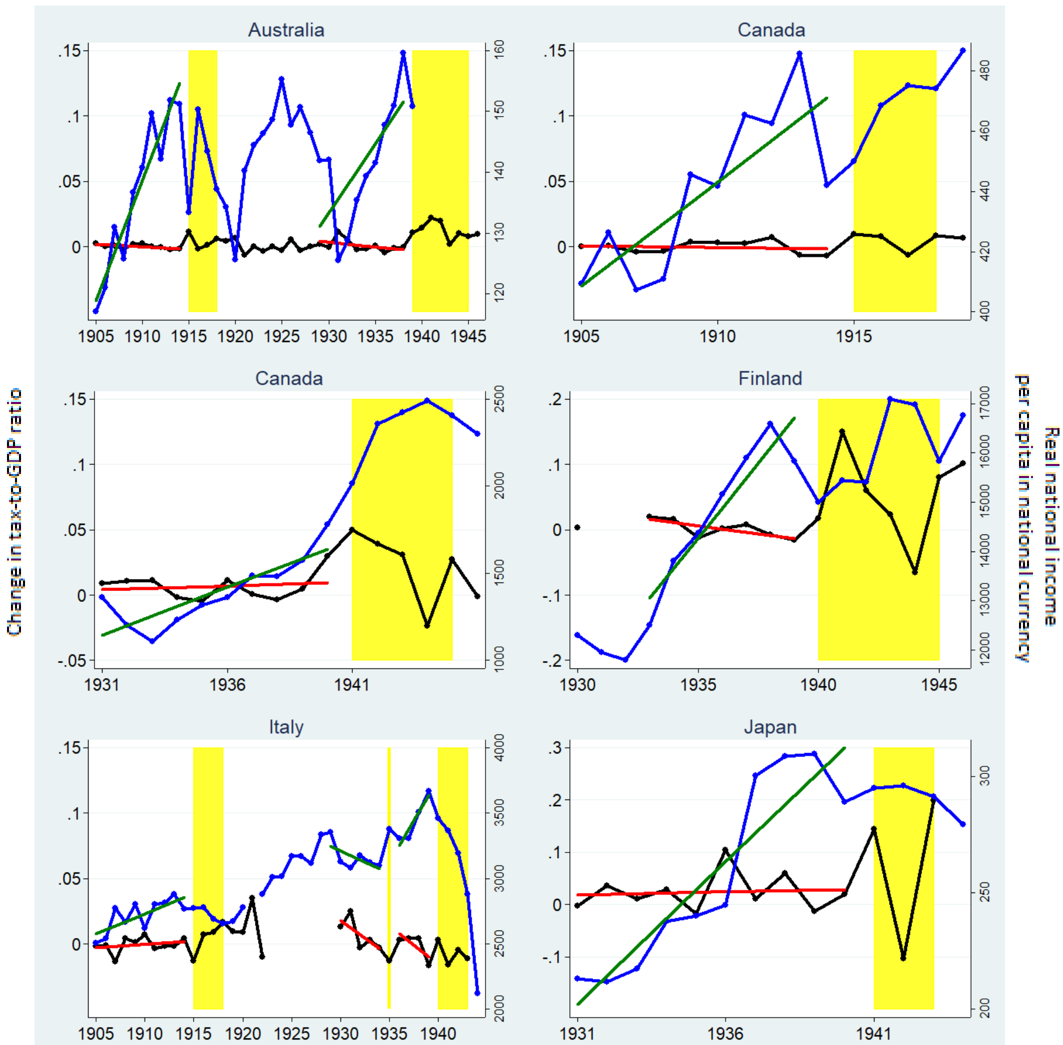


Figure 6: Change in tax-to-GDP ratio, real national income per capita, and war mobilization. (cnt.)

