Can market selection reduce anomalous behaviour in games?

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Abstract

We use an experiment to study whether market selection can reduce anomalous behaviour in games. In different treatments, we employ two alternative mechanisms, the random mechanism and the auction mechanism, to allocate the participation rights to the red hat puzzle game, a well-known logical reasoning problem. Compared to the random mechanism, the auction mechanism significantly reduces deviations from the equilibrium play in the red hat puzzle game. Our findings show that under carefully designed incentives, market competition can indeed reduce anomalous behaviour in games.

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1 Introduction

Anomalous behaviour in experimental game-theory suggests that people are less than fully rational. People are susceptible to psychological biases (e.g., Rabin, 1998; Conlisk, 1996; DellaVigna, 2009; Gabaix, 2017) and are not always able to perform the types of deductive reasoning demanded by the equilibrium solution (e.g., Crawford et al., 2013).[1] This might not be surprising to economists who recognise the limits of individual rationality in their day-to-day interactions, even when this is not always reflected in their professional work (Fehr and Tyran, 2005). Yet, it is not immediately obvious whether individual level bounded rationality necessarily falsifies the applicability of models that assume fully rational players.[2]

Markets are often used to allocate the rights (e.g., permits, licences, contracts) for performing the types of economic tasks modelled in games (i.e., situations involving strategic interactions)—the *market selects* players into the game.[3] A long-standing convention in economics is that markets divert resources to where they are valued the most. If the expected payoffs from the game are higher for the rational players relative to their less rational counterparts, then markets should result in the allocation of participation rights (i.e., the rights to play the game) to the former. Hence, behaviour in the game may be well approximated by models assuming fully rational players even when this is clearly not the case at the population level.[4]

However, it is not clear whether the above logic will naturally hold. Evidence from the economic and psychological literature suggest that human decision-makers are susceptible to a range of cognitive biases. A prominent example is *focusing failures* (e.g., Tor and Bazerman, 2003; Idson et al., 2004) such as when players fail to realise how their market behaviour should depend on their expected payoffs from the game. Even if all players anchor their market behaviour on the game, the less rational players may exhibit higher degrees of *over-confidence* (e.g., Camerer and Lovallo, 1999; Hoelzl and Rustichini, 2005) with respect to the expected payoffs from

[1]Where possible, we prefer to direct the reader to reviews of the established literature. Also, see Crawford (2013), Harstad and Selten (2013) and Rabin (2013) for discussions as to how bounded rationality can be incorporated into economic theory.

[2]Some economists such as Aumann (1985) take the "instrumental view" that the purpose of theory is to contribute to our comprehension of knowledge. Whilst we do not disagree with this view, we believe that it is also important to ask whether theory matches actual behaviour.

[3]Economists often build simple models to focus on the main economic interaction of interest. Indeed, the map paradox (Carroll, 1894) illustrates why a simpler model is sometimes more useful. Markets here can be interpreted as some un-modelled pre-game stage which determines players' participation into the game. This pre-game stage is irrelevant in equilibrium if all players are assumed to be fully rational and the selection outcome does not affect behaviour in the game.

[4]Inversely, markets may exacerbate the influence of the less rational players if their expected payoffs from the game are higher.

the game. Finally, people are also vulnerable to probability inference errors (e.g., Tversky and Kahneman, 1974). As such, it is unclear whether people's willingness to pay—to enter the game—will reflect their expected payoffs from the game.

In many economic situations, people's behaviour—and consequently, their expected payoffs from the game—depend on their beliefs about the behaviour of others. As Keynes (1974, p. 88) puts it "they are concerned, not with what an investment is really worth to a man who buys it 'for keeps', but with what the market will value it at under the influence of mass psychology ...". Even if the expected payoff from the game (in equilibrium) is higher for the rational relative to the less rational players, in the absence of common knowledge of rationality, rational players may not believe that other players entering the game will also be rational. Hence, it is not obvious whether well-designed markets will select rational players and drive behaviour (in games) closer to the equilibrium predictions.

In this paper, we study whether market selection can reduce anomalous behaviour (i.e., deviations from the equilibrium behaviour) in *games* (i.e., strategic interaction situations) where the equilibrium behaviour depends not only on players' ability to employ rational logical reasoning, but also their beliefs about the rational logical reasoning capabilities of other players in the game. This type of epistemological reasoning (i.e., reasoning about the reasoning of others) underlines many economic interactions. This task is well suited to laboratory experiments where the confounding forces such as liquidity constrains, beliefs and experiences can be carefully controlled. We consider an environment where auctions are used to select players into a game—auctions are one of the simplest and most common market mechanism used to allocate economic tasks (e.g., spectrum auctions, government projects).

We base our experiment on the three-player *Red Hat Puzzle* (RHP) game, a close variant of Littlewood's (1953) "Dirty faces problem".[5,6] We use the RHP game since the equilibrium, when modelled as a dynamic Bayesian game with incomplete information, requires players to apply strategic and epistemological reasoning—such reasoning is common in many economic interactions. Indeed, Weber (2001), Bayer and Chan (2007) and Bayer and Renou (2016a,b) use this feature of the RHP game to study "steps of individual bounded rationality".[7] Also, the equilibrium solution in

[5]Variations of the RHP game are often found in game theory textbooks (e.g. Myerson, 1991; Fudenberg and Tirole, 1991; Maschler et al., 2013), discussions about common knowledge (e.g., Geanakoplos, 1992; Samuelson, 2004) and epistemological reasoning (e.g., Fagin et al., 2004). Littlewood tells it as follows. "Three ladies, A, B, C in a railway carriage all have dirty faces and are all laughing. It suddenly flashes on A: why doesn't B realise C is laughing at her? — Heavens! I must be laughable."

[6]Choo et al. (2019a) show that prediction markets can help solve the red hat puzzle game by allowing traders to bet on the uncertain state of the nature in the game.

[7]The previous experiments primarily use the RHP game to study *k-level* (e.g., Nagel, 1995; Stahl and Wilson, 1994; Camerer et al., 2004) reasoning behaviour. Though behaviour in this study may

the game is Pareto optimal (in expectations) for all players. This ensures that the expected payoffs in the game are always higher for players who know the equilibrium solution and believe that all other players in the game will also play the equilibrium solution.[8]

To study whether market selection can reduce anomalous behaviour (i.e., non equilibrium behaviour) in the RHP game, we consider two mechanisms to select players into the RHP game: *auction* versus *random* mechanisms. However, players' behaviour in the RHP game, and consequently their auction bidding behaviour, may depend on their beliefs about the "rationality" of the other players who enter the RHP game. We therefore also vary the types of players with whom the selected players (i.e., those who enter the RHP game) will interact in the game: *computer-players* programmed to play the equilibrium behaviour versus *human* players. It should be clear that the strategic uncertainty element of the game is removed in the computer treatment and in doing so, reduces the RHP game into an individual decision task. Also note that the equilibrium solution in the RHP game neither depends on the selection mechanism nor on the type of opponents (computer or human). The above considerations result in a 2×2 experimental design.

We find that market selection (i.e., the use of auction as opposed to the random mechanism) significantly reduces deviations from the equilibrium behaviour in the RHP game—the reduction is largest when players face the most complex permutation of the RHP game. This holds when players interact with other human players as well as computer-players who are programmed to play the equilibrium. We show that this is because the auction mechanism selects "rational" players (i.e., those who know the equilibrium solution) into the RHP game more frequently than the random mechanism. Finally, behaviour in the RHP game do not seem to differ significantly depending on whether selected players interact with human or computer-players.

The experiment also enables us to study how players' "rationality" and strategic uncertainties are reflected in the auction bidding behaviour—we used a second-price auction. When the RHP game is transformed into an individual decision task such as in the computer-player treatment, we find that rational players often submit the equilibrium bid—the bidding behaviour of the less rational players are substantially

also involve some elements of the k-level model, such discussions will be omitted as they divert from the main research agenda.

[8] In a preliminary study, Choo (2014)—one of the authors in this study—embedded asset markets into the RHP game to study the influence of market interaction on anomalous behaviour. The experiment suggests that asset markets can in fact exacerbate anomalous behaviour in the RHP game. However, subsequent discussions with Zhou— the other author in this paper—revealed certain design limitations in the former study that made it difficult to evaluate the effects of markets on behaviour in the RHP game—we will elaborate on these limitations in the conclusion section. The discussions motivated the experimental design in this paper.

more “noisy”. Moving from the computer to human treatments, we observe that rational players “shade” their bids to compensate for the strategic uncertainties in the RHP game. In contrast, strategic uncertainties does not seem to affect the bidding behaviour of the less rational players.

This paper contributes to several strands of literature. The most relevant relates to the growing body of experimental evidence highlighting the intricacies between individual level bounded rationality and aggregate level outcomes.

In a related study, Kluger and Wyatt (2004) investigate whether individual probability judgement errors are reflected in asset market prices. To do so, they embed the “Monty Hall problem” into asset markets—traders trade assets which give them the rights to switch doors in the Monty Hall problem.[9] They find that market prices are close to the predicted equilibrium when the market consists of at least two *unbiased* Bayesian players, as determined by their behaviour in the Monty Hall problem. However, such occurrences are relatively rare, occurring only in around 25% of markets (Fehr and Tyran, 2005).

Like the Monty Hall problem, the RHP game requires players to perform Bayesian updating. However, the RHP game additionally requires players to apply strategic and epistemological reasoning. In addition to the different types of task studied (Monty Hall problem versus RHP game), there is also an important difference between our paper and Kluger and Wyatt (2004). Their paper mainly focuses on whether market prices will converge to the equilibrium.[10] In this paper, we focus on the allocation outcome of assets, that is whether the rational players get the assets more frequently than their bounded-rational counterparts. In fact, we also show that subjects’ “rationality” are reflected in their auction bidding behaviour.

A closely related concept to our study is the *market selection hypothesis* (e.g Alchian, 1950; Friedman, 1953) which posits that in the long-run, the evolutionary forces of the market will eventually drive out the less rational players.[11] Kendall and Oprea (2018) test the hypothesis in a laboratory experiment. They base their experiment on

[9]The Monty Hall Problem is inspired by a popular TV game show, where the host Monty hides a winning prize behind one of three closed doors. Contestants are invited to open a door. However, before the door is actually opened, Monty is pre-committed to opening a non-prize door and then offers each contestant the chance to switch her choice to the other unopened door. The dominant strategy in this problem is to always switch as it offers the contestant a 2/3 chance of picking the prize door.

[10]Even in markets where prices converge to the equilibrium, Kluger and Wyatt (2004) experiment suggests that the non-Bayesian players still hold a fair proportion of the assets.

[11]The market selection hypothesis is the economics analogy of natural selection. However, DeLong et al. (1991) and Blume and Easley (2006) show that the hypothesis does not always hold. Also, Fehr and Tyran (2005) argue that the market selection hypothesis may not be applicable in many economic situations. For example, it is difficult to see why a consumer who makes sub-optimal consumption decisions will be driven out of the consumer market.

the Blume and Easley (1992) multi-period model: at each period, players distribute their wealth over consumption and investment opportunities.[12] To trigger individual biases, returns on investments are linked to a Monty Hall type problem. Consistent to the market selection hypothesis, they find that unbiased Bayesian subjects are more likely to survive in the long-run.[13]

A key difference between this study and that of Kendall and Oprea (2018) is that in the latter, there are no markets where players compete for commodities: players' task is to allocate wealth between consumption and investment. In this view, our paper shows that the market selection hypothesis can also be supported in the short-run when players are allowed to compete for the rights to make investments.