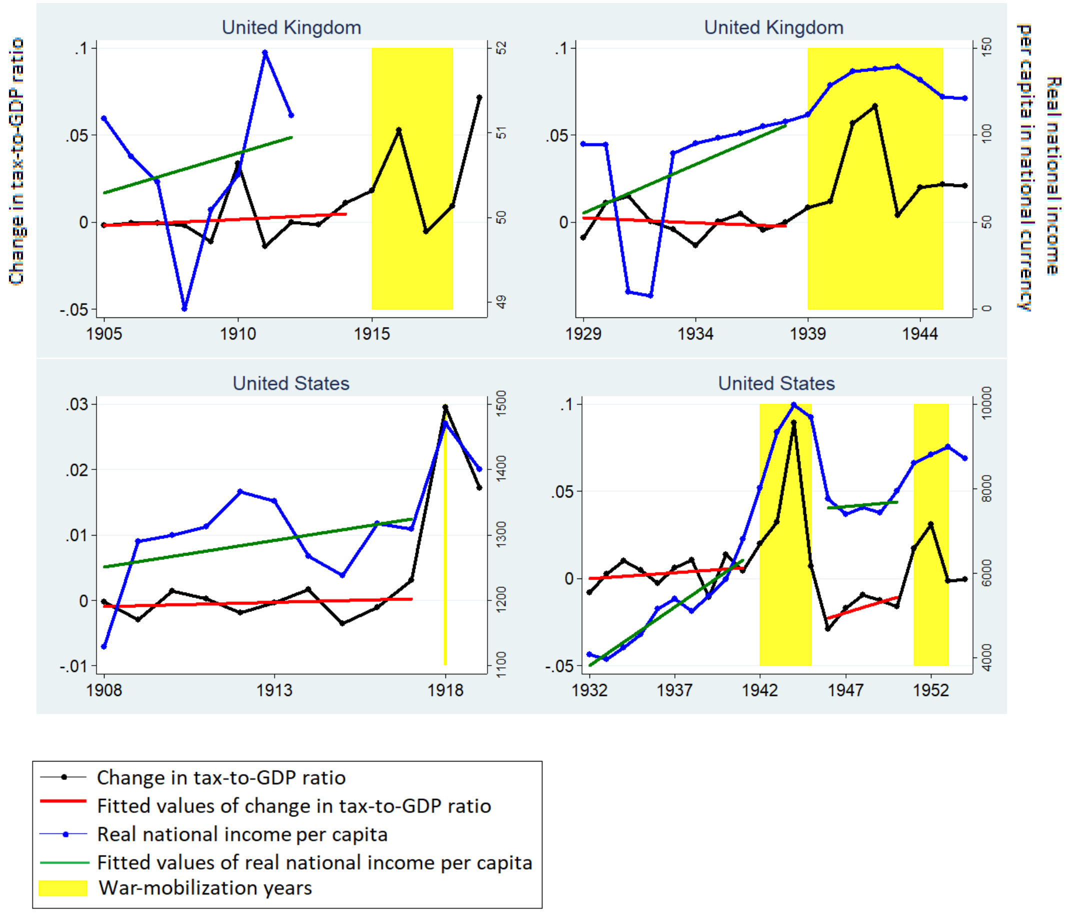


Figure 6: Change in tax-to-GDP ratio, real national income per capita, and war mobilization. Change in tax-to-GDP ratio in a year is calculated as the difference of that year's tax-to-GDP ratio from the previous year's, as measured by Rogers et al. (2018). Real national income per capita is calculated using the real national income and the mid-year-estimated population values given in Palgrave Macmillan Ltd. (2013). A country's war-mobilization years covers the time period in which it mobilized a certain amount of its population for a war with other countries, as measured by Rogers et al. (2018).

# B The Third- and Second-Period Choices of the Social Planner

The social planner aims to maximize the sum of expected utilities in the society over the three periods. I start by finding her optimal choices in the third period. Then taking them as given, I find her second-period choices.

I assume that the social planner is at equal distance to both groups. So, she does not transfer any income between them.

**The Third Period.** In the third period, the social planner's problem is given as

$$\max_{t_3, g_3} \omega(1-t_3) + \alpha_1 G_3$$

subject to the constraints $0 \leq g_3$, $t_3 \leq \tau_3$,

$$t_3 \omega = g_3,$$

and

$$G_3 = g_3 + (1-d)G_2.$$

By Assumption 1, the social planner does not collect any tax if $\alpha_3 = \alpha_L$. If $\alpha_3 = \alpha_H$, she imposes the highest tax rate and spends the whole tax revenue on public-good investment. So, we have the following result.

**Lemma 1.** *Given Assumption 1, if $\alpha_3 = \alpha_L$, then $t = g_3 = 0$. Otherwise, $t = \tau_3$ and $g_3 = \tau_3 \omega$.*

By her utility function and Lemma 1, we can write the social planner's third-period value function as

$$V_3(\tau_3, G_2; \alpha_3) = \begin{cases} \omega + \alpha_L(1-d)G_2 & \text{if } \alpha_3 = \alpha_L, \\ \omega(1-\tau_3) + \alpha_H\left(\tau_3\omega + (1-d)G_2\right) & \text{if } \alpha_3 = \alpha_H. \end{cases} \tag{5}$$

**The Second Period.** The social planner's problem in the second period is given as

$$\max_{t_2, g_2, \tau_3} \omega(1-t_2) + \alpha_2 G_2 + EV_3(\tau_3, G_2; \alpha_3)$$

subject to the constraints $0 \leq g_2$, $\tau_2 \leq \tau_3$, $t_2 \leq \tau_2$,

$$t_2\omega = g_2 + F(\tau_3 - \tau_2)$$

and

$$G_2 = (1-d)G_1 + g_2.$$

where

$$EV_3(\tau_3, G_2; \alpha_3) = \phi V_3(\tau_3, G_2; \alpha_H) + (1-\phi)V_3(\tau_3, G_2; \alpha_L)$$

is the social planner's third-period expected value function.

By Assumption 1, the social planner does not collect any tax if $\alpha_3 = \alpha_L$. If $\alpha_3 = \alpha_H$, she imposes the highest tax rate and spends the whole tax revenue on public-good investment. So, we have the following result.

**Lemma 2.** *Given Assumption 1, if $\alpha_2 = \alpha_H$, then $t = \tau_2$. Otherwise, $t = \frac{F(\tau_3 - \tau_2)}{\omega}$.*

Using Lemma 2, we can rewrite the social planner's second-period problem as

$$\begin{aligned}\max_{\tau_3} \quad & (\alpha_2 + E(\alpha)(1-d))\,(1-d)G_1 \\ & + \omega(1-\tau_2) + \lambda_2\left[\omega\tau_2 - F(\tau_3 - \tau_2)\right] \\ & + \omega(1-\tau_3) + E(\lambda_3)\omega\tau_3\end{aligned}$$

subject to the constraint $\tau_2 \leq \tau_3$ where

$$\lambda_2 = \max\{\alpha_2 + E(\alpha)(1-d), 1\}$$

is the value of public funds in the second period, and

$$E(\lambda_3) = \phi\alpha_H + (1-\phi)$$

is the expected value of public funds in the third period for the social planner.

Then, it is easy to show that the social planner's fiscal capacity investment, $\tau_3^s - \tau_2$, satisfies the first-order conditions

$$\omega\left[E(\lambda_3) - 1\right] + \mu = \lambda_2 F_\tau(\tau_3^s - \tau_2), \tag{6}$$

$$\mu(\tau_3^s - \tau_2) = 0$$

and

$$\mu \geq 0$$

where $\mu$ is the Lagrange multiplier.

Because, $E(\lambda_3) > 1$, it is clear that the social planner makes a positive investment in fiscal capacity—that is, $\tau_3^s > \tau_2$.

**Lemma 3.** *Given Assumption 1, we have* $\tau_3^s > \tau_2$.

By her utility function and Lemma 2, the social planner's second-period value function is given as

$$\begin{aligned}V_2(\tau_2, G_2; \alpha_L) = & \left[\alpha_L + E(\alpha)(1-d)\right](1-d)G_1 \\ & + \omega(1-\tau_2) + \left[\omega\tau_2 - F(\tau_3^s(\alpha_L) - \tau_2)\right] \\ & + \omega(1-\tau_3^s(\alpha_L)) + \left[\phi\alpha_H + (1-\phi)1\right]\omega\tau_3^s(\alpha_L),\end{aligned}$$

and

$$\begin{aligned}V_2(\tau_2, G_2; \alpha_H) = & \left[\alpha_H + E(\alpha)(1-d)\right](1-d)G_1 \\ & + \omega(1-\tau_2) + (\alpha_H + E(\alpha)(1-d))\left[\omega\tau_2 - F(\tau_3^s(\alpha_H) - \tau_2)\right] \\ & + \omega(1-\tau_3^s(\alpha_H)) + \left[\phi\alpha_H + (1-\phi)1\right]\omega\tau_3^s(\alpha_H).\end{aligned}$$

Then, the social planner's second-period expected value function is

$$EV_2(\tau_2, G_2; \alpha_2) = \phi V_2(\tau_2, G_2; \alpha_H) + (1-\phi) V_2(\tau_2, G_2; \alpha_L). \tag{7}$$

# C Derivation of the First-Order Conditions of the Social Planner in the First Period

By her first-period problem in Section 3, second-period expected value function in Equation 7, and policy choices in Proposition 1, the social planner's problem in the first period can be rewritten as following.

If $\alpha_1 = \alpha_H$, the social planner's problem is

$$\begin{aligned}
\max_{\tau_2} \; & \left[\alpha_H + E(\alpha)(1-d) + E(\alpha)(1-d)^2\right](1-d)G_0 \\
& + \omega(1-\tau_1) + \left[\alpha_H + E(\alpha)(1-d) + E(\alpha)(1-d)^2\right]\left[\tau_1\omega - F(\tau_2 - \tau_1)\right] \\
& + \omega(1-\tau_2) \\
& + \phi\left(\alpha_H + E(\alpha)(1-d)\right)\left[\omega\tau_2 - F(\tau_3^s(\alpha_H) - \tau_2)\right] \\
& + (1-\phi)\left[\omega\tau_2 - F(\tau_3^s(\alpha_L) - \tau_2)\right] \\
& + \phi\left[\omega(1-\tau_3^s(\alpha_H)) + \left[\phi\alpha_H + (1-\phi)\right]\omega\tau_3^s(\alpha_H)\right] \\
& + (1-\phi)\left[\omega(1-\tau_3^s(\alpha_L)) + \left[\phi\alpha_H + (1-\phi)\right]\omega\tau_3^s(\alpha_L)\right]
\end{aligned}$$

subject to $\tau_1 \leq \tau_2$.

Notice that in Equation 6, we have $\mu = 0$ by Lemma 3. Then, by applying the implicit function theorem in the same equation and using $F_{\tau\tau} \neq 0$, we have $\frac{\partial \tau_3^s}{\partial \tau_2} = 1$. So, if $\alpha_1 = \alpha_H$, the social planner's first-order conditions are

$$\omega\left[\phi\left(\alpha_H + E(\alpha)(1-d)\right) + (1-\phi) - 1\right] + \omega\left[\phi\alpha_H + (1-\phi) - 1\right] + \mu$$

$$= \left[\alpha_H + E(\alpha)(1-d) + E(\alpha)(1-d)^2\right] F_\tau(\tau_2^s - \tau_1),$$

$$\mu(\tau_2^s - \tau_1) = 0$$

and

$$\mu \geq 0.$$

If $\alpha_1 = \alpha_L$, the social planner's problem is

$$\begin{aligned}
\max_{\tau_2} & \left[\alpha_L + E(\alpha)(1-d) + E(\alpha)(1-d)^2\right](1-d)G_0 \\
& + \omega\left(1 - \frac{F(\tau_2 - \tau_1)}{\omega}\right) \\
& + \omega(1-\tau_2) \\
& + \phi\left(\alpha_H + E(\alpha)(1-d)\right)\left[\omega\tau_2 - F(\tau_3^s(\alpha_H) - \tau_2)\right] \\
& + (1-\phi)\left[\omega\tau_2 - F(\tau_3^s(\alpha_L) - \tau_2)\right] \\
& + \phi\left[\omega(1-\tau_3^s(\alpha_H)) + \left[\phi\alpha_H + (1-\phi)\right]\omega\tau_3^s(\alpha_H)\right] \\
& + (1-\phi)\left[\omega(1-\tau_3^s(\alpha_L)) + \left[\phi\alpha_H + (1-\phi)\right]\omega\tau_3^s(\alpha_L)\right]
\end{aligned}$$

subject to $\tau_1 \leq \tau_2$.

Again in Equation 6, we have $\mu = 0$ by Lemma 3. Then, by applying the implicit function theorem in the same equation and using $F_{\tau\tau} \neq 0$, we have $\frac{\partial \tau_3^s}{\partial \tau_2} = 1$. So, if $\alpha_1 = \alpha_L$, the social planner's first-order conditions are

$$\omega\left[\phi\left(\alpha_H + E(\alpha)(1-d)\right) + (1-\phi) - 1\right] + \omega\left[\phi\alpha_H + (1-\phi) - 1\right] + \mu = F_\tau(\tau_2^s - \tau_1),$$

$$\mu(\tau_2^s - \tau_1) = 0$$

and

$$\mu \geq 0.$$

We can combine the above two results as following

$$\omega\left[E\lambda_2 + E\lambda_3 - 2\right] + \mu = \lambda_1 F_\tau(\tau_2^s - \tau_1), \tag{8}$$

$$\mu(\tau_2^s - \tau_1) = 0$$

and

$$\mu \geq 0$$

where

$$E\lambda_2 = \phi\left(\alpha_H + E(\alpha)(1-d)\right) + (1-\phi)$$

and

$$E\lambda_3 = \phi\alpha_H + (1-\phi)$$

are the expected values of public funds in the second and third periods, respectively,

$$\lambda_1 = \max\{\alpha_1 + (1-d)E(\alpha) + (1-d)^2 E(\alpha), 1\}$$

is the value of public funds in the first period for the social planner.