In a price-setting game, Fehr and Tyran (2005, 2008) show that the effects of *money illusion* on aggregated prices depend on the strategic environment (Haltiwanger and Waldman, 1985, 1989).[14] They find that when the environment is one of strategic *complementarity*, prices adjust slowly in response to anticipated monetary shocks. In contrast, the adjustment rate is extremely quick (i.e., prices converge to equilibrium fairly quickly) when the environment is one of strategic *substitutability*.

This paper also contributes to the growing body of experiment literature that investigates the influence of selection into games. For example, auctioning participation rights can improve contributions in the threshold public goods game (Broseta et al., 2003), help people coordinate on the efficient equilibrium in coordination games (Van-Huyck et al., 1993; Crawford and Broseta, 1998), and affect behaviour in the Ultimatum game (e.g., Güth and Tietz, 1986; Shachat and Swarthout, 2013). In these games, the auctions can potentially function as a "coordination device" (i.e., people learn about what others will do from the market price) and help with equilibrium selection. As will be clearer in the experimental design, auctions in our study function purely as a "selection device".

Finally, we contribute to the body of literature which finds markets to be useful in guiding and solving complex problems. For example, Maciejovsky and Budescu (2005, 2013) show that market prices can help improve subjects' performances in the Wason (1966) selection task, a well-known test of deductive reasoning. Meloso et al. (2009) show that a market-based system of compensation can promote intellec-

[12]In Kendall and Oprea (2018) experiment, subjects make static consumption and investment decisions at the start of each period, and the computer program will simulate hundreds of rounds of draws to calculate their payment.

[13]Kendall and Oprea (2018) devise a "survival index" that is based on subjects' relative share of the wealth. The presumption here is that since wealth is required to make investment decisions, then a test of the market selection hypothesis is whether the less rational people will run out wealth sooner than their rational counterparts.

[14]Money illusion is a cognitive bias associated with confusing nominal and real variables.

tual discovery (modelled by solving the *knapsack problem*) better than a patent-based system.

Taken together, our findings suggest that individual level bounded rationality does not necessarily falsify the applicability of models that assume fully rational players. In fact, market selection can sometimes reduce anomalous behaviour in games. However, our findings do not lead us to conclude that markets can *always* correct or offset individual anomalous behaviour. Firstly, there are some situations (e.g., consumer market) where market competition for participation rights simply does not exist—players cannot be excluded from the interaction. Secondly, market competition can also exacerbate anomalous behaviour if the bounded-rational players value participation rights more than their rational counterparts.[15] For example, Choo et al. (2019b) show that it is possible to incentivise the less rational players to bid higher in auctions and hence increase deviations from the equilibrium in the p-beauty contest game (Nagel, 1995). Finally, markets may not always be desirable. For example, Offerman and Potters (2006) find that auctioning participation rights increases collusion in oligopoly markets. Kogan et al. (2011) find that asset market prices can drive people to coordinate on a less efficient equilibrium.

The rest of this paper is structured as followed. Section 2 details the RHP game. Building on this, Section 3 details our experiment design. Section 4 summarises our experimental findings. Finally, Section 5 concludes.

2 The red hat puzzle (RHP) game

There are three players each wearing a coloured hat that can be **red** or **black** with equal chance. Each player observes all other players' hats but her own. Players also receive the public signal that "*there are no red hats*" or "*there is at least one red hat*". The public signal depends on the total number of red hats and is always truthful. The above is common knowledge.

There are $t = 1, 2, 3, 4$ periods. At each period $t < 4$, players are asked "*Do you know your hat colour?*" and they can respond with the actions "*My hat is red*" (a_R), "*My hat is black*" (a_B) or "*I don't yet know*" (a_N). A player ends the RHP game whenever a_R or a_B is chosen and only progresses to the next period if she chooses a_N. At each period, players also observe the previous periods' actions of every other player. Finally, players at period 4 are asked the same question but can only respond

[15] For example in games that resemble the centipede game, it is possible for the rational player (i.e., one that plays the sub-game perfect equilibrium) to value the participation right less than the bounded-rational players (i.e., one that uses some non-equilibrium model of behaviour).

with a_R or a_B.[16]

A player's payoff depends on whether she is correct (e.g., choosing a_R when hat is red) and the period that she ends the RHP game ($\bar{T}$), and is derived as:

$$\pi = \begin{cases} 1100 - 100\bar{T} & \text{if the player is correct,} \\ 400 - 100\bar{T} & \text{if the player is incorrect.} \end{cases} \tag{1}$$

Hence, a player incurs a deduction of 100 each time a_N is chosen and a further deduction of 700 if she is incorrect about her hat colour—no deductions for being correct. Therefore, all players should always seek to correctly resolve their own hat colour in the soonest possible period.

2.1 Equilibrium

For each player, let $r \in \{0, 1, 2\}$ denote the total number of red hats that she observes. When modelled as a Bayesian game with incomplete information, the *indirect communication equilibrium* (Geanakoplos and Polemarchakis, 1982) is for each player to choose a_N at periods $t < r + 1$ and at period $t = r + 1$, choose a_R (resp. a_B) if her hat is red (resp. black)—henceforth known as the *equilibrium behaviour*. Each player resolves her hat in period $r + 1$ and the equilibrium payoff is $\pi^* = 1000 - 100r$.

To illustrate the equilibrium, consider the case where all hats are red.[17] Here, each player observes two red hats (i.e., $r = 2$) and assigns an equal posterior that her hat is red or black. Furthermore, despite players' private information and the public signal that there is at least one red hat, Aumann (1976) agreement theorem shows that the only common knowledge fact is that there is at least one red hat.

- *Period 1*: Players choose a_N when asked about their hats.
- *Period 2*: Each player learns that there is at least two red hats—otherwise someone would have chosen a_R in period 1. However, they already know this and there is no revision to their posterior—they will again choose a_N. Nevertheless, if it is common knowledge that all players performed the same reasoning, it becomes common knowledge that there is at least two red hats.

[16]Suppose that player 1 chooses a_B in period 1 whilst players 2 and 3 both choose a_N in period 1. Only players 2 and 3 progress to period 2. In addition, players 2 and 3 observe the previous period's actions of all other players (e.g., player 2 sees that player 1 chose a_B in period 1 and that player 3 chose a_N in period 1).

[17]We refer the reader to Chapters 9 and 10 of Maschler et al. (2013) for a detailed exposition of the equilibrium solution.

- *Period 3*: Each player deduces that there must be three red hats—otherwise the other two red hat players would have chosen a_R in period 2. Each player now chooses a_R.

Notice from the above that players who observe $r > 0$ need only learn about their own hat colour through the period r actions of those other players they observe to be under a red hat. Furthermore, choosing a_N at period $t' < r + 1$ is only optimal for an uncertain player if she expects to learn about her hat colour from the period r actions of the other red hat players—otherwise she should randomise between a_B and a_R at period t'. In equilibrium, all players always expect to resolve their hats.

The equilibrium solution is trivial when $r = 0$. When $r > 0$, the equilibrium solution requires players to apply logical and epistemological reasoning (i.e., reasoning about the reasoning of others). Intuitively, the complexity of reasoning involved is increasing with r. Experiments find that people are heterogeneous in their ability to perform the necessary logical and epistemology reasoning demanded by the equilibrium solution in the RHP game. For example, Weber (2001) finds that only 65% and 45% of experiment subjects , from the undergraduate and graduate cohort in Caltech, adhered to the equilibrium behaviour when observing $r = 1$ and $r = 2$, respectively—all did so when $r = 0$.[18] The corresponding $r = 1$ and $r = 2$ proportions in non-market experiments of Choo (2014) are 0.70 and 0.13, respectively—Choo uses undergraduate students from a middle tier UK university. The differences in the two studies suggest that performances in games that involve logical reasoning may vary across different subject pools (e.g., Chou et al., 2009).

In dynamic games, a player's deviation from the equilibrium path can be triggered by their opponents' prior deviations or *strategic uncertainty* (i.e., uncertainties with regards to the purposeful nature of others' actions). To control for the above concerns, Bayer and Renou (2016a,b) conduct a version of the RHP game where experiment subjects play against computer players programmed to always play the equilibrium (i.e, adherence to the equilibrium behaviour solely depended on subject's knowledge of the equilibrium solution).[19] They find that only 75% and 44% of subjects adhered to the equilibrium behaviour when observing $r = 1$ and $r = 2$, respectively.

[18] In Weber's experiments, players can only choose a_R or a_N at each period and the game ends for all players upon any player choosing a_R.

[19] Subjects in their study were primed that there exists a logical solution in the game. To avoid "logical inconsistencies" their design also includes a condition whereby the RHP game ends whenever a subject deviates from the equilibrium path.

3 Experiment design

We investigate whether market selection can reduce anomalous behaviour in the RHP game (i.e., deviations from the equilibrium behaviour). To do so, we use auctions to allocate the participation rights to the three-player RHP game—this will be compared against our control treatments where the participation rights are randomly allocated. To motivate our experiment design, consider the following thought experiment.

Thought experiment. There are three coloured hats. Two sisters, Ann and Eva, are under the same hat and observe $r' > 0$. The other two hats are worn by Charlie and Dale, respectively. A second-price auction is used to decide which of the two sisters, Ann or Eva, will be selected to play the RHP game against Charlie and Dale (i.e., both sisters bid for rights to play the RHP game). Both sisters always seek to maximise their own payoffs but only Ann knows the equilibrium solution in the RHP game. Consider the ex-ante expected payoff from the RHP game for each sister. If selected into the RHP game, Eva will randomise between a_B and a_R in period 1 with the expected payoff of $\underline{\pi} = 0.5(1000) - 0.5(1000 - 700) = 650$—she has an equal chance of being correct—and through backward deduction, bid $\underline{b} = \underline{\pi}$.[20] Ann's expected payoff is more subtle as it depends on whether she expects to resolve her hat in period $r + 1$. If this is positive, her expected payoff is $\pi^* = 1000 - 100r'$ and she bids $b^* = \pi^*$, where $b^* > \underline{b}$ for all $r' > 0$. If it is instead negative, Ann's expected payoff is similar to Eva and she bids $\underline{b}$.

Ann's bidding behaviour thus depends on whether she believes that she can deduce her hat colour from the period r' actions of the other red hat players. We therefore first consider a treatment where Ann can always expect to resolve her hat. To do so, players in the RHP game are always paired with *computer players* who are programmed to play the equilibrium.

Thereafter, we study behaviour in the RHP game when the auction winner (i.e., Ann or Eva) plays against other human players who are themselves also selected by the auction mechanism. In contrast to above treatment, Ann cannot be sure that she will always resolve her hat in period $r' + 1$. Nevertheless, Ann may value the participation rights more than Eva if she anticipates that the auction will also frequently select other "Ann like individuals" into the RHP game.

[20] Choosing a_N is strictly dominated for Eva as she incurs a deduction of 100 with no obvious benefits.

3.1 Design overview

The experimental design is summarised in Figure 1 and involves four treatments: *Random computer* (RCOM; $n = 54$ subjects), *Market computer* (MCOM; $n = 54$ subjects), *Random human* (RHUM; $n = 54$ subjects) and *Market human* (MHUM; $n = 45$ subjects). The experiment was conducted in June 2019 at the Shanghai Jiao Tong University, Smith Experimental Economics Research Center. Inexperienced subjects were recruited via ORSEE (Greiner, 2015). The experiment was programmed with zTree (Fischbacher, 2007).

Subjects interact in fixed-matching groups of 9 participants. The instructions are detailed in Appendix C and the experimental data and software are available upon request.

Each treatment consists of two independent parts (I and II) for which the instructions are only available at the start of that part. Part I involves five independent rounds and one random round is payoff relevant. Part II involves fifteen independent rounds and three random rounds are payoff relevant.[21] Experimental earnings in each round are denoted in "points".

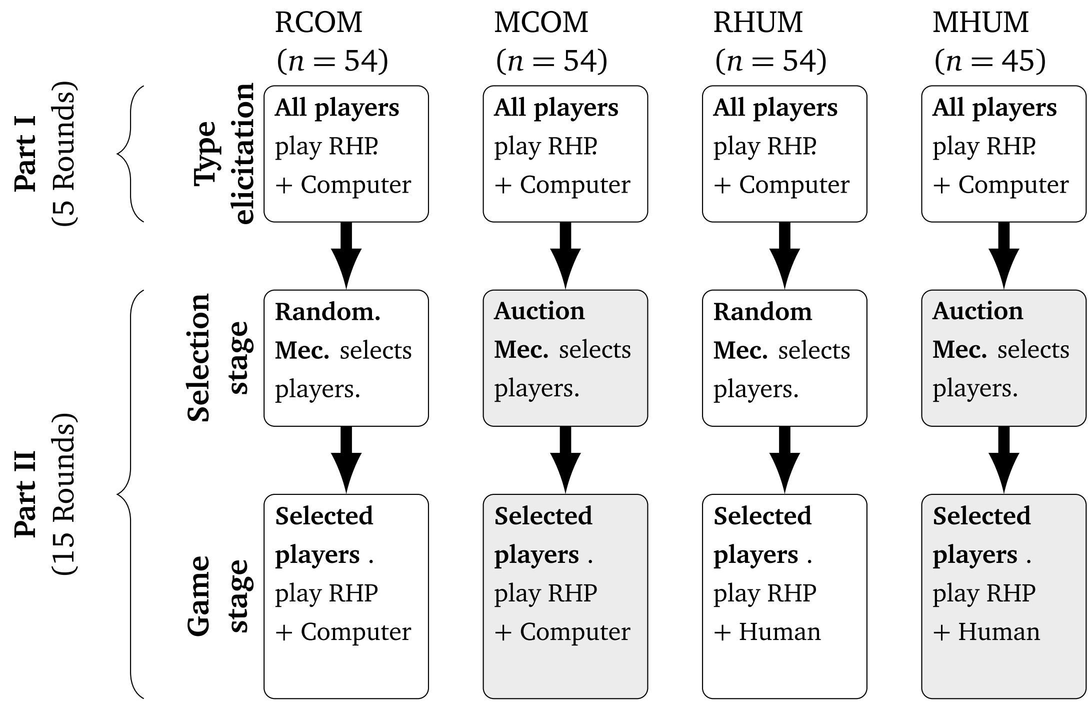
Part I is identical across all treatments. We use part II to differentiate between the *human* (RHUM and MHUM) and *computer* (RCOM and MCOM) treatments. The following subsections detail each part.

3.1.1 Part I

At each round, subjects play the three-player RHP game without feedback against *computer players* (i.e., a computer program makes decisions for the two other hat players). The computer players are programmed to always best respond to the actions of others at each period t. Appendix B details the computer players' programmed rules.[22] We use subject's behaviour in part I to elicit their understanding of the RHP game equilibrium solution.

[21] We chose to pay three random rounds as opposed to one random round in part II to "smooth" subjects' cash earnings (see Charness et al., 2016).

[22] Subjects are informed that the computer players are programmed to always maximise their own points and to always resolve their own hat colour in a logical manner. Furthermore, subjects are reminded that the computer can never "cheat" and will always base its decisions on the public signal, its observations of the other hat colours and the decisions of the other players in the RHP game. As such, computer players may also be incorrect about their hat colour when the subject deviates from the equilibrium path.



Note. There are four treatments each consisting of two independent parts. In each round of part II, subjects interact in fixed matching groups of nine players—part I involves individual decision-making task. We recruited 54 subjects (6 matching groups) in each of the RCOM, MCOM and RHUM treatments and 45 subjects (5 matching groups) the MHUM treatment.
+ Computer: All players in part I and selected players in part II play the RHP game against computer-players who are programmed to play according to the equilibrium solution.
+ Human: Selected players in part II play the RHP game against other selected players.

Figure 1: Summary of experiment design.

3.1.2 Part II

Each round involves three hats with three players under each hat. We refer to the players in the same hat as a *silo*. Players in the same matching group are randomly assigned to one of three silos in each round.

Players in each silo observe the public signal and the other two hat colours—players of the same silo observe the same r. Each silo selects one player to participate in the RHP game. The treatments differ on the selection mechanism.

- *RHUM and RCOM (random mechanism)*. Each silo randomly selects one player to enter the RHP game—equal chance for each player.

- *MHUM and MCOM (auction mechanism)*. Each silo uses a second-price auction to select one player for the RHP game — players are endowed with 1500 points and submit their bids for participating in the RHP game after observing r and the public signal.[23] Finally, auction winners only observe the selling price in their own silo. This prevents players from learning about their hats from the prices of other markets.

The selection process is common knowledge. The selected players in the human (RHUM and MHUM) treatments play the RHP game as described in Section 2—each selected player plays the RHP game against two other human players who are themselves selected by the auction or random mechanism. The selected players' hat colours correspond to their silo's hat colour.[24,25] In contrast to the human treatments, the selected players in the computer (RCOM and MCOM) treatments play the RHP game against two other computer players which are programmed as in part I.

The end of round payoffs for the non-selected and selected players are as follows

$$\Pi = \begin{cases} 1500 & \text{if not selected by auction or random mechanism,} \\ 1500 - z + \pi & \text{if selected by auction mechanism,} \\ 620 + \pi & \text{if selected by random mechanism,} \end{cases} \tag{2}$$

where $z \in [0, 1500]$ is the auction transaction price and π is the player's payoff from the RHP game (see equation (1)). In the MHUM and MCOM treatments, non-selected players keep their endowment of 1500 points and selected players pay z for the rights

[23] In the event of a tie, a random mechanism determines the auction winner.

[24] Suppose that players in silo A are under a red hat and observe two other red hats. The selected player in silo A will play the version of the RHP game where she is under a red hat and $r = 2$.

[25] Whilst the RHP game was ongoing in the experiment, the computers screens were blank for subject who were not selected by the auction or random mechanisms.

to participate in the RHP game. In RHUM and RCOM treatments, non-selected players receive a fixed payment of 1500 points and selected players receive a lower fixed payment of 620 points—as described in the following paragraph, this is lower to keep the average payoff consistent across all treatments.

Equilibrium. The selection mechanism does not affect the equilibrium in the RHP game. All players will expect to resolve their hats in period $r+1$ and bid $b^* = 1000 - 100r$. The equilibrium payoffs for selected players in the MHUM and MCOM treatments are therefore 1500 points. The payoffs for the selected players in the RHUM and RCOM treatments are also on average 1500 points.[26] Note that the random and auction mechanism are equivalent in equilibrium since players in the auction treatments will be randomly selected if all bid b^*.

Comment. We chose to model market selection with the second-price auction, as opposed to the first-price or all-pay auction, as we wanted to study bidding behaviour—it is a weakly dominant strategy for players to bid their valuations in the second-price auction, regardless of their risk preferences. This implies that players in the MCOM treatment who know the equilibrium solution should bid $b^* = 1000 - 100r$ since they would always play against computer players. Relative to the MCOM treatment, players in the MHUM treatment may shade their bids when $r > 0$ to "price in" the possibility that they would not correctly learn about their hats from the actions of others. Nevertheless, market selection may still reduce equilibrium deviations in the RHP game if players who know the equilibrium solution bid more than those who don't.

3.2 Other information

Subjects in part II interacted in fixed matching groups of nine participants—we pre-generated a sequence of states of nature (i.e., hat colours) and administered the same

[26]The equilibrium payoff in the RHP game depends on r. A player has a 2/8, 4/8 and 2/8 chance of observing $r = 0$, $r = 1$ and $r = 2$ red hats, respectively. The expected earnings in the RHUM and RCOM treatments will therefore be

$$\Pi^* = 620 + \frac{2}{8}(1000) + \frac{4}{8}(900) + \frac{2}{8}(800) = 1520$$

which is very close to the equilibrium payoff for selected players in the MHUM and MCOM treatments. Notice here that selected players in the RCOM and RHUM treatments will on average earn 20 points more than those who were not selected into the RHP game—the small difference is to compensate selected players for their effort. The rationale for the compensation is because subjects are randomly chosen to play the game in stead of participating voluntarily.

sequence to each matching group. At the end of each round in part II, subjects also received full feedback.[27] The experiment sessions took about 120 minutes and the mean earnings in the RCOM, MCOM, RHUM and MHUM treatments were \$19.80, \$20.20, \$19.90 and \$20.10 USD, respectively.[28] In the post experiment survey, subjects also completed the non-incentivised three question cognitive reflective test (CRT, Frederick, 2005) and were asked about any prior familiarity with the RHP game or similar puzzles. Around, 42%, 48%, 52% and 47% (Fisher exact, $p = 0.889$) of subjects in the RCOM, MCOM, RHUM and MHUM treatments, respectively, indicated that they have heard about the RHP game or similar puzzles.[29] We find no significant between treatment differences in CRT (Fisher exact, $p = 0.721$; mean 2.45).[30]

4 Results

4.1 Preliminaries

Define a player to be in *agreement* if she adheres to the equilibrium behaviour in the RHP game given r and her own hat colour.[31] We also use the R_0, R_1 and R_2 short-hand to denote instances where players observe $r = 0$, $r = 1$ and $r = 2$, respectively.

Part I is identical across all treatments: all subjects played 5 rounds ($1 \times R_0$ round, $2 \times R_1$ rounds, $2 \times R_2$ rounds) of the RHP game, without feedback, against computer-players. Here, 100% and 92% of subjects were always in agreement in the R_0 and R_1 rounds, respectively. We hence use their behaviour over both R_2 rounds to classify them into *sophisticated* and *unsophisticated* types:

- *Sophisticated* type: Subjects who were always in agreement over both R_2 rounds (44% of subjects) in Part I or only during the second R_2 round (12% of subjects).[32]

[27]Subjects were informed about the experimental payoff and their earnings for the round.

[28]The experiments were conducted in China. Subjects' earnings in parts I (one random round) and II (three random rounds) were converted to cash at the exchange rate of 1 point to 0.02 yuan. In addition, subjects also received a 10 yuan show up payment. The currency exchange rate during the period of the experiment was around USD\$1 to 6.67 yuan.

[29]Due to a software glitch, we failed to conduct the survey for 2 matching groups (i.e., 18 subjects) in the RCOM treatment. Nevertheless, the glitch did not affect the main experiment.

[30]Mean CRT score in our sample is higher than the original findings by Frederick (2005). This may be because the test is increasingly used in experiments. Indeed, Haigh (2016) find that performances in the CRT increases with prior exposure to the test. We find no significant correlation between prior familiarity (i.e., heard of the puzzle) and performances in the cognitive reflective test (spearman $\rho = 0.051$, $n = 189$, $p = 0.479$).

[31]A player who deviates from the equilibrium path is never defined as being in agreement even when it is a logical best response. This restriction helps comparability across the computer and human treatments.

[32]The latter condition allows for a minority of subjects to learn about the equilibrium through re-

- *Unsophisticated* type: Subjects who were never in agreement over both R_2 rounds (36% of subjects) or only during the first R_2 round (8% of subjects)—the latter condition assumes that sophisticated types do not make mistakes.

Both types are assumed to only differ in their knowledge of the R_2 equilibrium solution—they know the solution when R_0 and R_1 are observed.[33] Column (1) of Table 1 details the observed proportion of sophisticated types in each matching group. We make the following observations.