# D The Third- and Second-Period Equilibria

**The Third Period.** The problem of the third period's incumbent, $I_3$, is given as

$$\max_{t_3, r^{I_3}, r^{O_3}, g_3} \omega(1-t_3) + r^{I_3} + \alpha_3 G_3$$

subject to the constraints $0 \le g_3$, $t_3 \le \tau_3$,

$$0 \le r^{J_3} \text{ for each } J_3 \in \{I_3, O_3\},$$

$$t_3 \omega = g_3 + \frac{r^{I_3} + r^{O_3}}{2},$$

and

$$G_3 = (1-d)G_2 + g_3.$$

As it does not benefit her, the incumbent does not transfer any tax revenue to the opponent group. So, we have $r^{O_3} = 0$. Additionally, because her utility function is linear, the incumbent spends the whole tax revenue either on transfers to her group or on public-good investment. Therefore, by Assumption 2, if $\alpha_3 = \alpha_H$, she chooses the highest rate and spends the whole tax revenue on public-good investment. If $\alpha_3 = \alpha_L$, she again chooses the highest tax rate but spends the whole tax revenue on transfers to her own group. We can collect these results as following.

**Lemma 4.** *Given Assumption 2, we have $t_3 = \tau_3$. If $\alpha_3 = \alpha_H$, then $r^{I_3} = r^{O_3} = 0$ and $g_3 = \tau_3 \omega$. Otherwise, $r^{I_3} = 2\tau_3 \omega$ and $r^{O_3} = g_3 = 0$.*

Then, in the third period, the incumbent's value function is given as

$$V_{I_3}(\tau_3, G_2; \alpha_3) = \begin{cases} \omega(1-\tau_3) + \alpha_H \left(\tau_3 \omega + (1-d)G_2\right) & \text{if } \alpha_3 = \alpha_H, \\ \omega(1-\tau_3) + 2\tau_3 \omega + \alpha_L (1-d) G_2 & \text{if } \alpha_3 = \alpha_L, \end{cases} \tag{9}$$

and the opponent's is given as

$$V_{O_3}(\tau_3, G_2; \alpha_3) = \begin{cases} \omega(1-\tau_3) + \alpha_H \left(\tau_3 \omega + (1-d)G_2\right) & \text{if } \alpha_3 = \alpha_H, \\ \omega(1-\tau_3) + \alpha_L (1-d) G_2 & \text{if } \alpha_3 = \alpha_L. \end{cases} \tag{10}$$

**The Second Period.** The problem of the second period's incumbent, $I_2$, is given as

$$\max_{t_2, r^{I_2}, r^{O_2}, g_2, \tau_3} \omega(1-t_2) + r^{I_2} + \alpha_2 G_2 + p_{I_2} EV_{I_3}(\tau_3, G_2; \alpha_3) + (1-p_{I_2}) EV_{O_2}(\tau_3, G_2; \alpha_3)$$

subject to the constraints $0 \leq g_2$, $t_2 \leq \tau_2$, $\tau_2 \leq \tau_3$,

$$0 \leq r^{J_2} \text{ for each } J_2 \in \{I_2, O_2\},$$

$$t_2 \omega = g_2 + \frac{r^{I_2} + r^{I_2}}{2} + F(\tau_3 - \tau_2),$$

and

$$G_2 = (1-d)G_1 + g_2$$

where $p_{I_2} = p_I$ if $I_2 = I_1$, and $p_{I_2} = 1 - p_I$ otherwise. Additionally,

$$EV_{I_3}(\tau_3, G_2; \alpha_3) = \phi V_{I_3}(\tau_3, G_2; \alpha_H) + (1-\phi) V_{I_3}(\tau_3, G_2; \alpha_L)$$

is the incumbent's third-period expected value function if she is the incumbent in the third period, and

$$EV_{O_2}(\tau_3, G_2; \alpha_3) = \phi V_{O_2}(\tau_3, G_2; \alpha_H) + (1-\phi) V_{O_2}(\tau_3, G_2; \alpha_L)$$

is her third-period expected value function if she is the opponent in the third period.

Following a similar reasoning as in the third period, we get equilibrium policies for the second period's incumbent.

**Lemma 5.** *Given Assumption 2, we have* $t_2 = \tau_2$. *If* $\alpha_2 = \alpha_H$, *then* $r^{I_2} = r^{O_2} = 0$ *and* $g_2 = \tau_2 \omega - F(\tau_3 - \tau_2)$. *Otherwise,* $r^{I_2} = 2\left[\tau_2 \omega - F(\tau_3 - \tau_2)\right]$ *and* $r^{O_2} = g_2 = 0$.

Then, we can rewrite the second-period incumbent's problem as

$$\begin{aligned} \max_{\tau_3} \quad & \left(\alpha_2 + E(\alpha)(1-d)\right)(1-d)G_1 \\ & + \omega(1-\tau_2) + \lambda_2^{I_2}\left[\omega \tau_2 - F(\tau_3 - \tau_2)\right] \\ & + \omega(1-\tau_3) + E(\lambda_3^{I_3})\omega \tau_3 \end{aligned} \tag{11}$$

subject to the constraint $\tau_3 \geq \tau_2$ where

$$\lambda_2^I = \max\left\{\alpha_2 + E(\alpha)(1-d), 2\right\}$$

is the value of public funds in the second period, and

$$E_{I_2}(\lambda_3^I) = \phi \alpha_H + (1-\phi) p_{I_2} 2$$

is the expected value of public funds in the third period for the second period's incumbent. So, the fiscal-capacity investment of the second period's incumbent, $\tau_3^* - \tau_2$, satisfies the first-order conditions

$$\omega\left[E_{I_2}(\lambda_3^I) - 1\right] + \mu = \lambda_2^{I_2} F_\tau(\tau_3^* - \tau_2), \tag{12}$$

$$\mu(\tau_3^* - \tau_2) = 0,$$

$$\mu \geq 0,$$

where $\mu$ is the Lagrange multiplier. Then, it is easy to show the following result.