Observation 1 *There is some heterogeneity across the matching groups. The proportion of sophisticated types vary from 22% of subjects in some groups to around 78% of subjects in some other groups.*

The above suggests that it is important to disentangle the treatment effect (i.e., the selection mechanism) from the prior distribution of sophisticated types in the matching groups. This is because the treatment effect is based of the idea that the auction mechanism will select sophisticated types more frequently than the random mechanism.[34] Hence, in the absence of learning, it will be important to control for the proportion of sophisticated types in each matching group when we compare outcomes in the RHP game—the learning phenomenon here refers to the unsophisticated types realising the R_2 equilibrium solution through repeated play in part II.

To verify that little learning takes place in part II, consider the RCOM and RHUM treatments. Aggregating over all rounds in part II, the proportion of agreement subjects in the RCOM (resp. RHUM) treatment when R_0, R_1 and R_2 are 100%, 97% and 58% (resp. 98%, 93% and 50%), respectively.[35] Focusing on the R_2 case, we unsurprisingly find agreement likelihood (i.e., the probability of being in agreement) in the RCOM and RHUM treatments to be significantly ($p = 0.025$) higher for sophisticated relative to unsophisticated types—the econometric estimates are reported on Table A1 of the Appendix. There is no evidence of learning for unsophisticated types: agreement likelihood for sophisticated ($p = 0.971$) and unsophisticated ($p = 0.632$)

peated play even when there is no feedback. The results in this paper also hold if we will use a stricter criteria that sophisticated types are only those who were in agreement over both R_2 rounds.

[33] Cognitive reflective test scores are significantly higher for sophisticated relative to unsophisticated types (Mann-Whitney, $n = 189$, $p < 0.001$). Surprisingly, we find no significant between type differences in self-declared prior familiarity with the RHP game (z-test, $n = 189$, $p = 0.105$).

[34] If there are no sophisticated types in the market, then the auction and random mechanism are equivalent.

[35] The proportion of agreement subjects in part II of the RCOM and RHUM treatments are substantially higher than those reported by Bayer and Renou (2016b,a) and Weber (2001) when R_1 is observed—the R_2 rates are similar. Perhaps our subjects were better able to concentrate in experiments.

types do not increase significantly with rounds. These findings also carry forward to the MCOM and MHUM treatments.[36] This leads us to the second observation.

Observation 2 *We find no evidence for learning in the red hat puzzle game. As such, subjects' behaviour in the red hat puzzle game is strongly dependent on their type (sophisticated vs. unsophisticated).*

4.2 Market Selection

We now focus on behaviour in Part II. For each matching group, define the R_0, R_1 and R_2 *agreement rates* as the relative proportion of agreement subjects (pooling over all rounds) in the RHP game when R_0, R_1 and R_2, respectively, are observed—each matching group contributes three independent data-points.

Columns (2), (3) and (4) of Table 1 detail the R_0, R_1 and R_2 agreement rates, respectively, in each matching group. For example, of the 13 instances in group 3 where subjects in the RHP game observed R_2, 7 instances (54%) corresponded to them being in agreement.

As can be expected given subjects' behaviour in part I, R_0 and R_1 agreement rates are close to unity.[37] We therefore focus on the R_2 case.

Eyeballing column (4) of Table 1, R_2 agreement rates are often higher in the MCOM relative to RCOM treatments (Mann-Whitney, $n = 12$, $p = 0.194$) and in the MHUM relative to RHUM treatments (Mann-Whitney, $n = 11$, $p = 0.021$), though the differences are only significant in the latter comparison.[38] The insignificant difference between the MCOM and RCOM treatments is mainly due to the low proportion of sophisticated types in matching group one, which belongs to the MCOM treatment (column (1) of table 1). If we drop this matching group from the analyses, R_2 agreement rates are significantly higher in the MCOM relative to RCOM treatments (Mann-Whitney, $n = 11$, $p = 0.051$). Therefore, to see the between-treatments differences more clearly, it is necessary to control for the proportion of sophisticated

[36]Agreement likelihood in the MHUM and MCOM treatment is significantly ($p = 0.033$) higher for sophisticated relative to unsophisticated types. For unsophisticated ($p = 0.367$) and sophisticated ($p = 0.301$) types, agreement likelihood does not increase significantly with rounds.

[37]There were one and two instances in the RHUM (group 14) and MHUM (group 21) treatments, respectively, where subjects were not in agreement when observing R_0. This inevitably resulted in their peers (i.e., those observing R_1) also not being in agreement.

[38]We find a positive correlation between the proportion of sophisticated types and the corresponding R_2 agreement rates for matching groups in the RCOM (spearman $\rho = 0.831$, $n = 6$, $p = 0.0401$), MCOM (spearman $\rho = 0.955$, $n = 6$, $p = 0.003$) and MHUM (spearman $\rho = 0.811$, $n = 5$, $p = 0.095$)—there is no significant correlation in the RHUM treatment (spearman $\rho = -0.590$, $n = 6$, $p = 0.216$).

	matching group	(1) % sophisticated types	(2) agreement rates R_0	(3) R_1	(4) R_2	(5) sophistication rates R_0	(6) R_1	(7) R_2
RCOM	3	0.33	1.00	1.00	0.54	0.38	0.42	0.46
	4	0.78	1.00	0.79	0.69	0.88	0.71	0.77
	5	0.56	1.00	1.00	0.38	0.63	0.63	0.46
	6	0.56	1.00	1.00	0.69	0.38	0.75	0.62
	7	0.33	1.00	1.00	0.46	0.38	0.42	0.23
	8	0.78	1.00	1.00	0.69	0.88	0.96	0.92
MCOM	1	0.22	1.00	1.00	0.38	0.13	0.21	0.31
	2	0.56	1.00	1.00	0.77	0.75	0.46	0.92
	9	0.67	1.00	0.96	0.77	0.75	0.63	0.69
	10	0.78	1.00	1.00	1.00	0.50	0.92	1.00
	11	0.44	1.00	1.00	0.85	0.63	0.46	0.92
	12	0.33	1.00	0.96	0.46	0.38	0.50	0.46
RHUM	13	0.67	1.00	1.00	0.54	0.88	0.54	0.62
	14	0.44	0.88	0.88	0.92	0.38	0.46	0.31
	15	0.56	1.00	1.00	0.31	0.38	0.50	0.54
	16	0.67	1.00	0.83	0.31	0.25	0.71	0.62
	17	0.33	1.00	0.92	0.38	0.50	0.29	0.54
	18	0.56	1.00	0.96	0.54	0.63	0.67	0.38
MHUM	19	0.67	1.00	0.96	0.62	0.63	0.67	0.77
	20	0.67	1.00	0.96	0.69	0.50	0.54	0.85
	21	0.67	0.75	0.79	1.00	0.88	0.58	1.00
	22	0.56	1.00	1.00	0.92	0.38	0.46	0.77
	23	0.78	1.00	0.96	1.00	0.50	0.54	1.00
♯ of obs. per group		9	8	24	13	8	24	13

Note. Experimental subjects interact in fixed matching groups of 9 subjects. Sophisticated types are those who were in agreement over both R_2 rounds in part I or only the second R_2 round in part I. Column (1) reports the proportion of *sophisticated* types (determined by Part I behaviour) in each matching group. Columns (2), (3) and (4) report the *agreement rates* (i.e., proportion of agreement subjects in the RHP game) when R_0, R_1 and R_2, respectively, are observed. Columns (5), (6) and (7) report the *sophistication rates* (i.e., proportion of sophisticated types in the RHP game) when R_0, R_1 and R_2, respectively, are observed.

Table 1: Proportion of sophisticated types, agreement rates and sophisticated rates in Part II.

Dep. variable	R_2 agreement rate			R_2 sophistication rate		
	(1)	(2)	(3)	(4)	(5)	(6)
Treatments	COM	HUM	COM & HUM	COM	HUM	COM & HUM
Auction	0.17^{**} (0.07)	0.38^{**} (0.16)	0.27^{**} (0.11)	0.20^{**} (0.08)	0.30^{***} (0.07)	0.24^{***} (0.08)
Computer			0.06 (0.10)		0.05 (0.07)	
Computer × Auction			−0.12 (0.16)		−0.05 (0.11)	
% Sophisticated	0.73^{***} (0.19)	0.27 (0.66)	0.53^{*} (0.25)	1.09^{***} (0.23)	0.58 (0.33)	0.98^{***} (0.18)
Constant	0.16 (0.12)	0.64 (0.37)	0.21 (0.15)	−0.03 (0.14)	0.18 (0.19)	−0.03 (0.11)
n	12	11	23	12	11	23
R^2	0.65	0.46	0.45	0.72	0.81	0.74

Note. Each matching group constitutes one data-point in this analysis. We control for the proportion of *sophisticated* type in the matching group. Robust standard errors are reported in parenthesis.
***: $p < 0.01$, **: $p < 0.05$, *: $p < 0.10$.

Table 2: OLS regression estimates.

types in each matching group. The relevant OLS regression estimates are reported on columns (1) and (2) of Table 2.

Result 1 *Market selection can reduce anomalous behaviour in the red hat puzzle game when two red hats are observed. This holds when human players interact with other human players as well as computer-players who are programmed to play the equilibrium.*

Support for Result 1: The estimates on columns (1) and (2) of Table 2 show that controlling for the proportion of sophisticated types in each group, R_2 agreement rates are approximately 0.17 ($p = 0.049$) units higher in the MCOM relative to RCOM treatments and 0.38 ($p = 0.044$) units higher in the MHUM relative to RHUM treatments. The differences are significant at the 5% level.[39]

Given the RHP game payoff structure (see equation (1)), players should immediately randomise between a_R and a_B in the RHP game if they do not expect to correctly resolve their hat colour. Indeed, a substantial proportion of the non-agreement subjects (70%, 73%, 44% and 50% in the RCOM, MCOM, RHUM and MHUM treatments, respectively) chose to end the RHP game in period 1 when R_2 is observed.[40] The high

[39]The same conclusions are offered if the Fractional Logistic regression model (e.g., Papke and Wooldridge, 1996, 2008) is used. Controlling for the proportion of sophisticated types in the R_2 silos, as opposed to the proportion of sophisticated types in the matching groups, does not change the conclusions.

[40]Amongst the RCOM, MCOM, RHUM and MHUM non-agreement subjects who ended the RHP game

attrition rate may explain why the R_2 agreement rates do not seem to differ much across the computer and human treatments. This is confirmed by the OLS regression on column (3) of Table 2 which finds no significant differences in R_2 agreements rates between the MCOM and MHUM treatments ($p = 0.467$) as well as the RCOM and RHUM treatments ($p = 0.540$).

A natural explanation to Result 1 is that the auction mechanism selects sophisticated types more frequently than the random mechanism when R_2 is observed. For each matching group, we define the R_0, R_1 and R_2 *sophistication rates* as the proportion of sophisticated types selected by random or auction mechanisms to enter the RHP game when R_0, R_1 and R_2, respectively, are observed. Columns (5), (6) and (7) of Table 1 detail the corresponding sophistication rates in each matching group. For example, the 6 (46%) out of the 13 subjects who entered the RHP game in matching group 3 were identified as a sophisticated type.

There are no discernible between treatment differences in the R_0 and R_1 sophistication rates. This can be expected since sophisticated and unsophisticated types are assumed to know the equilibrium solution when R_0 and R_1 are observed—the auction should not discriminate in favour of sophisticated types. We will again focus on the R_2 case.

Eyeballing column (7) of Table 1, we observe that R_2 sophistication rates are often higher in the MCOM relative to RCOM treatments (Mann Whitney, $n = 12$, $p = 0.329$) and in the MHUM relative to RHUM treatments (Mann Whitney, $n = 11$, $p = 0.005$), though the differences are again only significant for the former comparison. We again use the OLS regression model to control for the proportion of sophisticated types. The relevant estimates are reported on columns (4) and (5) of Table 2.

Result 2 *The auction mechanism selects sophisticated types more frequently than the random mechanism when two red hats are observed in the red hat puzzle game. This holds when human players interact with other human players as well as computer-players who are programmed to play the equilibrium.*

Support for Result 2: The estimates on columns (4) and (5) of Table 2 illustrate that controlling for the proportion of sophisticated types in each group, R_2 sophistication rates are approximately 0.20 ($p = 0.048$) units higher in the MCOM relative to RCOM treatments and 0.30 ($p = 0.005$) units higher in the MHUM relative to RHUM treatments. The differences are significant at the 5% level.[41]

in period 1 when R_2 are observed, 87%, 77%, 100% and 80%, respectively, are unsophisticated types. The high proportion of unsophisticated types lend weight to our conjecture that players who do not expect to resolve their hats will randomise in period 1.

[41] We also note that the R_2 sophisticated rates are not significantly different from the prior distribution

Again, the estimates on column (6) of Table 2 show that R_2 sophistication rates do not differ significantly within the auction ($p = 0.660$) or random ($p = 0.461$) treatments depending on whether subjects play the RHP game against computer or human players.

In Table A2 of the Appendix section, we show that Results 1 and 2 also hold at the subject level analysis: relative to subjects in the random treatments, subjects in the auction treatments are significant more likely to be in agreement and be classified as sophisticated type when R_2 is observed.

Comment. When R_2 is observed, the optimal strategy is for the unsophisticated types is to end the RHP game in period 1. If the sophisticated types in MHUM and RHUM treatments anticipate this, then it may be possible for them to learn about their own hat colour in period 2 through the period 1 deviations of others when all three hats are red (i.e., the sophisticated type sees $r = 2$).[42,43] There were 9 and 3 instances where a sophisticated type observing R_2 in the RHUM and MHUM treatments, respectively, ended the RHP game in period 2 after observing that one or more of her opponents had ended the RHP game in period 1. Here, 88% and 67% of the above instances in the respective treatments resulted in the sophisticated type choosing a_R, a strategy that is consistent with resolving one's hat colour through the deviations of others—such instances will not be classified as being in agreement. However, such occurrences were relatively rare and do not affect the conclusions from Result 1. For example, if subjects following such a strategy were also classified as being in agreement, the R_2 case agreement rates will still be significantly higher ($p < 0.001$) in the MHUM relative to RHUM treatments.

4.3 Bidding behaviour

To better understand Result 2, we turn our attention to the bidding behaviour—subjects in the MCOM and MHUM treatments independently submitted their bids after observing r. For each subject, we compute her average bids (over all relevant

of sophisticated types for groups in the RCOM (Signrank, $n = 6$, $p = 0.601$) and RHUM (Signrank, $n = 6$, $p = 0.344$). In contrast, the R_2 sophisticated rates are significantly higher than the prior distribution of sophisticated types for groups in the MCOM (Signrank, $n = 6$, $p = 0.027$) and MHUM (Signrank, $n = 5$, $p = 0.043$) treatments.

[42]Suppose that a sophisticated type (say Ann) observing $r = 2$ notices that one other red hat player had chosen a_B or a_R in period 1. Ann immediately deduces that the player must have randomised. However, if Ann also believes that the player randomises because she sees two other red hats, Ann will immediate deduce her hat to be red and chooses a_R in period 2.

[43]Such concerns will not be relevant in the MCOM and RCOM treatments as subjects interact with computer-players programmed to always play the equilibrium.

rounds) when R_0, R_1 and R_2 are observed.[44] Figure 2 details the boxplot distribution of average bids for sophisticated and unsophisticated types in the MCOM (top row) and MHUM (bottom row) treatments.[45] In each panel, we use the horizontal dash lines to indicate the equilibrium bid price.

Result 3 *Sophisticated types bid higher than the unsophisticated types when two red hats are observed in the red hat puzzle game.*

Support for Result 3: Average bids are significantly higher for sophisticated relative to unsophisticated types in MCOM (Mann-Whitney, $n = 54$, $p < 0.001$) and MHUM (Mann-Whitney, $n = 45$, $p < 0.001$) treatments when R_2 is observed. We observe no significant between type differences (Mann-Whitney, $p \geq 0.395$ in each comparison) when R_0 and R_1 are observed—both types are assumed to know the equilibrium solution.

Taken together Results 1-3 suggest that because sophisticated types bid more than unsophisticated types when R_2 is observed, the auction mechanism was able to select the sophisticated types more frequently than the random mechanism. As a consequence, the R_2 agreement rates are higher in the auction treatments.

It is a weakly dominant strategy for subjects in the MCOM treatment to bid their valuation since they always face computer opponents in the RHP game. Figure 2 shows that unsophisticated and sophisticated types bid very closely to the equilibrium price, presumingly their valuation, when less than two red hats are observed. When R_2 is observed, only the sophisticated types bid closely to the equilibrium price.[46,47]

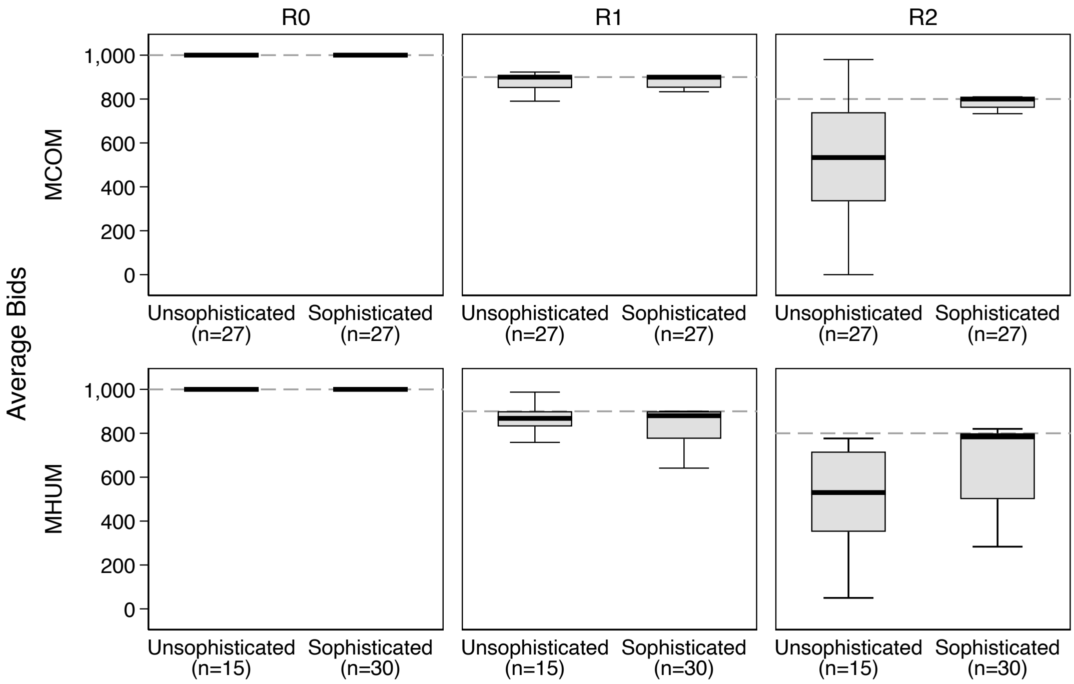
Subjects in the MHUM treatment interact with other human subjects in the RHP game. Hence, players in the MHUM treatment may shade their bids to compensate for the possibility of not correctly resolving their hat colour in the RHP game when R_2 is observed—by definition, all players expect to resolve their hats when R_0 and R_1 are observed. If so, we can expect sophisticated types to bid lower in the MHUM

[44] Across the 15 rounds, each subject had at least once observed R_0, R_1 and R_2. We chose to focus on the average bid to minimise the "noise" at the subject level.

[45] Excluding outliers (defined as being outside 1.5 times the interquartile range), the whiskers, box and thick line in the boxplot graph report the min/max value, interquartile range (i.e., 25th to 75th percentile) and median of the distribution, respectively. We chose to report the boxplot distribution as it shows how the data clusters around certain values.

[46] The average bid is less than 10 points away from the equilibrium price for 83% and 56% of MCOM subjects when R_0 and R_1, respectively, are observed—the bidding strategy space is 1500 points. The corresponding proportions when for unsophisticated and sophisticated types in the MCOM treatment are 4% and 63%, respectively, when R_2 is observed.

[47] We are aware that experiments with randomly induced valuations often find that subjects overbid in the second-price auction (e.g., Kagel and Levin, 1993). However, note that our design is different given that subjects' valuation in the auction are not randomly induced (i.e., depends on their expected payoffs in the RHP game).



Note. We report the average bids (denoted in points) by unsophisticated and sophisticated subjects in the MCOM (top row) and MHUM (bottom row) treatments. The dash horizontal line denote the equilibrium bid price. Outliers are omitted.

Figure 2: Boxplot distribution of average bids

relative to the MCOM treatment when R_2 is observed. In contrast, unsophisticated types should bid equally in both treatments when R_2 is observed since strategic uncertainties should only affect bidding behaviour if subjects know the equilibrium.

Result 4 *When two red hats are observed in the red hat puzzle game, there is some evidence that sophisticated human players "shade" their bids when they know that they will interact with other human players as opposed to computer-players.*

Support for Result 4: Average bids are marginally higher (Mann-Whitney, $n = 57$, $p = 0.092$) for sophisticated types in the MCOM relative to MHUM treatments when R_2 is observed—no significant differences when R_0 (Mann-Whitney, $n = 42$, $p = 0.942$) and R_1 (Mann-Whitney, $n = 42$, $p = 0.121$) are observed. We find no significant between treatment differences in the average bids of unsophisticated types when R_0, R_1 and R_2 (Mann-Whitney, $n = 42$, $p \geq 0.636$ in all comparisons) are observed.

We provide further econometric support for Results 3 and 4 in the Appendix.

5 Conclusion

In this paper we study whether market selection can reduce anomalous behaviour in games. To do so, we use the red hat puzzle (RHP) game where anomalous behaviour are linked with players' inability to perform the necessary steps of logical and epistemological reasoning. We use an auction mechanism to allocate the participation rights into the RHP game. Our experiment shows that auctions significantly decrease deviation from equilibrium play in the RHP game—this holds independently of whether players in the RHP game interact with human players or computer-players who are programmed to behave rationally.

Taken together, this experiment shows that market selection can sometimes reduce individual anomalous behaviour in games. Our findings suggest that individual level bounded rationality does not necessarily falsify the applicability of models that assume fully rational players. This is because with carefully designed incentives, market competition can result in the allocation of decision-making rights (i.e., the rights for performing economic task) to the rational players.

A fruitful line for future research is the influence of market institutions on anomalous behaviour in games. Indeed, a recent study by Deck et al. (2020) find that assets prices closely tract the fundamental value in the English Dutch Auction mechanism, while the price is much higher in Double Dutch and Double auction.

In a preliminary study, Choo (2014) embedded continuous double auction (CDA) asset markets into the RHP game. Here, players trade assets which represent the rights to play a variation of the three hat RHP game—a trader with $x > 1$ will play the RHP game "as if" he was playing on behalf of x players.[48] The experiment finds that deviations from the equilibrium in the RHP game are more frequent in the asset market treatment relative to the *standard* or baseline RHP game. Also, Choo finds indirect evidence that the inferior performances of the asset market treatment could be linked to "price bubbles" as assets were often transacted above the equilibrium price.