**Lemma 6.** *Given Assumption 2, we have $\tau_3^* - \tau_2 > 0$ if and only if $E_{I_2}(\lambda_3) > 1$.*

Then, in the second period, the incumbent's value function is given as

$$\begin{aligned} V_{I_2}(\tau_2, G_1; \alpha_2) = {} & (\alpha_2 + E(\alpha)(1-d))\,(1-d)G_1 \\ & + \omega(1-\tau_2) + \lambda_2^I\left[\omega\tau_2 - F(\tau_3^* - \tau_2)\right] \\ & + \omega(1-\tau_3^*) + E_{I_2}(\lambda_3^I)\omega\tau_3^* \end{aligned} \tag{13}$$

and the opponent's is given as

$$\begin{aligned} V_{O_2}(\tau_2, G_1; \alpha_2) = {} & (\alpha_2 + \delta E(\alpha)(1-d))\,(1-d)G_1 \\ & + \omega(1-\tau_2) + \lambda_2^O\left[\omega\tau_2 - F(\tau_3^* - \tau_2)\right] \\ & + \omega(1-\tau_3^*) + E_{O_2}(\lambda_3^I)\omega\tau_3^* \end{aligned} \tag{14}$$

where

$$\lambda_2^O = \begin{cases} \alpha_H + E(\alpha)(1-d) & \text{if } \alpha_2 = \alpha_H, \\ 0 & \text{if } \alpha_2 = \alpha_L \end{cases}$$

is the value of public funds in the second period, and

$$E_{O_2}(\lambda_3^I) = \phi\alpha_H + (1-\phi)(1-p_{I_2})2.$$

is the expected value of public funds in the third period for the second period's opponent.

So, the second-period incumbent's expected value function is given as

$$EV_{I_2}^I(\tau_2, G_2; \alpha_2) = \phi V_{I_2}(\tau_2, G_2; \alpha_H) + (1-\phi)V_{I_2}(\tau_2, G_2; \alpha_L), \tag{15}$$

and the second-period opponent's is given as

$$EV_{O_2}^I(\tau_2, G_2; \alpha_2) = \phi V_{O_2}(\tau_2, G_2; \alpha_H) + (1-\phi)V_{O_2}(\tau_2, G_2; \alpha_L). \tag{16}$$

# E Derivation of the First-Order Conditions of the First Period's Incumbent

By her first-period problem in Section 4, the second-period incumbent's and opponent's expected value functions in Equations 15 and 16, respectively, and her policy choices in Proposition 3, the first-period incumbent's problem can rewritten as

$$
\begin{aligned}
\max_{\tau_2} & \left[\alpha_H + E(\alpha)(1-d) + E(\alpha)(1-d)^2\right](1-d)G_0 \\
& + \omega(1-\tau_1) \\
& + \lambda_1^{I_1}\left[\tau_1\omega - F(\tau_2-\tau_1)\right] \\
& + \omega(1-\tau_2) \\
& + p_I\phi\Big[(\alpha_H + E(\alpha)(1-d))\Big[\omega\tau_2 - F(\tau_3^{I,*}-\tau_2)\Big]\Big] \\
& + (1-p_I)\phi\Big[(\alpha_H + E(\alpha)(1-d))\Big[\omega\tau_2 - F(\tau_3^{O,*}-\tau_2)\Big]\Big] \\
& + p_I(1-\phi)\Big[2\Big[\omega\tau_2 - F(\tau_3^{I,*}-\tau_2)\Big]\Big] \\
& + p_I\left\{\omega(1-\tau_3^{I,*}) + p_I\phi\alpha_H\omega\tau_3^{I,*} + (1-p_I)\phi\alpha_H\omega\tau_3^{I,*} + p_I(1-\phi)2\omega\tau_3^{I,*}\right\} \\
& + (1-p_I)\left\{\omega(1-\tau_3^{O,*}) + p_I\phi\alpha_H\omega\tau_3^{O,*} + (1-p_I)\phi\alpha_H\omega\tau_3^{O,*} + p_I(1-\phi)2\omega\tau_3^{O,*}\right\}
\end{aligned}
$$

subject to the constraint $\tau_1 \leq \tau_2$ where $\tau_3^{I,*}$ and $\tau_3^{O,*}$ are the third-period fiscal-capacity choices of the first period's incumbent and opponent, respectively, if they get the political power in the second period, as characterized in Equation 12. Specifically, $\tau_3^{I,*} = \tau_3^*$ in Equation 12 when $I_2 = I_1$, and $\tau_3^{O,*} = \tau_3^*$ in the same equation when $I_2 = O_1$.