An innovation of the asset market treatment is that there can be more than one player under each of the three hats making decisions in the RHP game—at each period t players choose their actions independently and observe the $t-1$ actions of each other players. Choo shows that the equilibrium behaviour in the RHP game is independent of the number of players under each hat and players under the same hat should always choose the same action at each period. However, this is rarely observed in the experiment as players under the same hat often chose different actions. In hindsight, increasing the number of players under each hat might therefore complicate players' ability to learn about their own hat colour from the actions of others. Also, subjects may not view the the RHP game decision task in the asset market treatment to be similar to the baseline game. Furthermore, the right resell in CDA markets provide opportunities for *speculative behaviour*.[49]

The previous study raised pertinent questions as to whether the inferior performances of the asset market treatment relative to the baseline game capture the failure of market selection, the distortionary effects of speculation opportunities or the possibility that increasing the number of players under each hat confuses subjects. This motivated us to conduct a "cleaner" study where we use auctions without resale opportunities to allocated the participation rights to the standard RHP game. We also elicit players' types to get a better understanding of their logical reasoning abilities. In doing so, we show that market competition can reduce anomalous behaviour in the RHP game. The experiment also offers a path forward as to how asset markets can be incorporated into the RHP game whilst keeping the decisional problem identical to the baseline game—a trader with $x > 1$ assets will play the baseline game x times. This will be an ambition for future research.

[48] The experimental design involved three coloured hats with 6 players under each hat. The CDA markets commence after players under each hat observe the other two hat colours. Players are each endowed with one asset and trade assets with the other players under the same hat. At the end of the market, ownership of an asset allows a player to enter the RHP game.

[49] Harrison and Kreps (1978) write that "investors exhibit speculative behavior if the right to resell [an] asset makes them willing to pay more for it than they would pay if obliged to hold it forever".

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ONLINE APPENDIX

A Econometric regressions

A.1 How do subjects' types affect their behaviour in part II?

We use the Panel Logistic model to study how subjects' types—determined by their decisions in part I—affect their "agreement likelihood" (i.e., likelihood of being in agreement) when observing R_2 in part II. The estimates are reported on Table A1.

Dependent variable: Agreement in part II when R_2 is observed.

Treatment	RCOM+RHUM	MCOM+MHUM
Reference group: *unsophisticated type in the human treatment*		
sophisticated	3.15^{**} (1.40)	5.27^{**} (2.47)
computer	-0.75 (1.30)	-1.48 (1.25)
round	0.03 (0.06)	0.08 (0.09)
sophisticated× computer	2.54^{*} (1.29)	1.94 (1.44)
sophisticated× round	0.00 (0.09)	-0.16 (0.16)
Constant	-2.00^{**} (0.79)	-1.63^{***} (0.26)
n obs.	156	143
n subjects	89	56

Note. "sophisticated" and "computer" are situation dummy variables. Standard errors clustered at the matching group level.
***: $p < 0.01$, **: $p < 0.05$, *: $p < 0.10$.

Table A1: Panel logistic estimates: How behaviour in part II depend on subjects' type.

We see that agreement likelihood are significantly higher for sophisticated relative to unsophisticated types in the random ($p = 0.025$) and auction ($p = 0.033$) treatments. To test for learning, we examine whether the agreement likelihood increases with rounds for the unsophisticated type—sophisticated types are assumed to already know the equilibrium solution. We find no evidence for learning as the coefficient estimates for "rounds" and "rounds×sophisticated" are not significant ($p > 0.301$) at any reasonable levels. If strategic uncertainty influences behaviour in the RHP game, we should expect sophisticated types' agreement likelihood to be higher in the computer relative to human treatments—unsophisticated types should not be affected since they do not know the equilibrium. We see some evidence for this in the random treatments: for sophisticated types, agreement likelihood is significantly lower

($p = 0.050$) in the RHUM relative to RCOM treatments. In contrast, agreement likelihood for sophisticated types in the MCOM and MHUM treatments are not significantly different ($p = 0.180$).

A.2 The influence of Market selection

We use the panel logistic model to study whether selected subjects in the auction treatments (relative to those in the random treatments) are more likely to be (i) in agreement and (ii) classified as sophisticated types when R_2 is observed—it is difficult to control for the proportion of sophisticated types when studying behaviour at the individual level. The estimates are reported on Table A2.

Dependent variable	Agreement	Sophisticated
auction	3.26^{*} (1.18)	6.69^{*} (3.57)
computer	1.10 (1.14)	1.02 (0.99)
auction×computer	−2.44 (1.58)	−4.95 (3.78)
Constant	−0.13 (0.70)	0.05 (0.36)
n obs.	299	299
n subjects	145	145

Note. "auction" and "computer" are situation dummy variables. Standard errors clustered at the matching group level.
***: $p < 0.01$, **: $p < 0.05$, *: $p < 0.10$.

Table A2: Panel logistic estimates: The influence of market selection on agreement likelihood and the types of subjects selected into the RHP game.

The estimates find that subjects in the auction treatments are significantly more likely to be in agreement ($p = 0.060$) and be classified as sophisticated types ($p = 0.061$). In addition to the above, playing against computer as opposed to human players has not significant marginal influences for subjects in the auction ($p \geq 0.190$) or random ($p \geq 0.294$) treatments.

A.3 Bidding behaviour

All subjects in the MCOM and MHUM treatments independently submitted their bids at the start of the round. We use the random-effects GLS to study the bidding behaviour of sophisticated and unsophisticated types. The estimates are reported on Table A3.

Treatment	MCOM			MHUM		
	R_0	R_1	R_2	R_0	R_1	R_2
Reference group = Unsophisticated type						
Sophisticated type	-20.82 (15.77)	-9.71 (9.56)	229*** (62)	-51.10 (41.96)	- 9.15 (69.35)	139 ** (65.26)
Constant	998 *** (9.08)	864 *** (12.53)	525 *** (52.31)	1017*** (17.85)	820 *** (56.05)	520*** (25.96)
# of obs.	144	432	234	120	360	195
# of subjects	54	54	54	45	45	45
# of groups	6	6	6	5	5	5

Note. Standard errors clustered at the matching group level.
***, ** and * indicate $p < 0.01$, $p < 0.05$ and $p < 0.10$, respectively.

Table A3: Random-Effects GLS estimates: Do sophisticate types bid higher than unsophisticated types?

We find no significant between type differences in bids ($p \geq 0.197$) when subjects observe R_0 and R_1. When subjects observe R_2, we find bids in the MCOM ($p < 0.001$) and MHUM ($p = 0.033$) to be significantly higher for sophisticated relative to unsophisticated types.

We also study the bidding behaviour of sophisticated and unsophisticated types across the MCOM and MHUM treatments. The estimates are reported on Table A4. We find no significant between treatment differences ($p \geq 0.119$) in the bids of sophisticated and unsophisticated types when subjects observe R_0 and R_1. When subjects observe R_2, the results show that sophisticated types bid significantly lower ($p = 0.085$) in the MHUM relative to MCOM treatments. In contrast, we find no significant differences ($p = 0.995$) in the bids for unsophisticated types.

Treatment	Unsophisticated types			Sophisticated types		
	R_0	R_1	R_2	R_0	R_1	R_2
Reference group = MCOM						
MHUM	17.72 (18.90)	-44.07 (54.54)	-4.669 (55.78)	-13.37 (39.39)	-43.39 (27.81)	-95.08* (55.28)
Constant	999 *** (8.70)	864 *** (12.10)	525 *** (50.10)	977 *** (12.71)	854 *** (17.11)	755 *** (18.95)
# of obs.	106	342	182	158	450	247
# of subjects	42	42	42	57	57	57
# of groups	11	11	11	11	11	11

Note. Standard errors clustered at the matching group level.
***, ** and * indicate $p < 0.01$, $p < 0.05$ and $p < 0.10$, respectively.

Table A4: Random-Effects GLS estimates: Do unsophisticated and sophisticate types bid differently aacross treatments?

B Computer rules

The equilibrium behaviour in the RHP game is for each player to choose a_N at periods $t < r+1$ and at period $t = r+1$, choose a_R (resp. a_B) if her hat is red (resp. black). The computer-players are programmed with the following rules:

Rule-1: Choose a_B in period 1 if $r = 0$ and the public signal is "there are no red hats".

Rule-2: Choose a_R in period 1 if $r = 0$ and the public signal is "there is at least one red hat".

Rule-3: Choose a_N in period 1 if $r > 0$.

Rule-4: Choose a_N in period $1 < t < r+1$ if $r > 0$ and the other red hat player(s) chooses a_N in period $t-1$.

Rule-5: Choose a_R in period $t = r+1$ if $r > 0$ and the other red hat player(s) chooses a_N in period $t-1$.

Rule-6: Choose a_B in period $t = r+1$ if $r > 0$ and the other red hat player(s) chooses a_R in period $t-1$.

Rule-7 Uniformly randomise between a_B and a_R at period $t > 1$ if the above rules cannot be accomplished.

Rules 1–6 imply that the computer-players will best-respond at each period t given their observations r, the public signal and the previous period's actions of the other players. The computer-players can be incorrect about their own hat colour if the human player deviates from the equilibrium path in a manner that is undetectable. Table B1 shows one such example. Here, there is only one red hat and each computer player observes $r = 1$. The human player chooses a_N in period 1 (as opposed to a_R). This incorrectly informs the computer players that their hats are red — they had choose a_R in period 2. In period 3, the human ends the RHP game — he observes that the two other computer-players choose a_R in period 2.

Rule 7 implies that the computer-player will randomise if it detects that some player had deviated from the equilibrium path. Table B2 provides an example. Here, the human player chooses a_R in period 1 (as opposed to a_N). This deviation is undetected by the centre computer-player who responds by choosing a_B in period 2. In contrast, the right computer-player knows that the human-player has deviated from the equilibrium path. Anticipating that it will no longer be able to resolve its own hat colour, it therefore randomises between a_R and a_B.

Players	Human	Computer	Computer
Hat	Red	Black	Black
Observe	$r = 0$	$r = 1$	$r = 1$
Period 1	a_N	a_N	a_N
Period 2	a_N	a_R	a_R
Period 3	a_R	-	-
Period 4	-	-	-

Table B1: Example 1

Players	Human	Computer	Computer
Hat	Red	Red	Black
Observe	$r = 1$	$r = 1$	$r = 2$
Period 1	a_R	a_N	a_N
Period 2	-	a_B	randomise
Period 3	-	-	-
Period 4	-	-	-

Table B2: Example 2

Players	Human	Computer	Computer
Hat	Red	Red	Red
Observe	$r = 2$	$r = 2$	$r = 2$
Period 1	a_N	a_N	a_N
Period 2	a_R	a_N	a_N
Period 3	-	randomise	randomise
Period 4	-	-	-

Table B3: Example 3

In Example 3 (Table B3), we see that the human player chooses a_N in period 1 and a_R in period 2. In period 3, the computer-players immediately know that someone had deviated from the equilibrium since it cannot be that one red hat player chooses a_N and the other chooses a_R. The computer players thus randomise in period 3.

C Experiment instructions.

The following subsections detail the translated version of the instructions (the instructions were written in Mandarin). The experiment consists of two parts (I and II) and subjects only received the instructions for part II at the end of part I.

Where necessary in part II, we use "*text*" and "text" to distinguish between sentences that are unique to the computer (RCOM and MCOM) and human (RHUM and MHUM) treatments, respectively. We also use "text" and "text" to distinguish between sentences that are unique to the random (RCOM and RHUM) and auction (MCOM and MHUM) mechanism treatments, respectively.

Subjects received the instructions for parts I and II on their desk. The experimenter also read the instructions together with the subjects. To ensure that all subjects understood the experiment design, they had to correctly answer a series of control questions at the start of each part.

For readability, all references to Tables, Figures and Section in the instructions will be labelled by the appendix order.

C.1 Instructions Part I

First of all, thank you for your participation! Please note that you are not allowed to talk with other participants during this experiment. If you have a question, please raise your hand and we will answer you privately. In order to minimise distractions, please turn off your mobile phone and put away anything that could distract you from the experiment (e.g. books, study notes or electronic devices). You are only allowed to use the computer for the purposes of this experiment. Note that violation of the laboratory rules may lead to an immediate exclusion from the experiment.