Notice that in Equation 12, when $\tau_3^* - \tau_2 > 0$ and so that $\mu = 0$, by applying the implicit function theorem and using $F_{\tau\tau} \neq 0$, we get $\frac{\partial \tau_3^*}{\partial \tau_2} = 1$. Then, it is easy to show that the first-period incumbent's fiscal-capacity investment, $\tau_2^* - \tau_1$, satisfies the first-order conditions

$$\omega\left\{E_I(\lambda_2) - 1 + p\left[E_I(\lambda_3) - 1\right]\right\} = \lambda_1^I F_\tau(\tau_2^* - \tau_1), \tag{17}$$

$$\mu(\tau_2^* - \tau_1) = 0$$

and

$$\mu \geq 0$$

where

$$E_I(\lambda_2) = \phi\left(\alpha_H + E(\alpha)(1-d)\right) + (1-\phi)p_I 2$$

and

$$E_I(\lambda_3) = \phi\alpha_H + (1-\phi)p_I 2$$

are the expected value of public funds in the second and third periods, respectively, and

$$\lambda_1^I = \max\{\alpha_1 + (1-d)E(\alpha) + (1-d)^2 E(\alpha), 2\}$$

is the value of public funds in the first period for the incumbent,

$$p = \begin{cases} 1 - p_I & \text{if } E_{I_2}(\lambda_3) < 1 \text{ and } E_{O_2}(\lambda_3) > 1, \\ 1 & \text{if } E_{I_2}(\lambda_3) > 1 \text{ and } E_{O_2}(\lambda_3) > 1, \\ p_I & \text{if } E_{I_2}(\lambda_3) > 1 \text{ and } E_{O_2}(\lambda_3) < 1 \end{cases} \tag{18}$$

and $\mu$ is the Lagrange multiplier. Note that to simplify my analysis and avoid non-generic cases, I do not consider the cases where $E_{I_2}(\lambda_3) = 1$ or $E_{O_2}(\lambda_3) = 1$. Additionally, it is not possible to have both $E_{I_2}(\lambda_3) < 1$ and $E_{O_2}(\lambda_3) < 1$.

# F Proof of Propositions

## F.1 Proof of Proposition 5

First, assume that $\alpha_1 = \alpha_H$. Then, if the social planner invests in the second period's fiscal capacity, by Equation 3 and Assumption 1, we can write her first-order condition (FOC) as

$$\omega\left[\phi\left(\alpha_H + E(\alpha)(1-d)\right) + (1-\phi) - 1 + \phi\alpha_H + (1-\phi) - 1\right]$$

$$= [\alpha_H + (1-d)E(\alpha) + (1-d)^2 E(\alpha)]F_\tau(\tau_2^s - \tau_1),$$

and, if the incumbent invests in the second period's fiscal capacity, by Equation 4 and Assumption 2, we can write her FOC as

$$\omega\left\{\phi\left(\alpha_H + E(\alpha)(1-d)\right) + (1-\phi)p_I 2 - 1 + p\left[\phi\alpha_H + (1-\phi)p_I 2 - 1\right]\right\}$$

$$= [\alpha_H + (1-d)E(\alpha) + (1-d)^2 E(\alpha)]F_\tau(\tau_2^* - \tau_1).$$

Because $F$ is increasing and convex, $F_\tau$ is increasing. Thus, the incumbent's fiscal-capacity investment is higher than the social planner's if and only if the left-hand side of the incumbent's FOC is larger than the social planner's:

$$\phi\left(\alpha_H + E(\alpha)(1-d)\right) + (1-\phi) - 1 + \phi\alpha_H + (1-\phi) - 1$$

$$< \phi\left(\alpha_H + E(\alpha)(1-d)\right) + (1-\phi)p_I 2 - 1 + p\left[\phi\alpha_H + (1-\phi)p_I 2 - 1\right],$$

that is,

$$0 < (1-\phi)(2p_I - 1) + \phi\alpha_H(p-1) + (1-\phi)(2pp_I - 1) - (p-1). \tag{19}$$

If we show that the derivative of the right-hand side of Inequality 19 with respect to $p_I$ is positive, then the proof when $\alpha_1 = \alpha_H$ is done. In the following, I show it.

The derivative of the right-hand side of the Inequality 19 with respect to $p_I$ is

$$2(1-\phi) + \phi\alpha_H p' + (1-\phi)2[p + p'p_I] - p',$$

which is equal to

$$2(1-\phi)(p+1) + [(1-\phi)2p_I + \phi\alpha_H - 1]p'. \tag{20}$$

If $(1-\phi)2p_I + \phi\alpha_H - 1 < 0$, then $p = 1 - p_I$ by Equation 18. So, $p' < 0$ and Expression 20 is positive.

If $(1-\phi)2p_I + \phi\alpha_H - 1 > 0$, then $p = 1$ or $p = p_I$ by Equation 18. So, $p' \geq 0$ and again Expression 20 is positive.

Second, assume that $\alpha_1 = \alpha_L$. Then, if the social planner invests in the second period's fiscal capacity, by Equation 3 and Assumption 1, we can write her FOC as

$$\omega\left[\phi\left(\alpha_H + E(\alpha)(1-d)\right) + (1-\phi) - 1 + \phi\alpha_H + (1-\phi) - 1\right] = F_\tau(\tau_2^s - \tau_1)$$

and, if the incumbent invests in the second period's fiscal capacity, by Equation 4 and Assumption 2, we can write her FOC as

$$\omega\left\{\phi\left(\alpha_H + E(\alpha)(1-d)\right) + (1-\phi)p_I 2 - 1 + p\left[\phi\alpha_H + (1-\phi)p_I 2 - 1\right]\right\} = 2F_\tau(\tau_2^* - \tau_1).$$

As in the first case, incumbent's fiscal-capacity investment is higher than the social planner's if and only if the left-hand side of the incumbent's FOC is larger than the social planner's:

$$\phi\left(\alpha_H + E(\alpha)(1-d)\right) + (1-\phi) - 1 + \phi\alpha_H + (1-\phi) - 1$$

$$< \frac{\phi\left(\alpha_H + E(\alpha)(1-d)\right) + (1-\phi)p_I 2 - 1 + p\left[\phi\alpha_H + (1-\phi)p_I 2 - 1\right]}{2},$$

that is,

$$\phi(\alpha_H + E(\alpha)(1-d)) - 1 < (1-\phi)(2p_I - 2) + \phi\alpha_H(p-2) + (1-\phi)(2pp_I - 2) - (p-2). \tag{21}$$

Again, the result follows from the fact that the derivative of the right-hand side of Inequality 21 with respect to $p_I$ is positive, which can be shown similarly to the first case.