At no time during this study will you learn the identity of the other participants you interact with. Also, no other participants will learn about your experimental earnings: At the end of the study, the amount of money you have earned will be paid out to you privately. If you follow the instructions and apply them carefully, you can earn some money in additional to the 10 yuan show-up fee which we will give you in any case.

The experiment will consist of two parts (Part I and Part II). Your earnings in this experiment will depend on your decisions in Parts I and II. In the following, we present the instructions for Part I. The Part II instructions will be available at the end of Part I.

C.1.1 Part I.

Part I of the experiment will consist of one practice (non-paying) round and five decision-making rounds. At each round, you will participate in the guessing game and earn points. The amount of points earned will depend on your decisions in the guessing game. At the end

Outcome	$O1$	$O2$	$O3$	$O4$	$O5$	$O6$	$O7$	$O8$
Player A's Hat	Black	Black	Black	Red	Black	Red	Red	Red
Player B's Hat	Black	Black	Red	Black	Red	Black	Red	Red
Player C's Hat	Black	Red	Black	Black	Red	Red	Black	Red

Table C1

of Part I, the computer will randomly pick one of the five decision-making rounds for payment. Your points in that round will be converted into cash at the exchange rate of 1 point = 0.023 yuan. The instructions are organised as follows:

- Section C.1.2 will detail the guessing game.
- Section C.1.3 will provide further information as to the other participants you will interact with in the guessing game.
- Section C.1.4 is a set of control questions to ensure that you understand the experiment design.

C.1.2 The guessing game.

There are 3 players (player A, player B and player C). Each player is given a hat which could either be Red or Black with equal chance. Each player does not observe his own hat colour. Each player observes the hat colour of the other two players. Each player receives a hint about the total number of red hats. There are two possible hints

- Hint 1: There is at least one red hat (i.e., one or more of the players has a red hat).
- Hint 2: There are no red hats (i.e., all players have black hats).

Table C1 provides a summary of all possible outcomes in the guessing game—each outcome is equally likely. Here are some examples to help you better understand the setup of the guessing game.

Example: Suppose that all hats are red (outcome $O8$ on Table C1). Then all players will be informed that “there is at least one red hat". Player A observes that B's and C's hats are red. Player B observes that A's and C's hats are red. Player C observes that A's and B's hats are red.

Example: Suppose that only players A and B have red hats (outcome $O7$ on Table C1). Again, all players will be informed that “there is at least one red hat". Player A observes that B's hat is red and C's hat is black. Player B observes that A's hat is red and C's hat is black. Player C observes that A's and B's hats are red.

Example: Suppose that all hats are black (outcome $O1$ on Table C1) and all players are informed that “there are no red hats". Player A observes that B's and C's hats are black. Player B observes that A's and C's hats are black. Player C observes that A's and B's hats are black.

	Player A (you) Hat colour: ??	Player B Hat colour: Red	Player C Hat colour: Black
Decisions in Period 1	(WAIT)	(RED)	(WAIT)

Table C2

** Reminder. You may sometimes learn about your own hat colour through the hint that receive and your observations about the other players' hats. Note that there is always an equal chance that your hat is red or black.**

Your task in the guessing game is to guess the colour of your own hat. There are 4 guessing opportunities which we call periods (i.e., period 1, period 2, period 3 and period 4). At each period, you will see the question:

"What is the colour of your hat?"

and you can respond with the following choices: (RED) My hat is Red, (BLACK) My hat is Black, (WAIT) I dont yet know the colour of my hat.

The rules are as follows:

- You end the guessing game when (RED) or (BLACK) is chosen. Example: If you choose (RED) in period 1, you end the guessing game in period 1.
- You only proceed to the next period when (WAIT) is chosen. Example: If you choose (WAIT) in period 1, you will proceed to period 2 where you will again see the same question "What is the colour of your hat".
- The choices of all players are public information in the following period. Example: If you are in period 2, you will observe the period 1 choices of the other two players.
- You can only choose between (RED) or (BLACK) in period 4.

Here are some examples to help you understand your task in the guessing game.

Example: Suppose that you are player A and you see that Bs hat is red and C's hat is black. In period 1, you see the question "what is your hat colour" and you choose (RED). You ended the guessing game in period 1.

Example: Suppose that you are player A and you see that B's hat is red and C's hat is black. In period 1, you see the question "what is your hat colour" and you chose (WAIT). In period 2, you will again see the question "what is your hat colour". In addition, you are provided information about the choices of the other players in period 1. Table C2 is an example of what you might observe. You see that player B chose (RED) in period 1. You also see that player C chose (WAIT) in period 1.

Example: Suppose that you are player A and you see that B's hat is red and C's hat is black. In period 1, you see the question "what is your hat colour" and you chose (WAIT). In period 2, you will again see the question "what is your hat colour" and the choices of the

	Player A (you) Hat colour: ??	Player B Hat colour: Red	Player C Hat colour: Black
Decisions in Period 1	(WAIT)	(RED)	(WAIT)
Decisions in Period 2	(WAIT)	-	(BLACK)

Table C3

	You chose (RED) or (BLACK) and your guess is correct	You chose (RED) or (BLACK) and your guess is incorrect
End guessing game in Period 1	1000 points	300 points
End guessing game in Period 2	900 points	200 points
End guessing game in Period 3	800 points	100 points
End guessing game in Period 4	700 points	0 points

Table C4

other players in period 1. Suppose that you chose (WAIT). In period 3, you will again see the question "what is your hat colour" and the choices of the other players in period 2. Table C3 is an example of what you might observe in period 3. Here, you see that player B did not participate in period 2 (i.e., he ended the guessing game in period 1) and player C chose (BLACK) in period 2.

** Reminder: At every period, you will see the decisions of the other players from the previous period. You may possibly learn about your own hat colour through the decisions of others.**

The points that you earn in the guessing game will depend on: (a) The period that you end the guessing game, (b) Whether you were correct or incorrect about your own hat colour. Table C4 summarises how your points are computed. You can see that you start the guessing game with 1000 points and receive a 100 points deduction each time you choose (WAIT). When you end the guessing game, you receive no deductions i f you guessed correctly. However, you receive a deduction of 700 points if you guessed incorrectly. Here are some examples to help you understand how your points are computed.

Example: Suppose that your hat is Black. You choose (WAIT) in period 1 and (BLACK) in period 2. You receive $1000 - 100 = 900$ points.

Example: Suppose that your hat is Black. You choose (WAIT) in period 1 and (RED) in period 2. You receive $1000 - 100 - 700 = 200$ points.

Example: Suppose that your hat is Black. You choose (WAIT) in period 1 and (WAIT) in period 2 and (BLACK) in period 3. You receive $1000 - 100 - 100 = 800$ points.

Example: Suppose that your hat is Black. You choose (WAIT) in period 1 and (WAIT) in period 2 and (RED) in period 3. You receive $1000 - 100 - 100 - 700 = 100$ points.

** Reminders. Notice from Table C4 that each player maximises his own points by correctly guessing his own hat colour in the soonest possible period. Hence, it may be possible to learn about your hat colour through the decisions of other players. In setting up the guessing game, there is always an equal chance that your hat is red or black. This means that if you do not know your hat colour but chose (BLACK) in period 1, there is an equal chance that you are correct or incorrect.**

C.1.3 Information about the other participants that you interact with in the guessing game.

At each round, you will be interacting with computer-players in the guessing game. This means that:

- If you are player A, then players B and C are computers.
- If you are player B, then players A and C are computers.
- If you are player C, then players A and B are computers.

The computer players are programmed to be in a similar position as yourself and can never cheat. This means that they observe your hat colour and the hat colours of their fellow computer players but not their own hat colour. Their decisions at each period will depend on the number of black and red hats that they observe, your decisions in the guessing game and the decision of the other computer player in the guessing game.

The computer-players are programmed to: (i) To always maximise their own points, (ii) To always guess their own hat colour in a logical manner. This means that it will base its decisions on the hint received, its observations of the other hat colours and the decisions of the other players in the guessing game.

C.1.4 Control questions

Please answer the following control questions on the computer.

Q1. What is the probability that you are given a red hat? (25%; 50%, 75%)

Q2. Suppose that outcome $O3$ (see Table C1) is chosen. Player A will observe a total of ___ red hat(s).

Q3. Suppose that outcome $O8$ (see Table C1) is chosen. Player A will observe a total of ___ red hat(s).

Q4. If you choose (BLACK) in period 1, you will proceed to period 2. (True; False)

Q5. If you choose (WAIT) in period 1, you will proceed to period 2. (True; False)

Q6a. Suppose that you are Player A and you chose (WAIT) in period 1 and (WAIT) in period 2. In period 3, you observe the following (see Table C5). Player C chose (WAIT) in period 1 and (RED) in period 2. (True; False)

	Player A (you)	Player B	Player C
Decisions in Period 1	(WAIT)	(BLACK)	(WAIT)
Decisions in Period 2	(WAIT)	-	(RED)

Table C5

Q6b. Player B did not proceed to period 2. (True; False)

Q6c. Player C will proceed to period 3. (True; False)

Q7. Suppose that your hat is red. You chose (WAIT) in period 1 and (RED) in period 2. You receive ___ points.

Q8. Suppose that your hat is red. You chose (WAIT) in period 1 and (BLACK) in period 2. You receive ___ points.

Q9. Suppose that your hat is red. You chose (WAIT) in period 1, (WAIT) in period 2 and (RED) in period 3. You receive ___ points.

Q10. Suppose that your hat is red. You chose (WAIT) in period 1, (WAIT) in period 2 and (BLACK) in period 3. You receive ___ points.

Q11. At each round of the guessing game, the other participants that you interact with are computer-players. (True; False)

Q12. If you are player A, then Players B and C are computers. (True; False)

Q13. At each period of the guessing game, the computer players are programmed to always submit the most logical decision. (True; False)

C.2 Part II.

Part II of the experiment consists of 1 practice (nonpaying) rounds and 15 decision-making rounds. At each round, you will earn points. The amount of points earned in each round will depend on your decisions in that round. At the end of Part II, the computer will randomly pick 3 of the 15 rounds for payment. Your points in that round will be converted into RMB at the exchange rate of 1 point = 0.023 yuan. The following instructions are organised as follows:

- Section C.2.1 will detail the experimental design of a round.
- Section C.2.2 will provide further information as to the other participants you will interact with in each round in the guessing game.
- Section C.2.3 is a set of control questions to ensure that you understand the experiment design.

C.2.1 Description of an experimental round.

You and two other participants in this room will be randomly paired together to form a group (each group consists of three participants). One participant in your group will receive a ticket. The ticket enables the owner to:

- Participate in the guessing game. That is, only the ticket owner will be one of the Hat-players in the guessing game.
- Receive points from participating in the guessing game.

** Reminder: The guessing game consist of exactly three hat-players. The participant in your group who is allocated the ticket will be one of the three hat-players. We will provide further information about the other hat-players in Section C.2.2 of the instructions. **

There are four stages in each round:

- Stage 1: You are provided some information about the guessing game.
- Stage 2: One participant in your group will receive the ticket.
- Stage 3: The participant with the ticket will participate in the Guessing game.
- Stage 4: The payoffs for all participants are computed.

In the next few pages, we will explain each stage in detail.

STAGE I. Information about the guessing game

As in part I of the experiment, the relevant information about the guessing game is the number of other red hats that you observe and the hint that you receive as a Hat-player in the guessing game. There are three hats (hats A, B and C). Each hat is assigned a colour which can be red or black with equal chance. Your group will be randomly assigned to one of the hats. (remember that you and two other participants in this room will be paired together to form a group).