$\square$

## F.2 Proof of Proposition 6

Assume that $\alpha_1 = \alpha_H$. Then, if there is positive investment in the second period's fiscal capacity, by Equation 4 and Assumption 2, we can write the first-order condition (FOC) of the first period's incumbent as

$$\omega \left\{ \phi \left( \alpha_H + E(\alpha)(1-d) \right) + (1-\phi) p_I 2 - 1 + p \left[ \phi \alpha_H + (1-\phi) p_I 2 - 1 \right] \right\}$$
$$= [\alpha_H + (1-d)E(\alpha) + (1-d)^2 E(\alpha)] F_\tau(\tau_2^* - \tau_1). \tag{22}$$

Note that because $F$ is increasing and convex, $F_\tau$ is increasing. An increase in income, $\omega$, or in political stability, $p_I$, increases the left-hand side of Equation 22. This implies that $\tau_2^*$ increases because $F_\tau$ is increasing.

A decrease in $F_\tau$ decreases the right-hand side of Equation 22. Then, $\tau_2^*$ should increases because $F_\tau$ is increasing.

$\square$

## F.3 Proof of Proposition 7

If there is positive investment in second period's fiscal capacity and $\alpha_1 = \alpha_H$, by Equation 4 and Assumption 2, we can write the incumbent's first-order condition (FOC) as

$$\omega \frac{\phi^2 a + \phi b + c}{\phi e + f} = F_\tau(\tau_2^* - \tau_1) \tag{23}$$

where

$$a = \frac{1}{y}(\alpha_H - \alpha_L)(1-d),$$

$$b = \left( \frac{1}{y}\alpha_H - r \right)(1+p) + \frac{1}{y}\alpha_L(1-d),$$

$$c = (r-1)(1+p),$$

$$r = p_I 2(1-\theta) + (1-p_I)2\theta,$$

$$e = \frac{1}{y}(\alpha_H - \alpha_L) \sum_{t=1}^{2} (1-d)^t$$

and

$$f = \frac{1}{y}\alpha_1 + \frac{1}{y}\alpha_L \sum_{t=1}^{2}(1-d)^t.$$

Because $F$ is increasing and convex, $F_\tau$ is increasing. Thus, an increase in the external-war probability, $\phi$, increases fiscal-capacity investment, $\tau_2^* - \tau_1$, if and only if it increases the left-hand side of Equation 23; in other words, if and only if the derivative of the left-hand side of Equation 23 with respect to $\phi$ is positive. Then, taking the derivative of the left-hand side of Equation 23 with respect to $\phi$, we have

$$\omega \frac{(2\phi a + b)(\phi e + f) - e(\phi^2 a + \phi b + c)}{(\phi e + f)^2}$$

which is positive if and only if

$$ae\phi^2 + 2af\phi + bf - ec > 0.$$

Then, the result follows form the fact that $ae > 0$ and $af > 0$.

$\square$

## F.4 Proof of Proposition 8

If there is positive investment in second period's fiscal capacity and $\alpha_1 = \alpha_H$, by Equation 4 and Assumption 2, we can write the incumbent's FOC as

$$\omega \frac{md + n}{ud^2 + vd + z} = F_\tau(\tau_2^* - \tau_1) \tag{24}$$

where

$$m = -\frac{1}{y}\phi E(\alpha),$$

$$n = \phi\frac{1}{y}\left[\alpha_H + E(\alpha)\right] + (1-\phi)r - 1 + p\left[\phi\frac{1}{y}\alpha_H + (1-\phi)r - 1\right],$$

$$r = p_I 2(1-\theta) + (1-p_I)2\theta,$$

$$u = \frac{1}{y}E(\alpha),$$

$$v = -\frac{1}{y}3E(\alpha),$$

and

$$z = \frac{1}{y}\left[\alpha_1 + 2E(\alpha)\right].$$

Because $F$ is increasing and convex, $F_\tau$ is increasing. Thus, an increase in the deprecation rate of the public good, $d$, increases fiscal-capacity investment, $\tau_2^* - \tau_1$, if and only if it increases the left-hand side of Equation 24; in other words, if and only if the derivative of the left-hand side of Equation 24 with respect to $d$ is positive. Then, taking the derivative of the left-hand side of Equation 24 with respect to $d$, we have

$$\omega \frac{m(ud^2 + vd + z) - (2ud + v)(md + n)}{(ud^2 + vd + z)^2}$$

which is positive if and only if

$$-mud^2 - 2nud + mz - vn > 0. \tag{25}$$

The result follows from the fact that the left-hand side of Inequality 25 is decreasing in $d$.

□

## F.5 Proof of Proposition 9

If there is positive investment in second period's fiscal capacity and $\alpha_1 = \alpha_L$, by Equation 4 and Assumption 2, we can write the incumbent's first-order condition as

$$\omega \left\{ \phi \left( \alpha_H + E(\alpha)(1-d) \right) + (1-\phi)p2 - 1 + p \left[ \phi \alpha_H + (1-\phi)p2 - 1 \right] \right\} = 2F_\tau(\tau_2^* - \tau_1). \tag{26}$$

Note that because $F$ is increasing and convex, $F_\tau$ is increasing. An increase in income, $\omega$, or in political stability, $p_I$, or in external-war probability, or a decrease in the public good's depreciation rate, $d$, increases the left-hand side of Equation 26. This implies that $\tau_2^*$ increases since $F_\tau$ is increasing.

A decrease in $F_\tau$ decreases the right-hand side of Equation 26. This implies that $\tau_2^*$ increases since $F_\tau$ is increasing.

□

## F.6 Proof of Proposition 10

Given the public good's price be $y > 0$, we can show that the problem of the first period's incumbent is given as

$$\max_{t_1, r^{I_1}, r^{O_1}, g_1, \tau_2} \omega(1 - t_1) + r^{I_1} + \alpha_1 G_1 + p_I EV^I_{I_2,y}(\tau_2, G_1; \alpha_2) + (1 - p_I) EV^I_{O_2,y}(\tau_2, G_1; \alpha_2)$$

subject to the constraints $0 \leq t \leq \tau_1$, $0 \leq r^{I_1}$, $0 \leq r^{O_1}$, $0 \leq g_1$, $\tau_1 \leq \tau_2$,

$$t_1 \omega = y g_1 + \frac{r^{I_1} + r^{O_1}}{2} + F(\tau_2 - \tau_1),$$

and

$$G_1 = (1 - d)G_0 + g_1$$

where $EV^I_{I_2,y}(\tau_2, G_2; \alpha_2)$ is the expected utility of the incumbent if she gets the political power in the second period when the public good's price is $y$, fiscal capacity is $\tau_2$, and the public-good stock is $G_1$. A similar interpretation also applies to $EV^I_{O_2,y}(\tau_2, G_1; \alpha_2)$ if the incumbent is the opponent in the second period. These functions are obtained from the equilibrium choices in future in a similar way as in Appendix D.[15]

Then, following a similar reasoning in Section 4, we can state the following result.

**Lemma 7.** *Given Assumption 3, the first-period incumbent's policies are given as following.*

*(i) If $\lambda^I_{g,y} < \lambda^I_r$, then $t = \tau_1$, $r^{I_1} = 2\left[\tau_1\omega - F(\tau_2 - \tau_1)\right]$, and $r^{O_1} = g_1 = 0$.*

*(ii) If $\lambda^I_{g,y} > \lambda^I_r$, then $t = \tau_1$, $r^{I_1} = r^{O_1} = 0$, and $g_1 = \frac{1}{y}\left[\tau_1\omega - F(\tau_2 - \tau_1)\right]$.*

By her problem and Lemma 7, we can show that the incumbent's fiscal-capacity investment, $\tau_2^* - \tau_1$, satisfies the first-order conditions (FOC)

$$\omega\left\{E_I(\lambda_{2,y}) - 1 + p\left[E_I(\lambda_{3,y}) - 1\right]\right\} + \mu = \lambda^I_{1,y} F_\tau(\tau_2^* - \tau_1), \tag{27}$$

$$\mu(\tau_2^* - \tau_1) = 0$$

and

$$\mu \geq 0,$$

where

$$E_I(\lambda_{2,y}) = \phi\frac{1}{y}\left[\alpha_H + E(\alpha)(1-d)\right] + (1-\phi)\left[p_I 2(1-\theta) + (1-p_I)2\theta\right]$$

and

$$E_I(\lambda_{3,y}) = \phi\frac{1}{y}\alpha_H + (1-\phi)\left[p_I 2(1-\theta) + (1-p_I)2\theta\right]$$

are the expected value of public funds for the incumbent in the second and third periods, respectively,

$$\lambda^I_{1,y} = \max\left\{\lambda^I_r, \lambda^I_{g,y}\right\}$$

is the value of public funds for the incumbent in the first period,

$$p = \begin{cases} 1 - p_I & \text{if only } O_1 \text{ invests in } \tau_3 \text{ in the second period,} \\ 1 & \text{if both } I_1 \text{ and } O_1 \text{ invests in } \tau_3 \text{ in the second period,} \\ p_I & \text{if only } I_1 \text{ invests in } \tau_3 \text{ in the second period,} \end{cases}$$

and $\mu$ is the Lagrange multiplier.[16]

[15] The formula and derivation of these functions are available from the author upon request.

[16] Derivation of the incumbent's first-order conditions when the public good's price is $y$ is similar to their derivation in Appendix E and available upon request from the author.

Assume that $\alpha_1 = \alpha_H$ so $\lambda_1^I = \lambda_{g,y}$. Then, we can rewrite the incumbent's FOC as

$$\omega \left( \frac{1}{y} A + B \right) = \frac{1}{y} C F_\tau(\tau_2^* - \tau_1)$$

where

$$A = \phi \left[ \alpha_H + E(\alpha)(1-d) \right] + p\phi\alpha_H,$$

$$B = (1+p) \left[ (1-\phi) p_I 2 - 1 \right],$$

and

$$C = \alpha_H + E(\alpha)(1-d) + E(\alpha)(1-d)^2.$$

Then, an increase in $y$ increases fiscal-capacity investment if and only if it increases $\frac{\frac{1}{y}A+B}{\frac{1}{y}C} = \frac{A+By}{C}$. Since, $C > 0$, $y$ increases $\frac{A+By}{C}$ if and only if $B = (1+p)\left[(1-\phi)p_I 2 - 1\right] > 0$; in other words, if and only if $(1-\phi)p_I 2 - 1 > 0$.

□

## F.7 Proof of Proposition 11

The first-period incumbent's first-order conditions (FOC) are given by Equation 27 in the Proof of Proposition. Then, if $\alpha_1 = \alpha_L$, we have $\lambda_1^I = \lambda_{g,y}$. So, we can rewrite the incumbent's FOC as

$$\omega \left( \frac{1}{y} A + B \right) = 2 F_\tau(\tau_2^* - \tau_1) \tag{28}$$

where

$$A = \phi \left[ \alpha_H + E(\alpha)(1-d) \right] + p\phi\alpha_H,$$

and

$$B = (1+p) \left[ (1-\phi) p_I 2 - 1 \right].$$

Note that because $F$ is increasing and convex, $F_\tau$ is increasing. An increase in $y$ decreases the left-hand side of Equation 28. This implies that $\tau_2^*$ decreases because $F_\tau$ is increasing.

□

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