** Reminder. After this assignment, the hat colour of each hat is fixed for the round. **

You cannot observe your own hat colour but can observe the colour of the other hats. In addition, you will also receive the hint:

- Hint 1: There is at least one red hat amongst the three hats.
- Hint 2: There are NO red hats amongst the three hats.

Suppose that your group is assigned to hat A. This means that all participants in your group (including yourself) will observe the colour of hats B and C (but not hat A). Also, all participants in your group will receive the same hint (either hint 1 or hint 2). Table C6 provides a summary of all possible outcomes.

Example: Suppose that your group is assigned to hat A and the computer chooses $O5$. All participants in your group will observe that hats B and C are red, and receive the hint that

Outcome	$O1$	$O2$	$O3$	$O4$	$O5$	$O6$	$O7$	$O8$
Hat A	Black	Black	Black	Red	Black	Red	Red	Red
Hat B	Black	Black	Red	Black	Red	Black	Red	Red
Hat C	Black	Red	Black	Black	Red	Red	Black	Red

Table C6

“there is at least one red hat amongst the three hats". However, no participant in your group observes the colour for hat A.

Example: Suppose that your group is assigned to hat B and the computer chooses $O2$. All participants in your group will observe that hat A is black and hat C is red. All participants in your group will receive the hint that “there is at least one red hat amongst the three hats". However, no participant in your group observes the colour for hat B.

STAGE II. Allocating the Ticket

After all participants in your group have received the information about the guessing game, the ticket is allocated to one of the three participants within your group.

The tickets will be randomly assigned in your group. In other words, the computer randomly picks one of the three participants within your group and gives him the ticket. Please note: Team members who do not receive tickets will receive 1500 points. The team member who gets the ticket will receive 620 points, in addition to his earnings in the guessing game.

We will use an auction to sell the ticket. Each participant is given 1500 points. Thereafter, each participant in your group submits a bid. A bid is the maximum amount you are willing to pay for the ticket.

** Important: The minimal bid amount you can submit is 0 points. The maximum bid amount you can submit is 1500 points. **

After all participants in your group have submitted their bids, we will rank the 3 bids and sell the ticket to the participant with the highest bid. The “ticket owner" (i.e., the participant who receives the ticket) is therefore the participant with the highest bid in your group. However, the ticker owner will only need to pay the second highest bid amount. If there are more than one participants with the same highest bids, the ticket owner will be randomly selected amongst the participants who submitted the highest bids. Table C7 shows three examples to help you better understand how the tickets are sold. In all examples, your group has been allocated to hat A. We therefore label participants A1, A2 and A3 as the first, second and third participants in the group, respectively.

Example: Participants A1, A2 and A3 bid 100 points, 200 points and 300 points, respectively. The ticket owner is A3 as he had submitted the highest bid (i.e., 300 points). The second highest-bid is 200 points. In this example, participant A3 only pays 200 points for the ticket.

	Example 1			Example 2			Example 3		
Participants in your group	A1	A2	A3	A1	A2	A3	A1	A2	A3
Bid (points)	100	200	300	100	200	100	100	200	200
Ranks within each group	3	2	1	2	1	2	2	1	1
ticker owner	A3			A2			A3		
Price paid by ticket owner	200 points			100 points			200 points		

Table C7: Only applicable to the auction mechanism treatments

Example: Participants A1, A2 and A3 bid 100 points, 200 points and 100 points, respectively. The ticket owner is A2. The second highest-bid is 100 points. In this example, participant A2 only pays 100 points for the ticket.

Example: Participants A1, A2 and A3 bid 100 points, 200 points and 200 points, respectively. Here, the highest bids are by participants A2 and A3–there is a draw and the computer will randomly determine whether participant A2 or A3 will be the ticket owner. This also means that the highest and second highest bids are both 200 points. In this example, participant A3 is randomly selected by the computer to be the ticket owner. Participant A3 purchases the ticket at 200 points.

** Reminder: Note that ticket holder will never pay more than his bid. In fact, the ticket holder may even sometimes pay less than his bid (i.e., if the second highest bid is less than the ticket holder's bid). **

STAGE III. Participating in the Guessing game

Only the participant with a ticket will participate in the guessing game (i.e., he will be a hat player in the guessing game).

Example: Suppose that participants 1, 2 and 3 are randomly assigned to hat A, and the ticket is sold randomly given to participant 2. This means that participant 2 will be hat-player A in the guessing game.

Example: Suppose that participants 1, 2 and 3 are randomly assigned to hat B, and the ticket is sold randomly given to participant 2. This means that participant 2 will be hat player B in the guessing game.

STAGE IV. Computing payoffs

Your points will depend on (a) Whether you received a ticket and (b) If you received a ticket, whether you correctly guess your hat colour in the guessing game. If you did not receive a ticket, your payoff is 1500 points. If you received a ticket, your payoff depends on your decision in the guessing game with regards to your hat colour i.e., Payoff = 620 + (Points from the guessing game) Table 2 below summaries the possible earnings in the guessing game.

	You chose (RED) or (BLACK) and your guess is correct	You chose (RED) or (BLACK) and your guess is incorrect
End guessing game in Period 1	1000 points	300 points
End guessing game in Period 2	900 points	200 points
End guessing game in Period 3	800 points	100 points
End guessing game in Period 4	700 points	0 points

Table C8

Your points will depend on (a) Whether you purchased a ticket and (b) If you purchased a ticket, whether you correctly guess your hat colour in the guessing game. If you did not purchase a ticket, your payoff is simply your endowment (i.e., 1500 points). If you purchased a ticket, your payoff depends on the purchase price and your decision in the guessing game with regards to your hat colour i.e., Payoff = 1500-(Ticket purchase price)+(Points from the guessing game)

Table C8 again details the ticket holder's (i.e., the hat player) payoff from the guessing game which depends on the period that the ticket holder ends the guessing game and whether he is correct. Here are some examples to help you better understand how your payoff is computed.

Example: Suppose that participants 1, 2 and 3 are randomly assigned to hat A. Participant 2 enters the guessing game as hat player A (receives the ticket). In the guessing game, hat player A (participant 2) chooses (WAIT) in period 1 and (RED) in period 2. If hat A is red, the payoffs to participants 1, 2 and 3 are: (participant 1) 1500 points, (participant 2) 620+900=1520 points, (participant 3) 1500 points. Note: participants 1 and 2 receive 1500 points as they did not participate in the guessing game.

Example: Suppose that participants 1, 2 and 3 are randomly assigned to hat A, and the ticket is sold to participant 2 at the price of 500 points. Participant 2 enters the guessing game as hat player A. In the guessing game, hat player A (participant 2) chooses (WAIT) in period 1 and (RED) in period 2. If hat A is red, the payoffs to participants 1, 2 and 3 are: (participant 1) 1500 points, (participant 2) 1500-500+900=1900 points, (participant 3) 1500 points. Note: participants 1 and 2 receive 1500 points as they did not participate in the guessing game.

Example: Suppose that participants 1, 2 and 3 are randomly assigned to Hat A. Participant 2 enters the guessing game as hat player A (receives the ticket). In the guessing game, participant 2 chooses (WAIT) in period 1 and (BLACK) in period 2. If hat A is red, the payoffs to participants 1, 2 and 3 are: (participant 1) 1500 points, (participant 2) 620+200=820 points, (participant 3) 1500 points.

Example: Suppose that participants 1, 2 and 3 are randomly assigned to Hat A, and the ticket is sold to participant 2 at the price of 500 points. Participant 2 enters the guessing game as hat player A. In the guessing game, participant 2 chooses (WAIT) in period 1 and (BLACK) in period 2. If hat A is red, the payoffs to participants 1, 2 and 3 are: (participant 1) 1500 points, (participant 2) 1500-500+200=1200 points, (participant 3) 1500 points.

C.2.2 Information about the other hat players.

As in part I of the experiment, the other players that are assigned to the hats, besides your group's hat, are computer players.

Example: If your group is randomly assigned to hat A, then the hat A player in the guessing game will be the ticket holder from your group. In contrast, the hat B and hat C players in the guessing game will be computer players.

Example: If your group is randomly assigned to hat B, then the hat B player in the guessing game will be the ticket holder from your group. In contrast, the hat A and hat C players in the guessing game will be computer players.

*** Reminder. The computer players are programmed to be in a similar position as you. They observe your hat colour and the hat colours of their fellow computer players but not their own hat colour. Their decisions at each period will depend on the number of black and red hats that they observe and the choices that all hat players make across the different periods. Also, the computer hat players are programmed to: (a) Where possible, to always maximise their own payoffs, (b) Where possible, to always predict their own hat colour in a logical manner, (c) The computer hat players cannot "cheat". They will try to correctly predict their own hat through a logical manner. ***

Please note that unlike the first part of the experiment, the other two players who participate in the guessing game are not computer players, but are selected from their respective groups. They are chosen in a similar manner as you: they are players in the other two groups that are [randomly selected by the program to get tickets] / [purchased a ticket].

Example: If your group is randomly assigned to hat A, then the hat A player in the guessing game will be the ticket holder from your group. The hat B player: the [randomly selected participant] / [participant who purchased a ticket] amongst the group of three participants under hat B. The hat C player: the [randomly selected participant] / [participant who purchased a ticket] amongst the group of three participants under hat C.

Example: If your group is randomly assigned to hat B, then the hat B player in the guessing game will be the ticket holder from your group. The hat A player: the [randomly selected participant] / [participant who purchased a ticket] amongst the group of three participants under hat A. The hat C player: the [randomly selected participant] / [participant who purchased a ticket] amongst the group of three participants under hat C.

C.2.3 Control questions.

Please answer the following control questions

1. At each round you will be randomly matched with two other participants in this room to form a group. (True/False)

2. In each round, your group will be randomly assigned to one hat. (True/False)
3. In each round, what is the probability that your group is assigned to a black hat is ___ .
4. You see the colour of your group's hat. (True/False)
5. You see the colour of the other groups' hats (True/False)
6. The other two participants in your group have the same information about the guessing game as you have. (True/False)
7. Only one participant in your group will receive a ticket. (True/False)
8. If you purchased a ticket, can the purchase price be higher than your bid? (Yes/No)
9. If you purchased a ticket, can the purchase price be lower than your bid? (Yes/No)
10. Suppose that participants A1, A2 and A3 (participants in Hat A) bid 20, 20 and 30 points, respectively.
    - Who will receive the ticket?
    - How much will that participant have to pay for the ticket?
11. Suppose that participants A1, A2 and A3 (participants in Hat A) bid 15, 25 and 20 points, respectively.
    - Who will receive the ticket?
    - How much will that participant have to pay for the ticket?
12. Suppose that participants A1, A2 and A3 (participants in Hat A) bid 5, 30 and 50 points, respectively. Suppose that there are three participants in Hat A (A1, A2 and A3) and participant A3 is randomly chosen to receive the ticket. In the guessing game, the Hat A player chooses (WAIT) in period 1 and (RED) in period 2. Suppose that hat A is red.
    - Participant A1 points: ___.
    - Participant A2 points: ___.
    - Participant A3 points: ___.
13. Suppose that participants A1, A2 and A3 (participants in Hat A) bid 5, 30 and 50 points, respectively. Suppose that there are three participants in Hat A (A1, A2 and A3) and participant A3 is randomly chosen to receive the ticket. In the guessing game, the Hat A player chooses (WAIT) in period 1 and (BLACK) in period 2. Suppose that hat A is red.
    - Participant A1 points: ___.

- Participant A2 points: ___.
- Participant A3 points: ___.

14. If your group is randomly allocated to Hat B, then Hat player A and Hat player C in the guessing game will be computer players. (True/False)